Linear Regression Assumptions in Python

In [ ]:

In [ ]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from sklearn import datasets

import statsmodels.api as sm
from statsmodels.stats.outliers_influence import variance_inflation_factor
from statsmodels.stats.stattools import durbin_watson

import scipy.stats as st
```

In [ ]:

- All assumptions of linear regression

- What are the consequence if each is violated

- How to test each assumption in python

- How to fix each violation

Assumptions of Linear regression

- Linearity
- Autocorrelation
- Multicollinearity
- Heteroskedasticity
- Normality

These assumptions will be tested against diabetes dataset from the standard sklearn dataset.

Import data and build linear regression model

In [2]:
```
diabetes = datasets.load_diabetes()
X = diabetes.data
y = diabetes.target

X2 = sm.add_constant(X)
est = sm.OLS(y, X2)
est2 = est.fit()
print(est2.summary())
```

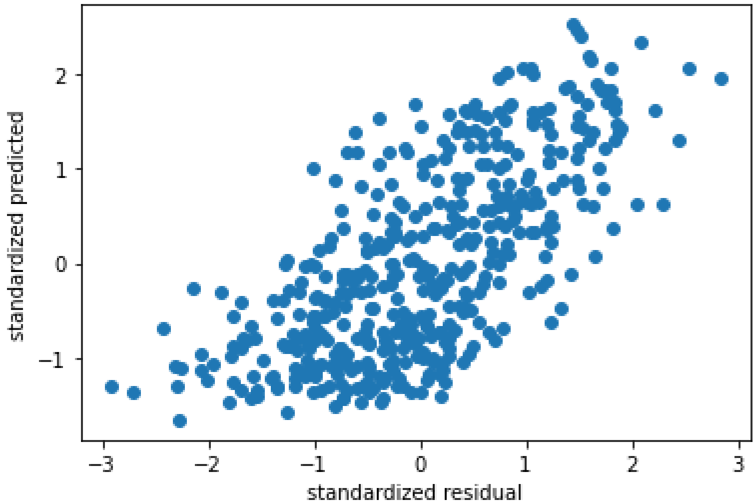
```
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.518
Model:                            OLS   Adj. R-squared:                  0.507
Method:                 Least Squares   F-statistic:                     46.27
Date:                Fri, 04 Dec 2020   Prob (F-statistic):           3.83e-62
Time:                        07:12:15   Log-Likelihood:                -2386.0
No. Observations:                 442   AIC:                             4794.
Df Residuals:                     431   BIC:                             4839.
Df Model:                          10
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const        152.1335      2.576     59.061      0.000     147.071     157.196
x1           -10.0122     59.749     -0.168      0.867    -127.448     107.424
x2          -239.8191     61.222     -3.917      0.000    -360.151    -119.488
x3           519.8398     66.534      7.813      0.000     389.069     650.610
x4           324.3904     65.422      4.958      0.000     195.805     452.976
x5          -792.1842    416.684     -1.901      0.058   -1611.169      26.801
x6           476.7458    339.035      1.406      0.160    -189.621    1143.113
x7           101.0446    212.533      0.475      0.635    -316.685     518.774
x8           177.0642    161.476      1.097      0.273    -140.313     494.442
x9           751.2793    171.902      4.370      0.000     413.409    1089.150
x10           67.6254     65.984      1.025      0.306     -62.065     197.316
==============================================================================
Omnibus:                        1.506   Durbin-Watson:                   2.029
Prob(Omnibus):                  0.471   Jarque-Bera (JB):                1.404
Skew:                           0.017   Prob(JB):                        0.496
Kurtosis:                       2.726   Cond. No.                         227.
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

$Linearity$:

- Expected value of the dependent variable is straight-line function of each individual independent variable.

- If this is violated, then extrapolating beyond the range of the sample data would give unrelable results
- This can be tested if predicted values and residual have pattern then relationship between dependent and independent variable is non linear.
- This can be fixed by using polynomial transformation of variables one by one manually or polynomial regression.
- If equation is $y = (\beta 1 * x1) + (\beta 2 * x2)$, after polynomial transformation it will become $y = (\beta 1 * x1) + (\beta 2 * x2) + (\beta 3 * x1 * x2) + (\beta 4 * x1^2) + (\beta 5 * x2^2)$

In [3]:
```
#step 1) calculate residual
residuals = y-est2.fittedvalues
#step 2) plot standardized residual vs actual
plt.scatter(st.zscore(residuals),st.zscore(y))
plt.xlabel('standardized residual')
plt.ylabel('standardized predicted')
plt.show()
```



It looks like there is relationship between predicated values and residual.

- Theoritically speaking, model should predict values higher than actual and lower than actual with equal probability.
- We need to plot dependent vs independent variable to identify polynomial relationships and add the terms in the model.

$Normality$:

- Residuals should be normally distributed.

- If it is violated, it causes problems with calculating confidence intervals and various significance tests for coefficients.

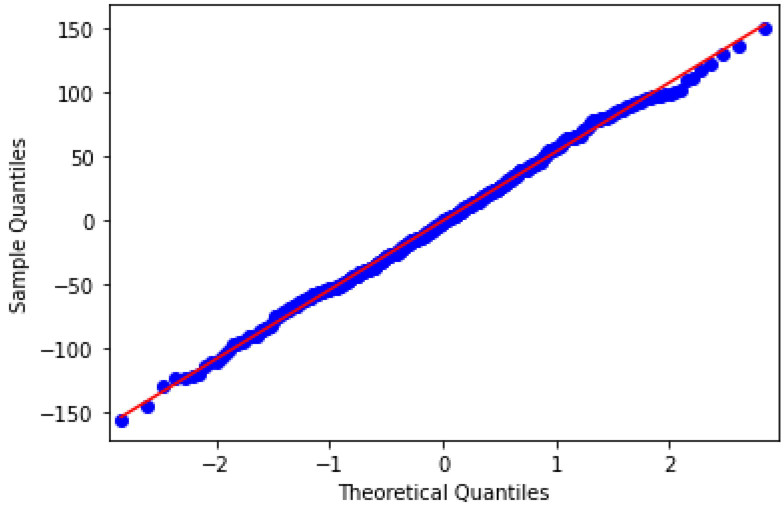
- Q-Q plot and Shapiro-Wilk test can help find if it is met or violated.

- Q-Q plot compares the quantiles of a data distribution with the quantiles of a standardized theoretical distribution. Majority of the middle part of the line should fall on a line.

- It can be fixed by outlier treatment and nonlinear transformation such as log

In [4]:
```
#normality through QQ plot
fig4=sm.qqplot(residuals, line='r')

#perform Shapiro-Wilk test
_,p=st.shapiro(residuals)
if p>0.05:
    print('Null hypothesis rejected. Residuals are normally distributed')
```

```
Null hypothesis rejected. Residuals are normally distributed
```



Multicollinearity

- All independent variables are independent of each other and have no correlation amongst each other.
- If it is violated, coefficient estimates will be unstable and can cause the coefficients to switch signs.
- It can be fixed by either 1) removing variables with VIF larger than 10 Or 2) by doing PCA

In [5]:
```
variables = est2.model.exog
vif = [variance_inflation_factor(variables, i) for i in range(variables.shape[1])]
vif
```

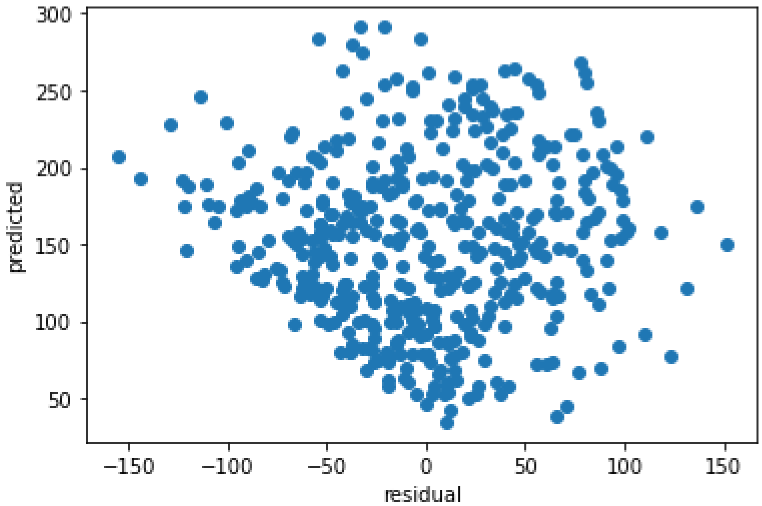
Out[5]:
```
[1.0,
 1.2173065764321338,
 1.278072545982699,
 1.5094458375317008,
 1.4594285821794584,
 59.20378568651294,
 39.19437938862489,
 15.402352175616604,
 8.890986224976269,
 10.076221589049265,
 1.4846225872940024]
```

Heteroskedasticity

- Error term should have equal variance across different observations. In the case of heteroskedasticity this is violated.
- Heteroskedasticity leads to wrong standard errors of the coefficients, which results in wrong t-statistic and p-values
- It can be identified if residual vs fitted values has specific pattern, it means has non-constant variance
- in such case, we can use variance stabilizing transformation, such as log and sqrt. We can also solve this through quantile regression

In [6]:
```
plt.scatter(residuals,est2.fittedvalues)
plt.xlabel('residual')
plt.ylabel('predicted')
print('It clearly shows that heteroskedasticity assumptions are violated')
```

```
It clearly shows that heteroskedasticity assumptions are violated
```



Autocorrelation

- If errors are correlated and not independent, its said to have auto-correlation. In other words, error of t instance bears an impact from $t-1$
- If residual has autocorrelation, then we might get coefficient for certain variables as significant, whereas these are actually non significant. In addition F test and R square will be unreliable
- It happens mostly with time series data.
- It can be tested by durbin-watson test. If it is between 1.5 to 2.5, its good. less tahn 1.5 is negative autocorrelation, more than 2.5 is positive autocorrelation. Although 2 is considered as ideal value.
- It can be fixed by adding a lag variable. and seasonality related dummy varriables.

In [7]:
```
durbin_watson(residuals)
```

Out[7]: 2.0285418752535134

In [ ]: