Linear Programming Problem

"Resources are scarce & property of the Society and their abuse is a social evil"

Linear Programming is a technique for determining an optimum schedule of interdependent activities in view of the available resources.

Programming is also known as planning : The process of determining a perticular plan of action from amongst several alterenatives.

Linear : All the relationships involved in a perticular problem are linear.

Mathematical formulation of the Problem

Mathematical formulation of a LPP consists of following Steps :

① Study the given situation to find the key decisions to be made.

② Identify the variables involved and designate them by symbols x_j $(j = 1, 2, 3, \ldots)$

③ State the feasible alternatives which generally are;
$x_j \geq 0$, for all j.

④ ~~Sta~~ Identify the constraints in the problem and express them as linear equations/inequalities.

⑤ Identify the objective function and express it as a linear function of the decision variables.

~~20~~ Ex ① A company has three operational departments (weaving, processing and packing) with capacity to produce three different types of clothes namely suitings, shirtings and woollens yielding a ~~weaving~~ profit of Rs. 2, Rs. 4 and Rs. 3 per metre respectively. One metre of suiting, requires 3 minutes in weaving, 2 minutes in processing and 1 minute in packing. Similarly one metere of shirting requires 4 minutes in weaving, 1 minutes in processing & 3 minutes in packing. One ~~minutes~~ meter of wollen requires 3 minutes in each dept. In a week total run time of each dept. is 60, 40 & 80 hours for weaving, processing and packing respectively.

Formulate the linear programming problem to find the product mix to maximize the profit.

Ans

Mathematical Formulation.

	Weaving (in min)	Processing (in min)	Packing (in min)	Profit (in Rs)
Suitings	3	2	1	2
Shirtings	4	1	3	4
Woollens	3	3	3	3
Availability	60X60	40X60	80X60	

St. 1 The key decision is to determine the weekly rate of production for the three types of clothes.

2. Let us designate the weekly production of suitings, shirtings & woollens by x_1 meters, x_2 meters & x_3 meters respectively

3. Since it is not possible to produce -ve quantities, feasible alternatives are sets of values of x_1, x_2 & x_3 satisfying $x_1 \geq 0, x_2 \geq 0$ & $x_3 \geq 0$

St. (4) The constraints are limited availability of three operational departments. 1 mtr. of suiting requires 3 mins of weaving i.e. total $3x_1$ mins Similarly $4x_2$ mins, $3x_3$ mins for shirtings & woollens respectively. Thus the total requirement of weaving will be $3x_1+4x_2+3x_3$ which would not exceed 3600 mins . i.e. $3x_1+4x_2+3x_3 \leq 3600$

Similarly other constraints are

$$2x_1+x_2+3x_3 \leq 2400$$

& $x_1+3x_2+3x_3 \leq 4800$ respectively for processing & packing.

St. (5) The objective is to maximize the total profit from sales . Assuming that whatever is produced is sold in the market, the total profit is given by the linear relation $z = 2x_1+4x_2+3x_3$.

So the Mathematical Formulation is given by

Obj. Fn. Max. $Z = 2x_1+4x_2+3x_3$

Subject to constraints

$$3x_1+4x_2+3x_3 \leq 3600$$
$$2x_1+x_2+3x_3 \leq 2400$$
$$x_1+3x_2+3x_3 \leq 4800$$

Non-ve restrictions.

$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0$

EXAMPLE 2.6-3 (Blending Problem)

A firm produces an alloy having the following specifications:

(i) specific gravity ≤ 0.98,

(ii) chromium $\geq 8\%$,

(iii) melting point $\geq 450°C$.

Raw materials A, B and C having the properties shown in the table can be used to make the alloy.

TABLE 2.3

Property	*Properties of raw material*		
	A	*B*	*C*
Specific gravity	0.92	0.97	1.04
Chromium	7%	13%	16%
Melting point	440°C	490°C	480°C

Costs of the various raw materials per ton are: Rs. 90 for A, Rs. 280 for B and Rs. 40 for C. Formulate the L.P. model to find the proportions in which A, B and C be used to obtain an alloy of desired properties while the cost of raw materials is minimum.

[P.U.B.E. (E. and Ec.) 1998]

Formulation of Linear Programming Model

Let the *percentage contents* of raw materials A, B and C to be used for making the alloy be x_1, x_2 and x_3 respectively.

Objective is to minimize the cost

i.e., $$\text{minimize } Z = 90x_1 + 280x_2 + 40x_3.$$

Constraints are imposed by the specifications required for the alloy.

They are

$$0.92x_1 + 0.97x_2 + 1.04x_3 \leq 0.98,$$
$$7x_1 + 13x_2 + 16x_3 \geq 8,$$
$$440x_1 + 490x_2 + 480x_3 \geq 450,$$

and $$x_1 + x_2 + x_3 = 100,$$

as x_1, x_2 and x_3 are the percentage contents of materials A, B and C in making the alloy.

Also $$x_1, x_2, x_3, \text{ each} \geq 0.$$

EXAMPLE 2.6-6 (Product Mix Problem)

A chemical company produces two products, X and Y. Each unit of product X requires 3 hours on operation I and 4 hours on operation II, while each unit of product Y requires 4 hours on operation I and 5 hours on operation II. Total available time for operations I and II is 20 hours and 26 hours respectively. The production of each unit of product Y also results in two units of a by-product Z at no extra cost.

Product X sells at profit of Rs. 10/unit, while Y sells at profit of Rs. 20/unit. By-product Z brings a unit profit of Rs. 6 if sold; in case it cannot be sold, the destruction cost is Rs. 4/unit. Forecasts indicate that not more than 5 units of Z can be sold. Formulate the L.P. model to determine the quantities of X and Y to be produced, keeping Z in mind, so that the profit earned is maximum.

[P.U.B. Com. April, 2006; Jammu U.B.E. (Mech.) 2004; P.T.U.B.Tech. 2000; R.E.C. Hamirpur, 1998]

Formulation of L.P. Model

Let the number of units of products X, Y and Z produced be x_1, x_2, x_z, where

$$\begin{aligned} x_z &= \text{number of units of Z produced} \\ &= \text{number of units of Z sold + number of units of Z destroyed} \\ &= x_3 + x_4 \text{ (say).} \end{aligned}$$

Objective is to maximize the profit. Objective function (profit function) for products X and Y is linear because their profits (Rs. 10/unit and Rs. 20/unit) are constants irrespective of the number of units produced. A graph between the total profit and quantity produced will be a straight line. However, a similar graph for product Z is non-linear since it has slope +6 for first part, while a slope of –4 for the second. However, it is piece-wise linear, since it is linear in the regions (0 to 5) and (5 to 2Y). Thus splitting x_z into two parts, *viz.* the number of units of Z sold (x_3) and number of units of Z destroyed (x_4) makes the objective function for product Z also linear.

Thus the objective function is

$$\text{maximize } Z = 10x_1 + 20x_2 + 6x_3 - 4x_4.$$

Constraints are

on the time available on operation I: $3x_1 + 4x_2 \le 20$,

on the time available on operation II: $4x_1 + 5x_2 \le 26$,

on the number of units of product Z sold: $x_3 \le 5$,

on the number of units of product Z produced: $2Y = Z$

or $2x_2 = x_3 + x_4$ or $-2x_2 + x_3 + x_4 = 0$,

where x_1, x_2, x_3, x_4, each ≥ 0.

Advantages of L.P.P.

1. It helps in attaining the opt. use of productive factors. (manpower, machines etc)
2. It improves the quality of decisions.
3. It also helps in providing better tools for adjustments to meet changing conditions.
4. Most business problems involve constraints like raw materials availability, market demand etc. which must be taken into consideration.
5. It highlights the bottlenecks in the production processes.

Limitations of L.P.P.

1. For large problems having many limitations and constraints the computational difficulties are enourmous.
2. It may yield fractional valued answers for the decision variables.
3. It is applicable to only static situations since it does not take into account the effect of time.
4. It is used in case of certainty. But probabilistic cases avoided.
5. In some cases obj. func^n & constraints are not in linear form.
6. Single objective vs multiple objective.