

## Priority Queue

### Complexities

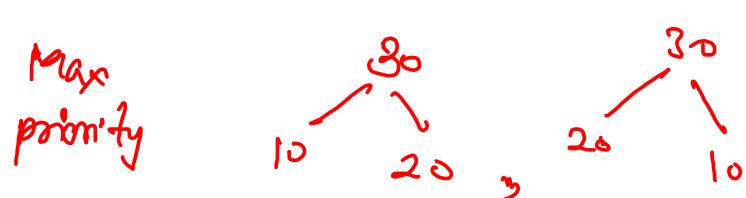
① add →  $O(n)$

② remove →  $O(n)$

③ peek →  $O(1)$

④ size →  $O(1)$

⑤ Display →  $O(n)$

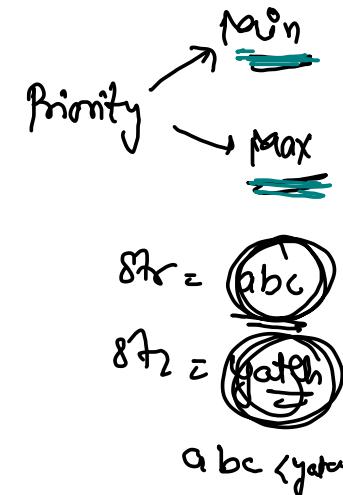
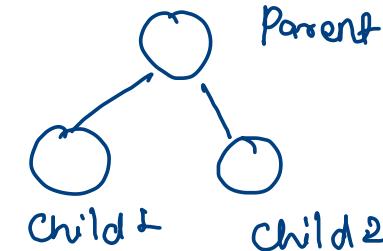


$\text{priority}(30) > \text{priority}(10, 20)$

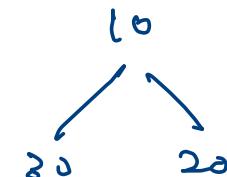
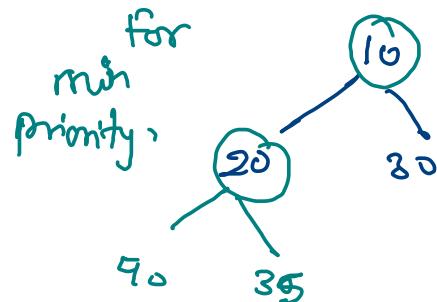
- ☛ ① Heap order property.
- ☛ ② Complete Binary Tree.

### Heaps order property

Min Heap →



$\text{priority}(\text{Parent}) > \text{priority}(\text{child}_1, \text{child}_2)$



AL →

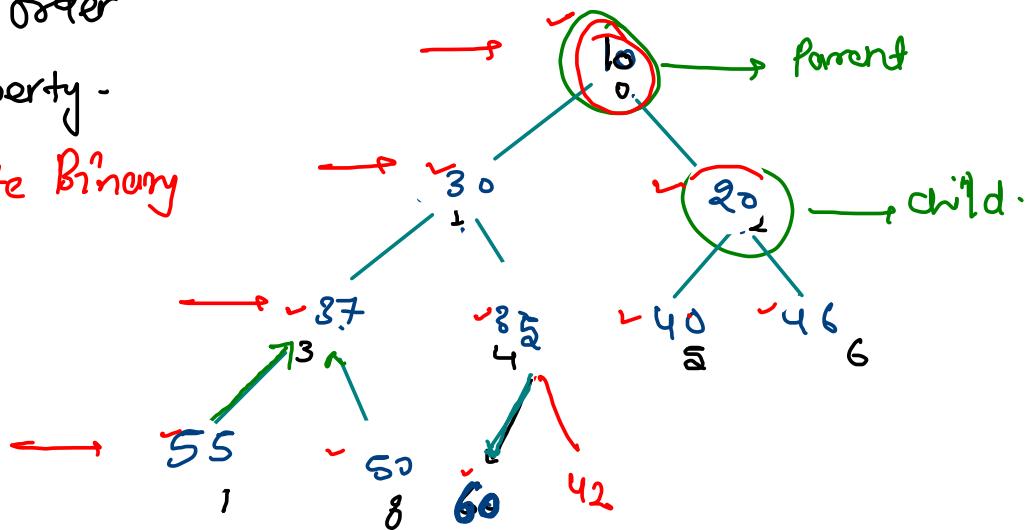
0	1	2	3	4	5	6	7	8	9	10
10	20	30	35	37	40	46	50	55	60	42

PQ → min.

# Heap Order

(property -

# Complete Binary Tree



Indirect connection

b/w parent and

child. →

parent index = pi

# left child =  $2 \times pi + 1$

# right child =  $2 \times pi + 2$

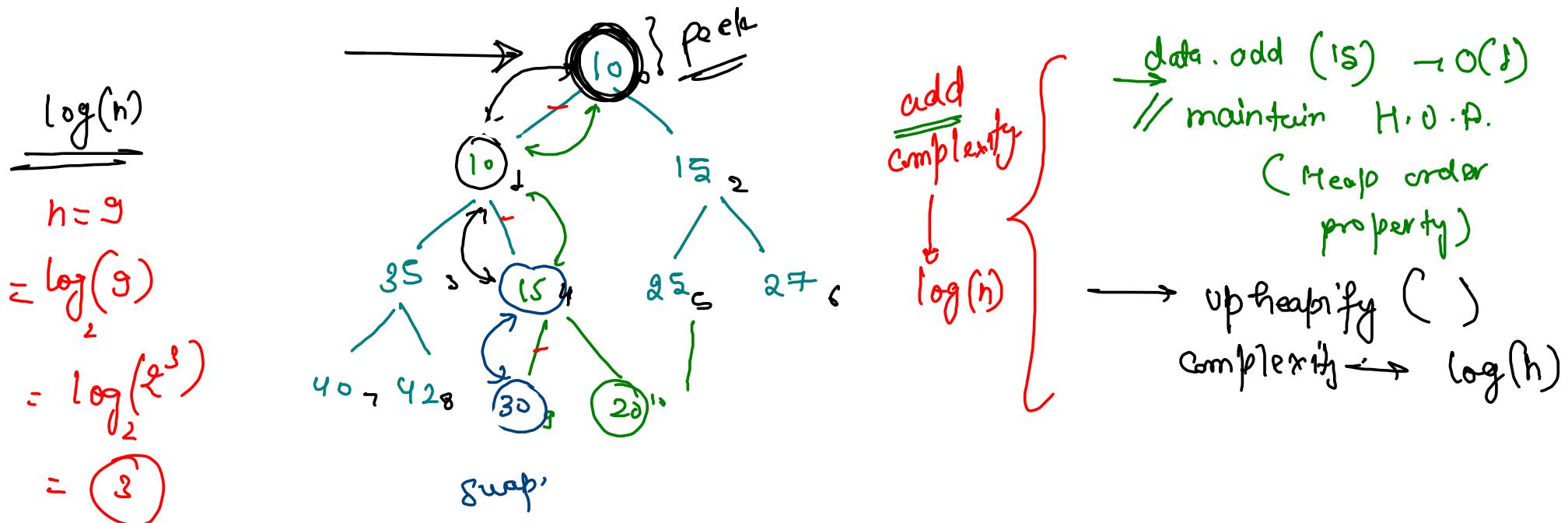
In ArrayList, we can move up or down too →

child index = i

parent index =  $(i-1)/2$ :

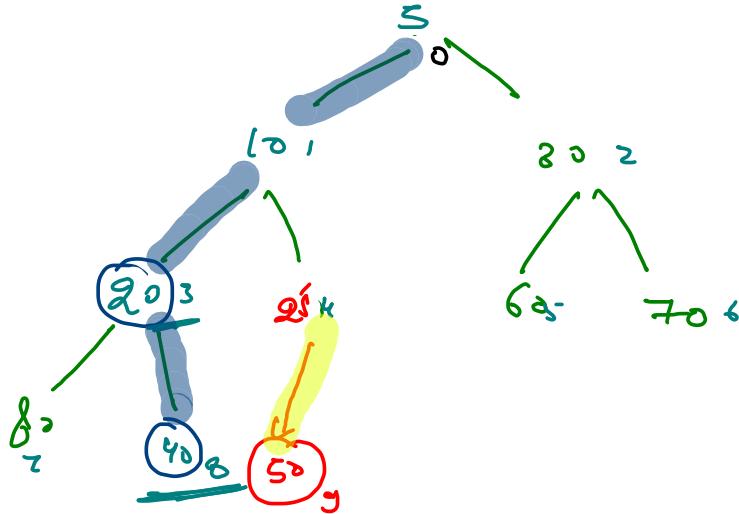
In Tree → we can move in downward direction only.

0	1	2	3	4	5	6	7	8	9	10
10	15	15	35	20	25	27	40	42	30	10



peak  $\rightarrow$  data.get(0);

up heapify:



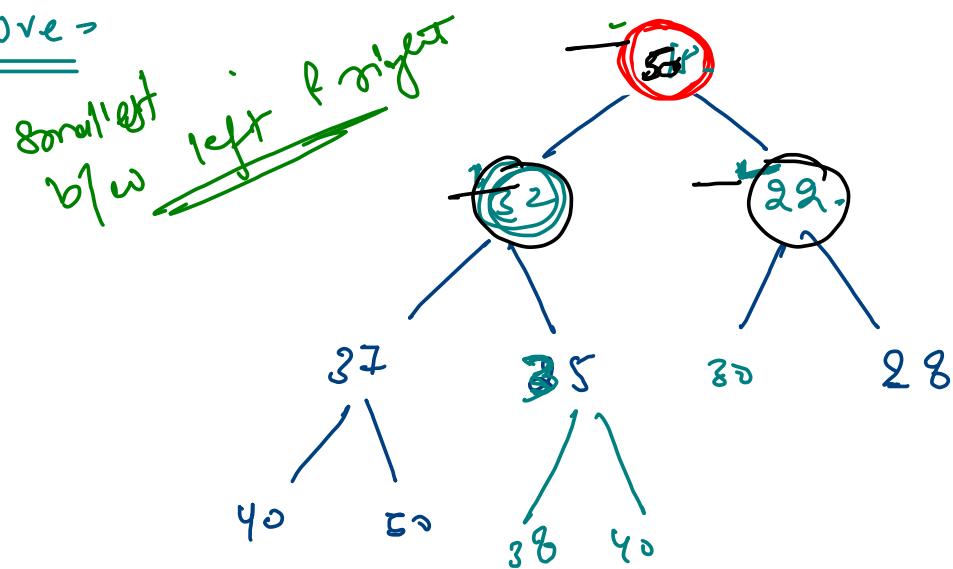
data.add(5)  
data.add(25)

index = 8  
pi = 4

Heap order property is  
maintained

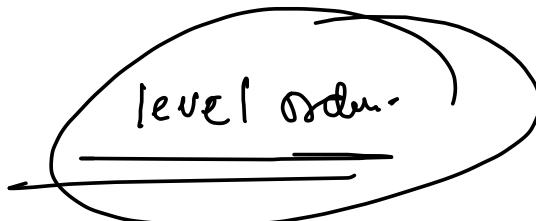
```
upheapify(index) {  
    → if(index == 0) return;  
    → pi = (index-1)/2;  
    → if(data.get(index) < data.get(pi)) {  
        swap(data, index, pi);  
        → upheapify(pi);  
    }  
}
```

Remove =



downheapify

minIndex



peek() → data[0]

pq. remove() →  $\log(n)$

~~data.remove(0) → O(n)~~

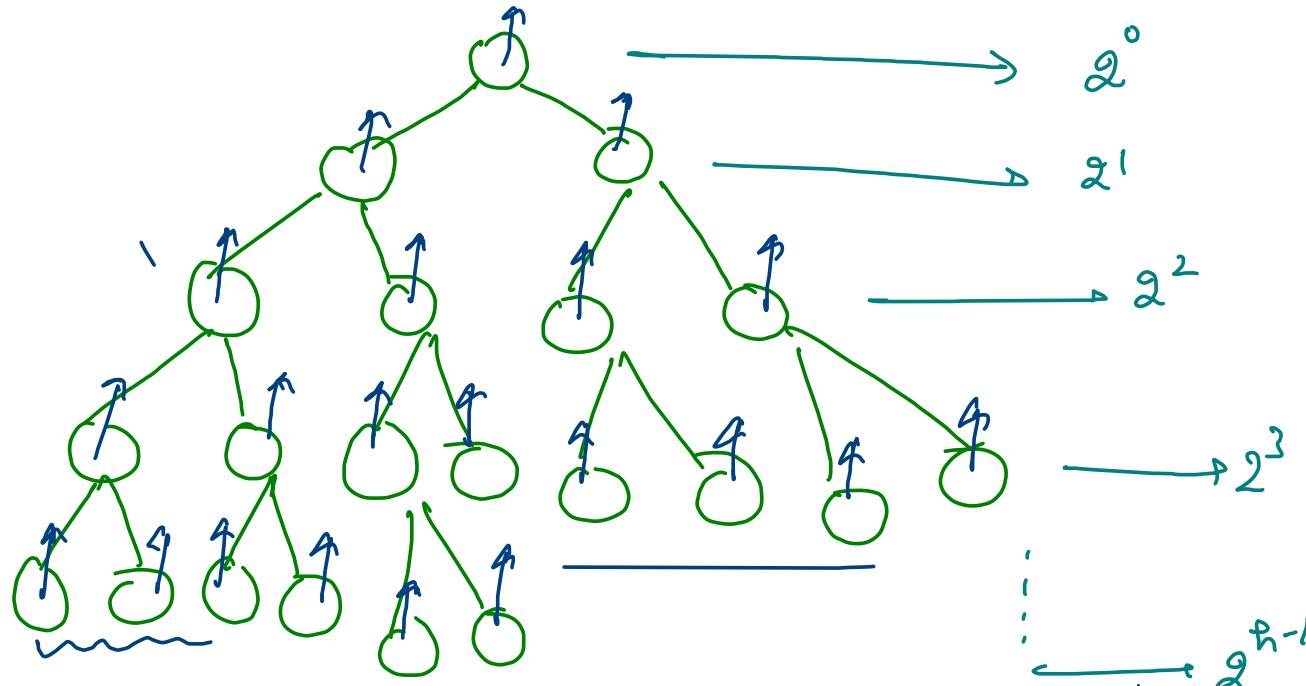
remove

swap(0, data.size-1) ↗  
int val = data.remove(size);

// maintain heap Ord. prop.  
downheapify(0);

return val;

## array passing through constructor



$$T(n) = 2^0 \times 1 + 2^1 \times 2 + 2^2 \times 3 + \dots + 2^{h-1} \times h$$

$$T(n) = 2^{h-1} \times h + \dots + 2^0 \times 1$$

$$\begin{aligned} &= \frac{2^h}{2} \times h + \dots + 2^0 \times 1 \\ &= \frac{2^h}{2} \times \log n \quad \xrightarrow{\text{if we consider only first term, it is } n \log n} \\ &\approx \frac{n}{2} \times \log n \end{aligned}$$

```
private void processArray(int[] arr) {
    for(int i = 0; i < arr.length; i++) {
        add(arr[i]);
    }
}
```

Upheapify

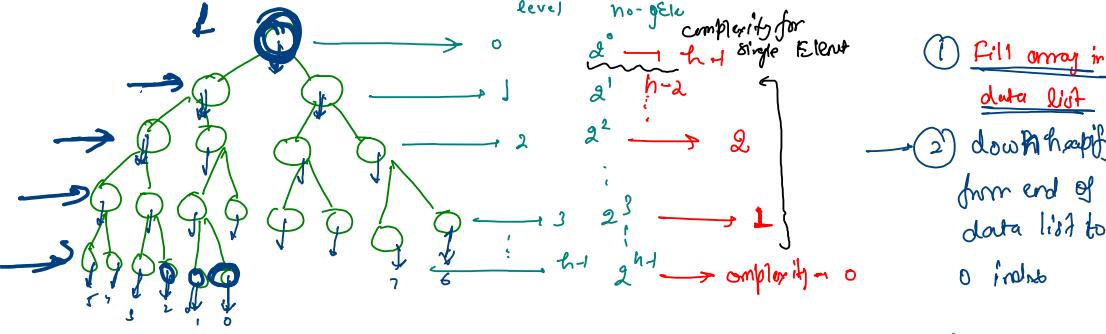
Time complexity

Required for some element = 'h'

h-levels

$$h = \underline{\underline{\log n}}$$

$$a^{\log_a b} = \underline{\underline{b}}$$



complexity for single Element  
 $2^0 \xrightarrow{h+1} h+2$   
 $2^1 \xrightarrow{h-2}$   
 $2^2 \xrightarrow{2}$   
 $\vdots$   
 $2^3 \xrightarrow{1}$   
 $2^{h-1} \xrightarrow{0}$  complexity ~ 0

- ① Fill array in data list
- ② downheapify from end of data list to 0 index

$$T(n) = 2^0 \times (h-1) + 2^1 \times (h-2) + 2^2 \times (h-3) + \dots + 2^{h-1} \times (0) \rightarrow ①$$

$$2 \times T(n) = 2^1 \times (h-1) + 2^2 \times (h-2) + (2^3 \times (h-3)) + \dots + 2^{h-1} \times 1 + 2^h \times 0 \rightarrow ②$$

eq ② - eq ①

$$\begin{aligned} 2T(n) - T(n) &= -2^0 \times (h-1) + 2^1 (\cancel{h-1} - \cancel{h+2}) + 2^2 (\cancel{h-2} - \cancel{h+3}) + 2^3 (\cancel{h-3} - \cancel{h+4}) \\ &\quad + \dots + 2^{h-1} (1 - 0) + 2^h \times 0 \end{aligned}$$

$$T(n) = -h + 1 + 2^1 + 2^2 + 2^3 + \dots + 2^{h-1} + 0$$

sum of G.P =  $\frac{\alpha(r^n - 1)}{r - 1}$

$$T(n) = -h + \frac{1(2^h - 1)}{2 - 1}$$

$$T(n) = -h + 2^h - 1$$

$h = \log n$

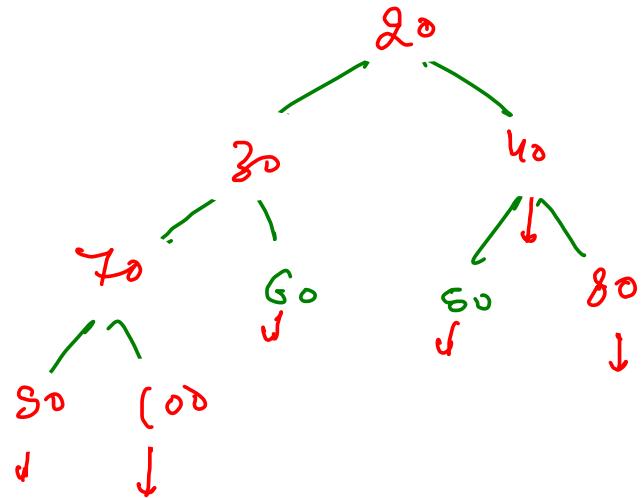
$$T(n) = 2^h - (h+1)$$

$$T(n) = 2^{\log n} - (\log n + 1)$$

$$T(n) = n - (\log n + 1)$$

$$\underline{T(n) = O(n)}$$

100	90	80	70	60	50	40	30	20
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→ ② Down heapify from start in arraylist

Topic cover till now → all the question