

Prefix Sum - Notes

Prefix Sum - Notes

Definition:

Prefix Sum is an array where each element at index 'i' is the sum of all elements from index 0 to i in the original array.

$$\text{Prefix}[i] = \text{arr}[0] + \text{arr}[1] + \dots + \text{arr}[i]$$

Time Complexity to build: $O(n)$

Time to get sum of any subarray (i to j): $O(1)$

Subarray Sum Formula:

$$\text{sum}(i, j) = \text{prefix}[j] - \text{prefix}[i - 1] \text{ (if } i > 0 \text{)}$$

$$\text{sum}(i, j) = \text{prefix}[j] \text{ (if } i == 0 \text{)}$$

Example:

Input: $\text{arr}[] = \{2, 4, 1, 3, 5\}$

Prefix Sum: $\{2, 6, 7, 10, 15\}$

To find sum of subarray $\text{arr}[1..3] \Rightarrow 4 + 1 + 3 = 8$

Using prefix sum: $\text{prefix}[3] - \text{prefix}[0] = 10 - 2 = 8$

Use Cases:

1. Fast subarray sum queries.
2. Range update problems (combined with difference array).

3. Helpful in problems like:

- Equilibrium index
- Maximum subarray sum (optimized brute force)
- Number of subarrays with sum K (prefix sum + hashmap)

Code Example (Prefix Sum Construction):

```
vector<int> prefixSum(vector<int>& arr) {  
    int n = arr.size();  
    vector<int> prefix(n);  
    prefix[0] = arr[0];  
    for (int i = 1; i < n; i++) {  
        prefix[i] = prefix[i - 1] + arr[i];  
    }  
    return prefix;  
}
```