Options on Foreign Exchange

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Options on Foreign Exchange

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For Julia DeRosa

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Preface

It is well known that foreign exchange is the world's largest financial market. What is less well known is that the market for currency options and other derivatives on foreign exchange is also massively large and still growing. Currency options are less visible than options on other financial instruments because they trade in the main in the private interbank market. Sadly, the field of foreign exchange is not popular with authors of technical business books. The attention that is given to foreign exchange pales in comparison to the vast outpouring of books on the bond and stock markets.

This book has been written for end-users of currency options, newcomers to the field of foreign exchange, and university students. I employ the real-world terminology of the foreign exchange market whenever possible so that readers can make a smooth transition from the text to actual market practice.

I use this book as the textbook for a course entitled "Foreign Exchange and Its Related Derivative Instruments" that I teach in the IEOR department of the Fu Foundation School of Engineering and Applied Science at Columbia University. I taught forerunners of this course (using the previous editions) at the Yale School of Management and University of Chicago's Booth School of Business. Students may be interested in a companion volume to this book that I edited for John Wiley & Sons. That book, *Currency Derivatives,* is a collection of scientific articles that have had an important impact on the development of the market for derivatives on foreign exchange.

This is the third edition of *Options on Foreign Exchange*. The foreign exchange market has undergone major transformations since the first edition came out in 1992 and this is especially the case since the second appeared in 2000. During the decade of 2000–2010 one could say there has been at least three remarkable developments in the foreign exchange market, each of which is has been incorporated in this new edition. The first is that the size of the foreign exchange market has grown enormously; by one count \$4 trillion of foreign exchange changed hands in a day in 2010 (compared to \$1.2 trillion in 2001). A substantial portion of this growth has to be ascribed to the success of electronic trading platforms and computerized dealing networks. Second, market stresses during the turmoil of 2007–2008 revealed anomalies in the foreign exchange market, both in the forward market and in the market for options on foreign exchange. Third, these abnormal market conditions have been the impetus for acceleration in the development in new and advanced option models.

WHAT'S NEW TO THIS EDITION

This edition has a substantial amount of new material, mostly included in reaction to market experience and the general development in the theoretical and applied understanding of currency options.

I have included new discussions of the volatility surface and the Vanna-Volga method. There are also new sections on static replication, numerical methods, and advanced models (stochastic and local volatility varieties). The materials on barrier, binary, and other exotic options are greatly expanded. There are a great number of new numerical examples in this edition.

BEFORE YOU BEGIN

I am fairly certain that nobody can become fully versed in the topics of currency options without becoming involved in the market. This book offers the next best thing. To that end it is important to start out learning about these products in the context of correct market terminology and protocol. That is why I always attempt to introduce and use trading room vernacular in this book. On the other hand, a certain level of mathematical understanding is also required. Some math is unavoidable, but its level of difficulty is easily overestimated. True enough, there a lot of equations in this book. However most of the important concepts can be grasped with little more than working knowledge of algebra and elementary calculus.

David DeRosa
www.derosa-research.com

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I thank my staff at DeRosa Research and Trading, Inc., for assistance in writing, analysis, and proofreading throughout the project. These include Devin Brosseau, Peter Halle, Anu Khambete, and Jason Stemmler. I extend very special thanks to John Goh for excellent research assistance.

I wish to thank Ron Marr and Ed Lavers for allowing me to reprint a page of their Euromarket Dayfinder Calendar. Also I am in indebted to Bloomberg Finance, LP for data and allowing me to reprint some of their exhibits.

Finally I wish to acknowledge Pamela van Giessen and Emilie Herman of John Wiley & Sons for their support and patience throughout this project.

Options on Foreign Exchange

CHAPTER 1

Foreign Exchange Basics

I start with some basic knowledge about foreign exchange that the reader will want to have before tackling currency options.

THE FOREIGN EXCHANGE MARKET

An exchange rate is a market price at which one currency can be exchanged for another. Exchange rates are sometimes called pairs because there are always two currencies involved. If the exchange rate for Japanese yen in terms of U.S. dollars is 90.00, it is meant that yen can be traded for dollars—or dollars traded for yen—at the rate of $1 for 90.00 yen.

A spot foreign exchange transaction (or deal)[1] is an agreement to exchange sums of currencies, usually in two bank business days' time. This transaction is the core of the foreign exchange market. A forward transaction is a deal done for settlement, or value, at a time beyond spot value day. There are two kinds of forwards. Forward outrights are similar to spot deals. The exchange rate is agreed when the deal is done on the trade date, but currencies settle at times in the future further out on the settlement calendar, say in a week, or a month, or in many months. A forward swap is the combination of a spot deal and a forward deal done in opposite directions. Forward outrights and forward swaps will be covered in detail later in this chapter.

It is well known that the foreign exchange market is a very large market, but exactly how large is hard to say. Our single best source as to the size and structure of the worldwide foreign exchange market is an extensive survey of trading done by the Bank for International Settlements (BIS) in

[1]Legal definitions of the vocabulary of foreign exchange dealing can be found in International Swaps and Derivatives Association, Inc. (1998).

conjunction with the central banks of 50 or so nations.[2] The most recent survey, published in 2010 (BIS 2010), documented the virtual explosion in foreign exchange trading since the previous surveys done in 2007, 2004, and 2001. After adjustments for double counting,[3] $4 trillion of foreign exchange changed hands per day in April 2010 compared to $3.3 trillion, $1.9 trillion, and $1.2 trillion in April of 2007, 2004, and 2001, respectively. These statistics cover transactions in spot, forward outright, forward swaps, currency swaps, and options (Exhibit 1.1).[4] There are at least two other recent central-bank-sponsored surveys covering specific segments of the foreign exchange market, both dating from October 2009. A Bank of England survey[5] of the London market (BOE 2009) estimated $1,430 billion in total daily turnover (including spot, outright forwards, non-deliverable forwards, and foreign exchange swaps). A Federal Reserve Bank of New York (NYFED 2009) survey[6] of the New York market estimated $679 billion of trading the same instruments.

Foreign exchange trading is done practically everywhere there is a banking center. According to the BIS 2010 survey, the largest centers by share of total world turnover were the United Kingdom (37 percent), the United States (18 percent), Japan (6 percent), Singapore (5 percent), Switzerland (5 percent), Hong Kong (5 percent), and Australia (4 percent). Not to be forgotten are the emerging markets nations where recently published data (BIS; Mihaljek and Packer 2010) (Exhibit 1.1) show to be rapidly expanding centers for foreign exchange trading.

There are well more than 100 currencies. As a general rule practically every country has its own currency[7] (with the European countries in the

[2]The practical reality is that the BIS and the central banks are in a unique position to accumulate such information because foreign exchange is an over-the-counter market that is conducted by commercial banks around the world. Unlike equities, for example, there is no central "tape" where trades are publicly posted.

[3]Every trade involves two counterparties. The BIS survey adjusts for double counting, meaning that a trade counts only once. For example, suppose Bank A buys 100 million dollar/yen from Bank B. Adjusting for double counting means that this would be counted as a single trade of 100 million of dollar/yen.

[4]For comparison, BIS (2010) reports that turnover in interest rate forward rate agreements and interest rate swaps were $600 billion and $1,275 billion, respectively in 2010.

[5]The Bank of England (BOE 2009) sponsored the Foreign Exchange Joint Standing Committee's survey of 31 institutions active in the foreign exchange market.

[6]The Federal Reserve Bank of New York (NYFED 2009) sponsored the Foreign Exchange Committee's survey of 25 participating institutions.

[7]See DeRosa (2009).

	Total Global Turnover (1)					Turnover in Emerging Markets (2)		
	1998	2001	2004	2007	2010	2004	2007	2010
Spot transactions	568	386	631	1,005	1,490	119	188	203
Outright forwards	128	130	209	362	475	21	47	73
Foreign exchange swaps	734	656	954	1,714	1,765	125	231	277
Currency swaps	10	7	21	31	43	3	4	7
Options and other products	87	60	119	212	207	10	18	24
Total	1,527	1,239	1,934	3,324	3,981	279	489	585
Memo: Turnover at April 2010 Exchange Rates	1,705	1,505	2,040	3,370	3,981			
Exchange-traded derivatives	11	12	26	80	168			
Global turnover by counterparty								
With reporting dealers	961	719	1,018	1,392	1,548			
With other financial institutions	299	346	634	1,339	1,900			
With non-financial customers	266	174	276	593	533			

EXHIBIT 1.1 Global Foreign Exchange Market Turnover (Daily Averages in April, in Billions of U.S. Dollars)
Source: (1) BIS (2010) and (2) Mihaljek and Packer (2010).

euro zone being a prominent, but not unique, exception). Yet trading in the foreign exchange market is remarkably concentrated in a handful of exchange rates (Exhibit 1.2). What is noteworthy is that the sum of trading in the dollar against the euro, yen, and sterling (in order of volume) made up 51 percent of all foreign exchange trading in 2010. In one sense, the foreign exchange market is largely the price of the dollar, inasmuch as in 2010 the dollar was on one side of 84.9 percent of all trades[8,9] (followed by the euro (39.1 percent), the yen (19.0 percent), sterling (12.9 percent), and the Australian dollar (7.6 percent).[10] But even a currency with a small share of total turnover can have a large volume of trading because the overall size of the market is enormous.

Foreign exchange dealing has become steadily more concentrated among a handful of powerful dealing banks. Indeed, according to the BIS, the top five dealers captured more than 55 percent of the market by 2009, up from a little more than 25 percent in 1999 (see Gallardo and Heath 2009).[11] At the same time that trading in foreign exchange has been growing, the number of banks doing large-scale foreign exchange trading has been shrinking. Roughly speaking, the number of money center banks that account for 75 percent of foreign exchange turnover has roughly dropped by two-thirds in the period between 1998 and 2010 (BIS 2010). On a geographic basis, the number of such banks shrunk from 24 to 9 in the U.K., from 20 to 7 in the United States, from 7 to 2 in Switzerland, from 19 to 8 in Japan, and from 23 to 10 in Singapore during this decade. This is probably best seen as an outcome of the general trend of consolidation in the financial services industry. In the meantime the development of electronic trading has materially altered the nature of the foreign exchange market. The lower section of Exhibit 1.1 shows global foreign exchange turnover by counterparty to the reporting banks. Note that the historical pattern is for dealing banks

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[8] The percentage share of the dollar was 85.6 and 88.0 in the 2007 and 2004 surveys, respectively.

[9] The BIS (2007) survey addressed the question of the euro's challenge to the dollar's dominance: "Expectations that the euro might challenge the U.S. dollar's dominance in the FX market have not been borne out. While dollar/euro remained the most important currency pair traded, accounting for 27% of total turnover measured in notional amounts, only 8% of all trades involved the euro and a currency other than the dollar" (page 15).

[10] The BIS (2007) survey estimated that 23 emerging-markets currencies tracked in the survey were 19.8 percent and 15.4 percent of trading in 2007 and 2004, respectively.

[11] Gallardo and Heath (2009) present a graph from which I have taken approximate numbers as to degree of concentration of foreign exchange dealing. See their Graph 1, left-hand Panel, their page 85.

EXHIBIT 1.2 Reported Foreign Exchange Market Turnover by Currency Pair (Daily Averages in April, in Billions of U.S. Dollars and Percent)

	2001		2004		2007		2010	
	Amount	**% Share**	**Amount**	**% Share**	**Amount**	**% Share**	**Amount**	**% Share**
U.S. dollar/euro	372	30%	541	28%	892	27%	1101	28%
U.S. dollar/yen	250	20%	328	17%	438	13%	568	14%
Sterling/U.S. dollar	129	10%	259	13%	384	12%	360	9%
Australian dollar/U.S. dollar	51	4%	107	6%	185	6%	249	6%
U.S. dollar/Swiss franc	59	5%	83	4%	151	5%	168	4%
U.S. dollar/Canadian dollar	54	4%	77	4%	126	4%	182	5%
U.S. dollar/Swedish krona	6	0%	7	0%	57	2%	45	1%
U.S. dollar/Other	193	16%	300	16%	612	18%	705	18%
Euro/yen	36	3%	61	3%	86	3%	111	3%
Euro/Sterling	27	2%	47	2%	69	2%	109	3%
Euro/Swiss franc	13	1%	30	2%	62	2%	72	2%
Euro/other	22	2%	44	2%	123	4%	162	4%
Other currency pairs	28	2%	50	3%	139	4%	149	4%
All currency pairs	1,239	101%	1,934	100%	3,324	100%	3,981	100%

Source: BIS (2010).

(i.e., "reporting" in the language of the BIS surveys) to trade primarily with other dealing banks. That pattern began to change as early as 2001. An explanation is that electronic trading has resulted in dealing banks now trading less with other dealing banks and more with other financial institutions that are not themselves dealing banks. The 2010 survey is the first time that the volume of trading between dealers and nondealers was reported to have been greater in volume than trading within the dealer community. The BIS category of nonreporting financial institutions includes smaller banks, mutual funds, money market funds, insurance companies, pension funds, hedge funds, currency funds, and central banks, among others.[12] The magnitude of this shift is remarkable when one considers that 85 percent of the increase in the global turnover in foreign exchange originated from dealers trading with this category of other financial institutions.

THE INTERNATIONAL MONETARY SYSTEM

Bretton Woods and the Smithsonian Period

For the first quarter century after the Second World War, the international monetary system consisted of a program of fixed exchange rates. Fixed exchange rates were established under the Bretton Woods agreement signed by the Allied powers in 1944 in advance of the end of the Second World War. The Bretton Woods agreement required all member central banks to keep their foreign exchange reserves in U.S. dollars, pounds Sterling, or gold. More importantly, member countries agreed to stabilize their currencies within a 1 percent band around a target rate of exchange to the U.S. dollar. The dollar, in turn, was pegged to gold bullion at $35 per ounce. Parts of the system lasted until 1971.

Periodically, currencies had to be revalued and devalued when market pressures became too great for central banks to oppose. Cynics dubbed the Bretton Woods a "system of creeping pegs." In 1971, after a series of dramatic "dollar crises," the dollar was devalued against gold to $38 an ounce,[13] and a wider bandwidth, equal to 2.25 percent, was established. This modification to the system, called the Smithsonian Agreement, postponed the collapse of the system of fixed exchange rates for two years.

[12]King and Rime (2010, p. 28).

[13]The devaluation of the dollar was mostly symbolic because the United States closed the gold window at the same time in 1971.

In 1973, President Richard Nixon scrapped the entire structure of fixed exchange rates that had begun with Bretton Woods. Since that time, exchange rates for the major currencies against the dollar have been floating.

The Euro

On January 1, 1999, 11 European nation members of the European Monetary Union, Austria, Belgium, Finland, France, Germany, Ireland, Italy, Luxembourg, Netherlands, Portugal, and Spain, adopted a new common currency, called the euro. The legacy currencies of these eleven nations, such as the German mark and French franc, circulated in parallel to the euro for a time but were exchangeable to the euro at fixed exchange rates. Total conversion to the euro happened on January 1, 2002, at which time the European Central Bank issued euro notes and coins. Additional countries have joined the euro since that time: Greece in 2001, Slovenia in 2007, Cyprus and Malta in 2008, Slovakia in 2009, and Estonia in 2011. At the current time 17 countries have adopted the euro. Noteworthy by their absence are the United Kingdom and Denmark. Switzerland is not part of the European Monetary Union.

The road to the creation of the euro was difficult. For nearly two decades, starting with the creation of the European Monetary System in March 1979, parts of Europe experimented with a fixed exchange rate system that was known as the Exchange Rate Mechanism (ERM). Under the ERM, member countries agreed to peg their currencies to a basket currency called the European Currency Unit (ECU). Currencies were allowed to move in relation to the ECU within either the narrow band of plus or minus 2.25 percent or the wide band of plus or minus 6 percent.

The ERM was a costly experiment in fixed exchange rate policy. In its 20 years of operation, from 1979 to 1999, ERM central rates had to be adjusted over 50 times. More spectacular yet were the two major ERM currency crises, one in September 1992 and the other in August 1993, each of which involved massive central bank losses in the defense of the fixed exchange rate grid. Finally after the second crisis, fluctuation bands were widened to plus or minus 15 percent, a move that effectively neutered the ERM.[14]

[14]The ERM still exists. For example, under "ERM II" the Danish krone is stabilized within a plus/minus 15 percent zone around a central rate. The Danish central bank further restricts movements in the unit to plus/minus 2.25 percent around the krone's central rate.

Fixed Exchange Rate Regimes

A great variety of fixed exchange rate regimes have come and gone in the twentieth century, especially with respect to the minor currencies and emerging market currencies. Only a handful of fixed exchange rate systems have been worth the trouble. One success story was the Austrian shilling, which remained faithfully pegged to the German mark for nearly 20 years before joining the ERM in January 1995.

But there were a great many other cases of fixed exchange rate regimes that ended badly.[15] History shows that pegged exchange rates are astonishingly explosive and damaging when they fail. The examples of the Mexican peso in 1994, Thai baht, Czech koruna, Indonesian rupiah in 1997, and the Russian ruble in 1998 are cases in point.

Fixed exchange rate regimes in their most simple form consist of a currency being pegged outright to the value of another currency.[16] A few fixed exchange rate regimes are operated under the framework of a currency board, such as the one that is in place for the Hong Kong dollar. Under the workings of a currency board, the government commits to maintaining a reserve of foreign exchange equal to the outstanding domestic base money supply and to exchange domestic and foreign reserve currency at the pegged exchange rate upon demand.

Basket peg systems are another fixed exchange rate regime. The Thai baht was operated as a basket peg currency prior to its spectacular collapse in July 1997. Under the basket regime, the Bank of Thailand pegged the baht to a basket of currencies made up of U.S. dollars, German marks, and Japanese yen, though the exact makeup of the basket was never revealed.

Another species of a fixed exchange rate regime pegs the currency, but permits gradual depreciation over time. Examples are the Mexican peso prior to the December 1994 crisis and the Indonesian rupiah before it collapsed in July 1997.

Still other currencies fit somewhere between floating and pegged exchange rate regimes. Singapore, for example, operates what at times has been described as a managed floating regime.

Exchange Rate Intervention

Since the end of the Bretton Woods–Smithsonian regimes, the value of the U.S. dollar against the currencies of America's major trading partners

[15]See DeRosa (2001) for discussions of exchange rate crises.
[16]See DeRosa (2009) for discussion of the variety of fixed exchange rate regimes in emerging markets nations.

has been determined by the forces of free-market supply and demand. This is a bit of an exaggeration because all exchange rates have at times been subject to manipulation through intervention by governmental bodies.

Intervention had a large presence in the foreign exchange market for a time in the 1980s. A predecessor of the current G-7 council,[17] called the G-5 council, initiated the Plaza intervention[18] in September 1985 (see Funabashi 1989). At that time, the council decided that a lower value for the dollar was warranted. Accordingly, its member nations' central banks launched a massive program to sell the dollar. The Plaza maneuver is remembered in foreign exchange history as the most successful coordinated intervention; the dollar fell by more than 4 percent in the first 24 hours. Two years later the council refocused its attention at the variability of exchange rates at another historic meeting, this time at the Louvre in February 1987.

But the appetite for intervention on the part of governments and their central banks ebbs and flows with economic circumstances and political leanings. For example, the administration of President George W. Bush seemed to have had no interest in foreign exchange intervention, whereas that of his predecessor, President Clinton, aggressively used intervention in an attempt to maintain what it called a strong dollar.

While most major central banks have given up on intervention, at least in current times, Japan remains convinced of the need to use intervention to manage the value and the volatility of the yen. Central banks of emerging markets nations regard foreign exchange intervention as an important tool to be used in parallel with monetary policy.

Exchange Rate Crises

Exchange rate crises are primarily manifestations of fixed exchange rate arrangements coming to their end. These are brief periods of spectacular volatility, not only of exchange rates but also of associated interest rates, bond prices, and stock prices. Their history is important to traders and risk managers, not to mention economists.

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[17]The G-7 stands for the Group of Seven industrialized nations, which is composed of the United States, Japan, Canada, the United Kingdom, Italy, Germany, and France. The G-5 did not include Italy and Canada. Today one hears of the G-8 which is the G-7 plus Russia.

[18]Curiously, these historically important accords tend to be named after either hotels (Bretton Woods and the Plaza) or museums (Smithsonian and Louvre).

The granddaddy of all foreign exchange crises was the aforementioned collapse of the Bretton Woods system of fixed exchange rates in August 1971.[19] The next-most-memorable crisis was in September 1992 during the ERM period before the launch of the euro. This was the episode that ended Great Britain's participation in the ERM and earned famed speculator George Soros the reputation for having "broke the Bank of England." August of 1993 is when the second ERM crisis occurred, principally involving the French franc's role in the ERM; 1994 saw the Mexican peso blow out of its crawling peg arrangement.

The Southeast Asian currencies experienced tremendous volatility in the summer of 1997. Two currencies, the Thai baht and the Indonesian rupiah, abandoned long-held fixed exchange rate regimes. The Malaysian ringgit and Philippine peso suffered steep losses in value against the U.S. dollar. The Korean won, a currency that was not fully convertible, also was devalued. One of the only convertible currencies in Asia not to be devalued was the Hong Kong dollar.

After the fact, basic macroeconomic analysis can explain this remarkable series of currency crises with a simple set of causal factors that relate to the fundamental domestic conditions in each of these countries. Many of the affected countries had banking systems that were on the verge of total breakdown before the exchange rate problems became manifest. Moreover, several countries were running enormous and unsustainable current account imbalances, and every one of the afflicted countries had managed to run up staggering foreign currency–denominated debts. Speaking of excessive debt, there are Russia (1998) and Argentina (2002) to consider. These were compound crises, in the sense that their fixed exchange rate regimes exploded at the same time their governments announced defaults on maturing sovereign debt.

Nonetheless, in some quarters, the blame for these episodes has been put on hedge funds and currency speculators. It is widely held that capital mobility invites disaster, mistaken though that belief is. No matter what ultimately one chooses to believe was the cause of the crisis or where one enjoys placing the blame, the history of fixed exchange rate regimes clearly demonstrates that exchange rates are capable of making violent and substantial—if not outright discontinuous—movements over short periods of time.

[19]One measure of how disruptive this crisis was is Root's (1978) report that after Nixon closed the gold window on August 15, 1971, West European governments kept their foreign exchange markets closed until August 23rd.

SPOT FOREIGN EXCHANGE AND MARKET CONVENTIONS

Spot Foreign Exchange

The spot exchange rate is a quotation for the exchange of currencies in two bank business days' time (except in the case of the Canadian dollar versus the U.S. dollar, where delivery is in one bank business day).

Foreign exchange settlement days are called value dates. To qualify as a value date, a day must not be a bank holiday in either currency's country and in almost all circumstances must not be a bank holiday in the United States as well.[20] Many traders rely on a specialized calendar called the Euromarket Day Finder published by Copp Clark Professional. A sample page of this calendar for trade date December 21, 2010, is displayed in Exhibit 1.3. Note that the value date for spot transactions on December 21, 2010, is December 23, 2010. An exception is Japan. Because December 23rd is an official holiday (the emperor's birthday), the value date for trades done on December 21, 2010, involving the yen is December 24, 2010.

The foreign exchange week commences on Monday morning at 6 A.M. Sydney time when New Zealand and Australian dealers open the market. Later, Tokyo, Singapore, and Hong Kong join the fray to constitute the Austral-Asian dealing time zone. Next, the center of the market shifts to London as it opens, but Frankfurt, Paris, Milan, Madrid, and Zurich also conduct currency dealing. New York is the capital of foreign exchange dealing in the Western Hemisphere. At 5 P.M. New York time, the day ends as trading seamlessly advances to the next value day.

Quotation Conventions

Dealers make spot exchange rate quotations as bid-ask quotations. For example, a quote on $10 million dollar/yen of 89.98/90.00 means that a dealer is willing to buy dollars and sell yen at the rate of 89.98 yen per dollar or sell dollars and buy yen at the rate of 90.00 yen per dollar. The quantity of $1 million dollars is sometimes simply called 1 dollar. Also, $1 billion is sometimes called 1 yard of dollars.

[20]Because the deepest parts of the interbank forward market are quoted against the U.S. dollar, it can be difficult to calculate accurate cross-currency settlement on a U.S. dollar holiday. Thus while settlement is technically possible on U.S. holidays, it is generally avoided, especially for smaller, less-traded currencies.

Tuesday

21

355
Day Number
10
Days Remaining

- ▼ AUD (Sydney)
- ▲ CAD (Toronto)
- ✚ CHF (Zurich)
- € EUR (TARGET)
- ■ GBP (London)
- ⧗ HKD (Hong Kong)
- ● JPY (Tokyo)
- ▬ NZD (Wellington/Auck.)
- ♦ SGD (Singapore)
- ★ USD (New York)

December 2010

SUN	MON	TUE	WED	THU	FRI	SAT
			1	2	3	4
5	6	7 K	8 M	9	10	11
12	13	14	15	16	17	18
19	20	21	22 1	23 ●	24 F 3	■♦€⧗✚▬▼▲★ BD PK LF SM
€■⧗▲▼▬✚ FD LP M	27 D ■▬▼⧗▲	28 D ▼■▲▬	29 D 8	30 9	31 F ●	

November 2010

S	M	T	W	T	F	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30				

- B Brussels
- D Dublin
- F Frankfurt
- K Kuala Lumpur
- L Luxembourg
- M Milan
- P Paris
- S Seoul

1 Jan 2011

S	M	T	W	T	F	S
						▼⧗€♦✚▲▬■●★
▬●✚	●■▼▲▬	4 ▬	5 15	6 16	7 17	8
9	10 ●	11 21	12 22	13 23	14 24	15
16	17 ★	18 28	19 29	20 30	21 31	22
23	24 ▬	25 35	26 ▼	27 37	28 38	29
30	31 ▬					

2 Feb 2011

S	M	T	W	T	F	S
		1 42	2 43	3 ♦⧗	4 ♦⧗	5 ⧗
6 ▬	7 48	8 49	9 50	10 51	11 ●	12
13	14 55	15 56	16 57	17 58	18 59	19
20	21 ★	22 63	23 64	24 65	25 66	26
27	28 69					

3 Mar 2011

S	M	T	W	T	F	S
		1 70	2 71	3 72	4 73	5
6	7 76	8 77	9 78	10 79	11 80	12
13	14 83	15 84	16 85	17 86	18 87	19
20	21 ●	22 91	23 92	24 93	25 94	26
27	28 97	29 98	30 99	31 100		

4 Apr 2011

S	M	T	W	T	F	S
					1 101	2
3	4 104	5 ⧗	6 106	7 107	8 108	9
10	11 111	12 112	13 113	14 114	15 115	16
17	18 118	19 119	20 120	21 121	▼⧗€▬✚■✚▲	⧗▼
24	▼€⧗■✚▬	26 ▼	27 127	28 128	29 ●	30

5 May 2011

S	M	T	W	T	F	S
€⧗✚♦	2 ■♦⧗	3 ●	4 ●	5 ●	6 136	7
8	9 139	10 ⧗	11 141	12 142	13 143	14
15	16 146	17 ♦	18 148	19 149	20 150	21
22	23 ▲	24 154	25 155	26 156	27 157	28
29	30 ★■	31 161				

6 Jun 2011

S	M	T	W	T	F	S
			1 162	2 ✚	3 164	4
5	6 ⧗▬	7 168	8 169	9 170	10 171	11
12	13 ▼✚	14 175	15 176	16 177	17 178	18
19	20 181	21 182	22 183	23 184	24 185	25
26	27 188	28 189	29 190	30 191		

Swaps/Mid-term table

THIS DATE IN

1-10 years forward	11-20 years forward	21-30 years forward
2011 is a Wednesday	2021 is a Tuesday	2031 is a **Sunday**
2012 is a Friday	2022 is a Wednesday	2032 is a Tuesday
2013 is a **Saturday**	2023 is a Thursday	2033 is a Wednesday
2014 is a **Sunday**	2024 is a **Saturday**	2034 is a Thursday
2015 is a Monday	2025 is a **Sunday**	2035 is a Friday
2016 is a Wednesday	2026 is a Monday	2036 is a **Sunday**
2017 is a Thursday	2027 is a Tuesday	2037 is a Monday
2018 is a Friday	2028 is a Thursday	2038 is a Tuesday
2019 is a **Saturday**	2029 is a Friday	2039 is a Wednesday
2020 is a Monday	2030 is a **Saturday**	2040 is a Friday

The Swaps & Options section at the end of this publication provides grids and day-counts through 2041.

Count-back table

TO DETERMINE DAYS BACK FROM TODAY, DEDUCT DATES IN

Nov-10	from 51	**May-10**	from 235
Oct-10	from 82	**Apr-10**	from 265
Sep-10	from 112	**Mar-10**	from 296
Aug-10	from 143	**Feb-10**	from 324
Jul-10	from 174	**Jan-10**	from 355
Jun-10	from 204	**Dec-09**	from 386

See count-back calendars at end of 2010 main calendar for past dates beyond one year.

Tuesday DECEMBER 21

Holiday observances are subject to change.

EXHIBIT 1.3 Euromarket Day Finder

Tuesday DECEMBER 21, 2010
Week 52

ACI THE FINANCIAL MARKETS ASSOCIATION

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7 Jul 2011

S	M	T	W	T	F	S
					1 ⧗▲	2
3	4 ★	5 196	6 197	7 198	8 199	9
10	11 202	12 203	13 204	14 205	15 206	16
17	18 ●	19 210	20 211	21 212	22 213	23
24	25 216	26 217	27 218	28 219	29 220	30
31						

8 Aug 2011

S	M	T	W	T	F	S
	1 ▼✚▲	2 224	3 225	4 226	5 227	6
7	8 230	9 ♦	10 232	11 233	12 234	13
14	15 237	16 238	17 239	18 240	19 241	20
21	22 244	23 245	24 246	25 247	26 248	27
28	29 ■	30 252	31 ♦			

9 Sep 2011

S	M	T	W	T	F	S
				1 254	2 255	3
4	5 ★▲	6 259	7 260	8 261	9 262	10
11	12 265	13 ⧗	14 267	15 268	16 269	17
18	19 ●	20 273	21 274	22 275	23 ●	24
25	26 279	27 280	28 281	29 282	30 283	

10 Oct 2011

S	M	T	W	T	F	S
						1 ⧗
2	3 ▼	4 287	5 ⧗	6 289	7 290	8
9	10 ★●▲	11 294	12 295	13 296	14 297	15
16	17 300	18 301	19 302	20 303	21 304	22
23	24 ▬	25 308	26 ♦	27 310	28 311	29
30	31 314					

11 Nov 2011

S	M	T	W	T	F	S
		1 315	2 316	3 ●	4 318	5
6	7 ♦	8 322	9 323	10 324	11 ★▲	12
13	14 328	15 329	16 330	17 331	18 332	19
20	21 335	22 336	23 ●	24 ★	25 339	26
27	28 342	29 343	30 344			

12 Dec 2011

S	M	T	W	T	F	S
				1 345	2 346	3
4	5 349	6 350	7 351	8 352	9 353	10
11	12 356	13 357	14 358	15 359	16 360	17
18	19 363	20 364	21 365	22 366	23 ●	24
▼€⧗♦✚▲▬■★	▼⧗€▬♦▲■✚★	⧗▲■▼▬	28 372	29 373	30 374	31 ●

Moscow (RUB)

2010	2011
1 Jan	1 Jan
2 Jan	2 Jan
3 Jan	3 Jan
4 Jan	4 Jan
5 Jan	5 Jan
6 Jan	6 Jan
7 Jan	7 Jan
8 Jan	10 Jan
23 Feb	23 Feb
8 Mar	8 Mar
3 May	2 May
10 May	9 May
14 Jun	13 Jun
4 Nov	4 Nov

Riga (LVL)

2010	2011
1 Jan	1 Jan
2 Apr	22 Apr
5 Apr	25 Apr
1 May	1 May
4 May	4 May
23 Jun	23 Jun
24 Jun	24 Jun
18 Nov	18 Nov
24 Dec	24 Dec
25 Dec	25 Dec
26 Dec	26 Dec
31 Dec	31 Dec

Tallinn (EEK)

2010	2011
1 Jan	1 Jan
24 Feb	24 Feb
2 Apr	22 Apr
1 May	1 May
23 Jun	23 Jun
24 Jun	24 Jun
20 Aug	20 Aug
24 Dec	24 Dec
25 Dec	25 Dec
26 Dec	26 Dec

Vilnius (LTL)

2010	2011
1 Jan	1 Jan
16 Feb	16 Feb
11 Mar	11 Mar
5 Apr	25 Apr
1 May	1 May
24 Jun	24 Jun
6 Jul	6 Jul
15 Aug	15 Aug
1 Nov	1 Nov
25 Dec	25 Dec
26 Dec	26 Dec

Tel Aviv (ILS)

2010	2011
28 Feb	20 Mar
30 Mar	19 Apr
5 Apr	25 Apr
20 Apr	10 May
19 May	8 Jun
20 Jul	9 Aug
9 Sep	29 Sep
10 Sep	30 Sep
17 Sep	7 Oct
18 Sep	8 Oct
23 Oct	13 Oct
30 Oct	20 Oct

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Tuesday DECEMBER 21

EXHIBIT 1.3 (*Continued*)

A pip is defined as the smallest unit of quotation for a currency. Therefore, a quote on dollar/yen of 89.98/90.00 is said to be two pips wide.

Currency trading is a fast-moving business. Dealing room errors can be disastrously expensive. For that reason, the foreign exchange community has developed rules on how quotations and trading instructions are given. The most basic rule is that the first currency in an exchange rate pair is the direct object of the trade. By that I mean to buy $10 million dollar/yen is to buy $10 million dollars against yen. The hierarchy of exchange rates is as follows:

EUR	Euro
GBP	Sterling
AUD	Australian Dollar
USD	U.S. Dollar
Non-Euro	Other European Currencies
JPY	Japanese yen

The rule in the professional market is that the higher currency on the grid is the one that deals. For example, the euro deals against all currencies (EUR/GBP; EUR/USD; EUR/JPY).

Unfortunately, two conventions for the quotation of spot foreign exchange have evolved. In the American convention, currency is quoted in terms of U.S. dollars per unit of foreign exchange (for example, sterling quoted at 1.7000 means that it takes 1.7000 U.S. dollars to equal one pound). The pound, Australian dollar, New Zealand dollar, and the euro are quoted American. Other currencies are quoted "European," which means they are expressed in the number of units of foreign exchange equal to one U.S. dollar (i.e., 90.00 yen per one U.S. dollar or 1.0850 Swiss francs per one dollar).

One additional matter that makes things even more confusing is that most exchange-traded currency futures and options quote currencies American, even for currencies that are quoted European in the spot market.

FOREIGN EXCHANGE DEALING

Two-Way Prices

Foreign exchange dealers stand ready to make bid-ask quotes on potentially very large amounts of currency to customers as well as to other banks. There is a distinction in the interbank market between reciprocal and nonreciprocal trading relationships. In a reciprocal trading relationship, two banks agree to supply each other with "two-way" (i.e., bid-ask) quotations upon demand. Reciprocal trading relationships, meaning those between money center banks, constitute the core of the foreign exchange market. A nonreciprocal trading relationship is merely a customer trading facility that happens to be between a small bank and a large money center–dealing bank. The dealer agrees to quote foreign exchange to the smaller bank, but the reverse is never expected to happen.

It is the custom in the foreign exchange market for the party soliciting a quote to reveal the size of the transaction at its onset. In the example in Exhibit 1.4, a hedge fund called Ballistic Trading is soliciting Martingale

EXHIBIT 1.4 Hypothetical Foreign Exchange Dealing Conversation: Martingale Bank New York and Ballistic Trading

	TO	**MARTINGALE NY**	**0130GMT 030110**
	HIHI FRIENDS		
1	JPY 10 PLS		
2	# 98 00		
3	I BUY		
4	#VALUE 3MARCH10		
5	#TO CONFIRM 10 MIO AGREED AT 90.00 I SELL USD		
6	# MY JPY TO MARTINGALE TOKYO		
7	#THANKS AND BIBI		
8	TO CONFIRM AT 90.00 I BUY 10 MIO USD		
9	VALUE 3MARCH10		
10	MY USD TO DIFFUSION BANK NY		
	THANKS AND BIBI		
	ENDED AT 0132GMT		

Explanation
Ballistic Trading is soliciting a quote from Martingale's NY desk.

Line 1 identifies this as a USD/JPY trade on $10 mio USD/JPY.

Line 2 is Martingale's quote in pips equal to 89.98/90.00 as the the "big figure" of 90 is understood.

Line 3 is Ballistic agreeing to buy the dollars at 90.00.

Line 4 confirms Martingale sells $10 mio USD/JPY at 90.00.

Lines 4–7 Margingale is confirming the trade and instructs Ballistic to deliver the yen to Martindale's Tokyo branch.

Lines 8–10 Ballistic confirms the trade details and instructs Martingale to deliver dollars to Diffusion Bank NY.

EXHIBIT 1.5 Sample Foreign Exchange Confirmation

Martingale Bank
Foreign Exchange Department
New York

March 1, 2010

Ballistic Trading, Inc.
Greenwich, CT
Account: 44-3309-2234

We confirm to you the following foreign exchange trade:

Trade Number	8660-071403
Trade Date	March 1, 2010
Value Date	March 3, 2010
Exchange Rate	90.00
Currency We Sold	U.S. Dollar
Amount Sold	\$10,000,000.00
Currency We Purchased	Japanese Yen
Amount Purchased	¥900,000,000.00

If you have any question about this transaction or have reason to question its accuracy contact us at once. This transaction is governened by a master trading agreeement signed by you and Martingale Bank.

Bank for a quote for dollar/yen in the amount of \$10 million—meaning Martingale's bid-ask quote on \$10 million.

This conversation (between these fictional counterparties) is being conducted over an electronic dealing network. The upshot is that Ballistic buys \$10 million against yen at 90.00. It is understood that this is a spot trade. Martingale would send Ballistic a written confirmation to memorialize the trade (Exhibit 1.5).

For large orders, say anything above \$100 million in a major currency (but less than that in a minor currency), the dealer might inquire whether the order is the "full amount." If the party indicates that the order is indeed the full amount, it means that the indicated size will not be immediately followed by similar transactions of the customer's own initiation. Why would it matter to the dealer? The answer is that under usual circumstances, the dealer will seek to rebalance its dealing book once a customer order is filled. For example, if the customer buys \$100 million against yen, then the dealing bank, having made short dollars, would immediately be in the market, buying dollars, using its reciprocal trading counterparties. The problem with a nonfull amount order is that it could put the customer and the dealer in competition with each other in the after-market just as when the dealer is trying to reconstitute its book from the effect of the original customer transaction.

Limit Orders and Stop-Loss Orders

Foreign exchange dealers accept limit orders and stop loss orders. A limit order gives a precise price at which a customer is willing to buy or sell foreign exchange. A stop loss order is a more complicated instruction. Stop loss orders are designed to liquidate bad trades in a timely manner so as to avoid steep losses. The trader who bought dollar/yen at 90.00 in the earlier example might have left instructions to "stop him out" at 88.00. This would mean that if the dollar were to trade at 88.00 or lower against the yen, the dealer would begin to sell the $10 million position as a market order. Banks accept stop loss orders only on a best-efforts basis. This means that there is never a guarantee that a stop loss order will be executed exactly at the stop level, in this case at 88.00.

The question becomes what happens if the dollar drops down to nearly 88.00. Stop orders present dealers with a chance to make some serious money if they are able to get the feel of the market correctly. Returning to the example, say that when dollar/yen was trading at 90.00, a customer gives a dealer a stop order to liquidate a position of long $10 million at 88.00. Suppose that the dollar falls to the 88.15 level. If the dealer has a good hunch that it will still go lower and trade at the 88.00 level, he, the dealer, will sell $10 million immediately for his own account in anticipation of being able to fill the customer's stop loss order later when 88.00 trades. If 88.00 does in fact trade, the dealer will have a profit of 15 pips, equal to the difference between where he sold dollars for his own account (at 88.15) and where he bought dollars (at 88.00) from the customer to fill the stop loss order. In reality, it will likely be a bit better than this for the dealer as the customer will almost certainly be filled below the level of the stop. But the whole trade is not without risk for the dealer. Consider that the dealer could have sold dollars at 88.15 only to see the dollar rebound upward, leaving him short dollars in a rising market and unable to fill the customer's stop loss order.

Knowledge of the placement of limit orders and stop loss orders, collectively called the order board, is valuable information for the dealer. One would hope this information would be kept completely confidential for the customer's sake. Sometimes the order board for a large dealer yields clues as to near-term movement of currencies. Stop loss orders can cause sudden, large movements in exchange rates, especially in cases where the market runs past an important level where there are large quantities of stop loss orders. Nonetheless, despite all the problems, there is no getting around the need for stop loss orders.

In recent years, stop loss orders have become a larger factor in the foreign exchange market because of the growing popularity of exotic options. Exotic option risk management for dealers and customers often depends on efficient execution of stop loss orders.

Direct Dealing, Brokers, and Electronic Trading

Foreign exchange dealers traditionally communicate with each other through computer messaging services, a facility that is called direct dealing. Each dealer has the ability to conduct a brief text conversation with counterparts at other dealing banks for the purpose of conducting foreign exchange trading. The actual text of a foreign exchange dealer conversation is usually highly abbreviated and assumes a close knowledge of market conditions.

Dealers make prices directly to other foreign exchange dealing banks but sometimes use the assistance of specialized foreign exchange brokers. Voice brokers work exclusively with the interbank market. They communicate with their client dealing banks via private direct phone lines and through computers. At all times, the job of the voice broker is to know who is making the highest bid and lowest ask for each currency. Brokers work their clients' orders in strict confidence, never revealing the name of a dealing bank until a trade has been completed. Brokers supply an important function in the foreign exchange market in that they collect and distribute price information.

In the 1990s, voice brokers began to have competition from electronic platforms such as the Electronic Broking System (EBS) and Reuters Matching 2000/2. According to the BOE (2009) survey, 18 percent of London market is done with voice brokers and 24 percent with electronic brokers. The NYFED (2009) survey showed 17 percent electronic broker and 20 percent voice broker trading.

Electronic broking is only one way that computers have changed the foreign exchange market. The most sophisticated dealing banks use computers to generate bid and ask quotations. "Auto-dealer" (the industry nickname) trades are usually sized in the small millions of dollars per ticket. The big trades remain the task of the flesh and blood traders, though. The auto-dealer process is not entirely robotic, as no sensible bank would let a machine run its dealing books without supervision and occasional intervention by humans. But the advantage of the auto dealer is that it can generate a more or less continuous stream of bid and ask prices.

The largest impact that computers have made is in the area of electronic trading. These platforms not only distribute bids and asks upon demand but allow clients to execute actual foreign exchange transactions online. Computerized trading and dealing has made the overall market deeper, faster, and less prone to dealing room errors. There are two main institutional varieties of electronic trading platforms. Multibank dealing systems[21] allow foreign exchange traders to compare and act on live bids and asks supplied by dealing banks. The second category consists of single-bank proprietary trading platforms.[22] This is the carriage trade of the foreign exchange market. Only a handful of the largest foreign exchange dealers have proprietary platforms, and those dealers make their platforms available only to select clients. On the other end of the spectrum are the retail foreign exchange companies[23] that focus on small speculative trading clients. Often these companies allow trading in currencies in smaller lots than the $1 million basic round lot size of the interbank market. These platforms are called foreign exchange aggregators because they combine odd-lot sized trades across their customer base, on one side, and lay off the positions in the round lots in the interbank electronic trading market.

The spread of electronic trading is one reason why the overall volume of foreign exchange trading has grown so large so fast. This was recognized in the BIS (2007) survey, where it was estimated that the median share of trades executed electronically in the interbank market was 34 percent. Electronic broking platforms were a particular success in Germany and Switzerland where they captured 55 percent and 44 percent, respectively, of all interbank trading. These shares rose to 67 percent and 58 percent for those countries when electronic trading was added to the mix. But in other countries, such as Belgium, the share was as low as 10 percent. The Bank of England (2009) survey found that 15 percent of all the London market volume was electronic trading. By contrast, the Federal Reserve Bank of New York (2009) survey found that 26 percent of New York trading was electronic.

All of these electronic trading platforms have allowed for the spread of what is sometimes called algorithmic (or just algo) trading. The term has been borrowed from electronic trading in the equity markets, but applications to the foreign exchange market are of a different nature. Some

[21]The Federal Reserve Bank of New York October 2009 survey gives examples of FXAll, Currenex, FXConnect, Globalink, and eSpeed.
[22]See King and Rime (2010).
[23]King and Rime (2010, page 39) cite examples of U.S.-headquartered FXCM, FX Dealer Direct, Gain Capital, and OANDA; European-based Saxo Bank and IG Markets; and Japanese-based Gaitame.com.

algorithmic trading is done on a high-frequency basis with the goal of stripping out what amounts to small profit opportunities—an example being pricing differentials between three exchange rates. Other methods are based on estimation of comovements in currency pairs. Another is rapid-fire execution triggered by computer analysis of macroeconomic data releases. Still others fit into the generic category of pattern recognition.

Electronic trading has greatly enhanced the price discovery process in foreign exchange. A consequence is a great narrowing of the width of the bid-ask spread, at least in cases of the major currencies. Gallardo and Heath (2009) present a graph of 22-day moving averages of bid-ask spreads (U.S. dollar per counterparty currency) of the EUR/USD, GBP/USD, JPY/USD, and AUD/USD for the period 1996–2008. Bid-ask spreads were in the vicinity of 4 to 5 pips for three of those currencies and around 8 pips for the AUD/USD in the period 1996–2001. Thereupon they began to fall, bottoming out at 2 to 3 pips for three of the currencies and 4 pips for AUD/USD. Interestingly, bid-ask spreads rocketed up following the Lehman crisis in September 2008 to the highest levels of the period in the Gallardo and Heath study, reaching over 8 pips for the three currencies and 12 pips for the AUD/USD.

Generally speaking, anything that reduces the cost of trading, deepens market liquidity, and brings new users to a market ought to be greeted with great enthusiasm. Electronic trading appears to have done all these things for the foreign exchange market. And the collateral benefit is a general cost reduction to the manufacturing of related instruments, such as all forms of options on foreign exchange. Still, electronic trading is not without controversy. But some commonly held thoughts about it may be unfounded. There is a worry that computer traders are absorbing market liquidity or that they create extra volatility. While it is true that electronic trading tends to hit the market in clusters over time, there is no evidence to date that it soaks up liquidity from the market. Moreover, at least one major study by economists at the Federal Reserve Board fails to find an association between volatility in exchange rate movements and electronic trading.[24]

Settlement of Foreign Exchange Trades

Interbank spot foreign exchange transactions settle on value date with the physical exchange of sums of currencies. Every foreign exchange deal is a

[24]Chaboud (2009) writes: "Despite the apparent correlation of algorithmic trades, there is no evident causal relationship between algorithmic trading and increased exchange rate volatility. If anything, the presence of more algorithmic trading is associated with lower volatility" (p. 1).

cross-border transaction in the sense that settlement involves the transfer of bank deposits in two countries. Suppose Bank A buys 10 million euros against dollars from Bank B at the rate of 1.400. On value day, Bank A will receive ten million euros from Bank B perhaps in a Frankfurt account. On the same day, Bank B will receive 14 million dollars from Bank A perhaps in a New York account.

Switch back now to the previous example of the dollar/yen trade with my fictional Martingale Bank and its customer Ballistic Trading. Ballistic buys $10 million at 90.00 against yen (Exhibit 1.5 was my prototypical confirmation of this trade). Suppose this takes place in the New York morning trading session and by the afternoon, the dollar has risen. When the dollar is trading at the 91.00 level, Ballistic decides to liquidate the position to realize the profit. It solicits a quote and is told by Martingale that yen is now trading at 90.95/90.97. Ballistic immediately sells its dollars at 90.95.

Now Ballistic could settle the transaction and realize the profit in either dollars or yen. The initial transaction created a long position of $10 million and a short position of 900,000,000 yen. If Ballistic did the second transaction to sell exactly 10 million dollars, the profit would consist of the residual 9,500,000 yen. If Ballistic wanted the profit in dollars, it could tell the dealer to buy 900,000,000 yen at the dealer's price of 90.95. This would leave Ballistic with a profit equal to $104,453 (=9,500,000 yen/90.95).

But not all foreign exchange deals are settled with physical exchange of currencies, especially with nonbank customer trades. Some dealing banks offer their customers the convenience of settling on the basis of the net profit or loss on a deal. Using net settlement is also a way for a bank to do business in size with customers who do not ordinarily have strong enough balance sheets to support qualifying to make large foreign exchange settlements.

INTEREST PARITY AND FORWARD FOREIGN EXCHANGE

The Forward Outright

A forward exchange rate is a quotation for settlement or value at a date in the future beyond the spot value date. Forward rates can be negotiated for any valid future spot value date, but indications are usually given for one week, one month, three months, six months, and one year in the future. The Euromarket Day Finder (Exhibit 1.3) is the customary arbiter of what is a valid forward value date.

The forward exchange rate, called the outright, is usually quoted in two parts, one being the spot bid or ask, and the other a two-way quote on forward points. Forward points are either added or subtracted from the spot rate to arrive at the forward outright.

Suppose that spot euro/dollar is quoted as 1.3998—1.4000 and that three-month forward points are quoted 0.0068—0.0070. Forward points are always quoted in foreign exchange pips. Forward points in this case are added to the spot level to obtain the three-month outright:

Dealer's Bid	Dealer's Ask
1.3998	1.4000
+0.0068	+0.0070
1.4066	1.4070

Interest Parity

When the previous example was created, forward points for the euro happened to be positive because euro interest rates were below dollar interest rates. Had the euro interest rates been above dollar rates, the forward points would have been negative. This relationship between spot exchange rates, forward points, and interest rates is called the covered interest parity theorem. This theorem, which explains forward exchange rates, plays an important role in currency option theory.

The basic concept is that the market sets the forward outright in relation to spot in order to absorb the interest rate spread between two currencies. It is a no-free-lunch idea: One cannot hop between currencies, picking up yield advantage, and lock up a guaranteed profit by using the forward market to hedge against currency risk. The forward outright is the spoiler.

For example, suppose a euro-based investor were attracted by comparatively higher yields in U.S. dollar instruments. Suppose that the yield on 90-day euro paper is 4.00 percent, the yield on 90-day dollar paper is 6.00 percent, and that the spot euro/dollar is equal to 1.4000.

The investor might consider converting euros to dollars for the purpose of investing in high-yielding dollar paper. Parenthetically, this is the *raison d'être* for a whole host of strategies known as carry trades. The problem is that there is no way for the investor to capture some of the yield spread between the dollar and the euro without taking risk on the future direction of the spot exchange rate. If the dollar were to subsequently decline against the euro, some or all of the prospective yield pickup would be lost. If the dollar were to fall sufficiently, there might be a net capital loss on the transaction. On the other hand, if the dollar were to rise, the investor would make a profit to an extent greater than the indicated yield spread of 200 basis points.

Hedging cannot get around the problem. Consider that the investor might contemplate hedging the foreign exchange risk by selling dollars forward against the euro. The key question is what forward rate for euro/dollar would be available in the marketplace. The only forward rate that makes sense from an overall market perspective is 1.4069. Any other forward outright would imply riskless arbitrage would be possible. To see this, suppose that the investor were to start out with 1 million euros. If the investor buys 90-day euro paper, the sum would grow to 1,010,000 euros. On the other hand, if the investor converts the euros to dollars, the investor would receive \$1,400,000. Invested for 90 days in dollar paper, the sum would grow to \$1,421,000. The only arbitrage-free forward outright[25] is then

$$\frac{1{,}421{,}000}{1{,}010{,}000} = 1.4069$$

At this forward rate, the investor would be indifferent between dollar-denominated paper and euro-denominated paper on a fully hedged basis.

This concept is known as the covered interest parity theorem. It can be written either in terms of American or European convention:

Interest Parity: American Convention

$$F = S\frac{(1 + R_d)^\tau}{\left(1 + R_f\right)^\tau}$$

Interest Parity: European Convention

$$F' = S'\frac{\left(1 + R_f\right)^\tau}{(1 + R_d)^\tau}$$

where the current time is t and the maturity date of the investment is T. The time remaining to maturity is denoted as τ, which is equal to

$$\tau = (T - t)$$

F is the forward rate quoted American convention for settlement at time T; S is the spot rate quoted American; R_d is the domestic interest rate, R_f is the foreign interest rate. F' and S' are the forward and spot rates quoted European convention. Both the domestic and foreign interest rates are simple

[25] I am assuming a 360-day year, simple interest, and that the euros would be sold spot and bought forward both at the dealer's asking rate.

interest rates in this formulation, but a more useful mathematical form can be had from working in terms of continuously compounded rates:

Interest Parity: American Convention

$$F = Se^{(R_d - R_f)\tau}$$

Interest Parity: European Convention

$$F' = S'e^{(R_f - R_d)\tau}$$

Specialized Forward Transactions

Dealers and other traders use the forward market to postpone or "roll" a maturing foreign exchange deal out on the calendar to a future value date. Two specialized forward transactions that accomplish this purpose are the spot/next and tom/next swap deals. These are examples of what I referred to previously as forward swaps. They are best understood in the context of an original spot foreign exchange deal. Suppose a trader buys 10 million euros against dollars spot at 1.1700. As I have described, spot transactions are for value in two bank business days.

Suppose that on the dealing day, the trader decides to extend the value date by one day. She could accomplish this by doing a spot/next swap transaction. A spot/next deal is actually a package of two trades that are bundled together. In the first part of the swap, the trader sells 10 million euros against dollars for normal spot value (hence the *spot*). Simultaneously, the trader buys 10 million euros against dollars for tomorrow's spot value date (hence the *next*). This accomplishes the stated goal of delaying settlement by one day because the settlement from the original spot deal crosses and is the opposite direction to the settlement in the first component of the spot/next swap. Some residual cash flows may yet occur on the original spot value date because of movements in the spot exchange rate between the time that the spot deal and when the spot/next roll is executed. Ordinarily this would be small in magnitude unless a violent movement in exchange rates has taken place.

Tom/next is practically the same thing as spot/next except that it is done on the day following the original spot deal (dealing date plus one day). Continuing with the example, in the *tom* part of the trade, the trader sells 10 million euros for dollars for value tomorrow. Tomorrow corresponds to the original deal's value date, which makes the settlement deliveries cross each other. The second part of the tom/next is a spot transaction to buy

10 million euros against dollars for the *next* value date—meaning the regular spot value date that occurs as usual in two bank business days.

Either way, using spot/next or tom/next, the trader in the example is able to maintain a long position in 10 million euro/dollar for one extra day without having to make physical delivery of the underlying sums of currency. Theoretically, the trader could continue to roll the value date using spot/next or tom/next transactions and keep the position on for an indefinite period of time. Alternatively, a trader could roll the position out for more than one day by doing a single swap transaction of spot against a forward outright for a specific term in the future. Squeezes in spot/next and tom/next are notorious, but not actually common in occurrence. Yet emerging market central banks have been known to engineer squeezes to flush speculators from taking or maintaining short positions in their currency.

Non-deliverable Forward Transactions

Forward transactions can be done on a non-deliverable basis. In a non-deliverable forward (NDF), counterparties agree to settle a forward transaction with the payment of a settlement amount payable on a forward value date. *Fixing date* is defined as two bank business days before forward value date. In a popular arrangement, the *fixing rate* will be defined as the observed spot exchange rate posted by a central bank on the fixing date. Settlement amount is defined as

$$Settlement\ Amount = Notional - \left[\frac{Notional \times ForwardOutright}{Fixing\ Rate}\right]$$

where the notional amount is another term for the forward face amount.

Non-deliverable forwards are essential for trading in currencies that are not fully convertible because delivery of physical currency may not be feasible. Sometimes NDF markets spontaneously spring up in cases where a government suspends currency convertibility or enacts capital controls, as did Malaysia in 1997. The Bank of England's survey of the foreign exchange market as of October 2009 covers trading in NDF contracts in London. The bulk of trading is the U.S. dollar against the Brazilian real, South Korean won, Russian ruble, Chinese yuan, and Indian rupee. There are also a much smaller number of trades done against the euro, the pound Sterling, and other currencies. The survey estimates that $26.5 billion a day are done in the London NDF market.

DEPARTURES FROM COVERED INTEREST PARITY IN 2007–2008

Covered interest parity is an arbitrage relationship that works well in normal market conditions. Taylor (1986) reached this conclusion in a carefully constructed empirical test of covered interest parity over a three-day period in November 1985. He collected contemporaneous data—by telephone conversations with dealers—at 10-minute intervals on GBP/USD and USD/DEM, observing spot, forward, and LIBOR deposit rates for 1-, 3-, 6-, and 12-month expirations. The results were a clear-cut affirmation of covered interest parity, meaning Taylor failed to find any evidence of profitable arbitrage.

Taylor (1989) did a second study that covered four other periods that he described as being times of market turbulence: the November 1967 Sterling devaluation, the June 1972 flotation of Sterling, the 1979 UK General Election, and the 1987 UK General Election. This time Taylor found evidence in at least some of the periods of profitable arbitrage opportunities. This was more the case in the earlier periods than the later ones, a finding that Taylor attributes to increases over time in the number of market participants and advances in information technology.

The finding that covered interest parity works in normal markets, Taylor's first study, but sometimes not in turbulent markets, his second study, is plainly echoed in the 2007–2008 period. This period was an exceptional time of financial crisis. Exhibit 1.6 was constructed with data from Bloomberg showing 3-month dollar LIBOR compared to the implied dollar interest rate extracted from the forward market for EUR/USD. Covered parity was working (as is normal) in 2006. But thereafter, as the crisis began to build, LIBOR rates were less than forward-implied rates. The climax of the crisis was in the middle of September 2008 when the investment banking firm of Lehman Brothers failed—here one can see forward implied LIBOR rates extending hundreds of basis points over LIBOR rates. Hui, Genberg, and Chung (2010) theorize that financial institutions, strapped for dollars, began to effectively borrow by doing massive forward swaps of foreign currency to get dollars.

This can be understood in the context of a huge surge in demand for dollar funding in the crisis period, dating from at least as early as the middle of 2007. Non-dollar-based institutions needing dollars apparently found it less costly to borrow in local currency in combination with doing forward swaps: They borrowed in local currency, then, in the first leg of the forward swap exchanged local currency for dollars on a spot basis, and finally, in the second leg of the forward swap simultaneously sold dollars forward for local currency. The excess demand for dollars showed up as the forward

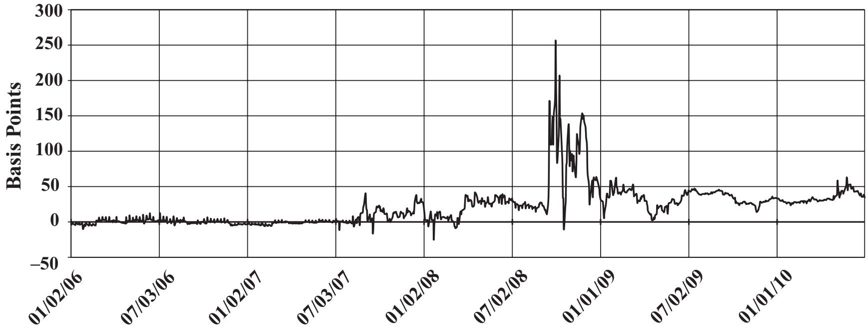

EXHIBIT 1.6 Deviations from Covered Interest Parity
Data source: Bloomberg Finance L.P.

points being bid far away from what covered interest parity would have suggested.[26]

Numerically speaking 2007–2008 appears to be a plainly obvious violation of covered interest parity. Yet, despite what the historical data show, there may not have been a way to actually make money in trying to arbitrage LIBOR against the forward market. Trading lines became scarce as counterparties scaled down or even closed trading relationships because credit fears had gripped the marketplace. And it is here that Taylor (1989) defines what truly matters: "The essence of covered interest parity is that a true deviation from it represents a potential profit opportunity to a market trader at a point in time" (p. 429). In other words, the deviation from covered interest parity may not have been a true arbitrage opportunity, but rather an illusion of potential profit, in that there was no actual way to execute the suggested trade. But there may have been yet another reason. Observers of the 2008 crisis report that some banks may have been quoting LIBOR rates below actual market levels—by one theory banks feared quoting higher, correct, LIBOR rates would have created suspicion that they were either in trouble or were simply desperate for funds.[27] Either way, although there were measured deviations from covered interest parity in this extraordinary period, it is not clear that those anomalies constituted actual profit opportunities by Taylor's criterion.[28]

[26]See Baba, Packer, and Nagano (2008).
[27]See Finch and Gotkine (2008).
[28]There are well-known covered interest parity anomalies in China's RMB market, which Wang (2010) believes can be attributed to the presence of official capital controls.

CHAPTER 2

Trading Currency Options

I now proceed to the main topic of this book, the specialized field of options on foreign exchange. I focus on vanilla European puts and calls on foreign exchange, as well as on some of the more popular exotic varieties of currency options that are covered in the final chapters.

THE INTERBANK CURRENCY OPTION MARKET

Commercial and investment banks run the currency option market. The same money-center dealers that constitute the core of the spot and forward foreign exchange market are the most powerful market makers of currency options. For this reason, this book generally uses the conventions and terminology of the interbank foreign exchange option market.

Currency options are used by currency hedgers, traders, speculators, portfolio managers, and, on occasion, by central banks.

Modern trading in currency options began in the 1970s and 1980s in the venue of the listed futures and options markets of Chicago, Philadelphia, and London. Trading was concentrated in options and futures options on only a handful of major exchange rates. A structural change occurred in the 1990s, when the bulk of trading in currency options migrated "upstairs" to bank dealing rooms.

Once installed in the domain of the interbank foreign exchange market, option trading exploded in volume. What is more, currency options began to key off of the full gamut of exchange rates. In the mid-1990s, trading in exotic currency options began to develop at a rapid pace. Today, dealers routinely supply two-way bid-ask prices for a wide spectrum of exotic currency options. However, the largest appetites for exotic currency options are for barrier options. The market for nonbarrier exotic options, including basket options, average rate currency options, compound currency options, and quantos options, is smaller yet not insignificant.

EXHIBIT 2.1 Currency Option Turnover

Year	Billons of U.S. dollars/day
1998	87
2001	60
2004	119
2007	212
2010	207

The currency option market can rightfully claim to be the world's only truly global, 24-hour option market. The currency option market is among the largest of the option markets by trading volume. The market for currency options, like the market for foreign exchange itself, is primarily an over-the-counter interbank market. The BIS (2010) survey I cited in the previous chapter has data for currency options—and these numbers show a rapid growth of currency option trading similar to that of trading in foreign exchange spot and forwards. The turnover (adjusted for double counting) in currency options revealed by the survey is shown in Exhibit 2.1.

According to the BIS (2010) survey, the outstanding notional amount of currency options in June 2010 was \$12.1 trillion with an estimated market value of \$456 billion. The notional amounts and market value were \$13.7 trillion and \$279 billion, respectively, in June 2007.[1]

The 2007 survey showed the importance of the dollar to trading in currency options, as 75 percent of all OTC options are the dollar against another currency. The largest trading (Exhibit 2.2) is done in options in the dollar against the euro, the yen, and Sterling. Though the size of trading in other currencies against the dollar was relatively smaller, it was nonetheless significant in absolute size for many currency pairs.

Exhibit 2.3 shows the findings of the BOE (2009) and NYFED (2009) surveys by currency pair of traded interbank foreign exchange options in their respective banking centers.

This chapter now turns to aspects of the mechanics of trading currency options.

[1]The BIS (2007) survey reported that the 2007 notional value of OTC interest rate options was \$56.575 trillion with outstanding market value of \$766 billion. Though larger by notional and market value outstanding, turnover in 2007 in interest rate options was \$215 billion, a number that was comparable to turnover in foreign exchange options (\$212 billion).

EXHIBIT 2.2 Reported Foreign Exchange Options Turnover by Currency Pair (Daily Averages in April, in Billions of U.S. Dollars)

	Apr-01	Apr-04	Apr-07
U.S. dollar vs. other currencies	48	92	158
Euro	16	31	43
Japanese yen	17	27	38
Pound sterling	3	9	19
Swiss franc	2	3	6
Canadian dollar	3	6	9
Australian dollar	3	8	9
Swedish krona	—	—	0
other	3	10	32
Euro vs. other currenices	10	20	37
Japanese yen	6	10	16
Pound sterling	2	3	4
Swiss franc	1	4	8
Canadian dollar	0	0	0
Australian dollar	0	1	1
Swedish krona	—	—	2
other	1	3	7
Japanese yen vs. other currencies	0	1	6
Other currency pairs	4	4	10
All currency pairs	60	117	212

Source: BIS Surveys.

OPTION BASICS

A currency put confers the right, but not the obligation, to sell a sum of foreign currency to the option seller (called the writer) at a fixed exchange rate called the option strike on or before the option expiration. A currency call confers the right but not the obligation to buy a sum of foreign currency from the option writer at the option strike on or before the option expiration. If there are no special features, such as out- or in-barriers, then the option is called a vanilla option. A call option on one currency is simultaneously a put option on a second currency. An unambiguous but somewhat redundant method identifies currency options by referring to both currencies. For example, the option on the left-hand column in Exhibit 2.4 is referred to as a USD put/JPY call—which could also be called a yen call. Similarly, the option in the right-hand column is a USD call/JPY put—which also could

EXHIBIT 2.3 Volume in Foreign Exchange Option Trading in New York and London, October 2009 (Daily in Millions of U.S. Dollars; Adjusted for Double Counting)

	London
U.S. Dollar versus	
Euro	$ 27,484
British pound	$ 5,327
Japanese yen	$ 11,373
Swiss franc	$ 3,261
Australian dollar	$ 5,136
Canadian dollar	$ 2,248
Norwegian krone	$ 215
Swedish krona	$ 191
New Zealand dollar	$ 452
South African rand	$ 208
Mexican peso	$ 1,964
Polish zloty	$ 88
Singapore dollar	$ 228
Russian ruble	$ 1,204
Turkish lira	$ 1,204
Brazilian real	$ 4,118
South Korean won	$ 2,417
Chinese yuan	$ 591
Indian rupee	$ 3,195
All other currencies	$ 3,769
EURO versus	
British pound	$ 2,881
Japanese yen	$ 3,145
Swiss franc	$ 4,490
Swedish krona	$ 1,204
Norwegian krone	$ 901
Polish Zloty	$ 882
Canadian dollar	$ 122
Australian dollar	$ 151
All other currencies	$ 3,414
STERLING versus	
Japanese yen	$ 647
Swiss franc	$ 150
Australian dollar	$ 385
Canadian dollar	$ 119
All other currencies	$ 498
ALL OTHER CURRENCY PAIRS	$ 7,403
TOTAL	$101,063

	New York
U.S Dollar versus	
Euro	$ 6,479
Japanese yen	$ 3,358
British pound	$ 1,370
Canadian dollar	$ 2,033
Swiss franc	$ 594
Australian dollar	$ 1,413
Argentine peso	$ 1
Brazilian real	$ 1,780
Chilean peso	$ 24
Mexican peso	$ 801
All other currencies	$ 2,578
EURO versus	
Japanese yen	$ 563
British pound	$ 541
Swiss franc	$ 712
ALL OTHER CURRENCY PAIRS	$ 2,839
Total	$25,086

Figures may not sum to totals due to rounding.
Sources: BOE (2009) and NYFED (2009).

EXHIBIT 2.4 Prototypical Calls and Puts on USD/JPY

Currency Pair	USD/JPY	USD/JPY
Put/Call	USD Put/JPY Call	USD Call/JPY Put
Face USD	$1,000,000	$1,000,000
Face JPY	89,336,700	89,336,700
Strike	89.3367	89.3367
Days to Expiry	90	90
Market Data		
Spot	90.00	90.00
Forward Outright	89.3367	89.3367
Option Pricing		
USD Pips	0.00030658	0.00030658
Total USD	**$27,389**	**$27,389**
JPY Pips (four digits)	2.4650	2.4650
Total JPY	2,464,996	2,464,996
Percentage of Face Amount	2.74%	2.74%

be called a yen put. USD and JPY are the interbank codes for the U.S. dollar and the Japanese yen, respectively.

Face Values

In Exhibit 2.4 the face value in dollars of each option is $1,000,000. The options are struck at dollar/yen equal to 89.3367. The face value of the options in yen can be found by multiplying the face in dollars by the strike, which is equal to 89,336,700 yen.

Exercise Conventions

The majority of interbank currency options are European exercise convention, meaning that they can only be exercised on the last day of their life. Strictly speaking, the holder of an option has a 24-hour window in which to exercise a European option.

A smaller number of interbank currency options are American exercise, which means they can be exercised at any time in their life.

The most popular option expiration cut-off time is 10:00 A.M. New York time (called the New York Cut). Some options expire at 3:00 P.M. Tokyo time (Tokyo Cut).

Exercise Mechanics

Upon exercise, a currency option turns into a spot foreign exchange deal done at the option strike for settlement on the spot value date. For example, suppose that the holder of the yen call in Exhibit 2.4 elected to exercise on the expiration date that happened to be a Monday. The exercise would obligate the holder of the option to deliver $1,000,000 to the option writer's account in a U.S. bank and obligate the option writer to deliver 89,336,700 yen to the option holder's account in a bank for value in two bank business days' time, normally on Wednesday if there were no intervening holidays.

The holder of the yen put displayed in Exhibit 2.4 would be rational to exercise the option at expiration if the spot exchange rate exceeded the option strike of 89.3367. Suppose that the spot exchange rate on exercise day were equal to 92.00. Exercise would mean that the option holder would receive delivery of the dollar face amount, equal to $1,000,000, and would be obligated to make delivery of the yen face amount, equal to 89,336,700 in two bank business days' time.

The exercise of the option leaves the option holder with a position that is long dollars and short yen. Normally, the exerciser of an option seeks to realize value by doing a spot transaction to liquidate the cash flow consequences of the exercise. For example, the exerciser of the yen put could sell the dollars and buy exactly 89,336,700 yen. The residual sum of dollars would be equal to $28,948.91:

$$\$1{,}000{,}000 - \frac{89{,}336{,}700\ yen}{92.00} = \$28{,}948.91$$

Alternatively, the option holder could sell exactly $1,000,000 dollars against yen. This would leave a net amount of 2,663,300 yen:

$$\$1{,}000{,}000 \times 92.00 - 89{,}336{,}700 = 2{,}663{,}300\ yen$$

Option Prices

Currency option dealers quote prices to customers in a variety of equivalent ways. Options can be quoted in either currency or as a percentage of face value. The USD put/JPY call in Exhibit 2.4 is quoted as .00030658 dollars per unit of yen face.[2] To arrive at the exact number of dollars, multiply the

[2]When calculated to a sufficient number of decimal digits, the premiums on the yen call and yen put in Exhibit 2.4 are slightly different in value, yet they are both struck at-the-money forward. This would seem to contradict a well-known theorem that the value of an ATMF call must be equal to that of an ATMF put (Chapter 4).

dollars per unit of yen face times the yen face amount:

$$0.00030658\left(\frac{USD}{JPY}\right) \times 89{,}336{,}700\ JPY = \$27{,}389$$

Alternatively, the quote in yen per unit of dollar face, 2.4650, can be multiplied by the dollar face amount to arrive at the price of the option in yen:

$$2.4650\left(\frac{JPY}{USD}\right) \times \$1{,}000{,}000 = 2{,}464{,}996\ JPY$$

The second method, using yen per unit of dollar face, is more familiar because 2.4650 corresponds to the usual way that dollar-yen is quoted. In the language of the spot dealers, the option costs approximately 246 yen pips.

A third method, and most simple way to quote the option, is as a percentage of face value. The USD call/JPY put in Exhibit 2.4 is quoted as 2.74 percent of face, as can easily be calculated:

$$\frac{\$27{,}389}{\$1{,}000{,}000} = 2.74\%$$

Delta and Option Identification

The interbank currency option market is one of the most professional of all trading markets. Not surprisingly it has developed its own set of convenient rules for identifying currency options. Instead of using strikes to tell one option from another, traders use deltas.

The delta of a currency option is the partial derivative of its price with respect to the underlying spot exchange rate. The concept of delta is one of the most important and useful inventions in the original Black and Scholes (1973) paper. That is true for a number of reasons, not the least of which is that delta is the key element in hedging and replicating options—this will be discussed at length in later chapters. For now, we will talk about delta's handiness in identifying options.

The delta of a standard (i.e., non-exotic) currency option is never less than zero nor greater than unity in absolute value. Unless otherwise noted, I will adopt the practice common among traders to multiply call deltas by 100—hence a mathematical delta equal to .50 is called 50 delta on the trading desk. Mathematically, put deltas are negative, ranging from zero to

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The reason for the discrepancy is that the strike price, which is calculated from the interest parity theorem, is rounded to two decimal digits, following the convention of the interbank market to quote dollar/yen to a precision of two pips.

negative one. In the trading world, put deltas are multiplied by negative 100. Hence a put with a mathematical delta of negative one-half is identified as being 50 delta.

An option that is significantly out-of-the-money will have a low delta. Examples would be 15 or 10 delta options. An example of an option deep-in-the-money would be one with 75 delta.

Four Important Trades

Of special interest are options struck at the prevailing forward outright that matures on the option expiration date. These are known as at-the-money forward options (ATMF), and they are approximately 50 delta. They have special properties that will be extolled in later chapters.

Straddles are combinations consisting of a same-strike call and put. The ATMF straddle is made up of a call and a put both struck ATMF. At inception such a straddle would have no delta because the ATMF call delta and the ATMF put delta cancel each other.

A risk reversal is made up of a long position in a call and a short position in a put with approximately equal delta (but unequal strike). Or it could be a short call and a long put with approximately equal absolute value delta (but unequal strike). The "25 delta risk reversal" consists of long and short positions in 25 delta calls and puts. The 25 delta risk reversal has an initial delta of 50 (getting 25 delta from each option).

Other common risk reversals are 15 delta and 10 delta. Another important trading strategy is the butterfly. This consists of four options. For example, in a 25 delta butterfly, two ATMF calls might be bought and one each of a 25 delta and 75 delta calls sold short. Butterflies are important in estimation of volatility surfaces.

All four of these important trades, meaning ATMF options, straddles, risk reversals, and butterflies, are routinely quoted by dealers for standard expirations: one week, one month, three months, six months, and one year.

A Sample Interbank Option Confirmation

Exhibit 2.5 displays a sample interbank foreign exchange option confirmation that can be regarded as industry standard in several important respects:

- The option premium is due two bank business days after trade date and is payable in dollars.
- Notice of exercise on expiration day must be given before 10:00 A.M. New York time.
- The option cannot be assigned to third parties.

EXHIBIT 2.5 Example of OTC Confirmation

Martingale Bank

New York, New York

March 1, 2010

Ballistic Trading Partners
Greenwich, CT

Ladies and Gentlemen:

The purpose of this confirmation is to confirm the terms and conditions of the Foreign Exchange Option entered into between Martingale Bank and Ballistic Partners.

General Terms

Trade date:	March 1, 2010
Buyer:	Ballistic Trading Partners
Seller:	Martingale Bank
Call Currency:	JPY
Put Currency:	USD
Option Style:	European
Notional Amount USD:	$1,000,000
Notional Amount JPY:	¥89,336,700
Expiration Date:	June 8, 2010
Strike Price:	89.3367
Premium:	$27,389

For a premium of 27,389 USD, receipt of which is due on June 8, 2010, you may call on the undersigned for up to 89,336,700 yen, at a JPY per USD rate of 89.3367. You may exercise this European Style option up to 10:00 A.M. New York time only on June 8, 2010 (the "Expiration Date") by telephone notice to the undersigned, confirmed in writing, indicating the amount of the currency you have elected to purchase, whereupon delivery and settlement shall be made on June 10, 2010. If the Expiration Date is not a New York business day, the Expiration Date shall be the next succeeding business day.

This option may not be assigned by either party and any such assignment shall be void and of no force or effect.

Unless you instruct us otherwise by 10:00 A.M. New York time on the Expiration Date, this option shall be deemed to be automatically exercised if at such time on such date it is in-the-money.

Acceptance by you of the premium constitutes your representation that you have substantial assets and/or liabilities denominated in the currency underlying this option, are in the business of trading or investing in such currency, in options in such currency or in assets and/or liabilities denominated in such currency or are otherwise a commercial user of such currency, and that you will be entering into such transaction for purposes related to your business in such currency.

The parties hereby consent to the jurisdiction of a state or federal court situation in New York City, New York in connection with any dispute arising hereunder. This option shall be governed and construed in accordance with the laws of the State of New York with regard to the principles of conflict of laws.

Very truly yours,

Martingale Bank

There is a measure of conformity in interbank option confirmations, as there is in the nomenclature of dealing. Prudent traders know that confirmations are parts of legally binding contracts, and as such, should be carefully examined.

Margin Practices for Interbank Currency Options

In most cases, customers who trade interbank currency options are required to have a preexisting spot dealing facility with their dealing bank. This is because the exercise of an option creates a spot-like transaction. Also, this rule can be seen as being part of the effort by a bank to screen for client suitability for option trading. Spot dealing lines are not usually granted except to creditworthy and sophisticated institutions.

In some instances, interbank option dealers permit clients without spot dealing facilities to purchase non-deliverable options. A non-deliverable option pays an option holder the value of any in-the-money exercise at expiration without invoking the physical exchange of currencies. This is similar in concept to the non-deliverable forward transaction described in the previous chapter.

Purchasers of interbank currency options must pay for their option in full two bank business days after the option trade date.

Some dealers credit their customers' trading lines for holding options that are deep in-the-money in a practice called delta release.

Short-sellers of interbank currency options must make margin arrangements with their dealer. The actual terms will depend on the nature of the relationship with the short-seller's bank, but in most instances, the option short-seller is required to post an initial bond and agree to top-up the collateral if the option appreciates in value. A common practice is to margin these positions using option analytics, such as the delta applied to the face value of the option. More elaborate are margin rules based on value-at-risk calculations.

Although the largest market for currency option trading is the interbank market, there is also trading in listed currency options on the Philadelphia Stock Exchange and listed currency futures options on the Chicago Mercantile Exchange.

LISTED OPTIONS ON ACTUAL FOREIGN CURRENCY

The Philadelphia Stock Exchange (PHLX) lists options on actual sums of foreign currencies. The Options Clearing Corporation is the counterparty to

EXHIBIT 2.6 The Philadelphia Stock Exchange Currency Options: Standardized Currency Options Contract Specifications

Currency	Size	Premium Quotations	Minimum Premium Change	Strike Intervals
Dollar-Based				
Australian dollar	10,000 AUD	100 dollars per unit	.01 = $1.00	.5 cent
Canadian dollar	10,000 CAD	100 dollars per unit	.01 = $1.00	.5 cent
Euro	10,000 EUR	100 dollars per unit	.01 = $1.00	.5 cent
Japanese yen	1,000,000 JPY	100 dollars per unit	.01 = $1.00	.005 cent
Swiss franc	10,000 CHF	100 dollars per unit	.01 = $1.00	.5 cent
Mexican peso	100,000 MXN	100 dollars per unit	.01 = $1.00	.05 cent
New Zealand dollar	10,000 NZD	100 dollars per unit	.01 = $1.00	.5 cent
South African rand	100,000 ZAR	100 dollars per unit	.01 = $1.00	.05 cent
Swedish krona	100,000 SEK	100 dollars per unit	.01 = $1.00	.05 cent

every option buyer and seller. The U.S. Securities and Exchange Commission regulates trading in PHLX options.

Contract Specifications

Contract specifications for currency options listed on the Philadelphia Stock Exchange are displayed in Exhibit 2.6. All of the options are calls and puts on currencies against the U.S. dollar, and all are European exercise. PHLX options have standardized strike prices, as is the case with other listed options on shares of stock, equity indexes, bonds, and currency. The PHLX currency options market is open from 9:30 A.M. to 4:00 P.M. eastern standard/eastern daylight time, Monday through Friday.

Quotation Conventions

PHLX options that are dollar-based are quoted in dollars that are standardized with 1 point equal to $100. A PHLX option on the euro (face 10,000 euros) quoted as 2.13 would cost

$$\$2.13 \times \$100 = \$213$$

A PHLX yen option (face 1,000,000 yen) quoted as 2.70 would cost

$$\$2.70 \times \$100 = \$270$$

Expiration

Expiration of the PHLX currency options is on the Saturday following the third Friday of the expiration month. This third Friday is the last trading day of the expiring contract. The currency expiration cycle offered maintains six expiration months at all times, the fixed quarterly months of March, June, September, and December and two additional near-term months. Standardized options expire at 11:59 P.M. eastern time on expiration day.

Exercise

In-the-money options are automatically exercised at expiration and cash-settled in U.S. dollars.

Position Limits

The PHLX establishes a position limit on the maximum number of contracts in an underlying currency that can be controlled by a single entity or individual. Currently, position limits are set at 600,000 contracts on each side of the market (long calls and short puts or short calls and long puts) for the Australian dollar, British pound, Canadian dollar, Japanese yen, New Zealand dollar, and Swiss franc. The limit is 1,200,000 contracts for the euro, and 300,000 for the Mexican peso, Swedish krona, and South African rand. Options involving the U.S. dollar against other currencies are aggregated for purposes of computing position limits.

PHLX Flex Options

PHLX Flex options are U.S. dollar-settled currency options for foreign currencies, which may be tailored to specific strike prices as well as for expiration dates.

CURRENCY FUTURES CONTRACTS

Currency futures are listed on the Chicago Mercantile Exchange's International Monetary Market (CME [IMM]) and on several other exchanges around the world. As measured by trading volume, the CME (IMM) is the most important currency futures exchange. This book conducts the discussion of currency and currency futures options in the framework of the CME (IMM) derivatives.

CME (IMM) currency futures are traded on the CME Globex electronic trading platform and in pits in an open outcry environment similar to futures contracts on agricultural and other financial commodities.

The Clearing House of the Chicago Mercantile Exchange clears all trades in the exchange's listed currency futures and futures options. The CME clearing house interposes itself between each buyer and seller of every currency futures contract in order to act as a guarantor of contract performance. This practice allows traders to operate on a net basis no matter that their long and short positions might have been initiated against different counterparties. Trading in U.S. currency futures and futures options is regulated by the Commodities Futures Trading Commission.

Contract Specifications

Listed futures contracts have fixed specifications with respect to expiration date, size, and futures tick value (Exhibit 2.7). For example, the CME yen futures contract has a notional value of 12,500,000 yen. CME regular trading hours are 7:20 A.M. to 2:00 P.M. central standard time. This applies to the currency futures and currency futures options. Trading is done on the electronic medium Globex from 5:00 P.M. to 4:00 P.M. the next day on weekdays and on Sundays.

Quotation Conventions

The CME (IMM) quotes futures American quotation style. Exhibit 2.7 displays the value of one futures tick for each contract.

Expiration

The last day of trading in CME (IMM) currency contracts is the second business day immediately preceding the third Wednesday of the contract month. The last day for Canadian dollar futures is the business day preceding the third Wednesday of the contract month. The last day for Brazilian real futures is the last business day of the contract month. The last trading day for Chinese yuan futures is the first Beijing business day preceding the third Wednesday of the contract month. The last trading day for the Russian ruble is the fifteenth day of the month, if not a business day, the next business day for the Moscow interbank foreign exchange market.

Margin Requirements

The clearing house of the CME sets minimum initial and maintenance margin rules for trading in currency futures. The exchange rules differentiate

EXHIBIT 2.7 Listed Currency Futures and Futures Options: Chicago Mercantile Exchange

Currency	Size	(Futures Tick Size) Futures Tick Value	Exchange symbol Globex*/ Open Outcry	Option Strike Intervals	Examples of Strike Intervals	Maximum Price Fluctuation
Dollar-Based						
Australian dollar	100,000 AUD	($.0001) $10.00	6A/AD	$0.0050	$0.8800 and $0.8850	$0.0060
Brazilian real	100,000 BRL	($.00005) $5.00	6L/BR	$0.0050	$0.5400 and $0.5450	$0.0030
British pound	62,500 GBP	($.0001) $6.25	6B/BP	$0.010	$1.5600 and $1.5700	$0.0060
Canadian dollar	100,000 CAD	($.0001) $10.00	6C/CD	$0.0050	$0.9550 and $0.9600	$0.0060
Chinese renminbi	1,000,000 CNY	($.00001) $10.00	RMB/—	$0.00100	$0.13900 and $0.14000	$0.00060
Czech koruna	4,000,000 CZK	($.000002) $8.00	CZK/CKO	$0.010000	$5.2200 and $5.2300	$0.000250
Euro	125,000 EUR	($.0001) $12.50	6E/EC	$0.0050	$1.3600 and $1.3650	$0.0060
Japanese yen	12,500,000 JPY	($.000001) $12.50	6J/JY	$0.000050	$0.011100 and $0.011150	$0.000060
Mexican peso	500,000 MXN	($.000025) $12.50	6M/MP	$0.000625	$0.077500 and $0.078125	$0.001500
New Zealand dollar	100,000 NZD	($.0001) $10.00	6N/NE	$0.0050	$0.6950 and $0.7000	$0.0060
Polish zloty	500,000 PLN	($.00002) $10.00	PLN/PLZ	$0.00100	$0.33700 and $0.33800	$0.00250
Russian ruble	2,500,000 RUB	($.00001) $25.00	6R/RU	$0.000250	$0.032750 and $0.033000	$0.0060
Swiss franc	125,000 CHF	($.0001) $12.50	6S/SF	$0.0050	$0.9250 and $0.9300	$0.0060
Cross rates						
Euro/British pound	125,000 EUR	(£.00005) £6.25	RP/UE	$0.00250	$0.86750 and $0.87000	£0.00300
Euro/Japanese yen	125,000 EUR	(¥.01) ¥1,250	RY/UH	¥0.50	¥122.00 and ¥122.50	¥0.60
Euro/ Swiss franc	125,000 EUR	(CHF.0001) CHF12.50	RF/UA	CHF 0.0025	CHF1.4650 and CHF1.4675	CHF 0.0060

Note: Futures contracts on various other cross-rates are also traded on the CME (www.cmegroup.com/trading/fx).
Source: Chicago Mercantile Exchange.

speculators from hedgers. The CME (IMM) has special margin rules for intracurrency spreads such as long one March, short one June Swiss franc futures. Intercurrency spread, such as a position that is long Australian dollar futures and simultaneously short Canadian dollar futures also have exceptions. Initial margin can be met with cash or U.S. Treasury securities, but in the case of the latter, a haircut is applied, meaning that the security will count as something less than 100 cents on the dollar.

Daily variation margin operates to pay and collect the gains and losses on futures every day. Variation margin is based on successive changes in the daily settlement price. In theory, the settlement price will be the last bona fide price at the close of a trading session. However, in practice, determination of a fair settlement price can be difficult because of the nature of the open outcry system where many trades can occur simultaneously at the close. In these cases, the settlement price can be the average of the highest and lowest trades that are done at the close. When no trade is done at the close, the clearing house is permitted to use special procedures that take into account the historical relationship between contract months. The procedures that govern settlement prices are specified in Exchange Rule 813.

Long positions in futures contracts receive positive variation margin and pay negative variation margin. Short futures positions do just the opposite; they pay positive variation margin and receive negative variation margin.

On the day that a position is opened, variation margin is based on the difference between the traded price and the settlement price that day. Thereafter, the daily variation margin is based on the difference between the settlement price that day and the settlement price on the previous day. On the day when the position is closed, the variation is based on the difference between the traded price and the previous day's settlement price.

Position Limits

The IMM enforces limits on the size of the position that any one investor may accumulate in a single currency. The limit combines the investor's position in both IMM futures and futures options. Futures options are counted on a delta-adjusted basis. The delta of an option is the dollar amount by which its price ought to change when the price of the underlying asset changes by one unit. The concept of delta will be covered in later chapters. But for now, if the delta of a call option is .5, the CME Market Surveillance Department counts the option as .5 futures contracts in the calculation of an investor's position limit.

Speculative limits on certain currencies are subject to the Position Accountability Rule that was approved by the Commodities Futures Trading Commission on January 2, 1992. Under this rule, any market participant who holds or controls a position in the euro, Japanese yen, British pound, or the Swiss franc futures and futures options contracts that exceed 10,000 contracts net long or short across combined contract months "shall provide, in a timely fashion, upon request by the Exchange, information regarding the nature of the position, trading strategy, and hedging information if applicable."

Exchange for Physical Transactions

Currency futures and spot foreign exchange are linked through a specialized trading market in exchange for physical (EFP) transactions. In an EFP trade, a long or short position in a currency futures contract is exchanged for an equivalent face position in spot foreign exchange. EFP trades are referred to as ex-pit transactions because they are executed at off-market prices. EFPs are quoted as two-way prices for either buying futures/selling spot or selling futures/buying spot. In normal markets, the EFP market closely follows the level of the swap points in the forward market for value on futures expiration.

One important function of the EFP market is to allow currency futures traders to unwind cash market stop-loss orders or limit orders that were executed outside of floor trading hours. The purpose of an EFP trade is to swap the futures and cash positions so as to flatten both positions.

The Concept of Basis

The difference between the futures price and the spot exchange rate is called the basis. The basis in currency futures is analogous to the forward points in a currency forward transaction. In theory, the basis is a function of the time to expiration, the level of the spot rate, and the spread between the interest rates of the currencies in question. The term *calendar basis* means the spread in prices between two futures contracts on the same currency but with different expiration dates.

LISTED CURRENCY FUTURES OPTIONS

A currency futures option delivers a long or a short position in a currency futures contract upon exercise.

Contract Specifications

The Index and Options Market of the Chicago Mercantile Exchange (CME [IOM]) lists American-style put and call options on their currency futures contracts (Exhibit 2.7). Trading on the IOM is regulated by the Commodities Futures Trading Commission.

Quotation Conventions

The rule for quoting futures options is similar to the procedures of the Philadelphia exchange for currency options. Quotations for futures options are given in U.S. dollars, which are multiplied by the deliverable futures contract's quantity of foreign currency. A futures option on the euro quoted at \$0.0124 would be worth:

$$\$0.0124 \times 125{,}000\ EUR = \$1{,}550.00$$

Expiration

The CME (IOM) lists currency futures options for quarterly expiration (that follow the March quarterly cycle), for monthly expiration for serial months (i.e., months that are not in the March quarterly cycle), and for weekly expiration. Quarterly and monthly expiration options cease trading on the second Friday immediately preceding the third Wednesday of the contract month (with special rules for options on futures on the Brazilian real, South African rand, Chinese yuan, and Russian ruble).

Margin Requirements

The CME (IOM) determines futures option margin requirements using a proprietary software program, which it calls SPAN (Standard Portfolio Analysis of Risk). SPAN sets margin requirements according to estimates of the maximum one-day loss that a position might suffer. SPAN generates daily margin requirements based on portfolio risk analysis and scenario models of changing market conditions. SPAN arrays are provided daily to clearing firms to calculate minimum margin requirements.

Exercise

CME (IOM) futures options may be exercised on any business day that the option is traded. To exercise an option, the clearing member representing the buyer must present an exercise notice to the clearing house by 7:00 P.M. of the day of exercise.

Exercise notices that are accepted by the clearing house are assigned by a process of random selection to clearing members that have open short option positions. Clearing firms then assign exercise to one or more of their clients who have short positions in the particular futures option that has been exercised.

The deliverable asset underlying a currency futures option is a currency futures contract. Following exercise, the delivered futures position becomes a live position on the trading day immediately following the exercise. Exercise creates long and short futures contract positions in the accounts of the option owner and writer, according to the following rule:

Exercise of a Futures Option	**Call Option**	**Put Option**
Owner of the Option	Long Future	Short Future
Writer of the Option	Short Future	Long Future

The deliverable futures contract is the one that is next after expiration in the March-June-September-December cycle. The exercise of an October futures options would result in the delivery of a December futures contract. The spread between the option strike and the futures settlement price is thereupon reflected in the accounts of the clearing firms and in their clients' accounts. If on a day in May, the owner of a June EUR call struck at 133.00 were to exercise, he would be credited with a long June futures position and a mark-to-market equal to

$$(1.34 - 1.33) \times 125{,}000 = \$1{,}250$$

where 1.34 is the settlement price of the June futures contract on that day. Initial and variation margin rules for the futures contract immediately come into force.

CHAPTER 3

Valuation of European Currency Options

This chapter discusses the valuation of European currency options. A European currency option is a put or a call on a sum of foreign currency that can be exercised only on the final day of its life. These options are sometimes called *vanilla* options because they have no exotic features, such as out-barriers. The chapter begins with various arbitrage and parity theorems and then advances to the important Black-Scholes-Merton model for European currency options as adapted by Garman and Kohlhagen (1983).

The following conventions will be used throughout this book:

C is the value of a European currency call option.

P is the value of a European currency put option.

S is the spot exchange rate.

K is the option strike.

R_f is the interest rate on the foreign currency.

R_d is the interest rate on the domestic currency.

For the purpose of presenting theoretical material, it will be assumed that the deliverable underlying asset of a basic put and call is one unit of foreign exchange. Both S and K are denominated in units of domestic currency (i.e., expressed in American spot convention): One unit of foreign currency is worth S units of domestic currency.

In this framework, a call is the right but not the obligation to surrender K units of domestic currency to receive one unit of foreign currency. A put is the right but not the obligation to surrender one unit of foreign currency and receive K units of domestic currency. The current time is denoted as t. Option expiration occurs at time T. The remaining time to expiration is τ which by definition is equal to $T - t$.

ARBITRAGE THEOREMS

Arbitrage is defined as the simultaneous purchase and sale of two or more securities in an attempt to earn a riskless profit.

Arbitrage opportunities do exist in the market on occasion but usually only for brief periods of time. When they are discovered, traders with ready sources of capital are quick to take advantage. Whatever is cheap soon becomes more expensive, and whatever is expensive soon becomes cheaper. The process only stops when such opportunities are "arbitraged out" of the market. In equilibrium, no permanent arbitrage opportunities should exist—this is referred to as the no-arbitrage condition or the no-arbitrage rule. A central tenet of capital markets theory is that all assets, including foreign exchange and options on foreign exchange, should be priced in the market consistent with the no-arbitrage rule.

Four elementary theorems of currency option pricing follow from the no-arbitrage rule (see Gibson 1991 and Grabbe 1983).

Option Values at Expiration

The value of a call and a put at expiration are given by

$$C_T = Max\,[0, S_T - K]$$
$$P_T = Max\,[0, K - S_T]$$

where C_T, P_T, and S_T are the values of the call, the put, and the spot exchange rate at expiration time T, respectively.

Options Have Non-Negative Prices

That is

$$C \geq 0, P \geq 0$$

The rationale is that because an option confers the right but not the obligation to exercise, it can never have a negative value.

Upper Boundaries

The maximum value for a European call is the spot value of the underlying deliverable currency:

$$C \leq S$$

If this were not true, an arbitrage profit would exist from selling the option and buying the deliverable underlying foreign currency, the latter having value equal to S.

Likewise, the maximum value for a European put is the value of the deliverable domestic currency which has value equal to the strike:

$$P \leq K$$

Lower Boundaries

The greater lower boundaries for currency calls and puts are given by

$$C \geq e^{-R_f \tau} S - e^{R_d \tau} K$$
$$P \geq e^{R_d \tau} K - e^{-R_f \tau} S$$

The terms $e^{-R_f \tau}$ and $e^{-R_d \tau}$ represent the continuous-time present value operators for the foreign and domestic interest rates, respectively.

To verify the inequality for the call, consider the following transaction involving two portfolios: The first portfolio consists of a long position in a call plus a long position in a domestic currency zero coupon bond that pays the deliverable amount of domestic currency, equal to the strike, at expiration (its present value is equal to $e^{-R_d \tau} K$); the second portfolio consists of a zero coupon foreign currency bond that pays one unit of foreign currency at the option's expiration date (its present value would be equal to $e^{-R_f \tau} S$). The payoff matrix at expiration would be

	$S_T \leq K$	$S_T > K$
Portfolio #1		
Long Call	0	$S_T - K$
Long Domestic Bond	K	K
Total Value	K	S_T
Portfolio #2		
Long Foreign Bond	S_T	S_T
Total Value	S_T	S_T

Taking a long position in portfolio #1 and a short position in portfolio #2 would create a non-negative payoff at expiration

	$S_T \leq K$	$S_T > K$
Portfolio #1 – Portfolio #2	$K - S_T \geq 0$	0

Since the expiration value of the first portfolio is greater than or equal to that of the second portfolio, the no-arbitrage rule forces the value of the first portfolio to be greater than or equal to the value of the second portfolio before expiration:

$$C + e^{-R_d \tau} K \geq e^{-R_f \tau} S_t$$

which completes the proof. A similar proof can be constructed for European puts.

PUT-CALL PARITY FOR EUROPEAN CURRENCY OPTIONS

Put-call parity is an arbitrage linkage between the prices of put and call options. It states that at any time before expiration, the difference between the price of a European put and a European call, having the same strike and same expiration, must be equal to the difference between (a) the present value of the deliverable quantity of domestic currency (i.e., the strike) and (b) the present value of the deliverable quantity of foreign currency.

The trick to understanding put-call parity is to realize that if you were long a put and short a call, you would in effect have a long position in domestic currency and a short position in the foreign currency, regardless of the level of the exchange rate on expiration day. This is because, if the put finishes in-the-money, you would exercise, meaning deliver foreign currency and receive domestic currency. But the same thing would happen if the short call finishes in the money. The call would be exercised against you, and again you would be obligated to deliver foreign exchange and receive domestic. If both options finish at-the-money, both would be worthless, but on the other hand, the deliverable quantity of foreign exchange would exactly equal the value of the deliverable quantity of domestic currency.

More formally, European put-call parity can be demonstrated by considering two portfolios. Portfolio #1 consists of a long European put and short European call having the same strike and expiration. At expiration, the deliverable quantity of foreign currency upon exercise of either option is one unit, which will be worth S_T. Portfolio #2 consists of a long position in a zero coupon bond that pays the deliverable quantity of domestic currency upon exercise, which will be worth K, plus a short position in a foreign currency zero coupon bond that pays one unit of foreign exchange at expiration. The equivalence of portfolio #1 and portfolio #2 can be demonstrated with the following expiration-day payoff matrix.

	$S_T \leq K$	$S_T > K$
Portfolio #1		
Long Put	$K - S_T$	0
Short Call	0	$-(S_T - K)$
Total Value	$K - S_T$	$K - S_T$
Portfolio #2		
Long Domestic Bond	K	K
Short Foreign Bond	$-S_T$	$-S_T$
Total Value	$K - S_T$	$K - S_T$

Since at expiration the two payoff matrices are equal, the cost of creating the portfolios before expiration must be equal. The cost of portfolio #1 is the difference between the put and the call. The cost of portfolio #2 is the difference between the present values of the domestic bond that pays the strike at expiration and the foreign currency bond that pays one unit of foreign exchange at expiration. This completes the demonstration of put-call parity, which can be expressed algebraically as

Put-Call Parity (European Options)

$$P - C = e^{-R_d \tau} K - e^{-R_f \tau} S$$

One immediate implication of put-call parity is that the value of at-the-money forward European puts and calls that have a common expiration must be equal. This can be seen by substituting the value of the interest parity forward rate for the strike K in the put-call parity formula:

$$K = F = e^{(R_d - R_f)\tau} S$$

then

$$P - C = e^{-R_d \tau} e^{(R_d - R_f)\tau} S - e^{-R_f \tau} S = 0$$

Option traders have developed a convenient paradigm for decomposing the value of an in-the-money (relative to the forward) currency option using put-call parity. Consider a call that is in-the-money, meaning that the prevailing forward outright exceeds the option strike ($F > K$). According to put-call parity, that option is worth

$$C = P + e^{-R_f \tau} S - e^{-R_d \tau} K$$

which can be written as

$$C = P + e^{-R_d \tau} (F - K)$$

If C is in-the-money forward, then it follows that a same-strike put, P, is out-of-the-money forward. The value of such a put is pure optionality, so to speak, or what traders call volatility value. Traders call the absolute value of the expression $(F - K)$ parity to forward. The term $e^{-R_d \tau}$ is a present value operator. All together, the value of the call is the sum of its volatility value and the present value of its parity to forward.

THE BLACK-SCHOLES-MERTON MODEL

The European option model was developed in stages by several theoreticians. The genesis for the idea comes from the well-known Black-Scholes (1973) model that was developed for a European call option on shares of a common stock that does not pay dividends. Merton (1973) extended this model to the theoretical case of an option on a share of stock that pays dividends continuously. Finally, Garman and Kohlhagen (1983) adapted the model to work for European options on foreign currencies. We refer to this as the Black-Scholes-Merton or BSM model.[1]

Three Assumptions

Like all theoretical models, BSM requires some simplifying assumptions:

1. There are no taxes, no transaction costs, no restrictions on taking long or short positions in options and currency. All transactors are price takers. This means that no single economic agent can buy or sell in sufficient size so as to control market prices.

[1]The option-pricing model for currency options is called by a variety of different names, ranging from Black-Scholes to Black-Scholes-Garman-Kohlhagen to Garman-Kohlhagen. Yet with no disrespect to Garman and Kohlhagen's work, it is clear that the critical thought process originated from Black, Scholes, and Merton. Black's "How We Came Up With The Option Formula" (1989) gives an interesting account of the discovery of the model and on Merton's contribution. Emanuel Derman's 1996 article "Reflections on Fischer" has further insights into Black's thinking on how the model came into existence.

2. The foreign and domestic interest rates are riskless and constant over the term of the option's life. All interest rates are expressed as continuously compounded rates.
3. Instantaneous changes in the spot exchange rate are generated by a diffusion process of the form.

The BSM Diffusion Process

$$\frac{dS}{S} = \mu dt + \sigma dz$$

where μ is the instantaneous drift and dt is an instant in time. Said another way, the term μ represents the risk premium on the spot exchange rate; σ is the instantaneous standard deviation. The differential of a stochastic variable is dz; dz is normally distributed, its mean is zero, its standard deviation is the square root of dt, and its successive values are independent.

The first assumption is a standard one that appears in many financial models, sometimes called the frictionless markets condition.

The second assumption is the key modification to the original Black-Scholes model (which was crafted for puts and calls on non-dividend-paying common stock) to make it work for options on foreign exchange. The interest rate on the foreign currency plays an analogous role to that of the continuous dividend in the Merton version of the model for options on common stocks.

The third assumption specifies that the stochastic process that generates exchange rates is a diffusion process. This particular process implies that the spot exchange rate level, S_t, is distributed lognormal. The natural log return series

$$\ln \frac{S_t}{S_{t-1}}$$

is normally distributed with mean

$$\left(\mu - \frac{\sigma^2}{2}\right)$$

and standard deviation σ (see Hull, 2009).

Six parameters must be known to use the BSM model:

S: The spot exchange rate quoted in units of domestic currency

K: The strike quoted in units of domestic currency

τ: The time remaining to expiration measured in years

R_f: The foreign currency interest rate

R_d: The domestic currency interest rate

σ: The annualized standard deviation of the spot exchange rate

The Local Hedge Concept

The heart of the BSM model is the idea that it is theoretically possible to operate a dynamic local hedge for a currency option using long or short positions in foreign exchange. The local hedge must be rebalanced in response to infinitesimally small changes in exchange rates. Consider the following example of the yen call described in the previous chapter:

USD Put/JPY Call	
Face in USD	$1,000,000
Face in JPY	89,336,700
Strike	89.3367
Spot	90.00
Term	90 Days
Interest Rate (USD)	5.00%
Interest Rate (JPY)	2.00%
Volatility	14.00%
Value in USD	$27,389
Exercise Convention	European

The price of this option is $27,389 at a dollar/yen spot exchange rate of 90.00. Suppose that in an instant the spot exchange rate were to rise to 90.20. As a matter of fact, if nothing else were to change, the value of this option would fall to $26,277, or by $1,111.

Suppose that before the move in the exchange rate, one were to have hedged a long position in the option with a long position in spot dollar/yen with face equal to $1 million dollar/yen. The idea of hedging using a long position in dollar/yen to hedge a dollar put option may seem counterintuitive at first, but not when it is understood that the value of the option, being a put on the dollar or call on the yen, is bound to fall if dollar/yen rises. The hedge works by making money when dollar/yen rises. If dollar/yen rose by 20 pips, from 90.00 to 90.20, the exact profit on the hedge would be $2,217.

Yet the size of the hedge is clearly too large because the loss in the value of the option is only $1,111. A smaller-sized hedge, say in the amount of the ratio

$$\frac{1111}{2217} = .5011$$

which would be long $501,127 USD/JPY, is an almost perfect fit. A spot position of that size would gain $1,111 on the move from 90.00 to 90.20.

The ratio .5011 is a crude estimate for what in option theory is called the delta (δ). Delta is the change in value of the option when the exchange rate changes either up or down by one unit. Delta itself changes when the spot exchange rate changes (and when other variables change as well). Delta is bounded in absolute value by zero and one for vanilla European options.

An option that is far out-of-the-money has a delta near zero because a unit change in the spot exchange rate would not make much of a difference to the value of the option; except for a tremendous change in the spot rate, the option is likely to expire out-of-the-money. At the other extreme, an option that is deep in-the-money should move up and down in almost equal unit value (delta near to one in absolute value) with the underlying changes in the spot rate. This is because changes in the spot rate are likely captured in the option's value at expiration. An option with delta exactly equal to one in absolute value would move up and down in value in an equivalent way to a spot foreign exchange position with size equal to the option face.

Continuing with the local hedge, if at every point in time it were possible to recalculate delta and accordingly maintain the correct size of spot hedge, the position in the option would be perfectly protected against movements in the spot exchange rate. As such, the aggregate position in the option and the hedge would be riskless. Said another way, the process of dynamic hedging replicates the target option, in this case the USD put/JPY call. According to capital market theory, such a combination must earn no more and no less than the riskless rate of interest. Although the idea of a local hedge seems highly impractical, the mere theoretical possibility that it could be successfully operated is, in fact, a key element in the option pricing model.

The Model in Terms of Spot Exchange Rates

Under the assumption that it is possible to operate a perfect local hedge between a currency option and underlying foreign exchange, Garman and Kohlhagen, following Black, Scholes and Merton, derive the following partial differential equation.

The BSM Partial Differential Equation

$$\frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} - R_d C + \left(R_d S - R_f S\right)\frac{\partial C}{\partial S} + \frac{\partial C}{\partial \tau} = 0$$

This equation governs the pricing of a currency call option. When the expiration day payoff function

$$C_T = Max\,[0, S_T - K]$$

is imposed as a boundary condition, the partial differential equation can be solved to obtain the BSM value for the call. The derivation of the put follows along the same lines.

The BSM Model (Spot)

$$C = e^{-R_f\tau} SN\left(x + \sigma\sqrt{\tau}\right) - e^{-R_d\tau} KN(x)$$
$$P = e^{-R_f\tau} S\left(N\left(x + \sigma\sqrt{\tau}\right) - 1\right) - e^{-R_d\tau} K\left(N(x) - 1\right)$$
$$x = \frac{\ln\left(\dfrac{S}{K}\right) + \left(R_d - R_f - \dfrac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}}$$

where $N(\cdot)$ is the cumulative normal density function.[2]

[2]Abramowitz and Stegun's (1972) *Handbook of Mathematical Functions* (paragraph 26.2.17) gives the following polynomial approximation for the cumulative normal density function for variable x

$$y = \frac{1}{1 + .2316419x}$$
$$N(x) = 1 - Z(x)\left(b_1 y + b_2 y^2 + b_3 y^3 + b_4 y^4 + b_5 y^5\right) + e(x)$$
$$Z(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

where

$$b_1 = .319381530;\ b_2 = -0.356563782;\ b_3 = 1.781477937;$$
$$b_4 = -1.821255978;\ \text{and}\ b_5 = 1.330274429$$

If x is less than zero, $N(x) = 1 - N(x)$. the absolute value of the error term should be less than 7.5×10^{-8}.

The first derivative of the theoretical value of these options is the delta mentioned previously. The call delta is given by

$$\delta_{call} = e^{-R_f \tau} N\left(x + \sigma\sqrt{\tau}\right)$$

The next chapter contains extensive discussion of delta and other option partial derivatives.

The BSM Model in Terms of the Forward Exchange Rate

An alternative formulation of the model uses the forward outright to stand in the place of the spot exchange rate. The value date for the forward outright, F, is the option expiration date. The equations for calls and puts are

The BSM Model (Forward)

$$C = e^{-R_d \tau}\left[FN\left(y + \sigma\sqrt{\tau}\right) - KN(y)\right]$$

$$P = e^{-R_d \tau}\left[F\left(N\left(y + \sigma\sqrt{\tau}\right) - 1\right) - K\left(N(y) - 1\right)\right]$$

$$y = \frac{\ln\left(\dfrac{F}{K}\right) - \left(\dfrac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}}$$

where all the variables are as previously defined and the forward outright can be derived from the interest parity formula

$$F = Se^{\left(R_d - R_f\right)\tau}$$

The Cox-Ross Risk Neutral Explanation

Cox and Ross (1976) provide an insight into the unimportance of investor attitudes toward risk in BSM option pricing theory.

Cox and Ross note that the Black-Scholes partial differential equation contains no variable that is dependent on investor risk preferences (for example, the term μ, which can be thought of as the risk premium for foreign exchange, is not present in the option model). Therefore, an option should be equally valuable to a risk-averse investor and to a risk-neutral investor, provided that it is at least theoretically possible to construct a perfect local hedge.

Consider how a risk-neutral investor would value a call option on foreign currency. At expiration, there can be only two realizations: Either the call will be worthless (because it is at-the-money or out-of-the-money), or it will be worth the difference between the exchange rate and the strike. The first case would be ignored by the risk-neutral investor. Only the second case matters. The value of the option is equal to the present value, using the riskless interest rate, of the conditional expectation of the future spot exchange rate minus the strike. The mathematical expectation is conditional on the option being in-the-money at expiration. The probability of the option being in-the-money at expiration and the expected value of the spot exchange rate at expiration can be derived from the cumulative lognormal density function for the spot exchange rate. Following Gemmill (1993) and Jarrow and Rudd (1983), the option model can be decomposed into the following parts:

$$\left[e^{-R_d \tau}\right]\left[N(x)\right]\left[e^{\left(R_d - R_f\right)\tau} S \frac{N\left(x + \sigma\sqrt{\tau}\right)}{N(x)} - K\right]$$

The first term in brackets is the present value factor. The second term[3] is the risk-neutral probability that the option will finish in-the-money. The third term is the expected payoff at expiration conditional on the option finishing in-the-money.

The Geometry of the Model

Exhibit 3.1 shows a graph of the USD put/JPY call that has been the running example in this chapter's discussion. The option is displayed at four stages of its life: 90 days, 30 days, 7 days, and expiration. The horizontal axis is the spot exchange rate and the vertical axis is the theoretical value of the option in yen pips.

At expiration, the value of the option is given by

$$C_T = Max\left[0, S_T - K\right]$$

Graphically, the expiration locus is the familiar option "hockey stick." All three live options lie above the expiration locus.

[3]Among currency traders it is popular to speak of the probability of all kinds of events as "deltas." If the trader thinks something is likely to occur he might say "I am a 90 delta." Something that is unlikely might be a "15 delta." There is a pseudo-analytical explanation. Mathematically, the delta of a BSM call option,

$$e^{-R_f \tau} N\left(x + \sigma\sqrt{\tau}\right)$$

is close in value to $N(x)$, provided the foreign currency interest rate is not large. The term $N(x)$ is the risk neutral probability that the option will finish in-the-money.

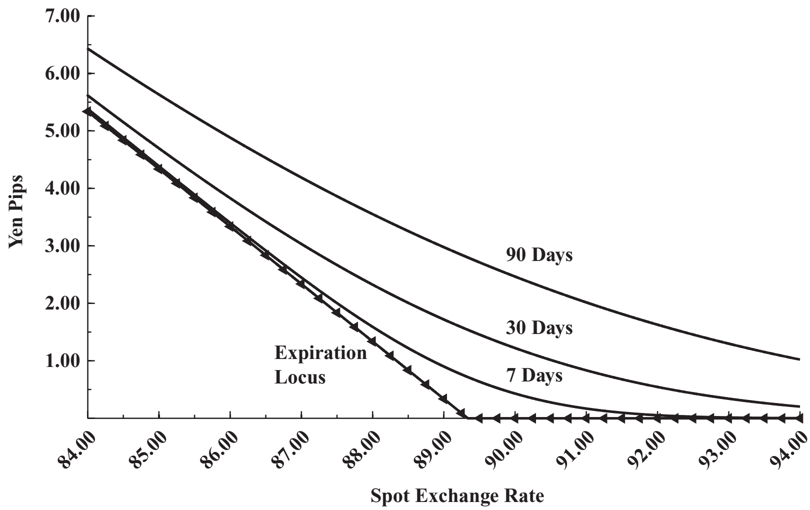


EXHIBIT 3.1 Graph of USD Put/JPY Call

The slope of the option line with respect to the spot exchange rate is the delta. As time remaining to expiration elapses, meaning that the option ages, the theoretical curve for the call shifts downward and to the left—in effect, the curve drops and sags (and therefore becomes convex, which is a topic for the next chapter). Finally, at expiration, the curve collapses on its expiration locus.

A Numerical Example

Consider a numerical example of how to calculate the value of a European currency option using the BSM model:

USD Put/JPY Call	
Face in USD	$1,000,000
Face in JPY	89,336,700
Strike	89.3367
Spot	90.00
Term	90 Days
Interest Rate (USD)	5.00%
Interest Rate (JPY)	2.00%
Volatility	14.00%
Exercise Convention	European

The BSM model classifies this option as a call (i.e., a yen call). To calculate the value of the option, first find the value of x:

$$x = \frac{\ln\left(\dfrac{\dfrac{1}{90.00}}{\dfrac{1}{89.3367}}\right) + \left(5.00\% - 2.00\% - \dfrac{(14\%)^2}{2}\right)\dfrac{90}{365}}{14\% \times \sqrt{\dfrac{90}{365}}} = -0.0347599$$

and the related expression

$$x + \sigma\sqrt{\tau} = x + 14\% \times \sqrt{\frac{90}{365}} = .034759$$

Next find the cumulative normal densities (see the approximation procedure in Note 3):

$$N(x) = .0.486136$$
$$N\left(x + \sigma\sqrt{\tau}\right) = 0.513864$$

Next find the value of the option, which in this case is a call option:

$$C = e^{-2.000\% \frac{90}{365}}\left(\frac{1}{90.00}\right) \times (.513864) - e^{-.5.00\% \frac{90}{365}}\left(\frac{1}{89.3367}\right)$$
$$\times (.4861355) = .00030658$$

This value when multiplied by the yen face amount of the option, 89,336,700, gives the dollar value of the option, \$27,389.

HOW CURRENCY OPTIONS TRADE IN THE INTERBANK MARKET

Professional interbank traders have developed a specialized system for quoting currency options that has its roots in option pricing concepts. Instead of quoting currency options in terms of dollars or other currencies, traders quote in units of volatility. Once quoted volatility is agreed, dealers work to arrive at an actual money price for the option using the BSM model.

Consider the following example. Suppose that an investor desires to do a transaction in a three-month, at-the-money forward USD put/JPY call. A check of the market reveals that one-month "yen" volatility (or just "vol") is being quoted by dealers at 14.00 percent bid – 14.10 percent ask.

To arrive at the money price of the option, the investor can use the BSM formula (Exhibit 3.2). Based on the indicated levels of volatility, the option is bid at $27,389 (where the investor can sell) and ask at $27,584 (where the investor can buy). Note that there are no commissions connected with buying and selling interbank currency options.

Sample volatilities for major currencies (observed in October 2009) are displayed in Exhibit 3.3. Note that there is a term structure for option volatility and that it can vary widely across exchange rates.

The volatility levels in Exhibit 3.3 correspond to at-the-money forward options. Out-of-the-money options can trade at higher levels of volatility, a phenomenon that is called the smile and is discussed in Chapter 5.

EXHIBIT 3.2 Dealer's Bid and Ask for USD Put/JPY Call

	Dealer's Bid	**Dealer's Ask**
Currency Pair	USD/JPY	USD/JPY
Put/Call	USD Put/JPY Call	USD Put/JPY Call
Customer Action	Sells	Buys
Dealer Actions	Buys	Sells
Face USD	$1,000,000	$1,000,000
Face JPY	89,336,700	89,336,700
Strike	89.3367	89.3367
Days to Expiry	90	90
Market Data		
Spot	90.00	90.00
Forward Outright	89.3367	89.3367
Interest Rate (USD)	5.0000%	5.000%
Interest Rate (JPY)	2.0000%	2.000%
Quoted Volatility	14.000%	14.100%
Option Pricing		
USD Pips	0.00030658	0.00030877
Total USD	**$27,389**	**$27,584**
JPY Pips (four digits)	2.4650	2.4826
Total JPY	2,464,996	2,482,604
Percentage of Face Amount	2.74%	2.76%
Dealer's Hedge		
Delta (times 100)	51.11	51.11
Hedge (Spot)	$511,336	–$511,435

EXHIBIT 3.3 Quote Volatilities of Selected Major Currencies at-the-Money Forward (as of 20 October 2009)

	1M	3M	6M
USD/CAD	15.065%	15.012%	15.175%
EUR/USD	10.507%	11.760%	12.558%
GBP/USD	13.335%	13.510%	13.580%
USD/CHF	10.850%	11.920%	12.575%
USD/JPY	13.830%	14.015%	14.420%
AUG/USD	16.042%	16.158%	16.335%
NZD/USD	17.540%	17.665%	17.850%
EUR/JPY	12.842%	13.050%	13.790%
EUR/GBP	12.035%	12.118%	12.132%
EUR/CHF	3.800%	4.175%	4.550%

Traders identify options in the first instance by their deltas to gauge the extent of their out-of-the-moneyness. As a rough rule of thumb, at-the-money forward puts and calls have deltas of approximately 50. A 25 delta option is out-of-the-money. A 15 delta option is even further out-of-the-money.

As a rule, option dealers buy and sell puts and calls only when the options are accompanied by hedging transactions consisting of spot foreign exchange deals. This convention allows the dealer to trade options on a delta-neutral basis. The size of the spot hedging transaction can be calculated by multiplying the delta by the face of the option, as can be seen at the bottom of Exhibit 3.2. The delta for the USD put/JPY call is calculated to be negative 51. The actual number is really 0.51 but traders multiply by one hundred. From the dealer's perspective, if he buys the option he needs to buy roughly $511,000 worth of dollar/yen. If he sells the option, he needs to sell the same amount of dollar/yen.

Customers buy and sell options either live (i.e., without a hedge) or hedged. In the latter case, it is customary for the customer to exchange spot foreign exchange with the dealer. For example, if the customer buys the USD put/JPY call option in Exhibit 3.2, the spot hedge would consist of the customer buying and the dealer selling $0.5 million dollar/yen.

REFLECTIONS ON THE CONTRIBUTION OF BLACK, SCHOLES, AND MERTON

It is no exaggeration to say that the work of Black, Scholes, and Merton fundamentally transformed the currency option market. Of course, Black-Scholes–type models greatly influenced the development of all derivatives markets. But the impact on the currency option market is one of their greatest enduring practical influences.

The basic paradigm of how trading in currency options operates is rife with Black-Scholes concepts. Options are identified in the first instance not by strikes but by their deltas. Traders might ask for the 50 delta or 15 delta options, for example. Currency options prices are not quoted in currency but in terms of volatility. The genius of quoting option prices in volatility is that price comparison across currencies, strikes, and term to expiration is instantly achieved. After an option is bought or sold, traders turn to the option pricing model to transform the volatility price into a price in currency (such as dollars and cents).

Perhaps the most revolutionary concept from Black-Scholes-Merton is their framework for risk analysis. Even in markets where financial mathematicians have produced second and third generation option models, the Black-Scholes-Merton vocabulary, such as delta, gamma, theta, vega, and rho still permeates the language of option modeling.

Options on Foreign Exchange, Third Edition
David F. DeRosa

CHAPTER 4

European Currency Option Analytics

The previous chapter developed the industry-standard Black-Scholes-Merton (BSM) model for European currency options. Attention now turns to using the model to understand the dynamics of option valuation and the analysis of option risk.

BASE-CASE ANALYSIS

In the BSM model, five factors contribute to the valuation of a currency option: The spot exchange rate, the market level of option volatility, the foreign interest rate, the domestic interest rate, and the time to expiration. One way to get a fast look at how these factors work is to examine the change in an option's value when each factor by itself is subject to a small change. Exhibit 4.1 does this experiment on the one-month dollar put/yen call from Chapter 3. Under the base-case assumptions, this option is worth $27,389. When the pricing factors change, the dynamics are as follows:

- A one-yen move in the spot exchange up from 90 to 91 removes $5,234 of value from the option.
- One day of time decay costs the holder of the option $190 as the time to expiration shrinks from 90 days to 89 days.
- A one percent increase in market option volatility, meaning a rise from 14 percent to 15 percent, adds $1,955 to the value of the option.
- An increase in the foreign interest rate by 1 percent, from 2 percent to 3 percent, subtracts $1,233 from the option value.
- An increase in the domestic interest rate by 1 percent, from 5 percent to 6 percent, adds $1,200 to the value of the option.

EXHIBIT 4.1 Option Sensitivity to Input Parameters

	Base-Case	Spot Exchange Rate	Time	Volatility	Foreign Interest Rate	Domestic Interest Rate
Currency Pair	USD/JPY	+1 yen	−1 day	+1%	+1%	+1%
Put/Call	USD put/JPY call					
Face USD	$1,000,000					
Face JPY	¥89,336,700					
Strike	89.3367					
Days to Expiry	90		89			
Market Data						
Spot	90.00	91.00				
Forward outright	89.3367					
Interest Rate (USD)	5.00%					6.00%
Interest Rate (JPY)	2.00%				3.00%	
Implied Volatility	14.00%			15.00%		
Option Pricing						
USD pips	0.0003066	0.0002480	0.0003044	0.0003285	0.0002928	0.0003200
Total USD	27,389	22,154	27,198	29,344	26,156	28,588
Change		**−$5,234**	**−$190**	**$1,955**	**−$1,233**	**$1,200**
Spot Equivalent						
Delta (times 100)	51.113	44.484	51.082	51.232	49.603	52.540
Delta times face	−$511,336	−$448,367	−$510,816	−$512,321	−$496,034	−$525,396

Yet the picture is more complex than the exhibit suggests because the sensitivity of the option to each of the pricing factors is dynamic over the course of the option's life and can be greatly affected by movements in the spot exchange rate toward and away from the option's strike. Moreover, the sensitivity of an option to one pricing factor, for example, to the spot exchange rate, is a function of all of the other pricing factors, volatility, time, and the two interest rates.

THE "GREEKS"

Exact decomposition of how an option changes when pricing factors change requires knowledge of the partial derivatives of the BSM model for both calls and puts. A partial derivative is the change in a function given an infinitesimally small change in one of its input parameters holding all other parameters constant. Market participants have decided to give these derivatives special names, all but one of which are letters in the Greek alphabet.

Delta

The most important partial derivative in option analysis is the delta (δ), which is defined as the partial derivative of the option price with respect to the spot exchange rate. Delta was introduced on a heuristic level in Chapter 3. The Appendix to this chapter derives delta using elementary calculus. The call and put deltas are given by

$$\delta_{call} \equiv \frac{\partial C}{\partial S} = e^{-R_f\tau} N\left(x + \sigma\sqrt{\tau}\right)$$
$$\delta_{put} \equiv \frac{\partial P}{\partial S} = e^{-R_f\tau} \left(N\left(x + \sigma\sqrt{\tau}\right) - 1\right)$$

The deltas of calls and puts are bounded as follows

$$0 \le \delta_{call} \le 1$$
$$-1 \le \delta_{put} \le 0$$

The delta of the dollar put/yen call in Exhibit 4.1 is equal to –.5113. In practice, currency option traders convert delta into units of one or the other face currencies, such as $0.5113 million dollars. What this delta would tell a trader is that this option behaves approximately like a short spot position with face equal roughly one half of $1 million for small movements in the exchange rate. Box 4.1 discusses the use of delta in practice.

BOX 4.1 USING DELTA TO HEDGE OPTIONS

The standard delta for a foreign exchange call is given by

$$\delta_{call} \equiv \frac{\partial C}{\partial S} = e^{-R_f\tau} N\left(x + \sigma\sqrt{\tau}\right)$$

which is obtained by taking the partial derivative of the BSM call with respect to the spot exchange rate.

A complication may arise where the premium is paid in foreign currency—in effect, the option's premium itself may represent an exposure to the exchange rate. For this reason traders sometimes adjust the delta of the option for the option premium. Bloomberg, for example, gives traders the choice option of adjusting delta for the option premium.

(*continued*)

A third delta is the delta to the forward. Most of the time option traders hedge exchange rate risk by trading spot foreign exchange. Sometimes, traders hedge options with forward foreign exchange. This is popular with options on emerging markets currencies where there is a risk of capital controls being imposed. If the forwards have the same expiration as that of the option then the delta comes straight from the forward version of the model:

$$\delta_{call}^{f} \equiv \frac{\partial C}{\partial F} = e^{-R_d \tau} N\left(x + \sigma\sqrt{\tau}\right)$$

Exhibit 4.2 shows the delta of the dollar put/yen call against the level of the spot exchange rate for at-the-money forward options with 30 and 90 days to expiration. The S-shaped appearance is accounted for by the presence of the cumulative normal density function in the equation for delta.

The deltas of calls and puts with the same strike and expiration are related in the following way

$$\delta_{call} - \delta_{put} = e^{-R_f \tau}$$

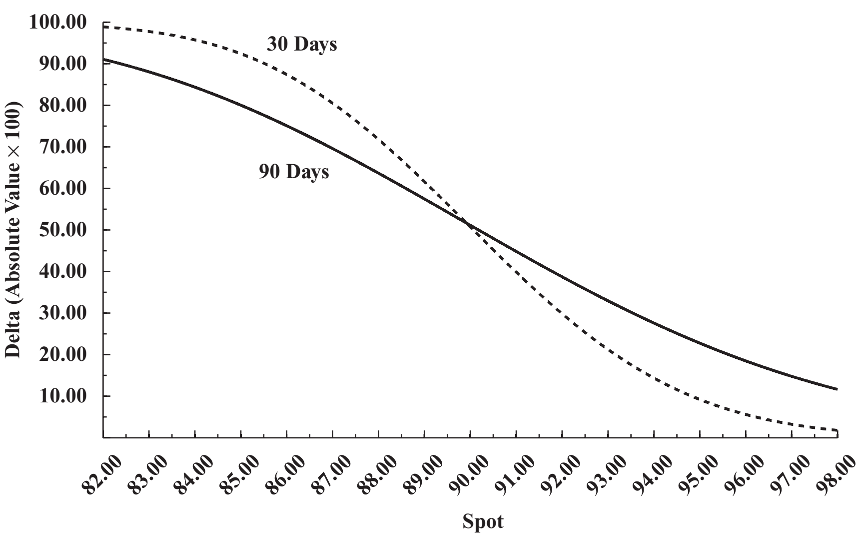


EXHIBIT 4.2 Delta
USD put/JPY call: 30- and 90-day ATMF; vol = 14.00%; $R_d = 5\%$; $R_f = 2\%$.

which can be obtained by taking partial derivative of the put-call parity formula with respect to the spot exchange rate.

Gamma

Delta is so widely used that its own behavior is studied. The partial derivative of an option's delta with respect to the spot exchange rate is called gamma (γ). Equivalently, gamma is the second-order partial of the option price with respect to the spot exchange rate. This formula is

$$\gamma_{call} = \gamma_{put} = \frac{\partial^2 C}{\partial S^2} = \frac{N'\left(x + \sigma\sqrt{\tau}\right) e^{-R_f \tau}}{S\sigma\sqrt{\tau}}$$

where N' is the normal density function

$$N'(z) = \frac{1}{\sqrt{2\pi}} e^{\frac{-z^2}{2}}$$

The fact that gamma is equal for calls and puts can be demonstrated by taking the second-order partial of the put-call parity formula with respect to the spot exchange rate.

According to the preceding formula, gamma for the dollar put/yen call in Exhibit 4.1 is 513.62—this is called raw gamma because it needs to be transformed into units that are meaningful. The better way to represent gamma is in terms of the movement in delta for a one-big figure (in this case one yen) movement in the spot exchange rate:

$$\gamma_{1\,big\ figure} = Face \times \left(\frac{1}{S_2} - \frac{1}{S_1}\right) \times \gamma$$

$$\gamma_{1\,big\ figure} = \$1mm \times \left(\frac{1}{91} - \frac{1}{90}\right) \times 513.62 = -\$62{,}713$$

This indicates that delta will drop in absolute value by approximately \$63,000 from its base level of \$511,336 if the spot exchange rate for dollar/yen rises to 91 from 90.

In panel A of Exhibit 4.3, the option with the maximum gamma is nearly at-the-money forward. In panel B of Exhibit 4.3, options with little time remaining to expiration are very rich in gamma. This is evident by comparing the two S-shaped delta curves in Exhibit 4.2. The 30-day

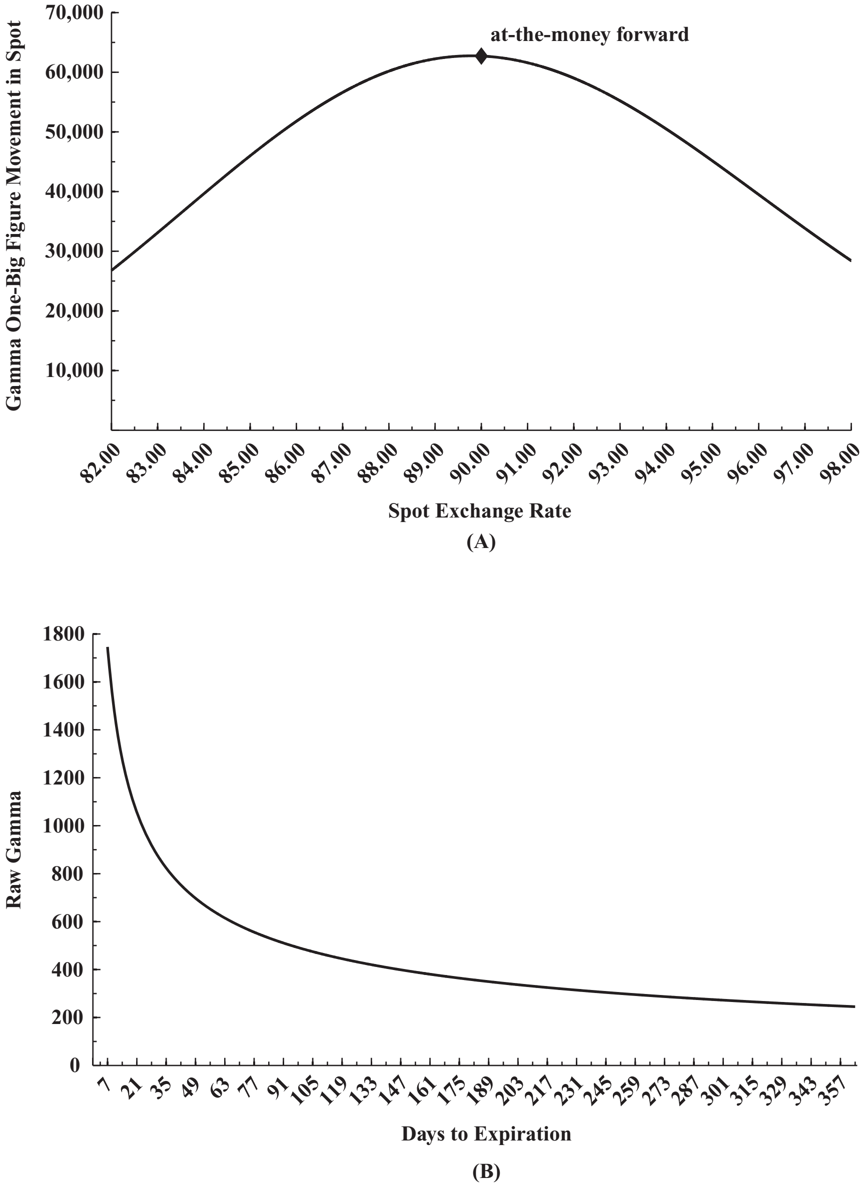


EXHIBIT 4.3 Gamma versus Spot and Gamma versus Time
USD put/JPY call: spot = 90.00; strike = 89.3367; vol = 14.00%; $R_d = 5\%$; $R_f = 2\%$.

option's delta curve has more curvature, or convexity, than does the 90-day option's delta.

Gamma is analogous to convexity in bonds. For example, when the dollar put/yen call option has much gamma, it means that the option's delta will rise quickly with downward movements in the dollar, thereby allowing the holder of the option to capture progressively more profit. Conversely, the same option's delta will fall sharply with a rise in the dollar, thereby limiting the size in the loss in the value of the option when spot moves adversely.

Theta

Theta (θ) is defined as the partial derivative of the option value with respect to the time remaining until expiration:

$$\theta_{call} \equiv \frac{\partial C}{\partial \tau} = R_f e^{-R_f \tau} SN\left(x + \sigma\sqrt{\tau}\right) - R_d e^{-R_d \tau} KN(x) - \frac{e^{-R_d \tau}\sigma}{2\sqrt{\tau}} KN'(x)$$

$$\theta_{put} \equiv \frac{\partial P}{\partial \tau} = R_f e^{-R_f \tau} S\left(N\left(x + \sigma\sqrt{\tau}\right) - 1\right) - R_d e^{-R_d \tau} K\left(N(x) - 1\right) - \frac{e^{-R_d \tau}\sigma}{2\sqrt{\tau}} KN'(x)$$

The convention of the market is to express theta as the rate of "time decay" that will be experienced with the passage of one day. In the case of the dollar put/yen call from Exhibit 4.1, the daily time decay amounts to

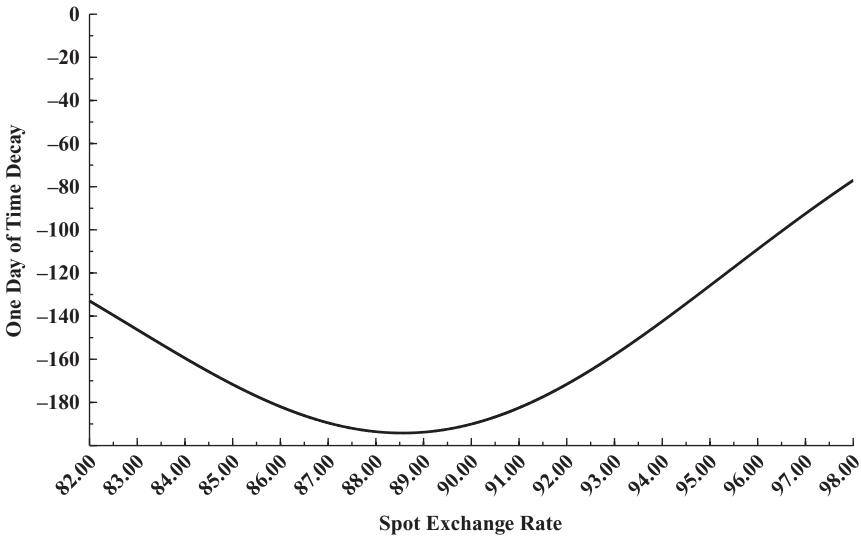
$$\theta_{1\ day} = \frac{\theta_{call}}{365} \times FACE$$

$$\theta_{1\ day} = -\frac{.0007765}{365} \times 89{,}336{,}700 = -\$190$$

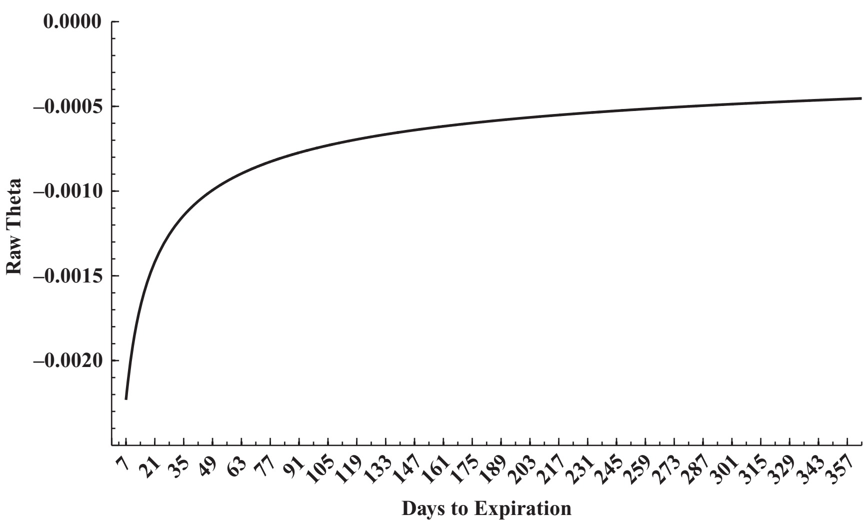
For a given time to expiration, theta takes on maximum value near the at-the-money forward strike option (Exhibit 4.4, panel A). Theta is sensitive to the amount of time remaining to expiration (Exhibit 4.4, panel B). Options that have little time remaining to expiration experience the fastest rate of decay.

The passage of time can be understood in terms of the locus of option values shown in Exhibit 3.1 in Chapter 3. With the passage of time, the option curve drops toward the expiration locus. In the process, the curve sags, meaning that the option acquires additional convexity.

Some European currency options experience positive time decay, meaning that they gain value as time remaining to expiration decreases. If the



(A)



(B)

EXHIBIT 4.4 Theta versus Spot and Theta versus Time
USD put/JPY call: spot = 90.00; strike = 89.3367; vol = 14.00%; $R_d = 5\%$; $R_f = 2\%$.

foreign interest rate is sufficiently small relative to the domestic interest rate, an in-the-money call could experience positive time decay, meaning that the option might increase in value with the passage of time.

An example of positive time decay would be a European USD call/JPY put that is deep-in-the-money and where the dollar interest rate is very large relative to the yen interest rate. Given sufficiently low volatility, this option could become more valuable as time passes because that would mean less time to wait before the option holder receives possession of the higher yielding domestic currency. Nonetheless, most European currency options experience negative time decay. Parenthetically, all American currency options experience negative time decay without exception because early exercise is permitted at any time in the option's life.

The passage of time can impact the value of a currency option in ways other than what theta would suggest. As time passes, an option is priced off a different part of the term structure of volatility, and foreign and domestic interest rates. For example, the one-month option in Exhibit 4.1 is originally priced using one-month volatility and one-month interest rates. But one week later, the option, having only three weeks left to expiration, would be priced using three-week volatility and three-week interest rates. This could be material depending on the steepness or flatness of the term structure of volatility and the term structure of interest rates.

Delta, Theta, and Gamma Are Related

Delta, theta, and gamma are related through the BSM partial differential equation. This equation (as introduced in Chapter 3) can be written

$$\frac{1}{2}\sigma^2 S^2 \gamma - R_d C + \left(R_d - R_f\right) S\delta + \vartheta = 0$$

where the symbols γ, δ, and θ replace the partial derivatives, with no change in meaning. At a given level of delta, gamma and theta are directly related. This equation is the theoretical basis of why options with high levels of gamma have fast time decay, or as traders say, "theta is the rent on gamma."

Vega

An increase in volatility leads to increases in the value of all vanilla European options. This is because the greater the perceived level of volatility, the greater the probability that the option will expire in-the-money. The partial derivative of the option price with respect to volatility is given by vega (not in the Greek alphabet!):

$$Vega_{call} = Vega_{put} \equiv \frac{\partial C}{\partial \sigma} = e^{-R_d \tau} K \sqrt{\tau} N'(x)$$

Put and call vegas are equal for a common strike and expiration. Vega tells the change in option value for a one percent change in volatility (e.g., rising to 21.25 percent from 20.25 percent). Multiply raw vega by the option face and divide by 100 to arrive at the dollar change in the option. The raw vega for the dollar put/yen call in Exhibit 4.1 is .002189, which makes the dollar sensitivity of the option equal to

$$\frac{vega}{100} \times Face$$
$$\frac{.002189}{100} \times 89{,}336{,}700 = \$1{,}955$$

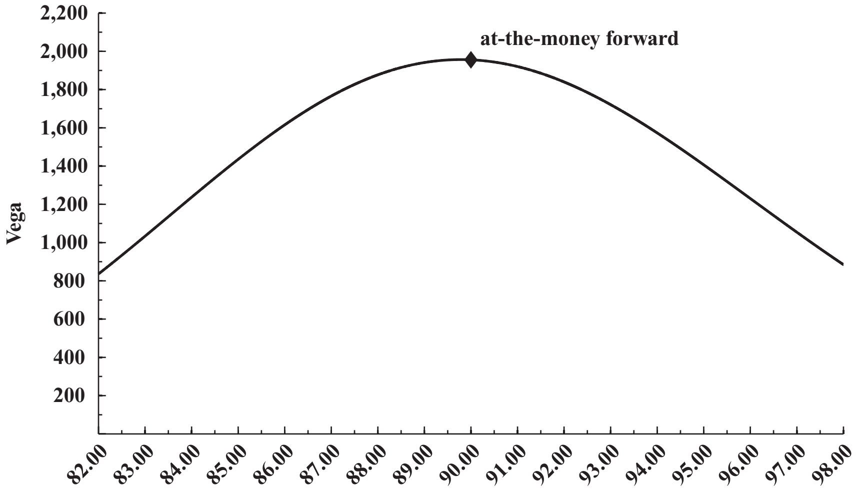
Vega is at maximum value for a given expiration around the at-the-money forward strike (Exhibit 4.5, panel A). Unlike gamma and theta, vega is an increasing function of time to expiration (Exhibit 4.5, panel B). The equation for vega indicates that option sensitivity to volatility increases with the square root of time to expiration.

Rho

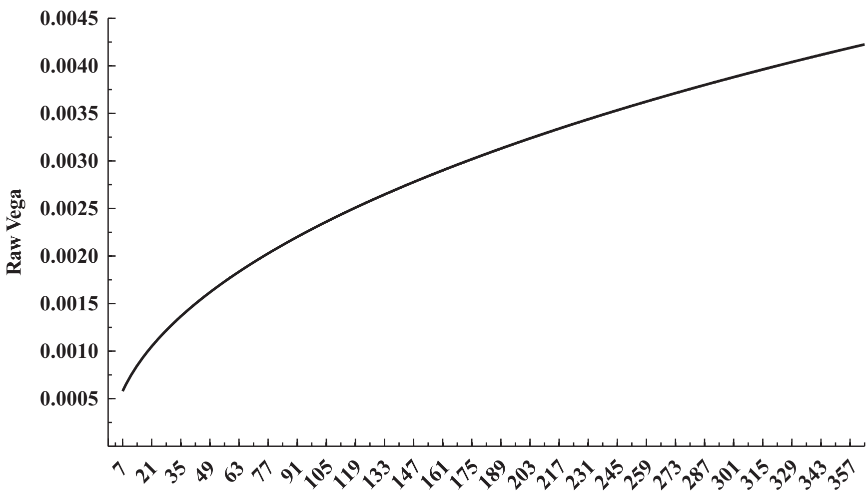
The impact of domestic and foreign interest rates on option premium is somewhat subtle. In the framework of the risk neutrality analysis, options are essentially probability-weighted discounted cash flows. Therefore an option's value is a function of the present value of the strike and the present value of the deliverable quantity of foreign exchange.

When the foreign currency interest rate rises, the present value of the underlying sum of foreign currency must fall. Therefore, an increase in the foreign currency interest rate lowers the value of a currency call option (which receives foreign exchange upon exercise) and raises the value of a currency put (which delivers foreign exchange upon exercise).

Likewise, a rise in the domestic interest rate lowers the present value of the underlying sum of domestic currency. Therefore, an increase in the domestic currency interest rate will raise the value of a call option (which must pay domestic currency upon exercise) and lower the value of a put option (which receives domestic currency upon exercise).



(A)



(B)

EXHIBIT 4.5 Vega versus Spot and Vega versus Time
USD put/JPY call: spot = 90.00; strike = 89.3367; vol = 14.00%; $R_d = 5\%$; $R_f = 2\%$.

EXHIBIT 4.6 Selected Higher-Order Partial Derivatives of the BSM model

Black-Scholes Value Sensitivities to Inputs				
Black-Scholes Value	**Spot (S)**	**Volatility (σ)**	**Time to Expiry (τ)**	**Interest Rate (r)**
Option Price (V)	Delta (δ)	Vega (ν)	Theta (θ)	Rho (ρ)
Delta (δ)	Gamma (γ)	Vanna	Charm	
Gamma (γ)	Speed	Zomma	Color	
Vega (ν)	Vanna	Volga	DvegaDtime	

By convention, the interest rate partial derivatives called rho (ρ). The partials are given by

$$\frac{\partial C}{\partial R_d} = \tau e^{-R_d \tau} KN(x) \geq 0$$

$$\frac{\partial P}{\partial R_d} = \tau e^{-R_d \tau} K\left(N(x) - 1\right) \leq 0$$

$$\frac{\partial C}{\partial R_f} = -\tau e^{-R_f \tau} SN\left(x + \sigma\sqrt{\tau}\right) \leq 0$$

$$\frac{\partial P}{\partial R_f} = -\tau e^{-R_f \tau} K\left(N\left(x + \sigma\sqrt{\tau}\right) - 1\right) \geq 0$$

Higher Order Partials

Higher-order partial derivatives can be used to refine the analysis of option risk (Exhibits 4.6 and 4.7). Some of them have acquired fanciful names, such as zomma, which is the partial derivative of gamma with respect to volatility. Sometimes these nicknames stick. Other times traders refer to them by phonetic calculus such as DgammaDvol, in this case.

Vanna and Volga

Vanna and volga are two second-order partial derivatives used in the construction of interpolated volatility surfaces (a topic for Chapter 5).

Vanna is the partial derivative of delta with respect to volatility (DdeltaDvol):

$$\frac{\partial}{\partial \sigma}\frac{\partial C}{\partial S}$$

Greeks		Calls	Puts
Vanna	$\frac{\partial^2 V}{\partial S \partial \sigma}$	$-e^{-R_f\tau} N'(x+\sigma\sqrt{\tau})\frac{x}{\sigma} = \frac{v}{S}\left(1 - \frac{x+\sigma\sqrt{\tau}}{\sigma\sqrt{\tau}}\right)$	
Charm	$\frac{\partial^2 V}{\partial S \partial \tau}$	$-R_f e^{-R_f\tau} N(N+\sigma\sqrt{\tau}) + e^{-R_f\tau} N'(x+\sigma\sqrt{\tau})\frac{2(R_d-R_f)\tau - x\sigma\sqrt{\tau}}{2\tau\sigma\sqrt{\tau}}$	$-R_f e^{-R_f\tau}[N(x+\sigma\sqrt{\tau})-1] + e^{-R_f\tau} N'(x+\sigma\sqrt{\tau})\frac{2(R_d-R_f)\tau - x\sigma\sqrt{\tau}}{2\tau\sigma\sqrt{\tau}}$
Speed	$\frac{\partial^2 V}{\partial S^2}$	$-e^{-R_f\tau}\frac{N'(x+\sigma\sqrt{\tau})}{S^2\sigma\sqrt{\tau}}\left(\frac{x+\sigma\sqrt{\tau}}{\sigma\sqrt{\tau}}+1\right) = -\frac{\gamma}{S}\left(\frac{x+\sigma\sqrt{\tau}}{\sigma\sqrt{\tau}}+1\right)$	
Zomma	$\frac{\partial^2 V}{\partial S^2 \partial \sigma}$	$e^{-R_f\tau}\frac{N'(x+\sigma\sqrt{\tau})(x(x+\sigma\sqrt{\tau})-1)}{S\sigma^2\sqrt{\tau}} = \gamma\left(\frac{x(x+\sigma\sqrt{\tau})-1}{\sigma}\right)$	
Color	$\frac{\partial^2 V}{\partial S^2 \partial \tau}$	$-e^{-R_f\tau}\frac{N'(x+\sigma\sqrt{\tau})}{2S\tau\sigma\sqrt{\tau}}\left[2R_f\tau + 1 + \frac{2(R_d-R_f)\tau - x\sigma\sqrt{\tau}}{\sigma\sqrt{\tau}}(x+\sigma\sqrt{\tau})\right]$	
Volga	$\frac{\partial^2 V}{\partial \sigma^2}$	$Se^{-R_f\tau}N'(x+\sigma\sqrt{\tau})\sqrt{\tau}\frac{x(x+\sigma\sqrt{\tau})}{\sigma} = v\frac{x(x+\sigma\sqrt{\tau})}{\sigma}$	
DvegaDtime	$\frac{\partial^2 V}{\partial \sigma \partial \tau}$	$Se^{-R_f\tau}N'(x+\sigma\sqrt{\tau})\sqrt{\tau}\left[R_f + \frac{(R_d-R_f)(x+\sigma\sqrt{\tau})}{\sigma\sqrt{\tau}} - \frac{1+x(x+\sigma\sqrt{\tau})}{2\tau}\right]$	

$$x = \frac{ln(S/K) + (R_d - R_f - \sigma^2/2)\tau}{\sigma\sqrt{\tau}}, \quad N'(z) = \frac{e^{\frac{-z^2}{2}}}{\sqrt{2\pi}}, \quad N(z) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{z} e^{\frac{-z^2}{2}} dz$$

EXHIBIT 4.7 Equations for Higher-Order Partial Derivatives of the BSM Model

which can be rearranged to be the partial derivative of vega with respect to spot.

Volga is the partial derivative of vega with respect to volatility (DvegaDvol):

$$\frac{\partial}{\partial\sigma}\frac{\partial C}{\partial\sigma}$$

The equations for vanna and volga are given in Exhibit 4.7.

SPECIAL PROPERTIES OF AT-THE-MONEY FORWARD OPTIONS

At-the-money forward (ATMF) options are the most heavily traded currency options. Not surprisingly, traders favor them because these options have pure volatility value. They have near-maximum values of gamma, theta, and vega among all options at their expiration. As it turns out, the BSM model for ATMF options can be reduced by approximation to a convenient computational shortcut (Brenner and Subrahmanyam, 1994). When an option is struck ATMF, it means that

$$K = F = Se^{\left(R_d - R_f\right)\tau}$$

which simplifies the arguments of the cumulative normal density function:

$$x = -\frac{\sigma}{2}\sqrt{\tau}$$

$$x + \sigma\sqrt{\tau} = \frac{\sigma}{2}\sqrt{\tau}$$

The value of the ATMF call or put becomes

$$C = P = e^{-R_f\tau}S\left[N\left(\frac{\sigma}{2}\sqrt{\tau}\right) - N\left(-\frac{\sigma}{2}\sqrt{\tau}\right)\right]$$

Kendall and Stuart (1943) state that the cumulative normal density $N(y)$ of variable y can be approximated as

$$N(y) = \frac{1}{2} + \frac{1}{\sqrt{2\pi}}\left(y - \frac{y^3}{6} + \frac{y^5}{40} - \cdots + \cdots\right)$$

For simplification to first approximation, we can drop the terms of third or higher order, whereupon

$$N(x) \cong \frac{1}{2} - .2\sigma\sqrt{\tau}$$

$$N\left(x + \sigma\sqrt{\tau}\right) \cong \frac{1}{2} + .2\sigma\sqrt{\tau}$$

The approximate value of the ATMF options becomes

$$C = P \cong 0.4e^{-R_f\tau}S\sigma\sqrt{\tau}$$

This approximation suggests that the value of the dollar put/yen call from Exhibit 4.1—which happens to be ATMF—is .000307453, which when multiplied by the yen face produces a total value of $27,466. This is reasonably close to the value of $27,389 from Exhibit 4.1. As an aside, note that the latter value was derived using a somewhat more precise yet less tractable approximation of the cumulative normal density function (see Note 2, Chapter 3).

A special rule of thumb for overnight options pops out of this approximation. Assuming that the foreign currency interest rate is not excessively high, the value of an overnight at-the-money call or put will be equal to one half of one percent of the face value of the option if the quoted volatility is equal to 24 percent.

This approximation works tolerably well in the usual range of experienced currency option volatility (say less than 50 percent). The same method provides simplifications for the ATMF deltas

$$\delta_{call} \cong e^{-R_f \tau}\left(\frac{1}{2} + .2\sigma\sqrt{\tau}\right)$$

$$\delta_{put} \cong e^{-R_f \tau}\left(.2\sigma\sqrt{\tau} - \frac{1}{2}\right)$$

Brenner and Subrahmanyam take matters one step further by approximating the normal density function

$$N'\left(x + \sigma\sqrt{\tau}\right) \cong 0.4$$

for values of the argument in the range normally experienced in option pricing. This allows for further simplification of ATMF gamma and vega:

$$\gamma \cong \frac{.4e^{-R_f \tau}}{S\sigma\sqrt{\tau}}$$

$$Vega \cong 0.4e^{-R_f \tau} S\sqrt{\tau}$$

These approximations have proven useful to floor traders and market makers where there was a need to "trade on your feet."

DIRECTIONAL TRADING WITH CURRENCY OPTIONS

Directional trading consists of taking positions consisting of spot and forward foreign exchange and options on foreign exchange that are designed to profit from correctly anticipating future movements in exchange rates. Directional trading is a risky business because foreign exchange rates can move violently and in unexpected ways. But when things work properly, directional trading can be explosively profitable.

Conventionally, trading in currencies is done in the spot market. A trader takes a position, either long or short, and waits for the anticipated move. Keeping exposure for longer than one foreign exchange trading day requires that the trader make arrangements to roll the position. To avoid having to make physical delivery, the position must be rolled out on the forward calendar using a forward swap transaction, such as tom/next or spot/next. Regardless of the exact method, being long a low interest rate currency (relative to the home currency) or short a high interest rate currency (relative to the home currency) involves paying interest-carrying charges. Conversely, being short a low interest rate currency (relative to the home currency) or long a high interest rate currency (relative to the home currency) means the trader receives interest-carrying charges.

Most spot traders use stop-loss orders to protect themselves from adverse moves in exchange rates. An almost universal belief among traders is that some form of risk control is necessary lest they be caught the wrong way when a violent move in exchange rates occurs.

Trading currency options to catch directional moves in exchange rates is more complex than trading spot. New dimensions have to be considered in constructing trading positions. There is the placement of the option strike to consider and the choice of the option expiration. Option volatility has to be factored into the strategy.

All things considered, successful directional option trading involves more than a good prognostication of whether an exchange rate is going to go up or down in the future. Timing is important. The greatest profits come to the trader who is able to anticipate where the exchange rate will be at a specific future point of time. The best directional traders think simultaneously in the dimensions of direction, time, and volatility with a healthy appreciation for risk management.

ATMF Options and Wing Options

A trader who has a directional view must choose which option or options to use. Suppose that a trader is bearish on dollar/yen. Should he buy an at-the-money USD put/JPY call option or rather a 25-delta yen call option (called the wing option) instead?

Exhibit 4.8 compares the instantaneous behavior of an at-the-money forward (ATMF) USD put/JPY call (strike equal to 89.3367) to that of a 25-delta yen call (strike equal to 85.0620) using sensitivity analysis across exchange rates (traders call this a slide). The slide gives a bird's-eye view of how the value of the position and its risk characteristics would change if spot were to move up or down. Exhibit 4.8 moves spot up and down in increments of two yen but this can be widened or shrunk.

EXHIBIT 4.8 Comparison of ATMF Option with 25-Delta Option ($1mm face, USD put/JPY call; vol = 14.00%; spot = 90; 90 days)

ATMF (strike 89.3367)					
Spot	**86**	**88**	**90**	**92**	**94**
Theoretical Value	$56,781	$40,358	$27,389	$17,697	$10,867
Delta	−$750,787	−$636,689	−$511,336	−$387,288	−$275,978
Gamma	$51,778	$60,179	$62,714	$59,018	$50,485
Vega	$1,615	$1,877	$1,956	$1,840	$1,573
Overnight decay	$182	$194	$190	$172	$142
25-delta (strike 85.0620)					
Spot	**86**	**88**	**90**	**92**	**94**
Theoretical value	$25,616	$16,034	$9,485	$5,297	$2,793
Delta	−$490,955	−$362,398	−$250,019	−$161,098	−$96,998
Gamma	$65,628	$60,407	$50,117	$37,737	$25,953
Vega	$1,949	$1,794	$1,488	$1,120	$770
Overnight decay	$188	$166	$134	$99	$67

The choice between the two options is not an easy one. What recommends the 89.3367 strike option is its relatively high levels of delta and gamma at the initial spot level. The trader must pay commensurately more for this option than for the 85.0620 strike option.

Volatility complicates the choice between the two options. As is shown in Exhibit 4.8, at the initial spot level, the at-the-money forward option has considerably more vega than the 25-delta option. In this exhibit, the quoted volatility for both the 89.3367 and 85.0620 strike options are assumed to be equal. Experience shows this to be a simplistic, if not misleading assumption because low delta options often command extra volatility.

Initially the ATMF option has more delta, gamma, theta, and vega than the 25-delta option at the starting spot level of 90.00. However, if spot goes to lower levels, the 25-delta option will gradually become the new ATMF option, and the 89.3367 strike option will lose some of its gamma, vega, and theta as it progressively goes deeper into-the-money.

The choice between the two options is not obvious but the slide analysis at least affords a preview of either trade at hypothetical future spot levels.

Risk Reversals

One of the most aggressive directional trades is the risk reversal. A risk reversal is created by the purchase of an out-of-the-money option and sale of a directionally opposite out-of-the-money option.

An example of a risk reversal is a long position in a 25-delta USD put/JPY call combined with a short position in a 25-delta USD call/JPY put.

Risk-Reversal	
Long 25-Delta 85.0620 USD Put/JPY Call	$ 9,485
Short 25-Delta 93.3735 USD Call/JPY Put	($11,164)
Net Premium	($ 1,680)

This trade is directionally bearish on the dollar. Assuming that the yen put and the yen call can be traded for approximately the same volatility, the risk reversal will be approximately zero cost initially.

If the view turns out to be correct, the value of the long yen call will rise and the short yen put will fall. To the unsuspecting this might look as though the trader had managed to acquire a yen call for free. Yet the experienced option trader would scoff at the idea of the risk reversal being a free trade, even though it requires little or no up-front payment. The trade is not free because the short yen put is an at-risk position. If dollar/yen were to rise, the risk associated with the short yen put could become ruinously large.

This case can be seen in the top panel of Exhibit 4.9 where there is a slide of the risk reversal trade. Note that at the onset, the delta of the risk reversal

EXHIBIT 4.9 Direction Trading with Currency Options

3-month 25-delta risk reversal (dollar bearish), $1mm face, strikes 85.0620 and 93.3735, vol 14.00%.

Spot	86	88	90	92	94
Value	$22,276	$9,644	−$1,680	−$12,706	−$24,256
Delta	−$583,177	−$521,782	−$500,056	−$520,865	−$576,691
Gamma	$38,323	$21,248	−$2	−$19,929	−$34,096
Theta	−$127	−$82	−$30	$15	$43
Vega	$1,059	$517	−$145	−$759	−$1,186

3-month vertical spread (dollar bearish), $1mm face, strikes 89.3367 and 85.0620, vol 14.00%.

Spot	86	88	90	92	94
Value	$31,165	$24,325	$17,904	$12,399	$8,073
Delta	−$259,832	−$274,291	−$261,317	−$226,190	−$178,980
Gamma	−$13,849	−$228	$12,596	$21,281	$24,532
Theta	$6	−$27	−$56	−$73	−$76
Vega	−$334	$83	$468	$720	$803

is approximately 50 in absolute value—the trade is short a 25-delta yen put and long a 25-delta yen call. The position is almost devoid of gamma, vega, and theta because of the offsetting effects of the long and short option positions.

When spot moves, the risk reversal begins to take on the personality of whichever option is favored. At lower levels of spot, this risk reversal becomes more like the long yen call, as the importance of the short yen put recedes. But at higher levels of dollar/yen, the short yen put dominates. Accordingly, at lower levels of spot, this risk-reversal will have positive vega but at higher levels of spot, it will have negative vega.

It is important to realize that if the risk reversal goes in reverse of what was anticipated, the trader will have a negative gamma position to hedge. Imagine the plight of a trader who tries to delta hedge this position when spot crisscrosses the strike of the short yen call. The trader will be caught having bought dollars right before the dollar drops and having to sell dollars right before the dollar rises. The only good outcome is the case where the trader buys dollars, preferably in the amount of the full face of the option, below the strike before the dollar rises without reversing. This raises the question whether it is ever advisable to use a risk reversal as a directional trade. The answer depends on one's risk appetite as well as one's trading agility. Professional foreign exchange traders do use risk reversals but they have been known to experience some large losses, as well as some large profits, with this trade.

Vertical Spreads

A far less dangerous directional combination of options is the vertical spread which consists of a long position in a put or a call plus a short position on the same option with a lower delta. Exhibit 4.9 (lower panel) contains a slide of the following vertical spread:

Vertical Spread	
Long ATMF 89.3367 USD Put/JPY Call	\$27,389
Short 25-Delta 85.0620 USD Put/JPY Call	(\$ 9,485)
Net Premium	\$17,904

This vertical spread is directionally bearish the dollar. The function of the short position in the 25-delta yen call is to lower the cost of the position. In turn, what is relinquished is any participation in directional gains below the 85.0620 strike. The maximum gross profit on a \$1,000,000 face spread is the difference between the strikes, which is equal to 4.27 yen, or \$50,254.

A directionally dollar bullish spread can be created by similar construction using yen puts.

Exhibit 4.9 shows that the vertical spread's greeks are muted at the initial spot rate of 90.00. Yet when spot approaches the vicinity of the short strike, the spread takes on the characteristics of the short option. This is a product simultaneously of the short strike option picking up sensitivity as it approaches its ATMF zone and the long strike option losing sensitivity as it departs from its ATMF zone. Of particular interest is the fact that at the initial spot level, the vega of the spread is positive but at lower spot levels the vega turns negative.

Vertical spreads are popular with technical traders who believe that they know precise levels where the spot exchange rate will meet support and resistance. The purpose of selling the lower-delta option is to unload an option that they think will expire worthless because spot will stop at a resistance level before the short strike. Non-technical traders use vertical spreads to capture small- and medium-sized movements in exchange rates. The full power of the trade can be realized if the proceeds of the sale of the low-delta option are used to buy a larger position in the vertical spread.

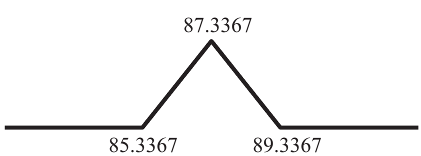
Butterflies

No directional trade illustrates the importance of the relationship between time and direction in trading better than the butterfly. Suppose that spot dollar/yen is trading at 90.00 and that a trader has a 90-day trading target of 87.3367, corresponding to a two-yen downward move from the three-month forward of 89.3367. A substantial amount of leverage with very little risk can be gained from the purchase of the following butterfly:

Butterfly

Long One 89.3367	USD Put/JPY Call	$27,389
Short Two 87.3367	USD Put/JPY Call	($34,898)
Long One 85.3367	USD Put/JPY Call	$10,270
	Net Premium	$ 2,761

The value at expiration is depicted in the following illustration.



The center or body of the butterfly is a short position in 87.3367 yen call in double the face of the other options. The long positions in the 89.3367 and 85.3367 options are called the "wings." The maximum expiration profit occurs at 87.3367 where the 89.3367 yen call is in-the-money by two yen, or $22,900 which is enormous compared to the investment of $2,761. Exhibit 4.10 shows a decomposition of the butterfly when 90 days remain to expiration.

To appreciate the role of time, consider the value of the butterfly at various times to expiration (Exhibit 4.11). Note that the butterfly with 90 days to expiration is relatively insensitive to the spot level. The butterfly picks up sensitivity to spot as time to expiration decreases and its final days are especially interesting. Right before expiration, the butterfly becomes highly sensitive to the level of the spot exchange rate when spot is anywhere close to the center strike.

Butterflies are peculiarly sensitive to volatility. When there is a great amount of time remaining to expiration, a butterfly will have an anemic level of vega (Exhibit 4.10).

The butterfly picks up vega when little time is left before expiration and when it is positioned over its "sweet spot," meaning that the spot exchange

EXHIBIT 4.10 One-Month USD/JPY Butterfly

	USD Put/ JPY Call	USD Put/ JPY Call	USD Put/ JPY Call	Butterfly
Position	1	−2	1	
Currency Pair	USD/JPY	USD/JPY	USD/JPY	
Face USD	$1,000,000	$1,000,000	$1,000,000	$1/$2/$1
Face JPY	¥85,336,700	¥87,336,700	¥89,336,700	
Spot	90.00	90.00	90.00	90.00
Strike	85.3367	87.3367	89.3367	85.3367/87.3367/ 89.3367
Days	90	90	90	90
Volatility	14.00%	14.00%	14.00%	14.00%
R_d	5.00%	5.00%	5.00%	5.00%
R_f	2.00%	2.00%	2.00%	2.00%
Value	$10,270	$17,449	$27,389	$2,761
Raw Delta (times 100)	26.49	38.37	51.13	0.90
Delta	−$264,949	−$383,655	−$511,336	−$8,975
Gamma	$ 51,645	$ 60,151	$ 62,714	−$5,944
Theta	−$ 139	−$ 171	−$ 190	$ 13
Vega	$ 1,538	$ 1,834	$ 1,956	−$ 173

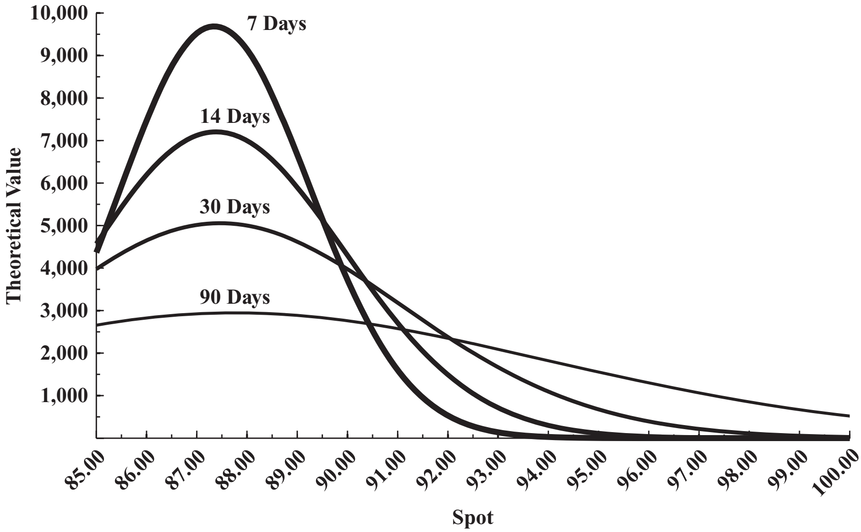


EXHIBIT 4.11 Butterfly (Dollar Bearish; Strikes at 85.3367, 87.3367, 87.3367)

rate is close to the center strike. Under this condition, the butterfly becomes a short volatility trade, meaning that it is inversely sensitive to volatility. This is because a large movement in the spot exchange rate could knock the butterfly out of the money.

HEDGING WITH CURRENCY OPTIONS

Traditional currency hedging is done with forward contracts. Examples would be an exporter who sells foreign exchange forward to hedge expected foreign currency receipts and a portfolio manager who decides to hedge his holdings of foreign stocks or bonds by executing a series of rolling forward contracts (see DeRosa 1996).

Hedging with forwards has several complications, starting with the fact that the forward encapsulates the spread between the domestic and foreign currency interest rates. Hence there is a cost to hedging a foreign currency that is at discount, meaning that the foreign interest rate is greater than the domestic interest rate. Conversely, a hedger is paid the interest rate spread when the foreign currency is at premium, meaning that the foreign currency has a lower interest rate than the domestic currency. This may be more of a concern to the exporter than to the portfolio manager because the latter's hedging costs might be recouped through future investment returns.

Operation of any currency hedging program involves the management of hedge-related cash flows. The accounting for foreign exchange hedging is the source of great confusion in many countries. For example, if a portfolio manager sells foreign exchange forward, and if the foreign currency being hedged appreciates, the hedging program will book a loss. No actual real loss will have occurred because the gains or losses in the hedging program will be matched by opposite currency translation losses and gains on the underlying portfolio. Yet not every country's accounting standards see it that way.

Hedging with currency options has similar concerns and requirements. Currency options, like forwards, are affected by interest rate spreads. Option hedging also has cash flow implications. Still options do have natural advantages for hedgers. The exporter could sell his future foreign exchange receipts by purchasing a currency put. The portfolio manager could hedge his international portfolio by buying a strip of puts to cover each currency exposure in the portfolio.

The advantage one gets with an option is protection from losses that a hedger needs but with no sacrifice of potential benefits from upside movements in a foreign currency. The disadvantage is that options cost money to purchase. At times, quoted volatility is cheap and at other times it is dear.

The question that preoccupies hedgers is how to economize on the cost of option protection. There is no such thing as a free lunch. But it is possible for the hedger to mitigate the cost of buying options by selling other options that are of little value or no use to him. This brings us to an application of the risk-reversal trade that is very popular with exporters.

Take the case of a Japanese exporter who is owed a certain amount of dollars at a future date. The exporter wants to convert the dollars to yen at a future time but is worried that dollar/yen might fall in the meantime. If the exporter is willing to sacrifice some upside profit potential from possible upward movements in dollar/yen, then the trade to do is to buy a yen call and sell a yen put, both options being out-the-money forward. The resultant structure, consisting of the long position in dollar/yen from the export activities, the long yen call, and the short yen put, is referred to as a cylinder or collar. Depending on the width of the spread between the option strikes, the exporter has room to participate in upward movements in dollar/yen but only as far as the strike of the yen put.

The most common risk-reversal construction is to use 25-delta puts and 25-delta calls. Depending on market conditions, the premiums on the two options may be so sufficiently close so as to offset each other completely. This transaction is often marketed to unsuspecting hedgers as being "cost-less." Although the collar transaction may be of zero premium, meaning no up-front payment of cash is required, it is not cost-less because the potential appreciation in dollar/yen above the yen put strike has been surrendered to pay for the yen call. Moreover, there is a potential for loss, though it is limited, if dollar/yen drops because the insurance coverage does not start until the exchange rate dips below the strike on the yen call.

None of this is to say that collar trades are entirely ill-advised. Rather the point is that there is no free lunch in option hedging. Chapter 10 discusses some non-barrier option strategies that may prove useful to hedgers.

APPENDIX 4.1 DERIVATION OF THE BSM DELTAS

Deriving the partial derivatives of BSM equations requires only basic knowledge of differential calculus.

Calculus Rules

1. A few elementary calculus rules are required.

   Polynomial Derivative Rule

$$\frac{d\,[ax^n]}{dx} = anx^{n-1}$$

   Product Rule

$$\frac{d[uv]}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

   Derivative of ln(x)

$$\frac{d\ln(x)}{dx} = \frac{1}{x}$$

   Derivatives of e^x

$$\frac{d\,e^x}{dx} = e^x \text{ and } \frac{d\,e^u}{dx} = e^u\frac{du}{dx}$$

   Derivative of the Cumulative Normal Density Function N(z)

$$\frac{d\,\mathrm{N}(z)}{dx} = \mathrm{N}'(z)\frac{dz}{dx}$$

where $N'(z)$ is the probability density function defined as

$$N'(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$$

Also note the following property:

$$N(-x) = 1 - N(x)$$

2. The BSM model is as follows.

$$C = e^{-R_f \tau} SN\left(x + \sigma\sqrt{\tau}\right) - e^{-R_d \tau} KN(x)$$
$$P = e^{-R_f \tau} S\left[N\left(x + \sigma\sqrt{\tau}\right) - 1\right] - e^{-R_d \tau} K\left[N(x) - 1\right]$$
$$x = \frac{\ln\left(\frac{S}{K}\right) + \left[R_d - R_f - \frac{\sigma^2}{2}\right]\tau}{\sigma\sqrt{\tau}}$$

3. Two intermediate partial derivatives.
The following derivatives are used repeatedly in the analysis that follows.

$$\frac{\partial x}{\partial S} = \frac{\partial}{\partial S}\left[\frac{\ln S}{\sigma\sqrt{\tau}} - \frac{\ln K}{\sigma\sqrt{\tau}} + \frac{\left(R_d - R_f - \frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}}\right] = \frac{1}{S\sigma\sqrt{\tau}}$$
$$\frac{\partial\left(x + \sigma\sqrt{\tau}\right)}{\partial S} = \frac{1}{S\sigma\sqrt{\tau}}$$

4. A Useful Result:

$$e^{-R_f \tau} SN'\left(x + \sigma\sqrt{\tau}\right) = e^{-R_d \tau} KN'(x)$$

This is easily proved as follows.

$$\frac{N'(x)}{N'\left(x + \sigma\sqrt{\tau}\right)} = \frac{e^{-\frac{x^2}{2}}}{e^{\frac{-\left(x+\sigma\sqrt{\tau}\right)^2}{2}}} = e^{x\sigma\sqrt{\tau} + \frac{1}{2}\sigma^2\tau} = e^{\ln\left(\frac{S}{K}\right) + (R_d - R)\tau} = \frac{Se^{-R_f \tau}}{Ke^{-R_d \tau}}$$

5. Finally, Solve for the Deltas.

Call Delta

$$\frac{\partial C}{\partial S} = e^{-R_f\tau} N\left(x+\sigma\sqrt{\tau}\right) + e^{-R_f\tau} SN'\left(x+\sigma\sqrt{\tau}\right)\frac{\partial\left(x+\sigma\sqrt{\tau}\right)}{\partial S}$$
$$-e^{R_d\tau} KN'(x)\frac{\partial x}{\partial S}$$

The partials in the second and third terms both equal

$$\frac{1}{S\sigma\sqrt{\tau}}$$

The second and third terms cancel, leaving

$$\frac{\partial C}{\partial S} = e^{-R_f\tau} N\left(x+\sigma\sqrt{\tau}\right)$$

Put Delta

$$\frac{\partial P}{\partial S} = -e^{-R_f\tau} N\left(-\left(x+\sigma\sqrt{\tau}\right)\right) - e^{-R_f\tau} SN'\left(-\left(x+\sigma\sqrt{\tau}\right)\right)$$
$$\times\frac{\partial\left(-\left(x+\sigma\sqrt{\tau}\right)\right)}{\partial S} + e^{R_d\tau} KN'(-x)\frac{\partial(-x)}{\partial S}$$

Similar to the case of the call delta, the second and third terms in this equation cancel, leaving

$$\frac{\partial P}{\partial S} = e^{-R_f\tau}\left(N\left(x+\sigma\sqrt{\tau}\right) - 1\right)$$

One can solve for all of the other BSM partial derivatives in a similar manner, using these basic rules of differential calculus.

CHAPTER 5

Volatility

As I noted in Chapter 3, volatility is a major element of modern option pricing theory. For the first decade and a half after the publication of the Black-Scholes paper, volatility was thought to be a constant, as the model did in fact assume. Then, the 1987 stock market crash occurred. That event convincingly disposed of the belief that volatility is a constant. Market participants were aghast to see quoted volatility on equities and equity-index options rise to staggeringly high levels. They also saw a pronounced excess demand send the quoted volatility of low-delta puts to even higher levels. Some of these same phenomena, now called smile and skew volatility, were subsequently detected in the pricing of currency options.

ALTERNATIVE MEANINGS OF VOLATILITY

The term *volatility* is ubiquitous in the options world. There are at least five distinct meanings that I will now introduce.

Theoretical Volatility

One of the core assumptions of Black-Scholes option pricing theory is that infinitesimal percentage changes in the spot exchange rate follow the diffusion process

$$\frac{dS}{S} = \mu dt + \sigma dz$$

The term dz is a Gaussian white-noise process that has zero mean and standard deviation equal to $\sqrt{dt}$; σ is assumed to be a known constant, which I call *theoretical volatility*, because it is that volatility that appears in the model.

Actual Volatility

For currency options, the term *actual volatility* means the estimated standard deviation of changes in the underlying spot exchange rate. This could mean *historic volatility*, if it is a measurement of past changes, or *future volatility* if it refers to what will happen in the future.

Actual volatility is measured as the sample standard deviation of percentage rates of return of the spot exchange rate over a specified period of time. The rate of return of the spot exchange rate, R_t, is calculated as the natural log difference in the spot rate:

$$R_t \equiv \ln(S_t) - \ln(S_{t-1}) = \ln\left(\frac{S_t}{S_{t-1}}\right)$$

where S_t and S_{t-1} are successive observations on the spot exchange rate (expressed in American convention). The unbiased sample standard deviation, $\hat{\sigma}$, is given by

$$\hat{\sigma} = \sqrt{\frac{1}{n-1}\sum (R_t - \overline{R})^2}$$

where there are n observations in the sample and $\overline{R}$ is the sample mean. An alternative statistic, which is more statistically efficient (but biased), is given by

$$\hat{\sigma}' = \sqrt{\frac{1}{n-1}\sum R_t^2}$$

Parkinson (1980) proposes an alternative extreme value estimator using the natural log ratio of the day's high and low spot exchange rates:

$$\hat{\sigma}_p = \sqrt{\frac{.361}{n}\sum \ln\left(\frac{High_t}{Low_t}\right)^2}$$

The annualized standard deviation is found by multiplying the daily sample standard deviation by the square root of the number of days in the period. Some practitioners use a 365-day count (calendar standard deviation), while others prefer to use the number of trading days in one year, roughly 252 days. The latter day count means that 1 percent daily fluctuations in price roughly equate to 16 percent annualized volatility (because the square root of 252 is roughly 16).

Implied Volatility

Theoretical volatility is an input parameter in the Black-Scholes-Merton model. In many cases the primary objective for using the model is the calculation of the theoretical value of an option. But in some trading environments, such as is the case for exchange-traded options, an actual market price of an option is observable. In such instances it is possible to use the BSM model in reverse, so to speak, to extract what volatility is implied by the known money price. One way is to use Newton's algorithm for finding the roots of a polynomial. The method starts with an initial guess of implied volatility. This value is plugged into the BSM model to arrive at an initial estimate of the option price, which can be compared to the actual known option price. The difference, which is called the error, is used to calculate a new candidate volatility using the following formula:

$$\sigma_{n+1} = \sigma_n + \frac{C - C(\sigma_n)}{vega(\sigma_n)}$$

where σ_{n+1} is the candidate volatility on the $(n + 1)$th iteration, σ_n is the candidate volatility from the previous iteration, C is the known value of the option, $C(\sigma_n)$ is the value of the call implied from the previous candidate volatility, and the option vega, meaning the partial derivative of the option price with respect to volatility, is given by

$$vega\ (\sigma_n) = e^{-R_d \tau} K \sqrt{\tau} N'(x)$$

In usual practice, this algorithm converges on a reasonably precise estimate of implied volatility in a small number of iterations.

In the special case of at-the-money forward options, implied volatility can be backed out from the Brenner and Subrahmanyam approximation presented in Chapter 4:

$$\hat{\sigma} \cong \frac{2.5C}{e^{-R_f \tau} S \sqrt{\tau}}$$

Quoted Volatility

Traders in the interbank market make currency option quotes in terms of volatility as opposed to in dollars and cents. *Quoted volatility* is subsequently changed into a money price using the BSM model. Quoted volatility is actually a form of implied volatility. But in the context of the interbank currency option market, the quoted volatility of an option is known in advance of the money price, hence the distinction "quoted" volatility. As is the practice among traders I will call quoted volatility "volatility" for simplicity.

The BSM model assumes volatility is a known constant. But I already provided plain evidence in Chapter 3 (Exhibit 3.3) that volatility varies across currencies.

Even for a single currency, volatility changes over time, as will be apparent from data that I present in this chapter. Big changes in option volatility come about whenever there is a good reason to believe that a sharp movement in underlying spot exchange rates is probable. Whenever this happens, the demand for short-dated options is apt to rise greatly. That is because short-dated options are relatively rich in gamma. Of course long-dated volatility is likely to rise as well, but usually not in such a pronounced fashion as the move in short-dated volatility.

At any given point in time, volatility can vary according to how much an option is in-the-money or out-of-the-money. As I mentioned previously, the moneyness dimension to volatility was first noted in the equity markets at the time of the October 19, 1987, crash. On this memorable day the U.S. stock market plunged over 22 percent as measured by the Dow Jones Industrial Index. For ever after it seems that out-of-the-money equity and equity-index puts have tended to be well bid relative to at-the-money options. The phenomenon is called the "skew." The foreign exchange market exhibits similar but not identical departures from the BSM-constant-volatility assumption. In foreign exchange the phenomenon is more often a *smile*, by which is meant both low-delta puts and low-delta calls have a somewhat symmetrical inverse relationship with volatility. But this is not always the case—sometimes in foreign exchange there is a skew, even for major currencies, especially the yen. Skew means there is relative imbalance between the demand for low-delta puts and low-delta calls—preference for one over the other produces a relative premium.

Volatility can be a function of the amount of time remaining in an option's life before expiration. The term structure of volatility can assume any number of shapes; for example it can be positive sloping, negative sloping, or flat. Xu and Taylor (1994) studied the behavior of implied volatility from the Philadelphia Stock Exchange currency options on pounds, marks, Swiss francs, and yen during the period 1985–1990.They report:

> *The term structure sometimes slopes upwards, sometimes downward, and its direction (up or down) frequently changes. The direction changes, on average, approximately once every two or three months. . . . The term structures of the pound, mark, Swiss franc and yen at any moment in time have been very similar.* (p. 73)

Another thing to consider is that volatility sometimes possesses subtle and ephemeral relationships with the level of the spot exchange rate. Traders have a nickname for this phenomenon, dollar-voler, because it was first identified as a relationship between the levels of the dollar and volatilities of dollar calls and puts.

Three Important Volatility Quotations

The location of the volatility surface and the degree to which it has smile and skew are captured in quotations of the three important trades introduced in Chapter 2:

At-the-money: σ_{ATM}

Risk reversal (25-delta): $\sigma_{RR25} = \sigma_{25\delta C} - \sigma_{25\delta P}$

and

Butterfly (Vega-weighted 25-delta): $\sigma_{VWB} = \frac{1}{2}\left[\sigma_{25\delta C} + \sigma_{25\delta P}\right] - \sigma_{ATM}$

I will refer to these as the *iconic volatility* quotations (some authors call them *volatility pillars*). Option dealers routinely supply quotations for these at standardized expirations (examples being one week, one month, two months, three months, six months, and one year. See further discussion in Boxes 5.1, 5.2, and 5.3). Dealers may also quote risk reversals and butterflies for 10-delta and 15-delta strikes in addition to 25-delta strikes.

BOX 5.1: AT-THE-MONEY VOLATILITY

The interest parity theorem states that the forward outright is given by

$$F = Se^{(R_d - R_f)\tau}$$

In Chapter 2, I showed that a European exercise put and call stuck at the forward outright have equal value (given a common expiration). I denoted the implied volatility of the at-the-money forwards strikes as σ_{ATM}.

(*continued*)

This is easily confused with the at-the-money volatility, σ_{ATM}. What is quoted in the market as σ_{ATM} is the implied volatility of a zero-delta ("delta neutral") straddle. A straddle is a put and a call with the same strike at a given expiration. For this particular straddle the component put and calls have the same strike *and* the same delta (but opposite sign). To make the call and put deltas to be equal in absolute value, the common strike K_{ATM} must be

$$e^{-R_f\tau} N\left(\frac{ln\frac{S}{K_{ATM}} + \left(R_d - R_f + \frac{1}{2}\sigma_{ATM}^2\right)\tau}{\sigma_{ATM\sqrt{\tau}}}\right)$$

$$= e^{-R_f\tau} N\left(-\frac{ln\frac{S}{K_{ATM}} + \left(R_d - R_f + \frac{1}{2}\sigma_{ATM}^2\right)\tau}{\sigma_{ATM\sqrt{\tau}}}\right)$$

which means that the at-the-money strike is different from the forward (unless volatility is zero) and the values of the put and call are unlikely to be equal to each other.

$$K_{ATM} = \left(Se^{\left(R_d - R_f + \frac{1}{2}\sigma_{ATM}^2\right)\tau}\right)$$

BOX 5.2: RISK REVERSAL VOLATILITY

A bullish risk reversal is created by buying a call and selling a put of the same expiration and same delta in absolute value. A bearish risk reversal sells the call and buys the put. The most common one is the 25-delta risk reversal but one also sees 15- and 10-delta risk reversals.

The convention is to quote the risk reversal as the bias for the call:

$$\sigma_{RR} = \sigma_{25\delta C} - \sigma_{25\delta P}$$

where the deltas are calculated from the spot exchange rate.

BOX 5.3: BUTTERFLY VOLATILITY

A butterfly (for credit) is bought with the sale of a straddle and the purchase of a strangle. The strangle consists of a purchase of out-of-the-money puts and calls with equal delta in absolute value. For example, the 25-delta strangle is composed of a 25-delta call and a 25-delta put. Hence the strangle portion is delta neutral at inception.

The straddle portion is also delta neutral at inception because it is struck at-the-money (K_{ATM}) (see Box 5.1).

The volatility of a butterfly is given by

$$\sigma_{VWB} = \frac{\sigma_{25\delta C} + \sigma_{25\delta P}}{2} - \sigma_{ATM}$$

By convention, this is sometimes referred to as the *vega-weighted* butterfly (hence the VWB). The butterfly, vega-weighted or not, is quoted just as the above formula indicates. However, the actual butterfly that would be traded likely would be vega-weighted. Since the vega of the straddle is greater than that of the strangle, the vega-weighted butterfly is created by doing unequal amounts of the straddle and the strangle. The weightings are chosen to make the total vega of the butterfly equal to zero. This allows the dealer to buy or sell the vega-weighted butterfly with no consequence to the vega of his dealing book. Note that the straddle and strangle are separately delta-neutral. Consequently, what looks at first to be a very messy trade for the dealer, involving four options [for example buying two options (the at-the-money put and call) and selling two options (the 25-delta put and call)], actually can enter the dealer's book flat with no delta or vega consequences.

The at-the-money volatility is an indicator of the volatility across the entire strike structure for a given expiration. The risk reversal volatility is a measure of the slope or skew of the volatility surface. As I mentioned, the risk reversal skew measures the relative demand for out-of-the-money calls over puts. The butterfly is a measure of the smile or curvature of the surface.

Smiles and skews affect option pricing but only within limits.[1] For one thing, the dollar value of a put must be positively related to strike, all other things constant. This is because the bearish put spread (consisting of buying

[1] See Hodges (1996).

a put at one strike and selling a similar put at a lower strike) must have a positive price. Since the payoff from this strategy is either zero or a positive amount, we know that the strategy must cost something. Hence the price of the higher strike put must exceed that of the lower strike put. We can write this as

$$\frac{\partial P}{\partial K} > 0$$

A similar argument for calls means

$$\frac{\partial C}{\partial K} < 0$$

There are also second-order conditions. Consider a butterfly. In the example of buying the 25-delta butterfly the trader sells an at-the-money straddle (i.e., sells a call and put that make up the body) and buys a 25-delta strangle (i.e., buys a 25-delta call and a 25-delta put that make up the wings). The strategy finishes either worthless out-of-the-money or in-the-money. Hence it must cost something to buy. This leads to a second-order condition on puts and calls:

$$\frac{\partial^2 C}{\partial K^2} > 0$$

$$\frac{\partial^2 P}{\partial K^2} > 0$$

These inequalities restrict how much skew or smile can influence option pricing. I note that they are strict money-based arbitrage conditions that actually have nothing to do with any formal model of option pricing.

Forward Volatility

As I discussed, quoted currency volatility has an observable term structure. It is natural to question whether expectations of future volatility play a part in how the market prices short-term and long-term dated options. There is a direct parallel between this question and classical expectations theory of the term structure of interest rates. "Forward volatility" can be extracted from the term structure of volatility much the same way as forward interest rates can be obtained from the term structure of interest rates.

Because volatility is linear in its square (i.e., variance), we can define the forward volatility in the following way: Suppose we observe volatility at various equally spaced points on the term structure. Let observed volatilities σ_1 and σ_2 correspond to maturities τ_1 and τ_2 with $\tau_1 < \tau_2$. The longer-term volatility can be decomposed as follows

$$\tau_2\sigma_{t_2}^2 = \tau_1\sigma_{t_1}^2 + (\tau_2 - \tau_1)\sigma_{t_1,t_2}^2$$

where σ_{t_1,t_2} is defined as the forward volatility. For example, if three- and six-month volatility were 10 percent and 15 percent, respectively, the three-month forward volatility would be

$$\sqrt{\frac{[.5 \times (15\%)^2 - .25 \times (10\%)^2}{.25}} = 18.71\%$$

Campa and Chang (1995 and 1998) investigate whether interbank forward quoted volatility has any predictive power for future quoted volatility. They study daily quoted volatility for options of up to one year in term on four major currencies over the period from December 1989 to May 1995. Campa and Chang cannot reject the expectations hypothesis that quoted volatility predicts future quoted volatility. In other words, they detect a relationship between current short-term volatility, current long-term quoted volatility, and future short-dated quoted volatility. They conclude in their 1995 paper:

> *In sharp contrast to the literature on the term structure of interest rates, we conclude that for all currencies and maturity pairs, current spreads between long-run and short-run volatility do predict the right direction of future short-rate and long-rate changes, even at horizons that are as brief as one month. (pp. 545–547)*

Campa and Chang perform out-of-sample tests to determine whether forward volatility is a good predictor of future quoted volatility in spot exchange rates. For most of their sample period, December 1989 to July 1993, forward volatility is an inferior predictor of future quoted volatility to forecasts constructed with Box-Jenkins time series models.

SOME VOLATILITY HISTORY

Secular Volatility

Exhibits 5.1, 5.2, 5.3, 5.4, and 5.5 show one-month and one-year quoted volatility on at-the-money forward options for a variety of major exchange rates during the period 1995–2010 and the Euro starting in 1999.

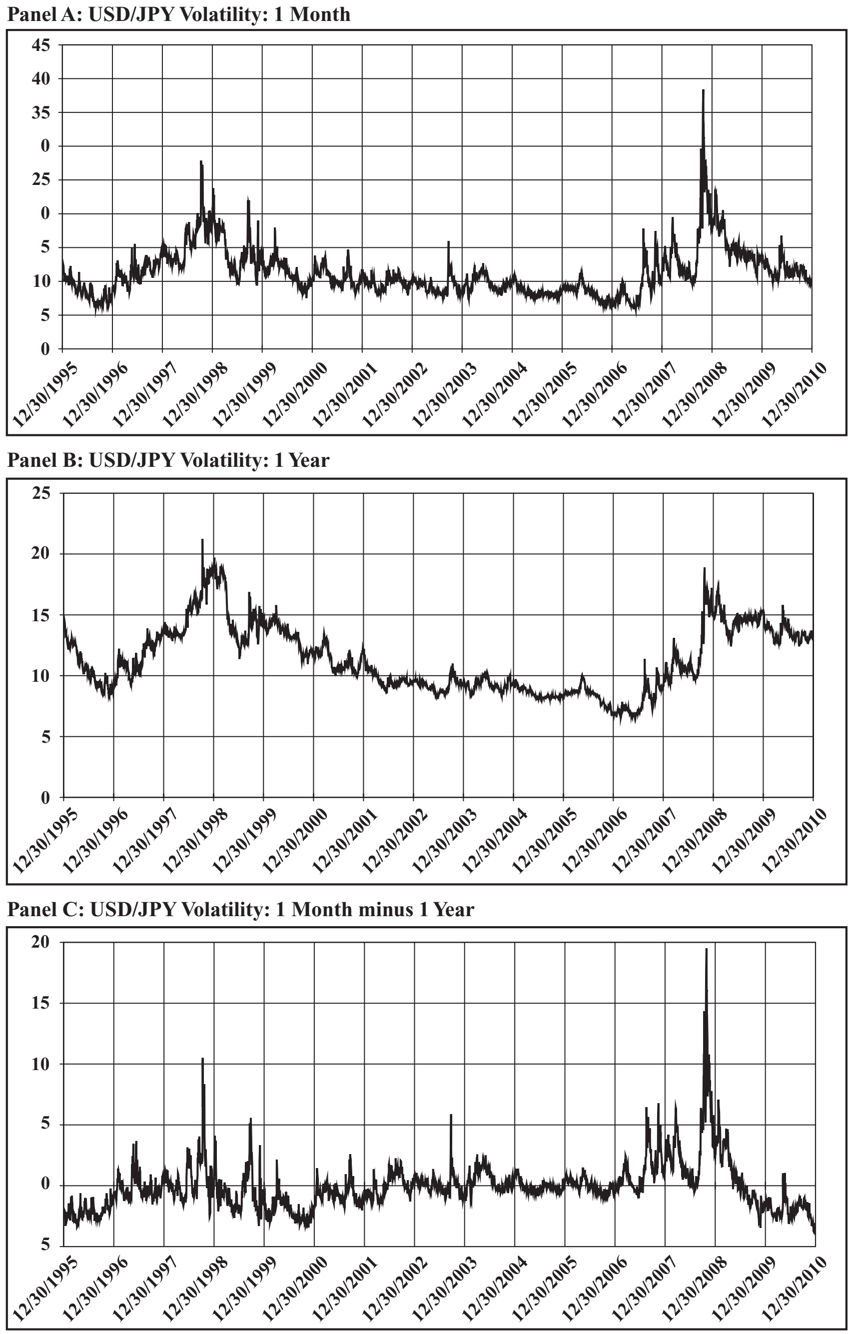


EXHIBIT 5.1 USD/JPY: Quoted Volatility on 1-Month and 1-Year Options, December 30, 1995–December 30, 2010, Daily
Data source: Bloomberg Finance L.P.

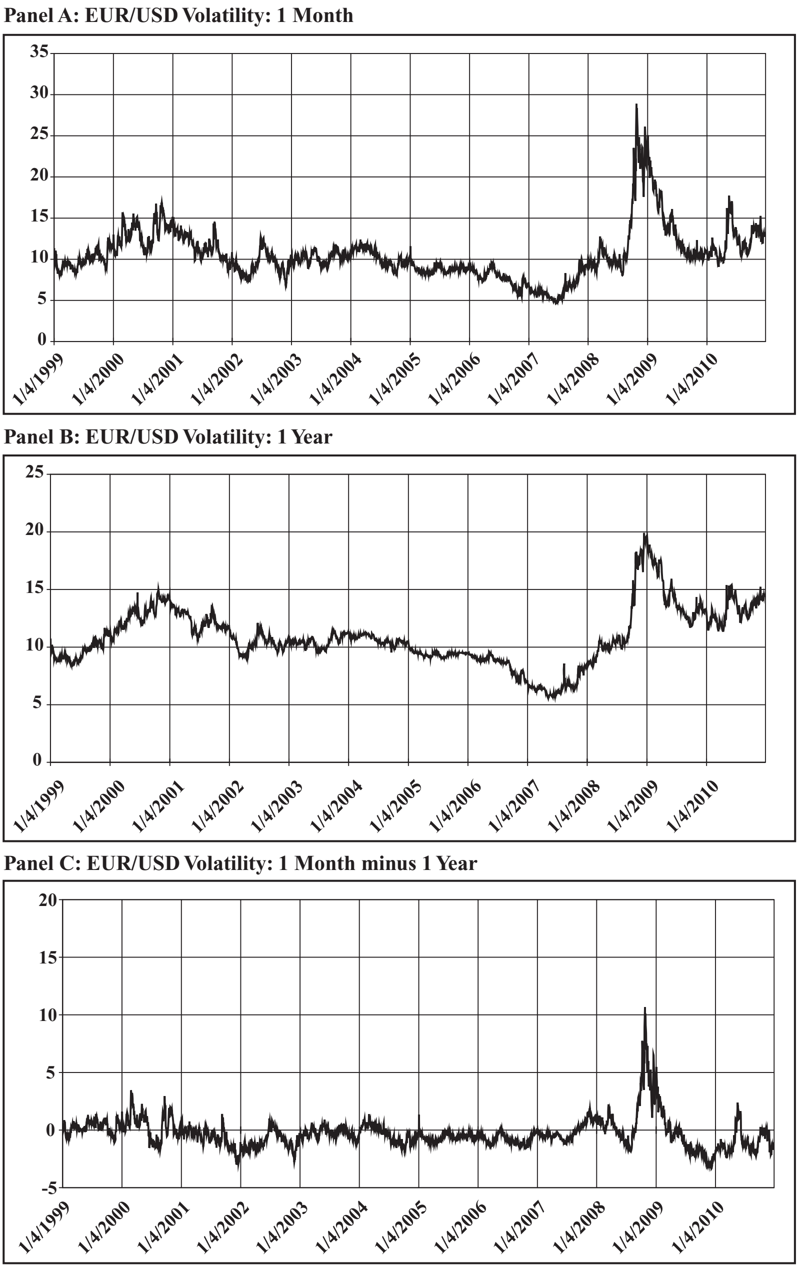


EXHIBIT 5.2 EUR/USD: Quoted Volatility on 1-Month and 1-Year Options, January 4, 1999–December 30, 2010, Daily
Data source: Bloomberg Finance L.P.

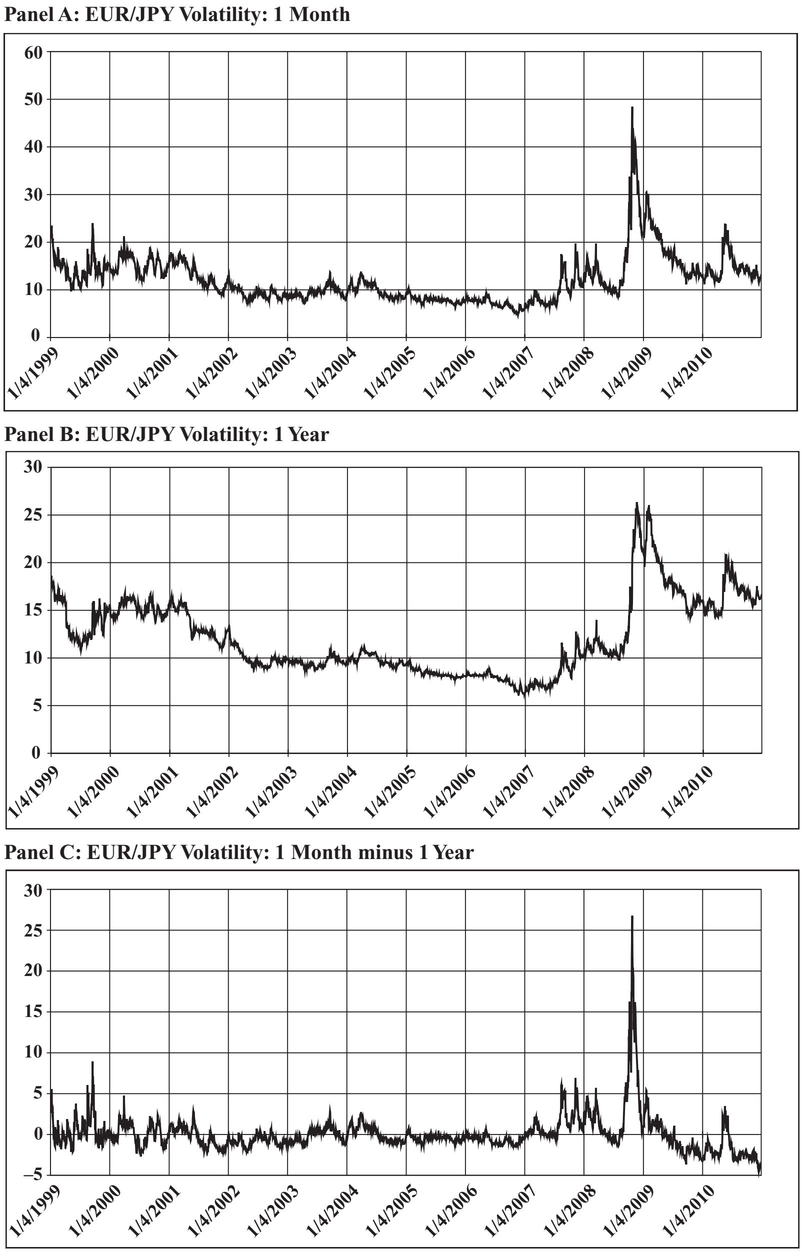


EXHIBIT 5.3 EUR/JPY: Quoted Volatility on 1-Month and 1-Year Options, January 4, 1999–December 30, 2010, Daily
Data source: Bloomberg Finance L.P.

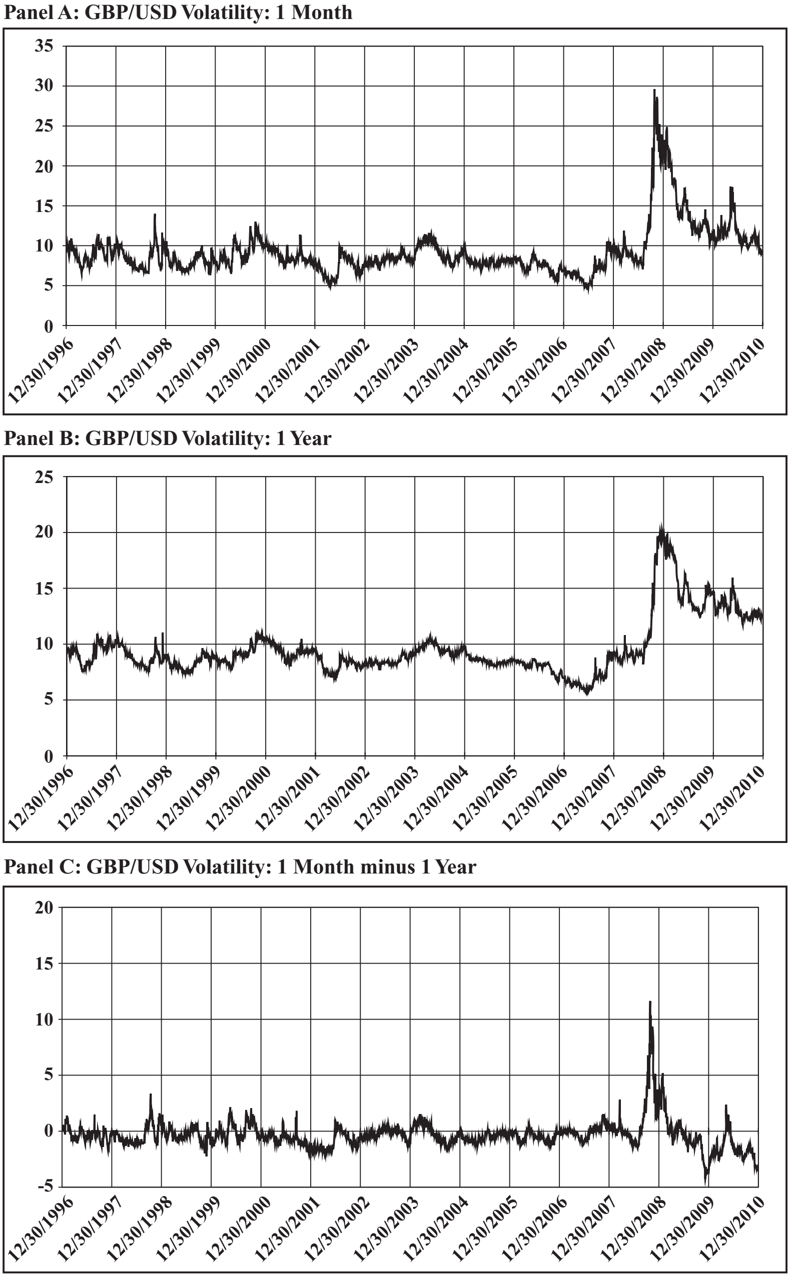


EXHIBIT 5.4 GBP/USD: Quoted Volatility on 1-Month and 1-Year Options, December 30, 1996–December 30, 2010, Daily
Data source: Bloomberg Finance L.P.

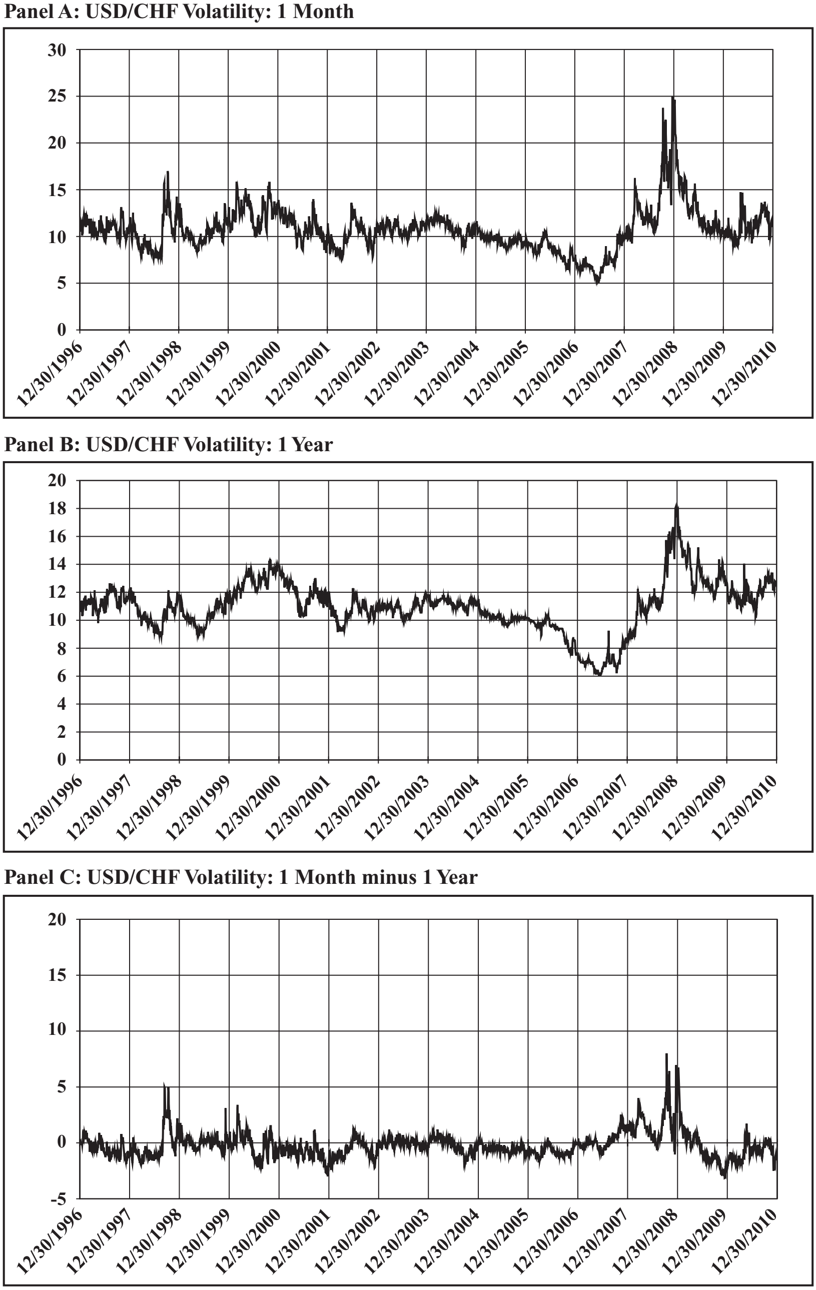


EXHIBIT 5.5 USD/CHF: Quoted Volatility on 1-Month and 1-Year Options, December 30, 1996–December 30, 2010, Daily
Data source: Bloomberg Finance L.P.

These exhibits illustrate that short-term volatility, in this case 1-month volatility, is usually more volatile than long-term volatility, 1-year volatility. The bottom panel of each exhibit shows the amount by which 1-month volatility exceeds 1-year volatility.

The exhibits also show that volatility is casually linked to identifiable market crises. An example is the August 1998 financial collapse of the Russian Federation. Russia simultaneously defaulted on a portion of its sovereign debt, de-pegged the ruble, and declared a moratorium on banks making payments on ruble forward contracts. The effect on financial markets was fairly dramatic, perhaps surprisingly so. For present purposes I note that volatility increased, or *went bid*, a term of art, as option prices incorporated the possibility of imminent large gyrations in exchange rates. Short-term volatility spiked upwards (but less so with long-term volatility). This is confirmed in Panels C of Exhibits 5.1 through 5.5. Other crises produced similar reactions in the options market, as I will now discuss.

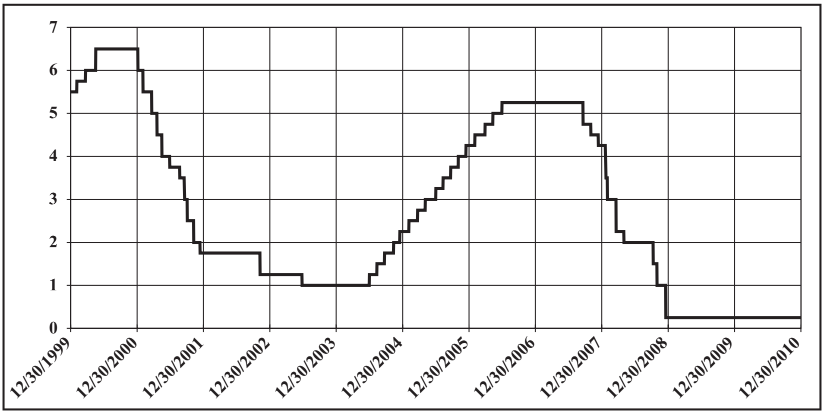
Option Volatility during 2007–2008

As I wrote in Chapter 1, aberrations in the credit and forward foreign exchange markets occurred in the period 2007–2008. I now turn attention to the behavior of currency option volatility in that remarkable period. As one would expect, given the magnitude of economic dislocations, volatility skyrocketed, especially in October 2008. This appears to have been the case for all of the exchange rates displayed in Exhibits 5.1 through 5.5. One-month volatility for USD/JPY was quoted as high as 35 percent, a level that at any time would be considered strikingly high for a major exchange rate. The collapse of Lehman Brothers in the previous month had a hand in this, but the story does not stop there. The interplay between the foreign exchange market and U.S. monetary policy in this period is interesting.

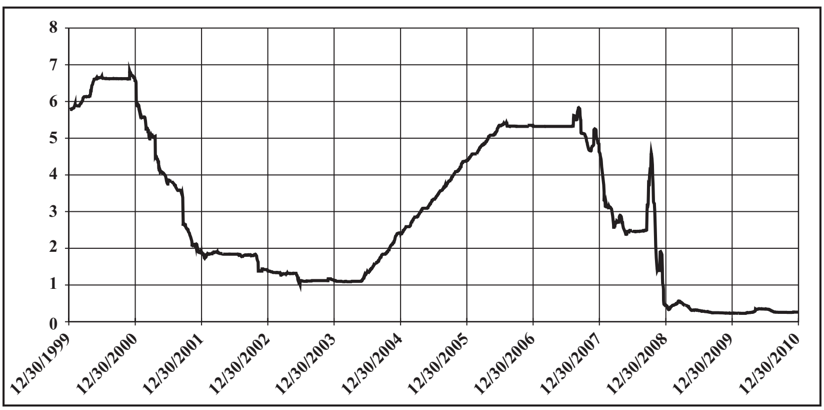
By way of background, the Federal Reserve (Fed) began to tighten credit markets starting on June 30, 2004, when it raised its target for overnight federal funds from 1 percent to 1.25 percent. (Exhibit 5.6 shows the Federal Reserve's target rate as well as 1-month LIBOR for U.S. dollars and Japanese yen).

Thereafter, the Fed continued to raise its target for the overnight federal funds rate in 25-basis-point increments ending with a final tightening to 5.25 percent on June 29, 2006. During all this time, the Fed's policy caused a widening of the gap between the static, near-zero Japanese interest rates and the U.S. interest rates. Fifteen months later, the Fed reversed course and began to cut its target rate. The first cut was on September 18, 2007, when the Fed lowered its target for overnight federal funds to 4.75 percent. The Fed cut rates aggressively in the coming months, reacting at first to slower

Panel A: Federal Reserve's Target Overnight Rate 2000–2010



Panel B: USD 1 Month Libor 2000–2010



Panel C: JPY 1 Month Libor 2000–2010

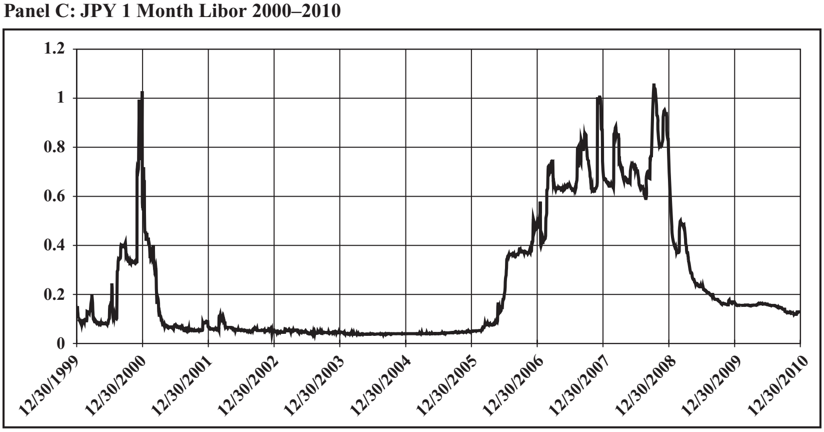


EXHIBIT 5.6 Federal Reserve's Target for Overnight Federal Funds, 1-Month U.S. Dollar LIBOR and 1-Month Japanese Yen LIBOR, December 30, 1999–December 30, 2010, Daily
Data source: Bloomberg Finance L.P.

economic growth (given no signs of a pickup in inflation). Soon though, the Fed was making cuts in response to the seriousness of the unfolding economic crisis. The Fed's final rate cut occurred on December 16, 2008, when it reduced its target rate to 0.25 percent.

Meantime, during this entire period, meaning 2004–2008, and for well before that time, Japanese short-term interest rates were nearly zero (practically under 1 percent). One well-known feature of the USD/JPY market is the carry trade. This trade is an out-and-out wager that uncovered interest parity (described in Chapter 1) does not work. By this I mean that investors and traders take positions that are either sales of yen, or at least borrowing in yen, against long positions in dollar-denominated deposits and fixed-income securities. In theory, the trade could earn the interest rate differential between dollars and yen. But this requires a constant value for USD/JPY. Of course, it could turn into a small bonanza if the dollar were to rise against the yen. Market observers in autumn 2008 reported that the fall in U.S. dollar short-term interest rates was causing yen carry traders to rethink their positions.[2] Certainly the environment was not one conducive to carry-trade optimism once the Fed began to aggressively cut its target interest rate. In the meantime, the Bank of Japan was keeping its overnight interest rates nearly at zero. Carry traders had to have been asking themselves what, if any, interest rate differential existed for them to earn. Worse yet, there were serious concerns that the dollar might continue to drop against the yen (Exhibit 5.7).

Fears of the dollar taking a serious tumble (against the yen) may well explain the anomalous pricing of USD/JPY risk reversals, which has to be considered one of the most remarkable parts of the entire carry-trade saga in this period. Exhibit 5.8 shows the USD/JPY 1-month 25-delta risk reversal.

It is not unusual to see the risk reversal quoted with USD puts/JPY calls at premium to USD calls/JPY puts. This phenomenon is usually attributed to the presence of Japanese exporters trying to protect long dollar positions (their unconverted dollar revenue from sales of goods to the United States) and to carry traders in general. A position that is long dollars against yen can be partially protected by purchasing a low-delta USD put/JPY call and selling a low-delta USD call/JPY put. This creates what is called in the equity markets a "collared" position. Greater concern about the dollar's possible fall could translate into greater pressure on the pricing of dollar puts compared to dollar calls. What is so remarkable about October 2008 is that the 1-month 25-delta risk reversal skew became so large in absolute

[2]Fackler, Martin, "In Japan, a Robust Yen Undermines the Markets," *New York Times*, October 27, 2008.

EXHIBIT 5.7 USD/JPY, December 30, 2006–December 30, 2010, Daily
Data source: Bloomberg Finance L.P.

value, as Exhibit 5.8 demonstrates. This gives support to the notion that the yen carry trade was indeed unwinding. If so it makes sense that carry traders would pay a premium for any protection they could get from the risk reversal market.

Yet all of this was specific to the dollar against the yen. In other parts of the foreign exchange market there was strong demand for dollars. This would be the more usual pattern, meaning the dollar to be well-bid in a crisis. The EUR/USD risk reversal (Exhibit 5.9) shows a preference for EUR puts/USD calls. In effect the market seems to have had a preference for selling dollars against yen but buying dollars against euros, as is confirmed by a

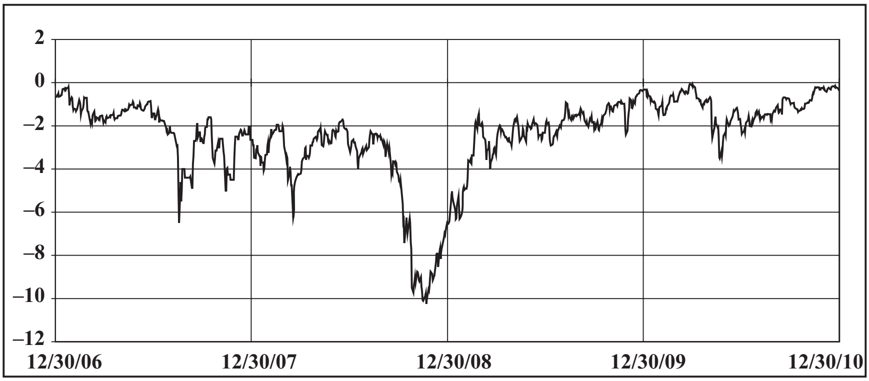


EXHIBIT 5.8 Quoted USD/JPY 1-Month 25-Delta Risk Reversal, December 30, 2006–December 30, 2010, Daily
Data source: Bloomberg Finance L.P.

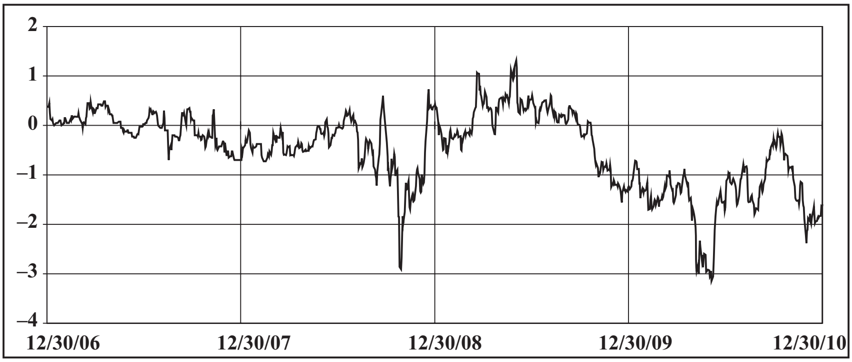


EXHIBIT 5.9 Quoted EUR/USD 1-Month 25-Delta Risk Reversal, December 30, 2006–December 30, 2010, Daily
Data source: Bloomberg Finance L.P.

monumental downward plunge in the EUR/JPY cross rate in October 2008 (Exhibits 5.10 and 5.11).

Volatility during Some Other Crises

A more recent crisis erupted in May 2010 when Greece, a member of the European Union and participant in the euro, declared its government to be insolvent without external aid. Fear of a possible sovereign default among the countries in the European Union caused sharp selling pressures on the euro. The option market responded by bidding up volatility on options on the euro, as can be seen in Exhibit 5.12. This exhibit depicts intraday

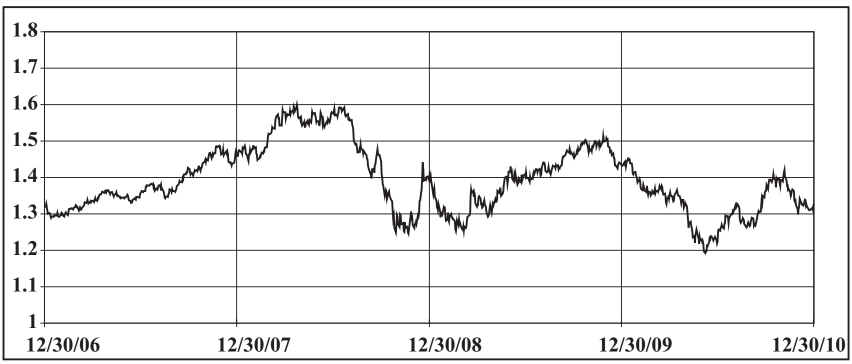


EXHIBIT 5.10 EUR/USD, December 30, 2006–December 30, 2010, Daily
Data source: Bloomberg Finance L.P.

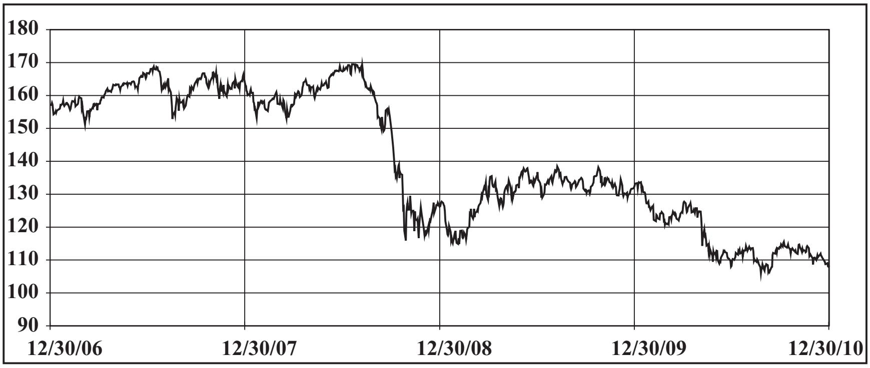


EXHIBIT 5.11 EUR/JPY January 1, 2007–December 31, 2010, Daily
Data source: Bloomberg Finance L.P.

EXHIBIT 5.12 EUR/USD: Quoted Volatility on 1-Month, Intraday May 3, 2010–May 7, 2010, Intraday
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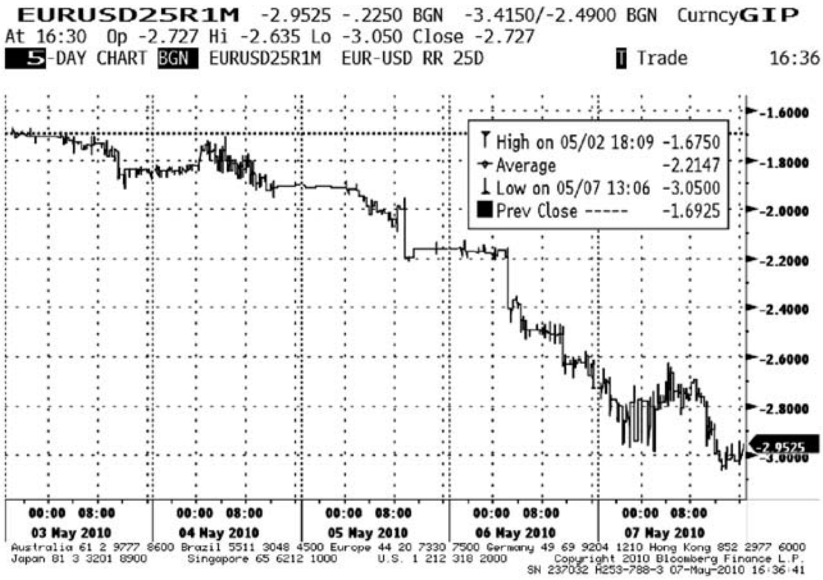

EXHIBIT 5.13 EUR/USD: Quoted 1-Month 25-Delta Risk Reversals May 3, 2010–May 7, 2010, Intraday
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at-the-money forward 1-month volatility on critical days in May 2010. Exhibit 5.13 shows the intraday 1-month 25-delta risk reversal skew (EUR calls – EUR puts) steepening for EUR puts as the crisis deepened.

It is also interesting to ask what happened to quoted volatility in some older but no less serious currency crises. In the spring and summer of 1997, the Southeast Asian currency crisis erupted, starting with a speculative attack on the Thai baht.[3] In May 1997, the Bank of Thailand reacted by aggressively selling dollars and buying baht. Moreover, it instructed domestic Thai banks to cease lending baht to overseas customers (presumed to be the speculators), thus creating a two-tier currency market for the baht. The bank won the immediate battle but soon lost the war. On July 2, 1997, the Bank of Thailand was forced to float the baht. Exhibit 5.14 shows 1-month quoted volatility on USD/THB (THB is the code for the Thai baht). Quoted volatility rose swiftly as the crisis became manifest. Volatility continued to

[3]See DeRosa (2001).

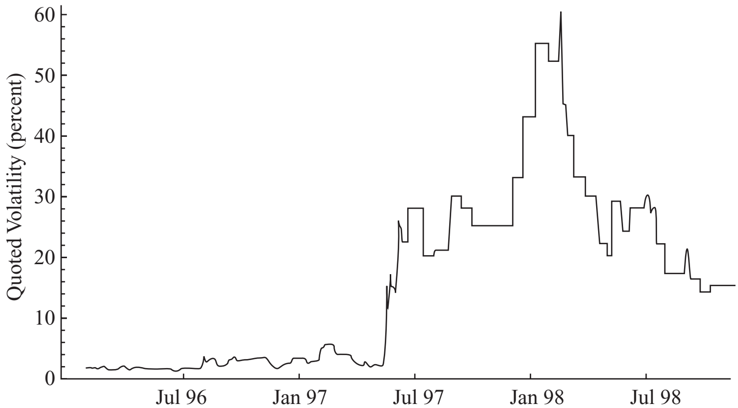


EXHIBIT 5.14 Quoted Volatility on Dollar/Baht (USD/THB), 1-Month, January 1995 to December 1998, Monthly Observations
Source: MSCI Inc.

climb as the crisis spread from one country to the next throughout the region and remained at relatively high levels until late 1998.

Another interesting period is the summer and autumn of 1992 when the European Monetary Union experienced the so-called Sterling crisis. Exhibit 5.15 displays the behavior of the Sterling-mark exchange rate taken from Malz (1996). Sterling entered the Exchange Rate Mechanism in October

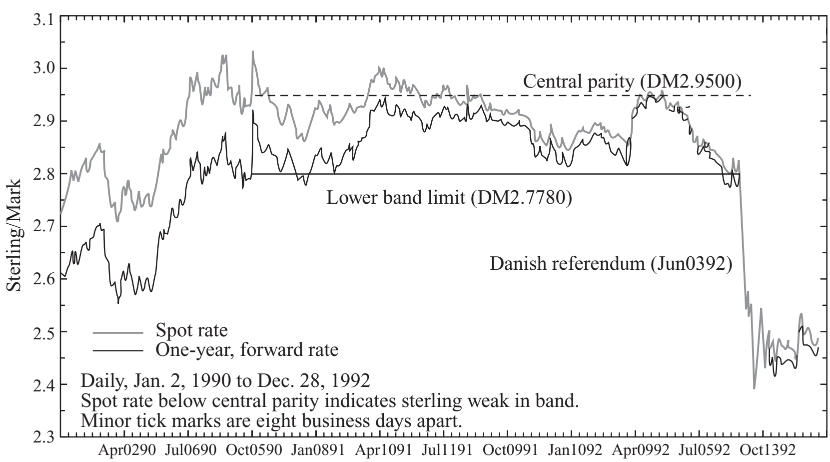


EXHIBIT 5.15 The Sterling/German Mark Exchange Rate in the ERM
Source: Reprinted from *Journal of International Money and Finance*, Allan Malz, 1996, with permission from Elsevier Science.

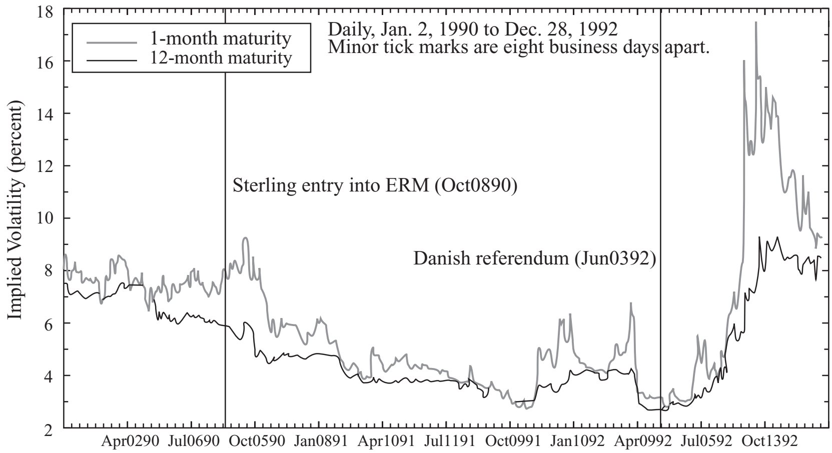


EXHIBIT 5.16 Sterling/Mark at-the-Money Implied Volatility
Source: Reprinted from *Journal of International Money and Finance*, Allan Malz, 1996, with permission from Elsevier Science.

1990 at a central rate against the mark of 2.95 marks. Sterling was permitted to fluctuate inside a bandwidth equal to plus or minus 6 percent around its ECU central rate. Crucial for Sterling was its relationship to the mark because the latter served as the anchor for the exchange rate grid. Sterling was violently forced out of the ERM in the September 2, 1992, crisis. Exhibit 5.16 (Malz 1996) shows the behavior of 1-month and 1-year Sterling-mark volatility. Volatility had declined following Sterling's admission to the ERM. However, quoted volatility began to rise sharply as early as October 1991, which later was recognized as a warning sign of trouble ahead. The crisis that followed began to take shape on June 3, 1992, when the Maastricht treaty, an essential component of the ultimate single currency project, was defeated in a Danish referendum. The Sterling crisis formed over the next several months, climaxing on September 2, 1992, when Sterling encountered irrepressible waves of selling. The story did not end there, however. Once free of the ERM constraints, Sterling continued to plunge violently against the mark in the course of the week that followed. Things finally settled down in October whereupon quoted option volatility began to drop.

CONSTRUCTION OF THE VOLATILITY SURFACE

The term *volatility surface* refers to a three-dimensional mapping of quoted volatility against option delta and term to maturity. An example

EXHIBIT 5.17 Volatility Surface for EUR/USD as of July 22, 2010

	ATM		25D Call EUR		25D Put EUR		10D Call EUR		10D Put EUR	
Exp	**Bid**	**Ask**	**Bid**	**Ask**	**Bid**	**Ask**	**Bid**	**Ask**	**Bid**	**Ask**
1D	15.541	16.781	15.059	16.813	16.500	18.200	15.118	17.548	17.818	20.068
1W	13.035	14.275	12.636	14.214	13.677	15.213	12.646	14.797	14.566	16.582
2W	12.805	13.750	12.364	13.571	13.559	14.726	12.339	13.993	14.489	16.018
3W	12.670	13.465	12.179	13.198	13.658	14.635	12.169	13.571	14.895	16.165
1M	12.525	13.125	11.956	12.729	13.595	14.330	11.911	12.979	14.888	15.842
2M	12.670	13.195	12.057	12.738	13.984	14.621	12.135	13.077	15.694	16.514
3M	12.850	13.330	12.197	12.823	14.370	14.950	12.350	13.218	16.416	17.157
6M	13.030	13.445	12.379	12.926	14.740	15.235	12.622	13.385	17.091	17.722
1Y	13.330	13.745	12.657	13.213	15.237	15.723	13.059	13.839	17.912	18.531
18M	13.070	13.495	12.493	13.067	14.799	15.291	12.911	13.721	17.318	17.950
2Y	12.925	13.375	12.358	12.972	14.537	15.053	12.772	13.641	16.910	17.578
3Y	12.495	12.970	11.942	12.598	13.826	14.364	12.227	13.165	15.821	16.526
5Y	11.920	12.400	11.399	12.076	13.003	13.532	11.418	12.404	14.406	15.121
7Y	10.996	11.584	10.464	11.306	11.936	12.575	10.313	11.560	13.150	14.025
10Y	10.220	10.970	9.647	10.733	11.036	11.834	9.339	10.966	12.092	13.193

ATM DNS | Spot Δ excl Prem | RR=EUR Call-Put | BF=(C+P)/2-ATMD
Data source: Bloomberg Finance L.P.

is given in Exhibit 5.17 for the EUR/USD volatility surface observed on July 22, 2010.

As I have mentioned, option dealers routinely quote iconic volatilities, at-the-money forward, risk reversals (25-delta and/or 10-delta), and butterflies (25-delta and/or 10-delta), at standard expirations (examples being overnight, 1 week, 1 month, 3 months, 6 months, and 1 year). These quotes are useful for many purposes, including trading, but they do not in and of themselves provide a continuous volatility surface. Having the complete surface—or at least being able to pick any point on the surface—is important because one can price or revalue practically any standard option. The task then can be seen as having to interpolate between and extrapolate from the iconic, standardized quotes to infer the volatility of any particular option, identified by strike or delta and term to expiration.

Interpolation methods are commonplace in applied mathematics and engineering disciplines, but their implementation requires careful treatment of the data. Even more important than the exact mathematical recipe is the selection of the data. Quotes must be timely and true indications of market levels of volatility if the end product is to have any usefulness. Most interpolation is done in the dimension of variance rather than standard deviation. Holidays and weekends are problematic because they occupy the same number of hours as a regular day, but price movements are less likely to occur because by definition the market is closed, or nearly closed.

THE VANNA-VOLGA METHOD

One problem with interpolated volatility surfaces is that they can falsely indicate the existence of profitable arbitrage opportunities. For this reason an alternative technique, called the Vanna-Volga Method, has become popular.

Recall that vega is the partial derivative of option value with respect to volatility. Vega itself has interesting partial derivatives. The partial of vega with respect to spot is called vanna; the partial of vega with respect to volatility is called volga. The idea behind the Vanna-Volga Method is to construct a series of replication portfolios that allow the extraction of any point on the entire volatility surface. These portfolios consist of out-of-the-money puts and calls plus at-the-money options. The replication portfolios can also include positions in the underlying currency (to flatten the delta). Consider a specific point on the surface corresponding to a theoretical option strike with a given time to expiration and delta. The replication portfolio must be constructed so as to have the same delta, gamma, vega, vanna, and volga as an option located at the relevant point on the surface. Points on the Vanna-Volga surface are arbitrage-free up to second-order partial derivatives.

The first step in using the Vanna-Volga Method is to obtain dealer quotes for iconic volatilities (see Exhibit 5.18).

It is essential to the integrity of the process that these quotations be fresh and accurate, though it has to be recognized that there is always some dispersion in the quotes from different dealers. In the next step, the quotes are easily transformed with simple algebra into volatilities for 25-delta calls and puts:

$$\sigma_{25\delta C} = \sigma_{ATM} + \sigma_{VWB} + \frac{1}{2}\sigma_{RR}$$

$$\sigma_{25\delta P} = \sigma_{ATM} + \sigma_{VWB} - \frac{1}{2}\sigma_{RR}$$

EXHIBIT 5.18 Iconic Quotations for EUR/USD Volatility (ATM, Risk-Reversals, and Butterflies), as of July 22, 2010

	ATM		25D RR		25D BF		10D RR		10D BF	
Exp	Bid	Ask	Bid	Ask	Bid	Ask	Bid	Ask	Bid	Ask
1D	15.529	16.769	–1.599	–1.237	0.251	0.714	–2.953	–2.275	0.437	2.517
1W	13.025	14.265	–1.105	–0.935	0.040	0.520	–2.025	–1.690	–0.080	2.065
2W	12.785	13.730	–1.240	–1.105	0.095	0.460	–2.215	–1.950	0.110	1.750
3W	12.640	13.435	–1.515	–1.400	0.195	0.505	–2.770	–2.540	0.445	1.820
1M	12.485	13.085	–1.665	–1.575	0.210	0.445	–3.010	–2.830	0.560	1.600
2M	12.670	13.195	–1.945	–1.865	0.315	0.520	–3.580	–3.415	0.970	1.875
3M	12.855	13.335	–2.190	–2.110	0.400	0.590	–4.080	–3.925	1.280	2.110
6M	13.030	13.450	–2.370	–2.300	0.500	0.665	–4.475	–4.330	1.610	2.325
1Y	13.330	13.745	–2.585	–2.505	0.590	0.750	–4.850	–4.695	1.945	2.650
18M	13.070	13.495	–2.305	–2.225	0.545	0.715	–4.405	–4.240	1.830	2.560
2Y	12.920	13.370	–2.175	–2.085	0.495	0.670	–4.130	–3.950	1.685	2.465
3Y	12.490	12.970	–1.875	–1.775	0.355	0.545	–3.575	–3.370	1.285	2.120
5Y	11.920	12.395	–1.585	–1.475	0.245	0.440	–2.960	–2.735	0.745	1.605
7Y	10.994	11.579	–1.444	–1.296	0.159	0.402	–2.797	–2.497	0.438	1.505
10Y	10.215	10.965	–1.345	–1.145	0.060	0.375	–2.690	–2.285	0.115	1.490

ATM DNS | Spot Δ excl Prem | RR=EUR Call-Put | BF=(C+P)/2-ATMD
Data source: Bloomberg Finance L.P.

These two options, plus the at-the-money option, are the building blocks of the replication portfolio (though a position in the underlying currency is needed to flatten the delta of the replication portfolio). The strikes for the three options can be obtained by

$$
\begin{aligned}
K_{ATM} \equiv K_2 &= Se^{\left(R_d - R_f + \frac{1}{2}\sigma_{ATM}^2\right)\tau} \\
K_{25\delta c} \equiv K_1 &= Se^{\alpha\sigma_{25\delta C}\sqrt{\tau} + \left(R_d - R_f + \frac{1}{2}\sigma_{ATM}^2\right)\tau} \\
K_{25\delta P} \equiv K_3 &= Se^{-\alpha\sigma_{25\delta P}\sqrt{\tau} + \left(R_d - R_f + \frac{1}{2}\sigma_{ATM}^2\right)\tau} \\
\alpha &= N'\left(\frac{1}{4}e^{R_f\tau}\right)
\end{aligned}
$$

where $K_1 < K_2 < K_3$. Now define $V(t;K)$ to be the vega $\left(\frac{\partial C}{\partial \sigma}\right)$ for an arbitrary option with expiration at time t and strike K. The objective is to build a portfolio of 25-delta calls, 25-delta puts, and ATM options that match the relevant first- and second-order partials (with respect to the spot exchange rate and volatility). It can be shown[4] that unique weights for the three respective options are given by

$$x_1 = \frac{V(t;K)}{V(t;K_1)} \frac{ln\frac{K_2}{K} ln\frac{K_3}{K}}{ln\frac{K_2}{K_1} ln\frac{K_3}{K_1}}$$

$$x_2 = \frac{V(t;K)}{V(t;K_2)} \frac{ln\frac{K}{K_1} ln\frac{K_3}{K}}{ln\frac{K_2}{K_1} ln\frac{K_3}{K_2}}$$

$$x_3 = \frac{V(t;K)}{V(t;K_3)} \frac{ln\frac{K}{K_1} ln\frac{K}{K_2}}{ln\frac{K_3}{K_1} ln\frac{K_3}{K_2}}$$

It should be noted that if the strike of the candidate option is equal to one of the Ks, say K_i, then $x_i = 1$ (and the other x's $=0$).

The Vanna-Volga price of an option (call) with strike K at a given expiration can then be produced by

$$C(K) = C^{BSM}(K) + \sum_{i=1}^{3} x_i(K)[C^{MKT}(K_i) - C^{BSM}(K_i)]$$

where the C^{BSM} stand for the unadjusted Black-Scholes-Merton values using the at-the-money volatility. The above formula can be thought of consisting of two parts, the BSM component, meaning the first term, and the weighted sum of the difference between the market cost of the replication portfolio and its BSM values. Implied volatilities can be extracted from these adjusted option values to complete the volatility surface. Castagna and Mercurio (2005) report two rules for approximating the Vanna-Volga volatility surface (Appendix 5.1).

The end product of this procedure is a volatility surface that can be put to good use. Options prices can be estimated for single options or for a whole book of options. The same is true for risk measures, like delta and vega. The need to do these sorts of tasks has made Vanna-Volga popular,

[4]Castagna and Mercurio (2009), Lipton and McGhee (2002), Wystup (2003), and Castagna and Mercurio (2005).

though it is noted that actual option volatility can only be known by doing a transaction.

THE STICKY DELTA RULE

The assumption behind any formulation of a volatility surface is that foreign exchange options obey what is known as the sticky delta rule.[5] *Sticky delta* means that quoted volatility and delta have an association: volatility "sticks" to the delta (as opposed to the strike) meaning that an option's volatility resets to a new point on the surface whenever its delta changes materially, such as when there is a big move in the spot exchange rate.

As an empirical matter, the volatility surface is relatively constant. Under normal circumstances, when the spot exchange rate moves, the delta of an option changes—and this causes the option to move to a new point on the surface where the option will acquire a new quoted volatility. Also, the passage of time causes the option to slide down the surface to a new volatility. This is not to say that the volatility surface cannot or does not change (remember the historical episodes I recounted earlier in this chapter). Good traders develop instincts as to how the surface would be likely to change when events and circumstances alter the supply and demand for options. Still, significant changes in the surface are more of an exception than an everyday occurrence.

RISK-NEUTRAL DENSITIES

The Cox-Ross risk-neutral approach presented in Chapter 3 establishes that the value of an option is equal to the present value of the conditional expectation of the spot exchange rate at expiration minus the strike. The mathematical expectation is conditional on the option being in-the-money at expiration. Observed market prices for currency options therefore imply a set of risk-neutral probability densities for the future spot exchange rate. The question is how to extract these densities. The answer, it turns out, is related to the butterfly spread.

Breeden and Litzenberger (1978)demonstrate that a risk-neutral density, p, is the second derivative of the option price with respect to strike (times

[5]See Derman (1999); Biseti, Castagna, and Mercurio (1997); and Castagna (2010).

the future value operator):

$$p(S, \tau, K) = e^{R_d \tau} \frac{\partial^2 C}{\partial K^2}$$

This partial derivative must be greater than or equal to zero—it is one of the arbitrage limits I discussed earlier in this chapter. Moreover, this term can be approximated by the butterfly spread centered around a strike K:

$$\frac{\partial^2 C}{\partial K^2} \approx \frac{C(S, K+h) - 2C(S, K) + C(S, K-h)}{h^2}$$

Malz (1996) explores risk-neutral densities of future exchange rates from observed quoted volatilities of currency options at differing terms and strikes. His work contains a museum-quality example of risk-neutral densities at a famous juncture in foreign exchange history. Exhibit 5.19 is Dr. Malz's risk-neutral density function of Sterling-mark during the Sterling Exchange Rate Mechanism crisis in 1992 which he describes as:

> *The distribution is tight and centered at high values of S_T in the spring of 1992. As Sterling weakens and the credibility of the target zone dissolves, the distribution becomes more dispersed and its centre moves lower. For days on which the market price of protection against realignment risk was high, but the estimated variance was low, the distribution is bimodal. (p. 735)*

Malz's comment on the bimodal nature of the implied distribution at the crescendo of the crisis, in September 1992, means that the relative price of deep out-of-the-money options on Sterling/mark became exorbitant as the market came to realize that Sterling's days in the ERM were about to meet an abrupt end.

DEALING IN CURRENCY OPTIONS

I can now return to the discussion about the functioning of the interbank currency options market that began in Chapter 3. Currency option market making is concentrated in a small group of money-center banks that have active foreign exchange desks. The dealer tries to capture an edge, meaning to earn the bid-ask spread, by making a market in options, hopefully without taking significant financial risk.

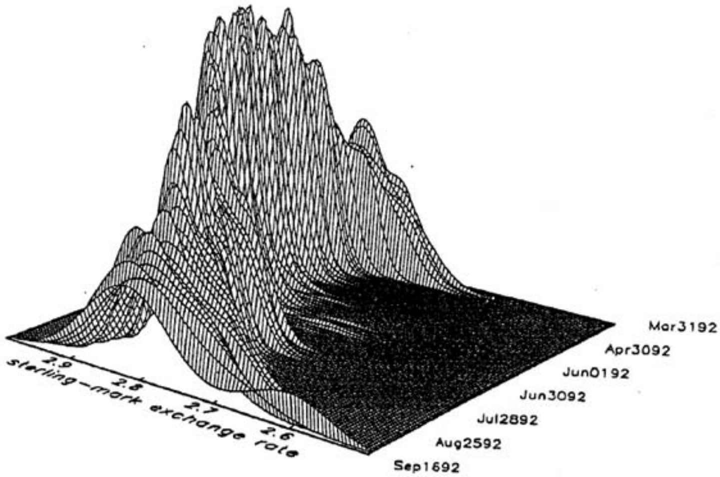


EXHIBIT 5.19 Probability Density Function of the Sterling/Mark Exchange Rate during the September 1992 Crisis
Source: Reprinted from *Journal of International Money and Finance*, Allan Malz, 1996, with permission from Elsevier Science.

Currency option dealers are not really in the business of matching (back-to-back) trades between buyers and sellers. In other words, a dealer who has sold an option to a customer doesn't count on being able to buy the same option for its own account from another dealer or another customer. Rather, dealers are in the business of buying and selling currency options, usually on a hedged basis. They warehouse their option trades as a cumulative option book that is carefully hedged, and subsequently rehedged as exchange rates change over time.

Dealers' books are sorted by currency pairs, for example USD/JPY or EUR/USD, for ease of hedging. Dealers manage directional risk of exchange rate movements in two stages. First, they buy and sell options with the accompaniment of spot (or forward) foreign exchange hedges, as was described in Chapter 3. The purpose is to make option trading into a series of delta-neutral events for the dealer. The second stage is an ongoing process of managing the subsequent directional risk of the option book as a whole by performing rebalancing transactions in spot (and maybe forward) foreign exchange. These rehedging trades are meant to keep the aggregate delta of the option book flat, thereby paring the risks associated with movements in the exchange rate. The aggregate delta, gamma, and theta of a currency option book are the summation of the delta, gamma, and theta of the individual component currency options for a specific currency pair.

The implied economies of scale for the business of dealing options should be noted. A large currency option book is likely to have a variety of puts and calls held as both long and short positions. Hence there is a natural consolidation of the option's book exposure by virtue of off setting positions. The practical implication is that there is less need to do rehedging trades, or at least do rehedging trades of a smaller size, in order to maintain the book in a flat delta state. This reason alone accounts for some of the oligopolistic industrial structure of the currency option business.

Vega is more complicated because it is not additive across option expiration. This is because the term structure of quoted volatility does not always move in parallel shifts. As I wrote earlier in this chapter, short-term volatility is more volatile than long-term volatility.

Some dealers get around this problem by slicing the option book into *time buckets*. Each bucket contains all of the options that fall into a narrow interval of term to expiration, such as in the range between six months and one year. Inside of each bucket, vega can be aggregated with little loss of precision.

Another method applies regression analysis to historical time series of implied volatility across term to expiration to estimate *weighted vega*. Weighted vega is centered around a chosen term to expiration, six months being popular. The purpose of this exercise is to standardize vega across the option portfolio. The vegas of longer-term options are thus deflated in proportion to the observed relative magnitude of the fluctuations in quoted volatility at their respective expirations. Weighted vega is additive across the currency option portfolio for a given currency pair, so it is possible to speak of the weighted vega of the option book taken as a whole.

In a similar manner, option risk associated with interest rates is complicated by nonparallel shifts in interest rate term structures. Fortunately, interest rate risk is not usually material for options with terms less than one year, at least not compared to the risk from movements in spot exchange rates and volatility. Yet interest rate risk, or *rho risk*, can dominate all other pricing factors in currency options that have multiyear lives. For this reason, it is common for dealing banks to assign long-term currency options to their interest rate product risk managers.

TRADING VOLATILITY

Option traders speak of being long or short volatility when they take positions hoping to correctly anticipate future quoted or actual volatility.

In the most basic sense, any position that is long a vanilla put or a call is long volatility. Options are long volatility in the sense that their value will rise if quoted volatility rises. Options are also long actual volatility because larger fluctuations in the spot exchange rate mean a greater chance of expiring in-the-money.

Straddles

The classical volatility trade is to go long volatility by buying a straddle or go short volatility by selling a straddle. A straddle is the combination of same-strike put and call. Parenthetically, a synthetic straddle can be created by adding a spot foreign exchange hedge to a put or a call in the opposite direction and in equal size to the option delta.

Long straddle positions benefit from large movements in the spot exchange rate in either direction. A sufficiently large movement could compensate for the cost of straddle premium. A separate but related issue is that long straddles also rise in value when quoted or implied volatility rises and fall in value when quoted or implied volatility falls.

A short straddle is a short volatility trade. It appreciates when there is little or no movement in the spot exchange rate and when quoted or implied volatility drops. However, a short straddle can produce dangerous losses if exchange rates suddenly exhibit violent fluctuations.

Gamma Scalping

Option traders trade actual volatility against implied volatility in what is known as gamma scalping. The concept is suggested by the Black-Scholes methodology. Suppose that the implied volatility of 1-month euro/dollar options is quoted at 14.00 percent but the trader is convinced that actual volatility in the euro over the next month will be much larger. This view can be expressed by purchasing a 1-month option (either a put or a call or even a straddle) and operating a dynamic hedging strategy using spot foreign exchange.

In theory, with no frictions or transactions costs, the dynamic hedging program would recoup the original option premium (at least to a first approximation) if the actual volatility over the option's life were to match the original 14.00 percent implied volatility. If actual volatility were to materialize at a higher level than 14.00 percent, there would be a profit to the strategy because the gains from the dynamic hedging program would exceed the cost of buying the option. Conversely, if actual volatility were to turn out to be lower than 14.00 percent, the program would produce a net loss.

EXHIBIT 5.20 Gamma Scalping

1. EUR/USD: Volatile Market

	Day 1	Day 2	Day 3	Total
Spot	1.3200	1.3400	1.3200	
Days to Expiry	30	29	28	
Option Value (USD)	$12,382	$19,724	$11,858	
Delta (USD)	–$431,114	–$580,794	–$428,335	
Daily Time Decay		$264	$267	$531
Hedge	$431,114	$580,794	$428,335	
Daily P&L Hedge		–$6,532	$8,669	$2,137
Daily P&L Option		$7,342	–$7,866	–$524
Total P&L		$810	$802	$1,612

2. EUR/USD: Calm Market

	Day 1	Day 2	Day 3	Total
Spot	1.3200	1.3250	1.3200	
Days to Expiry	30	29	28	
Option Value (USD)	$12,382	$13,809	$11,858	
Delta (USD)	–$431,114	–$467,635	–$428,335	
Daily Time Decay		$266	$267	$533
Hedge	$431,114	$467,635	$428,335	
Daily P&L Hedge		–$1,633	$1,765	$132
Daily P&L Option		$1,427	–$1,951	–$524
Total P&L		–$206	–$186	–$393

One-month EUR Call/USD Put $1mm USD face, strike=1.33, vol = 14%, *R* (USD) = 0.25% *R* (EUR) =0.50.

The reason why the hedging program generates cash can be understood from Exhibit 5.20 where a gamma scalping program is operated over three days in volatile and calm market environments. On day one, a $1 million face EUR call/USD put with 30 days to expiration is purchased for $12,382. This option is hedged by going long $431,114 in spot foreign exchange. In the top panel, the volatile market case, spot euro dollar rises overnight from 1.3200 to 1.3400 (by two big figures). At the new spot rate, the option rises by $7,342 but the spot hedge loses $6,532—there is a net profit overnight equal to $810. Now the delta has risen, and the hedge is expanded to $580,794. On day 3, the spot exchange rate falls back to its original level of 1.3200. The overnight loss on the option is equal to $7,866 but the hedge earns $8,669 to tally to a daily net profit of $802.

A very different picture is seen in the second panel where the overnight movement in the spot is more modest, from 1.3200 to 1.3250, or by 50 pips.

This magnitude of movement in the spot exchange rate is not sufficient to recover the overnight decay in the value of the option, and the program loses money over the two-day period.

Yet in both market environments, hedging in and of itself produces gains over the two-day period. More to the point, note that the spot exchange rate in Exhibit 5.20 does a round trip back to its original level. The reason why the hedging earns money is that the size of the hedge is dynamically adjusted to mimic the delta of the option. Delta changes, and by extension so does the size of the hedge, with movements in spot. These changes are predicted by the second derivative of the option theoretical value with respect to the spot exchange rate, the gamma. Hence the program is called gamma scalping.

In Black-Scholes terms, the process of operating the dynamic hedge amounts to synthetically selling the option at the actual volatility. Yet this is an exaggeration because the actual time path of the spot exchange rate is merely described but not uniquely determined by the standard deviation.

What is not shown in the exhibit is a case where implied volatility changes in the course of the program. In the volatile market case, it is not out of the question that implied volatility might rise if traders came to expect that more big moves in the spot exchange rate will follow. If implied volatility were to rise to the target level that the trader had established for actual volatility, then the trader would have an opportunity to terminate the program ahead of schedule. In effect, the rise in the value of the option would capture the profit that had been anticipated from the gamma scalping exercise. On the other hand, if implied volatility were to drop, the option profit and loss account would suffer.

MIXING DIRECTIONAL AND VOLATILITY TRADING

In some market environments, implied volatility is predictable by movements, or even levels, in the spot exchange rate. This can create very powerful trading opportunities where both a directional view and a volatility view can be combined in one strategy.

USD/JPY is a prime example, to continue on themes discussed earlier. Suppose that the level of spot USD/JPY is equal to 110 and that it is known that the Bank of Japan does not want the yen to appreciate below the level of 100 USD/JPY. Suppose that a trader has a three-month target for USD/JPY to fall to 100. Moreover, let's hypothesize that he thinks that the Bank of Japan will put up a fight before the dollar falls to 100. As a general rule, sudden intervention by a central bank causes option implied volatility to rise. Therefore, one way to express both views is to buy USD put/JPY call options with strikes around the 100 level and terms to expiration at three months or more. The trader stands to profit on both the directional move and the rise in implied volatility.

In a larger sense, every option trade that is not fully delta-hedged is a mixed direction and volatility trade. Sometimes, volatility movements can nullify directional gains or even compound directional losses on trades.

APPENDIX 5.1 VANNA-VOLGA APPROXIMATIONS

Castagna and Mercurio (2005) provide two approximations for Vanna-Volga.

Approximation 1

$$\sigma(K) \approx \sigma_1 \equiv \frac{ln\frac{K_2}{K} ln\frac{K_3}{K}}{ln\frac{K_2}{K_1} ln\frac{K_3}{K_1}}\sigma_{25\delta P} + \frac{ln\frac{K}{K_1} ln\frac{K_3}{K}}{ln\frac{K_2}{K_1} ln\frac{K_3}{K_2}}\sigma_{ATM} + \frac{ln\frac{K}{K_1} ln\frac{K}{K_2}}{ln\frac{K_3}{K_1} ln\frac{K_3}{K_2}}\sigma_{25\delta C}$$

σ_1 is a good approximation for the Vanna-Volga implied volatility if the option strike, K, is between K_1 and K_3.

Approximation 2

$$\sigma(K) \approx \sigma_2 \equiv \sigma + \frac{-\sigma + \sqrt{\sigma^2 + d_1(K)d_2(K)(2\sigma D_1(K) + D_2(K))}}{d_1(K)d_2(K)}$$

where

$$D_1 \equiv \sigma_1(K) - \sigma$$

$$\begin{aligned} D_2 \equiv\ & \frac{ln\frac{K_2}{K} ln\frac{K_3}{K}}{ln\frac{K_2}{K_1} ln\frac{K_3}{K_1}} d_1(K_1)d_2(K_1)(\sigma_{25\delta P} - \sigma)^2 \\ & + \frac{ln\frac{K}{K_1} ln\frac{K_3}{K}}{ln\frac{K_2}{K_1} ln\frac{K_3}{K_2}} d_1(K_2)d_2(K_2)(\sigma_{ATM} - \sigma)^2 \\ & + \frac{ln\frac{K}{K_1} ln\frac{K}{K_2}}{ln\frac{K_3}{K_1} ln\frac{K_3}{K_2}} d_1(K_3)d_2(K_3)(\sigma_{25\delta c} - \sigma)^2 \end{aligned}$$

$$d_1(x) = \frac{ln\frac{S}{x} + \left(R_d - R_f + \frac{1}{2}\sigma^2\right)\tau}{\sigma\sqrt{\tau}}$$

$$d_2(x) = d_1(x) - \sigma\sqrt{\tau}$$

$$x \in \{K, K_1, K_2, K_3\}$$

Approximation 2 is more involved than Approximation 1, but Catagna and Mercurio believe it to be the more accurate of the two.

Options on Foreign Exchange, Third Edition
David F. DeRosa

CHAPTER 6

American Exercise Currency Options

American currency options allow exercise at any time before expiration. Some interbank currency options and most listed currency options feature American exercise privilege.

ARBITRAGE CONDITIONS

American exercise privilege results in arbitrage conditions that differ from those implied by European exercise. Denote American currency calls and puts as C' and P', respectively.

Immediate Exercise Value

The minimum value of an in-the-money American option is equal to the value of immediate exercise:

$$C' \geq S_t - K$$
$$P' \geq K - S_t$$

for all times in the life of the option, $0 \leq t \leq T$.

Time Value

The value of an American option is a positive function of its remaining time to expiration, all other things being equal:

$$C'(T - t_0) \geq C'(T - t_1)$$
$$P'(T - t_0) \geq P'(T - t_1)$$

where $(T - t_0)$ and $(T - t_1)$ are times to expiration and $t_1 > t_0$. Since exercise is permitted at all times before expiration, incremental time to expiration has a non-negative effect on the value of American calls and puts. This result is not always true for European options.

American and European Exercise

The minimum value of an American option is the value of an otherwise identical European option. That is

$$C' \geq C$$

$$P' \geq P$$

A complete lower bound for American options is obtained by combining the immediate exercise value with the lower bound for European exercise options (from Chapter 3):

$$C' \geq Max\left[0, S - K, e^{-R_f \tau} S - e^{-R_d \tau} K\right]$$

$$P' \geq Max\left[0, K - S, e^{-R_d \tau} K - e^{-R_f \tau} S\right]$$

PUT-CALL PARITY FOR AMERICAN CURRENCY OPTIONS

The put-call parity theorem for European currency options states that

$$P - C = e^{-R_d \tau} K - e^{-R_f \tau} S$$

For American currency options put-call parity takes the form of an inequality:

$$C' + K - e^{-R_f \tau} S \geq P' \geq C' + e^{-R_d \tau} K - S$$

The left-hand side of the inequality

$$C' + K - e^{-R_f \tau} S \geq P'$$

must hold at expiration because a portfolio combination consisting of

(a) Long Call C'
(b) \$$K$ invested in a zero coupon bond that matures at option expiration with value $e^{+R_d\tau}K$
(c) A short position in a foreign currency zero coupon bond that pays one unit of foreign exchange at expiration that is worth S_T

will be worth more at expiration than the put by an amount equal to

$$e^{+R_d\tau}K - K \geq 0$$

regardless of where the spot exchange rate is relative to the strike. This can be verified with the following analysis of the values at expiration of going long the portfolio and short the put.

Value before Expiration	Value at Expiration	
	$S_T > K$	$S_T < K$
C'	$S_T - K$	0
$+K$	$e^{+R_d\tau}K$	$e^{+R_d\tau}K$
$-e^{-R_f\tau}S$	$-S_T$	$-S_T$
$-P'$	0	$-(K - S_T)$
	$e^{+R_d\tau}K - K$	$e^{+R_d\tau}K - K$

This guarantees that the left-hand side of the inequality must hold if the combination is held to expiration. Next what is needed is to show that it must hold allowing for the possibility of early exercise. If the inequality did not hold, the inverse condition,

$$C' + K - e^{-R_f\tau}S \leq P'$$

would be true, which would mean that we could sell the put for more than we would have to pay to buy the portfolio combination. This condition is impossible, even with early exercise. If the holder of the put did elect to exercise, we would be obligated to pay \$$K$ and receive one unit of foreign exchange worth S. To pay \$$K$, we could liquidate the domestic currency bond. This bond would be worth more than \$$K$, so we would be able to pocket some accrued interest. Also, the deliverable one unit of foreign exchange would give us more than enough to repay the short position in the foreign currency zero coupon bond, so again there would be a positive residual. On top of this, the long position in the call would have a non-negative value. All things counted, the inverse condition cannot be valid because it implies that a costless, riskless, and valuable position would be available in the market.

Likewise, the right-hand side

$$P' \geq C' + e^{-R_d\tau}K - S$$

must also hold at expiration because a portfolio combination consisting of

(a) Long Call C'
(b) A zero-coupon bond that matures at expiration with value \$$K$ (present value $e^{-R_d\tau}K$)
(c) A short position in a foreign currency zero coupon bond with present value equal to one unit foreign exchange

This portfolio will be worth less than the value of the put at expiration by the amount

$$e^{+R_f\tau}S_T - S_T \geq 0$$

A similar argument can be made as to why the right-hand side of the parity inequality must hold, even in the case of early exercise. If it were not valid, the inverse condition

$$P' \leq C' + e^{-R_d\tau}K - S$$

would hold, and it can be shown that this is impossible by virtue of the no-arbitrage condition. If the inverse condition were true, a profitable arbitrage trade could be done by buying the put and selling the more expensive combination, the later consisting of

(a) Short Call C'
(b) Short a zero-coupon bond that matures at expiration with value \$$K$ (present value $e^{-R_d\tau}K$)
(c) A long position in a foreign currency zero coupon bond with present value equal to one unit of foreign exchange

If the call were exercised prior to expiration, we would be obligated to deliver one unit of foreign exchange worth \$$S$ and receive domestic currency in the amount of the option strike, \$$K$. Since the short position in the domestic currency zero coupon bond has a present value less than K, we would pocket some domestic currency. Also, we would be able to liquidate the foreign currency zero coupon bond for more than one unit of foreign exchange and keep some accrued interest there as well. Furthermore, we own the put, which has a non-negative value. Altogether, the inverse condition of the right-hand side of the put-call inequality cannot hold because it implies that a costless, riskless but valuable trade is available in the market.

THE GENERAL THEORY OF AMERICAN CURRENCY OPTION PRICING

The Black-Scholes methodology was specifically developed for European exercise options, yet it does have some insights to offer about American exercise options. Working with the same set of assumptions that underlie the BSM European exercise currency option model, assume that

- There are no taxes, no transactions costs, and no restrictions on taking long or short positions in either options or currency. All transactors in capital and foreign exchange markets are price takers. This means that no single economic agent can buy or sell in sufficient size so as to control market prices.
- The foreign and domestic interest rates are riskless and constant over the term of the option's life. All interest rates are expressed as continuously compounded rates.
- Instantaneous changes in the spot exchange rate are generated by a diffusion process of the form

$$\frac{dS}{S} = \mu dt + \sigma dz$$

where μ is the instantaneous drift and dt is an instant in time; σ is the instantaneous standard deviation; dz is the differential of a stochastic variable that is normally distributed with mean zero and standard deviation equal to the square root of dt.

Option pricing theory states that these assumptions are sufficient to derive the BSM partial differential equations for any currency option, including American exercise currency puts and calls. The equations are:

BSM Partial Differential Equation for American Exercise

$$\frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C'}{\partial S^2} - R_d C' + (R_d - R_f) S \frac{\partial C'}{\partial S} - \frac{\partial C'}{\partial \tau} = 0$$

$$\frac{1}{2}\sigma^2 S^2 \frac{\partial^2 P'}{\partial S^2} - R_d P' + (R_d - R_f) S \frac{\partial P'}{\partial S} - \frac{\partial P'}{\partial \tau} = 0$$

For European options, the payoffs at expiration are governed by

$$C_T = \max[0, S_T - K]$$
$$P_T = \max[0, K - S_T]$$

As described in Chapter 3, Black and Scholes, and later Garman and Kohlhagen, used these expiration payoff functions as boundary conditions in the solution of the partial differential equations for European call and put options.

American options are governed by different boundary conditions because early exercise is permitted. American options are constrained to never be worth less than zero or less than their intrinsic value:

$$C'_t \geq \max[0, S_t - K]$$
$$P'_t \geq \max[0, K - S_t]$$

where t denotes the current time. These conditions must hold at every point in time in an American option's life. No analytic solution to the partial differential equations, using the American boundaries, is known to exist. Consequently, option theoreticians have turned to other classes of models. Some are numerical procedures, like the binomial models of Cox, Ross, and Rubinstein (1979), and others are analytical approximations, such as the compound option model of Geske and Johnson (1984), the Quadratic Approximation model of MacMillan (1986), Barone-Adesi and Whaley (1987), and Ho, Stapleton, and Subrahmanyam (1994). Alternatively, the method of finite differences can be used to approximate solutions to the BSM partial differential equation, as will be discussed later in this chapter.

THE ECONOMICS OF EARLY EXERCISE

American currency puts and calls are commonly exercised before expiration.

A sufficient condition for early exercise is that an American option would sell for less than its intrinsic value:

Sufficient Conditions for Early Exercise

$$C' < S_t - K$$
$$P' < K - S_t$$

Take the case of the American call. The opportunity cost of early exercise is the forgone interim interest that could have been earned by investing the present value of the strike for the days between a candidate exercise date and expiration. This is equal to

$$K\left(1-e^{-R_d\tau}\right)$$

On the other hand, the opportunity cost of not electing early exercise is the interim interest that could be earned by investing the present value of the deliverable one unit of foreign exchange for the days between a candidate exercise date and expiration. This is equal to

$$S\left(1-e^{-R_f\tau}\right)$$

The net difference of these two expressions can be thought of as the interest opportunity cost of the option.

Gibson (1991) notes the following necessary, but not sufficient, condition for early exercise of an American call:

Necessary Condition for Early Exercise

$$S\left(1-e^{-R_f\tau}\right) > K\left(1-e^{-R_d\tau}\right)$$

Early exercise depends not only on the option's in-the-moneyness but also on the spread between the domestic and foreign interest rates.

This analysis shows why early exercise of an American currency call is likely to be optimal for options deep in-the-money and where the foreign currency interest rate is high relative to the domestic interest rate. The condition is necessary, but not sufficient, because even if the interest opportunity cost were large, the option might still be worth keeping alive at sufficiently high levels of volatility. In other words, the interest opportunity cost might be more than offset by the chance that the option will be carried even deeper in-the-money by a big move in the exchange rate. This statement takes on added importance if spot exchange rates contain a jump process component.

For an American put, the necessary condition for optimal early exercise is given by the inequality:

Necessary Condition for Early Exercise: American Put

$$K\left(1-e^{-R_d\tau}\right) > S\left(1-e^{-R_f\tau}\right)$$

For American currency puts, optimal early exercise is likely to occur when the option is deep in-the-money and when the foreign currency interest is below the domestic interest rate. Optimal exercise of an American put requires that the interim interest that can be earned on the strike between a candidate exercise date and expiration must exceed the interim interest on the deliverable quantity of foreign exchange. This is a necessary but not sufficient condition because at sufficiently high levels of volatility, the chance that the put might get carried even deeper in-the-money might more than compensate for the interest opportunity.

The exact conditions under which it is optimal to choose early exercise depend on the in-the-moneyness of the option, the foreign and domestic interest rates, and volatility.

Currency option traders make early exercise decisions in terms of the following paradigm. Consider whether to exercise an in-the-money put. As was described in Chapter 3, the value of an in-the-money put can be decomposed into three parts: parity to the forward, a present value factor, and the volatility value of the option. The volatility value of the in-the-money put is equal to its pure *optionality*, which, according to the put-call parity theorem, is equal to the value of the same-strike out-of-the money call (which we will designate as $\overline{C}$).

Exercising the in-the-money put causes three things to happen:

1. The option holder receives the benefit of the interest differential between the two currencies from the time of exercise to the expiration date of the option (time interval τ). This is worth

$$K\left(1 - e^{-R_d\tau}\right) - S\left(1 - e^{-R_f\tau}\right)$$

2. The option holder is released from paying the cost of carry associated with the premium of the option,[1] which can be written

$$R_d\tau P'$$

3. The exercise kills the volatility value of the put, which is captured in the value of the same-strike call, $\overline{C}$.

If the sum of (a) and (b) is greater than (c), it is optimal to exercise the option at some point in time before expiration. In other words, if the value of early exercise is positive, exercise is indicated:

Value of Early Exercise: American Put

$$\left(K\left(1 - e^{-R_d\tau}\right) - S\left(1 - e^{-R_f\tau}\right)\right) + R_d\tau P' - \bar{C}$$

[1]I am using simple interest for simplicity.

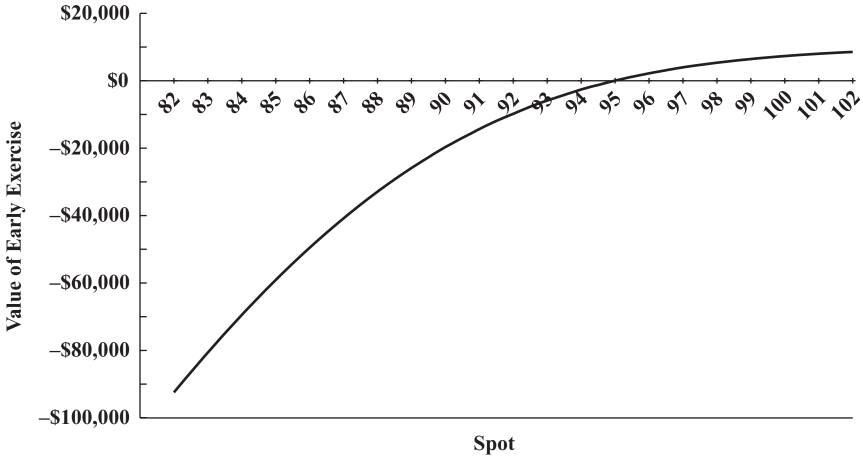


EXHIBIT 6.1 Early Exercise of American USD Call/JPY Put—Term Decision

The value of early exercise for a dollar call/yen put plotted against the spot exchange rate is displayed in Exhibit 6.1. The option parameters behind this exhibit are:

Option: USD Call/JPY Put
Face (USD): $1,000,000
Face (JPY): 89,336,700
Exercise: American
Strike: 89.3367
Days to Expiry: 90
USD Interest Rate: 5.00%
JPY Interest Rate: 2.00%
Volatility: 14.00%

Exhibit 6.1 indicates that early exercise at some time in the life of the option is optimal at spot exchange rates above 95.00—this is known as the term decision.

A second condition controls whether early exercise is immediately optimal. This takes into consideration the overnight level of market interest rates and overnight volatility value of option. At an assumed spot level of 95.00, the value of a 1-day dollar call/yen put represents the overnight volatility value of a 1-day in-the-money dollar put/yen call. To see if early exercise is immediately optimal, compare the overnight volatility value to the overnight

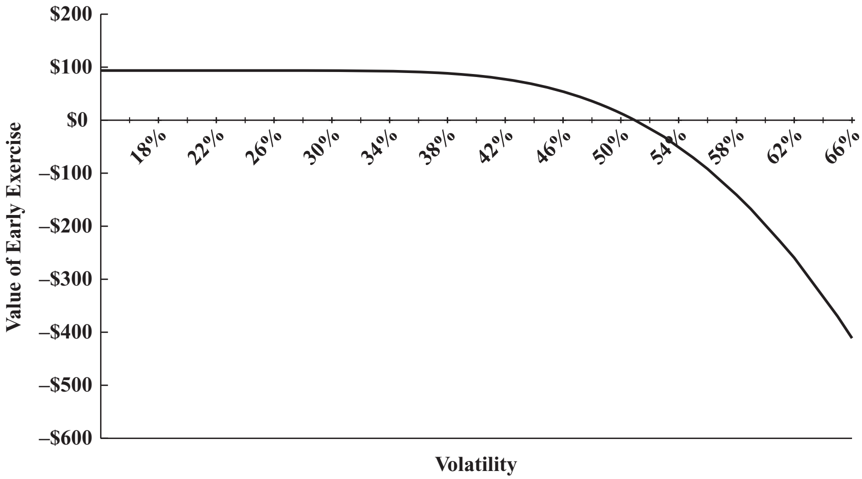


EXHIBIT 6.2 Early Exercise of American USD Call/JPY Put—Overnight Decision

interest advantage and the overnight carry on the option. If the condition for exercise is not met, the option would be better exercised at a later time.

Exhibit 6.2 demonstrates that the overnight exercise rule is a function of short-term market option volatility. At a high enough level of overnight volatility, over 51 percent, the dollar call/yen put is best left unexercised for one day—even though the term decision rule has established that early exercise is optimal at some point in time. This is particularly relevant around significant market event times when overnight volatility has been known to spike upward.

Under normal circumstances, early exercise that is indicated by the term decision will be confirmed by the overnight rule. But in some cases, where there is a potential market-rocking event, such as a central bank meeting, an election, or a G-8 meeting, early exercise could optimally be delayed. Taleb (1997) provides advanced discussion of the difficulties of implementing mechanical early exercise decision rules.

THE BINOMIAL MODEL

Cox, Ross, and Rubinstein (1979) proposes the binomial option model as an alternative to the Black-Scholes model. The binomial model can value both European and American options on common stocks that pay dividends, and it explicitly recognizes the possibility of early exercise. Bodurtha and Courtadon (1987) modify the binomial model to work on currency options.

The Binomial Paradigm

In the binomial model, the spot exchange rate at a point in time is constrained to jump in one of two possible mutually exclusive paths, one being upward and the other downward. During the remaining time to expiration, τ, the spot exchange rate must make a fixed number, N, of such jumps, which the user must specify. Practically speaking, the choice of N is a compromise between precision and speed of calculation. The size of each jump is a function of the domestic and foreign interest rates, the assumed volatility, and the number of jumps in the remaining time to expiration. The size of an up jump u and a down jump d are given by

$$u = e^{\left(R_d - R_f\right)\frac{\tau}{N} + \sigma\sqrt{\frac{\tau}{N}}}$$
$$d = e^{\left(R_d - R_f\right)\frac{\tau}{N} - \sigma\sqrt{\frac{\tau}{N}}}$$

Let the superscript that precedes a variable denote the sequence number of the jump. After the first jump, the initial spot rate S_0 will either be up, designated ${}^1S_u$, or down, designated ${}^1S_d$:

$$\begin{tikzcd} & {}^1S_u = uS_0 \\ S_0 \arrow[ur] \arrow[dr] & \\ & {}^1S_d = dS_0 \end{tikzcd}$$

For purposes of example, we will work with the paradigm of a USD put/JPY call:

Option: USD put/JPY call

Underlying Asset: 1 yen

Strike: .0111936 (= 1/89.3367)

Spot: .0111111(= 1/90.00)

Days to Expiry: 90

Interest Rate (USD): 5.00%

Interest Rate (JPY): 2.00%

Volatility: 14.00%

To begin with the simplest case, let the spot exchange rate be constrained to make only one jump between time 0 and day 90. This means that N equals

1 and that the magnitude of the up and down jumps, u and d, would be equal to

$$u = e^{(5.00\%-2.00\%)\frac{90}{365}+14\%\sqrt{\frac{90}{365}}} = 1.0799515$$

$$d = e^{(5.00\%-2.00\%)\frac{90}{365}-14\%\sqrt{\frac{90}{365}}} = 0.9397686$$

Accordingly, the spot exchange rate on day 90, S_T, would be either

$$S_T = {}^1S_u = uS_0 = 1.0799515 \times 0.0111111 = .0119995$$

or

$$S_T = {}^1S_d = dS_0 = 0.9397686 \times .0111111 = .0104419$$

Furthermore, on day 90, the value of the call would have only two possible expiration values, depending on whether spot had moved up or down:

Up

$${}^1C_u = \text{Max}\left[0, {}^1S_u - K\right] = 0.00080585$$

or

Down

$${}^1C_d = \text{Max}\left[0, {}^1S_d - K\right] = 0$$

where ${}^1C_u$ and ${}^1C_d$ are the values of the call that are conditional upon respective up and down movements in the spot exchange rate.

From these meager bits of information, the value of the call option on day 0 can be deduced without knowing in advance the direction of the jump of the spot exchange rate. This is because a portfolio with known market value at time 0 can be constructed to exactly replicate the value of the call at expiration. This portfolio consists of a combination of (1) a borrowed quantity, B, of dollars with repayment promised on day 90, and (2) a purchased quantity, D, of spot yen. Assuming that arbitrageurs can borrow and lend at the rates R_d and R_f, the future value of the borrowed dollars at expiration is equal to

$$Be^{R_d\tau}$$

The dollar value of the spot yen at time zero is S_0D. Interest on the yen will accrue at the rate of R_f. The future dollar value of the yen will either be equal to

$$^1S_u De^{R_f\tau} = uS_0 De^{R_f\tau}$$

or

$$^1S_d De^{R_f\tau} = dS_0 De^{R_f\tau}$$

where the only unknown is the future spot exchange rate.

The values of B and D that replicate the call are given by the following formulas

$$B = \frac{u\,^1C_d - d\,^1C_u}{(u-d)\,e^{R_d\tau}} = -.005336143$$

$$D = \frac{^1C_u - {}^1C_d}{(u-d)\,S_0 e^{R_f\tau}} = .514827350$$

The expiration-day value of this portfolio does in fact exactly match that of the call:

Call (^1C_u and ^1C_d)		**Portfolio**	
Up	.00080585	Dollar Loan $Be^{R_d\tau}$	(0.005402338)
		Value of Yen $S_u De^{R_f\tau}$	0.006208191
		Total	0.00080585
Down	0	Dollar Loan $Be^{R_d\tau}$	(0.005402338)
		Value of Yen $S_d De^{R_f\tau}$	0.005402338
		Total	0.0

This means that the value of the call at time zero must equal the known market value of the portfolio, otherwise riskless arbitrage would be profitable. The value of the call then must be

$$C_0 = B + S_0 D = 0.000384161$$

Seen another way, from the risk-neutral argument of Cox and Ross (1976), the value of C_0 can be represented as the risk-neutral expected present value of the binomial pair of outcomes

$$C_0 = e^{-R_d\tau}\left[p\,^1C_u + (1-p)\,^1C_d\right]$$

where p is the risk-neutral probability of an upward move and $(1 - p)$ is the risk-neutral probability of a downward move. The former can be obtained by rearranging the terms of $C_0 = B + S_0D$ equation with p defined as

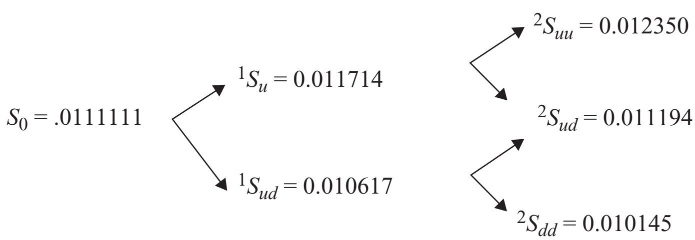
$$p = \frac{e^{\left(R_d - R_f\right)\tau} - d}{(u - d)}$$

The Binomial Tree Structure

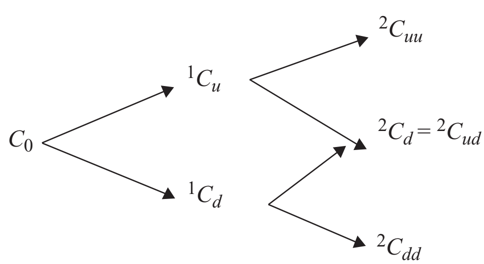
Now we will consider the case of two jumps before expiration ($N = 2$ and $\tau/N = 0.1232877$). The magnitude of the up and down jumps is given by

$$u = e^{(5\%-2\%)\frac{1}{2}\times\frac{90}{365}+14\%\times\sqrt{\frac{1}{2}\frac{90}{365}}} = 1.0542777$$
$$d = e^{(5\%-2\%)\frac{1}{2}\times\frac{90}{365}-14\%\times\sqrt{\frac{1}{2}\frac{90}{365}}} = 0.9555591$$

The tree structure for the spot exchange rate becomes



And the payoff pattern for the call in abstract can be depicted as



At expiration, there are exactly $(N + 1)$ possible terminal values for the call option. Each of them is either in-the-money, and worth $[S_T - K]$, or at- or out-of-the-money and worthless. In the $N = 2$ case there are three expiration values that are

$$
\begin{aligned}
{}^2C_{uu} &= .00115641 \\
{}^2C_{du} &= {}^2C_{ud} = 0 \\
{}^2C_{dd} &= 0
\end{aligned}
$$

The binomial model starts with expiration values and works backward toward time 0. First, the set of nodes that is one jump before expiration is evaluated. Each of these nodes has a binomial pair of outcomes. For example, at the ${}^1C_u$ node in the $N = 2$ case, the option will advance to either ${}^2C_{uu} = .00115641$ or ${}^2C_{ud} = 0$, which can be found by examining the fragment of the tree structure

$$
\begin{tikzcd}
 & {}^2C_{uu} = .00115641 \\
{}^1C_u \arrow[ur] \arrow[dr] & \\
 & {}^2C_{ud} = 0
\end{tikzcd}
$$

Using the procedure of replicating this payoff pattern by borrowing dollars and buying yen, we can calculate that B is equal to –0.0111248 and D is equal to .9975370. The value of ${}^1C_u$ is the greater of zero and the value of the portfolio consisting of short dollars and long yen or the immediate exercise value,

$$
\begin{aligned}
{}^1C_u &= Max\left[0, B + {}^1S_u D, {}^1S_u - K\right] = Max\,[0, .00056053, .00052059] \\
&= .000560534
\end{aligned}
$$

The same procedure must next be used for ${}^1C_d$, which finds this node's value equal to 0.

Finally, the value of C_0 can be found to be .00027170, utilizing the same replication procedure of borrowing dollars and purchasing yen now that we know the binomial pair of outcomes, ${}^1C_d$, and ${}^1C_u$.

$$
\begin{tikzcd}
 & {}^1C_u = .00056053 \\
C_0 \arrow[ur] \arrow[dr] & \\
 & {}^1C_d = 0
\end{tikzcd}
$$

The General Binomial Model

Two jumps of course are not enough to accurately value an option. A great many more are required for accuracy. Using 50 jumps, the binomial model values the American option at .0003050 (or $27,252 per $10 million of face). But no matter how large N is, the procedure is always the same: First evaluate the expiration values; next, move back one jump to the (N – 1)th node; continue until the entire binomial tree has been calculated to produce C_0.

In the generic case, say at node n, the call value at the node can be found as

$$Max\,[0, {}^{n}B + {}^{n}S^{n}D, {}^{n}S - K]$$

where the final term in the brackets, (${}^{n}S - K$), represents the sufficient condition for early exercise. At each generic node n, the values of ${}^{n}B$ and ${}^{n}D$ are given by

$$^{n}B = \frac{u\,^{n+1}C_d - d\,^{n+1}C_u}{(u-d)\,e^{R_d \frac{\tau}{N}}}$$

$$^{n}D = \frac{^{n+1}C_u - {}^{n+1}C_d}{(u-d)\,^{n}Se^{R_f \frac{\tau}{N}}}$$

American currency puts follow the same approach in which each node is evaluated

$$Max\,[0, {}^{n}B + {}^{n}S\,^{n}D, K - {}^{n}S]$$

where

$$^{n}B = \frac{u\,^{n+1}P_d - d\,^{n+1}P_u}{(u-d)\,e^{R_d \frac{\tau}{N}}}$$

and

$$^{n}D = \frac{^{n+1}P_u - {}^{n+1}P_d}{(u-d)\,^{n}Se^{R_f \frac{\tau}{N}}}$$

and ${}^{n+1}P_u$ and ${}^{n+1}P_d$ are the values of the put-nodes at jump (n + 1) for the up and down jumps, respectively, and all the other variables are defined the same as in the case of the call option. The term $(K - {}^nS)$ is the sufficient condition for early exercise.

THE BINOMIAL MODEL FOR EUROPEAN CURRENCY OPTIONS

As was mentioned earlier, the binomial model also works for European options. The only modification is that early exercise need not be considered. For a European option, however, a shortcut exists with which to derive the option's value directly from the final set of nodes and their associated binomial probabilities of occurrence. This approach invokes the risk neutrality argument of Cox and Ross that was presented in the development of the BSM model. The value of the European call is

$$C_0 = e^{-R_d \tau} \sum C_N p_N$$

where the C_N terms are the potential values at expiration and p_N are their risk-neutral probabilities of occurrence. This equation represents C_0 as the present value of the risk-neutral expected value of the payoff matrix at the final set of nodes. The assumption that spot exchange rates follow a multiplicative binomial process defines the probability of occurrence of each potential value at expiration. In the generalized case, the probability of an up move, denoted as p, is equal to

$$p = \frac{e^{\left(R_d - R_f\right)\frac{\tau}{N}} - d}{(u - d)}$$

Each potential cash flow at maturity has a number of paths. For example, with $N = 4$, the payoff that results from any combination of one up and three down moves could be achieved with four possible paths (namely, *uddd, dudd, ddud,* and *dddu*). If j is defined as the number of up moves then there are exactly

$$\frac{N!}{j!\,(N - j)!}$$

paths to any one expiration value, and the probability of any one path materializing is

$$p^j\,(1 - p)^{N-j}$$

The present value of the risk-neutral expected value of the terminal cash flows is then

$$e^{-R_d\tau}\sum\frac{N!}{j!\,(N-j)!}p^j\,(1-p)^{N-j}\;Max\left[0,u^j d^{N-j}S_0-K\right]$$

This equation completes the European binomial model, but it is usually transformed into its more recognizable form by defining the variable a as the minimum number of up moves that would be required to make the option finish in-the-money so that

$$\begin{aligned}&Max\left[0,u^j d^{N-j}S_0-K\right]=0,\quad \text{for}\quad j<a\\&Max\left[0,u^j d^{N-j}S_0-K\right]=u^j d^{N-j}S_0-K,\quad \text{for}\quad j\geq a\end{aligned}$$

then

$$C_0=e^{-R_f\tau}SZ\left(a;\mathrm{N};p'\right)-e^{-R_d\tau}KZ\left(a;\mathrm{N};p\right)$$

where Z is the cumulative density function of the binomial distribution and

$$p'=ue^{-\left(R_d-R_f\right)\frac{\tau}{\mathrm{N}}}p$$

It is important to note that as N is allowed to grow large, the binomial distribution converges at the limit on the lognormal distribution. Therefore, one can say that with a sufficiently large number, N, of binomial paths, the binomial solution for European options will closely approximate the BSM value for a European option.

AMERICAN CURRENCY OPTIONS BY APPROXIMATION

Several methods of valuing an American currency option by analytic approximation have appeared in the literature. The interest in these methods is that they achieve some degree of computational efficiency over the binomial model.

The finite differences method of Schwartz (1977), Parkinson (1977), Brennan and Schwartz (1977), and Brennan, Courtadon, and Subrahmanyam (1985) partitions the spot price of the underlying assets, here being foreign exchange, and the time to expiration into a fine grid that represents every possible time path.

The Quadratic Approximation Method

A second method originates with the compound option model that dates back to Geske (1979) and Geske and Johnson (1984). Barone-Adesi and Whaley (1987) report that the compound option method is considerably quicker than the finite differences method, but it has the disadvantage that it requires evaluation of the cumulative bivariate and trivariate density functions.

Barone-Adesi and Whaley (1987) propose a quadratic approximation method based on earlier work by MacMillan (1986). Similar to Jorion and Stoughton (1989a, 1989b), Barone-Adesi and Whaley define the early exercise premium on an American option as the difference between the value of an American option and that of a similarly specified European option. For a call option, this premium, denoted e_c, is defined as

$$C' - C$$

where C' and C are the values of the American and European options, respectively. The key insight of the quadratic model is that the exercise premium must also obey the Black-Scholes partial differential equation. In other words, the quadratic approach treats the American exercise feature itself as an option. Hence it must obey the same partial differential equation.

Partial Differential Equation for Early Exercise Premium

$$\frac{1}{2}\sigma^2 S^2 \frac{\partial^2 e_c}{\partial S^2} - R_d e_c + \left(R_d - R_f\right) S \frac{\partial e_c}{\partial S} - \frac{\partial e_c}{\partial \tau} = 0$$

If the early exercise privilege has value, then there must exist a critical spot exchange rate, S^*, at which early exercise becomes optimal. S^* is a function of time to expiration as well as volatility and the domestic and foreign interest rates, as noted earlier. The value of an American call for values $S > S^*$ would be equal to

$$C' = [S - K], \quad S \geq S^*$$

For values of $S < S^*$, the quadratic approximation of the American call is

$$C' = C + A_2 \left(\frac{S}{S^*}\right)^{q_2}, \quad S < S^*$$

where

$$
\begin{aligned}
A_2 &= \frac{S^*}{q_2}\left[1 - e^{-R_f\tau} N(d_1(S^*))\right] \\
q_2 &= \frac{1}{2}\left[-(N-1) + \sqrt{(N-1)^2 + 4\left(\frac{M}{B}\right)}\right] \\
N &= 2\frac{\left(R_d - R_f\right)}{\sigma^2} \\
M &= 2\frac{R_d}{\sigma^2} \\
B &= 1 - e^{-R_d\tau} \\
d_1(s^*) &= \frac{\ln\frac{S^*}{K} + \left(R_d - R_f + \frac{1}{2}\sigma^2\right)\tau}{\sigma\sqrt{\tau}}
\end{aligned}
$$

where *N(∗)* is the cumulative normal density function.

All of the preceding variables are directly knowable except S^*, the critical spot rate that triggers early exercise. Barone-Adesi and Whaley provide an algorithm that iteratively converges on S^* within a tolerable level for error. Fortunately, once S^* for one strike has been estimated, it is not necessary to repeat the iterative procedure for similar options that differ only in terms of strike. This is because there is a linear relationship between critical S^*s and strikes within a class of options:

$$S_2^* = \frac{S^*}{K}K_2$$

where S_2^* and K_2 are the critical spot rate and strike, respectively, for another option with otherwise similar specifications.

For American put options, the quadratic approximation is

$$
\begin{aligned}
P' &= P + A_1\left(\frac{S}{S^{**}}\right)^{q_1} \quad \text{for} \quad S > S^{**} \\
P' &= K - S \quad \text{for} \quad S \leq S^{**}
\end{aligned}
$$

where it is optimal to exercise the option at or below spot rate S^{**} and where

$$
\begin{aligned}
A_1 &= -\frac{S^{**}}{q_1}\left[1 - e^{-R_f\tau} N(-d_1(S^{**}))\right] \\
q_1 &= \frac{1}{2}\left[-(N-1) - \sqrt{(N-1)^2 + 4\frac{M}{B}}\right]
\end{aligned}
$$

and N, M, B, d_1, and $N(*)$ are defined the same as in the procedure for the American call. Also

$$S_2^{**} = \frac{S^{**}}{K} K_2$$

for a second put with everything the same except for strike.

The Double Exercise Method

A third approach for approximating the value of an American currency option was developed by Ho, Stapleton, and Subrahmanyam (HSS) (1994). HSS estimate the value of an American option by combining the values of (a) a European exercise option and (b) a twice-exercisable option. The latter is a compound option that can be exercised either at the midpoint of its life or at normal expiration time.

The valuation of options that can be exercised only on a number of given dates goes back to work by Geske and Johnson (1984). In the Geske and Johnson framework, a European option has one given date when exercise is permitted. An American option has an infinite number of exercise dates because it is continuously exercisable. HSS produce an exponential relationship between the value of an American option and the number of exercise points allowed up to option expiration. This relationship allows them to estimate the value of an American option as

$$\ln \hat{C}' = \ln C_2' + \left(\ln C_2' - \ln C\right)$$

where $\hat{C}'$ is the estimated value of the American option, C_2' is the value of the twice-exercisable option, and C is the value of the European option. The same formula works for put options. The value of the twice exercisable option comes from Geske and Johnson (1994) and works as follows:

$$C_2' = \lambda\,(K w_2 - S w_1)$$

where

$$w_1 = e^{-R_f \tau_1} N_1\left(-\lambda d_1'\right) + e^{-R_f \tau_2} N_2\left(\lambda d_1', -\lambda d_1''; -\rho\right)$$
$$w_2 = e^{-R_d \tau_1} N_1\left(-\lambda d_2'\right) + e^{-R_d \tau_2} N_2\left(\lambda d_2', -\lambda d_2''; -\rho\right)$$

where $\lambda = 1$ for a put option and $\lambda = -1$ for a call option, τ_1 is the time remaining until the first allowed exercise and τ_2 is the time until expiration,

N_1 is the cumulative univariate normal distribution, and N_2 is the cumulative bivariate normal distribution. The remaining parameters are defined as

$$d_1' = \frac{\ln\left(\frac{S}{S_1^*}\right) + \left(R_d - R_f + \frac{1}{2}\sigma^2\right)\tau_1}{\sigma\sqrt{\tau_1}}$$

$$d_1'' = \frac{\ln\left(\frac{S}{K}\right) + \left(R_d - R_f + \frac{1}{2}\sigma^2\right)\tau_2}{\sigma\sqrt{\tau_2}}$$

$$d_2' = d_1' - \sigma\sqrt{\tau_1}$$

$$d_2'' = d_1'' - \sigma\sqrt{\tau_2}$$

$$\rho = \sqrt{\frac{\tau_1}{\tau_2}}$$

HSS provide an efficient iterative method for estimating the critical price S_1^*.

The Method of First Passage

Yet another approach to American exercise valuation, proposed by Bunch and Johnson (1999), is based on the first passage of the critical price of the underlying where early exercise is optimal. Bunch and Johnson write about an American exercise put on shares of common stock, but the approach is readily generalized to American currency options.

Bunch and Johnson write the American put price as an integral involving the first-passage probability of the underlying stock price hitting the critical stock price level. The assumption is that the American put option will be exercised when the critical stock price level is hit. This makes the put price equal to the expectation of the discounted value of the exercise price less the critical stock price, or

$$P' = \underset{S_c}{Max} = \int_0^T e^{-R_d t}\,(K - S_c)\,f\;dt, \quad (S > S_c)$$

where S_c is the critical stock price. The first term on the right-hand side is the discount factor, the second is the payoff when exercise occurs, and the third term, f, is the first-passage probability, meaning the probability that the stock price will decline from S to S_c for the first time at time t.

Bunch and Johnson provide an exact calculation of S_c using a formula from Kim (1990) and by noting that when $S = S_c$, the put has no time decay:

$$\frac{\partial P'}{\partial \tau} = 0$$

This is because when the stock price declines to the critical price, it doesn't matter how much time is left to expiration. Alternatively, Bunch and Johnson describe this condition as where "the interest rate effect is exactly offset by the volatility effect." A direct implication of the Bunch and Johnson paper is that exercise of in-the-money puts should increase dramatically as maturity nears.

FINITE DIFFERENCES METHODS

The binomial and trinomial models are examples of numerical methods. These tree models approximate the movement in the underlying asset over time as a first step and next calculate the value of the subject option. Another class of numerical methods that has found application in the derivatives field is the method of finite differences. These techniques work for European exercise and American exercise vanilla options plus a wide variety of exotic options.

The finite differences method is used to approximate the Black-Scholes-Merton partial differential equation (BSM-PDE):

$$\frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} - R_d f + \left(R_d S - R_f S\right)\frac{\partial f}{\partial S} + \frac{\partial f}{\partial \tau} = 0$$

Here the value of the option, either a call or a put, is denoted as f. This equation assumes the usual conditions: constant interest rates, a spot diffusion process, and the existence of a known and constant volatility. A critical assumption is that the dynamic hedging paradigm can achieve a perfect hedge in the dimension of the spot exchange rate. In the case of a European exercise option, the objective is to solve the BSM-PDE for the value of the derivative, f, subject to the boundary constraint at expiration. For European exercise these constraints are given by

$$C_T = Max\,[0, S_T - K]$$
$$P_T = Max\,[K - S_T, 0]$$

for a call and put, respectively. What Black and Scholes (1973) discovered—that being what is universally identified as their option pricing model—is an analytical solution for the BSM-PDE, subject to the boundary constraints at expiration for a European call or put option on shares of a non-dividend-paying stock.

Unfortunately, analytical solutions are not available for all varieties of options, starting with those that feature American exercise. The BSM-PDE still holds, under the usual assumptions, but a nice, neat, closed-form solution is non existent. Here, certain numerical methods provide alternative methods for approximating solutions to the BSM-PDE.[2] One popular numerical method is the finite differences method. Three varieties of finite differences have been shown to be useful in option pricing: the implicit, the explicit, and the Crank-Nicolson finite-differences methods. I will now summarize these methods, though only briefly.[3]

The Implicit Finite Differences Method

This is the most robust of the three methods. The idea starts with the construction of a rectangular grid of points as compared to a binomial tree. See Exhibit 6.3.

The outer edges of the grid are assumed to be known. Next, work backward to estimate intermediate successive values for the option. The key is that a BSM partial differential equation must hold everywhere throughout the grid. The finite differences insight is to replace delta, theta, and gamma in the BSM-PDE with finite differences terms.

I can illustrate with a European put option on a unit of foreign exchange. The underlying currency will be quoted American, as is the case for the euro against the dollar. The option expires at time T. The goal is to price the option at time zero. Divide the life of the option into N equally spaced time intervals across the points on the grid. Moving horizontally, from left to right from one point to the next represents the passage of a unit of time equal to Δt. By definition, $\Delta t = T/N$. Time advances as we move from one point to the next as follows:

$$0, \Delta t, 2\Delta t, \ldots, T$$

The vertical axis of the grid is the spot exchange rate, S. Moving vertically up from one point to the next represents an advance in the spot

[2]See Schwartz (1977), Hull and White (1990), and Curtardon (1982). Of course, other methods, such as Monte Carlo techniques, also could be used.

[3]See Hull (2009) and Wilmott (1998) for more thorough discussions of finite differences methods.

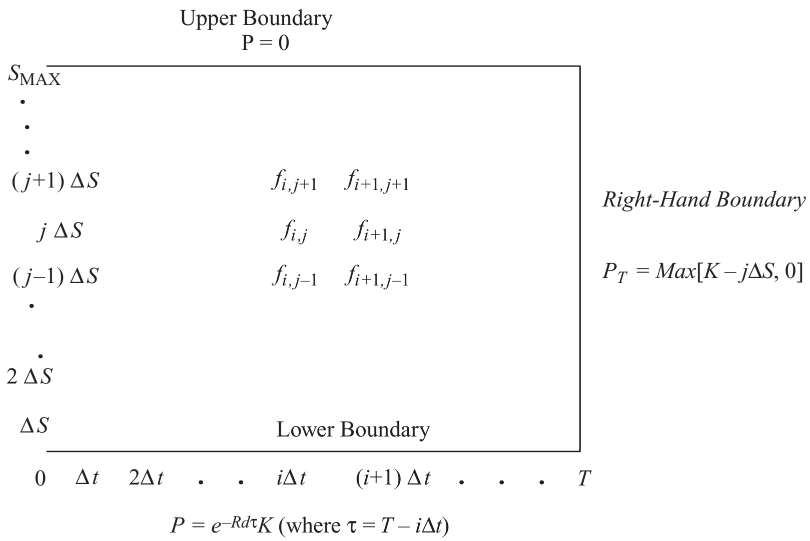


EXHIBIT 6.3 The Implicit Finite Differences Method Grid for a Put Option

exchange rate by one unit equal to an amount ΔS. There are a number M of such increments.

Importantly, a maximum value of S, denoted S_{MAX}, exists such that the value of the put is assumed to be zero when $S = S_{MAX}$.

Consequently $\Delta S = S_{MAX}/M$ and this can denoted as

$$0, \Delta S, 2\Delta S, \ldots, S_{MAX}$$

Consider an arbitrary interior point on the grid with coordinates (i,j)—with i representing the time counter and j the spot exchange rate counter. There will have been i increments of time and j increments of spot movement to reach this point. The corresponding entry into the grid at that point is the value of the derivative, denoted as $f_{i,j}$. The basic idea with the finite differences method is to replace the three partials, meaning delta, theta, and gamma, in the partial differential equation with approximations in discrete differences in f across the entire grid. For the implicit version, these are

$$\frac{\partial f}{\partial S} \approx \frac{f_{i,j+1} - f_{i,j-1}}{2\Delta S}$$

$$\frac{\partial f}{\partial t} \approx \frac{f_{i+1,j} - f_{i,j}}{\Delta t}$$

$$\frac{\partial^2 f}{\partial S^2} \approx \frac{f_{i,j+1} + f_{i,j-1} - 2 f_{i,j}}{\Delta S^2}$$

These finite differences expressions stand in place of the real partial derivatives in a finite differences version of BSM partial differential equation (with some rearrangement in the order of the terms):

$$\frac{f_{i+1,j} - f_{i,j}}{\Delta t} + \left(R_d - R_f\right) j \Delta S \frac{f_{i,j+1} - f_{i,j-1}}{2\Delta S}$$
$$+\frac{1}{2}\sigma^2 j^2 \Delta S^2 \frac{f_{i,j+1} + f_{i,j-1} - 2 f_{i,j}}{\Delta S^2} = R_d f_{i,j}$$

There is one of these equations for every point on the grid except at points corresponding to expiration and where $S = S_{\text{MAX}}$ (for $i = 0, 1, 2, \ldots, N - 1$ and $j = 1, 2, \ldots, M - 1$).

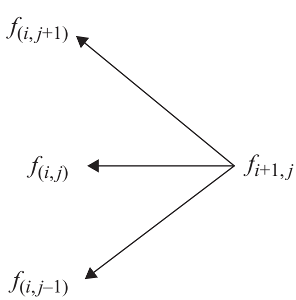
This can be rewritten as

$$a_j f_{i,j-1} + b_j f_{i,j} + c_j f_{i,j+1} = f_{i+1,j}$$

where

$$a_j = \frac{1}{2}\left(R_d - R_f\right) j \Delta t - \frac{1}{2}\sigma^2 j^2 \Delta t$$
$$b_j = 1 + \sigma^2 j^2 \Delta t + R_d \Delta t$$
$$c_j = -\frac{1}{2}\left(R_d - R_f\right) j \Delta t - \frac{1}{2}\sigma^2 j^2 \Delta t$$

which can be visualized as:



Next, three of the borders of the box are determined:

1. The right-hand border: At expiration, meaning after N time advances, the value of the put is worth the boundary constraint:

$$f_{N,j} = MAX\left[K - j\Delta S, 0\right], \quad \text{for} \quad j = 0, 2, \ldots, M$$

2. The bottom border: If the spot exchange rate is zero, the value of the put is the present value of the strike:

$$f_{i,0} = Ke^{-R_d\tau}, \quad \text{for} \quad i = 0, \ldots, N; \quad \tau = \mathrm{T} - i\Delta t$$

3. The top border: By construction, the put is worthless at S_{MAX}:

$$f_{i,M} = 0, \quad \text{for} \quad i = 0, 1, \ldots, N$$

The knowledge of values of the put at the grid boundaries permits the solution of the entire grid, performed one discrete time period at a time, working backward from the option expiration. At each time there is a system of $(M - 1)$ simultaneous equations in $(M - 1)$ unknowns. For example, starting with the time one period from expiration (i.e., $i = N - 1$) there are $(M - 1)$ values

$$f_{N-1,1}, \ldots, f_{N-1,M-1}$$

In this way the entire grid can be solved, working backward one time-step at a time. How good this approximation will be depends on the fineness of width of the time and spot intervals. The smaller the steps, the better is the approximation but at the cost of having to solve a greater number of equations. I should mention that the implicit method is convergent. By this I mean that as one decreases the size of the intervals between the points on the grid (i.e., making the grid finer), the method will indeed converge on what is the true value of the option.

The Explicit Finite Differences Method

The explicit finite differences method assumes that the delta and gamma (but not theta) are constant for a local movement in time from one point to the next with no change in spot. This simplification achieves a great reduction in the calculation efforts needed to use finite differences. However, one trade-off is that the explicit method is not as robust as the implicit method. This means that the explicit finite differences method is not always convergent as the number of points in the grid is increased with the hope of gaining precision. However, this can be managed, and this method does offer considerable economy in calculation.

It works like this. Start with the new assumptions:

$$\frac{\partial f_{i,j}}{\partial S} = \frac{\partial f_{i+1,j}}{\partial S} \approx \frac{f_{i+1,j+1} - f_{i+1,j-1}}{2\Delta S}$$
$$\frac{\partial^2 f_{i,j}}{\partial S^2} = \frac{\partial^2 f_{i+1,j}}{\partial S^2} \approx \frac{f_{i+1,j+1} + f_{i+1,j-1} - 2f_{i+1,j}}{\Delta S^2}$$

which are substituted into the BSM-PDE

$$\frac{f_{i+1,j} - f_{i,j}}{\Delta t} + \left(R_d - R_f\right) j\Delta S \frac{f_{i+1,j+1} - f_{i+1,j-1}}{2\Delta S}$$
$$+\frac{1}{2}\sigma^2 j^2 \Delta S^2 \frac{f_{i+1,j+1} + f_{i+1,j-1} - 2f_{i+1,j}}{\Delta S^2} = R_d f_{i,j}$$

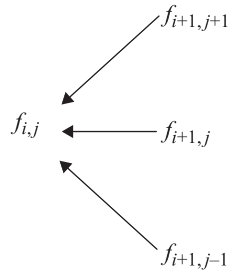
Similar to the preceding,

$$a_j^* f_{i+1,j-1} + b_j^* f_{i+1,j} + c_j^* f_{i+1,j+1} = f_{i,j}$$

where

$$a_j^* = \frac{1}{1+r\Delta t}\left(-\frac{1}{2}\left(R_d - R_f\right) j\Delta t + \frac{1}{2}\sigma^2 j^2 \Delta t\right)$$
$$b_j^* = \frac{1}{1+r\Delta t}\left(1 - \sigma^2 j^2 \Delta t\right)$$
$$c_j^* = \frac{1}{1+r\Delta t}\left(\frac{1}{2}\left(R_d - R_f\right) j\Delta t + \frac{1}{2}\sigma^2 j^2 \Delta t\right)$$

The explicit method formulation makes the value of the option a function of three "future" option values. The paradigm can be seen as



The process of implementing the explicit differences method begins with establishing the same borders that were identified in the case of the implicit finite differences method. But the trick of assuming no local changes in delta and gamma allowed the equations to be rewritten in a way that eliminates the cumbersome requirement of solving a great number of simultaneous equations.[4]

The Crank-Nicolson Method

Crank-Nicolson is a third finite differences method that has found applications to option pricing theory.[5] Crank-Nicolson is an average of the explicit and implicit methods. Adding the equations for the two methods together gets what I call the Crank-Nicolson equation:

$$\begin{aligned} a_j\, f_{i,j-1} &+ \left(b_j - 1\right) f_{i,j} + c_j\, f_{i,j+1} \\ &= -a_j^*\, f_{i+1,j-1} - \left(b_j^* - 1\right) f_{i+1,j} - c_j^*\, f_{i+1,j+1} \end{aligned}$$

Crank-Nicolson is the preferred finite differences method because it achieves faster convergence than either the implicit or explicit methods.[6] All three versions of the finite differences method are useful in that they do in fact solve for the value of an option. Their application is particularly important in working with American exercise options and certain exotic options (considered in Chapter 8). Indeed Wilmott (1998) writes that he uses these methods extensively in his analysis of derivatives.

[4]The explicit finite differences method can be interpreted as a trinomial model; see Hull (2009), Chapter 19.
[5]MATLAB code to implement implicit, explicit, and Crank-Nicolson finite differences is provided in Brandimarte (2006).
[6]See Wilmott (1998) and Brandimarte (2006).

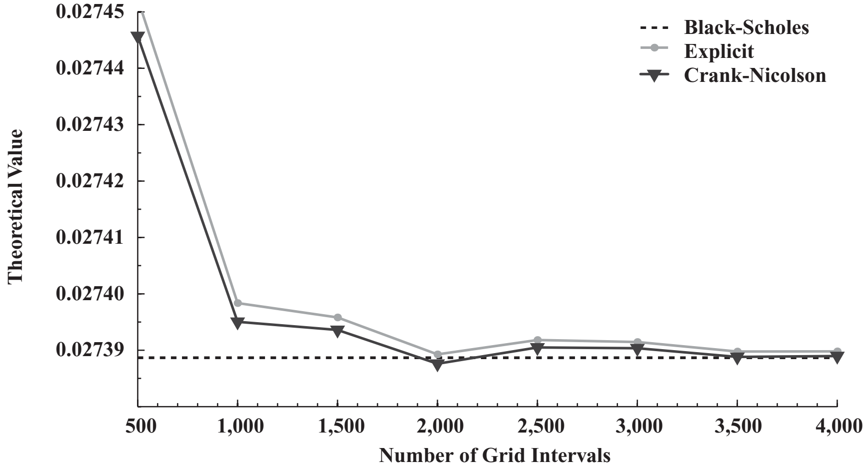


EXHIBIT 6.4 The Crank-Nicolson Method for a European USD Put/JPY Call Strike = 89.3367; spot = 90; 90 days; vol = 14%; $R_d = 5\%$; $R_f = 2\%$.

Finite Differences for Vanilla Options

European exercise options are easily valued by finite differences methods in the manner that I have presented. Exhibit 6.4 shows the convergence of both the explicit method and Crank-Nicolson for a vanilla European exercise USD put/JPY call.

As the number of grid intervals is increased, the values of both finite differences methods converge on the Black-Scholes-Merton values. However, Crank-Nicolson gets to the target faster, at least in this example.

In the case of American exercise, the grid has to be modified to allow for early exercise: The value at every point in the grid must be examined for the possibility of optimal early exercise. If early exercise is indicated, the value must be replaced by the difference between the strike and underlying spot. Simply stated, the value of an American option at all points in the grid cannot be less than the associated immediate exercise value. In the case of a put this means

$$P'(S,t) \geq Max[K - S(t), 0]$$

Exhibit 6.5 demonstrates the Crank-Nicolson for the value of the same option with American exercise.

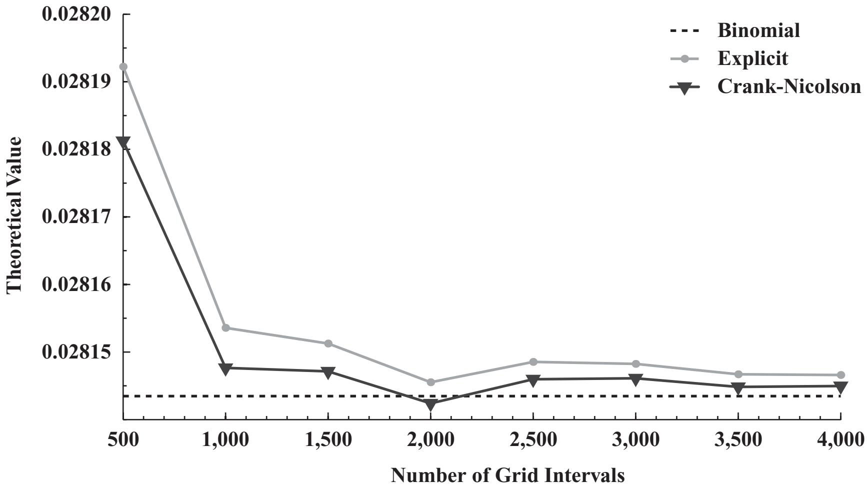


EXHIBIT 6.5 The Crank-Nicolson Method for an American USD Call JPY Put Strike = 89.3367; spot = 90; 90 days; vol = 14%; $R_d = 5\%$; $R_f = 2\%$.

Exhibit 6.5 carries out this exercise, using both the explicit and Crank-Nicolson methods. The benchmark is the binomial model. The results are similar to the European exercise case in the previous exhibit: Both finite differences methods converge, insofar as these examples, but Crank-Nicolson produces greater precision.

CHAPTER 7

Currency Futures Options

A currency futures option exercises into a currency futures contract, whereupon the in-the-money spread between the strike and futures price becomes an immediate credit or debit to accounts having long and short positions.

The practical aspects of trading currency futures options were covered earlier in Chapter 2. In this chapter, I begin with a discussion of the relationship between futures, spot, and forward prices. Next, I cover parity theorems for European and American currency futures options. Afterward, I present models for the valuation of futures options.

CURRENCY FUTURES AND THEIR RELATIONSHIP TO SPOT AND FORWARD EXCHANGE RATES

The Forward Outright

In Chapter 1, I introduced basic nomenclature for foreign exchange transactions. A spot foreign exchange deal is an agreement between two counterparties to promptly exchange sums of currencies. A forward contract is an agreement between two counterparties to exchange sums of currencies on a value day sometime in the future beyond the spot value date.

The exchange rate for a forward deal is called the outright. According to the Interest Rate Parity Theorem, the outright is equal to

$$F = Se^{(R_d - R_f)\tau}$$

where F is the outright, S is the spot exchange rate, R_d is the domestic currency interest, R_f is the foreign currency interest rate, and τ is the time remaining to settlement. All exchange rates are assumed to be American quotation style (i.e., U.S. dollars per one unit of foreign currency).

The difference between the outright and the spot exchange rate is called the forward points:

$$F - S = S\left(e^{(R_d - R_f)\tau} - 1\right)$$

Forward Contracts

In most instances, forward contracts are negotiated at outrights equal to the prevailing forward exchange rates; the initial value of such a forward contract is zero. Thereafter, the value of the contract assumes positive or negative values as a function of exchange rates, domestic and foreign interest rates, and the passage of time. On value day, T, the value of a forward contract to buy one unit of foreign exchange, denoted as V_T, is equal to

$$V_T = S_T - F_0$$

where S_T is the spot rate at value day T and F_0 is the outright that was established when the parties entered into the contract.

To exit a forward contract, one could do a second "closing" transaction that must be for the same value day. A closing transaction could be done at any time before the forward contract's value day. The idea of a closing transaction forms the basis of how to value a forward contract. The closing transaction for a forward contract to buy foreign currency consists of a second forward contract to sell foreign currency. The closing transaction for a forward contract to sell foreign currency consists of a second forward contract to buy foreign currency. The residual cash flow at the forward value date represents the profit or loss on the forward transaction. The net present value of this residual amount is the value of the forward contract at any time before expiration.

Consider an investor who wished to buy 1,000,000 euros forward against the U.S. dollar for value in 90 days. The initial spot exchange rate is equal to \$1.400 per euro, the 90-day interest rate in dollars is 6.00 percent, and the interest rate in euros is 4.00 percent. The 90-day forward outright, F_0 consistent with interest parity is

$$F_0 = 1.4000e^{(.06-.04)\frac{90}{360}} = 1.4070$$

The forward contract would consist of an agreement to receive 1,000,000 euros and deliver \$1,407,000 USD in 90 days' time. Now suppose, the spot exchange rate falls to 1.3900 instantaneously. What would the forward contract now be worth? To unwind the contract and settle in dollars, a hypothetical closing transaction can be considered: Sell 1,000,000 euros for value in 90 days' time. The number of dollars that would be received at settlement with respect to this transaction would depend on the new forward exchange rate, which I will assume to be the new interest parity outright:

$$F_t = 1.3900e^{(.06-.04)\frac{90}{360}} = 1.3970$$

assuming no change in the interest rates. The closing transaction would call for the investor to deliver 1,000,000 euros and receive \$1,397,000 USD. On value day the euros would net to zero but a net sum of \$10,000 would be deliverable to the counterparty. This sum has a present value of \$9,855 based on the 6 percent dollar interest rate. Accordingly, the forward contract would have a negative value equal to \$9,855.

This paradigm leads directly to a model of forward contract valuation. The algebra of valuation is most easily understood when the currency is quoted American and when the closing transaction is settled in the domestic currency.

Suppose on day t, when time $\tau = (T - t)$ remains before the forward value date, I wish to value an existing forward contract to buy one unit of foreign exchange. The initial forward contract and the closing contract have the following cash flows in domestic and foreign currency at value date T:

	Domestic Currency	**Foreign Currency**
Forward Contract	$-F_0$	$+1$
Closing Contract	$\underline{+F_t}$	$\underline{-1}$
Total	$F_t - F_0$	0

where F_0 is the outright when the original forward contract was struck and F_t is the outright for the closing contract. The net present value of $F_t - F_0$ is the value of the forward contract at time t with time τ remaining is:

$$V_t = e^{-R_d\tau}(F_t - F_0) = e^{-R_f\tau}S_t - e^{-R_d\tau}F_0$$

This equation demonstrates that the value of the forward contract to buy foreign currency is positively related to the spot exchange rate. Its sensitivity to changes in the spot rate, called the delta (the lower-case Greek letter delta is δ) of the forward contract, can be found as:

$$\delta_{forward\ contract} = \frac{\partial V_t}{\partial S_t} = e^{-R_f\tau} \leq 1$$

Since ($\tau = T - t$) is positive, δ has to be less than one (I assume R_f is not a negative number). This means that a one unit change in the spot exchange rate will cause the value of a forward contract to change by less than one unit. It is interesting to note that δ does not depend on the domestic interest rate. The process of taking present values washes the domestic interest rate out of the delta.

The greater the interest rate on the foreign currency, the smaller the delta of the forward contract for the unit change in spot:

$$\frac{\partial}{\partial R_f}\frac{\partial V_t}{\partial S_t} = -(T-t)\,e^{-R_f(T-t)} < 0$$

Also, when less time remains before value date, the delta of the forward contract will be larger because

$$-\frac{\partial}{\partial(T-t)}\frac{\partial V_t}{\partial S_t} = R_f e^{-R_f(T-t)} > 0$$

Note that $-\tau$ is, in effect, the shrinkage in the time to settlement as the contract ages.

To complete the model, the sensitivities of the value of the forward contract to movements in interest rates and to the passage of time need to be presented. Forward contracts are inversely related to the foreign interest rate and positively related to the domestic rate:

$$\frac{\partial V_t}{\partial R_f} = -(T-t)\,e^{-R_f(T-t)}S_t < 0$$

$$\frac{\partial V_t}{\partial R_d} = (T-t)\,e^{-R_d(T-t)}F_0 > 0$$

The value of the forward contract can change over time, even if spot rates do not move. The relationship between the value of the forward contract and time is ambiguous in sign:

$$-\frac{\partial V_t}{\partial(T-t)} = R_f e^{-R_f(T-t)}S_t - R_d e^{-R_d(T-t)}F_0$$

Exhibit 7.1 displays a numerical example of a valuation of a forward contract to buy 1 million euros against the dollar for 90-day value.

EXHIBIT 7.1 Analysis of Changes in Value to a Forward Contract (Forward Purchase of 1 Million EUR)

	Initial Levels	Theta	Delta	Rho R*d*	Rho R*f*	Total Change
Currency Pair	EUR/USD	−1 day	−0.01 USD	+1%	+1%	
EUR Notional	€ 1,000,000					
USD Notional	$1,407,018					
Spot	1.4000		1.3900			1.3900
Domestic Interest Rate, R_d	6.00%			7.00%		7.00%
Foreign Interest Rate, R_f	4.00%				5.00%	5.00%
Days to Expiry	90	89				89
Time to Expiration, τ	0.2500	0.2472				0.2472
Outright	1.4070	1.4069	1.3970	1.4105	1.4035	1.3969
Change		**–$77**	**–$9,900**	**$3,461**	**–$3,461**	**–$9,954**

The value of the contract at inception is zero as it executed at the market level of the forward outright.[1] The dynamics are as follows:

- One day (from day 90 to day 89) of time decay (theta) costs \$77.
- A drop in the euro/dollar spot exchange rate in the amount of one U.S. cent (from 1.4000 to 1.3900) removes \$9,900 of market value ("delta").
- An increase in the domestic interest rate (Rho R_d) of 100 basis points adds \$3,461 of market value.
- An increase in the foreign interest rate (Rho R_f) of 100 basis points subtracts \$3,461 of market value.

Currency Futures Contracts

A currency futures contract is different from a forward contract because profits and losses are settled between parties and the exchange's clearing house on a daily basis. That is to say, futures contracts have daily cash settlement of the mark-to-market because of daily futures price movements.

I will use the lowercase letter f to indicate a futures price. It is assumed that on expiration day T, the futures price converges on the spot exchange rate:

$$f_T = S_T$$

Before expiration, the difference between the futures price and the spot exchange rate

$$f_t - S_t$$

is defined as the futures basis.

The mark-to-market process, called variation margin, resets the value of the futures contract to zero each day.

The interest rate at which variation margin can be invested or borrowed is the focus in the theoretical distinction between futures and forward contracts. In the simplest case, assume that the interest rate is known with perfect certainty, meaning that it is non-stochastic. Cox, Ingersoll, and Ross (1981), building on earlier work by Black (1976), demonstrate that the futures price must equal the forward price if this assumption holds. Another

[1]The forward outrights in Exhibit 7.1 are derived from the interest parity equation with no rounding.

way to express this is to say that if the interest rate were known with perfect certainty, the futures price must obey the Interest Parity Theorem. This would make the futures basis equal to the outright forward points.

Cox, Ingersoll, and Ross's proof of this theorem demonstrates that a rolling series of futures contracts can perfectly duplicate a forward contract, at least in the non-stochastic interest rate case. Their argument introduces the concept of a rollover futures hedge. In a rollover futures hedge, the number of futures contracts is adjusted each day as a function of the known interest rate and the remaining time to expiration. The number of contracts is equal to

$$e^{-R_d \tau}$$

At time $t = 0$, the hedge consists of

$$e^{-R_d T}$$

contracts because T is then the remaining time to expiration. The next day, an incremental amount of contracts must be added, making the total number of contracts

$$e^{-R_d (T - \frac{1}{365})}$$

Finally, on expiration day, there would be exactly one whole contract because $t = T$ and

$$e^{-R_d (T-T)} = 1$$

The following explanation of the Cox, Ingersoll, and Ross proof paraphrases Stoll and Whaley (1986) and Whaley (1986). Consider two portfolios, A and B. Portfolio A consists of two parts. The first is a long position in a forward contract negotiated at time $t = 0$ and struck at the prevailing forward outright F_0 (quoted American style). The forward will receive one unit of foreign exchange on day T. The second part is a long position in riskless domestic currency zero coupon bonds that mature on day T. Just enough bonds are purchased so as to have maturity value equal to F_0; their initial present value is equal to

$$F_0 e^{-R_d T}$$

On day T, when the bonds mature and the forward contract settles, Portfolio A will be worth the spot exchange rate S_T. This is because the forward contract will be worth

$$(S_T - F_0)$$

and the bonds will mature to pay an amount equal to F_0.

Portfolio B also consists of two parts. The first is a rollover futures position that also expires on day T. The second part is a long position in riskless domestic currency zero coupon bonds that mature on day T. The initial futures price is denoted as f_0. A sufficient amount of bonds are purchased to make their maturity value equal to f_0. The initial present value of the bonds is equal to

$$f_0 e^{-R_d T}$$

The daily mark-to-market in the rollover program is invested or financed at the domestic interest rate R_d. The value of Portfolio B on expiration day T will be equal to S_T . This is because the profit or loss on the

$$e^{-R_d(T-t)}$$

futures contracts will be marked-to-market each day in an amount equal to

$$e^{-R_d(T-t)}(f_t - f_{t-1})$$

where f_t is the futures price at the end of trading on day t and f_{t-1} is the futures price from the previous day. The future value of this amount on day T will be equal to

$$e^{R_d(T-t)} e^{-R_d(T-t)}(f_t - f_{t-1}) = (f_t - f_{t-1})$$

and the sum of all the future values of the daily mark-to-market will be equal to

	$f_1 - f_0$
plus	$f_2 - f_1$
plus	...
plus	$\underline{S_T - f_{T-1}}$
which will equal	$S_T - f_0$

assuming that the futures price at expiration, f_T, converges on the spot rate, S_T. The value of portfolio B is equal to S_T, which is the sum of the values of the rollover hedge ($S_T - f_0$) and the zero coupon bonds (f_0). Under the no-arbitrage rule, Cox, Ingersoll, and Ross conclude that the futures price, f_0, must equal the forward exchange rate, F_0 because the value of portfolio A is equal to the value of portfolio B.

Cox, Ingersoll, and Ross also speak to the case of stochastic interest rates, based on a discovery by Margrabe (1976). Margrabe's insight is that uncertainty about the interest rate is immaterial to the market—meaning that it would not count as market-priced risk—unless it were correlated with the underlying spot exchange rate.

According to Margrabe, futures could have either a risk premium or a risk discount relative to the forward exchange rate, depending on the existence and sign of any correlation. As an empirical matter, studies by Cornell and Reinganum (1981) and Chang and Chang (1990) found no statistically significant difference between prices of currency futures and forwards. Moreover, Cornell and Reinganum also found no significant level of correlation between exchange rates and interest rates, a fact that is consistent with their other finding that currency futures in their sample have no risk premium or discount, in the Margrabe sense.

ARBITRAGE AND PARITY THEOREMS FOR CURRENCY FUTURES OPTIONS

Several arbitrage and parity theorems for currency options were discussed in Chapter 4. Similar relationships also exist for futures options.

Non-Negative Prices

Futures options have non-negative prices because the holder of the option is never obligated to exercise. This can be expressed as

European Currency Futures Options

$$C^f \geq 0; \quad P^f \geq 0$$

American Currency Futures Options

$$C^{f'} \geq 0; \quad P^{f'} \geq 0$$

where the superscript f denotes a futures option and the prime superscript denotes American exercise.

Properties of American Currency Futures Options

(a) The value of an in-the-money American futures option is worth at least as much as its immediate exercise profit. The no-arbitrage condition implies the following inequalities.

$$C^{f'} \geq f_t - K$$
$$P^{f'} \geq K - f_t$$

(b) The value of an American futures option is a positive function of the time remaining until expiration, all other things remaining equal.

$$C^{f'}(T - t_0) \geq C^{f'}(T - t_1)$$
$$P^{f'}(T - t_0) \geq P^{f'}(T - t_1)$$

where $t_1 > t_0$. An extension of the exercise date must not diminish from the value of an American futures option because immediate exercise is allowed at any time before expiration. This proposition is usually, but not always, true for European futures options.

(c) American exercise options are never worth less than equivalent European exercise options.

$$C^{f'} \geq C^f$$
$$P^{f'} \geq P^f$$

This follows simply from the fact that an American futures option with no optimal early exercise becomes the same thing as a European futures option.

European Currency Futures Options and Options on Actual Foreign Exchange

By definition, a European currency futures option can be exercised only on expiration day.

If the underlying futures contract also expires on option expiration day, and if it can be assumed that the futures price will converge on the spot rate at expiration, then there will be no difference at any point in time between the price of the European futures option and a European option on actual foreign currency:

$$C = C^f$$
$$P = P^f$$

This result does not hold when the futures expiration differs from the futures option's expiration.

Assume that the Interest Parity Theorem strictly holds for futures prices:

$$f = Se^{(R_d - R_f)\tau}$$

then the lower boundaries for call and put options on actual foreign exchange that were presented in Chapter 3:

$$C \geq Se^{-R_f\tau} - Ke^{-R_d\tau}$$
$$P \geq Ke^{-R_d\tau} - Se^{-R_f\tau}$$

hold as lower boundaries for currency futures options as

$$C^f \geq (f - K)\,e^{-R_d\tau}$$
$$P \geq (K - f)\,e^{-R_d\tau}$$

American Currency Futures Options and Options on Actual Foreign Exchange

For American futures options, the comparison to options on actual foreign exchange is less straightforward. For options with the same strike and expiration, the relative valuation depends on whether the foreign currency is at discount ($R_d < R_f$) or at premium ($R_d > R_f$). For a discount currency, the futures price must be less than the spot exchange rate. Hence an American futures call must be less valuable and an American futures put more valuable than corresponding American options on actual foreign exchange:

Discount Currencies

$$C^{f'} \leq C'$$
$$P^{f'} \geq P'$$

For a premium currency, the futures price must be greater than the spot exchange rate, which implies that an American futures call must be more valuable and American futures put must be less valuable than corresponding American options on actual foreign exchange:

Premium Currencies

$$C^{f'} \geq C'$$
$$P^{f'} \leq P'$$

Put-Call Parity for European Currency Futures Options

Stoll and Whaley (1986) provide a put-call parity theorem for generalized futures options that is adaptable to the case of European currency futures options. For a put and call struck at level K and with time τ remaining until expiration, the relationship is

Put-Call Parity for European Currency Futures Options

$$C^f - P^f = e^{-R_d \tau}(f - K)$$

Stoll and Whaley's proof of this relationship constructs a portfolio that consists of four parts:

1. A long currency futures rollover program, which consists of a position in futures equal to $e^{-R_d \tau}$ that is adjusted each day.
2. A long European futures put.
3. A short European futures call.
4. A long position in a riskless zero coupon domestic currency bond that matures on expiration day with value equal to $(f_0 - K)$.

where f_0 is the futures price that prevails in the market at the start of the arbitrage program. Stoll and Whaley demonstrate that the value of this portfolio is zero on expiration day no matter what the level of the futures price.

		Expiration Values	
Position	**Initial Value**	$f_T < K$	$f_T > K$
1. Rollover Futures	0	$f_T - f_0$	$f_T - f_0$
2. Long Put	$-P^f$	$K - f_T$	0
3. Short Call	$+C^f$	0	$-(f_T - K)$
4. Long Bonds	$-(f_0 - K)e^{-R_d \tau}$	$(f_0 - K)$	$(f_0 - K)$
Total	$C^f - P^f - (f_0 - K)e^{-R_d \tau}$	0	0

Under the no-arbitrage condition, the value must also be zero at all times before expiration, which proves the parity theorem.

Put-Call Parity for American Currency Futures Options

Stoll and Whaley's work can also be adapted to provide a put-call parity theorem for American currency futures options. As in the case of American currency options on physical foreign exchange, put-call parity takes the form of an inequality. For an American futures put and futures call struck at level K and with a common expiration date T, the relationship is:

Put-Call Parity for American Currency Futures Options

$$e^{-R_d\tau} f - K \leq C^{f'} - P^{f'} \leq f - e^{-R_d\tau} K$$

The proof involves constructing one arbitrage portfolio for the left-hand side, called the lower boundary, and another arbitrage portfolio for the right-hand side, called the upper boundary. Because these futures puts and futures calls are American exercise, there is the possibility that either arbitrage portfolio might experience assignment of exercise at some point in time before expiration that I will denote as t^*.

The lower boundary arbitrage portfolio (Exhibit 7.2) consists of the following:

1. A long position in an American currency futures call with strike equal to K and expiration at time T. The initial cost equals $C^{f'}$. At some intermediate time, t^*, the option is worth $C^{f'}_{t^*}$. At expiration, the futures price is assumed to converge on the spot exchange rate, and the option is worth

$$C^{f'}_T = MAX[0, S_T - K]$$

2. A short position in an American currency futures put with strike equal to K and expiration at time T. The initial proceeds of the sale equal $P^{f'}$. At some intermediate time, t^*, the option might be exercised, whereupon the portfolio would be debited the difference between the strike and the current futures price

$$-(K - f_{t^*})$$

Technically, the arbitrage portfolio would also receive delivery of one futures contract, which would have no instantaneous value.

3. A short rollover futures program, which takes a position of $e^{-R_d(T-t)}$ contracts with daily adjustment. On expiration day, the portfolio is

EXHIBIT 7.2 Put-Call Parity for American Currency Futures Options

Lower Boundary: $fe^{-R_dT} - K \leq C^{f'} - P^{f'}$				
			Expiration Value	
Position[1]	**Initial Value**	**Intermediate Value**	$S_T < K$	$S_T \geq K$
Long call	$-C^{f'}$	$+C^{f_{t^*}'}$	0	$(S_T - K)$
Short put	$+P^{f'}$	$-(K - f_{t^*})$	$-(K - S_T)$	0
Short rollover futures program[2]	0	$-(f_{t^*} - f_0)\,e^{-R_d(T-t^*)}$	$-(S_T - f_0)$	$-(S_T - f_0)$
Short bonds	$\left(f_0e^{-R_dT} - K\right)$	$-\left(f_0e^{-R_d(T-t^*)} - Ke^{R_dt^*}\right)$	$-\left(f_0 - Ke^{R_dT}\right)$	$-\left(f_0 - Ke^{R_dT}\right)$
Total	$-\left(C^{f'} - P^{f'}\right) + \left(f_0e^{-R_dT} - K\right)$	$C^{f_{t^*}'} + K\left(e^{R_dt^*} - 1\right)$ $+ f_{t^*}\left(1 - e^{-R_d(T-t^*)}\right) > 0$	$K(e^{R_dT} - 1) > 0$	$K(e^{R_dT} - 1) > 0$

[1]The position consists of a long call, short put, a short position in a rollover futures program, and an initial borrowing of $(f_0e^{-R_dT} - K)$ dollars.
[2]The short rollover futures program consists of a short position in $e^{-R_d(T-t^*)}$ contracts with daily rebalancing.

EXHIBIT 7.2 Put-Call Parity for American Currency Futures Options (*continued*)

	Upper Boundary: $C^{f'} - P^{f'} \leq f - Ke^{-R_d T}$			
			Expiration Value	
Position[1]	Initial Value	Intermediate Value	$S_T < K$	$S_T \geq K$
Short call	$+C^{f'}$	$-(f_{t^*} - K)$	0	$-(S_T - K)$
Long put	$-P^{f'}$	$+P^{f_{t^*}'}$	$(K - S_T)$	0
Long rollover futures program[2]	0	$(f_{t^*} - f_0)\, e^{R_d t^*}$	$(S_T - f_0)\, e^{R_d T}$	$(S_T - f_0)\, e^{R_d T}$
Long bonds	$-\left(f_0 - Ke^{-R_d T}\right)$	$\left(f_0 e^{R_d t^*} - Ke^{-R_d (T - t^*)}\right)$	$\left(f_0 e^{R_d T} - K\right)$	$\left(f_0 e^{R_d T} - K\right)$
Total	$\left(C^{f'} - P^{f'}\right) - \left(f_0 - Ke^{-R_d T}\right)$	$P^{f_{t^*}'} + f_{t^*}\left(e^{R_d t^*} - 1\right)$ $+K\left(1 - e^{-R_d (T - t^*)}\right) > 0$	$S_T\left(e^{R_d T} - 1\right) > 0$	$S_T\left(e^{R_d T} - 1\right) > 0$

[1]The position consists of a short call, a long put, a long position in a rollover futures program, and an investment in bonds with initial value $(f_0 - Ke^{-R_d T})$ dollars.
[2]The long rollover futures program consists of a long position in $e^{R_d t^*}$ contracts with daily rebalancing.

short one contract (because $t = T$ on that day). At expiration, this program will have value equal to

$$-(S_T - f_0)$$

4. A short position in bonds (i.e., borrowed funds) with initial value equal to

$$\left(e^{-R_d T} f_0 - K\right)$$

and value at maturity (i.e., repayment) on expiration day equal to

$$-\left(f_0 - e^{+R_d T} K\right)$$

The proof is given by the fact that the value of the arbitrage portfolio is always positive, both at expiration and in the intermediate case. At expiration, the value of the arbitrage portfolio is equal to

$$K\left(e^{R_d T} - 1\right)$$

which is positive as long as the interest rate is greater than zero. In the intermediate case, I assume involuntary exercise of the short put against the arbitrage portfolio; Exhibit 7.2 shows that the intermediate value of the arbitrage portfolio is also positive. Since both the intermediate value and expiration value are positive, no-arbitrage rule requires that the initial cost of the portfolio

$$\left(C^{f'} - P^{f'}\right) - \left(e^{-R_d T} f_0 - K\right)$$

must be positive, which completes the proof.

The proof of the upper boundary condition (Exhibit 7.2) runs along similar lines, except that the rollover futures program is always long $e^{R_d t}$ contracts.

BLACK'S MODEL FOR EUROPEAN CURRENCY FUTURES OPTIONS

This section develops a model for European futures options. In the case where the futures option and its underlying futures contract share a common expiration date, the value of the futures option should be the same as an option on physical foreign exchange. This is because the futures option has no intermediate cash flows associated with owning it beyond the payment of the premium and because the futures price ought to converge on the spot exchange rate at expiration.

In the general case, futures options have shorter lives than their associated deliverable futures contracts. For example, a futures option that expires in October delivers a December futures contract. This is why a special model for European futures options is needed.

Black (1976) adapted the Black-Scholes common stock option pricing model to work on a generic class of European commodity futures options. What follows is a presentation of the Black model as applied to currency futures options.

Assumptions for Black's Model

Black's European futures option model requires three assumptions:

1. There are no taxes, no transactions costs, no restrictions on taking long or short positions in futures options or futures contracts. All transactors are price takers.
2. The domestic interest rate is riskless and constant over the future option's life.
3. Instantaneous changes in the futures price are generated by a diffusion process of the form

$$\frac{df}{f} = \alpha dt + \sigma dz$$

where α is the drift term, dt is an instant in time, σ is the standard deviation of the process, and dz is a stochastic variable that is independent across time and is normally distributed with zero mean and standard deviation equal to square root of dt.

Under these assumptions, it is possible to construct a local hedge for a long position in a currency futures call option using a short position in some number of currency futures contracts. As is common in financial theory, the rate of return on the hedged position is assumed to be equal to the risk-free interest rate, R_d.

Given the assumptions, the price of a futures call must conform to the following partial differential equation:

Black's Partial Differential Equation for Currency Futures Options

$$R_d C^f - \frac{1}{2}\sigma^2 f^2 \frac{\partial^2 C^f}{\partial f^2} - \frac{\partial c^f}{\partial \tau} = 0$$

The boundary conditions for the value of European futures calls and puts at expiration are given as

$$C_T^f = \text{Max}[0, f_T - K]$$
$$P_T^f = \text{Max}[0, K - f_T]$$

That is to say, at expiration, the value of either futures option is the greater of zero and the amount by which the option is in-the-money.

Black produces a model for European futures options by solving the partial differential equation subject to the expiration boundary conditions:

Black's European Currency Futures Option Model

$$C^f = e^{-R_d\tau}\left[fN(h) - KN\left(h - \sigma\sqrt{\tau}\right)\right]$$
$$P^f = e^{-R_d\tau}\left[-f\left(N(-h)\right) + K\left(N\left(-h + \sigma\sqrt{\tau}\right)\right)\right]$$

where

$$h = \frac{\ln\left(\frac{f}{K}\right) + \frac{\sigma^2}{2}\tau}{\sigma\sqrt{\tau}}$$

It is interesting to note that the foreign currency interest rate does not explicitly appear in the Black futures option model, although it does factor into the futures-spot relationship. In a sense, the currency futures option model is blind with respect to the fact that what underlies the futures contract is foreign exchange; it might as well be a stock market index or an agricultural commodity for all that the model is concerned. All that matters is the form of the stochastic process that drives the futures price, that the domestic currency interest rate is riskless and constant, and that it is possible to operate the local hedge without any risk between futures and futures options.

Exhibit 7.3 contains the partial derivatives for Black's model. Exhibit 7.4 shows a numerical example for futures puts and calls on the Japanese yen.

EXHIBIT 7.3 Partial Derivatives for Black's Futures Option Model

Currency futures calls

$$\delta_{call} \equiv \frac{\partial C^f}{\partial f} = e^{-R_d\tau}N(h)$$

$$\gamma_{call} \equiv \frac{\partial^2 C^f}{\partial f^2} = \frac{e^{-R_d\tau}N'(h)}{f\sigma\sqrt{\tau}}$$

$$\theta_{call} \equiv \frac{\partial C^f}{\partial \tau} = R_d f e^{-R_d\tau}N(h) - R_d K e^{-R_d\tau}N\left(h-\sigma\sqrt{\tau}\right) - f e^{-R_d\tau}N'(h)\frac{\sigma}{2\sqrt{\tau}}$$

$$\kappa_{call} \equiv \frac{\partial C^f}{\partial \sigma} = e^{-R_d\tau} f\sqrt{\tau}N'(h)$$

$$\rho_{call} \equiv \frac{\partial C^f}{\partial R_d} = K\tau e^{-R_d\tau}N\left(h-\sigma\sqrt{\tau}\right)$$

Currency futures puts

$$\delta_{put} \equiv \frac{\partial P^f}{\partial f} = -e^{-R_d\tau}N(-h)$$

$$\gamma_{put} \equiv \frac{\partial^2 P^f}{\partial f^2} = \frac{e^{-R_d\tau}N'(h)}{f\sigma\sqrt{\tau}}$$

$$\theta_{put} \equiv \frac{\partial P^f}{\partial \tau} = -R_d f e^{-R_d\tau}N(-h) + R_d K e^{-R_d\tau}N\left(-(h-\sigma\sqrt{\tau})\right) - f e^{-R_d\tau}N'(h)\frac{\sigma}{2\sqrt{\tau}}$$

$$\kappa_{put} \equiv \frac{\partial P^f}{\partial \sigma} = e^{-R_d\tau} f\sqrt{\tau}N'(h)$$

$$\rho_{put} \equiv \frac{\partial P^f}{\partial R_d} = -K\tau e^{-R_d\tau}N\left(-\left(h-\sigma\sqrt{\tau}\right)\right)$$

EXHIBIT 7.4 Numerical Example of Black's European Futures Options Model: Japanese Yen Futures

Parameters		
Futures Price	90.01	
Strike	90	
Days	71	
Volatility	14.00%	
Interest Rate	5.00%	
Output	*Call*	*Put*
Price	2.200	2.190
Delta	0.508	–0.482
Gamma	0.071	0.071
Theta	–0.015	–0.015
Vega	0.157	0.157
Rho	0.085	–0.089

Note: Vega has been scaled down by 1/100 and theta by 1/365.

THE VALUATION OF AMERICAN CURRENCY FUTURES OPTIONS

Comments on Optimal Early Exercise

American futures options might be optimally exercised before expiration if they were sufficiently in-the-money. Whaley (1986) points out that the Black model reveals that the lower bound for a deep in-the-money European futures call is equal to

$$e^{-R_d\tau}(f - K)$$

This is because both of the terms $N(h)$ and $N\left(h - \sigma\sqrt{\tau}\right)$ in the Black model approach unity as the futures price rises above the strike. For the American futures option, however, the intrinsic value

$$(f - K)$$

can be realized at any time. This forms the lower bound for the option. Since the term $e^{-R_d\tau}$ is less than one $(R_d > 0)$, the lower bound for the in-the-money American futures option must be greater than the lower bound of a similar European futures option. This is because

$$(f - K) > e^{-R_d\tau}(f - K)$$

Therefore, early exercise of an American futures option could be theoretically optimal.

At some sufficiently high futures price, denoted as f^*, the value of the American call will be equal to its intrinsic value, whereupon the holder of the option will be indifferent to its early exercise (Exhibit 7.5).

For values of f less than f^*, the value of early exercise, written as epsilon (ε), is equal to the difference between the values of the American and European futures options:

$$\varepsilon = C^{f'} - C^{f} \quad \text{for } f < f^*$$

where $C^{f'}$ is an American futures call and C^{f} is a similarly specified European futures call.

At futures prices above f^* the value of epsilon is given by the difference between the intrinsic value of the American call and the value of the

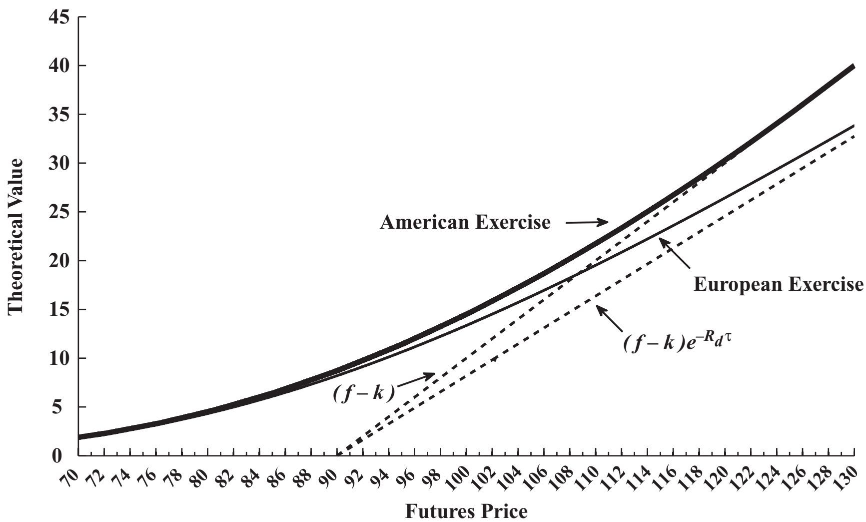


EXHIBIT 7.5 American and European Futures Calls on the Yen

European call:

$$\varepsilon = (f - K) - C^{f'} \quad \text{for } f \geq f^*$$

As f grows very large compared to the strike, the European call approaches its lower bound, equal to the present value of its intrinsic value,

$$(f - K)\, e^{-R_d T}$$

This term represents the interest income that could be earned on investing the proceeds from early exercise.

The Binomial Model for Futures Options

Hull (2009) shows that the binomial option pricing model can be adapted for the case of American futures options with slight modification. As in the case of Black's European futures option model, the binomial approach is blind to the fact that it is foreign currency that underlies the futures contracts. In other words, the binomial model for American currency futures options is the same model that would be appropriate for other types of futures options.

Hull's approach is to treat the American futures option in a similar way to what one would do for an American option on a dividend-paying common stock. The only modification is that the foreign interest rate stands in for the dividend rate. The resultant model is not substantially different from the binomial model described in Chapter 6. The up and down jump terms and the probability term p (the probability of an upwards jump), need to be redefined as

$$u = e^{\sigma\sqrt{\frac{\tau}{N}}}$$
$$d = e^{-\sigma\sqrt{\frac{\tau}{N}}}$$
$$p = \frac{1-d}{u-d}$$

THE QUADRATIC APPROXIMATION MODEL FOR FUTURES OPTIONS

Whaley (1986) adapted the Barone-Adesi and Whaley quadratic approximation method to handle American futures options. The model is

$$C^{f'} = C^{f} + A_2\left(\frac{f}{f^*}\right)^{q_2}, \quad f < f^*$$
$$C^{f'} = f - K, \quad f \geq f^*$$

where

$$A_2 = \left(\frac{f^*}{q_2}\right)\left(1 - e^{-R_d\tau}N(d_1(f^*))\right)$$
$$d_1(f^*) = \frac{\left(\ln\left(\frac{f^*}{K}\right) + \frac{1}{2}\sigma^2\tau\right)}{\sigma\sqrt{\tau}}$$
$$q_2 = \frac{\left(1+\sqrt{1+4b}\right)}{2}$$
$$b = \frac{2R_d}{\sigma\left(1-e^{-R_d\tau}\right)}$$

The value f^* can be found iteratively by solving

$$f^* - K = C^{f'}(f^*) + \left[1 - e^{-R_d\tau}N(d_1(f^*))\right]\frac{f^*}{q_2}$$

For American futures puts, the quadratic model is:

$$P^{f'} = P^{f} + A_1 \left(\frac{f}{f^{**}} \right)^{q_1}, \quad f > f^{**}$$

$$P^{f'} = K - f, \quad f \leq f^{**}$$

$$A_1 = -\left(\frac{f^{**}}{q_1} \right) \left(1 - e^{-R_d \tau} N(-d_1(f^{**})) \right)$$

$$q_1 = \frac{\left(1 - \sqrt{1 + 4b} \right)}{2}$$

where f^{**} is the optimal exercise futures price for the American futures put and all other terms are defined as in the formulation for the American futures call. The value of f^{**} can be found iteratively by solving

$$K - f^{**} = P^{f'}(f^{**}) - \left[1 - e^{-R_f \tau} N(-d_1(f^{**})) \right] \frac{f^{**}}{q_1}$$

Alternatively, the value of an American futures option can be approximated using the Ho, Stapleton, and Subrahmanyam approach discussed in Chapter 6 by setting the foreign interest rate (R_f) equal to the domestic interest rate (R_d). The intuition for this is that a futures option is analogous to an option on an asset that pays a yield equal to the risk-free rate of interest, meaning that it has no cost of carry.

Options on Foreign Exchange, Third Edition
David F. DeRosa

CHAPTER 8

Barrier and Binary Currency Options

Barrier options on foreign exchange are options designed to go out of existence or come into existence when the spot exchange rate trades at or through some predetermined barrier level before option expiration (a barrier event). Similar to barrier options are binary options. Binary options pay a fixed sum only if a barrier level is triggered (Exhibit 8.1).

Presently barrier and binary currency options are traded exclusively in the interbank market. All the major foreign exchange dealing banks offer these options.

For the most part, barrier options are constructed to be susceptible to barrier events at any time in their lives. For this reason they are said to be path-dependent, referring to the fact that the state of their very existence is affected by the spot exchange rate at points in time during their lives before expiration. In contrast, vanilla currency options are not path-dependent because their existence is not affected by intermediate values of the spot exchange rate. As a rule, currency barrier options, unlike barrier options in other markets, do not pay rebates. A rebate is a cash payment made when a barrier event causes an option to be extinguished.[1] The models in this chapter assume that barriers are monitored continuously. These models are useful even though in reality there are times when markets are closed. Still, the foreign exchange market is the closest among all markets to fulfilling the condition in that trading takes place 24 hours per day on weekdays.[2]

Binary options are a special case of barrier options. First, they pay a fixed sum conditional on a barrier event occurring. Some binaries are barrier-sensitive only at expiration while others can be triggered at any time in the option's life (such as the one-touch option). Binary options also

[1]See Haug (2007) for barrier options that feature rebates.
[2]See Haug (1996) for discussion of noncontinuous barrier monitoring.

EXHIBIT 8.1 Popular Barrier and Binary Options on Foreign Exchange

	Exercise	Placement of Barrier	Result of Barrier Event	If Barrier Event Does Not Occur	Barrier Event
Single Barrier Options					
Knock-Out	European	$K>H$ (Call)	Option Extinguishes	Option Functions as Vanilla	Any Time
Knock-In	European	$K>H$ (Call)	Option Becomes Vanilla	Option Worthless at Expiration	Any Time
Kick-Out	European	$K<H$ (Call)	Option Extinguishes	Option Functions as Vanilla	Any Time
Kick-In	European	$K<H$ (Call)	Option Becomes Vanilla	Option Worthless at Expiration	Any Time
Double Barrier	Eurpoean	$H1<S<H2$	Option Extinguishes	Option Functions as a Straddle	Any Time
Binary Options					
European	European	$S<H$ (Call)	Pays Cash	Option Worthless at Expiration	Expiration Only
European	European	$S>H$ (Put)	Pays Cash	Option Worthless at Expiration	Expiration Only
One-Touch	American	$S<H$ (Call)	Pays Cash	Option Worthless at Expiration	Any Time
One-Touch	American	$S>H$ (Put)	Pays Cash	Option Worthless at Expiration	Any Time
Double Barrier Binary Range Options	American	$H1<S<H2$	Pays Cash for Every Day There Is a Barrier Event	Option Worthless at Expiration	Any Time
Contingent Premium	European	$S>H$	Owner Pay Premium(s)	Option Functions as Vanilla	Any Time

include range options and contingent premium varieties, both of which are discussed in this chapter.

Why all the interest in barrier options? A simple answer is that barrier options afford market participants an opportunity to structure highly specific transactions with respect to the levels of future exchange rates. Moreover, barrier options can create highly leveraged directional trades that cost a fraction of the purchase price of a vanilla option. Also, they have interesting volatility properties. For example, it is possible to construct a short volatility trade with the purchase of barrier options. Being able to construct a fixed-premium short volatility position using barrier options is an improvement over far riskier conventional short volatility trades that require taking short positions in vanilla options. Hedgers too have found that some barrier currency options can be useful and cost effective in managing currency risk.

Many basic barrier options have BSM-style analytical models. These are introduced in this chapter. A clever adaptation of Vanna-Volga maps barrier options onto the volatility surface. In some cases binomial and trinomial solutions provide improvements over the analytical models. There are other numerical methods, such as Crank-Nicolson, that have been used successfully on barrier options. The topic of barrier options will continue in the following chapter, where I will present some of the newer models, including the local volatility models of Derman and Kani and Dupire, a barrier option version of Heston's model, and a stochastic local volatility model.

SINGLE BARRIER CURRENCY OPTIONS

Taxonomy of Single Barrier Options

A knock-out barrier option contains a cancellation feature that causes the option to extinguish when and if the spot exchange rate touches or trades through an out-strike barrier level. By definition, a knock option has an out-strike located out-of-the-money. Knock-out calls are called down-and-out calls; knock-out puts are called up-and-out puts.

Consider the example of a knock-out USD call/JPY put struck at 90 and out-struck at 85. Suppose USD/JPY were trading at 90. If USD/JPY were to trade at 85, the option would extinguish immediately. Otherwise, and for so long as the 85 level did not trade, the knock-out would function as a vanilla option.

A knock-in barrier option is an option that does not come into existence unless the spot exchange rate breaches a specified in-strike level. The in-strike for a knock-in option is located out-of-the-money. If the in-strike level trades, the knock-in permanently becomes a vanilla put or call. However, if the in-strike never trades, the knock-in will expire worthless at expiration, even if it is in-the-money. Continuing from the previous example, 85 would be a valid in-strike for a knock-in USD call/JPY put.

Kick-out and kick-in options are similar to knock-out and knock-in options except that the out-strike or in-strike is located in-the-money. An example of an up-and-out kick-out call would be a USD put/JPY call struck at 90 with an out-strike at 85. Similarly, a down-and-out kick-out put would be a USD call/JPY put stuck at 90 with an out-strike at 95.

The combination of an in-option and an out-option with the same strike, barrier, and expiration is equivalent to a vanilla option. This is because if the barrier is triggered, the out-option extinguishes but the in-option springs into life. Therefore

$$C_{out} + C_{in} = C$$

$$P_{out} + P_{in} = P$$

Knock Options

Haug (2007), following Rich (1994) and Reiner and Rubinstein (1991a), provides a convenient framework of analytical models for valuing barrier calls and puts. Analysis of barrier options dates back to early, prescient works by Merton (1973) and Black and Cox (1976). Haug defines the following equations that will be used repeatedly in this chapter:

$$Z1 = \phi e^{-R_f \tau} SN(\phi x_1) - \phi e^{-R_d \tau} KN\left(\phi x_1 - \phi\sigma\sqrt{\tau}\right)$$

$$Z2 = \phi e^{-R_f \tau} SN(\phi x_2) - \phi e^{-R_d \tau} KN\left(\phi x_2 - \phi\sigma\sqrt{\tau}\right)$$

$$Z3 = \phi e^{-R_f \tau} S\left(\frac{H}{S}\right)^{2(\mu+1)} N(\eta y_1) - \phi e^{-R_d \tau} K\left(\frac{H}{S}\right)^{2\mu} N\left(\eta y_1 - \eta\sigma\sqrt{\tau}\right)$$

$$Z4 = \phi e^{-R_f \tau} S\left(\frac{H}{S}\right)^{2(\mu+1)} N(\eta y_2) - \phi e^{-R_d \tau} K\left(\frac{H}{S}\right)^{2\mu} N\left(\eta y_2 - \eta\sigma\sqrt{\tau}\right)$$

$$\mu = \frac{R_d - R_f - \frac{1}{2}\sigma^2}{\sigma^2}$$

The symbol H refers to the barrier and

$$x_1 = \frac{\ln\left(\frac{S}{K}\right)}{\sigma\sqrt{\tau}} + (1+\mu)\,\sigma\sqrt{\tau}$$

$$x_2 = \frac{\ln\left(\frac{S}{H}\right)}{\sigma\sqrt{\tau}} + (1+\mu)\,\sigma\sqrt{\tau}$$

$$y_1 = \frac{\ln\left(\frac{H^2}{SK}\right)}{\sigma\sqrt{\tau}} + (1+\mu)\sigma\sqrt{\tau}$$

$$y_2 = \frac{\ln\left(\frac{H}{S}\right)}{\sigma\sqrt{\tau}} + (1+\mu)\sigma\sqrt{\tau}$$

Other terms are as were previously defined, and η and ϕ are index reference variables:

Knock-Out Calls ($K > H$) *and Puts* ($K < H$)

$$C_{down\,\&\,Out} = Z1 - Z3 \ with\ \eta = 1 \ and\ \phi = 1$$
$$P_{up\,\&\,out} = Z1 - Z3 \ with\ \eta = -1 \ and\ \phi = -1$$

Knock-In Calls ($K > H$) *and Puts* ($K < H$)

$$C_{down\,\&\,in} = Z3 \ with\ \eta = 1 \ and\ \phi = 1$$
$$P_{up\,\&\,in} = Z3 \ with\ \eta = -1 \ and\ \phi = -1$$

Why Knock-Out Options Are Popular

Exhibit 8.2 shows the theoretical value of a knock-out USD put/JPY call for comparison to a vanilla option.

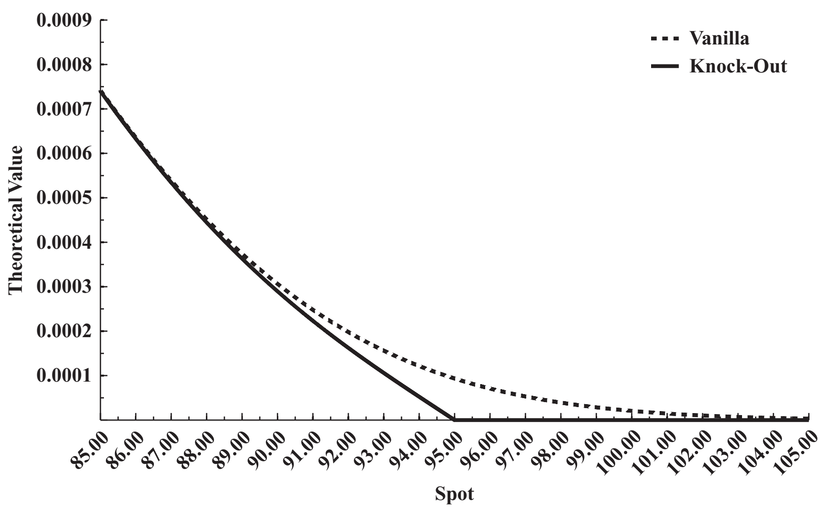


EXHIBIT 8.2 Knock-Out USD Put/JPY Call
Strike = 89.3367; out-strike = 95; 90 days; vol = 14%; $R_d = 5\%$; $R_f = 2\%$.

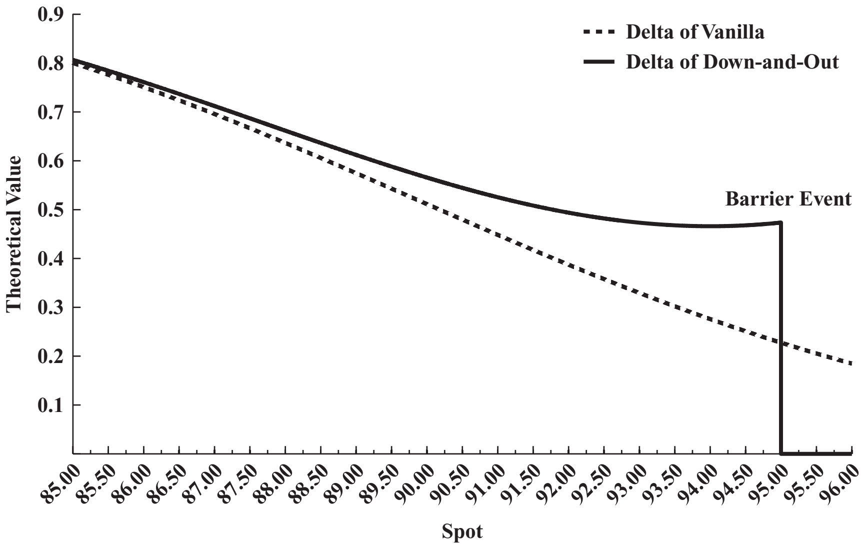


EXHIBIT 8.3 Deltas of Down-and-Out USD Put/JPY Call and Vanilla USD Put/JPY Call
Strike = 89.3367; out-strike = 95; 90 days; vol = 14%; $R_d = 5\%$; $R_f = 2\%$.

The option strike is 89.3367, and the barrier is 95. This exhibit demonstrates that the value of the barrier option is smaller than the vanilla in the proximity of the barrier. At progressively lower levels for the spot exchange rate, the probability of the barrier being knocked out diminishes, and the barrier option converges on the vanilla. This is reflected in the behavior of the barrier option delta relative to the delta of the vanilla (Exhibit 8.3).

When the barrier is hit, the delta of the barrier option instantly becomes zero. Other than in that case, the knock-out delta is considerably higher than the vanilla delta.

This explains part of the popularity of knock-out puts and calls on foreign exchange. These are low-cost, high-delta options and, as such, appeal to traders seeking to place speculative bets on currencies.

But knock-out options are also popular among hedgers, and legitimately so. Consider a Japanese exporter who is long dollars against yen at the 90 level. The knock-out USD put/JPY call might be cost effective as a hedge because it affords the downside protection that the exporter is seeking. The fact that the option is knocked out at 95 may not be of great concern to the hedger because the value of his dollars would have appreciated by five yen by the time that the knock-out is extinguished. However, it should be noted that the exporter either would have to buy a new knock-out option or devise a new hedging strategy were the 95-barrier to be broken. One simple way to stay continuously hedged through a barrier event would be to establish a stop-loss order to sell one-half of his dollars at the 95 barrier. If the barrier were subsequently triggered, the short dollar position would function as a short-run delta hedge in approximately the same way as an at-the-money forward USD put/JPY call. Moreover, the short dollar position could be later exchanged with a dealer as part of a transaction to buy a vanilla USD put/JPY call.

Kick Options

By definition, the out-strike for the kick family of barrier options is located in the zone where the option is in-the-money.

Kick-Out Options Consider a kick-out USD call/JPY put struck at 90 and out-struck at 95. The potential gains from owning a kick-out option are limited—in the example, the kick-out put could at best finish by something slightly less than five yen in-the-money. Accordingly, its price should be relatively small.

Following Haug (2007), and recognizing earlier work by Rich (1994) and Taleb (1997):

Kick-Out Calls and Puts

$$C_{up\,\&\,out} = Z1 - Z2 + Z3 - Z4 \text{ } with \text{ } \eta = -1 \text{ } and \text{ } \phi = 1$$

$$P_{down\,\&\,out} = Z1 - Z2 + Z3 - Z4 \text{ } with \text{ } \eta = 1 \text{ } and \text{ } \phi = -1$$

Consider the following kick-out call:

Option: USD put/JPY call
Face: $1,000,000
Initial spot: 90
Strike: 89.3367
Out-strike: 85
Interest Rate (USD): 5%
Interest Rate (JPY): 2%
Volatility: 14%
Term: 30 days

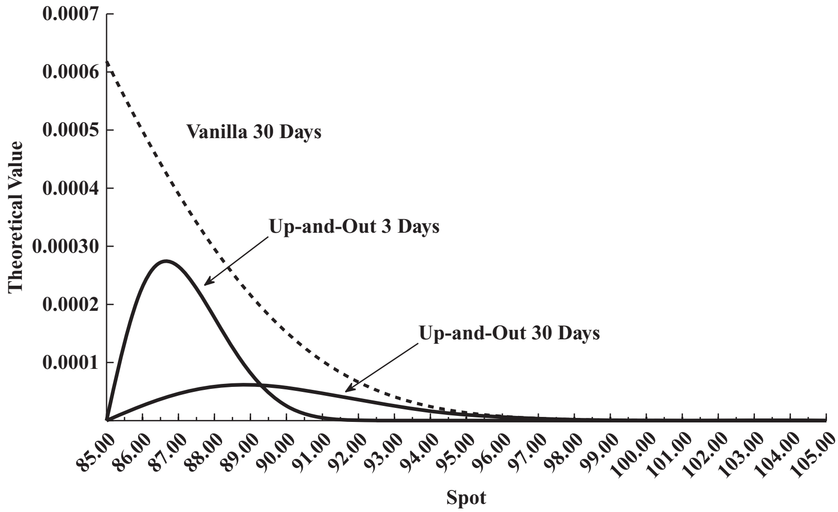


EXHIBIT 8.4 Kick-Out Up-and-Out USD Put/JPY Call
Strike = 89.3367; out-strike = 85; vol = 14%; $R_d = 5\%$; $R_f = 2\%$.

This option has a theoretical value of only $4,575. Yet it has the potential value of nearly $51,020 (i.e., 4.3367 yen of expiration value). That maximum value would be the theoretical case where the spot exchange rate comes as close as possible to the 85 level without actually touching it. Exhibit 8.4 shows the value of this option at 30 days to expiration and 3 days to expiration compared to a 30-day vanilla call.

With 30 days to expiration, the kick-out call has little or no delta, gamma, vega, or time decay. In effect, the option is caught between Scylla (meaning the strike) and Charybdis (the out-strike)—and being torn in opposite directions as the strike and out-strike crushes the option's sensitivity to movements in the spot exchange rate, time, and volatility.

A very different picture arises during the final days of the kick-out's life, as can be seen in Exhibit 8.4. The kick-out comes to life in a way that resembles the behavior of a butterfly spread near to expiration. Exhibit 8.4 demonstrates that when only three days remain before expiration, the kick-out assumes its maximum value where spot is midway between the strike and out-strike. Also note that the sign of the kick-out's delta changes somewhere in the region between the strike and out-strike.

Kick-out options are far less important to option end-users than are knock-out options. Kick-outs are of limited use in hedging—they might even be called a hazard to a hedger because they extinguish in the region where the hedger needs protection the most.

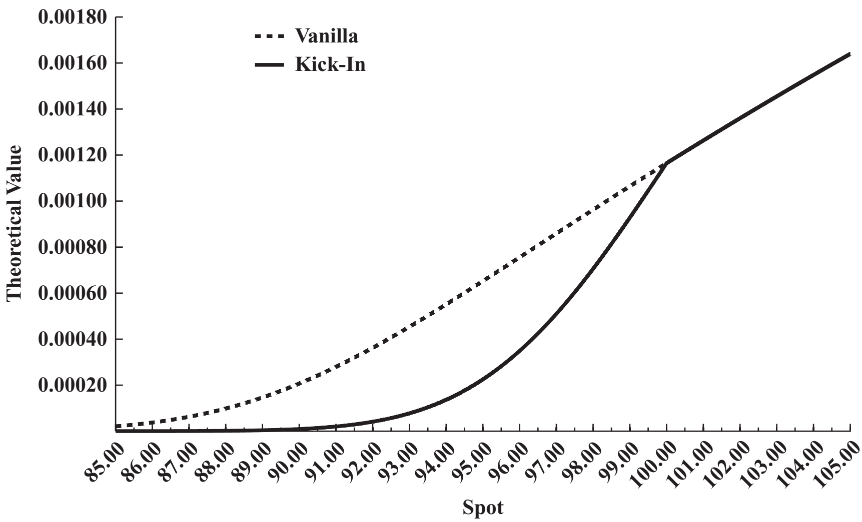


EXHIBIT 8.5 Kick-In USD Call/JPY Put
Strike = 89.3367; in-strike = 100; 30 days.

Kick-In Options Kick-in options come to life only when the option is sufficiently in-the-money so as to trigger the in-strike. The relative cheapness of these options comes from the fact that there is a range of in-the-moneyness, between the strike and the in-strike, which cannot be captured at expiration unless the in-strike is triggered. Exhibit 8.5 compares the theoretical values of a 30-day kick-in USD call/JPY put struck at 89.3367 and in-struck at 100 to a vanilla USD call/JPY put.

Notice that the kick-in always has a positive delta but naturally lags the vanilla in accumulating value as spot rises. Once the in-strike is triggered, the kick-in mutates into a vanilla option.

Kick-In Calls and Puts

$$C_{up\,\&\,in} = Z2 - Z3 + Z4 \textit{ with } \eta = -1 \textit{ and } \phi = 1$$

$$P_{down\,\&\,in} = Z2 - Z3 + Z4 \textit{ with } \eta = 1 \textit{ and } \phi = -1$$

Kick-ins are useful to traders who anticipate a large directional move. Hedgers like them when they want to buy cheap insurance against a large move in an exchange rate. Rich (1994) and Taleb (1997) provide discussion of kick-in option valuation.

Binomial and Trinomial Models

A considerable amount of work has been done on the use of the binomial model in the valuation of barrier options. One advantage of using a tree model is that it can work on the odd barrier option that features American exercise. Direct application of the binomial model requires that each node of the tree be adjusted for the occurrence of a barrier event.

Unfortunately, the binomial approach has been shown to have serious biases when applied to barrier options, even when the number of branches in the tree is large. Boyle and Lau (1994) show substantial errors can occur when a barrier is located between adjacent branches of the binomial tree. They suggest defining the partitioning of time so as to make the resulting tree branches as close as possible to the barrier.

Ritchken (1995) introduces a trinomial option model (see Boyle and Lau 1994) that is capable of valuing single and double barrier options. In the trinomial model, the spot exchange rate is constrained to move in one of three ways, up, middle, and down:

$$\begin{aligned} &\textit{Up}\colon \lambda\sigma\sqrt{\Delta T} \\ &\textit{Middle}\colon 0 \\ &\textit{Down}\colon -\lambda\sigma\sqrt{\Delta T} \end{aligned}$$

where ΔT is a partitioned unit of time. The probabilities of up, middle, and down moves are given by

$$\begin{aligned} p_u &= \frac{1}{2\lambda^2} + \frac{\mu\sqrt{\Delta T}}{2\lambda\sigma} \\ p_m &= 1 - \frac{1}{\lambda^2} \\ p_d &= \frac{1}{2\lambda^2} - \frac{\mu\sqrt{\Delta T}}{2\lambda\sigma} \end{aligned}$$

where

$$\mu = R_d - R_f - \frac{\sigma}{2}$$

and λ is the parameter that controls the width of the gaps between the layers of the lattice. Ritchken's trick is to set λ so as to guarantee a barrier event after an integer number of successive moves. Ritchken reports impressive computational efficiency with his trinomial model for barrier options.

Finite Differences for Barrier Options

Some academics and practitioners prefer to use finite difference methods over tree models for barrier options.[3] Implementation requires shrinking the grid by reestablishing either the upper or lower boundaries for barrier events. For example, a down-and-out option calls for the elimination of rows of the grid below the out-strike level.[4] Similarly, for an up-and-out option the upper boundary must be reset to the out-strike level. Exhibit 8.6 shows how Crank-Nicolson values the knock-out USD put/JPY call from Exhibit 8.1.

Note how Crank-Nicolson oscillates around and converges to the value produced by the analytical model as the number of points in the grid increases.

DOUBLE BARRIER KNOCK-OUT CURRENCY OPTIONS

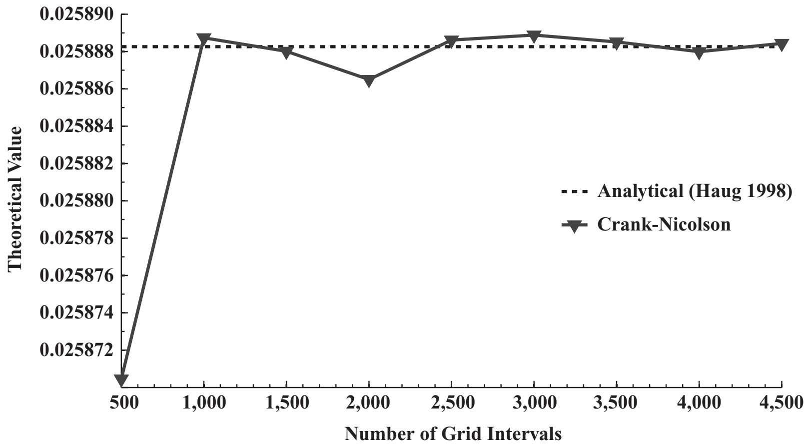
A double barrier option is a knock-out option that has two barriers, one that is in-the-money and another that is out-of-the-money. For example, at spot USD/JPY equal to 90, the double barrier option might be struck at 90 and have two out-strikes, one at 80 and the other at 100. A barrier event can occur at any time in the life of this option.

The valuation of double knock-out options is problematic. Kunitomo and Ikeda (1992) (also see Haug 2007) provide this model for a double knock-out call option where the upper barrier is denoted as U and the lower barrier as L:

$$
\begin{aligned}
C_{double} = e^{-R_f \tau} S \sum_{n=-\infty}^{\infty} &\left[\left(\frac{U^n}{L^n} \right)^{\mu} [N(d_1) - N(d_2)] - \left(\frac{L^{n+1}}{U^n S} \right)^{\mu} [N(d_3) - N(d_4)] \right] \\
&- e^{-R_d \tau} K \sum_{n=-\infty}^{\infty} \left[\left(\frac{U^n}{L^n} \right)^{\mu - 2} [N(d_1 - \sigma\sqrt{\tau}) - N(d_2 - \sigma\sqrt{\tau}) \right] \\
&- \left(\frac{L^{n+1}}{U^n S} \right)^{\mu - 2} [N(d_3 - \sigma\sqrt{\tau}) - N(d_4 - \sigma\sqrt{\tau})]]
\end{aligned}
$$

[3]Wilmott (1998) has extensive discussion on the application of finite difference methods to various classes of options, including barrier options. Some academics are critics of the use of finite differences for barrier options, notably Duffy (2004 and 2006).

[4]Some practitioners enhance the stability of Crank-Nicolson by placing knock-out levels of the spot exchange rate close to or on top of the grid points (an analogous procedure to what Boyle and Lau recommend for application of the binomial model to barrier options, as discussed previously). See Topper (2005) for applications of finite element models to the valuation of barrier options.



(A)

Number of Grid Intervals	Crank-Nicolson Value	Relative Error
500	0.0258705	0.0688%
1,000	0.0258887	0.0018%
1,500	0.0258880	0.0010%
2,000	0.258865	0.0068%
2,500	0.0258886	0.0014%
3,000	0.0258889	0.0024%
3,500	0.0258885	0.0010%
4,000	0.0258880	0.0010%
4,500	0.0258884	0.0007%

(B)

EXHIBIT 8.6 Finite Differences: Knock-Out USD Put/JPY Call
Strike = 89.3367; spot = 90; out-strike = 95; 90 days; vol = 14%; $R_d = 5\%$; $R_f = 2\%$; analytical value (Haug 2007) = 2.329943.

where

$$d_1 = \frac{\ln\left(\dfrac{SU^{2n}}{XL^{2n}}\right) + \left(b + \dfrac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}}$$

$$d_2 = \frac{\ln\left(\dfrac{SU^{2n}}{UL^{2n}}\right) + \left(b + \dfrac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}}$$

$$d_3 = \frac{\ln\left(\dfrac{L^{2n+2}}{KXU^{2n}}\right) + \left(b + \dfrac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}}$$

$$d_4 = \frac{\ln\left(\dfrac{L^{2n+2}}{SU^{2n+2}}\right) + \left(b + \dfrac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}}$$

$$\mu = \frac{2b}{\sigma^2} + 1$$

$$b = R_d - R_f$$

In practice, the series terms converge rapidly making it unnecessary to have to evaluate more than a few terms (you could restrict the range on n to go from –5 to +5, for example). Haug (2007) provides the companion model for double knock-out puts.

Exhibit 8.7 shows the valuation of a double knock-out USD put/JPY call at various levels of quoted volatility.

The exhibit gives visual confirmation to the obvious fact that double knock-out options are short volatility trades.

Geman and Yor (1996) provide a mathematically sophisticated alternative to the Ikeda and Kunitomo formulation based on excursions theory that

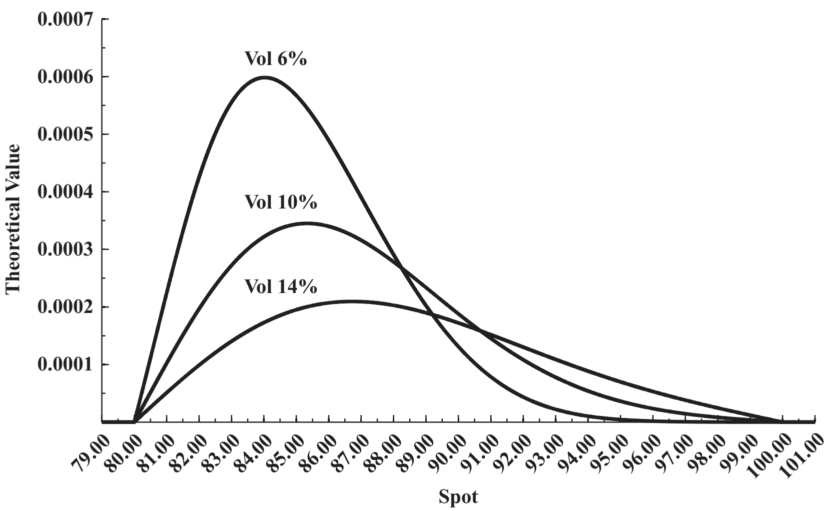
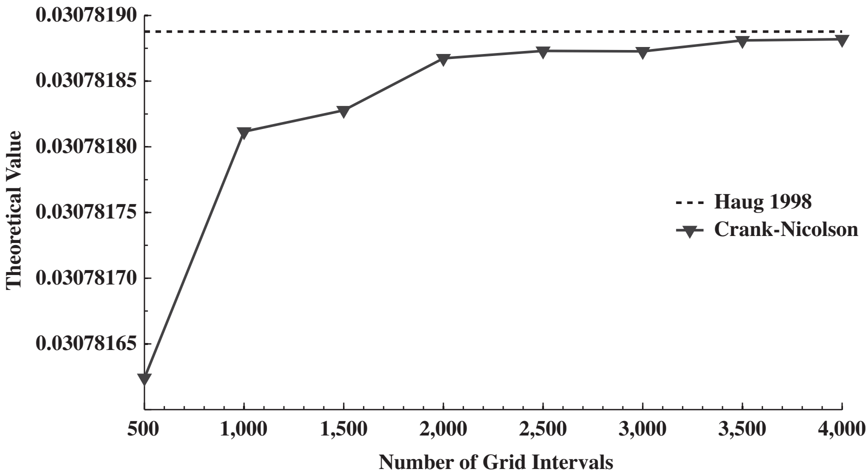


EXHIBIT 8.7 Double Barrier USD Put/JPY Call
Strike = 89.3367; 90 days; barriers at 80 and 100.



(A)

Number of Grid Intervals	Crank-Nicolson Value	Relative Error
500	0.030781624	0.000857%
1,000	0.030781812	0.000247%
1,500	0.030781828	0.000195%
2,000	0.030781867	0.000066%
2,500	0.030781873	0.000048%
3,000	0.030781873	0.000049%
3,500	0.030781881	0.000022%
4,000	0.030781882	0.000019%

(B)

EXHIBIT 8.8 Finite Differences: USD/JPY Knock-Out Straddle
Strike = 89.3367; spot = 90; barriers at 80 and 100; 90 days; vol = 14%; R_d = 5%; R_f = 2%; value (Haug 2008) = 0.030781888.

produces a Laplace transformation for double barrier options. Ritchken's trinomial model also handles double barrier options with some modification.

Alternatively, Exhibit 8.8 demonstrates the use of Crank-Nicolson in the valuation of the double knock-out straddle.

The grid has been modified to eliminate rows above the upper out-strike and below the lower out-strike.

One of the most popular double barrier trades is the double knock-out straddle. Purchasing a vanilla straddle (i.e., buying the same-strike put and call) is a long volatility trade. The owner of the vanilla straddle is hoping that actual volatility will be enormous so that either the put or the call will be driven deep into-the money. The owner of a vanilla straddle is also better off if quoted volatility rises. The cost of maintaining a long position in a straddle is the decay in premium over time.

By the same logic, taking a short position in a vanilla straddle is a short volatility trade. The writer of a vanilla straddle is hoping that the spot foreign exchange rate will stay somewhat fixed in place, making the straddle expire worthless. The writer benefits when quoted volatility drops because the straddle could be repurchased at a lower price. Along the way, the writer of the straddle collects time decay.

Things are totally reversed when a straddle is created with knock-out barriers. By market convention, breach of either barrier extinguishes both the put and the call that comprise the knock-out straddle. The owner of the knock-out straddle is shorting volatility, both in the sense of actual volatility and quoted volatility. The owner hopes that actual volatility will be low and that the knock-out straddle is never knocked out. He would be pleased to see quoted volatility drop so that the market price of the knock-out straddle would rise. In what may seem somewhat counterintuitive at first, the owner of a knock-out straddle collects time decay—as time passes, the probability that the knock-out straddle will be knocked-out drops, and the structure becomes worth more in value.

BINARY CURRENCY OPTIONS

A binary option, sometimes called a digital option, is defined to have a lump-sum payoff function. Binaries are quoted as the ratio of premium to payoff. For example, a binary quoted "3 to 1" means that the option buyer would receive three dollars for every one dollar in premium paid to the option writer if the option expires in-the-money.

European Binary Options

A European exercise vanilla binary option pays a lump sum of domestic currency at expiration provided the option is in-the-money. There are binary puts and binary calls. European binary options that pay one unit of domestic currency can be modeled as a fragment of the BSM model:

European Vanilla Binary Options

$$C_{Binary} = e^{-R_d \tau} N(x)$$
$$P_{Binary} = e^{-R_d \tau} (1 - N(x))$$

A European vanilla binary option is not path dependent because the only condition for the option to be alive and pay is that it be in-the-money at expiration. What happens between the time that the option is dealt and when it expires is irrelevant.

One-Touch Binary Options

A one-touch binary option pays a lump sum of domestic currency provided that a specific in-strike trades during the life of the option. This option is sometimes called a "bet" option. An example would be a binary option structured to pay a sum of $1 million provided that the spot level 100 trades at some time during the life of the option. Sometimes the owner of the option receives the lump sum payment on the spot when the trigger is hit. In other cases the owner has to wait until option expiration to receive the in-the-money payment.

Reiner and Rubinstein (1991) and Wystup (2006) offer a model to value a single barrier binary option that pays a lump sum W of domestic currency provided the barrier is hit:

One-Touch Binary Options

$$C_{OTB} = e^{-R_d \tau \omega} W \left[\left(\frac{H}{S} \right)^{\frac{a+b}{\sigma}} N(-\phi z_+) + \left(\frac{H}{S} \right)^{\frac{a-b}{\sigma}} N(\phi z_-) \right]$$

where

$$z_{\pm} = \frac{\pm \ln\left(\frac{S}{H}\right) - \sigma b \tau}{\sigma \sqrt{\tau}}$$

$$a = \frac{\left(R_d - R_f\right)}{\sigma} - \frac{\sigma}{2}$$

$$b = \sqrt{a^2 + 2(1-\omega) R_d}$$

where $\phi = 1$ for a put, meaning the option is "down and in" ($S > H$) and $\phi = -1$ for a call, meaning the option is "up and in" ($S < H$). The term $\omega = 1$ if the option promises to pay at expiration and $\omega = 0$ if it pays immediately whenever the barrier is triggered.

Exhibit 8.9 shows the theoretical valuation of a one-touch USD put/JPY call at differing levels of option volatility—the higher the volatility, the greater the value of the one-touch because the probability of the barrier being struck is greater.

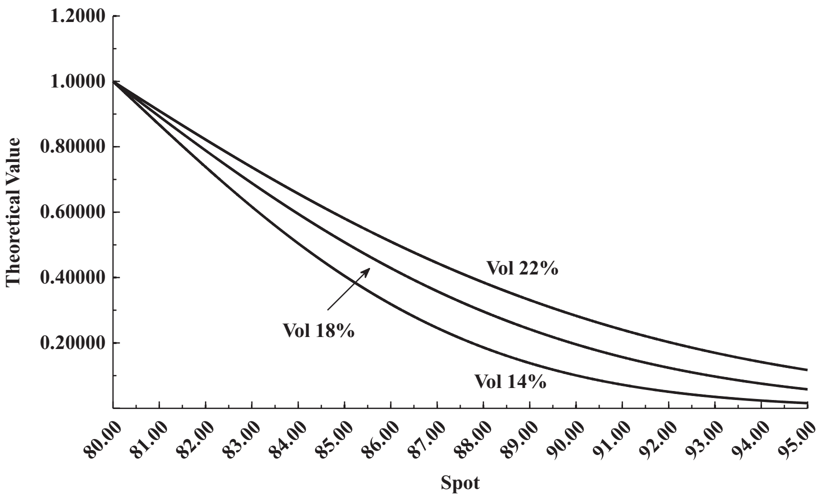


EXHIBIT 8.9 One-Touch Binary Options
Ninety days; barrier at 80.

Knock-Out Probabilities and Stopping Time

The one-touch model provides an insight into an important concept in barrier option pricing, called the knock-out probability. The question is this: what is the probability, p^H, that a certain barrier, H, will be crossed at any time in the option's life? The answer comes from backing out the present value of the payout (paid at expiration) from the one-touch:

$$p^H = \frac{1}{W} e^{R_d \tau} C_{OTB}$$

But when will that happen? Option models use time as a continuous variable. But the first time at which a barrier event occurs in the future is stochastic. Stopping time,[5] also known as first hitting time, is the solution to how much time will pass until a barrier event takes place. Wystup (2006) provides a density function for stopping time τ_H $(t > 0)$:

$$pr[\tau_H \in dt] = \frac{\frac{1}{\sigma}\ln\left(\frac{H}{S}\right)}{t\sqrt{2\pi t}} e^{\left[\frac{\left[\frac{1}{\sigma}\ln\left(\frac{H}{S}\right)-at\right]^2}{2t}\right]}$$

[5]See discussion of stopping time in Taleb (1997).

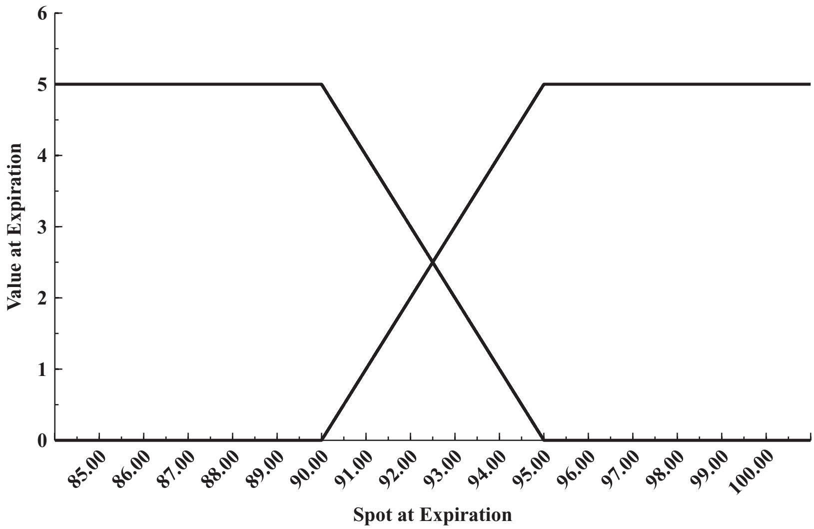


EXHIBIT 8.10 USD/JPY 90–95 Box

Double Barrier Binary Range Options

Perhaps the most interesting binary option is the double barrier binary range option. This option has two barrier strikes, for example, 90 and 95, to continue with the running example of USD/JPY barrier options. The option is an "out" option because it is structured to pay a lump sum at expiration provided neither 90 nor 95 is hit during the life of the option.

Double barrier binary range options are analytically the same thing as a "box" trade with barriers. In a box trade, the trader who goes long the box buys both a call spread and a put spread so as to create a fixed payoff regardless of the location of the spot exchange rate at expiration. For example, a five-yen wide box (Exhibit 8.10) could consist of the following four positions:

Long 90 USD call/JPY put
Short 95 USD call/JPY put
Long 95 USD put/JPY call
Short 90 USD put/JPY call

Essentially, boxes are financing devices. The buyer of a box is in effect lending the present value of the width of the box, in this case five yen, to the writer of the box. Vanilla boxes are creatures of the listed option markets. The box in the example could be transformed into a double-barrier binary range option by inserting barriers at 80 and 95, a structure unique to the interbank option market.

Double barrier binary range options are popular with traders who want to express short volatility views without being short vanilla options. The most common vanilla trade to express short volatility views is the sale of a straddle. The danger with the short straddle trade is that the writer creates an exposure to negative gamma. Negative gamma implies that the option writer is at risk to whipsaw moves in the spot exchange rate. What is alluring about the double barrier binary range option is that a trader can enter into a short volatility trade by paying a fixed premium.

Hui (1996) develops the theoretical value for a double barrier binary range option in a Black-Scholes environment:

Double Barrier Binary Range Option

$$C_{DBB} = \sum_{n=1}^{\infty} \frac{2\pi n R}{L^2} \left[\frac{\left(\frac{S}{H_1}\right)^{\alpha} - (-1)^n \left(\frac{S}{H_2}\right)^{\alpha}}{\alpha^2 + \left(\frac{n\pi}{L}\right)^2} \right] \sin\left(\frac{n\pi}{L} \ln \frac{S}{H_1}\right) e^{-.5\left[(n\pi/L)^2 - \beta\right]\sigma^2 \tau}$$

for $H_2 > H_1$ and where

$$\begin{aligned} L &= \ln\left(\frac{H_2}{H_1}\right) \\ \alpha &= -\frac{1}{2}(k_1 - 1) \\ \beta &= -\frac{1}{4}(k_1 - 1)^2 - \frac{2R_d}{\sigma^2} \\ k_1 &= 2\left(R_d - R_f\right)/\sigma^2 \end{aligned}$$

Hui establishes that his equation, though exact in its valuation of the double barrier binary, contains a convergent series that can be well approximated by evaluating only a few terms.

Exhibits 8.11 and 8.12 demonstrate that the double barrier binary is short volatility and benefits from the passage of time.

Note that the option depicted in these exhibits pays one dollar if neither of the barriers is struck—accordingly, the theoretical value can be interpreted as the premium expressed as the percentage of the payoff.

There are also double barrier binary range options that are "in" options, meaning that they pay the binary payoff provided that one or the other barriers are hit during the life of the option. This is a long volatility strategy

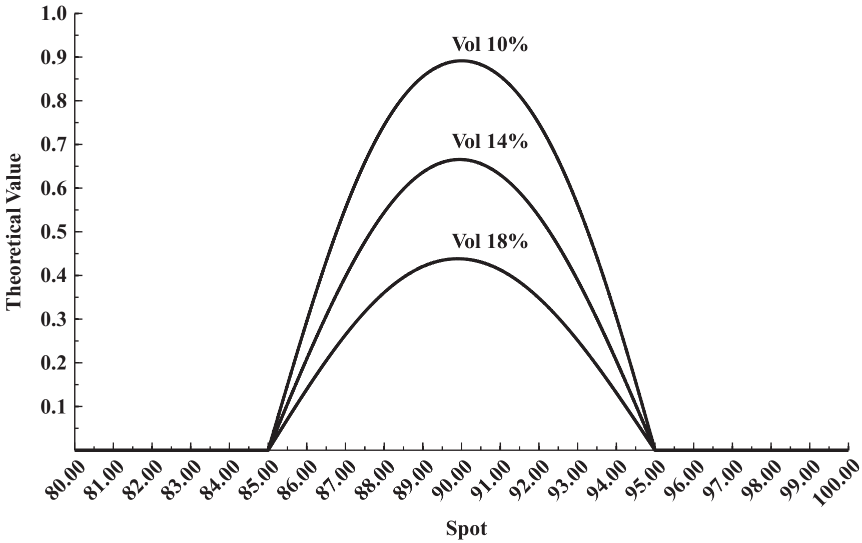


EXHIBIT 8.11 Double Barrier Binary Range Options at Various Implied Volatilities
Barriers at 85 and 95; 30 days.

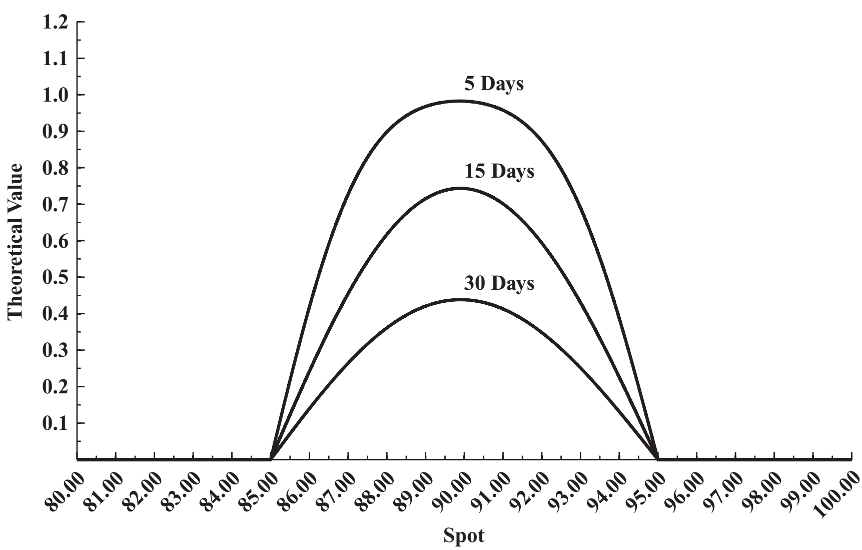


EXHIBIT 8.12 Double Barrier Binary Range Options at Various Terms to Expiration
Barriers at 85 and 95; 30, 15, and 5 days.

because the owner of the option is counting on a sufficiently large move in the spot exchange rate to cause one or the other barriers to be struck.

Finally, Wilmott (1998) gives the following equation for expected first exit time from the range $H_2 > S > H_1$:

$$\frac{1}{\frac{1}{2}\sigma^2 - \left(R_d - R_f\right)} \left[ln\frac{S}{H_1} - \frac{1 - \left(\frac{S}{H_1}\right)^{1-2\frac{R_d - R_f}{\sigma^2}}}{1 - \left(\frac{H_2}{H_1}\right)^{1-2\frac{R_d - R_f}{\sigma^2}}} \right] ln\frac{H_2}{H_1}$$

CONTINGENT PREMIUM CURRENCY OPTIONS

A contingent premium option is an option that can be bought for no up-front premium. The option has one or more "premium strikes." If, during the life of the option, a premium strike event happens to occur, meaning spot trades at or through a premium strike, the option holder is required to immediately make a premium payment to the writer of the option. Take the example of a 90-day USD call/JPY put contingent premium struck at 89.3367 on $1 million face. This option has two contingent premia, one at 87 and a second premium at 85. This option is easy to manufacture because it can be thought of as a vanilla option—the USD call/JPY put that the dealer issues to its customer, plus two one-touch options that the customer issues to the dealer. The one-touch options represent the contingent premia. Suppose that the USD call/JPY put parameters are specified as follows:

USD Call/JPY Put
Face: $1,000,000
Spot: 90
Strike: 89.3367
Expiry: 90 days
Volatility: 14%
Interest Rate (USD): 5%
Interest Rate (JPY): 2%
Value (percent Face): 2.739%

The value of 2.739 percent was obtained from the BSM model. The one-touch strikes are set at 87 and 85. From this information we can learn that each one-touch would require a payment equal to 2.541 percent of the vanilla's face. This can be deduced from the fact that the 87 and 85 one-touch options are worth .645 and .433 respectively per dollar of payoff. In the worst case, if USD/JPY hits both the 87 and 85 barriers, the customer would be obligated to pay a total of 5.08 percent of the vanilla face. On the other hand, if the dollar remains above 87, the customer would have succeeded in acquiring the vanilla option without making a cash outlay.

Contingent premium options are a useful substitute for the directional risk-reversal trade that many bank proprietary traders are fond of doing. As was described in Chapter 4, traders sometimes express a directional view that, for example, could be bullish by buying a 25-delta call and selling a 25-delta put. This trade can be established for zero or close-to-zero premium. Everything is wonderful if the spot exchange rate goes up, as predicted. But when the trade goes wrong, meaning that the spot exchange rate goes down, the trader is left with an active, short gamma position in the form of the short position in a dollar put option. The outcome from trying to hedge the short put is indeterminate. By comparison, a contingent premium option could be structured to express basically the same directional view. The advantages are that the downside is knowable in advance, meaning the worst that can happen is that one or more contingent premia have to be paid.

APPLYING VANNA-VOLGA TO BARRIER AND BINARY OPTIONS

The analytical models for barrier options and binary options that I have discussed require volatility to be specified as an input. What complicates matters is that what are observable in the marketplace are volatilities quoted on vanilla options. Where on the vanilla volatility surface should a barrier option be located? What does not work is to match the delta and expiration of the barrier option to a point on the surface—and that is because the barrier option's volatility risk complex is not likely to match that of the vanilla option with the same delta and expiration.

One answer is to put the Vanna-Volga methodology to work. Wystup (2006) and Bossens, Rayee, Skantzos, and Deelstra (2010) present a quick method for correcting the pricing of barrier options as follows:

$$X^{VV} = X^{BSM} + (1 - P^{H})\left[\frac{Vanna(X)}{Vanna(RR)}RR_{COST} + \frac{Volga(X)}{Volga(BF)}BF_{COST}\right]$$

where

$(1 - P^H)$ is the risk-neutral probability that the barrier event will not occur
X^{VV} is the adjusted price of the barrier option
X^{BSM} is the price of the barrier option based on the ATM volatility

$$
\begin{aligned}
RR_{COST} = & \left[call\left(K_c,\ \sigma(K_C)\right) - Put\left(K_P,\ \sigma(K_P)\right)\right] \\
& - \left[call\left(K_C,\ \sigma(K_{ATM})\right) - Put\left(K_P,\ \sigma(K_{ATM})\right)\right]
\end{aligned}
$$

and

$$
\begin{aligned}
BF_{COST} = & \frac{1}{2}\left[call\left(K_C,\ \sigma(K_C)\right) + Put\left(K_P,\ \sigma(K_P)\right)\right] \\
& - \frac{1}{2}\left[call\left(K_C,\ \sigma(K_{ATM})\right) + Put\left(K_P,\ \sigma(K_{ATM})\right)\right]
\end{aligned}
$$

This process adjusts the price of the barrier option for its Vanna and Volga components. The approximation uses risk reversal pricing for Vanna risk and butterfly pricing for Volga risk.[6]

WHAT THE FORMULAS DON'T REVEAL

All varieties of barrier and binary options I have discussed critically depend on the determination of whether a barrier has been struck. Herein lies one of the most contentious issues in derivatives. Industry practice is to have the dealing bank assume the role of the barrier determination agent (or, as sometimes described, calculation agent), despite the obvious conflict of interest.

Complicating the arrangement is the fact that foreign exchange transaction prices are not readily observable, except to the actual counterparties. Foreign exchange transactions are private conversations between two parties—no public record of trades exists as it does in the listed equity markets. What is more, there is the question of what constitutes a legitimate barrier event in terms of the defining trade. For example, it is logical that a transaction of de minimis size should not constitute a barrier event. Trading

[6]Obviously this is an approximation because risk reversals are not perfectly zero. Volga and butterflies do have some Vanna.

in thin markets is another problem—for example, trading done during the early hours on Monday mornings in Australia and New Zealand perhaps should not be counted. For some exchange rates, there is the question of whether a barrier event could be inferred from a cross rate—in other words, could a euro/yen option be knocked by implying a barrier event from the euro/dollar and USD/JPY exchange rates? Not surprisingly, a number of disputes have arisen between barrier counterparties—some of which have led to lawsuits.

A subtle conflict of interest exists between the writer and holder of barrier options. Consider the case of the knock-out USD put/JPY call having its strike at 90 and its out-strike at 95 when USD/JPY is trading at 90. The dealer of this option, like all dealers, tries to maintain delta-neutrality. Consequently, the dealer's hedge would consist of a dynamic short USD/JPY spot position. If the spot exchange rate were to move upward, in the direction of the out-strike, the dealer would be obliged to remove a portion of the short USD/JPY hedge by buying dollars. If the option were suddenly knocked out at 95, the dealer would remove the balance of short dollar position that was serving as the hedge. But, in fact, dealers sometimes liquidate their hedges in the anticipation that the spot exchange rate will subsequently breach the barrier. For example, suppose that USD/JPY were well bid at 94.80 with the option still alive. If the dealer feels 95.00 or even higher levels are imminent, he isn't going to wait to buy the dollars to remove the hedge. The conflict is that this transaction, referred to as an "anticipatory dehedging transaction," may materially raise the probability that the barrier will be struck.[7]

Many times dealers execute dehedging orders through the use of stop loss orders. The execution of such an order automatically knocks out the barrier option and at the same time removes any residual spot position from the dealer's hedging book. Spot traders will attest to the fact that the behavior of the underlying spot market at times reflects the presence of these dehedging stop orders, some of which are quite large in size.

I do not want to leave readers with the impression that barrier options are hopelessly biased in favor of market makers. Still, one should know what exact economic realities are built into an instrument that one considers trading. On the other side of the coin, one has to realize that many barrier options are more complicated to manufacture than what analytical formulas suggest. Binary options, in particular, can present ferocious hedging problems (see Taleb 1997).

[7]See Hsu (1997).

CHAPTER 9

Advanced Option Models

This chapter extends the discussion of option pricing to more advanced models that attempt to deal with the observed non-constancy of volatility in the foreign exchange market. As I documented in Chapter 5, quoted volatility rises and falls with market conditions. More perplexing yet are the phenomena of skew and smile volatility patterns. Derman (2007) remarked:

> *The Black-Scholes model has no simple way to obtain different implied volatilities for different strikes. The size of a building cannot depend on the angle from which you photograph it, except perhaps in quantum mechanics. Similarly, the volatility of a stock itself cannot depend upon the option with which you choose to view it. Therefore the stock's volatility should be independent of the option strike or time to expiration, because the option is a derivative that "sits above" the stock.* (p. 3)

Derman was writing about options on common stocks, but his remarks are germane to options on foreign exchange. Hence the interest in new models, ones that are designed to be less restrictive than BSM is in the treatment of volatility. Such is the case for the models that I will discuss in this chapter. They incorporate stochastic processes that are more complex than the BSM diffusion process. These models are of special importance to the pricing of barrier options.

However, there are costs associated with transitioning to new models. Many of these models are mathematically complex and relatively difficult to use. Some require onerous calibration work. All incorporate additional, and at least initially, unfamiliar parameters that can frustrate market experience and for which there may be little or no intuition. But the potential benefits to having a model or models that can accurately reflect the true nature of volatility are nontrivial.

The first up for discussion are the stochastic volatility models. Here Heston (1993) gets more attention because of its popularity among practitioners. Next I will address the mixed diffusion jump process model. This model allows for the exchange rate to take jumps, or noncontinuous departures from the underlying diffusion process. Next are the local volatility models of Derman and Kani (1994a and 1994b), Rubinstein (1994), and Dupire (1994). In recent times, practitioners have turned to new eclectic formulation, an example being stochastic local volatility models. These models are mostly based on the local volatility formulation but introduce various stochastic elements to improve functionality. I finish the chapter with material on static replication methods. These procedures are operational alternatives to dynamic hedging procedures.

STOCHASTIC VOLATILITY MODELS

As the name implies, stochastic volatility models treat volatility as a random variable. Pioneering work comes from Hull and White (1987). Variance is generated by a stochastic process all its own apart from the process that drives the spot exchange rate—though there is consideration of the interaction between the two processes. Heston's (1993) model is the most widely used stochastic volatility model[1] because it has a closed-form solution, at least for vanilla options. As such it has computational advantages over other stochastic volatility models (Chesney and Scott 1989).

Heston's model as applied to options on foreign exchange[2] assumes that the spot exchange rate and variance follow the forms

$$\begin{aligned} dS_t &= (R_d - R_f)S_t dt + \sqrt{v_t} S_t dZ_1 \\ dv_t &= -\lambda(v_t - \bar{v})dt + \eta\sqrt{v_t} dZ_2 \\ \langle dZ_1 dZ_2 \rangle &= \rho dt \end{aligned}$$

where

S is the spot exchange rate.

dZ_1 and dZ_2 are Weiner processes that are correlated at level ρ.

[1]Earlier work by Chesney and Scott (1989) applies to currency options a stochastic variance model originally developed by Hull and White (1987), Scott (1987), and Wiggins (1987).

[2]Yekutieli (2004).

v_t is the initial instantaneous variance.

$\bar{v}$ is the long-run variance.

λ is the rate of mean reversion which controls the speed at which v_t moves to $\bar{v}$.

η is the volatility of volatility.

When η and λ are set to zero, the model reverts to the BSM model. Note that volatility is mean reverting. All other terms for the Heston model are given in Appendix 9.1. The partial differential equation for a call option $C(S, v, t)$ is given by

$$\frac{\partial C}{\partial t} + \frac{1}{2}\nu S^2 \frac{\partial^2 C}{\partial S^2} + \rho\eta S v \frac{\partial^2 C}{\partial S \partial v} + \frac{1}{2}\eta^2 \nu \frac{\partial^2 C}{\partial v^2} + (R_d - R_f) S \frac{\partial C}{\partial S} - \lambda(v - \bar{v})\frac{\partial C}{\partial v} - R_d C = 0$$

Imbedded in this equation is consideration for the market price of risk for the presence of stochastic volatility. The price of a European call option is given by

$$C = \left[e^{-R_f \tau} S\Pi_1 - \mathrm{e}^{-\mathrm{R_d}\tau}\mathrm{K}\Pi_2\right]$$

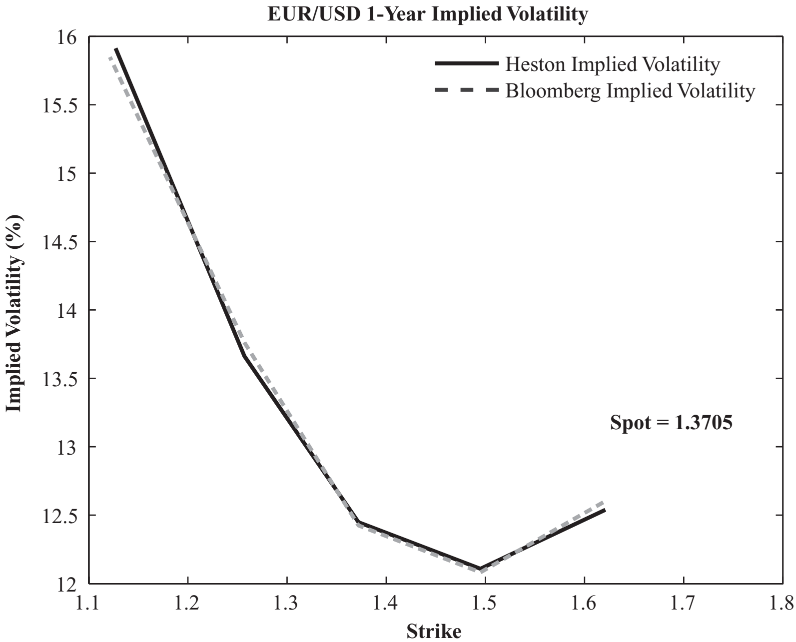
where

$$\Pi_j = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \mathrm{Re}\left\{\frac{\exp\left(B_j\,(u,\tau)\,\bar{v} + D_j\,(u,\tau)\,v + iux\right)}{iu}\right\} du, \quad j = 1, 2$$

The price of a put can be found through put-call parity.

In practice the Heston model requires the input of five variables beyond what the BSM requires: The long-run volatility ($\bar{v}$), the volatility of volatility (η), the correlation of the spot exchange rate and volatility (ρ), rate of mean reversion (λ), and the initial instantaneous variance of the spot exchange rate (v_t). Unlike the BSM model, the Heston model does not require volatility per se. Practitioners calibrate the Heston model by choosing a set of parameters that minimize the sum of the squared errors between the quoted market prices for specific options and the model's implied values.

Once these Heston parameters are determined, the model produces theoretical values for all strikes at the specific expiration. Exhibit 9.1 is an illustration of option pricing using Heston's model.



(A)

Option Strike	1.1270	1.2570	1.3720	1.4950	1.6210
Bloomberg Vol	0.1576	0.1376	0.1243	0.1208	0.1261
Heston Implied Vol (Mean Reversion=5.0)	0.1590	0.1366	0.1245	0.1211	0.1252
Heston Implied Vol	0.1591	0.1366	0.1245	0.1211	0.1254

Parameters	Vol of Variance	Correlation	Initial Variance	Long Run Variance	Mean Reversion
Bloomberg	0.52628	−0.3421	0.0136	0.0188	5.0000
Calibrated (Mean Reversion Constant)	0.6919	−0.2608	0.0195	0.0178	5.0000
Calibrated	0.6905	−0.2600	0.0155	0.0189	4.8726

(B)

EXHIBIT 9.1 The Heston Model for 1-Year EUR/USD European Options

Heston's model became popular because it has a closed-form solution for European options. It is capable of producing option pricing for options with terms over two months that incorporates volatility smiles. However, stochastic volatility models do not generate realistic values for short-term expirations.[3]

I do not know of closed-form variants of Heston for barrier and binary options. However, numerical methods can be brought to bear on pricing barrier and binary options using the Heston partial differential equation. The literature reports some success with non-uniform grid finite difference methods and finite elements methods (see Winkler, Apel, and Wystup 2001).

THE MIXED JUMP-DIFFUSION PROCESS MODEL

As I mentioned, one deficiency of the stochastic volatility models is their inability to produce volatility smiles for expirations at the short end of the calendar. This can be addressed with jump process models. Additionally, there is the well-known phenomenon of "fat tails" (more formally, leptokurtosis) in measured foreign exchange returns—this also calls for consideration of jump process.[4] In a mixed jump-diffusion process, the time series of rates of return follows a diffusion process, as in the BSM model, but has the added feature of being subject to random discontinuous jumps. These jumps themselves are governed by a stochastic process of their own. For example, the process governing exchange rates might be specified as the following

Mixed Jump-Diffusion Model

$$\frac{dS}{S} = \mu dt + \sigma dz_t + J dq$$

The first two terms on the right-hand side are the diffusion process components, and the third term is the jump process component where J is the height of a jump and dq is the stochastic number of jumps. The term dq can be modeled as a Poisson process; it could be described by the mean number of jumps that take place in a unit of time. The actual jumps, Y_i, are assumed to be independent and lognormally distributed.

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[3]Gatheral (2006, p. 42).

[4]Jorion (1988) finds evidence of the existence of significant discontinuous jumps in sample foreign exchange returns. Moreover, the stochastic variance model cannot account for these discontinuities.

In discrete time, the currency return can be written as

$$ln\frac{S_t}{S_{t-1}} = \mu\Delta t + \sigma z_t + \sum_{i=1}^{n_t} Y_i$$

where n_t is the actual number of jumps during a particular interval.

Merton's (1976) jump process option pricing model can be adapted for currency options. Merton's model, which assumes that jump risk does not command an economic risk premium in the market, is as follows (adapted here for currency options):

Merton's Jump Process Model

$$C_{Merton} = e^{-Rf\tau} \sum_{j=0}^{\infty} \frac{e^{-\lambda\tau e^{\theta+.5\delta^2}} (\lambda\tau e^{\theta+.5d^2})^J}{J!}$$

$$\times\ C_{BS}\left[S,\ \tau,\ R_d - R_f + J\,\frac{\theta+.5d^2}{\tau}\right.$$

$$\left. -\lambda(e^{\theta+.5d^2} - 1),\ \sqrt{\sigma_0^2 + J\,\frac{\delta^2}{\tau}},\ K\right]$$

where σ_0^2 is the variance of the diffusion component of the process, λ is the mean number of jumps per interval, θ is the mean jump size, and δ^2 is the variance of the jump size Y. Note that the total variance of the mixed jump-diffusion process is given by

$$\sigma^2 = \sigma_0^2 + \lambda\frac{\delta^2}{\tau}$$

One important question is whether either the stochastic volatility or mixed jump-diffusion models can explain the observed volatility smile. In theory, either model could account for the phenomenon of the symmetric smile. Taylor and Xu (1994) are able to trace a portion of the smile to stochastic volatility. Taking the analysis further, Bates (1994) creates a "nested" stochastic volatility-jump-diffusion model that he tests with trading data on the IMM deutschemark futures options. Bates reports:

> *The stochastic volatility model cannot explain the "volatility smile" evidence of implicit excess kurtosis, except under parameters implausible given the time series properties of implied volatilities. Jump fears can explain the "volatility smile," and are consistent with an 8% jump in the \$/DM futures price observed over 1984–91. (p. 69)*

LOCAL VOLATILITY MODELS

The concept known as local volatility[5] was discovered independently by Derman and Kani (1994), Rubinstein (1994), and Dupire (1994). In the Derman and Kani approach, the entire volatility surface is integrated into a flexible, arbitrage-free binomial lattice. Every node of the Derman and Kani tree has its own volatility. Derman and Kani achieve this by making volatility a flexible but deterministic function of the underlying price and time:

Local Volatility

$$\frac{dS}{S} = \mu dt + \sigma\left(S, t\right) dz$$

It is important to understand that in this formulation volatility, though a function of a stochastic variable, S, is itself not a stochastic variable. As such, volatility can be hedged with transactions in the underlying instrument. As a consequence, movements in volatility do not require the introduction of investor risk preferences. Moreover, the BSM partial differential equation must hold at each node in the Derman and Kani tree. However, there is no closed-form solution for the value of an option, as there is in the BSM framework. Hence practitioners must resort to numerical methods, such as binomial models and finite difference methods.

Dupire's (1994) work on local volatility is in a continuous time framework. Dupire's equation, as it is now known, allows for the extraction of local volatility from an established volatility surface across strikes and maturities:

Dupire's Equation

$$\sigma_{LV} = \left[\frac{\dfrac{\partial C}{\partial \tau} + \left(R_d - R_f\right) K \dfrac{\partial C}{\partial K} + R_f C}{\dfrac{1}{2} K^2 \dfrac{\partial^2 C}{\partial K^2}}\right]^{\frac{1}{2}}$$

I look at the local volatility approach as a procedure to extract information imbedded in the volatility surface about the strike structure and

[5]Chriss (1997), Berger (1996) are good sources for further discussion of these models.

term structure of volatility. It has the potential to be of great advantage in the pricing of options, and in particular, barrier options. For example, one would expect that a significant strike skew in favor of the low-delta options would allow a dealer to improve on the pricing of down-and-out knock-out options. Sadly, empirical tests[6] are not favorable to the local volatility models, as Dumas, Fleming, and Whaley (1998) found in tests on S&P 500 index options.

STOCHASTIC LOCAL VOLATILITY

A frequent criticism of the stochastic volatility and local volatility models is that they do not produce deltas precise enough for hedging purposes. A second complaint is that they may not be able to work under conditions of great market stress, an example being the chaotic market conditions in 2008, as I described in Chapter 5. That period, with its phenomenal volatility spikes sorely tested all option models. Theoreticians have responded by fortifying local volatility models with the addition of new stochastic properties. One new group of models allows volatility the freedom to rise or fall on its own with no associated change in the underlying exchange rate. Some of these models have come to be known as featuring "stochastic local volatility." At the present time, Bloomberg Finance, L.P., a major supplier of online data and analytical tools, recommends using a stochastic local volatility model of its own development for best pricing barrier currency options (Tataru and Fisher 2010).

There is no one single stochastic local volatility model. The most basic formulation calls for separate implementation of a stochastic volatility model and a local volatility model. Once calibrated, these two are combined with a "mixing rule," something as simple as a weighted average of the option prices implied by the two models. The mixing rule weights could be derived from some arbitrary rule, or they could be inferred from a historical goodness-of-fit analysis. A second way is to fit a local volatility model across alternative "volatility states," which then are combined according to assigned probabilities.

A more sophisticated example of a stochastic local volatility model comes from Ren, Madan, and Qian (2007). Three equations govern the spot

[6]Alexander and Nogueira (2004), Bochouev and Isakov (1997, 1999), and Avallaneda, Friedman, Holmes, and Samperi (1997).

exchange rate and volatility:

$$\frac{dS}{S} = \left(R_d - R_f\right) dt + \sigma\,(S,t)\,Z(t) dW_S(t)$$

$$dlnZ = \kappa\left(\theta\,(t) - lnZ\right) dt + \lambda dW_Z(t)$$

$$\theta(t) = -\frac{\lambda^2}{2\kappa}\left(1 + e^{-2\kappa t}\right)$$

The first equation almost makes the movements in the spot exchange rate into a local volatility process. But there is an extra variable, Z. It functions as an independent stochastic component. Z is driven by the second equation. The two Ws are Brownian motion processes. In the basic form of the model the Ws are uncorrelated. The rate of mean reversion is κ, and λ is the volatility of volatility. Z begins by the assumption $Z(0) = 1$. $\theta(t)$ is the long-term deterministic drift. Note that the third equation, for $\theta(t)$, forces the unconditional expectation of $Z(t)^2$ to be unity.

STATIC REPLICATION OF BARRIER OPTIONS

All of the models for pricing barrier and binary options that were introduced in Chapter 8 and the further models described in this chapter are commonly predicated on the ideal of being able to execute dynamic replication programs. Dynamic replication is widely used in practice, but it is not without its problems. In theory, a dynamic program requires continuous adjustment. In practice, that is impossible. Moreover every adjustment in a dynamic program incurs transaction costs, though these may not be explicitly accounted for in elementary models. Barrier options have special problems because dynamic hedging can be extremely expensive when gamma levels are large, an example being when spot is near to a barrier.

There is another way to replicate options, including barrier and binary options. It is called static replication or static hedging. The basic idea is to manufacture a targeted barrier option by constructing a portfolio of other options, usually well-traded vanilla options.[7]

I discuss three types of static replication with barrier options in mind. All three use vanilla options to replicate barrier options. The first is a special case called "put-call symmetry." This only works when the domestic and

[7]Derman (2007).

foreign interest rates happen to be equal. The second, called the method of Carr and Chou (1997), uses vanilla options of varying strikes but all with a common expiration date. Finally I introduce the approach of Derman, Ergener, and Kani (1995), or DEK, that uses vanilla options of one single strike but many expiration dates.

Put-Call Symmetry

Suppose the interest rates in the two currencies, R_d and R_f, happen to be equal. If the strike and barrier of the target option happen to be equal, certain barrier options can be replicated with simple forward contacts.

This works as follows. Suppose the target is a down-and-out knock-out USD call/JPY yen put. Assume spot (S) is currently above the barrier (H) and that the barrier is equal to the option strike (K). If the barrier is never triggered, then the option will pay this amount at expiration

$$S_T - K$$

This happens to be the same payoff as a forward contract struck at the option strike. Before expiration, this forward contract is worth

$$e^{-R_f \tau} S_t - e^{-R_d \tau} K$$

according to the forward contact valuation formula introduced earlier (with F_0 set equal to K). Interestingly, if the domestic and foreign interest rates happened to be equal, the value of the forward would be zero anytime the barrier is hit ($S_t = K$). This means a forward contract would constitute a static hedge for this down-and-out call. However, if the barrier were triggered, the down-and-out call would go out of existence, and the forward would have to be liquidated immediately—still, this qualifies as what is known as a static hedge, but only in the case that $R_d = R_f$ throughout the life of the option.

Now relax the assumption that the strike is set equal to the barrier. This makes forwards unusable as a static hedge. But there is still some insight to be gained about knock-out options from an artifact of option algebra called "put-call symmetry" (formally described by Carr 1994). But the assumption that the two interest rates are equal must remain. Put-call symmetry dictates that the following relationship must hold:

$$\frac{C}{\sqrt{K_C}} = \frac{P}{\sqrt{K_P}}$$

where

$$\sqrt{K_C K_P} = F$$

K_C and K_P refer to the strikes of the call and the put, respectively. The second equation is a condition that places the strikes of the call and the put equidistant from each other on opposite sides of the forward. One can think of put-call symmetry as saying that a call struck at twice the forward has the same value as a put struck at half the forward.

To illustrate, if the forward were 100, then a symmetrical pair of strikes for a call and a put would be 105 and 95.24, respectively. The symmetry principle says that the ratio of each of these option values to the square root of their strikes would have to be equal. However, one should note that unlike the put-call parity theorem, which is a true arbitrage relationship, put-call symmetry is merely an exercise in algebra conditioned on the assumption that there is no smile or skew in option volatility.

Carr, Ellis, and Gupta (1998) use put-call symmetry to replicate barrier options. Their basic model requires that the domestic and foreign currency interest rates be equal in the case of currency options. The replication of a down-and-out call consists of a long position in a vanilla call and a short position in some number of units of a vanilla put. The short put is struck symmetrically opposite to the call around the out-strike of the down-and-out call:

$$C_{D\&O} = C(K) - \sqrt{\frac{K}{K_P}}P(K_P)$$

where the put strike is defined as

$$K_P = \frac{H^2}{K}$$

Consider a down-and-out call with the following parameters:

Option: USD put/JPY call

Face: $1,000,000

Spot: 90

Strike: 85

Out-Strike: 95

Interest Rate (USD): 5%

Interest Rate (JPY): 5%

Volatility: 14%

Term: 3 Months

This knock-out call is worth $7,516 (equal to .00008842 times the face in yen) according to the barrier option pricing model that was presented in the previous chapter.

According to Carr, Ellis, and Gupta, the value of the knock-out call should be equal to the difference between a vanilla USD put/JPY call and some number of symmetrically struck USD call/JPY put options. Using put-call symmetry, the strike of the yen put is 106.18 and the number of puts is 0.8947. Using BSM—and assuming a constant volatility across all of these options—it can be shown that value of the 85-strike yen call is $7,710 and that the value of the 106.18-strike yen put is $217. Therefore:

	Units	Option Value	Total
USD Put/JPY Call (85 Strike) $7,710 less	1.00	$7,710	$7,710
USD Call/JPY Put (106.18 Strike) equals	0.8947	$217	($194)
Synthetic Knock-Out			$7,516

In the example, the symmetry replication appears to work, but there are some qualifiers. There is the problem that the 106.18-strike USD call/JPY put, in the example, being a low-delta option, might have a very different quoted volatility than the 85-strike USD put/JPY call. This was outside of the initial analysis because the notions of volatility smile and skew are non-existent in put-call symmetry. Moreover, the replication technique requires that the vanilla put and call be liquidated were the knock-out barrier, 95, to be struck. This might not be possible at reasonable costs. The wind-down of the replication vanilla options requires the repurchase of 0.8947 units of the 106.18-strike put and the sale of the 85-strike yen call—either of these options could be skewed, meaning that the outcome of the whole exercise is uncertain from the beginning.

The predicament of the replicator is aggravated by the necessity to do as many as four vanilla option trades. This means, of course, crossing as many as four option bid-ask spreads. Two trades are required to establish the replication portfolio and possibly two more to remove them if the barrier option is knocked-out. These costs could present an insurmountable argument against static replication. Proponents of the technique point out that static replication requires no dynamic rebalancing, hence no need to do periodic spot trading to achieve delta equality with the barrier option. However, this exaggerates the advantage of static hedging. In the context of running a sizeable option market-making operation, significant economies of scale in hedging exist because the book may contain positions that are naturally offsetting (being long some options, short others, or having positions in both puts and calls). Still, put-call symmetry and its associated replication are insights into barrier pricing.

The Method of Carr and Chou

Now consider more general methods that work regardless of whether the domestic and foreign interest rates are equal and the placement of the strike relative to the barrier. I start with Carr and Chou (1997) who produce a replication strategy based on a reformulation of Black-Scholes in the context of the famous Arrow-Debrue state-dependent claim framework.[8] They derive a pricing formula for a generic European payoff function $f(S_T)$ that replicates the claim with a portfolio of bonds, forwards, and a strip of vanilla puts and calls.

$$\begin{aligned} V_t = {} & f(\kappa)B(t, T) + f'(\kappa)\,(S_T - \kappa B(t, T)) \\ & + \int_0^{\kappa} f''(v)P(t, T, v)dv + \int_{\kappa}^{\infty} f''(v)C(t, T, v)dv \end{aligned}$$

where

$B(t, T)$ is the price of a zero coupon bond at time t with maturity T.

$P(t, T, v)$ is the price of a put at time t with maturity at T and strike v.

$C(t, T, v)$ is the price of a call at time t with maturity at T and strike v.

and $f(\kappa)$, $f'(\kappa)$, and $f''(\kappa)$ are the weights for the zero coupon bond, the forward contract, and the put and calls, respectively.

Their replication methodology works across strikes at the common expiration date. But their valuation formula requires the payoff function to be twice differentiable. This is clearly not the case for a barrier option, since the payoff function is discontinuous at the barrier.

Their solution is to create a theoretical European payoff function, called the adjusted payoff, that matches the value of the barrier option and is in

[8]Patel (2005).

EXHIBIT 9.2 Carr and Chou's Adjusted Payoff for Down Securities

Barrier Security	When $S_T > H$	When $S_T < H$
No-Touch Binary Put	1	$-(S_T/H)^p$
One-Touch Binary Put[1]	0	$1 + (S_T/H)^p$
Down-and-Out Call	$\max(S_T - K_c, 0)$	$-(S_T/H)^p \max(H^2/S_T - K_c, 0)$
Down-and-Out Put	$\max(K_p - S_T, 0)$	$-(S_T/H)^p \max(K_p - H^2/S_T, 0)$

[1]European Exercise
$p = 1 - (2(R_d - R_f)/\sigma^2)$

fact twice differentiable. This equivalence must hold at all points in the life of the barrier option, including at the barrier. Two things are important to understand. First, it is assumed that the replication portfolio will be immediately liquidated if the barrier is hit. This explains the seemingly impossible shape of the adjusted payoff function. Secondly, Carr and Chou rely totally on the BSM analytical valuation models of the sort in Chapter 8.

The adjusted payoffs for some common barriers are given in Exhibit 9.2.

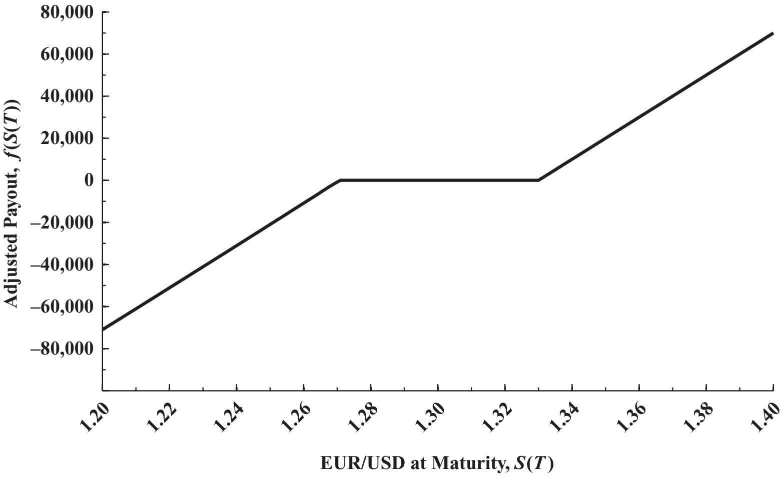
These payoffs are graphed in Exhibit 9.3.

Once the "adjusted" payoff security has been isolated, Carr and Chou are able to price the target barrier option by pricing the intermediate adjusted payoff given their aforementioned methodology. More to the point, they build portfolios of vanilla puts and calls to replicate the adjusted payoff, and by extension the target barrier option. Exhibit 9.4 shows the results of Carr and Chou static replication using an increasingly larger number of vanilla options in the portfolio.

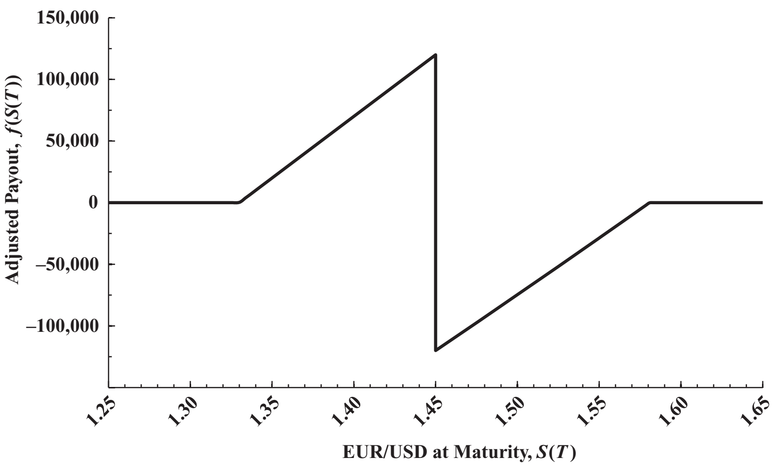
The portfolio can be made to converge more quickly on the analytic value when the strikes of the vanilla replication portfolio are deliberately chosen.

The Method of Derman, Ergener, and Kani

Derman, Ergener, and Kani (1994) (DEK) invented a static replication technique for barrier options that works by constructing a portfolio of vanilla options, all but one with strikes equal to the barrier but at varying times to expiration. One could say that DEK is more flexible than Carr and Chou in the fundamental sense because the latter is rooted in Black-Scholes methodology whereas the former is rooted in the wider context of local volatility frameworks (including Black-Scholes).

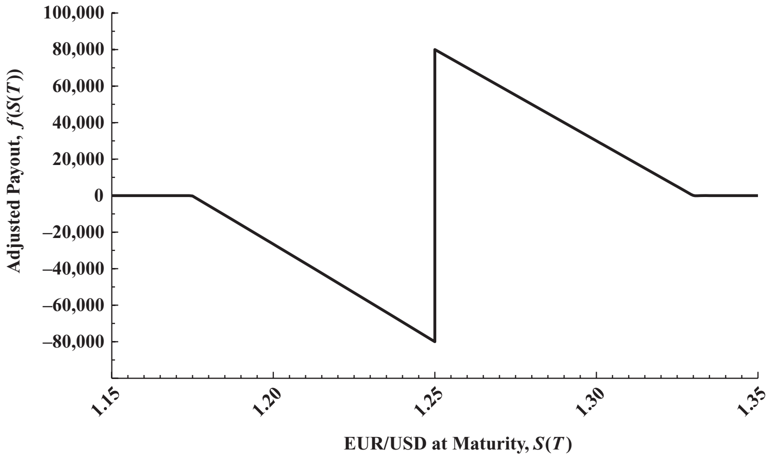


Panel A: Down-and-Out EUR Call/USD Put: 1 Year; strike = 1.3300; barrier = 1.3000; vol = 11.41%; R_d = 0.41%; R_f = 0.56%.

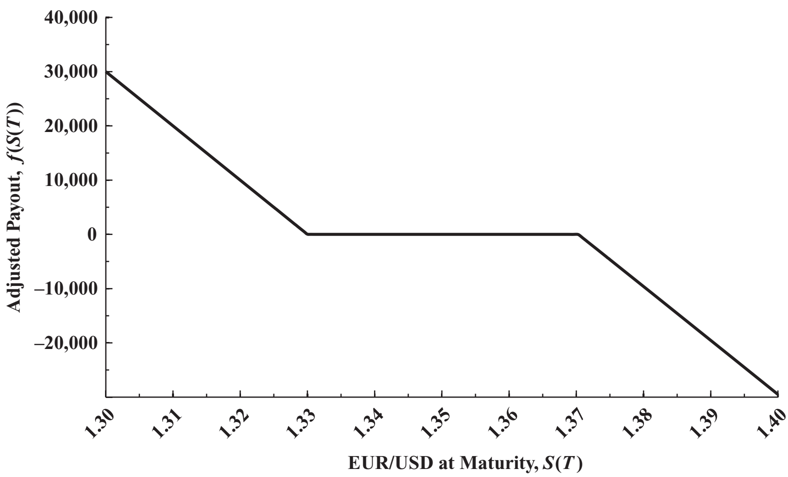


Panel B: Adjusted Payout for Up-and-Out EUR Call/USD Put: 1 Year; strike = 1.3300; barrier = 1.4500; vol = 11.41%; R_d = 0.41%; R_f = 0.56%.

EXHIBIT 9.3 Carr and Chou Adjusted Payoffs

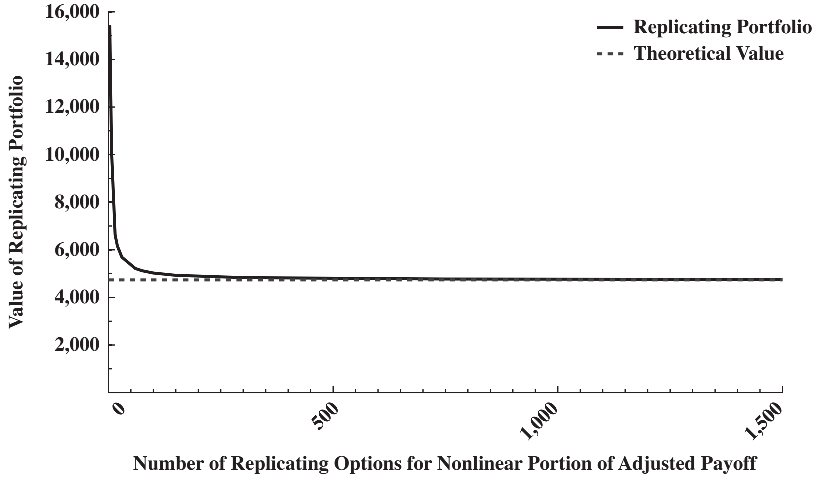


Panel C: Adjusted Payout for Down-and-Out EUR Put/USD Call: 1 Year; strike = 1.3300; barrier = 1.2500; vol = 11.41%; R_d = 0.41%; R_f = 0.56%.



Panel D: Adjusted Payout for Up-and-Out EUR Put/USD Call: 1 Year; strike = 1.3300; barrier = 1.3500; vol = 11.41%; R_d = 0.41%; R_f = 0.56%.

EXHIBIT 9.3 (*Continued*)



(A)

Number of Replicating Options (Strikes Evenly Distributed)	Value of Replicating Portfolio
3	15,434.78
7	10,159.30
15	6,627.53
20	6,159.45
30	5,690.99
60	5,215.46
75	5,119.77
100	5,023.94
150	4,927.82
300	4,831.46
750	4,773.56
1500	4,754.23

Theoretical Up-and-Out Call Value	4,734.90
15 replicating options (strikes not evenly distributed)	4,776.13

(B)

EXHIBIT 9.4 Carr and Chou Static Replication: Convergence Graph of Replicating Portfolio to Kick-Out EUR Call/USD Put
One year; strike = 1.3300; barrier = 1.4500; vol = 11.41%; $R_d = 0.41\%$; $R_f = 0.56\%$.

I go immediately to a simple example.[9] The purpose is to replicate a one-year kick-out EUR call/ USD put of these characteristics:

Face (EUR): 1,000,000
Spot: 1.3200
Strike: 1.3300
Barrier: 1.4500
R (USD): 0.41%
R (EUR): 0.56%
Term: 1 year
Volatility: 11.41%

Standard analytical models say this option is worth \$4,740 (equal to .00474 USD times the face in euros). This option "kicks out"—meaning extinguishes worthless—in the in-the-money region if the spot breaches 1.4500.

A vanilla option of the same basic terms—but without a barrier feature—would be worth \$54,201 (equal to .05420 USD times the face in euros). This option is called the "primary vanilla." It represents what the kick-out call would be worth if no barrier event were to take place. To take account of the barrier, I will construct successively larger portfolios that consist of the primary vanilla and the purchase and sale of other vanilla options all struck at the barrier but with expiration dates that span the space between when the kick-out is acquired and when it expires (assuming no barrier date). To illustrate this technique I will use only three other vanilla options. All three—call them Options A, B, and C—are struck at the barrier, 1.4500. Option A has an original tenor of one year. Options B and C have original tenors of two-thirds of one year and one-third of one year, respectively.

The first crack at building a replication portfolio consists of the purchase of the primary vanilla and the sale of some amount of Option A. This portfolio, called portfolio 1, is designed to have zero value at two-thirds of the way into the kick-out's life when spot is equal to the barrier 1.4500. This means there is one-third of a year remaining in the life of the primary vanilla and also in the life of Option A. On this date, and using all of the same inputs as I stated previously for the kick-out, I find that the Primary Vanilla is worth \$123,236. Option A, with a higher strike, is worth substantially less. However, if I sell 3,270,804 euros face of Option A, Portfolio One will have zero value on the date if spot trades at the barrier.

[9] Liljefors (2001).

I now move backward in time to the one-third marker. At this time the Primary Vanilla and Option A have two-thirds of one year of life. I now introduce Option B that at that time has one-third of a year remaining. Portfolio Two has the objective of being worthless at the one-third marker when spot is equal to the barrier 1.45000. This is accomplished by the purchase of 1,127,961 euros of face of Option B.

Finally, I move one more unit backward to the trade date. At this time the Primary Vanilla has one year, Option A has one year, Option B has two-thirds of a year, and a new option, Option C, has one-third of a year of life. Portfolio Three consists of Portfolio Two plus the purchase of 350,825 euros of face of Option C. If the spot exchange rate were to immediately rise to the barrier 1.4500, Portfolio Three would be worthless.

The value of a similar static replication Portfolio V at time t_0 with n options is then:

$$V(t_0) = C^*(t_0, K) + \phi C'(t_0, H)$$

where $C^*(t_0, K)$ is the price of the primary vanilla call at time t_0 with strike K and maturity T equal to that of the barrier option, and $\phi = (\phi_1, \phi_2, \ldots, \phi_n)$ and $C = (C_1, C_2, \ldots, C_n)$ are the portfolio and prices of vanilla calls with strike K equal to the barrier H, and maturities $T_1, T_2, \ldots T_n$ (where $T_i < T_{i+1}$ and $T_n < T$) respectively.

To solve for ϕ, we start first from the nth option (that is the option with the longest maturity) to get ϕ_n and work backward:

$$\phi_i = -\frac{C^*(t_i, K) + \sum_{j=i+1}^{n} \phi_j C_j(t_i)}{C_i(t_i)}$$

Exhibit 9.5 demonstrates the construction of such replication portfolios.

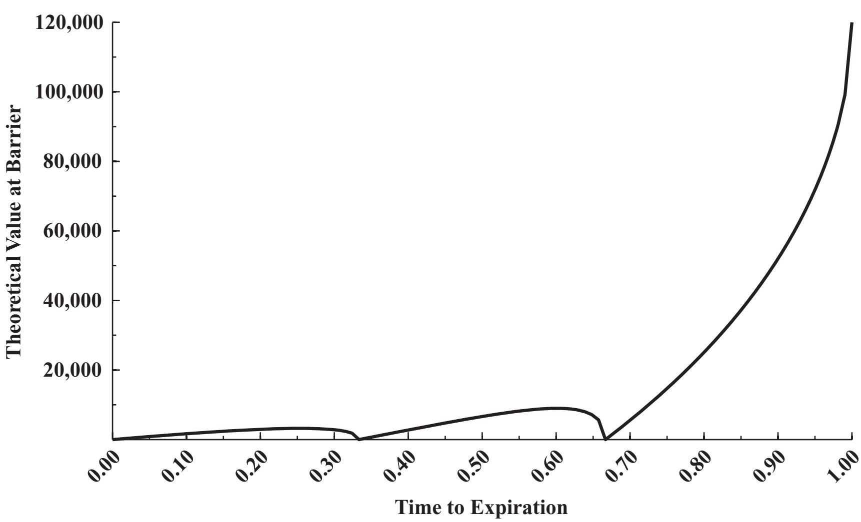
The idea is to show the effect of adding an arbitrary number of options, each one struck at the barrier but at different expiration dates. Note that although this is called a static hedge, should a barrier event occur, there would be an immediate need to liquidate any remaining vanilla options.

The Limitations of Static Hedging

Static hedging gives us new insights into barrier options. But it has its own set of problems. I focus on an out-barrier. If the barrier is hit, the target is immediately dead—but the replication portfolio is still alive until it is liquidated, presumably at once. The replication portfolio ought to be worthless, but events might have conspired to make that not be true. In practical fact it is unlikely to be true if for no other reason than that market conditions

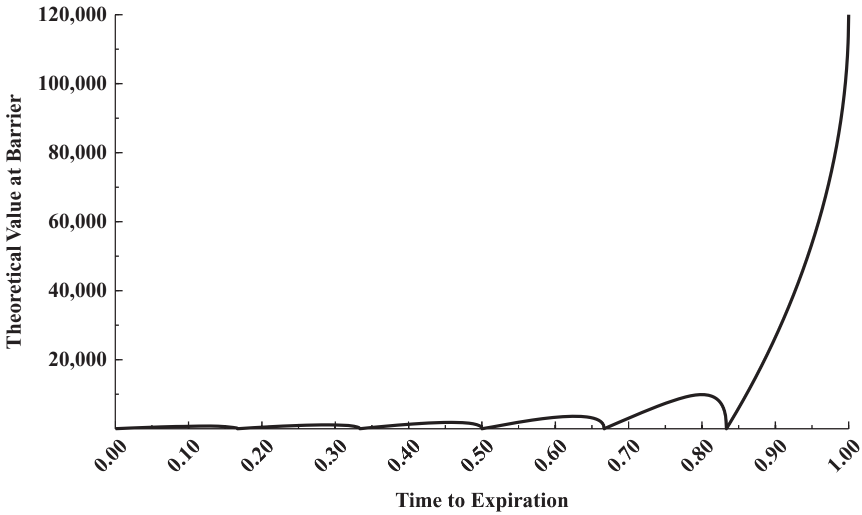
Panel A: Summary	
Replicating Portfolio	
Number of Options	Value
3	9,089.41
6	6,931.61
12	5,832.27
36	5,099.85
60	4,953.82
120	4,844.38
240	4,789.66
480	4,762.29
960	4,748.60
1920	4,741.76

Replicating Portfolio for Kick-Out EUR Call/USD Put: 1 Year; Spot = 1.3200; Strike = 1.3300; Barrier = 1.4500; Vol = 11.41%; $R_d = 0.41\%$; $R_f = 0.56\%$; Theoretical Value of Kick-Out Option = 4,734.90.

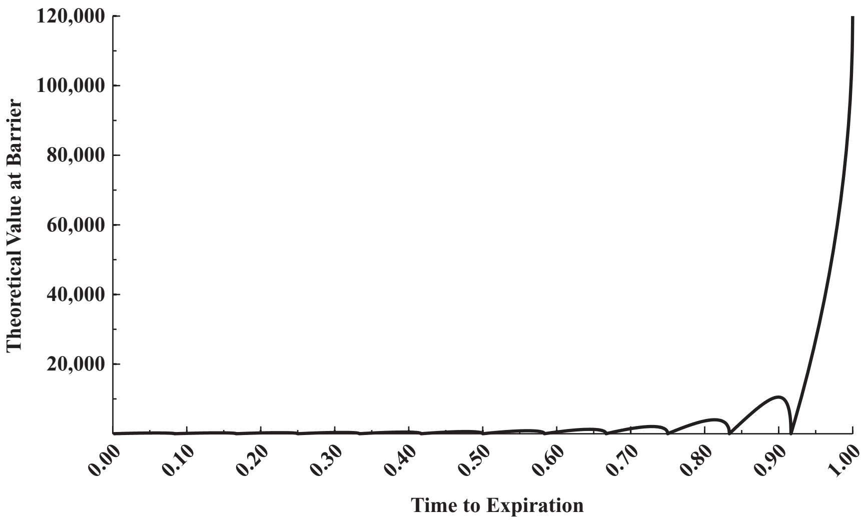


Panel B: Replicating Portfolio with 1 Primary Option and 3 Other Options

EXHIBIT 9.5 Derman, Ergener, and Kani Static Replication

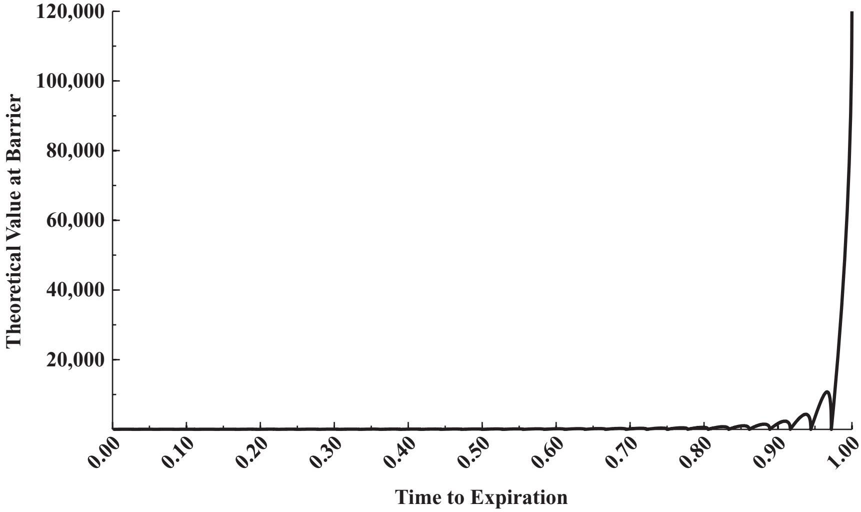


Panel C: Replicating Portfolio with 1 Primary Option and 6 Other Options

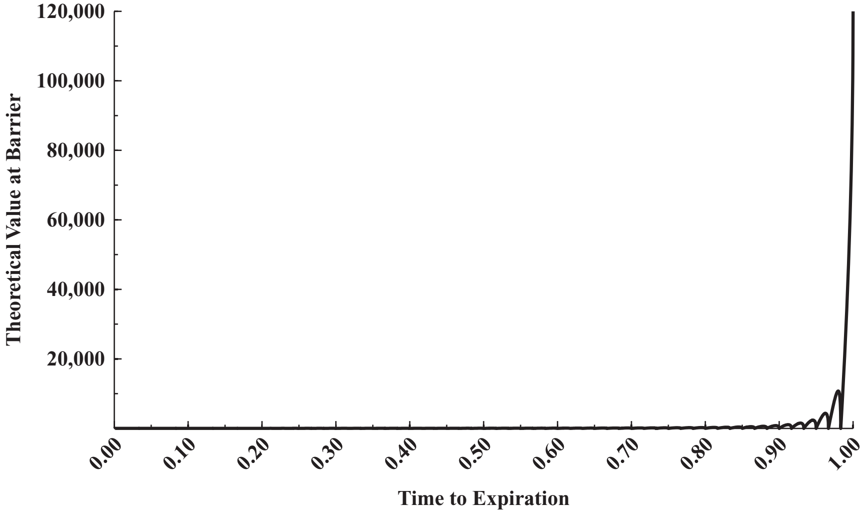


Panel D: Replicating Portfolio with 1 Primary Option and 12 Other Options

EXHIBIT 9.5 (*Continued*)

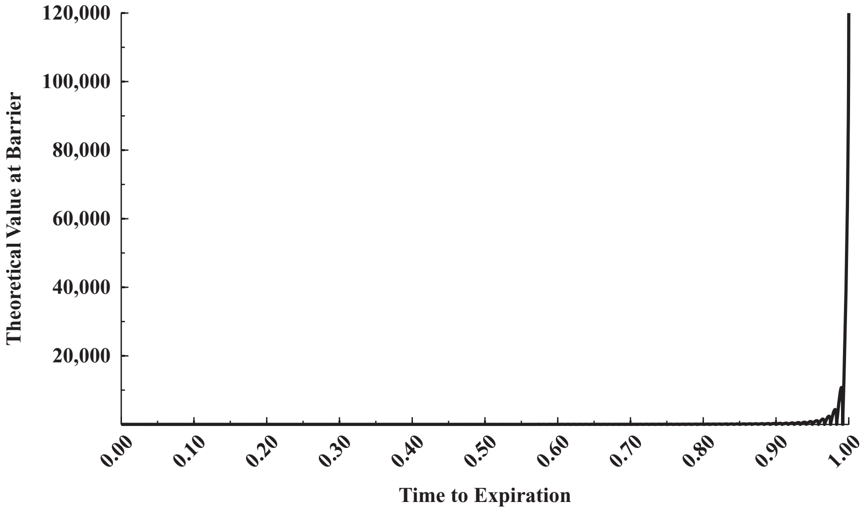


Panel E: Replicating Portfolio with 1 Primary Option and 36 Other Options

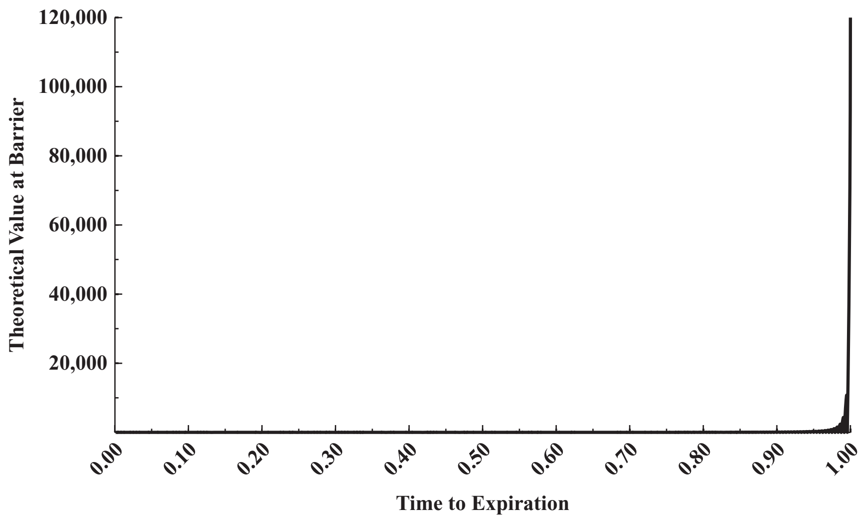


Panel F: Replicating Portfolio with 1 Primary Option and 60 Other Options

EXHIBIT 9.5 (*Continued*)

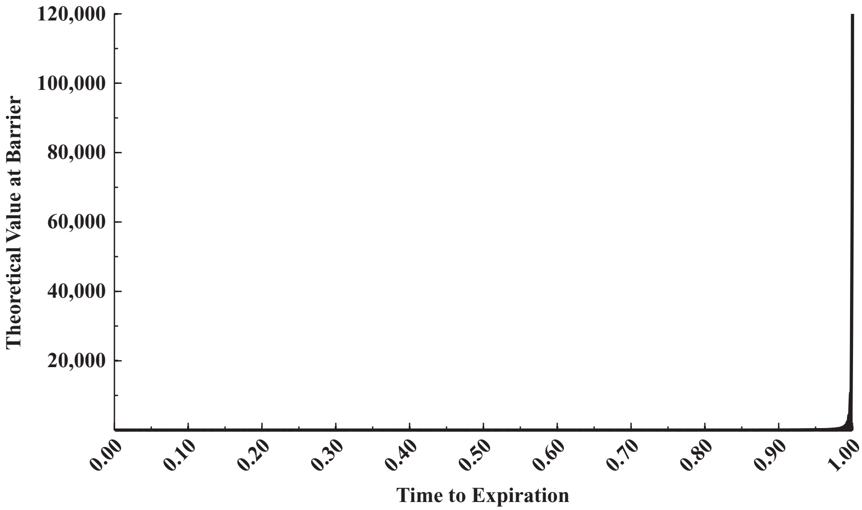


Panel G: Replicating Portfolio with 1 Primary Option and 120 Other Options

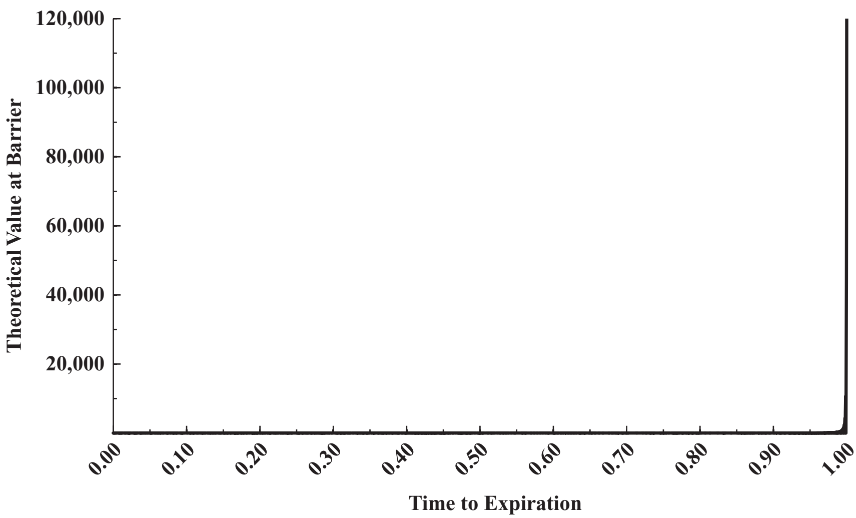


Panel H: Replicating Portfolio with 1 Primary Option and 240 Other Options

EXHIBIT 9.5 (*Continued*)

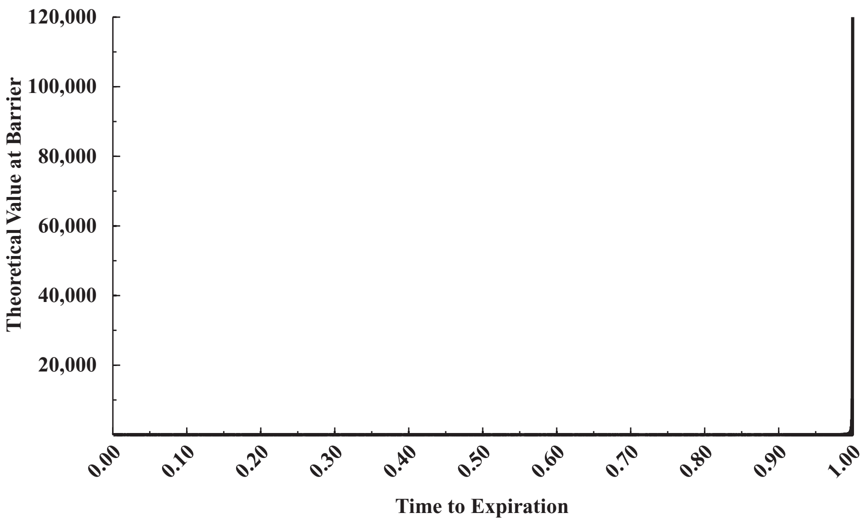


Panel I: Replicating Portfolio with 1 Primary Option and 480 Other Options



Panel J: Replicating Portfolio with 1 Primary Option and 960 Other Options

EXHIBIT 9.5 (*Continued*)



Panel K: Replicating Portfolio with 1 Primary Option and 1920 Other Options

EXHIBIT 9.5 (*Continued*)

with respect to volatility and interest rates have changed over the life of the option. The second problem is that the construction of a truly tight-fitting replication portfolio might involve initial trading in a great many options. Static replication is supposed to get away from the costs of trading via dynamic hedging but may only be substituting trading in options for trading in the underlying.

APPENDIX 9.1: EQUATIONS FOR THE HESTON MODEL

The Processes Driving S and V:

$$dS_t = \left(R_d - R_f\right) S_t dt + \sqrt{v_t} S_t dZ_1$$
$$dv_t = -\lambda \left(v_t - \bar{v}\right) dt + \eta \sqrt{v_t} dZ_2$$
$$\left\langle dZ_1, dZ_2 \right\rangle = \rho dt$$

where

v_t is the instantaneous volatility.
$\bar{v}$ is the long-term volatility.
dZ_1 and dZ_2 are two Weiner processes that are correlated at level ρ.

λ is the rate at which the instantaneous volatility moves to the long-term volatility.

η is the volatility of volatility.

All other terms are as previously defined.

The Heston Partial Differential Equation:

$$\frac{\partial C}{\partial t} + \frac{1}{2}\nu S^2 \frac{\partial^2 C}{\partial S^2} + \rho\eta S v \frac{\partial^2 C}{\partial S \partial v} + \frac{1}{2}\eta^2 \nu \frac{\partial^2 C}{\partial v^2} + R_d S \frac{\partial C}{\partial S}$$
$$- \lambda \left(v - \bar{v}\right) \frac{\partial C}{\partial v} - R_d C = 0$$

The Pricing Model for Option C

$$c = \left[e^{-R_f \tau} S \Pi_1 - \mathrm{e}^{-\mathrm{R_d}\tau} \mathrm{K} \Pi_2\right]$$

where:

$$\Pi_j = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \mathrm{Re}\left\{\frac{\exp\left(B_j\left(u, \tau\right)\bar{v} + D_j\left(u, \tau\right)v + iux\right)}{iu}\right\} du, \; j = 1, 2$$

$$x = \ln\left(\frac{S}{K}\right)$$

$$D_j\left(u, \tau\right) = r_- \frac{1 - e^{-d\tau}}{1 - ge^{-d\tau}}$$

$$B_j\left(u, \tau\right) = \lambda\left\{r_-\tau - \frac{2}{\eta^2}\ln\left(\frac{1 - ge^{-d\tau}}{1 - g}\right)\right\}$$

$$r_{\pm} = \frac{\beta \pm \sqrt{\beta^2 - 4\alpha\gamma}}{2\gamma}$$

$$d \equiv \sqrt{\beta^2 - 4\alpha\gamma}$$

$$g \equiv \frac{r_-}{r_+}$$

$$\alpha \equiv -\frac{u^2}{2} - \frac{iu}{2} + iju$$

$$\beta = \lambda - \rho\eta j - \rho\eta iu$$

$$\gamma = \frac{\eta^2}{2}$$

Options on Foreign Exchange, Third Edition
David F. DeRosa

CHAPTER 10

Non-Barrier Exotic Currency Options

Non-barrier currency options tend to be used more by hedgers than by currency traders. There is a great variety of non-barrier currency options, but the focus in the chapter will be on the options that are common in the marketplace. Those are average rate currency options, compound currency options, basket options, and quantos options.

AVERAGE RATE CURRENCY OPTIONS

Average rate currency options,[1] also called "Asian currency options," exist in the foreign exchange market but are more prevalent in physical commodities markets.

Average options are useful when a commodity or asset is susceptible to price manipulation. An average option makes it more difficult for either the buyer or writer of the option to be cheated because a series of prices during the averaging period would have to be rigged to materially disadvantage either party.

Average options are also useful in situations where hedgers are primarily concerned with the average price of a commodity or currency of which they must make regular purchases or sales.

Intuitively, one would expect that an average option would have to be less valuable than a vanilla option. This is because the standard deviation of an average of a series of prices that is not perfectly autocorrelated has to be less than the standard deviation of a single price in the series. In other

[1]For simplicity I will refer to average rate currency options simply as "average rate options."

words, the average options should sell for smaller implied volatilities than vanilla options (Kemna and Vorst 1990).

There are two types of average options: the average rate options (sometimes called average price) and the average strike option. As their names suggest, the average rate option is in- or out-of-the-money at expiration, depending on where the option's strike is relative to the average observed price over the averaging period. The payoff functions for average rate calls and puts are defined as

Average Rate Call

$$Max\,[0,\ A - K]$$

Average Rate Put

$$Max\,[0,\ K - A]$$

where A represents the average rate, and K represents the option strike price.

In the case of an average strike option, the strike is an average calculated from observed spot exchange rates. This means that the strike is actually not known until the end of the averaging period, which usually corresponds to the option's expiration. The payoff functions of average strike options are defined as

Average Strike Call

$$Max\,[0,\ S_T - A]$$

Average Strike Put

$$Max\,[0,\ A - S_T]$$

The average might be calculated from observations that span the option's entire life or from observations taken from a shorter period ending with the option's expiration day.

For either type of average option, the average is usually taken to be the arithmetic average, but other averages, such as the geometric mean, are sometimes used. The choice of the mathematical form of the average turns out to be of importance to the modeling of average options. This is because a closed-form solution for the geometric mean average option does exist, but there is no such solution for arithmetic mean average options.

Geometric Mean Average Options

The geometric mean of a series of discrete observations on the spot exchange rate is given by

$$G = \left(\prod_{i=1}^{n} S_i\right)^{\frac{1}{n}}$$

Kemna and Vorst (1990) derive a closed-form solution for a continuous geometric mean call option on a non-dividend-paying stock. Ruttiens (1990) presents the Kemna and Vorst model for geometric mean average currency calls:

Geometric Mean Average Rate Currency Call

$$C^{gmaro} = e^{-R_d\tau} e^{d^*} SN(d_1) - e^{-R_d\tau} KN(d_2)$$

where

$$d_1 = \frac{\left[\ln\left(\frac{S}{K}\right) + \frac{1}{2}\left(R_d - R_f + \frac{1}{6}\sigma^2\right)\tau\right]}{\sigma\sqrt{\frac{\tau}{3}}}$$

$$d^* = \frac{1}{2}\left(R_d - R_f - \frac{1}{6}\sigma^2\right)\tau$$

$$d_2 = d_1 - \sigma\sqrt{\frac{\tau}{3}}$$

This formulation assumes the option is at the beginning of its life and the averaging process to be conducted over the entire life of the option. Unfortunately, this model cannot revalue a geometric option after the averaging period has already begun.

The reason why this closed-form solution exists is because the geometric mean itself must be lognormally distributed when the underlying time series of random variables is lognormally distributed. This convenient property does not hold for the arithmetic mean.

Arithmetic Mean Average Options

As I mentioned, no closed-form solution exits for arithmetic mean average options under the standard assumption that spot exchange rates follow a lognormal diffusion process.

Kemna and Vorst (1990) propose an efficient control variate Monte Carlo simulation strategy for arithmetic mean average options. In their control variate approach for a call, for example, the value of the geometric mean counterpart option, whose value is known from the closed-form solution, is used as a lower bound to the arithmetic mean option. Because the geometric mean of any series is never greater than the arithmetic mean, the geometric mean average call establishes a lower bound for the arithmetic mean call.

Levy's Model

Levy (1990 and 1992) develops a different approach using an analytical closed-form approximation that proves to be reasonably precise. Levy's model is

Levy's Arithmetic Mean Currency Call

$$C^{amaro} = S_A N(d_1) - e^{-R_d \tau} K N(d_2)$$

where

$$S_A = e^{-R_d \tau} \frac{t}{T} S_{AV} + \frac{S}{Tg}(e^{-R_f \tau} - e^{-R_d \tau})$$

$$d_1 = \frac{1}{\sqrt{V}} \left(\frac{1}{2} \ln D - \ln K \right)$$

$$d_2 = d_1 - \sqrt{V}$$

$$V = \ln D - 2(R_d \tau + \ln S_A)$$

$$D = \frac{1}{T^2} \left[M + (tS_{AV})^2 + \frac{2tSS_{AV}}{g} (e^{g\tau} - 1) \right]$$

$$M = \frac{2S^2}{g + \sigma^2} \left[\frac{e^{(2g+\sigma^2)\tau} - 1}{(2g + \sigma^2)} - \frac{(e^{g\tau} - 1)}{g} \right]$$

$$g = (R_d - R_f)$$

In Levy's formulation, time starts at $t = 0$ and runs to expiration time T; time remaining to expiration, τ, is equal to $(T - t)$. The term S_{AV}

EXHIBIT 10.1 Arithmetic Mean Average Call Option—Levy's Model Compared to Kemna and Vorst Efficient Control Variate Monte Carlo Simulation

Spot	Levy's Model	MC Simulation	Standard Deviation	Absolute Difference
85.00	\$1,739	\$1,699	0.66	2.30%
86.00	\$3,130	\$3,087	0.71	1.39%
87.00	\$5,269	\$5,226	0.76	0.81%
88.00	\$8,336	\$8,303	0.78	0.39%
89.00	\$12,460	\$12,442	0.82	0.15%
90.00	\$17,693	\$17,693	0.87	0.00%
91.00	\$23,993	\$24,008	0.91	0.07%
92.00	\$31,237	\$31,262	1.01	0.08%
93.00	\$39,246	\$39,280	1.07	0.09%
94.00	\$47,813	\$47,845	1.16	0.07%
95.00	\$56,740	\$56,768	1.22	0.05%

50,000 simulations, daily observations; Face = \$1,000,000; USD put/JPY call; Strike = 89.3367; 90 days; Vol = 14.00% $R_d = 5\%$; $R_f = 2\%$.

represents the arithmetic average of the known spot rates. All other terms are as previously defined.

To arrive at the value of an arithmetic put, Levy suggests the following parity relationship:

Put-Call Parity for Arithmetic Mean Options

$$C_{amaro} - P_{amaro} = S_A - e^{-R_d \tau} K$$

An alternative approach would be to use Monte Carlo simulation to estimate the value of an arithmetic average currency option.[2] A comparison of the Levy model to simulation is contained in Exhibit 10.1. Appendix 10.1 describes the simulation methodology used in this exhibit.

COMPOUND CURRENCY OPTIONS

A compound currency option is an option that delivers another option upon exercise. A "ca-call" is a call option that delivers a vanilla call upon

[2]Turnbull and Wakeman (1991) and Curran (1992) also provide approximation formulas for average rate options.

expiration. A "ca-put" is a call option that delivers a vanilla put upon exercise. There are also put options that deliver vanilla calls and puts. Compound currency options were briefly mentioned in Chapter 6 as being the foundation of the Barone-Adesi and Whaley quadratic approximation model for American currency options.

Geske (1979) develops a model to value an option on a share of common stock as a compound option. His motivation stems from a theoretical concept from corporation finance that a share of stock is itself actually an option. In this paradigm, the firm in its totality consists of a series of claims to future cash flows. The ownership of the firm is divided into two classes. Bondholders have priority on the firm's cash flow up to some maximum level. This simple way to model the bonds is to think of them as zero coupon bonds. The common stock then becomes an option consisting of the right, but not the obligation, to purchase the entire firm from the bondholders for a strike price equal to the maturity value of the firm's debt. Declaration of bankruptcy would amount to the shareholders allowing their option to expire unexercised in a state of being out-of-the-money. If one were to assume that the total value of the firm (i.e., the combined value of all of the shares of the stock and all of the bonds) were to follow a diffusion process plus all of the other standard option pricing theory assumptions, then the value of the common stock would be given by the Black-Scholes model. But the value of an option on a share of stock would be an entirely different matter. It would be an option on an option. Also, the share of stock could not follow a diffusion process because it is an option on an asset, namely the firm, which is assumed to follow a diffusion process.

Compound options require the definition of some new variables: Let τ be the time to expiration of the underlying "daughter" vanilla option, and τ^* be the time to expiration of the "mother" compound option. Kc is the strike of the compound option.

To keep things compact, define the binary variables ϕ and η such that

Call on a Call	$\phi = 1$	and	$\eta = 1$
Call on a Put	$\phi = 1$	and	$\eta = -1$
Put on a Call	$\phi = -1$	and	$\eta = 1$
Put on a Put	$\phi = -1$	and	$\eta = -1$

The compound options under discussion are European exercise. At the time of expiration, T^*, the holder of the compound option has the right, but not the obligation, to exercise. Exercise of a compound call requires payment of the strike K_c to receive a vanilla currency call or put. Exercise of a compound put requires the delivery of a vanilla call or put in exchange for the strike K_c. This can be expressed as

Compound Option on Vanilla Call at Expiration

$$O_T^{compound} = MAX\,[0, \phi\,[C\,(S_{T^*}, K, T - T^*, \eta) - K_c]]$$

Compound options obey a put-call parity theorem:

Compound Option Put-Call Parity

$$O^{compound}\,(\phi = 1) - O^{compound}\,(\phi = -1) = \eta\,[C\,(S, \tau^*, \eta) - K_c]\,e^{-R_d\tau^*}$$

The first terms on the left-hand side of the equation are a compound call and compound put. The value of the vanilla call ($\eta = 1$) or put ($\eta = -1$) priced at compound expiration is represented by $C\,(S, \tau^*, \eta)$.

Following Geske (1979) and Briys, Bellalah, Mai, and Varenne (1998), the value of a compound option on a vanilla call or put on one unit of foreign exchange is given by:

Compound Currency Option

$$\begin{aligned} O^{compound} = {} & \phi\eta e^{-R_f\tau^*} SN_2\,(\phi\eta x, \eta y, \phi\rho) - \phi\eta e^{-R_d\tau^*} KN_2 \\ & \left(\phi\eta x - \phi\eta\sigma\sqrt{\tau}, \eta y - \eta\sigma\sqrt{\tau^*}, \phi\rho\right) \\ & -\phi K_c e^{-R_d\tau} N\left(\phi\eta x - \phi\eta\sigma\sqrt{\tau}\right) \end{aligned}$$

where

$$\rho = \sqrt{\frac{\tau}{\tau^*}}$$

$$x = \frac{\ln\left(\frac{e^{-R_f\tau}S}{e^{-R_d\tau}S_{cr}}\right)}{\sigma\sqrt{\tau}} + \frac{1}{2}\sigma\sqrt{\tau}$$

$$y = \ln\frac{\left(\frac{e^{-R_f\tau^*}S}{e^{-R_d\tau^*}K}\right)}{\sigma\sqrt{\tau}^*} + \frac{1}{2}\sigma\sqrt{\tau^*}$$

N_2 [a,b,ρ] is the cumulative bivariate normal distribution that covers the portion from minus infinity to a and from minus infinity to b and where ρ is the correlation coefficient. The value S_{cr} can be found by iteration of the following equation:

$$\eta S_{cr} e^{-R_f(\tau^* - \tau)} N(z) - \eta K e^{-R_d(\tau^* - \tau)} N\left(z - \sigma\sqrt{\tau^* - \tau}\right) - K_c = 0$$

where

$$z = \frac{\ln\left(\frac{e^{-R_f(\tau^*-\tau)} S_{cr}}{e^{-R_d(\tau^*-\tau)} K}\right)}{\sigma\sqrt{\tau^* - \tau}} + \frac{1}{2}\sigma\sqrt{\tau^* - \tau}$$

Exhibit 10.2 is the theoretical value of a compound option, a "ca-call" on EUR/USD graphed against the spot exchange rate.

The tricky aspect to trading compound options is the selection of the compound strike. Obviously, the higher the strike on the compound option, the more it costs to buy the vanilla through exercise of the compound option, and hence the smaller would be the initial value of the compound option. Yet a hedger must also balance the cost of the compound option against an alternative strategy of buying a vanilla option (with

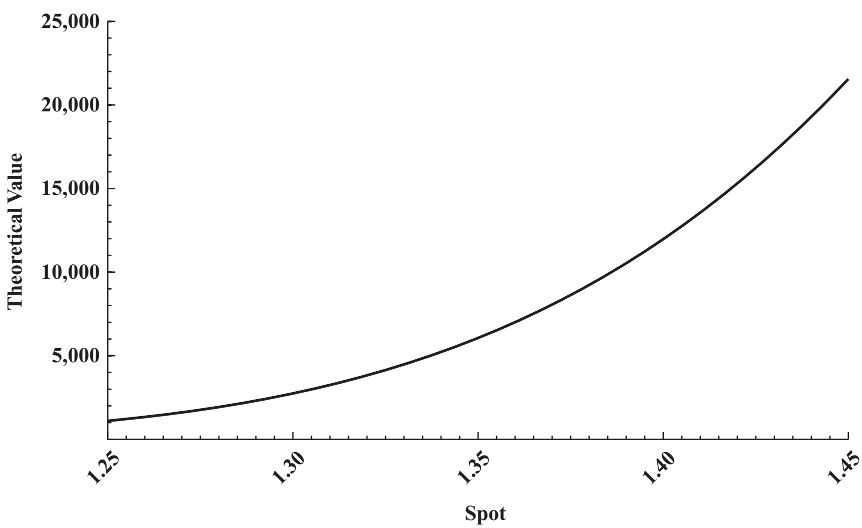


EXHIBIT 10.2 Compound Option "Ca-Call" on EUR/USD
EUR Face = 1,000,000; Compound strike = 0.25; "Daughter" strike = 1.33; Compound expiration = 1 Year; "Daughter" expiration = 1.5 years; Vol = 11.41%; $R_d = 0.41\%$; $R_f = 0.56\%$.

expiration at time T) that could be sold at the time of the compound option expiration if the option is not needed.

BASKET OPTIONS

A basket currency option is a put or a call on a collection of currencies taken together as a portfolio. By definition, a basket option is all-or-none exercise, meaning that there is no allowance for partial exercise of some of the currencies in the basket. Basket options can be cash or physically settled.

European basket options can be valued with the BSM model, but the user must have the implied volatility of the basket of currencies. This can be derived from the implied volatilities of each of the currencies with respect to the base currency, which are called the "leg" volatilities, as well as the implied volatilities of each of the associated cross exchange rates, which are called the "cross" volatilities. There are a total of $N(N+1)/2$ such terms for a basket that is comprised of N currencies. There are N leg volatilities and $N(N-1)/2$ cross volatilities.

Consider the example of a basket composed of euros, pounds, and yen where the base currency is the dollar. The implied volatility for the legs, USD/JPY, EUR/USD, and GBP/USD plus the implied for the crosses GBP/EUR, GBP/JPY, and EUR/JPY are needed to calculate implied volatility of the basket option. It is possible to derive a set of implied correlations from these volatilities. The implied correlation between currencies 1 and 2 is given by

$$\rho_{1,2} = \frac{\sigma_1^2 + \sigma_2^2 - \sigma_{1/2}^2}{2\sigma_1\sigma_2}$$

where σ_1^2 and σ_1^2 are the variances of exchange rates 1 and 2, and $\sigma_{1/2}^2$ is the variance of the cross rate of exchange between currencies 1 and 2. For example, the correlation between euro/dollar and dollar/yen is given by

$$\rho(\mathrm{EUR/USD}, \mathrm{USD/JPY}) = \frac{\sigma^2_{\frac{\mathrm{EUR}}{\mathrm{USD}}} + \sigma^2_{\frac{\mathrm{USD}}{\mathrm{JPY}}} - \sigma^2_{\frac{\mathrm{EUR}}{\mathrm{JPY}}}}{2\sigma_{\frac{\mathrm{EUR}}{\mathrm{USD}}}\sigma_{\frac{\mathrm{USD}}{\mathrm{JPY}}}}$$

The implied correlations can be used to create a set of covariance terms:

$$\mathrm{COV}(1,2) \equiv \sigma_{12} = \rho_{12}\sigma_1\sigma_2$$

that completes the variance-covariance matrix, ***V***

$$V = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_2^2 & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_3^2 \end{bmatrix}$$

The implied variance of the basket is equal to the variance-covariance matrix premultiplied by the row vector of the currency weights and postmultiplied by the column vector of the currency weights. The weight of each currency is defined as its percentage component in the basket. The implied volatility of the basket is equal to the square root of the implied variance. The value of a European basket option can be found directly from the forward exchange rate version of the BSM model.

Exhibit 10.3 contains a numerical example of the construction of a dollar-based basket option on euros, Sterling, and yen.[3]

The sensitivity of the price of the basket option to the various correlations between the currencies is shown graphically in Exhibit 10.4.

Basket options are favored by portfolio managers and corporate treasurers who seek to hedge a collection of currency exposures with a single option. The premium on a basket option is lower than the aggregate cost of purchasing separate options for each currency. This savings results from the fact that the implied volatility of the basket is less than the average of the separate currency implied volatilities (see Hsu 1995).

Said another way, the value of the basket option must be less than the value of a strip of vanilla currency options because there is a possibility that the basket option could expire out-of-the-money whereas one or more of the options in the strip could expire in-the-money. The principle can be understood in terms of correlation. Imagine a trader buying a basket option and simultaneously selling a strip of vanilla options on the component currencies in the basket. The net premium would be positive, as has been said. The combination would be *long correlation*. Conversely the combination of being short the basket and long the strip would be *short correlation* because the strip is more valuable when the correlation between the basket components breaks down.

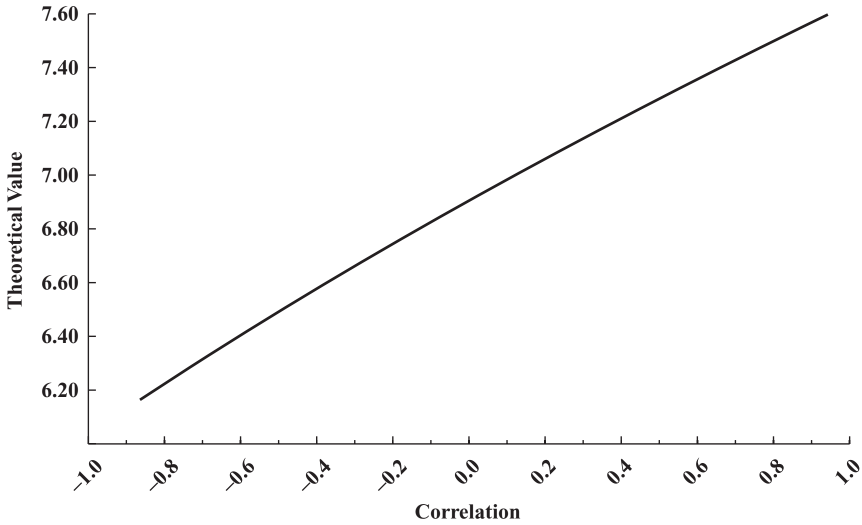
QUANTOS OPTIONS

The usual variety of quantos option is put or a call on a foreign stock index that features an implied fixed exchange rate. A second type is a quantos currency binary option.

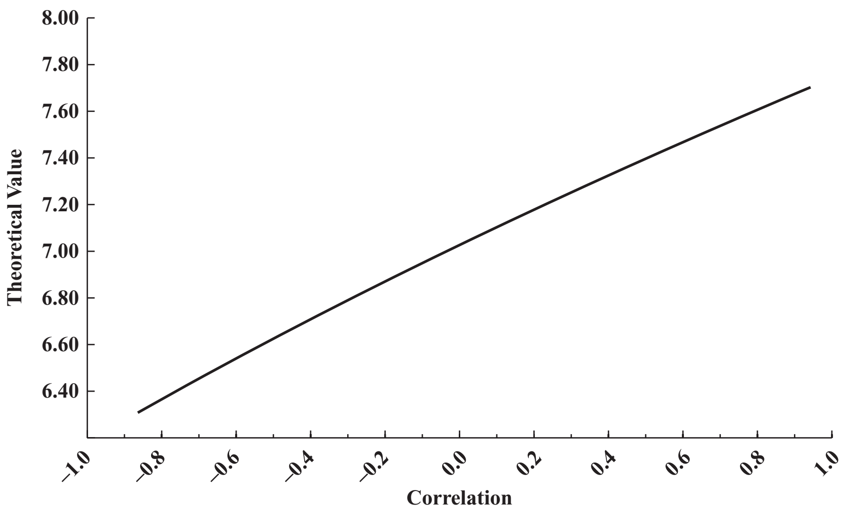
[3]See DeRosa (1996).

EXHIBIT 10.3 Currency Basket Option on EUR/USD, GBP/USD, and USD/JPY

	Spot	Forward Points	Forward Outright	Currency Amount	Spot Value	Forward Value	Weights
EUR/USD	1.3300	−0.0060	1.3240	9,000,000	$11,970,000	$11,916,256	33.59%
GBP/USD	1.5800	−0.0087	1.5713	7,500,000	$11,850,000	$11,785,004	33.22%
USD/JPY	85.00	−0.08	84.92	1,000,000,000	$11,764,706	$11,776,476	33.19%
					$35,584,706	$35,477,736	100.00%
Leg Volatilities			Cross Volatilities			Implied Correlations	
EUR/USD	14.00%		EUR/GBP	13.80%		EUR/USD, GBP/USD	54.90%
GBP/USD	15.00%		EUR/JPY	16.00%		EUR/USD, USD/JPY	39.29%
USD/JPY	15.00%		GBP/JPY	18.50%		GBP/USD, USD/JPY	23.94%
Option Parameters				Basket Options		Call	Put
Face Amount	$35,477,736			Total Cost		$2,596,867	$827,409
Forward Index Level	100			Percent of Forward		7.3197%	2.3322%
Strike Index Level	95						
Expiry (Days)	365						
Interest Rate (USD)	0.25%						
Basket Volatility	11.30%						

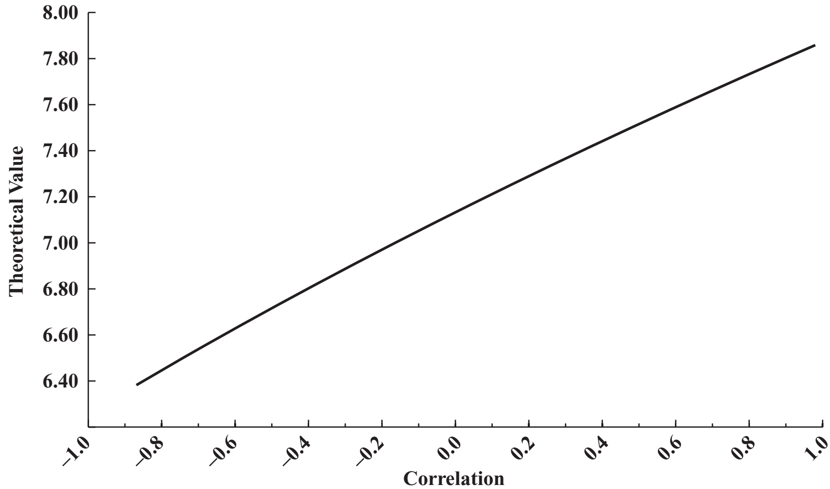


Panel A: Basket Option Call Price for Different Values of $\rho_{EUR/USD,GBP/USD}$



Panel B: Basket Option Call Price for Different Values of $\rho_{EUR/USD,USD/JPY}$

EXHIBIT 10.4 Sensitivity of Basket Option to Correlation



Panel C: Basket Option Call Price for Different Values of $\rho_{GBP/USD,USD/JPY}$

EXHIBIT 10.4 (*Continued*)

Quantos Options on Stock Indexes

A quantos option on stock index is the equivalent of an option on a foreign stock index, such as the Nikkei, that is denominated in an alternative currency, such as the U.S. dollar. The value of a quantos option is dependent upon the level of the foreign stock index but not upon the exchange rate.

The payoff function of quantos calls and puts at expiration time T is given by:

$$C_T^{quantos} = Max\left[\bar{S}Z_T - \bar{S}K, 0\right]$$
$$P_T^{quantos} = Max\left[\bar{S}K - \bar{S}Z_T, 0\right]$$

where $\bar{S}$ is the fixed level of the exchange rate and Z is the foreign stock index.

Derman, Karasinski, and Wecker (1990) and Dravid, Richardson, and Sun (1993) solve for the value of the European quantos options:

Quantos Options

$$C_t^{quantos} = \left[Z_t e^{(R_f - D')\tau} N(d_1) - KN(d_2)\right] \bar{S}e^{-R_d\tau}$$
$$P_t^{quantos} = \left[KN(-d_2) - Z_t e^{(R_f - D')\tau} N(-d_1)\right] \bar{S}e^{-R_d\tau}$$

where

$$d_1 = \frac{\ln\left(\frac{Z_t}{K}\right) + \left(R_f - D' + \frac{\sigma_z^2}{2}\right)\tau}{\sigma_z\sqrt{\tau}}$$

$$d_2 = d_1 - \sigma_z\sqrt{\tau}$$

$$D' = D + \sigma_{ZS}$$

and where D is the continuously compounded dividend yield on the foreign stock index, σ_S and σ_{ZS} are the volatility of the exchange rate and the covariance between the stock market index and the exchange rate, respectively.

The role of the covariance term σ_{ZS} is interesting. To place things in a more familiar context of correlation,

$$\sigma_{ZS} = \rho_{ZS}\sigma_Z\sigma_S$$

where ρ_{ZS} is the correlation coefficient between the stock market index and exchange rate. Consider the numerical example of European exercise puts and calls on the Nikkei stock market index that have the additional feature that each payoff function is on a fixed level for USD/JPY:

Quantos Puts and Calls on the Nikkei Index

Initial level of Nikkei	10,000
Strike	10,100
Fixed exchange rate (USD/JPY)	90.00
Days to Expiration	90
Interest rate (USD)	5%
Interest rate (JPY)	2%
Nikkei dividend rate	1%
Volatility of Nikkei	25%
Volatility of USD/JPY	14%
Correlation (Nikkei, USD/JPY)	–40%

Option Theoretical Values

Quantos Call	5.294
Quantos Put	5.645

The payoff function for these quantos puts and calls are given by

Quantos Call on Nikkei

$$\text{Max}\left[0, \frac{Nikkei}{90} - \frac{10{,}100}{90}\right]$$

Quantos Put on Nikkei

$$\text{Max}\left[0, \frac{10{,}100}{90} - \frac{Nikkei}{90}\right]$$

The value of these options is measured in Nikkei index points. Quantos calls are inversely related and quantos puts are positively related to the level of correlation between the stock market index and the exchange rate. This is shown in Exhibit 10.5 where the quantos options from the above numerical example are valued at alternative correlation assumptions.

Quantos options live in the environment of the over-the-counter market, although listed stock index warrants with quantos features have existed. Usually dealers create quantos options for their institutional asset manager

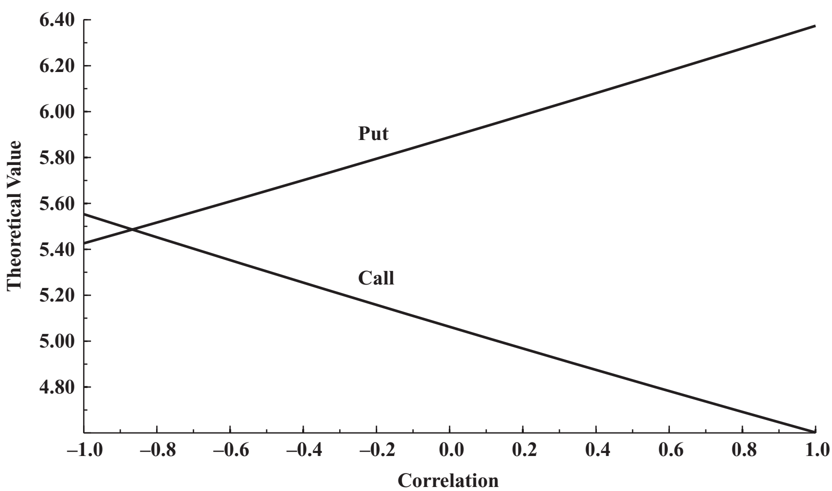


EXHIBIT 10.5 Quantos Call Option on the Nikkei Index Denominated in USD for Different Values of Correlation
$\rho_{nikkei,\ USD/JPY}$ Nikkei spot = 10,000; Strike = 10,100; Fixed Exchange Rate = 90.00; 90 Days; Nikkei Vol = 25%; USD/JPY Vol = 14%; $R_d = 5\%$; $R_f = 2\%$; Nikkei Div. Rate = 1%.

clients upon demand. The advantage to the end-user is that the quantos delivers currency hedging in precisely the correct face value—in effect the quantos currency face value rises and falls in precise proportion to the movements in the foreign stock index. On the other side of the transaction stands a dealer who is faced with having to adjust the currency hedge to movements in the foreign stock index. The major problem is that the correlation between the stock market index and the exchange rate might be sufficiently unstable so as to create the risk of significant under- or overhedging. As Piros (1998) points out, the end-user should expect to pay for this convenience.

Quantos Binary Currency Options

A quantos binary currency option is a binary currency option that has a payoff denominated in a third currency. For example, consider a digital option on EUR/GBP that has a payoff in USD:

Quantos Binary Currency Option	
Put/Call	EUR call/GBP put
Spot Exchange Rate	.8600
Strike	.8600
Payoff	100,000 USD
Expiration	90 days
Exercise	European
Volatility (EUR/GBP)	15%
Volatility (GBP/USD)	14%
Volatility (EUR/USD)	14%
Interest Rate (USD)	0.25%
Correlation (EUR/GBP, GBP/USD)	53.6%
Option Theoretical Values	
Quantos Binary Call	$48,979

Following Wystup (2008), the value of a quantos binary currency option 0^{QB} is given by

$$0^{QB} = Qe^{-r_Q T} N(\phi d_-)$$

$$d_- = \frac{\ln\left(S_0/K\right) + \left(\tilde{\mu} - \frac{1}{2}\sigma_{FOR/DOM}^2\right) T}{\sigma_{FOR/DOM}\sqrt{T}}$$

$$\tilde{\mu} = r_d - r_f - \rho\sigma_{FOR/DOM}\sigma_{DOM/QUANTO}$$

$$\rho = \frac{\sigma^2_{FOR/QUANTO} - \sigma^2_{FOR/DOM} - \sigma^2_{DOM/QUANTO}}{2\sigma_{FOR/DOM}\sigma_{DOM/QUANTO}}$$

where

ϕ equals 1 for calls and –1 for puts.
Q is the payoff of the option.
r_Q is the risk-free interest rate in the quanto currency.
S_0 is the spot exchange rate.
K is strike price of the foreign currency in domestic currency.
ρ is the correlation between the foreign currency in domestic currency and the domestic currency in quanto currency.

Exhibit 10.6 depicts the quantos binary EUR call/GBP put across alternative spot exchange rates.

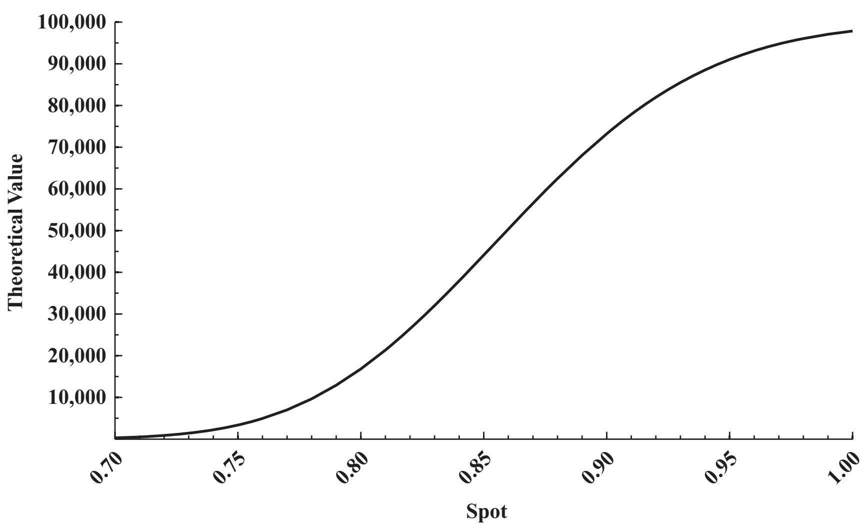


EXHIBIT 10.6 Quantos Digital EUR Call/GBP Put Option Denominated and Valued in USD
Strike = 0.86; Payoff = \$100,000; 90 days; EUR/GBP vol = 15%; GBP/USD vol = 14%; EUR/USD vol = 14%; $\rho_{EUR/GBP,GBP/USD} = -53.6\%$; $R_d = 0.25\%$.

COMMENTS ON HEDGING WITH NON-BARRIER CURRENCY OPTIONS

The world of exotic currency options is a dynamic environment where new options are always being invented. Many new exotic options are nothing more than mathematical whimsy. But occasionally, a useful new exotic currency option is born.

A good rule for when to use exotic options is to look for structures that do meet the hedging objectives at a significant cost advantage compared to what would have to be spent for vanilla currency options. This would seem to indicate that there is a free lunch imbedded in some varieties of exotic currency options. I am not implying this. Rather the point is that cost savings can materialize when a hedger works to selectively buy what protection he does in fact need without paying for forms of protection that are unwanted.

Average rate currency options are appropriate for the hedger who is interested primarily in the average exchange rate over a period of time. Because average rate options are cheaper than vanilla options, there will be a clear cost savings over buying a vanilla option.

In the same way, a basket option saves on hedging expenses provided that the objective is to hedge a portfolio of currencies as opposed to buying protection on one currency at a time.

Compound options can be effective where there is uncertainty about the need to hedge. Rather than commit to the purchase of a vanilla option, the hedger can pay a lower initial premium to buy a compound option and therefore lock in the cost of the hedge in the future if one is needed.

Finally, the quantos option costs money, but it can save money, time, and reduce risk. The defining attribute of a quantos option is that it delivers the correctly sized hedge for a foreign stock index that changes with market conditions.

APPENDIX 10.1 MONTE CARLO SIMULATION FOR ARITHMETIC MEAN AVERAGE OPTIONS

The Monte Carlo simulated arithmetic mean average call option price $C_{arithmetic}$ at time T_0, spot price $S(T_0)$ with strike K and maturity T is given by

$$\tilde{C}\left(S\left(T_0\right), T_0\right)_{arithmetic} = \frac{1}{N}\sum_{j=1}^{N} C_j\left(S\left(T_0\right), T_0\right)_{arithmetic}$$

$$C_j\left(S\left(T_0\right), T_0\right)_{arithmetic} = e^{-R_d(T-T_0)} \max\left(A_j\left(T\right) - K, 0\right)$$

$$A(T) = \frac{1}{k+1}\sum\nolimits_{i=0}^{k} S(T_i)$$

$$T_i = T_0 + i\,\Delta t$$

$$\Delta t = \left(T - T_0\right)/k$$

where N is the number of simulations and k is the number of discrete observation points from which the average spot price is determined.

$S(T_i)$ follows the Black-Scholes model, $S(T_{i+1}) = S(T_i)e^{\left(R_d - R_f - \frac{\sigma^2}{2}\right)\Delta t + \sigma\left(W_{i+1} - W_i\right)}$ and W_i is a standard Brownian motion.

Using the geometric mean average call option price $C_j\left(S\left(T_0\right), T_0\right)_{geometric} = e^{-R_d(T-T_0)} \max\left(G_j\left(T\right), 0\right)$ as a control variate, where $G(T) = \Pi_{i=0}^{k}\left[S(T_i)\right]^{\frac{1}{i+1}}$, a variance reduction of the factor $\rho_{\hat{C}_{arithmetic}\hat{C}_{geometric}}$ (correlation of the two option prices) is achieved.

$$C^*\left(S\left(T_0\right), T_0\right)_{arithmetic} = \frac{1}{N}\sum_{j}^{N}\left\{\begin{array}{l} C_j\left(S\left(T_0\right), T_0\right)_{arithmetic} - \\ a\left[C_j\left(S\left(T_0\right), T_0\right)_{geometric} - C_{geometric}^{closed\,form}\right] \end{array}\right\}$$

$$a = -\frac{Covariance(\hat{C}_{arithmetic,}\hat{C}_{geometric})}{Variance(\hat{C}_{geometric})}$$

$$Variance(C^*) = (1 - \rho_{\hat{C}_{arithmetic}\hat{C}_{geometric}})Variance(\hat{C}_{arithmetic})$$

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