FOREIGN EXCHANGE DERIVATIVES:
Effective Theoretical and Practical Techniques for Trading, Hedging and Managing FX Derivatives

by
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Philosophy is written in that great book whichever lies before our gaze — I mean the universe — but we cannot understand if we do not first learn the language and grasp the symbols in which it is written. The book is written in the mathematical language, and the symbols are triangle, circles and other geometrical figures, without the help of which it is impossible to conceive a single word of it, and without which one wonders in vain through a dark labyrinth.

Galileo Galilei (1564-1642)

Abstract

Instruments traded in the financial markets are getting more and more complex. This leads to more complex derivative structures that are harder to analyse and risk managed. These instruments cannot be traded or managed without the relevant systems and numerical techniques.

The global economy is becoming more and more interlinked with trading between countries skyrocketing. Due to the world trade, foreign exchange forwards, futures, options and exotics are becoming increasingly commonplace in today's capital markets.

The objective of these notes is to let the reader develop a solid understanding of the current currency derivatives used in international treasury management with an emphasis on the African continent. This will give participants the mathematical and practical background necessary to deal with all the products on the market.

Before I came here I was confused about the subject.
Having listened to your lecture I am still confused.
But on a higher level.

Enrico Fermi (1901-1954)

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But the creative principle resides in mathematics.
In a certain sense, therefore, I hold it true that
pure thought can grasp reality, as the ancients dreamed.

Albert Einstein

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Chapter 1

Introduction

The publication of the preference-free option pricing formula by *Fischer Black* and *Myron Scholes* in 1973 was a giant step forward in financial economics [BS 73]. However, options were traded a long time before the publication of the *Black & Scholes* model. Why was it then such a huge event? After 37 years the reasons seem obvious: the *Black & Scholes* option pricing model established the everyday use of mathematical models as essential tools in the world of finance, both in the classroom and on the trading floor. Since then option pricing theory has developed into a standard tool for designing, pricing and hedging derivative securities of all types. It simplified risk management in the financial markets by offering a methodology to predict the seemingly unpredictable by using the lessons of complex mathematics and probability theory to forecast stock valuations. Even though many traders do not comprehend all of the mathematics behind the model, it can be understood clearly from an intuitive perspective. The trading community thus accepted it as the *de facto* standard in derivatives pricing. In less than thirty years it has changed the course of economic theory and financial practice.

Traders accepted the model because it is easy to implement and the results are easy to understand. *Black & Scholes* showed one needs six input parameters to value a vanilla European option in an ideal market: the current stock price, the strike price, the time to expiry, the riskfree interest rate, the dividends and the volatility. Of these, the first three are known from the outset and the last three must be estimated. *Black & Scholes* assumed a perfect world in their analysis and took the last three as constants. In the real world, however, the correct values for these parameters are only known when the option expires. This means that the future values of these quantities need to be determined if an option is to be priced correctly [Wi 98]. If these parameters and the estimation thereof is not understood well, valuation risk management errors can occur.

So, Black & Scholes and Robert Merton received the Nobel prize for this famous formula, but what do you need derivatives for? The old saying is that there are two emotions that drive the market — greed and fear. These emotions then lead to two types of traders: speculators and hedgers. Derivatives are attractive financial products for both of these. Speculators are interested in derivatives because they can provide an inexpensive way to expose a portfolio to some market risk with a view to outperform the market. Hedgers are interested in derivatives because they allow investors to reduce market risk to which they are already exposed [MS 00]. I now list a few other benefits of derivatives:

- they form part of our universe of financial instruments - bigger choice;
- they are basic financial instruments - used to construct more complex instruments;
- Derivatives allow you to participate in price movements without committing large amounts of money needed to buy stock outright - gearing or leverage;
- Derivatives are used to hedge a stock position;
- Options allows one to acquire or sell stock at a purchase price more favorable than the current market price;
- In the case of writing (selling) options, one earns premium income;
- Derivatives give you options. You're not just limited to buying, selling or staying out of the market;
- With derivatives, you can tailor your position to your own financial situation and risk tolerance;
- Purchasing derivatives offer you the ability to position yourself accordingly with your market expectations. Many different kinds of strategies can be constructed;
- Options only has an indirect or limited influence on the underlying market.

Most options are traded over the counter (OTC) but exchange-traded options have many benefits including leverage, limited risk for buyers employing these strategies, and contract performance guaranteed by a Clearing House.

We start this book on derivatives by first looking at the history of derivatives trading and option valuation. In Chapter 2 we introduce the foreign exchange market. We look at the shear size of the market, the players and the risks in the market and the management thereof. We also introduce some important regulatory issues.

The capital markets in Africa are evolving rapidly and this is the topic of Chapter 3. We give an introductory overview of interest rate calculations that are important to understand when it comes to the pricing of derivatives. We start on the topic of derivatives by introducing the simplest derivative: the forward and future. This forces us to introduce the concept of arbitrage in the financial markets which will show us the way in calculating forward points using box diagrams.

Chapter 4 is a natural extension of this where we introduce nonlinear derivatives or options. We start by discussing the environment within which *Black & Scholes* derived their model in detail. We will not delve into the derivation of the formula but will rather concentrate on their arguments that enabled them to calculate a closed-form solution to the option pricing partial differential equation. Here we stress the importance of volatility and discuss the intuitions behind and the meaning of the formula and what it tells traders. The mechanics of the option prices will be touched on in Chapter 5 where we discuss the factors that affect option prices. We discuss the dynamics of option prices and introduce the Greeks or risk measures. We also discuss some useful symmetries and parity relationships.

The currency derivatives market is dominated by European options. We will, however, discuss American options and their valuation in Chapter 6. Having an understanding of numerical methods like binomial and trinomial trees are essential when it comes to the pricing of some exotic options.

In Chapter 7 we turn our attention to the important subject of volatility and the estimation thereof. We will show why volatility is so important and look at some different types of volatility. The volatilities that will be discussed are implied volatility, historical volatility, noncentered volatility, realized or actual volatility and extreme-value volatility. After these rather theoretical discussions, we get practical and start asking pertinent questions every trader and risk manager should ask about volatility. At the end of this chapter we will discuss the volatility skew or smile. This is a very important aspect of any option market.

In the last Chapter, Chapter 8 we ask the question: how does the market participants use FX options? The answer is that options are used to hedge the risks we introduced in Chapter 2. These hedges mostly take the form of option strategies that are just portfolio of options structured such that the outcome will minimise the risk associated with a FX exposure.

Note that we only concern ourselves with the currency market, although, most of the ideas could be implemented in the equity, commodity and interest rate markets as well.

1.1 Introduction

The key to understanding derivatives is the notion of a premium. Some derivatives are compared to insurance. Just as you pay an insurance company a premium in order to obtain protection on your car (if it is stolen) for instance, there are derivatives products that have a payoff contingent upon the occurrence of some event. This premium needs to be calculated. *Black & Scholes* developed a model for this. But, is such a model a true reflection of reality? Where does it come from? Options have been traded long before the publication of a model so why do we need a model? Isn't the price set by the market and isn't this market price "right" in the sense of the efficient market hypothesis?

Before we can fully understand the *Black & Scholes* model for the option price on a stock, we need to understand where it comes from and what the benefits are of having such a model at our disposal. We will discuss the history and benefits in this chapter.

1.2 A History of Derivatives

The history of derivatives is quite colorful and surprisingly a lot longer than most people think. I would like to first note that some of these stories are controversial. Do they really involve derivatives? Or do the minds of people like myself and others see derivatives everywhere?

To start we need to go back to the Bible. In Genesis Chapter 29, believed to be about the year 1700 B.C., Jacob purchased an option costing him seven years of labor that granted him the right to marry Laban's daughter Rachel. His prospective father-in-law, however, reneged, perhaps making this not only the first derivative but the first default on a derivative. Laban required Jacob to marry his older daughter Leah. Jacob married Leah, but because he preferred Rachel, he purchased another option, requiring seven more years of labor, and finally married Rachel, bigamy being allowed in those days. Jacob ended up with two wives, twelve sons, who became the patriarchs of the twelve tribes of Israel, and a lot of domestic friction, which is not surprising. Some argue that Jacob really had forward contracts, which obligated him to the marriages but that does not matter. Jacob did derivatives, one way or the other. Around 580 B.C., Thales the Milesian purchased options on olive presses and made a fortune off of a bumper crop in olives. So derivatives were around before the time of Christ.

Derivatives trades stem back many many years. The Greeks bought maize forward from the Egyptians some 3000 years ago. The first recorded instance of futures trading occurred with rice in 17th Century Japan. There are, however, some evidence that there may also have been rice "futures" traded in China as long as 6,000 years ago. With the rice coupon becoming an actively traded entity, the Dojima Rice exchange became the world's first futures exchange. It was established in 1697. The first "futures" contracts are generally traced to the Yodoya rice market in Osaka, Japan around 1650. Remember, futures trading is a natural outgrowth of the problems of maintaining a year-round supply of seasonal products like agricultural crops.

The Chicago Board of Trade was established in 1848 and in the 1870s and 1880s the New York Coffee, Cotton and Produce Exchanges were born. In 1878, a central dealing facility was opened in Chicago, USA where farmers and dealers could deal in 'spot' grain, i.e., immediately deliver their wheat crop for a cash settlement. Futures trading evolved as farmers and dealers committed to buying and selling future exchanges of the commodity. The biggest increase in futures trading activity occurred in the 1970s when futures on financial instruments started trading in Chicago. Over the counter (OTC) options also have a long history. It started with the Tulip-bulb craze in Holland in the early seventeenth century [Ma 90].

Tulips originates from central Asia in the mountain ranges near modern day Islamabad. Tulips were first introduced to Europe in the 1500s by the Turks during the great Ottoman Empire, also known as the Turkish Empire or Persia. It is known that the Turks were cultivating tulips as early as 1,000 AD. In 1593 Conrad Guestner brought the first tulip bulbs from Constantinople to Holland and Germany, and people fell in love with them.

The Dutch cultivation of tulips began, slowly but surely in very small private plots. At first the tulip was a rarity that only the very wealthy could afford. So rare and so beautiful were they that the rich clamored to have them. Tulips became a status symbol and wealthy Dutch and European aristocrats and newly-wealthy merchant classes had to have them! A buying mania evolved.

By 1624 things had progressed to to such a craze that one tulip, the renowned white and maroon "Rembrandt-type" tulip 'Semper Augustus', could command a price as high as 3,000 guilders per bulb — with only 12 bulbs available for sale! The equivalent of $1,500 U.S. today. Just a short time later, a similar bulb fetched a whopping 4,500 guilders ($2,250 U.S.), plus a horse and carriage [Bu 02]. During the peak of the mania in 1635, a single tulip bulb sold for the equivalent of $150,000. It progressed so far that by 1636, tulips were established on the Amsterdam stock exchange, as well as exchanges in Rotterdam, Harlem, Levytown, Horne and other forums in Europe. Options on tulips bulbs were bought and sold and speculators and gamblers drove prices. When prices started to fall the Dutch government tried to intervene - to no avail. By November 1636, it was all over with a lot of blood on the trading floors.

In April 1973, a major innovation in securities trading took place with the opening of the Chicago Board Options Exchange (CBOE). The CBOE began with call options on 16 heavily traded common stocks and has subsequently evolved into one of the largest exchanges in the world in terms of the value of securities traded. This was the first organized trading of options on an exchange.

The American Stock Exchange and the Philadelphia Stock Exchange started trading options in 1975, and the Pacific Stock Exchange introduced option trading in 1976. In 1977, put options became exchange listed and expanded the benefits of options. On March 11, 1983, the CBOE transformed options trading when it introduced index options in the form of the S&P 100 Index. The New York Stock exchange joined the others and introduced option trading in June 1985. Today, options are available on over 2,300 underlying securities and 60 indices [Sc 02].

Options on equities have been traded on the Johannesburg Stock Exchange (JSE) since the end of the 1800's. The first standardized option contract in South Africa was written in 1987 on E168 stock of Eskom (this still was an OTC option). This led to the formation of the South African Futures Exchange (SAFEX) in September 1988 where standardized option contracts on equity and interest rate futures contracts are traded.

But what about currency derivatives? For as long as trade between countries with different currencies has taken place, foreign exchange has been in existence. According to Julian Walmsley, author of The Foreign Exchange and Money Markets Guide, although foreign exchange has existed since before biblical times, a formal global market for foreign exchange did not develop until the 1800s with cable transfers taking place between London and New York[1].

Historically, governments attempted to set exchange rates themselves to improve a country's trade position. If a country set its exchange rate low relative to others, it could improve the country's trade position by making its exports more affordable and imports from other countries less affordable. Such policies led to trade wars as countries struggled to improve their trade positions. Since the early 1970s, however, most major currencies have been allowed to "float", which means allowing exchange rates to be determined by supply and demand on the currency markets. Most countries still fine tune exchange rates by keeping a reserve of gold or foreign currencies, known as foreign exchange reserves, which they buy and sell to stabilize their own currency when necessary.

Currency futures were first created at the Chicago Mercantile Exchange (CME) in 1972, less than one year after the system of fixed exchange rates was abandoned along with the gold standard. Some commodity traders at the CME challenged the banks by establishing the International Monetary Market (IMM) and launched trading in seven currency futures on May 16, 1972. Today, the IMM is a division of CME. In the fourth quarter of 2009, CME Group FX volume averaged 754,000 contracts per day, reflecting average daily notional value of approximately $100 billion. Currently most of these are traded electronically.

In 2005, the JSE started a new bourse called Yield-X where yield related instruments are listed and traded. In an extention to this, Rand currency derivatives started trading during June 2007. Currency futures pairs currently traded are ZAR/USD, ZAR/GBP, ZAR/EUR, ZAR/YEN, ZAR/CAD, ZAR/AUS AND ZAR/CNY. During August 2010, Mauritius launched its own derivatives exchange called Global Board of Trade (GBOT) where commodity and currency derivatives are traded. Currency futures pairs currently traded are USD/MUR, ZAR/USD, EUR/USD, GBP/USD, JPY/USD.

1.3 A History of Option Valuation

The previous section showed that options have been traded for many centuries without a simple way of determining the price. Modern option pricing techniques, with roots in stochastic calculus, are often considered among the most mathematically complex

[1] http://www.referenceforbusiness.com/encyclopedia/For-Gol/Foreign-Exchange.html

of all applied areas of finance. These modern techniques derive their impetus from a formal history dating back to 1877, when *Charles Castelli* wrote a book entitled *The Theory of Options in Stocks and Shares*. *Castelli's* book introduced the public to the hedging and speculation aspects of options, but lacked any monumental theoretical base.

Twenty three years later, in 1900 *Louis Bachelier* offered the earliest known analytical valuation for options in his doctoral mathematics dissertation "Théorie de la Spéculation" at the Sorbonne[2]. *Bachelier* valued options by assuming stock prices follow an arithmetic Brownian process. This process implies a normally distributed stock price behaviour and can lead to negative prices. He also assumed that the mean expected price change per unit time is zero [Sm 76]. *Bachelier* views the stock market as a fair gamble; he feels that competition will reduce the expected return to zero.

Since Bachelier people realized that stock prices are better described by a lognormal distribution and their behaviour can be modeled by geometric Brownian motion[3]. Research into finding better statistical ways to describe the market is still continuing[4]. Many different distributions have been proposed like the parabolic distribution. Most of these lead to complex equations that need to be solved numerically. Due to the complexity of this methodology, it has not gone down well with practitioners.

Bachelier's work interested a professor at MIT named Paul Samuelson, who in 1955, wrote an unpublished paper entitled "Brownian Motion in the Stock Market". Then, since the early sixties people seriously tried to calculate option prices. *Sprenkle* (1961), *Ayres* (1963), *Boness* (1964), *Samuelson* (1965) and *Chen* (1970) all produced valuation formulas similar in form to the *Black & Scholes* model but with constants which were either difficult to calculate (or to define) or had to be obtained empirically [Sm 76, Ha 07]. All these formulas had the *variance rate of return* on the stock prices as a parameter. *Black & Scholes* defined this in their paper as follows [BS 73]

> "the variance rate of return on a security is the limit, as the size of the interval of measurement goes to zero, of the variance of the return over that interval divided by the length of the interval."

[2]It is said that Louis Bachelier is to financial derivatives what Albert Einstein is to modern physics and Pablo Picasso is to modern art. Einstein and Picasso are justly well known. Bachelier, on the other hand, still remains an enigma. We do not even have a photograph of him so we don't know what he looks like. Bachelier's supervisor was the great French philosopher-mathematician Henri Poincaré. Bachelier died on 28 April 1946.

[3]This roughly means that the market maintains a constant expected percentage move.

[4]Some evidence, however, suggests that financial markets can be better described by a mixed process: arithmetic Brownian motion in the short term and geometric Brownian motion in the long run.

1.4 A Word on Modeling

In analyzing the world around us mathematically (whether it be atoms or economic systems), models have to be created. These models capture as many aspects as possible of the real system. In order to obtain models that can be analyzed mathematical rigorously, some simplifications will be necessary. Even then, most systems are still too complex to analyze exactly. These models usually take the form of differential equations that have to be solved to obtain physical answers.

Under uncertainty the situation is even more complex. The construction of models requires that we distinguish known from unknown realities and find some mechanisms to reconcile our knowledge with the lack of it. For this reason modeling is not merely a collection of techniques but an art in blending the relevant aspects of a problem and its unforeseen consequences with a descriptive, yet tractable, mathematical methodology. To model under uncertainty, we typically use probability distributions to describe quantitatively the set of possible events that may unfold over time [Ta 88]. From this, stochastic differential equations can be obtained. Specification of the structure of the probability distributions are important and based on an understanding of the process. Moments of such processes (particularly the mean and variance) tend to reflect the trend and the degrees to which we are more or less certain about events as they occur.

To test the validity or appropriateness of a certain option model is very difficult. Many academics validate their models against the market. This is not always appropriate due to issues like liquidity and market inefficiencies. A "good" theoretical model should have the following properties [Le 91]:

- it should be able to identify instances of market mispricing, thus providing opportunities for arbitrage;
- it should be able to generate a hedge ratio suitable for immunising the option position over certain well-defined time intervals; and
- it should replicate most of the everyday features of its target market well enough such that its underlying assumptions do not matter;
- it should be understood well enough by its users - do not treat your model as a "black box".

Always remember that a model is not reality, *but something that imitates reality at a certain scale* [Mo 91]. If one adheres to the above few rules, model risk will be minimised.

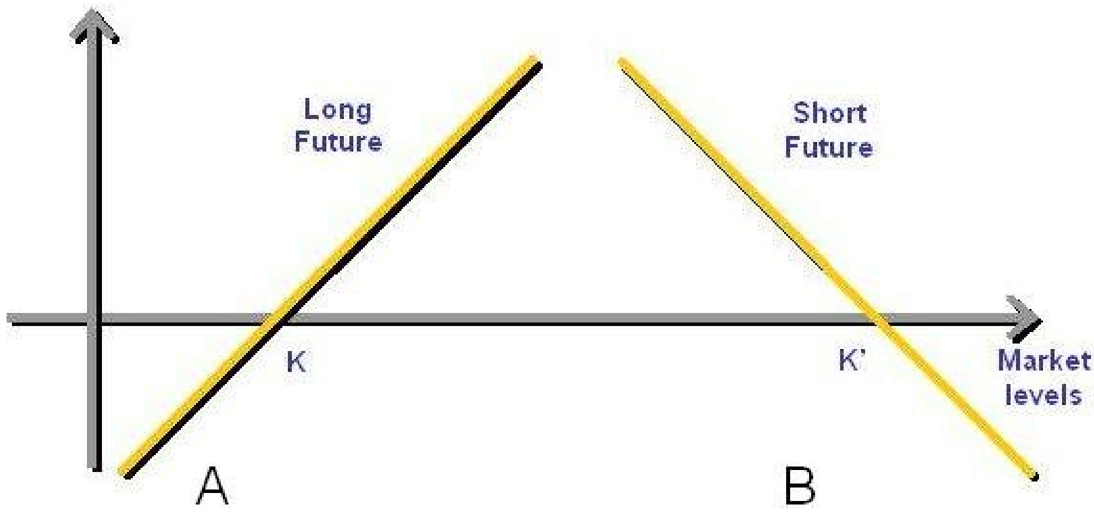


Figure 1.1: Payoff profiles for futures and forwards.

1.5 Payoff Diagrams

For most options and option structures, the important issue for investors is what the possible payoff profiles look like. A payoff profile shows the investor the payoff that would be received if the underlying instrument is at its current level when the option expires. It also shows the investor the risk associated with the strategy: a future has unlimited profit potential, but such a diagram also shows the potential losses [Ko 05].

These profiles are popular because they show the profit/loss scenario (possible risks) in a simple diagram. They are also easy to work with because they are additive meaning that we can add or subtract them from one another. This feature makes them useful in constructing more complex financial instruments or strategies [Hu 06]. For a derivative structure one will always look at the total or end profile. Figure 1.1 shows the payoff diagrams for long and short future's positions while Figure 1.2 shows similar profiles for call and put options. In Figure 1.3 we show the profile for a capped collar. This structure is obtained by buying a future and at-the-money put and by selling an out-the-money call (a call spread can give a similar profile). Structures like this will be investigated in Chapter 8.

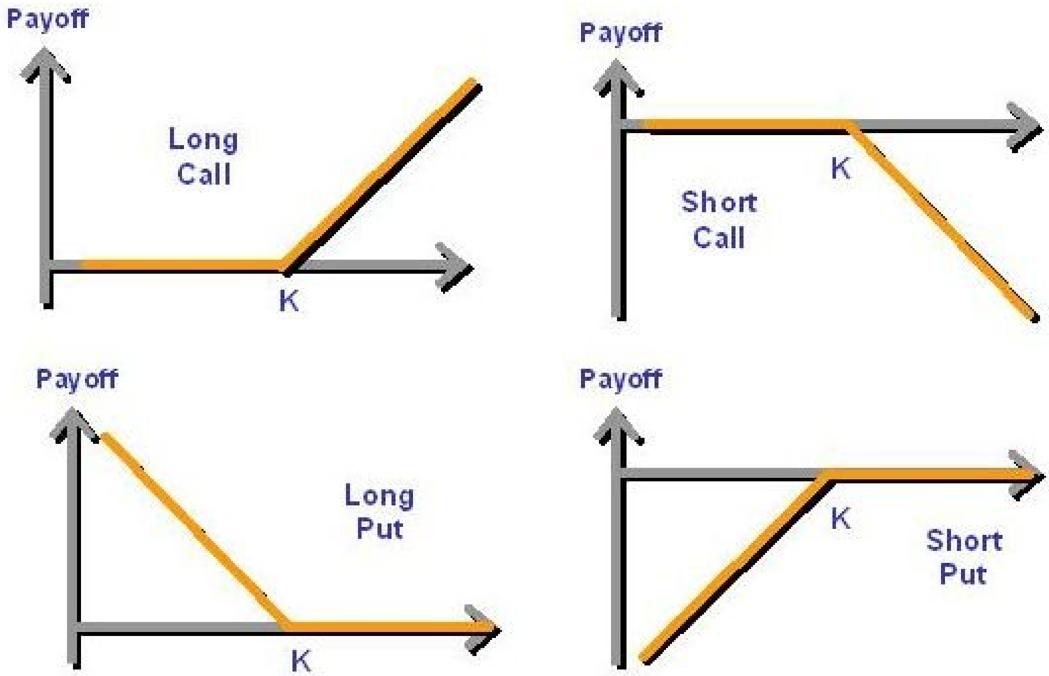


Figure 1.2: Payoff profiles for Calls and Puts.

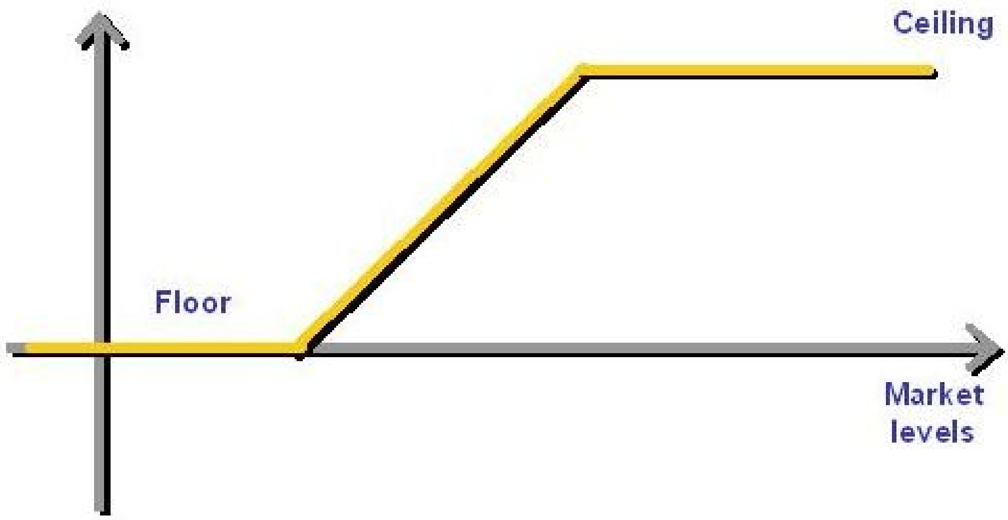


Figure 1.3: Payoff profile for a capped collar.

1.6 Understanding the Foreign Exchange Markets

The foreign exchange market (forex, FX, or currency market) is a worldwide decentralized over-the-counter financial market for the trading of currencies[5]. Financial centers around the world function as anchors of trading between a wide range of different types of buyers and sellers around the clock, with the exception of weekends. Today, the foreign exchange market determines the relative values of different currencies. This was not always the case.

1.6.1 The Gold Standard

Before WWI, the gold standard was in use. It is defined as "a commitment by participating countries to fix the prices of their domestic currencies in terms of a specified amount of gold". A county under the gold standard would set a price for gold, say $100 an ounce and would buy and sell gold at that price. This effectively sets a value for the currency; in our fictional example $1 would be worth 1/100th of an ounce of gold.

A true gold bullion standard came to fruition in 1925 with the passage of the Gold Standard Act in the UK. This was done in conjunction with South Africa and Australia (gold producing colonies of the UK). The gold standard effectively came to an end in 1933 when President Franklin D. Roosevelt outlawed private gold ownership (except for the purposes of jewelery). The Bretton-Woods System, enacted in 1946, created a system of fixed exchange rates that allowed governments to sell their gold to the United States treasury at the price of $35/ounce. Under this system, many countries fixed their exchange rates relative to the U.S. dollar. Implicitly, then, all currencies pegged to the dollar also had a fixed value in terms of gold

The Bretton-Woods system ended on August 15, 1971, when President Richard Nixon ended trading of gold at the fixed price of $35/ounce. At that point for the first time in history, formal links between the major world currencies and real commodities were severed. Nearly all nations then switched to full fiat money.

Fiat money is defined as "money that is intrinsically useless; is used only as a medium of exchange". The value of money is set by the supply and demand for money and the supply and demand for other goods and services in a country's economy. The prices for those goods and services, including gold and silver, are allowed to fluctuate based on market forces.

1.6.2 Uniqueness of the Market

The foreign exchange market is unique because of

- its huge trading volume, leading to extremely high liquidity;

[5] http://en.wikipedia.org/wiki/Foreign_exchange_market

- its geographical dispersion;
- the large number of, and variety of, traders in the market;
- its near continuous operation: 24 hours a day except weekends, i.e. trading from 20:15 GMT on Sunday until 22:00 GMT Friday;
- the variety of factors that affect exchange rates;
- the low margins of relative profit compared with other markets of fixed income; and
- the use of leverage to enhance profit margins with respect to account size.

As such, it has been referred to as the market closest to the ideal of perfect competition, notwithstanding currency intervention by central banks.

1.6.3 Size of the Market

According to the Bank for International Settlements (BIS) as of April 2010, average daily turnover in global foreign exchange markets is estimated at $3.98 trillion, a growth of approximately 20% over the $3.21 trillion daily volume as of April 2007 [BIS 10]. The market has more than doubled since 2004. However, the growth in the market between 2004 and 2007 was a staggering 72%. The $3.98 trillion break-down is as follows

- $1.490 trillion in spot transactions
- $475 billion in outright forwards
- $1.765 trillion in foreign exchange swaps
- $43 billion currency swaps
- $207 billion in options and other products.

The breakdown is shown Fig. 1.4. We see that more than 56% of the FX trade is in outright forwards and FX swaps. Figure 1.5 shows the regional breakdown of the global FX turnover. This shows that Africa is minute in FX trading. Figure 1.6 shows the breakdown of the currencies. South Africa is the only African country on this list. This data shows that FX trade in Africa (excluding South Africa) is incredibly small if compared to the global turnover. Potential for growth is enormous.

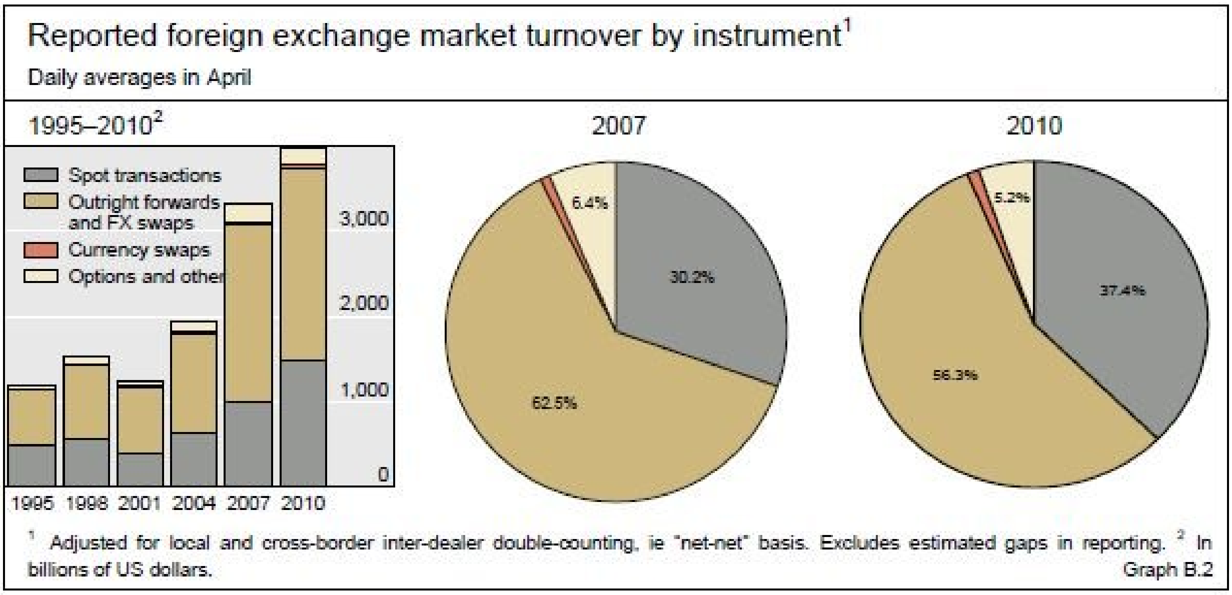


Figure 1.4: Foreign exchange instruments by turnover.

Regional FX turnover by instrument[1]

Daily averages in April 2010, in billions of US dollars

Region	Spot	Outright forwards and FX swaps	Currency	Options	Total
North America	491	423	10	42	966
Western Europe	916	1,672	29	163	2,780
Asia-Pacific	361	743	16	38	1,159
Eastern Europe	30	38	0	0	68
Latin America	21	20	1	1	42
Africa and Middle East	13	25	1	2	41
All regions ("net-gross")	1,832	2,921	57	246	5,056
Adjustment for cross-border double-counting	-342	-681	-14	-39	-1,075
Global turnover ("net-net") [2]	1,490	2,240	43	207	3,981

[1] Regional aggregates are adjusted for local inter-dealer double-counting, ie trades between reporting dealers located in the same countries were halved, ie "net-gross" basis. Regional aggregates are not adjusted for intraregional double-counting, ie trades between reporting dealers located in different countries of the same region were not halved. [2] Global aggregates are adjusted for both local and cross-border inter-dealer double-counting, ie "net-net" basis.

Table B.7

Figure 1.5: Regional breakdown of FX turnover by instrument.

Currency and instrument distribution of global foreign exchange market turnover[1]

Percentage shares of average daily turnover in April 2010

Currency	Spot	Outright forwards	Foreign exchange swaps	Currency swaps	Options and other instruments
US dollar	35.2	11.6	47.4	1.1	4.7
Euro	44.4	9.6	39.2	1.1	5.6
Japanese yen	39.7	15.2	36.9	0.9	7.2
Pound sterling	41.6	10.7	43.4	0.5	3.9
Swiss franc	36.4	7.5	50.2	0.7	5.3
Australian dollar	36.8	9.6	46.7	1.9	5.1
Canadian dollar	37.0	12.5	46.2	1.4	2.9
Swedish krona	21.5	9.8	64.4	0.8	3.4
Hong Kong dollar	19.9	4.0	74.0	0.4	1.8
Norwegian krone	23.4	11.7	60.0	1.2	3.6
Korean won	35.1	29.9	27.5	1.6	5.9
Mexican peso	36.3	10.8	47.5	0.7	4.6
New Zealand dollar	34.2	8.0	52.3	1.0	4.4
Singapore dollar	27.7	7.8	59.6	0.1	4.8
Danish krone	21.2	12.4	65.0	0.5	0.9
South African rand	31.7	9.9	54.3	0.5	3.6
Russian rouble	50.6	6.3	39.7	0.5	2.9
Polish zloty	22.4	11.1	59.4	0.6	6.5
New Taiwan dollar	31.9	35.9	25.0	0.5	6.7
Indian rupee	35.8	36.1	18.0	0.1	9.9
Brazilian real	31.3	47.3	2.9	1.4	17.1
Czech koruna	17.4	8.0	71.3	0.4	2.8
Thai baht	37.1	14.5	45.8	1.4	1.2
Hungarian forint	24.1	10.6	57.8	0.3	7.2
Chilean peso	38.8	48.2	9.0	3.5	0.5
Malaysian ringgit	38.0	37.9	20.2	1.2	2.7
Chinese renminbi	23.7	41.6	19.9	0.2	14.6
Israeli new shekel	23.1	7.3	58.6	1.3	9.6
Turkish lira	27.2	10.4	43.1	6.5	12.8
Philippine peso	33.5	35.7	17.7	2.8	10.2
Indonesian rupiah	40.9	44.2	11.1	1.1	2.7
Saudi riyal	50.6	11.6	37.8	0.0	0.1
Colombian peso	43.6	24.3	0.8	30.9	0.4
Argentine peso	85.0	13.9	0.3	0.0	0.7
Bulgarian lev	91.2	1.0	7.7	0.2	0.0
Bahraini dinar	31.1	22.2	45.6	0.7	0.4
Estonian kroon	39.5	4.6	55.9	0.0	0.0
Lithuanian litas	19.4	0.8	79.7	0.0	0.0
Latvian lats	18.8	2.4	78.8	0.0	0.0
Peruvian nuevo sol	64.6	34.6	0.3	0.3	0.2
Romanian leu	34.8	4.3	58.7	0.7	1.6
All currencies	**37.4**	**11.9**	**44.3**	**1.1**	**5.2**

[1] Adjusted for local and cross-border inter-dealer double-counting (ie "net-net" basis).

Table B.5

Figure 1.6: Turnover by currency pair.

Increase in global FX market turnover by counterparty[1]				
	Turnover in 2010[2]	Absolute change from 2007[2]	Growth since 2007 (%)	Contribution to FX market growth[3] (%)
Global FX market	3,981	657	20	100
By counterparty				
Reporting dealers	1,548	156	11	24
Other financial institutions	1,900	561	42	85
Non-financial customers	533	–60	–10	–9
By instrument				
Spot	1,490	485	48	74
Outright forwards	475	113	31	17
FX swaps	1,765	51	3	8
Currency options	207	–4	–2	–1
Currency swaps	43	11	36	2

[1] Adjusted for local and cross-border double-counting, ie "net-net" basis. [2] In billions of US dollars. [3] Percentage contribution to the total increase of $657 billion from 2007 to 2010.

Source: 2010 Triennial Central Bank Survey. Table 1

Figure 1.7: Increase in global FX market turnover by counterparty.

1.6.4 Growth in the FX Market

Interestingly is that the growth between 2007 and 2010 was driven by "other financial institutions" according to the BIS [KR 10]. This is shown in Fig. 1.7. While FX turnover grew by 20% between April 2007 and 2010, trading by corporations and governments fell by 10% over this period, possibly reflecting slower economic growth. Given that most of the growth in FX market activity since 2007 is due to increased trading by other financial institutions, the $4 trillion dollar question is: which financial institutions are behind this growth? The Triennial data do not break down trades within this category of counterparty.

Discussions with market participants, data from regional FX surveys and an analysis of the currency composition and location of trading activity provide some useful clues. Taken together, they suggest the increased turnover is driven by

1. greater activity of high-frequency traders;
2. more trading by smaller banks that are increasingly becoming clients of the top dealers for the major currency pairs; and
3. the emergence of retail investors (both individuals and smaller institutions) as a significant category of FX market participants.

An important structural change enabling increased FX trading by these customers is the spread of electronic execution methods. Electronic trading and electronic bro-

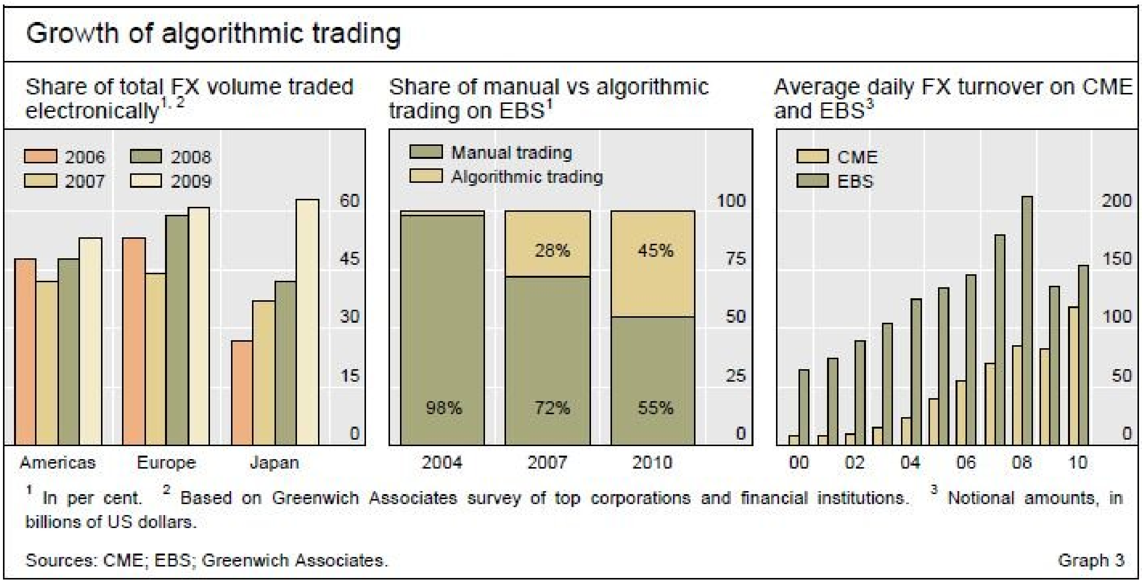


Figure 1.8: Growth in algorithmic trading.

kering are transforming FX markets by reducing transaction costs and increasing market liquidity. These changes, in turn, are encouraging greater participation across different customer types.

Continued investment in electronic execution methods has paved the way for the growth of algorithmic trading. In algorithmic trading, investors connect their computers directly with trading systems known as electronic communication networks (ECNs). Examples of ECNs in FX markets are electronic broking systems (such as EBS and Thomson Reuters Matching), multi-bank trading systems (such as Currenex, FXall and Hotspot FX) and single-bank trading systems. A computer algorithm then monitors price quotes collected from different ECNs and places orders without human intervention. High-frequency trading (HFT) is one algorithmic strategy that profits from incremental price movements with frequent, small trades executed in milliseconds. The growth in algorithmic trading of FX is shown in Fig. 1.8.

While banks engaged in FX markets below the top tier continue to be important players, the long-term trend towards greater concentration of FX activity in a few global banks continues (see the right-hand panel in Fig. 1.9). The largest dealers have seen their FX business grow by investing heavily in their single-bank proprietary trading systems. The tight bid-ask spreads and guaranteed market liquidity on such platforms are making it unprofitable for smaller players to compete for customers in the major currency pairs. Increasingly, many smaller banks are becoming clients of the top dealers for these currencies, while continuing to make markets for customers

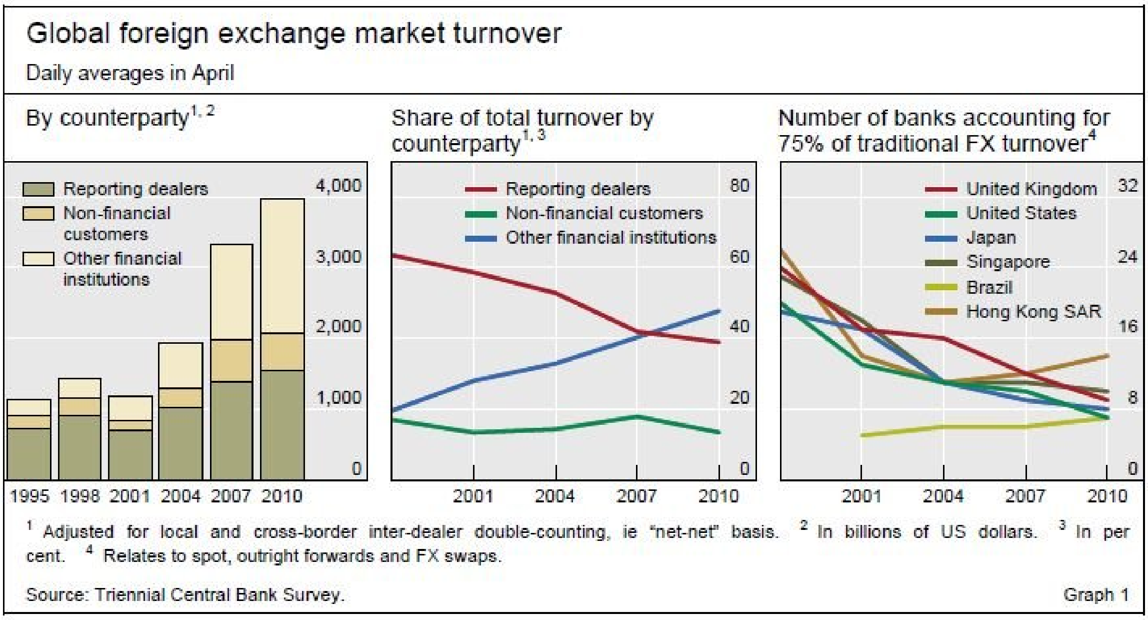


Figure 1.9: Graphs showing the increase in global FX market turnover.

in local currencies. This hybrid role allows smaller banks with client relationships to profit from their local expertise and comparative advantage in the provision of credit, while freeing them from the heavy investment required to compete in spot market-making for the major currency pairs.

Finally, greater FX trading activity by small retail investors has made a significant contribution to growth in spot FX, and this growth in activity was made possible by the spread of electronic execution methods.

1.6.5 Emerging Markets: Growth in Derivatives Activity

Derivatives markets in EMEs remain small compared to those in advanced economies. Average daily turnover of derivatives in 33 EMEs for which data are available was \$1.2 trillion in April 2010 (6.2% of those economies' GDP), compared to \$13.8 trillion (36% of GDP) in advanced economies. Though small, derivatives markets in EMEs have expanded rapidly: average daily turnover has increased by 300% since 2001, and by 25% over the past three years, despite the crisis in 200809 (Fig. 1.10, left-hand panel). This was higher than the growth of turnover in advanced economies (250% since 2001, and 22% since 2007).

OTC derivatives are relatively more important in emerging markets than in advanced economies. In EMEs, derivatives are traded in almost equal proportions over the counter and on exchanges (see Fig. 1.10, centre and righthand panels). By

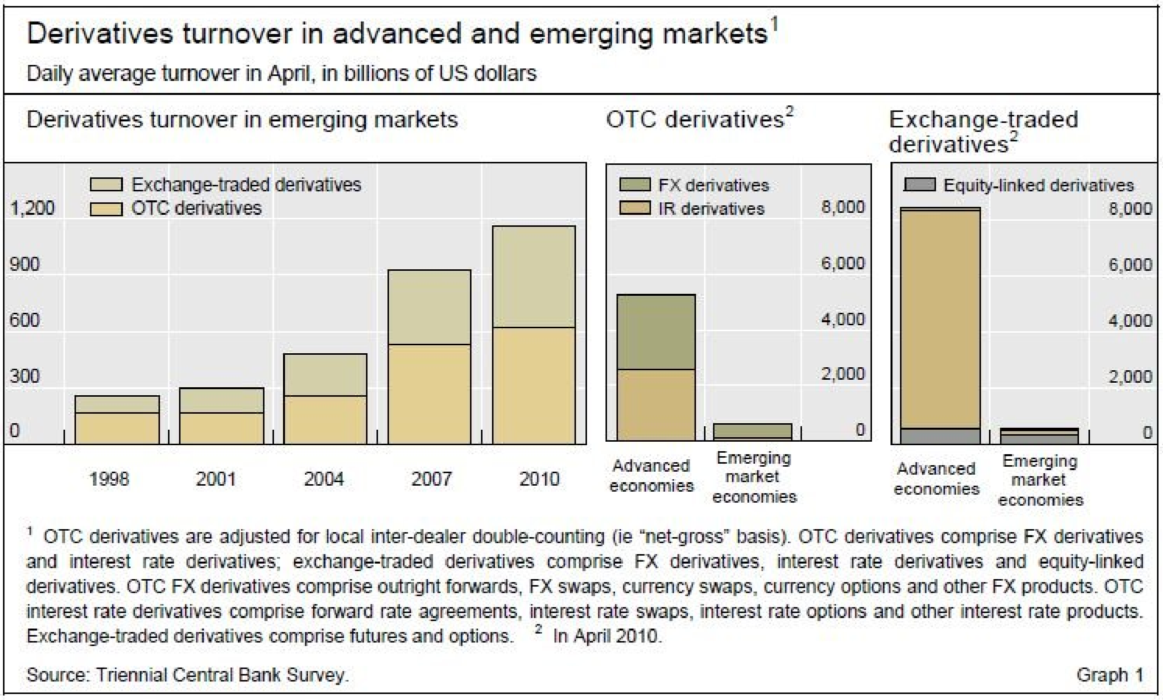


Figure 1.10: Derivatives turnover in advanced and emerging markets.

comparison, in advanced economies almost two thirds of derivatives are traded on exchanges (right-hand panel) and 38% over the counter (centre panel). Furthermore, the relative size of the exchange-traded derivatives market is distorted by two special cases with well developed derivatives exchanges, Brazil and Korea, which together account for nearly 90% of all emerging market turnover of exchange-traded derivatives [MP 10].

Derivatives in EMEs are used mainly to hedge or speculate on exchange rate and, to a lesser extent, equity market risk. FX derivatives account for 50% of total turnover in emerging markets, equity-linked derivatives for 30% and interest rate derivatives for the rest. By contrast, derivatives in advanced economies are used by and large to trade interest rate risk (77% of total turnover), with FX derivatives and in particular equity-linked derivatives being less important. These differences reflect above all the depth and liquidity of bond and money markets in developed countries, and the relatively limited concern with exchange rate risk in advanced compared to emerging market economies.

Over-the-counter derivatives represent the most developed segment of the derivatives market in EMEs. The average daily turnover of OTC derivatives in April 2010 was $625 billion, or roughly 3% of EMEs' (annual) GDP. By comparison, the average daily turnover of OTC derivatives in advanced economies was $5.3 trillion (13% of their GDP). The OTC market in EMEs is dominated by FX derivatives, which account for nearly 90% of total turnover, versus 50% in advanced economies. Despite these differences, trading of OTC derivatives in EMEs has converged towards advanced economy patterns in terms of instruments, counterparties and currencies being traded.

The turnover of OTC foreign exchange derivatives in EMEs — \$535 billion per day in April 2010 (see Table 1.1) — increased 24% between 2007 and 2010. This represents a slowdown compared to the previous three-year period, when turnover almost doubled, but was much faster than the growth in advanced economies (just 5.6%). No doubt the recent financial crisis has taken some of the shine off the use of OTC foreign currency derivatives in advanced economies, particularly FX swap markets, where growth over the entire three year period was only 0.3%. At the same time, the financial crisis had a relatively small impact on FX derivatives markets in emerging market economies.

In terms of FX instruments, the OTC markets in EMEs have already converged to the advanced economies' pattern. In both groups of countries, FX swaps comprise the lion's share of turnover (over 70%), followed by outright forwards (19%), options and currency swaps. The relative size of FX spot and derivatives markets has also converged. The ratio of FX derivatives to spot transactions increased in EMEs to 1.9 in 2010, continuing the steady rise evident since 1998. Meanwhile, the ratio of derivatives to spot transactions in advanced economies declined to 1.6 in 2010.

Turning to the question of who is trading derivatives in emerging markets, we see that trades with other financial institutions such as pension funds and hedge funds - increased the most, to 30% of total turnover in 2010. At the same time, the shares of trade with other reporting dealers (usually commercial and investment banks) and non-financial customers declined to 58% and 12%, respectively. The shift towards trading with financial customers represents the resumption of a trend that started in 1998, when the share of this counterparty type was as low as 15%. The trend is present across all foreign exchange instruments, especially the three largest categories [MP 10].

1.6.6 Currency Composition of OTC Derivatives in Emerging Markets

The US dollar remains the pre-eminent global currency in OTC derivatives markets of EMEs. In the FX derivatives markets, the dollar was one of the currencies in more than 95% of transactions in 2010 (see Table 1.2). This fraction was virtually unchanged from 2007, thus confirming the dollar's ongoing status as the leading currency for international financial transactions, paralleling its continued leading role in critical areas of international trade and finance [MP 10]. Another interesting development is that emerging market currencies gained share in EMEs' FX derivatives trading. The percentage of transactions in EMEs involving emerging market currencies on one side increased to 60% in 2010 from 55% in 2007 (out of a potential 200%).

Geographical distribution of OTC foreign exchange derivatives turnover[1]

Daily averages in April

	In billions of US dollars			Percentage share[2]
	2004	2007	2010	2010
Total emerging market economies	**222**	**430**	**535**	**100**
Total advanced economies	*1,546*	*2,546*	*2,689*	*503*
Asia	**184**	**354**	**442**	**83**
Hong Kong SAR	70	143	194	36
Singapore	91	153	175	33
China	...	1	11	2
India	3	24	14	3
Korea	10	18	25	5
Other	9	16	22	4
Latin America	**7**	**14**	**21**	**4**
Brazil	1	1	5	1
Mexico	5	11	12	2
Other	1	3	4	1
Central and eastern Europe	**19**	**43**	**50**	**9**
Poland	5	7	6	1
Russia	6	16	19	4
Turkey	2	3	11	2
Other	6	17	13	2
Other emerging market economies	**12**	**19**	**22**	**4**
South Africa	8	11	10	2
Other	4	8	12	2

[1] Outright forwards, FX swaps, currency swaps, currency options and other FX products. The category "other FX products" covers highly leveraged transactions and/or trades whose notional amount is variable and where a decomposition into individual plain vanilla components was impractical or impossible. Adjusted for local inter-dealer double-counting (ie "net-gross" basis). [2] As a percentage of total emerging market economies.

Source: Triennial Central Bank Survey. Table 1

Table 1.1: Geographical distribution of OTC foreign exchange derivatives turnover.

OTC foreign exchange derivatives turnover by currency[1]

Daily averages in April, percentage shares

	2004	2007	2010
US dollar	95.5	95.2	94.7
Euro	19.3	15.1	15.8
Japanese yen	16.6	14.0	9.7
Australian dollar	7.5	5.7	8.0
Pound sterling	7.9	6.7	4.3
Swiss franc	1.5	2.4	1.2
Hong Kong dollar	12.4	17.3	15.9
Korean won	6.3	6.2	8.3
Singapore dollar	4.9	6.2	6.7
Chinese renminbi	0.4	1.6	4.8
Indian rupee	2.0	4.5	4.4
Russian rouble	1.1	2.0	2.6
Mexican peso	1.9	2.7	1.8
South African rand	3.1	2.2	1.6
Brazilian real	0.7	0.2	1.0
Polish zloty	1.7	1.2	0.9
Emerging market currencies	**43.5**	**55.0**	**60.4**

[1] Outright forwards, FX swaps, currency swaps, currency options and other FX products. Because two currencies are involved in each transaction, the sum of the percentage shares of individual currencies totals 200% instead of 100%. Because not all of the currencies are listed in the table, the total of the listed percentage shares is less than 200%. Adjusted for local and cross-border inter-dealer double-counting (ie "net-net" basis).

Source: Triennial Central Bank Survey. Table 4

Table 1.2: OTC foreign exchange derivatives turnover by currency pair.

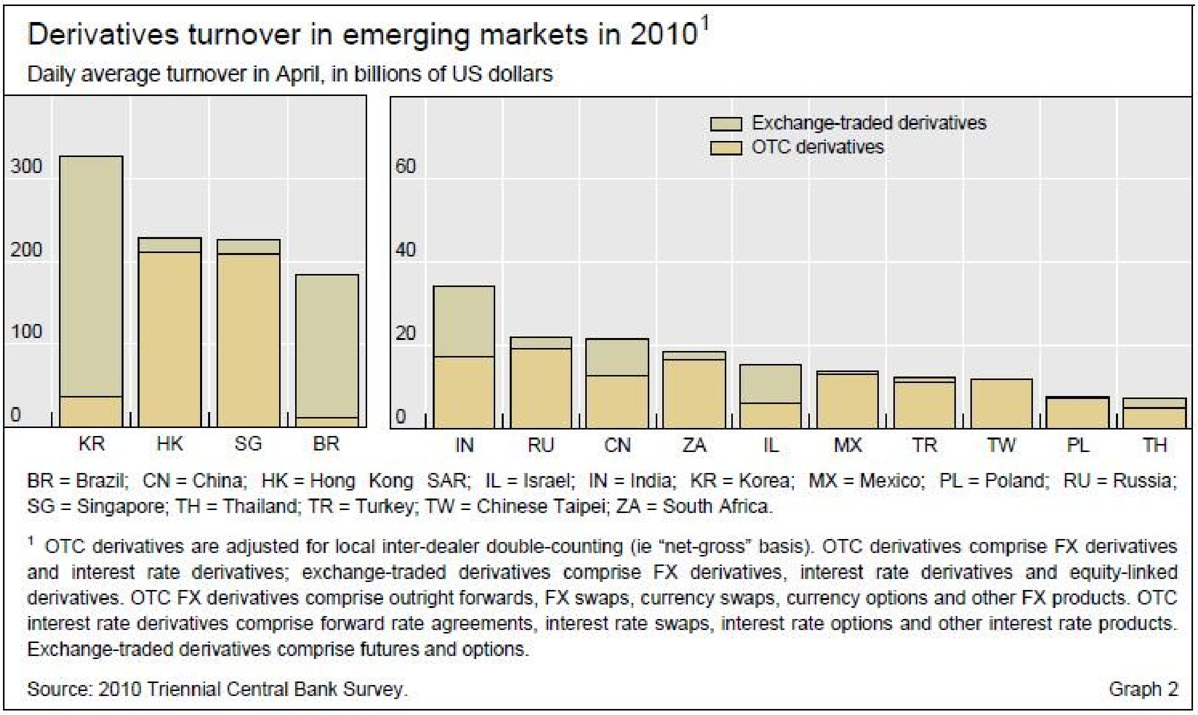


Figure 1.11: Derivatives turnover in EMEs during 2010 showing the ratio between OTC and exchange traded derivatives.

Four emerging market economies stand out in terms of the size and maturity of their derivatives markets: Korea, Brazil and the two Asian financial centres of Hong Kong and Singapore. Brazil and Korea are exceptional in terms of the size of their exchange-traded derivatives markets, and Hong Kong and Singapore in terms of their OTC derivatives markets (see Fig. 1.11, left-hand panel). In addition, no less than 10 EMEs now have total daily derivatives turnover of around \$10 billion or more (right-hand panel).

1.7 Why Trade FX?

The primary purpose of the foreign exchange market is to assist international trade and investment, by allowing businesses to convert one currency to another currency. For example, it permits a Kenyan business to import British goods and pay Pound Sterling, even though the business's income is in Kenyan Shilling (KES).

Another big part of the FX market is due to risk management or hedging. FX risk is the exposure to potential financial losses due to devaluation of the foreign currency against the local currency like KES — in simpler words, it is the risk that profits will change if FX rates change. Contracts like forwards, futures, options and swaps are then used in the process. In Fig. 1.6 we show the size of the derivatives market. From Fig. 1.4 we see that more FX swaps and forwards are traded than the spot.

It also supports speculation, and facilitates the carry trade, in which investors borrow low-yielding currencies and lend (invest in) high-yielding currencies, and which (it has been claimed) may lead to loss of competitiveness in some countries. It correlates with global financial and exchange rate stability and retracts in use during global liquidity shortages. The carry trade is often blamed for rapid currency value collapse and appreciation. It is estimated that the USD/YEN carry trade is more than $1 trillion — borrowing YEN at nearly zero interest and investing in the USA.

Unlike a stock market, the foreign exchange market is divided into levels of access. At the top is the inter-bank market, which is made up of the largest commercial banks and securities dealers. Within the inter-bank market, spreads, which are the difference between the bid and ask prices, are razor sharp and not known to players outside the inner circle. The difference between the bid and ask prices widens (for example from 0-1 pip to 1-2 pips for a currency such as the EUR) as you go down the levels of access. This is due to volume.

If a trader can guarantee large numbers of transactions for large amounts, they can demand a smaller difference between the bid and ask price, which is referred to as a better spread. The levels of access that make up the foreign exchange market are determined by the size of the "line" (the amount of money with which they are trading).

The top-tier interbank market accounts for 53% of all transactions. We show the current top currency trading banks in Fig. 1.12. After that there are usually smaller banks, followed by large multi-national corporations (which need to hedge risk and pay employees in different countries), large hedge funds, and even some of the retail FX market makers. Since 2000, Pension funds, insurance companies, mutual funds, and other institutional investors have played an increasingly important role in financial markets in general, and in FX markets in particular — hedge funds are becoming a big player in the FX market. Lastly, central banks also participate in the foreign exchange market to align currencies to their economic needs.

1.8 Where is the Spot Traded

There is no unified or centrally cleared market for the majority of FX trades, and there is very little cross-border regulation. Due to the over-the-counter (OTC) nature of currency markets, there are rather a number of interconnected marketplaces, where different currencies instruments are traded. This implies that there is not a single exchange rate but rather a number of different rates (prices), depending on what bank or market maker is trading, and where it is. In practice the rates are often very close, otherwise they could be exploited by arbitrageurs instantaneously. Due to London's dominance in the market, a particular currency's quoted price is usually the

Top 10 currency traders

% of overall volume, May 2010

Rank	Name	Market share
1	Deutsche Bank	18.06%
2	UBS AG	11.30%
3	Barclays Capital	11.08%
4	Citi	7.69%
5	Royal Bank of Scotland	6.50%
6	JPMorgan	6.35%
7	HSBC	4.55%
8	Credit Suisse	4.44%
9	Goldman Sachs	4.28%
10	Morgan Stanley	2.91%

Figure 1.12: Top 10 currency traders

London market price. A joint venture of the Chicago Mercantile Exchange (CME) and Reuters, called FxMarketSpace opened on 26 March 2007 and aspired but failed to the role of a central market clearing mechanism. It suspended operations on 17 October 2008.

In §1.6.4 we mentioned that the growth in the turnover of the FX market was driven by a structural change that enabled customers to execute FX transactions electronically. Their is a myriad of trading systems available[6]. Platforms like 360T, ICAP's EBS and FXall are used by many of the players in the market. In response to the increased competition, top FX dealers launched proprietary single-bank trading systems for their clients, such as Barclays' BARX in 2001, Deutsche Bank's Autobahn in 2002 and Citigroup's Velocity in 2006. According to data provided to the BIS, daily average trading volumes on the top single-bank trading systems have increased by up to 200% over the past three years. Electronic methods of execution now accounts for 41.3% of all transactions. This is shown in Fig. 1.13. However, there is a fundamental lack of functionality on some of these platforms, as there is no liquidity beyond G-10 currencies, no longer-term forwards and little to no options functionality, which is of interest to companies that manage cash flows (addressing FAS 133).

The main trading center is London, but New York, Tokyo, Hong Kong and Singapore are all important centers as well. Banks throughout the world participate. Currency trading happens continuously throughout the day; as the Asian trading session ends, the European session begins, followed by the North American session and then back to the Asian session, excluding weekends. Trading in London accounted

[6] Look at http://vcapfx.com/forex-portals/

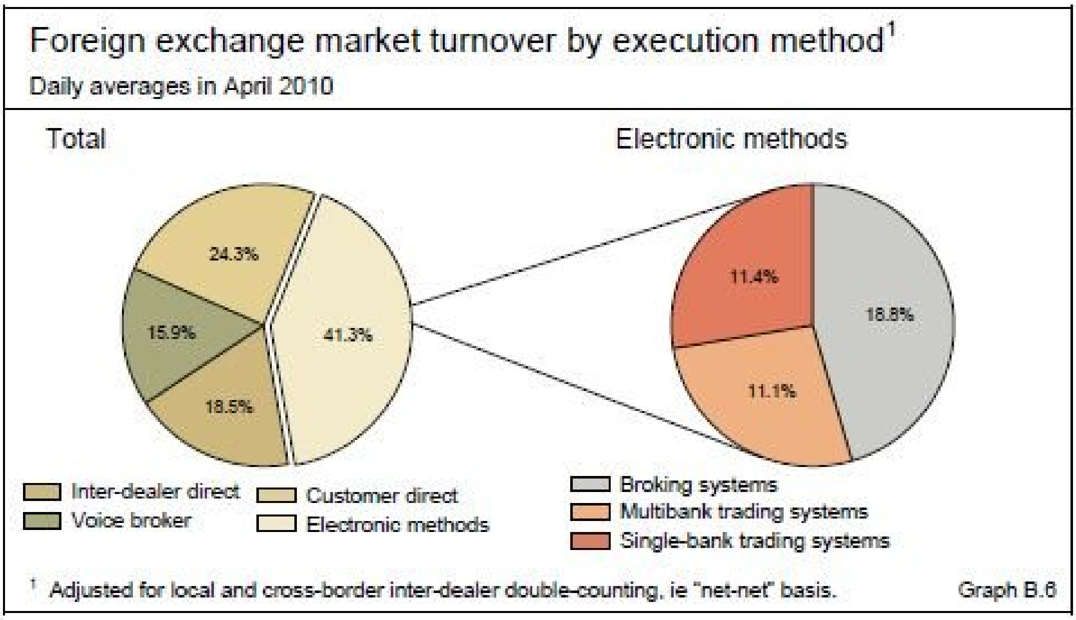


Figure 1.13: Execution method in FX trades

for 36.7% of the total, making London by far the most important global center for foreign exchange trading. In second and third places respectively, trading in New York City accounted for 17.9%, and Tokyo accounted for 6.2%. Note that more than 50% of the USDZAR is traded in London. In Fig. 1.14 we show the turnover of currencies traded out of South Africa as well as the turnover of currencies traded against the Rand.

As FX is now traded as an asset class in its own right, some segments of the market are pondering whether trades should be moved on-exchange[7]. The exchange-traded currency market is still relatively small compared to the OTC market. Turnover of exchange-traded foreign exchange futures and options have grown rapidly in recent years, reaching $166 billion daily in April 2010 (double the turnover recorded in April 2007). Exchange-traded currency derivatives represent 4% of OTC foreign exchange turnover — this amounts to $160 billion of notional value; still small compared to the OTC market.

FX futures contracts were introduced in 1972 at the Chicago Mercantile Exchange and are actively traded relative to most other futures contracts — about $100 billion of notional value per day (this is about 63% of the global value). The average daily trade of the USDZAR on the South African Yield-X platform is about $20 million of notional value per day (or $40 million in turnover). This is minuscule if compared to the $11 billion turnover in the OTC market.

[7]http://www.gfmag.com/archives/124-june-2010/10332-foreign-exchange-trading.html

A

All currencies agianst:

Daily averages, in millions of US dollars

US dollar	13,945.63
Euro	1,886.61
Yen	307.71
Pound sterling	800.37
Swiss franc	108.45
Canadian dollar	117.05
Australian dollar	150.29
Swedish krona	49.26
Currencies of other reporting countries[3]	11,205.26
Residual currencies[4]	178.68
TOTAL (USD)	**14,374.66**

B

Local currency against:

US dollar	10,763.82
Euro	203.87
Yen	14.18
Pound sterling	35.37
Swiss franc	5.63
Canadian dollar	6.74
Australian dollar	7.05
Swedish krona	0.74
Residual currencies[2]	18.26
TOTAL (USD)	**11,055.66**

Figure 1.14: A: OTC foreign exchange traded in South Africa. B: South Africa's Rand traded against other currencies.

1.9 Quotation Styles

In FX there is a natural numeraire currency for every currency pair traded. For the British Pound against the US Dollar (GBPUSD), the market standard quote is GPBUSD meaning the number of US Dollars per one Pound. The quote could have been the other way around e.g., USDGBP meaning the number of Pounds per one US Dollar. Using GBPUSD is purely due to convention. You need to understand this. For a currency pair quoted as

$$ccy1ccy2$$

the spot rate S_t at time t is the number of units of $ccy2$ (also known as the domestic currency, the terms currency or quote currency) required to buy one unit of $ccy1$ [Cl 11].

Note that the spot rate is dimensionally equal to units of $ccy2$ per $ccy1$. The GBPUSD quote is for US Dollars per Pound Sterling. We discourage writing the pairs as GBP—USD or GBP/USD because this can be read as "GBP per USD" which is not what the market quotes. The quote styles confused me at first, but the following heuristic rules help in that regard

- Precious metals are always $ccy1$ against any currency, e.g. XAGUSD
- Euro is always $ccy1$

- Emerging market currencies strongly tends to be *ccy*2, e.g. USDZAR or USDKES
- Currencies that, historically, were subdivided into nondecimal units such as GBP, AUD and NZD tend to be *ccy*1.
- For currency pairs where the spot rate would be either markedly greater than or markedly less than unity, the quote style tends to end up being the quote style that gives a spot rate significantly greater than unity, e.g. USDJPY and even ZARJPY.

1.10 Jargon: Nicknames

Many of the currency pairs are referred to using nicknames. Here are a few [We 06].

GBPUSD	Cable (from Trans-Atlantic cable between US & Britain)
EURUSD	Euro or Fiber
USDCHF	Swissy
USDCAD	Loonie (from bird on Canadian coin)
AUDUSD	Ozzie
EURJPY	Euppy (pronounced "Yuppy")
GBPJPY	Gopher
EURGBP	Chunnel (from channel tunnel between Britain & France)
USDJPY	Yen
USDNZD	Kiwi
USDZAR	Rand or Zar

1.11 Settlement Rules

In the purest contexts, a *spot price* refers to the price for an immediate exchange of an asset for its cash value. However, due to certain defined settlement dates, the exchange of cash only happens on the settlement date — also known as the value date or spot date. In the currency markets the settlement date for most currencies is two business days after the trade date. This is know as T+2. Note, there are some exceptions like USDCAD, USDTRY[8] and USDRUB[9] that settles on T+1. We give the settlement date for a few currencies in Fig. 1.15.

1.11.1 Banking Holidays

Please note that banking holidays are crucial. As an example, ZARUSD contracts settle in New York. Such a contract can thus settle when it is a South Africa public

[8]Turkey's currency.

[9]Russian Ruble

Tag	Currency	Settlement (T +)	Day count
AUD	Australian dollar	2	act/360
CAD	Canadian dollar	2	act/360
CHF	Swiss franc	2	act/360
CZK	Czech koruna	2	act/360
DKK	Danish krone	2	act/360
EUR	Euro	2	act/360
GBP	UK pound	0	act/365
HKD	Hong Kong dollar	2	act/365
JPY	Japanese yen	2	act/360
NOK	Norwegian kroner	2	act/360
NZD	New Zealand dollar	2	act/360
PLN	Polish zloty	2	act/360
SEK	Swedish krona	2	act/360
USD	US dollar	2	act/360
ZAR	South African rand	2	act/365

Figure 1.15: Settlement date and day count convention for some major currencies [Ca 10].

holiday but it cannot when it is a banking holiday in the USA. In general we can state the following

- For currency pairs $ccy1ccy2$, a day is only a good business day if it is a good business day for both $ccy1$ and $ccy2$.

1.11.2 Islamic Currencies

The situation with respect to currencies in the Islamic world is complicated and appears to vary by institution and country [Cl 11]. The reason that Friday is a particular holy day in the Muslim faith, influencing trading calendars. Most Muslim countries are now observing weekends on Friday and Saturday — Saudi Arabia is still observing weekends on Thursday and Friday though.

Due to the Muslim weekend, most Arab currencies cannot settle on a Friday and Saturday. Market convention in the interbank market for Arab currencies is that the spot date for Wednesday's trades is taken to be Monday. For AED, BHD, EGP, KWD, OMR and QAR, the spot date for Thursday's trades is also taken to be Monday, because this still leaves two working days for each currency in the pair (i.e. Friday and Monday for the USD, and Sunday and Monday for the Arab currency). This means that Tuesday is never a spot date in these currencies and can only be priced as a broken date. The exceptions to this rule are SAR and JOD, where the spot date for Thursday's trades is taken to be Tuesday, effectively making a three-day weekend (Friday, Saturday, Sunday) for value date purposes. Some banks, particularly Arab banks when trading with their customers, use split settlement for USD/Arab currency pairs, with USD settling on the Friday or Monday, and the Arab currency settling on the Sunday. In such cases of split settlement, the USD payment is always to the bank's advantage, whereby the bank receives USD from its customer on the Friday but pays USD to its customer on the Monday[10].

For reference let's get the codes right:

SAR	Saudi Riyal
AED	United Arab Emirates Dirhams
BHD	Bahrain Dinars
EGP	Egyptian Pound
KWD	Kuwaiti Dinars
OMR	Omani rials
QAR	Qatar riyals

1.12 Pips and Big Figures

FX rates are expressed as five-digit numbers for no regard for the number of decimals. On trading floors, the spot rate S_t is read out aloud with the word 'spot' used to indicate the decimal place, e.g. 'seven **spot** two four eight six' for a USDZAR rate of 7.2486. If there are no decimals and the Rand was at 7.00 to the Dollar, traders will call out 'seven the **figure**' [Cl 11].

If the Rand went from 7.2486 to 7.2487, we would say the price had gone up by one "pip"

A "pip" is the smallest quoted unit of the spot price.

This is very important because a pip is not always the same size. A one pip change in USDJPY might take you from 83.00 to 83.01.

If the Rand went up from 7.2400 to 7.2500, we would say it went up by 100 pips. There are two tings we need to point out here [We 06]. The first:

We refer to 100 pips as a "big figure" or "the figure".

If the Rand rose by three big figures it would go from 7.2400 to 7.2700. Note that for the USDJPY three big figures might take the Yen from 82.00 to 85.00.

The second point:

A big figure often relates to (the smallest quoted) currency unit in that currency.

In both South Africa and Kenya it is a 'cent' with 100 cents making R1 or Ks1 while in Japan it is a Yen.

[10] http://www.londonfx.co.uk/valdates.html

Most traded currencies

Currency distribution of reported FX market turnover

Rank	Currency	ISO 4217 code (Symbol)	% daily share (April 2010)
1	United States dollar	USD ($)	84.9%
2	Euro	EUR (€)	39.1%
3	Japanese yen	JPY (¥)	19.0%
4	Pound sterling	GBP (£)	12.9%
5	Australian dollar	AUD ($)	7.6%
6	Swiss franc	CHF (Fr)	6.4%
7	Canadian dollar	CAD ($)	5.3%
8	Hong Kong dollar	HKD ($)	2.4%
9	Swedish krona	SEK (kr)	2.2%
10	New Zealand dollar	NZD ($)	1.6%
	Other Currencies		18.6%
		Total[notes 1]	**200%**

Figure 1.16: Most traded currencies. Note that the total adds up to 200% due to each currency being a pair.

1.13 Major, Minor and Exotic Currencies

1.13.1 Most Heavily traded Currencies

On the spot market, according to the 2010 BIS Triennial Survey, the most heavily traded bilateral currency pairs were [BIS 10]

- EURUSD: 28%
- USDJPY: 14%
- GBPUSD: 9%.

However, the US currency was involved in 84.9% of all transactions, followed by the euro (39.1%), the yen (19.0%), and sterling (12.9%) (see Fig 1.16). Volume percentages for all individual currencies should add up to 200%, as each transaction involves two currencies called a pair. These three are thus the major currencies in the world. The South African Rand is about 20th on this list. No other African currency is mentioned in this survey. The BIS survey expanded with more detail on some emerging markets. The Rand was worth a mention as can be seen in Fig. 1.17.

Trading in the euro has grown considerably since the currency's creation in January 1999, and how long the foreign exchange market will remain dollar-centered is

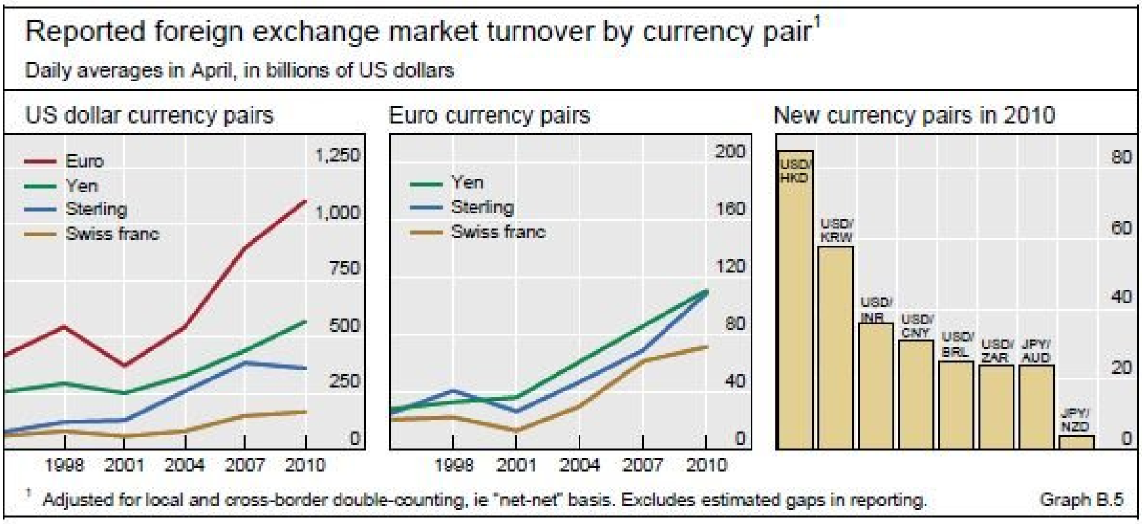


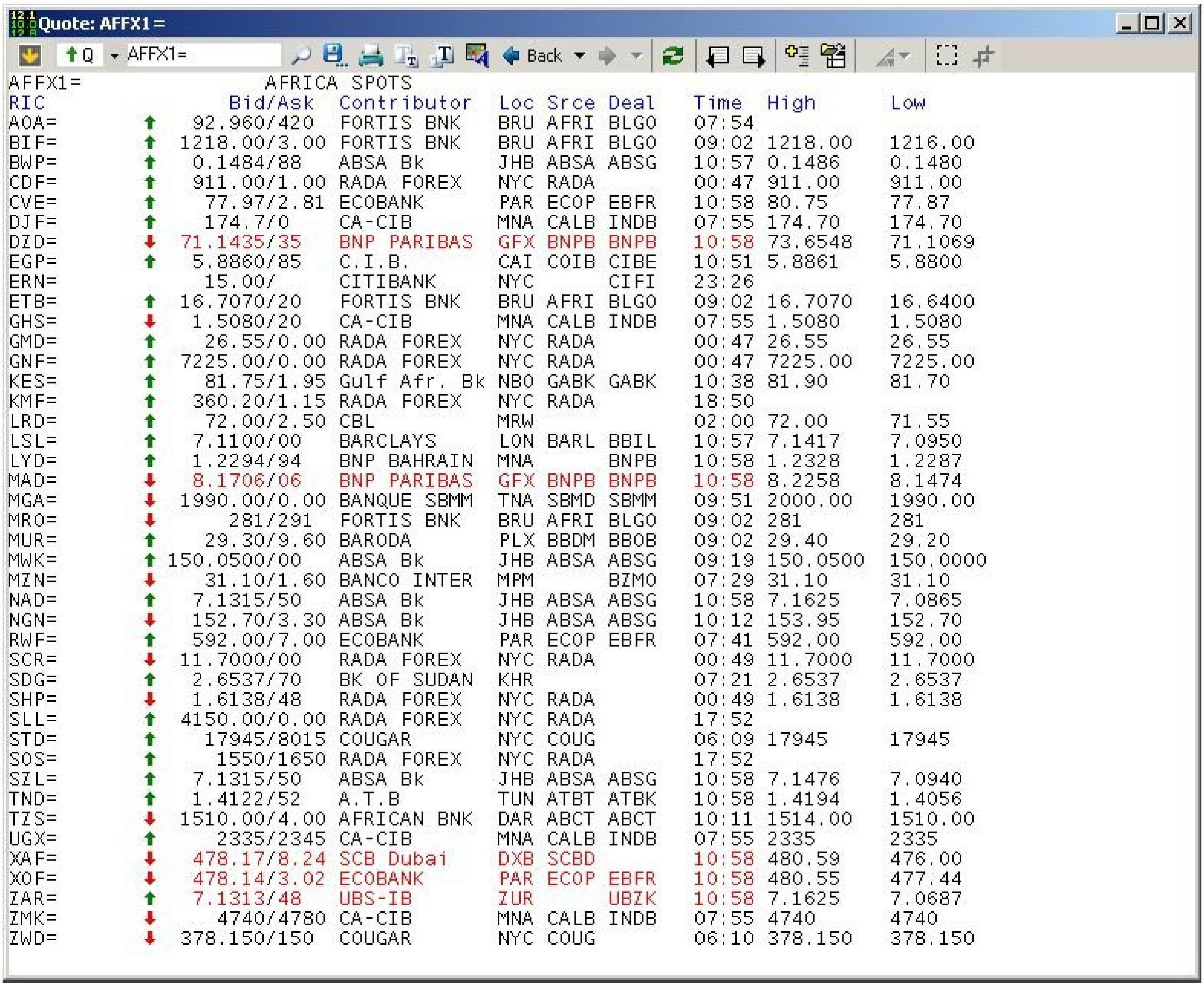
Figure 1.17: FX market turnover by currency pair.

open to debate. Until recently, trading the euro versus a non-European currency ZZZ would have usually involved two trades: EURUSD and USDZZZ. The exception to this is EURJPY, which is an established traded currency pair in the interbank spot market. As the dollar's value has eroded during 2008, interest in using the euro as reference currency for prices in commodities (such as oil), as well as a larger component of foreign reserves by banks, has increased dramatically. Transactions in the currencies of commodity-producing countries, such as AUD, NZD and CAD, have also increased. However, even today, there is no actively traded EURZAR market (and the Eurozone is South Africa's biggest trading partner). Every Rand-Euro trade involves two trades: USDZAR and then EURUSD.

1.13.2 Circulating Currencies

Does every country has its own currency? The answers is, no. Due to a lack of confidence in a currency some countries have abandoned their currencies. This happened in 2000 in Ecuador which now use the US Dollar. The most recent case is Zimbabwe that abandoned the Zimbabwe Dollar during January 2009 — one can now use the US Dollar, Rand or Pound in any transaction. One can find a "list of circulating currencies" on WikiPedia[11]. This list contains the 182 current official or de facto currencies of the 192 United Nations member states, one UN observer state, nine partially recognized states, one unrecognized state, and 36 dependencies. We show the African currencies prices as seen on Reuters 3000 in Fig. 1.18.

[11] http://en.wikipedia.org/wiki/List_of_circulating_currencies



Quote: AFFX1=

AFFX1= AFRICA SPOTS

RIC		Bid/Ask	Contributor	Loc	Srce	Deal	Time	High	Low
AOA=	↑	92.960/420	FORTIS BNK	BRU	AFRI	BLGO	07:54		
BIF=	↑	1218.00/3.00	FORTIS BNK	BRU	AFRI	BLGO	09:02	1218.00	1216.00
BWP=	↑	0.1484/88	ABSA Bk	JHB	ABSA	ABSG	10:57	0.1486	0.1480
CDF=	↑	911.00/1.00	RADA FOREX	NYC	RADA		00:47	911.00	911.00
CVE=	↑	77.97/2.81	ECOBANK	PAR	ECOP	EBFR	10:58	80.75	77.87
DJF=	↑	174.7/0	CA-CIB	MNA	CALB	INDB	07:55	174.70	174.70
DZD=	↓	71.1435/35	BNP PARIBAS	GFX	BNPB	BNPB	10:58	73.6548	71.1069
EGP=	↑	5.8860/85	C.I.B.	CAI	COIB	CIBE	10:51	5.8861	5.8800
ERN=		15.00/	CITIBANK	NYC		CIFI	23:26		
ETB=	↑	16.7070/20	FORTIS BNK	BRU	AFRI	BLGO	09:02	16.7070	16.6400
GHS=	↓	1.5080/20	CA-CIB	MNA	CALB	INDB	07:55	1.5080	1.5080
GMD=	↑	26.55/0.00	RADA FOREX	NYC	RADA		00:47	26.55	26.55
GNF=	↑	7225.00/0.00	RADA FOREX	NYC	RADA		00:47	7225.00	7225.00
KES=	↑	81.75/1.95	Gulf Afr. Bk	NBO	GABK	GABK	10:38	81.90	81.70
KMF=	↑	360.20/1.15	RADA FOREX	NYC	RADA		18:50		
LRD=	↑	72.00/2.50	CBL	MRW			02:00	72.00	71.55
LSL=	↑	7.1100/00	BARCLAYS	LON	BARL	BBIL	10:57	7.1417	7.0950
LYD=	↑	1.2294/94	BNP BAHRAIN	MNA		BNPB	10:58	1.2328	1.2287
MAD=	↓	8.1706/06	BNP PARIBAS	GFX	BNPB	BNPB	10:58	8.2258	8.1474
MGA=	↓	1990.00/0.00	BANQUE SBMM	TNA	SBMD	SBMM	09:51	2000.00	1990.00
MRO=	↓	281/291	FORTIS BNK	BRU	AFRI	BLGO	09:02	281	281
MUR=	↑	29.30/9.60	BARODA	PLX	BBDM	BBOB	09:02	29.40	29.20
MWK=	↑	150.0500/00	ABSA Bk	JHB	ABSA	ABSG	09:19	150.0500	150.0000
MZN=	↓	31.10/1.60	BANCO INTER	MPM		BZMO	07:29	31.10	31.10
NAD=	↑	7.1315/50	ABSA Bk	JHB	ABSA	ABSG	10:58	7.1625	7.0865
NGN=	↓	152.70/3.30	ABSA Bk	JHB	ABSA	ABSG	10:12	153.95	152.70
RWF=	↑	592.00/7.00	ECOBANK	PAR	ECOP	EBFR	07:41	592.00	592.00
SCR=	↓	11.7000/00	RADA FOREX	NYC	RADA		00:49	11.7000	11.7000
SDG=	↑	2.6537/70	BK OF SUDAN	KHR			07:21	2.6537	2.6537
SHP=	↓	1.6138/48	RADA FOREX	NYC	RADA		00:49	1.6138	1.6138
SLL=	↑	4150.00/0.00	RADA FOREX	NYC	RADA		17:52		
STD=	↑	17945/8015	COUGAR	NYC	COUG		06:09	17945	17945
SOS=	↑	1550/1650	RADA FOREX	NYC	RADA		17:52		
SZL=	↑	7.1315/50	ABSA Bk	JHB	ABSA	ABSG	10:58	7.1476	7.0940
TND=	↑	1.4122/52	A.T.B	TUN	ATBT	ATBK	10:58	1.4194	1.4056
TZS=	↓	1510.00/4.00	AFRICAN BNK	DAR	ABCT	ABCT	10:11	1514.00	1510.00
UGX=	↑	2335/2345	CA-CIB	MNA	CALB	INDB	07:55	2335	2335
XAF=	↓	478.17/8.24	SCB Dubai	DXB	SCBD		10:58	480.59	476.00
XOF=	↓	478.14/3.02	ECOBANK	PAR	ECOP	EBFR	10:58	480.55	477.44
ZAR=	↑	7.1313/48	UBS-IB	ZUR		UBZK	10:58	7.1625	7.0687
ZMK=	↓	4740/4780	CA-CIB	MNA	CALB	INDB	07:55	4740	4740
ZWD=	↓	378.150/150	COUGAR	NYC	COUG		06:10	378.150	378.150

Figure 1.18: African currency rates shown on Reuters 3000.

1.13.3 Groups of Currencies

In basic forex nomenclature, the different currency pairs of the world are divided into groups by the amount of daily trading activity and liquidity in each of them. The percentage of the daily turnover is shown in Fig. 1.16. The market refers to these groups specifically as the major, minor and exotic currencies. Below we list the different currencies that make up each group.

Major Currencies

- USD - U.S. Dollar
- EUR- European Union Euro
- JPY- Japanese Yen
- GBP - U.K. Pound Sterling
- CHF - Swiss Franc

As stated in §1.13.1, these currencies account for the vast majority of FX transactions.

Minor Currencies

The Commodity Currencies (or Com Dollars) are minor currencies that belong to countries which have a variety of natural resources which give their currencies intrinsic value. As a result, the values of these currencies are linked to the value of the commodities their economies produce, such as oil or gold, for example. These are

- AUD - Australian Dollar
- CAD - Canadian Dollar
- NZD - New Zealand dollar

Note that from Fig. 1.16 we ascertain that the Australian Dollar has replaced the Swiss Franc in position number 5. The AUD is becoming a major currency thus.

Exotic Currencies

The fact that these currencies are deemed exotic is because of the lack of liquidity in their foreign exchange markets, not because of the country's location or size. These currencies are usually from emerging markets or developing countries in Asia, the Pacific, Middle East and Africa. Some of the more active exotic currencies include

- HKD - Hong Kong Dollar

- MXN - Mexican Peso
- SGD - Singapore Dollar
- KRW - South Korean Won
- ZAR - South African Rand
- RUB - Russian Federation Ruble
- INR - Indian Rupee

The group of Exotic Currencies consists out of just about all the currencies not included in the Majors and the Minors.

One of the major challenges in trading an exotic currency is the market is usually illiquid, and has a high cost of carry. An illiquid market is one that has little volume traded. Cost of carry refers to cost of holding a currency position. If you buy the currency, the cost of carry is the interest paid on a margin account. If you short a currency the cost of carry is the cost paying dividends or opportunity cost. Illiquid markets are often hard to enter and exit without substantial cost. Higher costs and difficulty of entering and exiting exotic trades can reduce your profit potential and longevity of your trading. We show historical rates for some African currencies in Fig. 1.19.

1.14 What is a Hard Currency?

A reserve currency, or anchor currency, is a currency that is held in significant quantities by many governments and institutions as part of their foreign exchange reserves. It also tends to be the international pricing currency for products traded on a global market, and commodities such as oil, gold and copper. Policies of the World Central Banks regarding the holdings of foreign reserves favour long term views (20+ years). Since the USA moved off the gold standard in 1971, reserve currency holdings have been stable. This can be seen through the IMF COFER table shown in Table 1.3.

A hard currency is mandatory to transact international business. It enables goods and services to be bought by inspiring confidence that the exchanged paper money and coins will hold worth over time. These currencies are identified with home nations that promote free markets, technological innovation and strong legal systems that protect individual rights.

A hard currency can be defined as follows[12]

- Deardorff's Glossary of International Economics defines a hard currency as "a currency that is widely accepted around the world, usually because it is the

[12] http://www.ehow.com/about_5435731_hard-currency.html

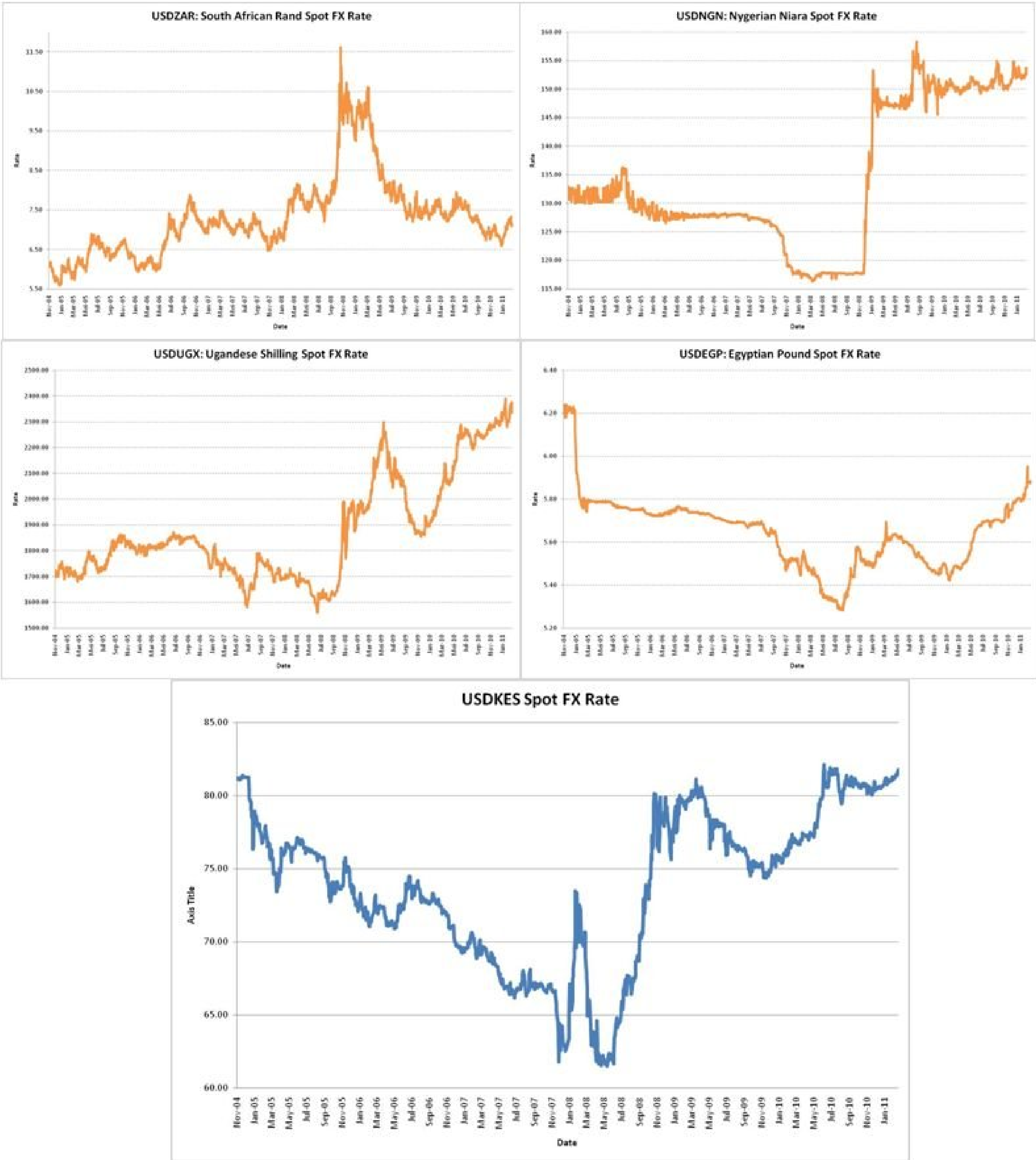


Figure 1.19: Historical FX rates for African currencies: ZAR, KES, NGN and EGP.

Currency composition of official foreign exchange reserves

	1995	2001	2002	2003	2004	2005	2006	2007	2008	2009	Q2 2010
US$	59.00%	70.70%	66.50%	65.80%	65.90%	66.40%	65.70%	64.10%	64.00%	61.50%	62.10%
Euro		19.80%	24.20%	25.30%	24.90%	24.30%	25.20%	26.30%	26.50%	28.10%	26.50%
DEM	15.80%										
GBP	2.10%	2.70%	2.90%	2.60%	3.30%	3.60%	4.20%	4.70%	4.10%	4.20%	4.20%
YEN	6.80%	5.20%	4.50%	4.10%	3.90%	3.70%	3.20%	2.90%	3.30%	3.00%	3.30%
FFR	2.40%										
SFr	0.30%	0.30%	0.40%	0.20%	0.20%	0.10%	0.20%	0.20%	0.10%	0.10%	0.10%
Other	13.60%	1.20%	1.40%	1.90%	1.80%	1.90%	1.50%	1.80%	2.00%	3.10%	3.80%

Sources: 1995-1999, 2006-2010 IMF: Currency Composition of Official Foreign Exchange Reserves

Table 1.3: IMF COFER Table showing 62% of global reserves are held in USD.

currency of a country with a large and stable market." Further, this definition implies that hard currency represents viable storage of value that will not fluctuate wildly with market conditions and politics.

A hard currency has the following characteristics

- Hard currency is always an accepted medium to transact business and deliver price quotes internationally. It is readily available at major banks for exchange, and the rates are the most often quoted by the financial media. Although exchange rates and values of competing currencies will fluctuate from day to day, hard currency buying power is relatively stable in the short term.

If we impose the two points above we the following currencies that are globally accepted as being "hard currencies": Canadian Dollar, Euro, Yen, British Pound and U.S. dollar, respectively.

The U.S. dollar is the language of business. Oil is quoted and delivered in dollars; multinational corporate annual reports are broken down into U.S. dollars and numerous emerging markets peg their home currencies to the dollar for legitimacy. However, deficits and inflationary government policy threaten to derail the U.S. Dollar as the world's premier hard currency. Already bloated twin U.S. budget and trade deficit that are constantly nearly $50 billion per month, will lead to a weaker Dollar. The USA has financial problems. As of December 31, 2010, the "Total public debt outstanding" was $14.03 trillion and was 96.5% of 2010's annual gross domestic product (GDP) of $14.5 trillion. Printing money and stoking inflation destabilizes the U.S. dollar, which is not the way to maintain hard currency appeal. However, Japan is not better off. Its ratio of public debt to GDP is nearly 200%. Table 1.4 lists the world

World Top 20			Africa Top 20		
World Rank	Country	% of GDP	World Rank	Country	% of GDP
1	Zimbabwe	282.60	1	Zimbabwe	282.60
2	Japan	189.30	10	Sudan	103.70
3	Saint Kitts and Nevis	185.00	13	Egypt	80.10
4	Lebanon	156.00	23	Kenya	66.70
5	Greece	126.80	28	Côte d'Ivoire	61.90
6	Jamaica	124.50	31	Mauritius	58.70
7	Italy	115.20	40	Ghana	55.20
8	Singapore	113.10	41	Morocco	55.10
9	Iceland	107.60	46	Tunisia	53.00
10	Sudan	103.70	65	Malawi	39.50
11	Belgium	97.60	80	Ethiopia	31.80
12	Sri Lanka	86.70	86	Senegal	29.80
13	Egypt	80.10	87	South Africa	29.50
14	Israel	78.40	88	Gabon	29.10
15	Hungary	78.00	93	Zambia	25.50
16	France	77.50	101	Angola	22.10
17	Portugal	76.90	103	Tanzania	21.40
18	Canada	38.10	104	Algeria	20.00
19	Germany	72.10	105	Uganda	19.70
20	Malta	69.40	107	Botswana	17.90

Table 1.4: Ranking country's GDP to debt ratio. We show the world Top 20 and also African Top 20.

Top 20 in the world's GDP to debt ratio[13]. Next to that we also show the African Top 20.

Brazil, Russia, India, and China (BRIC) are rapidly emerging nations that have yet to earn hard currency acclaim — South Africa has just been accepted as a member of BRIC; now BRICS. China has actually artificially pegged and devalued its yuan to the USD in order to boost its export economy.

1.15 Cross Currencies

Historically, forex transactions had to involve the US Dollar. When two non-USD currencies had to be traded, the trader was required to first convert the currency he/she was holding into US Dollars and then convert US Dollars into the desired currency. Cross currency trading bypasses this step. There are two types of Cross Currency pairs[14]

- Pairs involving the Euro as the reference currency, such as Euro/Japanese

[13]Data from the CIA's "The World Fact Book". See http://en.wikipedia.org/wiki/List_of_sovereign_states_by_public_debt

[14]http://www.economywatch.com/currency/cross-currency.html

Yen (EURJPY), Euro/Swiss Francs (EURCHF) and Euro/British Pound (EURGBP). Cross rates that involve the Euro are known as *Euro crosses*.

- Pairs that involve neither the Euro nor the US dollar, such as Malaysian Ringgit/Singapore Dollar (MYRSGD) and Singapore Dollar/Thai Baht (SGDTHB). These are called *cross currency pairs* or *cross rates*.

A cross-currency transaction is one which involves the simultaneous buying and selling of two or more currencies. The term is also used generically for any transaction that involves more than one currency, such as a currency swap. Most exotic currency rates are determined by cross rates against the USD. If a South African company exports goods to Kenya, a cross rate between ZAR and KES is obtained by the USDZAR and USDKES rates.

In Table 1.5 we list most of the African currencies with their ISO codes.

1.16 Currency Indices: Rand Currency Index

A currency index tracks the combined movement of a base currency against a basket of currencies. For example, the US dollar index measures the US dollar against 6 main currencies: Euro (EUR); Yen (JPY); Pound Sterling (GBP); Loonie (CAD) - Canada; Kronas (SEK) - Sweden and Swiss Francs (CHF). There are many other such indices like the Canadian-dollar effective exchange rate index (CERI)and the Bank of England's new Sterling exchange rate index (ERI). The BIS calculates effective exchange rate (EER) indices for a total of 58 economies (including individual euro area countries and, separately, the euro area as an entity). Nominal EERs are calculated as geometric weighted averages of bilateral exchange rates[15].

The South African Reserve Bank has been publishing a real effective Rand exchange rate since 1970 and a nominal effective Rand exchange rate since 1990. During 2010 the JSE decided to create its own Rand based currency index that will track the changes/movement of the Rand against a basket of currencies. The South African Rand Currency Index (RAIN) is an arithmetic index of five foreign currencies, weighted to reflect the relative competitiveness of SA goods in international markets. The currency weights in the index are determined according to the importance of a country in its trade with South Africa. The bigger the share of trade flows (inflows and outflows) a country has with South Africa the larger the weight of its currency in the index. The weight calculations are based on trade data provided by the South African Revenue Services (SARS) and is updated annually. The selected foreign currencies and their corresponding weights are reviewed and updated annually as data comes available, to ensure that the index adequately reflects ongoing developments in international trade patterns. For the past 5 years the 5 currencies that were and

[15] http://www.bis.org/statistics/eer/index.htm

Country	Currency	Sign	ISO code	Fractional unit	Number to basic
Algeria	Algerian dinar		DZD	Santeem	100
Angola	Angolan kwanza	Kz	AOA	Cêntimo	100
Benin	West African CFA franc	Fr	XOF	Centime	100
Botswana	Botswana pula	P	BWP	Thebe	100
Burkina Faso	West African CFA franc	Fr	XOF	Centime	100
Burundi	Burundian franc	Fr	BIF	Centime	100
Cameroon	Central African CFA franc	Fr	XAF	Centime	100
Cape Verde	Cape Verdean escudo	Esc or $	CVE	Centavo	100
Central African Republic	Central African CFA franc	Fr	XAF	Centime	100
Chad	Central African CFA franc	Fr	XAF	Centime	100
Comoros	Comorian franc	Fr	KMF	Centime	100
DRC	Congolese franc	Fr	CDF	Centime	100
Congo, Republic of the	Central African CFA franc	Fr	XAF	Centime	100
Côte d'Ivoire	West African CFA franc	Fr	XOF	Centime	100
Egypt	Egyptian pound	£	EGP	Piastre[F]	100
Equatorial Guinea	Central African CFA franc	Fr	XAF	Centime	100
Eritrea	Eritrean nakfa	Nfk	ERN	Cent	100
Ethiopia	Ethiopian birr	Br	ETB	Santim	100
Gabon	Central African CFA franc	Fr	XAF	Centime	100
Gambia, The	Gambian dalasi	D	GMD	Butut	100
Ghana	Ghanaian cedi	?	GHS	Pesewa	100
Hong Kong	Hong Kong dollar	$	HKD	Cent	100
Kenya	Kenyan shilling	Sh	KES	Cent	100
Lesotho	Lesotho loti	L	LSL	Sente	100
Liberia	Liberian dollar	$	LRD	Cent	100
Libya	Libyan dinar	?.?	LYD	Dirham	1,000
Madagascar	Malagasy ariary	Ar	MGA	Iraimbilanja	5
Malawi	Malawian kwacha	MK	MWK	Tambala	100
Mauritius	Mauritian rupee	Rs	MUR	Cent	100
Morocco	Moroccan dirham	?.?.	MAD	Centime	100
Mozambique	Mozambican metical	MTn	MZN	Centavo	100
Namibia	Namibian dollar	$	NAD	Cent	100
Niger	West African CFA franc	Fr	XOF	Centime	100
Nigeria	Nigerian naira	?	NGN	Kobo	100
Rwanda	Rwandan franc	Fr	RWF	Centime	100
Senegal	West African CFA franc	Fr	XOF	Centime	100
Seychelles	Seychellois rupee	Rs	SCR	Cent	100
Sierra Leone	Sierra Leonean leone	Le	SLL	Cent	100
Somalia	Somali shilling	Sh	SOS	Cent	100
Somaliland	Somaliland shilling	Sh	None	Cent	100
South Africa	South African rand	R	ZAR	Cent	100
Sudan	Sudanese pound	£	SDG	Piastre	100
Suriname	Surinamese dollar	$	SRD	Cent	100
Swaziland	Swazi lilangeni	L	SZL	Cent	100
Tanzania	Tanzanian shilling	Sh	TZS	Cent	100
Togo	West African CFA franc	Fr	XOF	Centime	100
Tunisia	Tunisian dinar	?.?	TND	Millime	1,000
Uganda	Ugandan shilling	Sh	UGX	Cent	100
Zambia	Zambian kwacha	ZK	ZMK	Ngwee	100
Zimbabwe	Botswana pula	P	BWP	Thebe	100
	British pound[C]	£	GBP	Penny	100
	Euro	l	EUR	Cent	100
	South African rand	R	ZAR	Cent	100
	United States dollar	$	USD	Cent[D]	100
	Zimbabwean dollar[M]	$	ZWL	Cent	100

Table 1.5: List of circulating currencies in Africa.

still are included in the index are: the Euro; the US Dollar; the Chinese Yuan; the UK Pound and the Japanese Yen.

The objective is to summarise the effects of Rand appreciation and depreciation against foreign currencies in the competitiveness of SA goods relative to goods produced by South Africa's main trading partners. Secondly, it tracks the value of the Rand against a basket of foreign currencies, so it can also be used to gauge the financial market pressure on the Rand. Finally, it provides investors with a new product for currency market participation and risk management.

The currency index will be calculated and distributed by the JSE on a daily basis and, if demand surfaces, future and option contracts can be listed on it[16]. The RAIN is inversely quoted against the Rand. That means that when the Rand appreciates against the basket of currencies, the index value goes down and when the Rand depreciates against the basket of currencies, the index value goes up.

The RAIN is calculated daily using an arithmetic weighted index calculation with the following formula

$$RAIN_t = \sum_{i=1}^{N} R_{i,t} \times ContZ_i \times NCont_i \tag{1.1}$$

where $RAIN_t$ is the index on date t;
N is the number of foreign currencies in the index;
$R_{i,t}$ is the Rand exchange rate per foreign currency;
$ContZ_i$ is the contract size or notional value for foreign currency i, i.e. number of the currency units traded for one currency futures contract and is the constant number set for the futures contract on the listing day. The current contract sizes are: 1,000 Euros; 1,000 US Dollars; 10,000 Chinese Renminbi; 1,000 British Pounds and 100,000 Japanese Yen.
$NCont_i$ is the fixed number of contract for currency i in the index.

These fixed numbers of contracts encompassing the currency weights are adjusted on the rebalancing dates, usually a day after currency futures close-out date, using the formula as follows:

$$NCont_i = \frac{(RAIN_T \times W_{i,T})}{R_{i,T} \times ContZ_i}$$

where $RAIN_T$ is the level of the index at inception or the rebalancing date T;
W_i is the weight of currency i in the index on the rebalancing date T;
$\sum_{i=1}^{N} W_{i,T} = 1$;
$R_{i,T}$ is the SA Rands per foreign currency unit i at inception or the rebalancing date T; and
$ContZ_i$ is the contract size for foreign currency i.

[16]http://www.jse.co.za/Products/Currency_Derivatives_Market.aspx

Figure 1.20: RAIN since inception.

We plot the RAIN in Fig. 1.20 showing the Rand weakening up to September 2008 when the global financial crisis hit the world. Also seen is a strengthening Rand since the crisis.

Chapter 2

Foreign Exchange Risk and Hedging

Foreign exchange requirements (FER) arise from the commercial transactions of an organisation, including purchases from suppliers and vendors to customers in other countries and currencies. They also arise as a result of foreign currency assets and liabilities. In addition, FER may be influenced by exposure to commodity prices.

Because foreign exchange exposure arises from many of the commercial transactions of an organisation, it is sometimes possible to reduce exposure and therefore risk [Ho 06]. We are now going to delve deeper into the risks associated with the foreign exchange markets.

2.1 Introduction

In Fig. 2.1 we show a diagram outlining the risks corporates are exposed to on any given day [Wa 08]. In these notes we focus on foreign exchange risk — one of many risks for companies.

A firm maximizes profit by taking risks in *areas in which it has unique expertise and experience* only; e.g.

- IBM: computer technology and related services
- Microsoft: software technology
- Wal-Mart: retail distribution.

A non-financial firm minimizes risks in areas in which it has no unique competitive advantage, e.g., in foreign exchange risk and interest rate risk. Active risk management creates shareholder value through better stock price performance by

- Stabilizing earnings and dividends

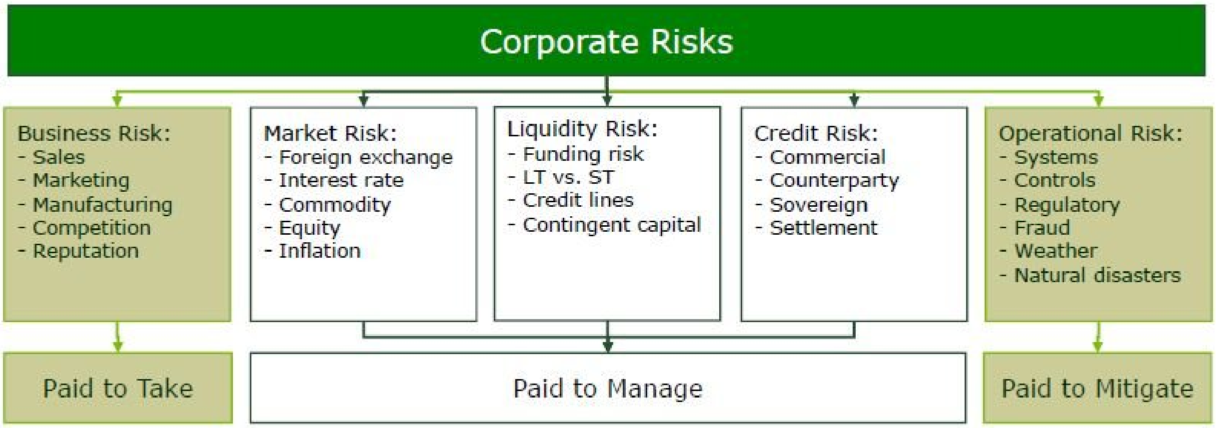


Figure 2.1: Corporate risks.

- Communicating clear expectations and risk profile to the stock market
- Lower discount rate and related weighted average cost of capital (WACC).

In these notes we focus on risks due to foreign exchange exposure — risks taken on by any company that imports or exports goods or services.

Understanding the FX market involves exchanging one country's currency for the currency of another country, it is all about trading money [We 06]. Why is trading money so important? In 1752, David Hume[1] wrote

> Money is not, properly speaking, one of the subjects of commerce; but only the instrument which men have agreed upon to facilitate the exchange of one commodity for another. It is none of the wheels of trade: It is the oil which renders the motion of the wheels more smooth and easy.

So, why is the FX market so big? International trade, and (more important) international money and capital movements, form the basis of foreign exchange dealings. The French economist *Pirou* said that foreign exchange transactions spring from "the coexistence between the internationalism of trade and the nationalism of currencies" [PF 91]. International capital movements are enormous, thus an enormous FX market.

[1] http://en.wikipedia.org/wiki/David_Hume

2.2 FX Market Participants

Most foreign exchange flows will go through a bank. Bank's therefore acts in the market on behalf of many counter parties and clients. From this, 5 groups of market participants emerge [We 06]

- Corporate or multinational companies doing business all around the world
- Majour investment, pension and mutual funds
- Hedge funds
- High Net Worth individuals falling under the "Wealth Management" or "Private Banking" hubs — many FX-based retail structured products are distributed to these clients
- B4B (Bank-for-Banks) — the treasury of the bigger banks acts as a wholesaler of FX products and services to smaller banks and also the business banking arm of the bank where all branches will bring their FX business from smaller companies and individuals.

2.3 Risks in Forex Trading

The foreign exchange market is arguably the largest and most liquid of the international markets, and large and rapid movements in exchange rates are commonplace — see §1.13.1. In order to minimise the possibility of financial loss, it is therefore essential that deposit takers (hereinafter referred to as bank or banks as applicable), importers and exporters identify, measure and manage their foreign exchange risk effectively.

Foreign exchange risk is not confined to proprietary positions taken by a bank and client driven transactions, but can also arise from known profit flows in foreign currency, and provisions for bad debts denominated in foreign currency. Some other reason why there might be foreign exchange exposures are

- Trade — Drawdown and Repayment of Import/Export Foreign Currency Loans and payments of Import/Export Bills denominated in foreign currencies
- Inward and Outward Remittances denominated in foreign currencies
- Overseas Dividends, e.g. repatriating overseas profit home and Overseas Operating Expenses, e.g. paying overseas employees' salary expenses
- Overseas Assets, e.g. surplus cash balances of overseas subsidiaries and Overseas Liabilities, e.g. foreign currency borrowing

It is important that these exposures are identified and, where necessary, hedged, on a timely basis.

Who should be concerned?

- Companies repatriating Overseas Dividends and/or paying Overseas Operating Expenses
- Importers/Exporters
- Companies liquidating Overseas Assets and/or repaying Overseas Liabilities

Do you know whether your business is exposed to FX risk?

- Estimate the total value of all your business components that are exposed to foreign exchange risk
- Then calculate what would happen to your profitability when there are changes in the respective exchange rates.
- Also, consider the timing of your payables and receivables and estimate the potential impact of exchange rate fluctuation on your profit and loss over such time (example 30, 60 or 90 days)

The major currency risks are

- Market Risk or Transaction Risk: This is a short term risk but the only type of risk most traders think about — how daily fluctuations of currency values affect your positions.
- Economic or Business Risk: This relates to the impact of exchange rate changes on a company's long term competitive strength. A depreciation of the local currency may involve import replacement and larger exports.
- Translation Risk: this arise due to the periodic consolidation of the financial statements of a parent and its affiliates for the purpose of uniform reporting to shareholders.
- Political Risks: Political policy changes, major economic emergencies and governing authority intervention can all have an impact on a country's currency value.
- Country Risk: There is the risk that a country won't have the money to meets it's financial commitments, and will default. When this happens the effects trickle down to all other financial instruments in the country and the other countries it's doing business with.

- Broker Risk: A broker is a business like any other, and as such they can face the same problems any regular business can, including bankruptcy. In 2005 a broker in the USA called Refco went bankrupt and they were one of the world's largest investment and brokerage firms involved in forex. Be sure you do your due diligence when selecting a broker.

- Technology Risk: In a trading world run almost entirely on computers, the effects of a hard drive crash, power loss or Internet connection drop out can be drastic. At a bare minimum you should have daily backups of your computer on a separate hard drive or other backup system. And if you are a serious trader consider investing in a surge protector, power generator and backup dial-up Internet connection. It might seem like overkill now but may just save your skin in an emergency.

2.4 Modern Corporate Risk Management

Although we will look in depth into currency risk management, we always have to understand that that is only one facet of a company's risk management process. Any multinational corporate is exposed to may risks as we saw in Fig. 2.1. Risk management has to be taken seriously and a modern risk management program is sustained over a period of time with these objectives

- Raising the awareness level of key risks in the business (risk exposure reporting)

- Proactively mitigating significant risks so as not to exceed senior management's defined worst case (risk tolerance level)

- Incorporating risk management into capital allocation decisions (risk-adjusted returns)

- Strong controls and meaningful reporting to senior management (governance)

These objectives are shown diagrammatically in Fig. 2.2. Hedging should stabilise earnings and modify the risk profile of a company. This is shown in Fig. 2.3.

2.5 What is FX Risk Management?

Day-to-day currency risk management is mostly related to transaction risk. For all practical purposes we can define currency risk as the risk of a mismatch between foreign receivables and foreign payables. In economic terms, the aim of currency risk management is to determine the appropriate mismatch, or imbalance, between maturing foreign assets and liabilities, given certain basic information such as current

Figure 2.2: Objectives of Risk Management.

and expected exchange rates, interest rates (both locally and abroad) and the risk-return profile acceptable to a company's management [PF 91].

FX risk management is thus designed to preserve the value of currency inflows, investments and loans, while enabling international businesses to compete abroad. Although it is impossible to eliminate all risks, negative exchange outcomes can be anticipated and managed effectively by individuals and corporate entities. Businesses do so by becoming familiar with the typical foreign exchange risks, demanding hard currency, diversifying properly and employing hedging strategies.

To summarise, why should you manage your foreign exchange risk? We list a few reasons

- Changes in exchange rates induce changes in the value of a firm's assets, liabilities and cash flows, especially when these are denominated in a foreign currency.
- Therefore, fluctuations in the currency markets have an impact on your outgoing import payments and incoming export funds
- Your foreign exchange risk is influenced by many factors such as length of exposure and currency volatility
- By managing the risk, you could maximize profits or minimize the risk

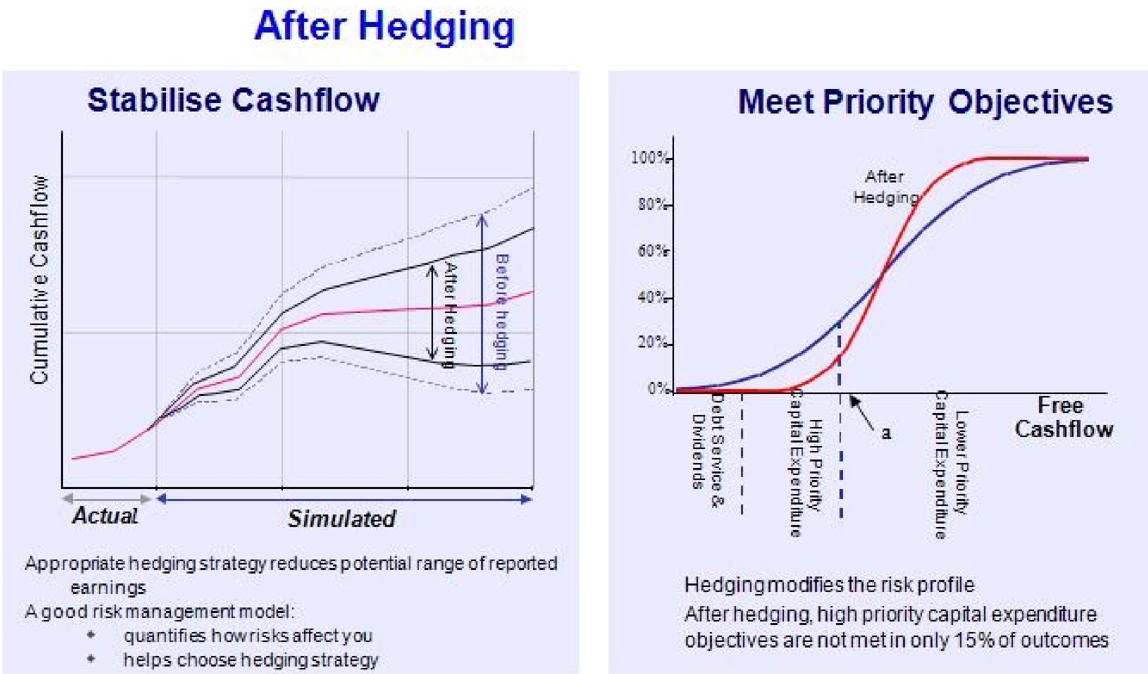


Figure 2.3: Hedging should stabilise cash flows.

2.6 The Great Equiliser

Trading in the financial markets can be summarised by the phrase, “You just don’t know what you don’t know.” Today, the flow of information and the speed of its dissemination are simply astounding. In the electronic age, the markets have evolved from a disjointed fragmentation of phone calls and hand signals to a symphony of speed and synchronisation [Sh 10].

Yet, despite all the growth and development of the financial markets, there remains a great equiliser. There is an element, as applicable to an eighteenth-century bourse trader as to the twenty-first-century day trader, enveloping himself with television screens, statistical forecasts and computer monitors.

The great equiliser — the factor that surpasses both time and knowledge — is volatility. Volatility is the ultimate unknown. No matter what is said, modeled, or written about it, volatility simply cannot be forecasted. Volatility and risk can be construed synonymously, and both terms are derived from uncertainty. In terms of financial markets, *uncertainty generates volatility, and volatility results in risk.*

So, what is risk then? Risk is the direct result of a random event which has a quantifiable probability. The probability of an event can be determined by using either practical observance of the frequency of past events or theoretical forecasting models. Risk as we define it here is market risk or transaction risk.

What is uncertainty? The concept of uncertainty is more intricate than that of risk. Risk can be observed and quantified, uncertainty cannot. Uncertainty is, however, the conduit to volatility and should not be dismissed by any investor.

2.7 How to Minimise Foreign Exchange Risks?

We minimise our FX market risks by hedging the exposure. Various hedging instruments are available: forwards, futures, currency swaps and currency options. Derivative instruments provide the means to achieve this risk transfer. Precisely how derivatives are used is a matter of discretion that requires an assessment of an assortment of strategies available to the firm.

In essence, the well-run company has two choices: Either it can use derivatives in a manner that allows it to shift any unwanted risk to a willing third party; or, if derivatives are not viable for shifting risk, negotiated contracts should reflect the inherent risk involved in the transactions. Put another way, the firm should require a risk premium from its customers (or a risk discount from its suppliers) to compensate for bearing risks that have to be maintained. Success in corporate FX risk management, or even speculative trading for that matter, often hinges upon

- Defining your objective(s)
- Devising a plan or strategy to achieve the objective(s)
- Executing the strategy and
- Very importantly, remaining focused on the objective

2.7.1 Foreign Exchange Forward Contracts

Foreign exchange forwards or futures can be used to

- Protect your business against adverse movements in foreign exchange rates.
- Make financial planning easier as you can now budget using a guaranteed exchange rate.

There are two types of forward contracts: Deliverable and non-deliverable forwards (NDF). A derivable forward will be exchanged into the spot currency at the expiry of the forward. An NDF is handy when you need to hedge currency exposures from countries that have foreign exchange control where access to the local forward markets are restricted to domestic companies only. A NDF, unlike the deliverable contracts, only settles the difference between the onshore official fixing and the NDF rates at contract maturity date. Customer either pays to the Bank or receives from the Bank the difference depending on the onshore official fixing at contract maturity. The customer could then buy/sell the same currency in the onshore market to fulful the physical currency requirement. Effectively, the customer is buying/selling the onshore currency at the offshore NDF contract rate.

2.7.2 Currency Options

Options are handy when

- you want to hedge against adverse exchange rates movement but you do not want to miss the potential gain if the future currency movement is in your favor.
- you want to have the right to deal but not the obligation to deal.

Options are combined into structures that give specific payoff profiles and exposures. We will later explain this in more detail.

2.7.3 Cross Currency Swaps

A cross-currency swap is similar to an interest rate swap but where each leg of the swap is denominated in a different currency. A cross currency swap therefore has two principal amounts, one for each currency. Normally, the exchange rate used to determine the two principals is the then prevailing spot rate although for delayed start transactions, the parties can either agree to use the forward FX rate or agree to set the rate two business days prior to the start of the deal. With an Interest Rate Swap there is no exchange of principal at either the start or end of the transaction as both principal amounts are the same and therefore net out. For a cross currency swap it is essential that the parties agree to exchange principal amounts at maturity. The exchange of principal at the start is optional).

Cross currency swaps are handy when

- a company wishes to raise capital in a country other than their primary country of operation.

Such swaps can result in a lower funding cost and can also be used to lock in the exchange rate for a series of firm commitments.

2.8 Hedging FX Exposures in Emerging Markets

Before we look at option strategies, we need to know whether options are actually used as hedging instruments. The use of financial hedging instruments (the traditional approach) in emerging markets are different than those in developed countries and, therefore, the need emerges to study alternatives. These different conditions can be seen in that

- currencies from emerging countries could be difficult to hedge due to the high costs or the non-existence of either forward or currency options;
- swap arrangements may be difficult to set up because there is no active market for swaps in a specific currency;
- emerging markets' currency positions may involve investments in brand development or other intangible assets;
- loans in local currencies may affect the performance and the profitability of companies trading with these currencies due to higher rates of interest in emerging economies compared to those in developed countries;
- the volatility of emerging countries' currencies makes it riskier to borrow long-term as companies could face a maturity mismatch; and
- companies borrowing in strong currencies could suffer a currency mismatch paying the capital and services of the loan with revenues in a devalued currency.

A study that was done during 2006-2007 looking at the FX hedging practises of multinational Indian firms [SS 08]. The results are given in Table 2.1. It is evident that most Indian firms use forwards and options to hedge their foreign currency exposure. This implies that these firms chose short-term measures to hedge as opposed to foreign debt. This preference is possibly a consequence of their costs being in Rupees, the absence of a Rupee futures exchange in India and curbs on foreign debt. It also follows that most of these firms behave like Net Exporters and are adversely affected by appreciation of the local currency. There are a few firms which have import liabilities which would be adversely affected by Rupee depreciation. However it must be pointed out that the data set considered for this study does not indicate how the use of foreign debt by these firms hedges their exposures to foreign exchange risk and whether such a strategy is used as a substitute or complement to hedging with derivatives.

Who are the natural suppliers of FX derivatives? A study done in Chile found that pension funds are the main providers of foreign exchange hedging to corporate end-users. Foreign exchange derivatives in Chile are traded mainly in the over-the-counter market, and banks have a major role as market makers.4 Domestic banks and financial institutions can write a variety of derivatives instruments, and are responsible for matching corporate end users and institutional investors' needs to cover exchange rate risk.

As of end-December 2003, pension funds held 24 percent of their assets in foreign assets, most of them denominated in U.S. dollars. Minimum coverage requirements of foreign assets makes pension funds the natural providers of foreign currency hedging to corporate end-users since they have an incentive to take the foreign currency paying leg of a derivatives transaction. Furthermore, the sizable foreign asset holdings of

Instruments	Currency(mn)	Rs (Cr)	Nature of exposure
Reliance Industries			
Currency Swaps		1064.49	Earnings in all businesses are linked to USD. The key input, crude oil is purchased in USD. All export revenues are in foreign currency and local prices are based on import parity prices as well.
Options Contracts		2939.76	
Forward Contracts		5764.10	
Maruti Udyog			
Forward Contracts	6411 (INR-JPY) 70 ($-INR)		Import/Royalty payable in Yen and Exports Receivables in dollars.
Currency swaps	124.70(USD -INR)		Interest rate and forex risk.
Mahindra and Mahindra			
Forward Contracts	350 (INR-JPY) 2(INR-EUR) 27.3($-INR)		Trade payables in Yen and Euro and export receivables in dollars.
Currency Swaps	5390 (JPY-INR)		Interest rate and foreign exchange risk.
Arvind Mills			
Forward Contracts	152.98 ($-INR) 2.25 (GBP-INR) 5 (INR-$)	703.67 21.88	Most of the revenue is either in dollars or linked to dollars due to export.
Option Contracts	122.5 ($-INR)	547.16	
Infosys			
Forward Contracts	119 ($-INR)	529	Revenues denominated in these currencies.
Options Contracts Range barrier options	4 ($-INR) 8 (INR-$) 2 ($-INR) 3 (Eur-INR)	18 36 971	
Tata Consultancy Services			
Forward Contracts	15 (Eur-INR) 21 (GBP-INR)	265.75	Revenues largely denominated in foreign currency, predominantly US$, GBP, and Euro. Other currencie include Australian $, Canadian $, South African Rand, and Swiss Franc
Option Contracts	830 ($-INR) 47.5 (Eur-INR) 76.5 (GBP-INR)	4057	
Ranbaxy			
Forward Contracts		2894.589	Exposed on accounts receivable and loans payable. Exposure in USD and Jap Yen
Dr. Reddy's Labs			
Forward Contracts	398 ($-INR) 11(Eur $)		Foreign currency earnings through export, currency requirements for settlement of liability for import of goods.
Options Contracts	30 (EUR-$)		

Note:

1. $-INR Forward contracts denote selling of USD forwards to convert revenues to INR. INR-$ Forward contracts denote buying of USD forwards to meet USD payment obligations.
2. $-INR Option contracts are Put options to sell USD. INR-$ are Call options to buy USD

Table 2.1: Hedging FX risk by Indian firms.

pension funds (14 percent of GDP) implies there is no shortage of foreign exchange hedging to meet corporate end users' needs. Indeed, by end-December 2003, institutional investors had an outstanding dollar-paying position of $7.7 billion compared to the outstanding dollar-buying position of $2.9 billion of corporations.

Pension funds however, only supply derivatives contracts with short maturities. Pension funds and exporters take foreign currency paying positions in derivatives contracts with maturities of three months or less, according to market analysts. Furthermore, analysts also note that pension fund managers do not always cover their long foreign currency positions fully since carrying naked dollar positions during periods of dollar appreciation is profitable. Banks, therefore, are the suppliers of foreign exchange hedging for maturities of one year and above. Banks hedge the foreign exchange exposure arising from these long-term forward contracts with dollar and dollar-linked bonds issued by the Central Bank [Ch 05]. This study found that the onshore option market is very thin and rather expensive compared to offshore options.

Companies operating in emerging markets should take the following salient points in FX exposure management into account

- Make sure operational mechanisms have been exhausted before considering hedging with financial derivatives;
- Try to implement natural hedges before engaging in financial derivatives;
- Hedging should be coordinated and executed at the corporate/group level even when done on behalf of particular operations;
- Anticipate accounting implications related to hedging;
- Make sure pricing of customized exotic products is transparent;
- Most of all make sure that cost of hedging is justifiable;
- It is always easier to hedge transactional exposure than translation exposure in an emerging market environment through financial hedging instruments.

2.9 Hedging and Profit Opportunities

Besides hedging, the availability of derivative instruments also permits firms to be more aggressive in expanding their profit opportunities. In the simplest case, a company may be able to use derivatives to fix both the cost of inputs and the price of outputs, thereby creating a predetermined profit [Ka 01].

Clearly, if the pricing of the relevant derivatives ensures a generous markup, proactive position-taking with these tools would ensure generous profitability in coming periods. Such opportunities may not be persistent, however, so firms need a systematic way of monitoring and assessing these market opportunities, so that they will be able to capitalize on them when they do arise.

In this context, a company may view its "product" in somewhat of a different way. A product is not just the goods a company offers its customers, per se, it is the process by which customer needs are satisfied. Essentially, companies may decide to take on the risks that their customers want to avoid, and the resulting profit derives from being paid to take these risks and managing these risks in a disciplined way. This business model works, and ultimately, it may be adopted on a more widespread basis.

Risk management is not a panacea. The approach outlined here is based on the expectation that a firm that can couple an understanding of its risks with a competency in using derivative instruments may have a critical advantage over competitors lacking such capabilities. But a firm cannot expect to be successful if risk management decisions are made in an ad hoc manner.

Rather, it takes a disciplined approach to assemble information in a way that enables management to compare alternative courses of action and then make judicious choices.

2.10 Fundamental en Technical Analysis

As in the stock market, there are two basic areas of forex trading strategy, which are Technical analysis and Fundamental analysis. However Technical analysis is by far the most commonly used forex trading strategy by individual forex traders. Here is a brief overview of both forms of analysis and how they directly apply to forex trading[2].

Fundamental analysis in the forex market is often an extremely difficult one, and it's usually used only as a means to predict long-term trends. However it is important to mention that some traders do trade short term strictly on news releases. There are a lot of different fundamental indicators of the currency values released at many different times. Here are a few of them to get you started

- Non-farm Payrolls
- Purchasing Managers Index (PMI)
- Consumer Price Index (CPI)
- Retail Sales
- Durable Goods

[2]http://www.investopedia.com/university/forexmarket/forex6.asp

You need to know that these reports are not the only fundamental factors that you have to watch. FX carry trades, mentioned in §1.7, are done after a thorough fundamental analysis of the economic situations in both countries.

Just like their counterparts in the equity markets, technical analysts of the forex trading market analyze price trends. The only real difference between technical analysis in forex and technical analysis in equities is the time frame that is involved in that forex markets are open 24 hours a day.

Due to this, some forms of technical analysis, that factor in time, have to be modified so that they can work with the 24 hour forex market. Some of the most common forms of technical analysis used in forex are

- The Elliott Waves
- Fibonacci studies
- Parabolic SAR (Parabolic Stop and Reverse)
- Pivot points (a pivot point is a price level of significance)

2.11 Hedging Policy Principles

In order to strengthen the very process of corporate FX risk management, every FX risk manager needs to devise a hedging policy. Many companies have adopted a "Currency Risk Management Policy" at the board level. However, a policy is not the same as a process. While a "policy" is more philosophical, outlining the overall guiding principles of an activity, a "process" is more practical involving the nuts-and-bolts issues of the day to day performance of the activity. We summarise the principles of a 'policy" and "process" in Fig. 2.4.

Why should all companies have hedging policies? Due to the following

- Business-generated financial exposures and (ancillary) commodity exposures should be hedged effectively
- Put measures in place to measure the effectiveness of hedges including those outlined in FAS 133
- To implement pragmatic, performance-based standards
- To enforce the following: hedges must mitigate risk.
- To ensure that no speculation takes place — not hedging significant risks is speculative and hedges that increase risk are speculative

HEDGING POLICY	POCESS
* Philosophical in nature * Lays down overall guidelines and overall Do's and Don't's, such as "no leverage allowed" * Provides authority to officers and lays down reporting protocols	* Practical in nature * Provides answers to the following questions: What to hedge? How much to hedge? When to hedge? For what period to hedge? How to hedge? Which products to use? Why are we hedging in the first place? How do we measure performance?

Figure 2.4: Principles of a Policy and Process.

The Foreign Exchange Risk Management Policy should clearly define instruments in which the company is authorised to trade, risk limits commensurate with the company's activities, regularity of reports to management, and who is responsible for producing such reports. The policy should be reviewed on a regular basis, normally at least annually, to ensure that it remains appropriate.

The main points that need to be considered when drawing up a policy are given below

- Open position limits commensurate with customer driven turnover, and the company's appetite for market risk.
- Separate limits should be allocated for each currency, together with an "overall cap" limit. Banks that assume risk on a proprietary trading basis should also introduce measures to limit intraday risk (normally a maximum of five times the overnight cap limit).
- Settlement and country limits should be addressed and clearly defined.
- Forward foreign exchange mismatch limits.
- List of approved instruments.
- Use of foreign exchange derivatives.
- The expertise and experience of authorised personnel.
- Authority to trade with counterparties other than group companies (where appropriate).

- Monitoring and reporting systems.
- Recording and follow up of limit excesses.
- Impact on P&L of an adverse 10% movement in exchange rates on maximum permitted exposure.
- Imposition of a "stop loss" limit to restrict or prevent any further trading other than client deals and hedging.
- All compliance issues.
- Segregation of duties.
- Trading mandates for authorised personnel.
- Limitation on out of hours trading.
- List of authorised brokers (if applicable).
- Code of Conduct for authorised personnel.

A former treasurer of a multinational US company stated in an interview that

> "A company's currency risk management policy is the foundation upon which its currency risk management programme is built. Developing and implementing it is the single most important thing a company must do in managing FX risk. It defines what the company expects to achieve, how risks are defined, controls over processes and risks, risk tolerances, accountabilities and performance measurement. Without a comprehensive and well thought out policy, there is the potential for chaos."

Read this GTNews interview in Appendix A.

2.12 Regulatory and Accounting Initiatives

Regulation for exchange traded instruments has always been tight and onerous. However, what the financial crises during 2008 highlighted was the lack of oversight for OTC products. This has prompted regulators to put pen on paper and draft regulation after regulation.

Regulatory risk, a term describing the problems arising from new or existing regulations, is now one of the greatest threats to business. In the eyes of many corporate leaders, regulatory risk is now a greater source of concern than country risk, market and credit risk, IT and people risks, or terrorism and natural disasters.

How did regulation, much of which is designed to reduce business risks, become a major source of risk in its own right? Most companies accept the need for rules to govern business, and are used to working within regulatory constraints. But a spate of new regulations in recent years has had major — and some would argue unforeseen — consequences for business. For companies with international operations, in particular, the cost and complexity of ensuring compliance have risen sharply. So have the penalties, direct and indirect, of non-compliance.

Due to the big exposure banks have to the financial markets, they are seen as special cases and are treated as such. Individual banks risks create Systematic risk, i.e., the risk that the whole banking system fails. Systematic risk results from the high interrelations between banks through mutual lending and borrowing commitments. The failure of single institution generates a risk of failure for all banks that have ongoing commitments with the defaulting bank. Systematic Risk is a major challenge for any regulator.

2.12.1 Basel Accords

The main enforcement of such regulations is Capital Adequacy. That is by enforcing a capital level in a level in a line with risks, regulators focus on pre-emptive (in-anticipation) actions limiting the risk of failure. Guidelines are defined by a group of regulators in Basel at the Bank for International Settlements (BIS), Switzerland (hence the name Basel Accord). The process attempts to reach a consensus on the feasibility of implementing new guidelines by interacting with the industry. Basel guidelines are subject to some implementation variations from one country to another according to the view of local supervisors (RBI in case of our country).

However, Capital Adequacy requirements are primarily to meet the following objectives

- To ensure survival of the institution to protect it against the risk of insolvency.
- To absorb unanticipated losses with enough margin to inspire stakeholders confidence and enable the institution to continue as a going concern.
- To protect depositors, bondholders, creditors in the event of insolvency and liquidation.

The first Accord, known as Basel I, focused on Credit Risk and was implemented in 1988. Basel II was introduced in 2004 and it recommended rules for enhancing credit risk measures, extending the scope of capital requirements to operational risk, providing various enhancements to the existing accord and detailing the supervision and market discipline. Basel II comprised of 3 pillars

- Pillar 1 - Minimum Capital Requirements.
- Pillar 2 - Supervisory Review Process.
- Pillar 3 - Market Discipline.

Basel III is the latest version of the regulatory reforms introduced by the Basel Committee. In response to the failure of regulation and supervision in the run-up to the subprime crisis, and the subsequent banking crisis, the Basel Committee on Banking Supervision[3] (BCBS) was back in action to drastically revise and tighten the various aspects of banking sector risk management. Basel III will replace Basel II in 2012. Draft regulations include[4]

- the quality, consistency, and transparency of the capital base will be raised
- the risk coverage of the capital framework will be strengthened
- tighter definitions of common equity
- a framework for counter-cyclical capital buffers,
- measures to limit counterparty credit risk, and,
- short and medium-term quantitative liquidity ratios.

As of September 2010, the most significant reforms are being implemented in the definition of bank capital, and risk capital requirements. The proposals includes many other enhancements to risk management measures and regulatory procedures, which will only be realized gradually, with the whole package being activated by 2018.

From a currency traders point of view, the Basel III framework has two major implications[5]. One of them relates to the impact of stricter capital requirements on growth. The obvious goal of the new regulations is smoothing out the business cycle as much as possible, and while stricter standards are certainly beneficial over the longer term, short term costs of adjustment may lead to less leveraged money floating around, which may in turn cause of some markets in the emerging market world to suffer unexpectedly sharp corrections. It is also far from clear that the Eurozone will be able to adopt the new rules without causing a worsening of the existing gaps between various regions. The response of the German, Iberian, and Italian banking sector to the tighter standards, may diverge strongly, which is not good for the Euro, or the stability of the region.

For the latest find the following BIS document "Basel III: A global regulatory framework for more resilient banks and banking systems" by Basel Committee on Banking Supervision — December 2010

[3]The Basel Committee on Banking Supervision is an institution created by the central bank Governors of the Group of Ten nations. It was created in 1974 and meets regularly four times a year.

[4]http://en.wikipedia.org/wiki/Basel_III

[5]http://www.forextraders.com/forex-news/basel-iii-rules-and-regulations-for-the-banking-sector-and-the-implications-for-fx.html

2.12.2 Regulatory Reform of OTC Derivatives

As a consequence of the 2008 financial crises, the G20 leaders called for improvements to the functioning, transparency and regulatory oversight of over-the-counter (OTC) derivatives markets. They realised that while markets in certain OTC derivatives asset classes continued to function well throughout the crisis, the crisis demonstrated the potential for contagion arising from the interconnectedness of OTC derivatives market participants and the limited transparency of counterparty relationships.

21 recommendations were made addressing implementation of the G20 commitments concerning standardisation, central clearing, organised platform trading, and reporting to trade repositories

- **Standardisation**: The proportion of the market that is standardised should be substantially increased. Authorities should work with market participants to increase standardisation, including through introducing incentives and, where appropriate, regulation.

- **Central clearing**: All standardised derivatives should be centrally cleared in order to mitigate systemic risk. The report sets out the factors that should be taken into account when determining whether a derivative contract is standardised and should be centrally cleared. The recommendations also address mandatory clearing requirements; robust risk management requirements for the remaining non-centrally cleared markets; and supervision, oversight and regulation of central counterparties (CCPs).

- **Exchange or electronic platform trading**: IOSCO will complete an analysis by end January 2011 identifying the actions that may be needed to fully achieve the G20 commitment that all standardised products be traded on exchanges or electronic trading platforms, where appropriate.

- **Reporting to trade repositories**: Authorities must have a global view of the OTC derivatives markets, through full and timely access to the data needed to carry out their respective mandates. All OTC derivatives transactions must be reported to trade repositories. Trade repository data must be comprehensive, uniform and reliable and, if from more than one source, provided in a form that facilitates aggregation on a global scale.

A key message of these recommendations is the need to improve the availability of data on the OTC derivatives market as an input to policymaking to promote financial stability, as well as for monitoring whether targets to bring all standardised derivatives into central clearing are being met. Trade repositories will help to fill this gap, but a high level of coordination is necessary to ensure accessibility and usefulness of data on a global scale.

As a consequence of these recommendations, on July 21, 2010 USA president Barack Obama signed into law the "Restoring American Financial Stability Act of 2010". The bill contains a number of provisions targeting the OTC derivatives industry, including the Volcker Rule, which would bar banks from trading with their own funds. The bill also requires most derivatives to be traded on swap execution facilities (SEFs) or exchanges and be cleared through clearinghouses, and would require banks to spin off their OTC derivatives businesses into affiliates.

Due to the pressure to centrally clear OTC derivatives, the Basel Committee issued a consultative document[6] entitled "Capitalisation of Bank Exposures to Central Counterparties" dated December 2010. This consultative paper seeks comments from banks, central counterparties (CCPs) and other stakeholders on the proposed Basel III reforms reflected in the proposed regulatory capital adequacy rules. These changes seek to require banks to more appropriately capitalise their exposures to CCPs, including both trade and default fund exposures to CCPs. It is stated in the paper

> "Further, with the important efforts being made to increase the use of CCPs, the Committee recognises that banks, and financial systems, will be increasingly reliant on CCPs. *Consequently, it is important that banks maintain sufficient capital for their exposures to CCPs.* Finally, the Committee is mindful that regulatory provisions requiring banks to capitalise their exposures to CCPs in a risk-sensitive way inevitably leads to the creation of certain incentives for banks and CCPs to structure their financial resources. For this reason, the Committee wishes to ensure that such incentives are appropriate and do not have unintended consequences."

Banks in South Africa has studied this paper and commented on the proposals due to the impact it might have on clearing houses and the listed derivatives market in South Africa.

2.12.3 Accounting Reforms

International accounting and reporting standards for derivatives continue to evolve toward marking-to-market and fair value [Ho 06]. Many organisations have incurred costs in the shift in accounting treatment of derivatives. The USA introduced FASB statement No. 133 in 1998 and FASB statement No. 157 in 2006. These regulate accounting for derivative instruments and hedging activities. FASB 133 was amended later in statements Nos. 137 and 138. Similar rules in Europe falls under IAS 39. One of the biggest problems in implementing derivative and hedge accounting rules is identifying a transaction as a derivative and, if applicable, a hedge. Derivatives are difficult to identify because, by definition, there are no significant expenditures or recording of assets or liabilities.

[6] http://www.bis.org/publ/bcbs190.htm

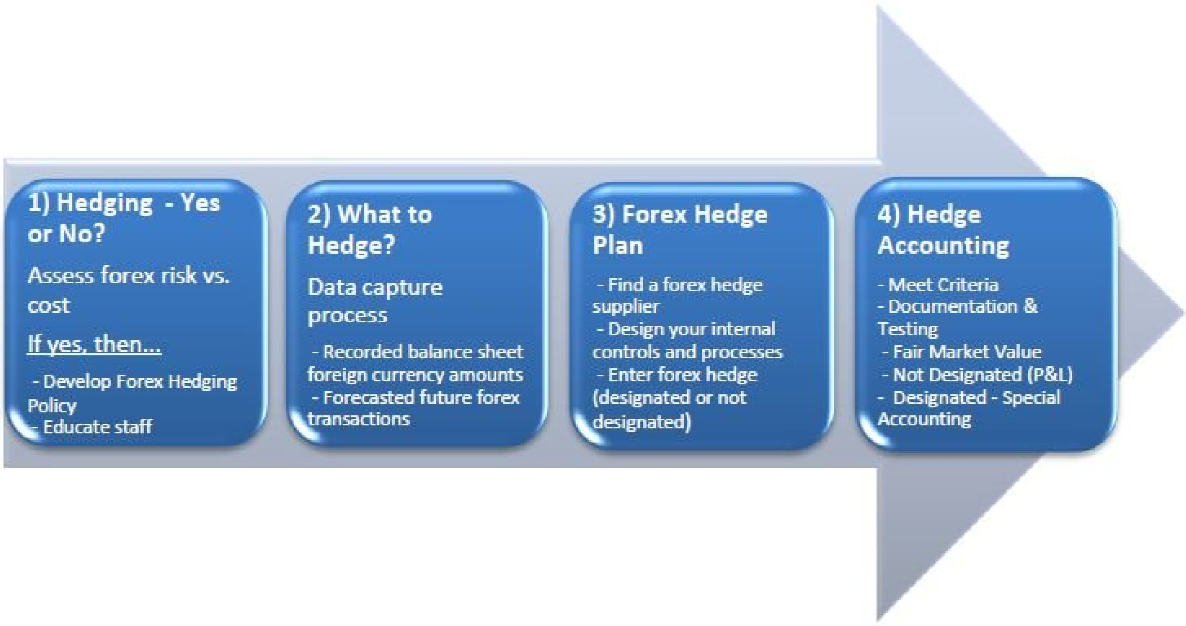


Figure 2.5: Steps to hedge accounting.

The steps to hedge accounting is shown in Fig. 2.5. Note that the policies and processes need to be in place before proper accounting can take place[7]

However, the most sweeping reforms, regarding reporting and disclosure standards, in the USA happened when they introduced the Sarbanes-Oxley (SOX) Act in 2002. The bill was enacted as a reaction to a number of major corporate and accounting scandals including those affecting Enron, Tyco International, Adelphia, Peregrine Systems and WorldCom. These scandals, which cost investors billions of dollars when the share prices of affected companies collapsed, shook public confidence in the nation's securities markets. Note that it does not apply to privately held companies.

[7]To get a good overview of hedge accounting for currency derivatives, download and read the document at http://fxconsulting.oanda.com/documents/Forex_Hedge_Accounting_Treatment.pdf.

Chapter 3

Capital Markets in Africa

There is a growing body of research which points towards capital market development and financial deepening in general and stock markets development in particular making positive contribution to economic growth. An array of financial instruments including stock market quoted shares and the bond market is almost certainly going to enhance the overall level of savings in an economy.

Capital markets are the markets for long-term loanable funds as distinct from the money markets, which deals in short-term funds. However, there is no clear-cut distinction between the two markets. In principle, capital market loans are used by industry and commerce mainly for fixed investment. The capital market is an increasingly international one and in any country the market is not one institution but all those institutions that generate the supply and demand for long-term capital and claims on capital. In this respect, stock exchanges could be defined as the central point of the capital market.

3.1 Introduction

Those who have watched financial crises in over the past 10 years would have noticed two things. First, 10 years ago, emerging markets had problems. Second, it is now the developed world's turn. Ten Years ago we had problems in Argentina, Brazil, Turkey, and Uruguay. For instance, on 14 November 2002 Argentina defaulted on a $726 million payment due to the World Bank, the largest such default in World Bank History. Currently we sit with the "PIGS" — Portugal, Ireland, Greece and Spain. As the examples above show, such debt crises are not new.

How was this debt generally structured? Usually it takes the form of a bond where either a rich country or foreign bank, lends money to the developing country. The developing country pays interest in the form of annual or biannual coupons and the principle must be paid back at some date in the future.

One has to remember that there is nothing inherently unsound in borrowing or lending. A developing country should tap the capital of rich nations to advance from a poor, largely agricultural society into a wealthier, industrial economy. Without loans or foreign investments, a developing nation is limited to its own resources, its own savings; it must pay for imports of fertiliser exclusively from the coffee, rubber, tin, copper or other commodities it can sell abroad.

The extra inflow of hard currency serves another vital function. It can add to the stock of local savings that pay for investment in roads, power, port improvements and other vast projects that may yield faster growth. To escape from dependence on raw materials, to become an exporter of steel, textiles, electronic products, and more, a nation must accumulate more hard currency, more Dollars and Euros, than its exports will earn. Aid from governments, credits from foreign suppliers, loans from international institutions can all help, provided, these are not misdirected by suppliers or misused by recipients.

So, what went wrong with these loans? A country needs to earn hard currency from exports of services and goods to enable it to repay a foreign loan — it is very difficult for a country with a weak currency to finance a loan out of its own resources. In most cases, the banks that advanced the loans to the developing countries did not understand for what the loans would be used. Their risk management practises let them down and they lent more than what they should have. The borrowers also borrowed more than what they could repay. A useful rule of thumb holds that a borrowing nation should limit its debt so that the payments due in any one year are one fifth of its earnings from the export of goods and services. This is called the *debt-service ratio*. Thailand, for example, has enacted this 20% rule into law, limiting the foreign debt it can amass. This law has helped Thailand tremendously during the past three decades — even the Asian crises. As a matter of fact, this rule has helped most of the Asian "tigers" to grow at enormous rates during the past three decades. This rule tells a country there is a day of reckoning, that earnings from trade should cover its debt repayment costs five times.

Forwards and futures are some of the simplest derivative instruments. In this chapter we discuss how forward prices and futures prices are related to the price of the underlying asset. We use simple arbitrage arguments to understand this. However, before we can do just that we need some basic knowledge about interest rates: what they mean, how to use them and the time value of money.

3.2 Time Value of Money

The notion that money has a time value is one of the basic concepts in the analysis of any financial instrument. Money has time value because of the opportunities for investing money at some interest rate. *Interest* is a fee which is paid for having the use of money and the *interest rate* specifies the rate at which interest accumulates.

Convention	**Instruments**
actual/365	South Africa (all markets); US treasury notes US treasury bonds; UK gilts; German bunds
actual/364	Kenya; Zimbabwe
actual/360	US commercial paper; US and Euro money markets
actual/actual	New Euro bonds; LIFFE UK bond futures; LIFFE German bund futures
30/360	Eurobonds; US corporate bonds

Table 3.1: Day count conventions

3.2.1 Day Count

This determines the length of the "interest rate year" and the method of determining the number of days during the period. Day count conventions differ from market to market and they differ in assumptions on the number of days in a year as well as number of days in a month. Table 3.1 shows the different day count conventions used around the globe[1]. For currencies also refer back to Fig. 1.15.

3.2.2 Simple Interest Rate Calculations

If we invest a certain amount of money (called the *principle*) in an interest bearing security, we would like to know what amount of interest would we earn over time. Interest that is paid solely on the amount of the principle is called *simple interest*. The general formula for calculating the total amount of interest to be received at the end of a period, based on simple interest, is

$$\text{Interest} = \text{Principle Amount} * \text{Interest Rate} * \text{Length of Time (years)}.$$

The total amount we will receive at expiry (future value) can be stated as

$$F = P(1 + rt) \tag{3.1}$$

where P is the original amount invested (present value), r is the "simple" interest rate (in per annum format) and t is the period of the investment. How does one annualise a rate? By dividing by the fractional time period. A simple rate of 3% that is valid for 3 months is 12% annualised — beware, this is for a 30/360 day count. If we assume 3 months has 91 days then the annualised rate is 12.033%.

[1]http://www.slidefinder.net/d/day_count_conventions/21189180; http://en.wikipedia.org/wiki/Day_count_convention

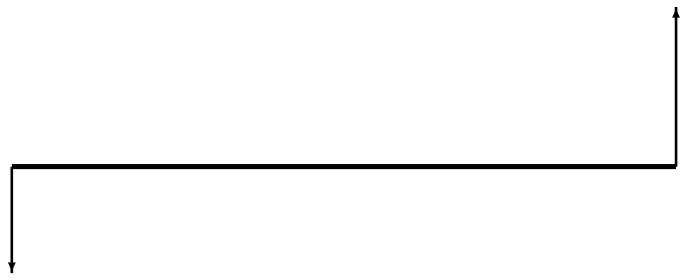


Figure 3.1: Cash flow diagram for a zero-coupon bond.

The equation in (3.1) can be inverted to calculate the yield over a period given the present value P and the future value F

$$r = \frac{\left(\frac{F}{P} - 1\right)}{t} \tag{3.2}$$

3.2.3 Zero-Coupon Bond

A zero-coupon bond is a bond that makes no coupon or interest payments over its lifetime. It is purchased at an initial price, and the interest earned is determined by the bond's payoff at maturity. The face value of the bond is the amount paid at maturity, one KES. A cash flow diagram is shown in Fig. 3.1. Most money market instruments around the globe are zero coupon instruments (maturities of 1 year or less). A fixed deposit at a bank is also a zero coupon instrument. The Treasury Bills issued by the Kenyan government are zero-coupon instruments.

3.2.4 Discount Factors

Some instruments such as bankers acceptances and Treasury bills do not pay interest on the principal P, but are sold at a discount to the face value. The amount of the discount reflects the interest that will be earned at maturity. When you buy a security at a discount, you pay an amount less than the principle but you receive the principle back at maturity.

The rate quoted on these instruments is a "discount rate" and the present value of a discount instrument is obtained by turning the previous exercise around: what do we need to invest today to receive KES1 million in 91 days? The answer is

$$\frac{1000000}{1 + 13\%\frac{91}{365}} = 968606.53$$

The term

$$\frac{1}{1 + 13\%\frac{91}{365}} = 0.96860653$$

is called the discount factor (*df*).

The discount factors *df* are defined as follows

$$\begin{aligned} df &= \frac{1}{1+rt} \approx 1 - r * t && \text{for simple rates} \\ df &= \left(1 + \frac{r}{m}\right)^{-mt} && \text{for compound rates} \\ df &= e^{-rt} && \text{for continuous rates} \end{aligned} \tag{3.3}$$

which is 0.96758904 for our exercise above.

The role of a discount factor is very important as it permits the comparison of cash flows that occur at different points in time where

$$C_t * df = C_0, \tag{3.4}$$

i.e., a cash flow at a certain future point in time t, times the discount factor equals the value of that cash flow today — *time value of money.*

Instruments traded on a discount include BA's, treasury bills, promissory notes and Land Bank bills. Commercial papers are similar to treasury bills but are issued by companies and institutions.

3.2.5 Compounding Periods

A compound period is the length of time after which interest earned is capitalised; i.e. the interest is free to earn interest. The compounding period is characteristic of an instrument, e.g. NCD's pay interest at the end of the period; most bonds pay interest semi-annually and compounding is thus semi-annual; mortgages usually have monthly compounding.

3.2.6 Compounded Rates

Consider an amount P invested for t years at an interest rate of r per annum. Let's assume that after the first year, the interest is added to the principle and we now earn interest on this total amount. After the second year we again add the interest to the total to earn interest on this larger amount. This is called compounding of interest[2] (the interest earned is reinvested periodically and we earn interest on interest). If the interest rate is compounded once per annum, the terminal value of the investment is

$$P(1+r)^t.$$

If it is compounded m times per annum, the future value F of the investment is

$$F = P\left(1 + \frac{r}{m}\right)^{mt}. \tag{3.5}$$

[2] Einstein said that the most powerful force on earth is compound interest!

Compounding Frequency	**Rate Type**
Daily (m=365)	Notional Amount Compounded Daily (NACD)
Weekly (m=52)	Notional Amount Compounded Weekly (NACW)
Monthly (m=12)	Notional Amount Compounded Monthly (NACM)
Quarterly (m=4)	Notional Amount Compounded Quarterly (NACQ)
Semi-annually (m=2)	Notional Amount Compounded Semiannually (NACS)
Annually (m=1)	Notional Amount Compounded Annually (NACA)

Table 3.2: Types of compounding

Equation (3.5) can be inverted to give the implied interest rate if the present and futures values are known

$$r = \left[\left(\frac{F}{P}\right)^{1/mt} - 1\right] \times m$$

In theory we can let the factor m tend to infinity. This exercise leads us to *continuous compounding*. We then obtain

$$F = P\left(1 + \frac{r}{m}\right)^{mt} \lim_{m\to\infty} Pe^{rt} \tag{3.6}$$

with e the mathematical natural antilogarithm[3] equal to 2.7182818284. Using continuous compounding, our KES100 would grow to KES113.88. In option pricing theory it is easier to work with continuous rates than any other type of rate.

The important point is, that one should know what type of rate one is working with. The different types are set out in Table 3.2.

Note:

When a sum of money is invested, there may be a desire to know how long it will take for the principle to grow by a given percentage [Bu 88]. Suppose we want to know how long it will take for an investment P to double itself given that it receives compound interest of r. From Eq. (3.5) we have

$$\frac{F}{P} = \left(1 + \frac{r}{m}\right)^{mt}.$$

Here, F/P equals the compound-amount factor. An investment will double when $F/P = 2$ such that

$$\left(1 + \frac{r}{m}\right)^{mt} = 2.$$

[3]For common logarithms and antilogarithms the base is 10. Natural logarithms and antilogarithms has a base = e. Here, $\log_e x = \ln x$ and $y = e^{\ln y}$.

This can be solved for t where

$$t = \frac{\ln 2}{m \ln\left(1 + \frac{r}{m}\right)}.$$

3.2.7 Forward Rates

In the previous section we learnt that we can invest money today and earn interest on it. The rates we can invest at is usually the spot rates, i.e., rates that start from today. A question that we need to ask is, if we know we are going to receive an amount of money at a certain point in the future, and we want to invest it at that point, can we fix the interest rate of that investment today?

The answer is yes. This is done by fixing a forward rate. A *forward rate of interest* is the rate of interest, implicit in currently quoted spot rates, that would be applicable from one time point in the future to another time point in the future [BM96]. In the market, a forward rate is quoted as $6X12$ for instance. This means it is a rate that is applicable in 6 months time and finishes in 12 month's time. A time line explanation is given in Fig. 3.2.

Forward rates are not usually quoted and has to be implied from spot rates through an arbitrage argument. Let's assume we want to invest A for 12 months. There are two alternative ways we can do this:

- invest at the 6 month rate and then reinvest at the $6X12$ forward rate; or
- invest at the 12 month rate.

Because the total maturity of both investments is the same, if there are no arbitrage opportunities, we should end up with the same amount of money. Thus (using simple rates)

$$A(1 + r_6 \times 0.5)(1 + r_{6|12} \times 0.5) = A(1 + r_{12}).$$

This equation can be rearranged to obtain $r_{6|12}$. Using money market spot rates r_6 and r_{12} are known and $r_{6|12}$ can thus be calculated. In general we obtain

$$r_{1,2} = \frac{1}{t_{1,2}}\left[\frac{1 + r_2 t_2}{1 + r_1 t_1} - 1\right] \tag{3.7}$$

where $r_{1,2}$ is the forward rate effective from time t_1 to t_2 in the future and $t_{1,2} = t_2 - t_1$.

This rearrangement is messy for compound rates but if the rates were continuous rates we have

$$r_{1,2} = \frac{r_2 t_2 - r_1 t_1}{t_{1,2}}. \tag{3.8}$$

The explanation is given in Figure 3.2.

Now, let's turn the previous argument around: let's use discounts instead. The 12 month discount factor is 0.88161485 and the 6 month discount factor is 0.93706746.

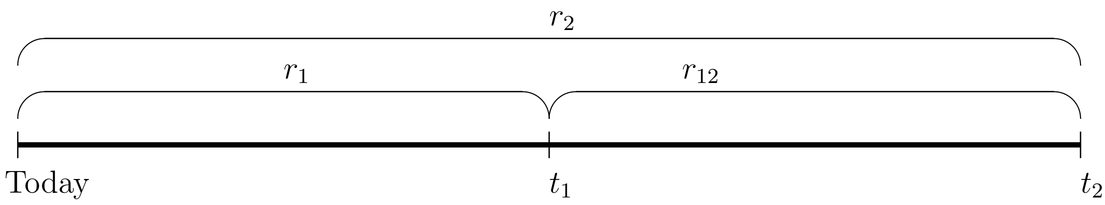


Figure 3.2: Determining the forward rate.

- KES1 million in 12 months time is worth KES881,614.85 today;
- KES1 million discounted back from 12 to 6 months (using the forward rate) and then discounted back using the 6 month discount factor should also be worth KES881,614.85.

From this we can calculate the forward rate $F_{6|12}$. In general we obtain

$$F_{t_1|t_2} = \frac{1}{t_2 - t_1}\left(\frac{df_{t_1}}{df_{t_2}} - 1\right) \tag{3.9}$$

with df the discount factor. In our example above we obtain $F_{6X12} = 12.57978\%$ simple with a discount factor of 0.94082324.

Note: Cash market (money market) uses deposit rates to calculate the forward rates while the swap market uses discount factors.

3.2.8 Converting between Rates

Derivatives traders need to aware that interest rates are quoted in different formats. In general money market rates are simple rates, swap rates are quarterly rates and bond rates are semi-annual. To convert between the different type of rates, we use the formulas as listed in Table 3.3. Here, m_1 is the compounding frequency of the reference rate r and m_2 is the compounding frequency of the rate we convert r to.

When compounding is done more frequently than annually, an *effective annual interest rate* can be determined. This is just the specific compounded rate converted to an NACA rate. Also, an NACA rate for one year equals the simple rate for one year.

3.2.9 Nominal and Effective Rates

It is often useful to compare two interest rates which are for the same investment period, but with different interest payment frequencies. It is common to calculate an equivalent annualized rate. This is the rate with interest paid annually which would

NAC to NAC	$m_2\left[\left(1+\frac{r}{m_1}\right)^{\frac{m_1}{m_2}}-1\right]$
NAC to Continuous	$m_1\left[\ln\left(1+\frac{r}{m_1}\right)\right]$
NAC to Simple	$\frac{1}{t}\left[\left(1+\frac{r}{m_1}\right)^{tm_1}-1\right]$
NAC to Discount	$\frac{1}{t}\left[1-\left(1+\frac{r}{m_1}\right)^{-tm_1}\right]$
Simple to NAC	$m_2\left[(1+rt)^{\frac{1}{m_2 t}}-1\right]$
Simple to Continuous	$\frac{1}{t}\ln(1+rt)$
Simple to Discount	$\frac{1}{t}\left(1-\frac{1}{1+rt}\right)$
Continuous to NAC	$m_2\left(e^{r/m_2}-1\right)$
Continuous to Simple	$\frac{1}{t}\left(e^{rt}-1\right)$
Continuous to Discount	$(1-e^{-rt})$
Discount to NAC	$m_2\left[(1-rt)^{\frac{-1}{m_2 t}}-1\right]$
Discount to Simple	$\frac{1}{t}\left(\frac{1}{1-rt}-1\right)$
Discount to Continuous	$\frac{1}{t}\log\left(\frac{1}{1-rt}\right)$

Table 3.3: Converting between different rate formats

give the same compound return at the end of the year ($t = 1$) as the rate we are comparing. From this it follows [St 98]

$$A\left(1+\frac{r}{m}\right)^m = A(1+r^\star)$$

where A is the principle, r is the compound rate compounded m times per annum and $r^\star$ is the equivalent annual rate. From this we obtain $r^\star$ as being

$$r^\star = \left[\left(1+\frac{r}{m}\right)^m - 1\right]$$

This equivalent annual interest rate $r^\star$ is known as the *effective* rate. The rate r from which it is calculated is known as the *nominal* rate.

3.3 Modern Capital Markets

In a traditional MBA Finance Program one is taught that capital markets allocate funds effectively to creditworthy businesses at reasonable costs to fund operating activities and strategic investments. Is this view still true today? Developments in IT and the financial markets are causing significant alterations in the traditional roles of financial institutions and corporates. The internet and new financial instruments

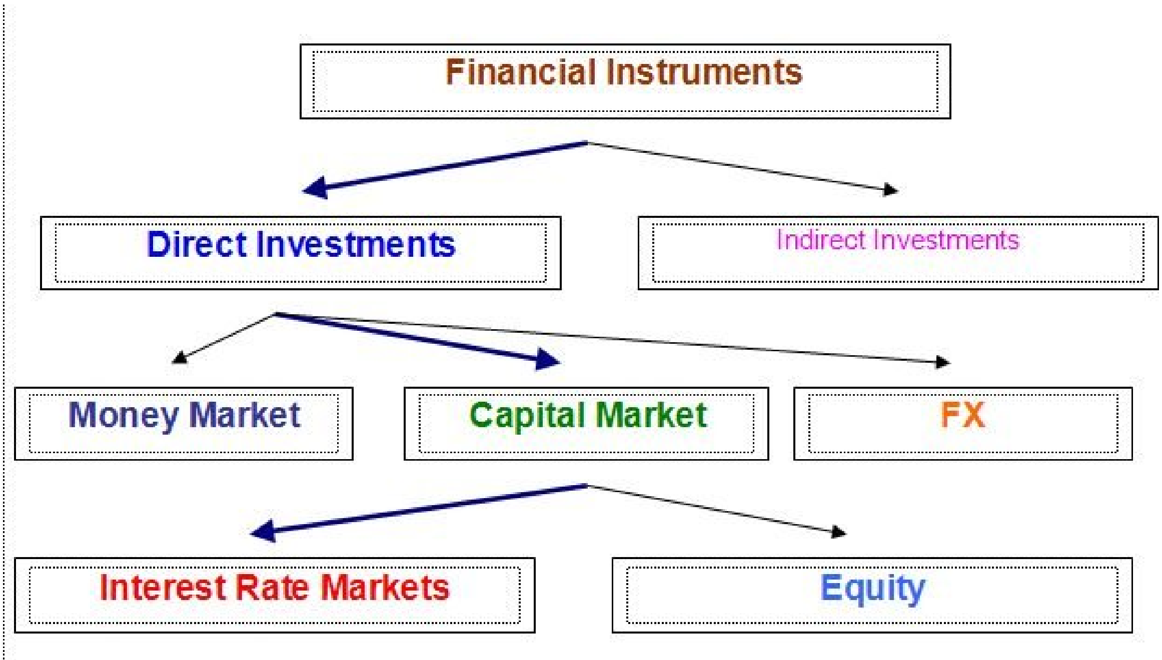


Figure 3.3: Traditional View of the Financial Markets: markets are stand alone silos.

have also brought the retail market into the picture. The masses are starting to be a significant player on the traditional wholesale playground!

The traditional view of the financial markets is depicted below in Fig. 3.3. However, easy classification of the marketplace is a thing of the past [Ko 04b]. Boundaries between seemingly different markets are blurred. For instance, a blanket reference to bonds being a safer investment than equities is no longer true! Just the opposite might be true.

Markets are becoming more and more complex due to globalisation, research, more instruments, transparency, IT and a global network of undersea cables[4]. In Appendix F we show how African countries are being connected with one another and the world. The trading environment has now become a highly specialised area and a clear understanding of the interrelationships among markets is essential. The liquidity on the BM&FBOVESPA (located in São Paulo, Brazil) went through the roof after the following announcement

> "CME Group, the world's largest and most diverse derivatives exchange, and BM&FBOVESPA, the largest exchange in Latin America, have announced that the order routing of BM&F derivatives products on CME Globex is scheduled to begin September 30 2008.

[4]In the year 2000, Africa's total international capacity was just 200Gbits/s or 0.2Tbit/s. By 2012 it will be 20.2Tbit/s.

Figure 3.4: Micro oriented considerations in the financial markets.

> The order routing linkage will enable customers in more than 80 countries using the CME Globex electronic trading platform to now trade BM&FBOVESPA products directly, including futures and options on One Day Inter-Bank Deposits, the Bovespa Stock Index, which is pending regulatory approval, and commodities such as Arabica coffee, live cattle and corn."

Such a linkup was unthinkable 2 decades ago — all made possible because of the global network of high speed cables linking countries.

Tools such as probability theory, historical analysis, stress testing and scenario analysis must be utilised to guide informed and prudent decision making. Similarities between the big three products (equities, bonds and currencies) are much more dominating and persuasive than any differences [Be 04]. We have to embrace the notion of how similar financial products are, rather than believing they are so different. The JSE is in talks with various international exchanges to facilitate a similar model.

Taking this new view into account, we state that the financial markets are underpinned by three brought building blocks: products, cash flows and credit. This is depicted in Fig. 3.4.

This leads to a wider view of the markets that are shown in Fig. 3.5. This is a triangular description of the financial markets. We are going to focus on the "Product Structure" triangle — this is also known as the "Cash Flow" triangle.

3.4 African Equity and Bond Markets

A bond market is one of the most fundamental financial markets — textbooks normally take it for granted that governments borrow money by issuing government or sovereign bonds and that these bonds are freely traded; basic economic theory on monetary supply assumes that a bond market exists; risk pricing and investment valuation models rely on data from bond markets. Yet for most African countries

Figure 3.5: Financial markets: a new paradigm.

the bond market is insignificant or non-existent, even though Africa has some of the most heavily indebted countries in the world. The lack of a functioning bond market could be a fundamental contributor to these African countries' persistent poverty and indebtedness — preventing proper debt management systems from forming, blocking the functioning of corrective market forces, and slowing investment flows and the formation of more complex financial markets.

After several major debt relief initiatives, many countries are left with large but 'affordable' debt stocks at highly concessional rates. It would be difficult, if not impossible, for these countries to be able to issue further debt at market related prices in order to create a functioning bond market. As most of the debt of these countries is concentrated in multilateral debt with the World Bank/International Development Association (IDA), the World Bank would be an integral part in any initiative to create a bond market. The World Bank should pursue a programme to convert a large portion of its debt with these African countries into tradeable bonds and assist the countries in creating a market in which these bonds can be traded [Ni 08].

This is an enormous task. In theory only privately held debt is tradeable. South Africa has a very well-developed local bond market and almost its entire debt is held by private creditors, mostly in the form of bonds. For the rest of Sub-Saharan Africa, excluding South Africa, the debt is split largely between multilateral (45%) and bilateral (43%) debt with the minority as private (12%). This means, only 12% of Africa's debt is tradable.

Are things progressing?

The evolution of capital markets in Africa in recent years has been rather

> dramatic, as countries have sought not only to mobilize domestic resources but also to attract foreign direct investment. Accordingly, activity in a number of capital markets that had been dormant for years picked-up significantly and a number of new markets have emerged. In a number of established stock exchanges, activity has been boosted by increased listings of companies; mostly made possible by privatization of state-owned enterprises[5]. At present, there are about twenty stock exchanges in the continent.

These remarks stem from 1999! Rather upbeat it seems. Has anything changed? There seems to be improvement: there are currently 29 exchanges in Africa, representing 38 nations' capital markets. Table 3.4 list the current exchanges. However, in an article that appeared in theCitizen[6] on 8 May 2010, a journalist, *Samuel Kamndaya*, wrote "African capital markets need to be redesigned to cater for the continent's massive financing needs, experts advised on Thursday." The journalist is sceptical whether African stock exchanges are large enough to cater for its financial needs. It goes further and states "Most of these stock exchanges were under-developed as domestic banks, insurance companies and pension funds in the region make up the majority of their investment. Africa has to attract international investors in order to grow though some countries will require massive reforms to win investors' confidence." A report published by Deutsche Bank in July 2009 noted that automatisation of trading systems, regional integration and increased primary market activity will boost size and liquidity in future.

A big step forward for all markets in Africa would be if they implemented world class information dissemination platforms. Access to markets is very important. In the previous section we mentioned the ever shrinking world due to a network of high speed cables linking countries and continents. Access is becoming easier but investors need information. Lots of information on markets, economic indicators and reserve bank activities. Without information, no-one will leap into a market. The consequence is that banks, asset managers, exchanges, governments and all players in the capital markets will have to increase spending on computer systems and networks.

3.4.1 South Africa

South Africa's debt market when measured in terms of debt issued comprises but a fraction of the world's debt markets combined, yet it constitutes the lion's share of the African debt market. The size of the South African debt market, compared to some other emerging markets are shown in Fig. 3.6. It boasts sophistication and efficiency

[5] Extract from "THE PROMOTION OF CAPITAL MARKETS IN AFRICA: Assessment of needs in Capital Markets Development Southern, Western and Central Africa"; 1-3 November 1999 Addis Ababa, Ethiopia

[6] http://www.thecitizen.co.tz/business/13-local-business/1813-experts-redesign-africas-capital-markets.html

Country	Exchange	Location	Founded	Listings
West African Regional Stock Exchange	Bourse Régionale des Valeurs Mobilières	Abidjan	1998	39
Algeria	Bourse d'Alger	Algiers	1997	7
Botswana	Botswana Stock Exchange	Gaborone	1989	44
Cameroon	Douala Stock Exchange	Douala	2001	2
Cape Verde	Bolsa de Valores de Cabo Verde	Mindelo		
Egypt	Egyptian Exchange	Cairo	1883	378
Ghana	Ghana Stock Exchange	Accra	1990	28
Kenya	Nairobi Stock Exchange	Nairobi	1954	48
Libya	Libyan Stock Market	Tripoli	2007	7
Malawi	Malawi Stock Exchange	Blantyre	1995	8
Mauritius	Stock Exchange of Mauritius	Port Louis	1988	40
Morocco	Casablanca Stock Exchange	Casablanca	1929	81
Mozambique	Bolsa de Valores de Moçambique	Maputo	1999	
Namibia	Namibia Stock Exchange	Windhoek	1992	
Nigeria	Abuja Securities and Commodities Exchange Nigerian Stock Exchange	Abuja Lagos	2001 1960	 223
Rwanda	Rwanda Over The Counter Exchange	Kigali	2008	2
South Africa	JSE AltX Bond Exchange of SA; merged with JSE 2009 JSE Limited JSE SAFEX; SA Futures Exchange JSE Commodity Derivatives	Johannesburg Johannesburg Johannesburg Johannesburg Johannesburg	2003 1989 1887 1990 1995	51 400 472
Sudan	Khartoum Stock Exchange	Khartoum		
Swaziland	Swaziland Stock Exchange	Mbabane	1990	10
Tanzania	Dar es Salaam Stock Exchange	Dar es Salaam	1998	11
Tunisia	Bourse des Valeurs Mobilières de Tunis	Tunis	1969	56
Uganda	Uganda Securities Exchange	Kampala	1997	14
Zambia	Agricultural Commodities Exchange of Zambia Lusaka Stock Exchange	Lusaka Lusaka	 1994	 16
Zimbabwe	Zimbabwe Stock Exchange	Harare	1993	81

Table 3.4: African Stock Exchanges.

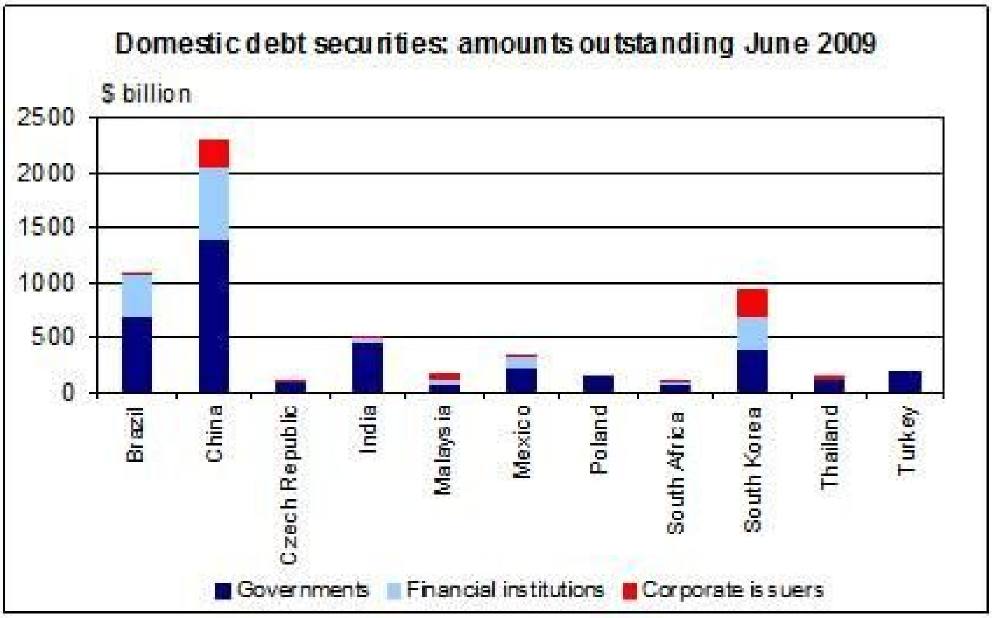


Figure 3.6: Domestic debt of some emerging markets.

that match those of many of the bigger debt markets in the developed world. It is thus both a David and a Goliath. In its former role, it is often a taker of global financial market developments that from time to time ripple out globally, while in its latter role it is a leader on the continent, and possibly even among emerging markets elsewhere[7].

Turnover in the South African debt market is relatively large, and is dominated by repo transactions which account for two-thirds of total turnover. In 2009, turnover amounted to $2 trillion (ZAR14.9 trillion) while in 2008 the bond market registered a record turnover of $2.3 trillion (ZAR21.3 trillion) due to a surge in trading during the height of the global financial crisis. The turnover in 2010 seems to be 15% higher than that of 2009. The daily turnover is nearly ZAR87 billion ($12.5 billion) per day. The turnover is shown in Fig. 3.7 A. The total market capitilisation of the bond exchange is ZAR1.265 trillion ($180.67 billion). At the end of March 2010, South Africa's total outstanding foreign debt was $81.7 billion amounting to 26% of GDP — well within prudent levels.

Research conducted by the Bank for International Settlements in 2007 using 2005 data also concluded that the South African bond market was the sixth most liquid bond market in the world, when measured in terms of turnover ratio. We show the value of bonds traded in Fig. 3.7. This shows that South Africa plays above its league in this regard. Most of this liquidity, admittedly, is concentrated in a handful of government issues because of their size. Indeed, the lack of liquidity

[7]http://www.world-exchanges.org/news-views/views/david-and-goliath-south-african-debt-market

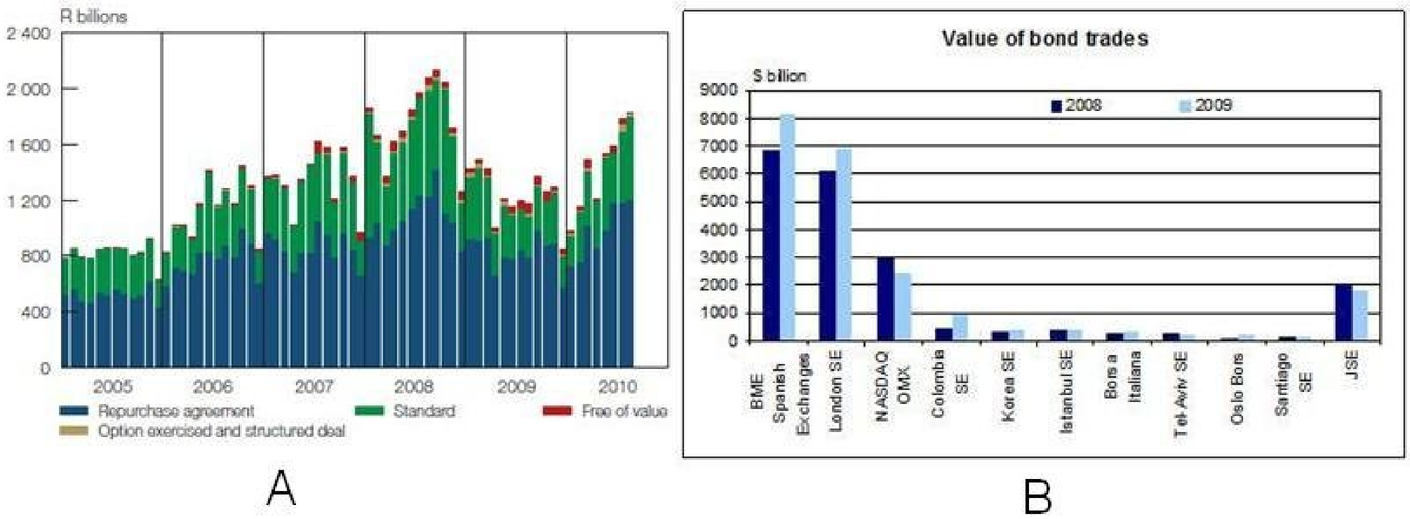


Figure 3.7: A. RSA turnover in the secondary bond market by trade type. B. Value of bonds traded on some exchanges.

in corporate bonds due to the relative dearth of issuance and, consequently, buy-and-hold preferences, in truth often creates pricing complications when trades do occur. Fig. 3.8 shows the composition of the South African bond market. But these phenomena tend to be characteristic of bond markets generally. The JSE is working hard at making the market more liquid. They are busy implementing a central order book trading system. This will enhance the flow of information and hence the reach of the exchange.

The evolution of the South African debt market has, in many respects, resembled that of others globally, with government debt issuance giving rise to pricing and risk benchmarks against which private sector issuers could enter the market. The first issuers of bonds in South Africa were the National Treasury and the parastatals Eskom (electricity utility), Telkom (telecommunications utility) and Transnet (transport utility). The first private sector bond was issued in 1992. In the early 1980s, the parastatals made a market in their own paper in order to spur liquidity and lower prices, and the government followed suit in the early 1990s. As the market began to take shape in the 1980s, a Commission of Enquiry was established to investigate the best way to regulate it. Self-regulation by market participants was proposed as a viable option, and this ultimately laid the foundations for the establishment of a formal exchange-based marketplace. Bond issuers, intermediaries, banks, brokers and investors became members of the Bond Market Association (BMA) in 1989 but it was

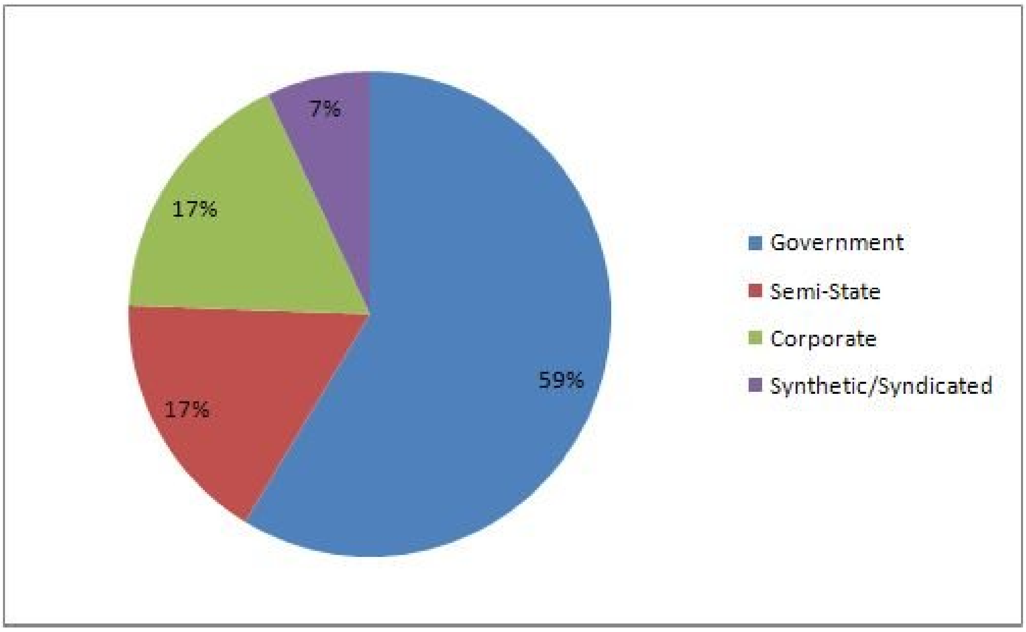


Figure 3.8: Composition and size of the South African bond market.

only on 15 May 1996 that the Association was formally licensed as an exchange.

During the past five years, South Africa has relied heavily on portfolio inflows into equity and bond markets to finance the current account deficit. This is only possible due to the South African bond and equity markets that are liquid and barriers to entry are relative low. During the first 8 months of of 2010, foreigners bought ZAR68.6 billion of South African bonds. That is about $10 billion that was invested into South Africa.

The market capitalisation of the shares listed on the JSE is ZAR5,800 billion ($830 billion). Currently more than 50% of all trades on the JSE originate in London. During 2009, the 407 JSE listed companies raised, in total, ZAR107 billion ($15 billion) in equity capital. During 2010, the average monthly turnover on the JSE's equity market was ZAR253 billion — this translates into a daily turnover of ZAR12 billion ($1.7 billion) per day[8]. During 2009 foreigners bought ZAR75.4 billion ($10.77 billion) of shares on the JSE. The activity of foreigners on the JSE's equity market is shown in Fig. 3.9.

3.4.2 Kenya

At the beginning of February 2011, the Kenyan Shilling FX rates were: $1 = Ksh81; EUR 1 = Ksh110 and ZAR 1 = Ksh11. Daily rates are published on the central banks' web site: http://www.centralbank.go.ke/.

[8]South African statistics from the SARB's annual economic report found at http://www.reservebank.co.za/.

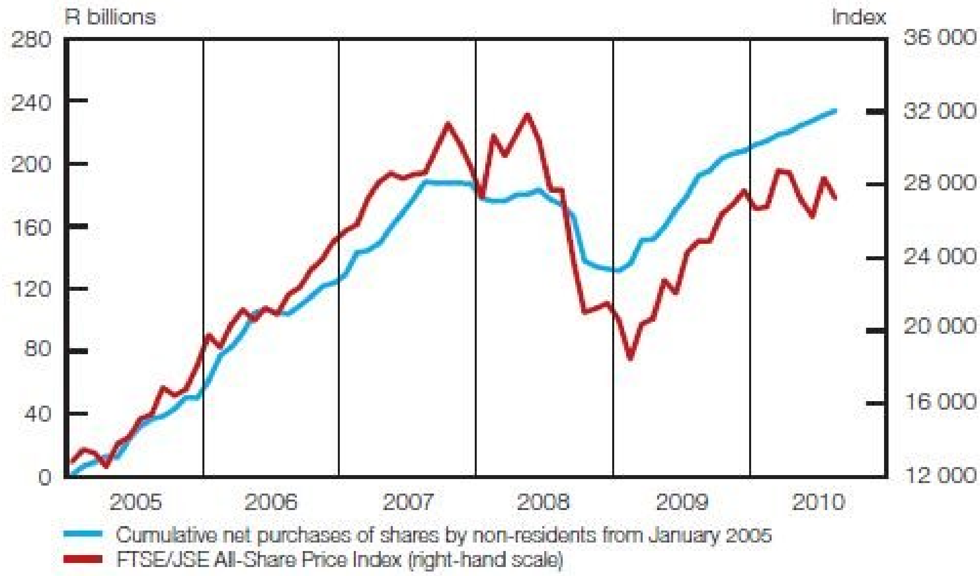


Figure 3.9: Activity of foreigners on the JSE's equity market.

The Central Bank of Kenya has a very nice web site with loads of information: http://www.centralbank.go.ke/. Kenya's outstanding treasury bonds[9] were Ksh 498.1 billion ($6.15 billion) at the end of October 2010. Commercial Banks hold 48.6% of this, pension funds 18.2% and insurance companies 12.3%. The outstanding treasury bills and bonds are shown in Fig. 3.10.

The Nairobi Stock Exchange is very well developed. There is currently 47 listed companies with a combined market capitalisation of Khs1,221.85 billion ($15 billion). It seems the secondary bond market is gaining momentum with Khs34 billion traded during October 2010. Even the interest rate derivatives market is starting with the first ever Forward Rate Agreement (FRA) being traded on 10 August 2010[10].

3.4.3 Nigeria

At the beginning of February 2011, the Niara FX rates were: $1 = N153; EUR 1 = N207.5 and ZAR 1 = N21.

The Nigerian Stock Exchange is very active. It has 264 listed companies with a market capitalisation of N7,800 billion ($60 billion). Fig. 3.11 shows the value and volume of securities traded on the Nigerian Stock Exchange. The fall in volumes is probably still due to the 2008 financial crisis.

The Nigerian bond market seems very active. The Nigerian Central Bank's third quarter economic report for 2010 states the Over-the-Counter (OTC) bond market,

[9] http://www.centralbank.go.ke/downloads/publications/mer/2010/Oct10.pdf
[10] http://af.reuters.com/article/investingNews/idAFJOE6790O320100810

	2009				2010							
	June	%	Dec	%	Jun	%	Aug	%	Sept	%	Oct	%
91-Day	23.523	4.926	24.784	4.56	23.663	3.8977	28.259	4.4135	30.702	4.7801	29.709	4.6254
182-Day	93.271	19.532	87.414	16.083	85.337	14.056	81.368	12.708	76.049	11.84	74.98	11.674
364-Day	0	0	28.621	5.266	49.494	8.1523	42.169	6.586	42.169	6.5653	39.474	6.1457
1-Year	14.789	3.097	6.7271	1.2377	0	0	0	0	0	0	0	0
2-Year	45.205	9.4664	43.843	8.0667	46.577	7.672	42.924	6.7038	55.665	8.6664	55.665	8.6664
3-Year	12.798	2.6799	5.7808	1.0636	1.7808	0.2933	1.7808	0.2781	1.7808	0.2773	1.7808	0.2773
4-Year	12.914	2.7043	7.3069	1.3444	3.3837	0.5573	3.3837	0.5285	3.3837	0.5268	3.3837	0.5268
5-Year	52.787	11.054	66.698	12.272	86.582	14.261	86.582	13.522	86.582	13.48	86.582	13.48
6-Year	38.77	8.1188	37.076	6.8217	59.114	9.737	70.789	11.056	70.789	11.021	70.789	11.021
7-Year	24.154	5.0579	24.154	4.444	21.353	3.5171	30.053	4.6936	30.053	4.6789	26.957	4.197
8-Year	17.944	3.7576	17.944	3.3016	29.573	4.8712	29.573	4.6188	29.573	4.6043	29.573	4.6043
9-Year	12.615	2.6417	12.615	2.3211	17.76	2.9254	27.732	4.3312	27.732	4.3176	27.732	4.3176
10-Year	44.415	9.3008	57.038	10.494	69.09	11.38	69.09	10.79	69.09	10.757	69.09	10.757
11-Year	4.0314	0.8442	4.0314	0.7417	4.0314	0.664	4.0314	0.6296	4.0314	0.6276	4.0314	0.6276
12-Year	28.492	5.9665	47.39	8.7193	20.066	3.3052	20.066	3.1339	20.066	3.1241	20.066	3.1241
15-Year	42.303	8.8585	51.723	9.5166	61.935	10.202	61.935	9.6729	61.935	9.6426	61.935	9.6426
20-Year	9.5262	1.9948	20.361	3.7462	20.361	3.3538	20.361	3.18	20.361	3.17	20.361	3.17
25-Year					7.0082	1.1543	20.193	3.1537	20.193	3.1438	20.193	3.1438
Total	477.54	100	543.51	100	607.11	100	640.29	100	650.15	101.22	642.3	100

Source: Central Bank of Kenya

Figure 3.10: Kenya's outstanding debt instruments.

Figure 3.11: Volume of shares traded on the Nigerian Stock Exchange.

DOMESTIC DEBT STOCK OUTSTANDING AS AT 31ST DECEMBER, 2010

INSTRUMENTS	AMOUNT IN NAIRA	% OF TOTAL
FGN BONDS	2,901,600,329,000.00	63.75
NIGERIAN TREASURY BILLS	1,277,101,559,000.00	28.06
TREASURY BONDS	372,900,500,000.00	8.19
DEVELOPMENT STOCK	220,000,000.00	0.005
TOTAL	**4,551,822,388,000.00**	**100.00**

Figure 3.12: Composition of Nigerian bond market.

had a turnover of 3.31 billion units, worth N3,420 billion ($22.4 billion), in 32,761 deals in the review quarter, compared with 3.4 billion units, valued at N3.22 trillion, in 28,276 deals recorded in the second quarter of 2010. The most active bond was the 10.00% FGN July 2030 Bond followed by the 4.00% FGN April 2015 Bond.

The value of the traded Federal Government Bonds and Preference Shares remained constant at N1,930 billion ($12.6 billion) and N4.6 billion $30 million), respectively. The share of Federal Government, sub-national and corporate bonds in aggregate market capitalization increased by 0.5, 0.9 and 2.1 per cent, respectively, while the share of preference shares remained constant at 0.1 per cent. At the end of 2010, the total debt market had a market capitalisation of N4,500 billion ($29.5 billion). Fig. 3.12 shows the composition of the Nigerian bond market[11].

3.4.4 Mauritius

At the beginning of February 2011, the Rupee FX rates were: $1 = MUR28.8; EUR 1 = MUR39 and ZAR 1 = MUR3.94.

At the end of 2009, the Stock Exchange of Mauritius has 40 listed companies with a market capitalisation of MUR151.2 billion ($4.8 billion). In Fig 3.13 we give a sectoral view of the market.

The structure of the Mauritian government debt is shown in Fig 3.14. Its debt

[11] http://www.dmo.gov.ng/index.php

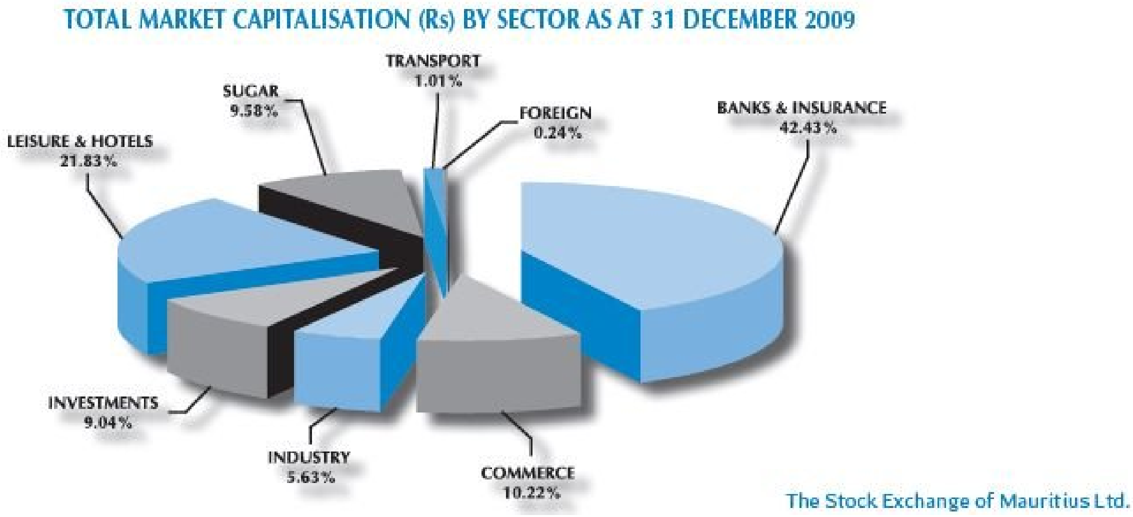


Figure 3.13: Sectoral breakdown of the Mauritian stock market.

market is very short term orientated with small bond issues up to 20 years of maturity.

3.4.5 Egypt

The number of companies listed on the Egyptian Exchange[12] went down to 219 at the end of March 2010, from 333 at the end of June 2009, and so did their nominal capital to £E130.7 billion, from £E149.6 billion. Likewise, the market value of corporate shares dropped by 0.7 percent during the period, £E460.7 billion at the end of March 2010. It seems that the ripple effects of the 2008 crisis could still be felt in 2010.

We show the structure of the Egyptian bond market in Fig. 3.15

3.5 The Zero-Coupon Yield Curve

There is no single interest rate for an economy. The interest rate that a borrower pays depends on a number of factors such as the term of the loan and the credit-worthiness of the borrower. A government, for instance, is seen as being default free and such should be able to borrow at the best possible rates in the market — government rates are thus risk free and the bench mark rates. Governments borrow by issuing debt instruments with a wide range of maturities.

[12] http://www.cbe.org.eg/public/EconomicVol50-3.pdf

	Treasury Bills	Treasury Notes	MDLS/GOM Bonds	TOTAL
2010-11	23,072	9,280	2,004	34,356
2011-12	8,843	15,152	4,516	28,511
2012-13	-	13,974	5,835	19,809
2013-14	-	4,320	8,328	12,648
2014-15	-	1,297	9,538	10,835
2015-16	-	-	3,611	3,611
2016-17	-	-	1,363	1,363
2017-18	-	-	369	369
2018-19	-	-	1,610	1,610
2019-20	-	-	1,826	1,826
2020-21	-	-	3,409	3,409
2021-22	-	-	851	851
2022-23	-	-	636	636
2023-24	-	-	-	-
2024-25	-	-	-	-
2025-26	-	-	3,513	3,513
2026-27	-	-	784	784
2027-28	-	-	838	838
2028-29	-	-	988	988
2029-30	-	-	579	579
TOTAL	31,915	44,023	50,598	126,536

Note: Figures may not add up to totals due to rounding.
Source: Accounting and Budgeting Division.

Figure 3.14: Mauritian government debt.

Bonds Listed on the EGX

(Value in LE mn)

End of	June 2009		March 2010	
	Listed	%	Listed	%
Total	**97586**	**100.0**	**152911**	**100.0**
Government Bonds	**92625**	**94.9**	**141890**	**92.8**
- Treasury bonds	92500	94.8	141767	92.7
- Housing bonds	115	0.1	114	0.1
- US dollar development bonds	10	0.0	9	0.0
Corporate Bonds	**3096**	**3.2**	**4365**	**2.9**
Securitization Bonds	**1865**	**1.9**	**6656**	**4.4**

Source: EGX.

Figure 3.15: The Egyptian bond market.

In finance, the yield curve is the relation between the interest rate (or cost of borrowing) and the time to maturity of the debt for a given borrower in a given currency [Ko 03]. This is also termed the "term structure of interest rates."

A yield curve is a graphical representation of the term structure of interest rates for instruments of a similar credit rating. In any market there are interest instruments with different maturities. In the money market we have instruments with maturities of less than one year, and in the capital market instruments with maturities longer than one year. I f we convert these interest rates to the same format and type, and we plot them against their maturities (time), we obtain a yield curve. Yield curves can have different shapes. In Fig. 3.16 we show the current yield curves in South Africa, Kenya and Nigeria.

South Africa has an active bond and interest rate swap market. We can thus generate both bond and swap zero curves. Swap curves as mostly used in pricing equity and currency derivatives. However, due to the dominance of the repo market[13], repo curves are used to price short dated derivatives. Note, a repo is linked to a specific bond. There should thus be a repo curve for every bond. Fig. 3.17 shows the repo curves as posted on Reuters by some players in the South African market.

Yield curves are normally upward sloping — the interest rates for more distant maturities are normally higher the further out in time. Why? First, because lenders fear a depreciating monetary unit: price inflation. To compensate themselves for this expected (normal) falling purchasing power, they demand a higher return. Second, the risk of default increases the longer the debt has to mature.

Zero-coupon yield curves are very useful because

- A zero-coupon yield is unambiguous: it is simply a measure of the relationship between a single future value and its present value.

- A zero-coupon yield curve enables us to answer the question: at what rate must I discount a single cash flow in the future to calculate its value today? The answer to this question is important because any interest rate product can be broken down into a portfolio of zero-coupon instruments.

3.6 From Spot to Derivatives

Any financial asset can be decomposed into one or more of the following cash flows: spot, forwards and futures, and options [Be 04]. This is known as the "Product Structure" or "Cash Flow" triangle and is shown in Fig.3.18.

[13] Also known as the carry and sell-buyback market.

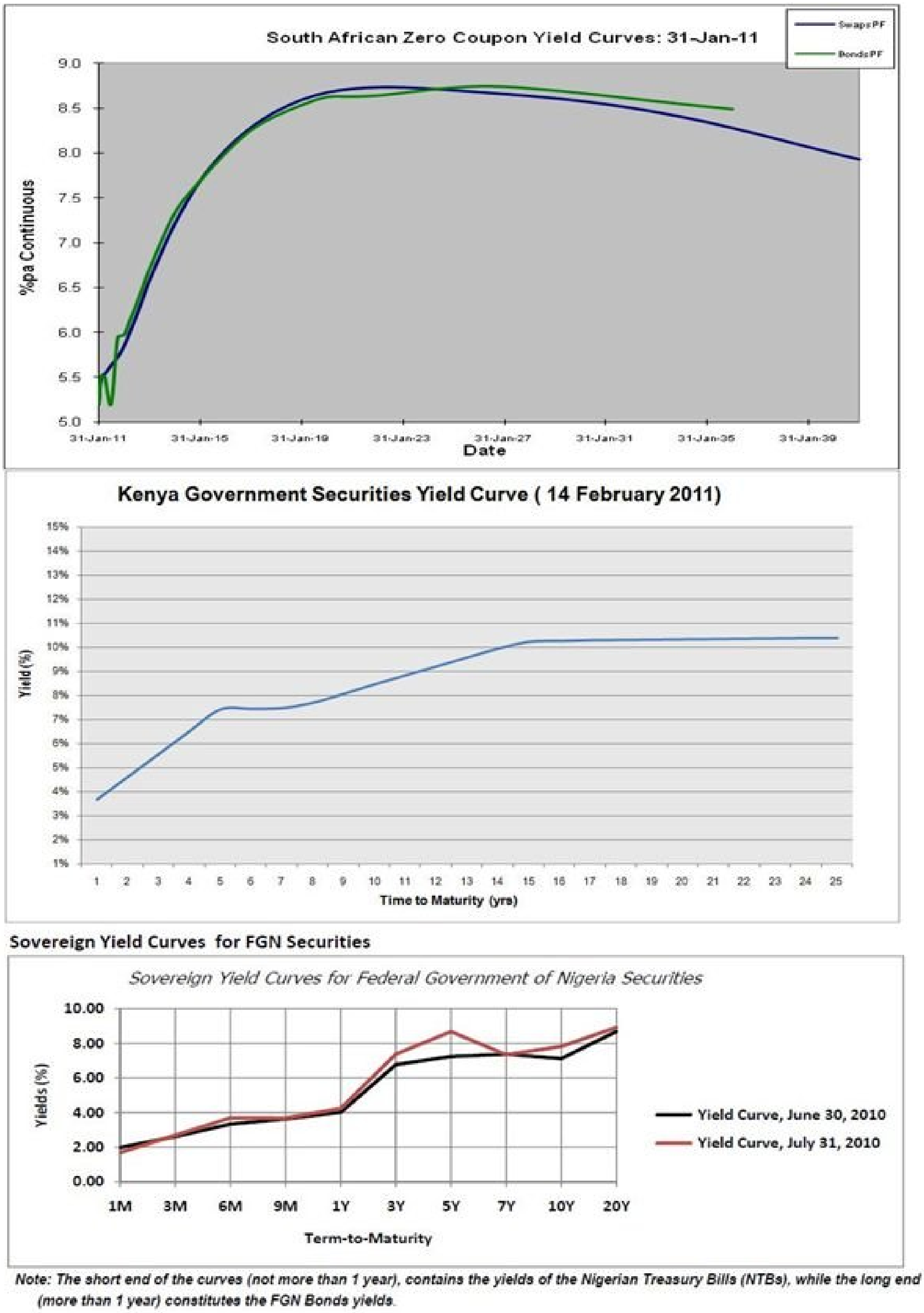


Figure 3.16: African Yield Curves.

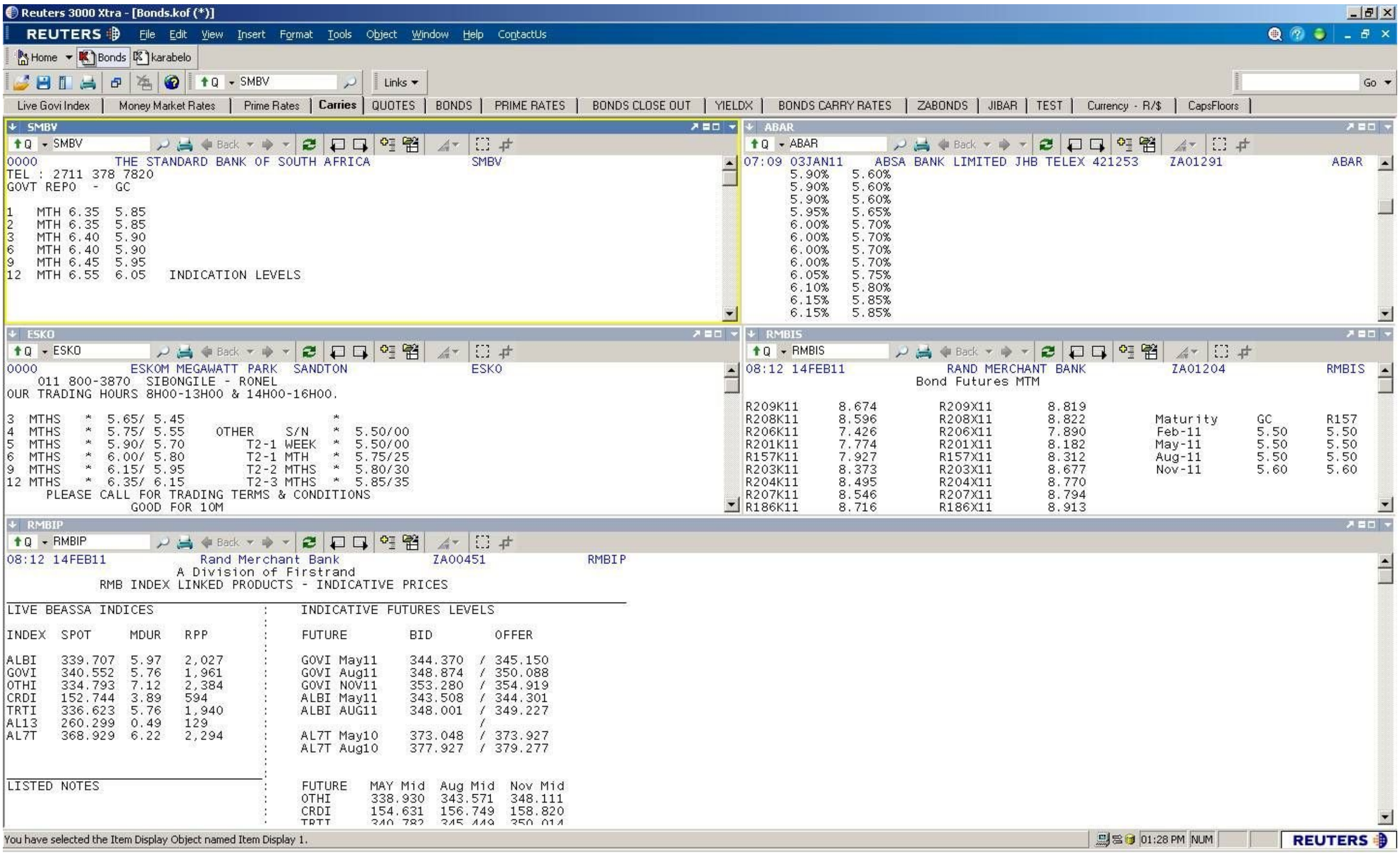


Figure 3.17: Reuters window showing quotes on bond repurchase agreements on 14 February 2011.

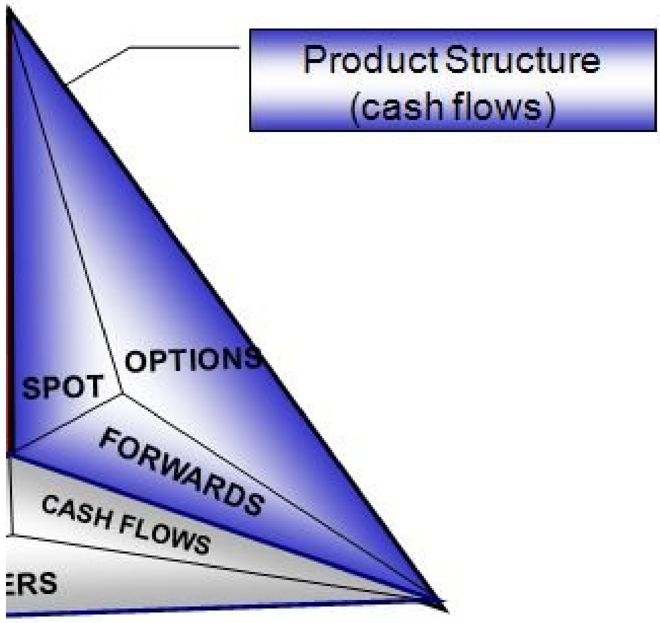


Figure 3.18: The cash flow or product structure triangle.

3.6.1 The Spot Price

Let us begin with spot. We've talked about the spot price in §1.8 where we asked the question: where is the spot traded? But, what is this "spot" we talk about? What is the spot price of a financial instrument? "Spot" simply refers to today's price of an asset — it is the market's view of what the price should be. Remember, many issues are considered in formulating a "fair" spot price. This can be quite involved for currencies where liquidity and a whole country's credit risk needs to be taken into account.

What makes the currency market different to say, for instance, the gold market or share market, is that one quotes the price of one currency in terms of another currency — we call this a currency pair. Thus, the USDKES rate is the number of Kenyan Shillings per one US Dollar. The settlement rules are also very important. Please refer back to Sections 1.9-1.12.

3.6.2 Simple Derivatives

What is a derivative instrument? Shakespeare wrote

"*Neither a borrower nor lender be*".

He foresaw the explosion in off-balance sheet "derivative" products 400 years later.

A derivative is a treasury/capital markets synthetic off-balance sheet instrument derived from or bears a close relation to a cash instrument. It is important to remember that no derivative can exist without a cash instrument. The Group of Thirty defines a derivative as follows[14]

"... a contract whose value depends on (or derives from) the value of an underlying asset, reference rate or index."

Why do we need derivatives? There are two emotions that drive markets

GREED and FEAR.

There are thus two types of traders

SPECULATORS and HEDGERS.

Now, speculators use derivatives to expose portfolios to some market risk and hedgers use derivatives to reduce the market risk they are exposed to. In technical terms we can say that derivatives make a market more "complete" i.e., investors have a bigger choice of instruments to invest in [Ko 02, Ko 03].

3.6.3 The Forward Contract

A forward is one small step away from spot. A forward is the simplest and perhaps oldest form of a financial derivative. We mentioned in §1.2 that there is evidence that the Chinese traded rice forwards as far back as 4000 B.C. Also, the Greeks bought there maize from the Egyptians on a forward contract basis at around 2000 B.C. The biggest increase in futures trading activity occurred in the 1970s when futures on financial instruments started trading in Chicago.

The definition of a forward contract[15] is:

> "it is an agreement to buy or sell an asset for a certain pre-determined price, at a certain future time – at some point beyond the relevant settlement date."

There is thus no cash flow when this deal is struck. We can state it differently: a forward contract is one where the buyer and the seller agree on a price, but the actual transfer of payment for property is deferred until a later time. With such purchases, the buyer and the seller agree on the price but delivery of the goods and payment comes later — perhaps in a week, perhaps in a month, or perhaps even later than that. Forward contracts are arranged between two principals with complete flexibility as to exactly what property is being transferred and when the transfer will occur [Ko 10].

One of the parties to a forward contract assumes a *long position* and agrees to buy the underlying asset on the specified future date for the specified price. The other

[14] http://www.group30.org/

[15] In the FX market a forward is also called an 'outright'.

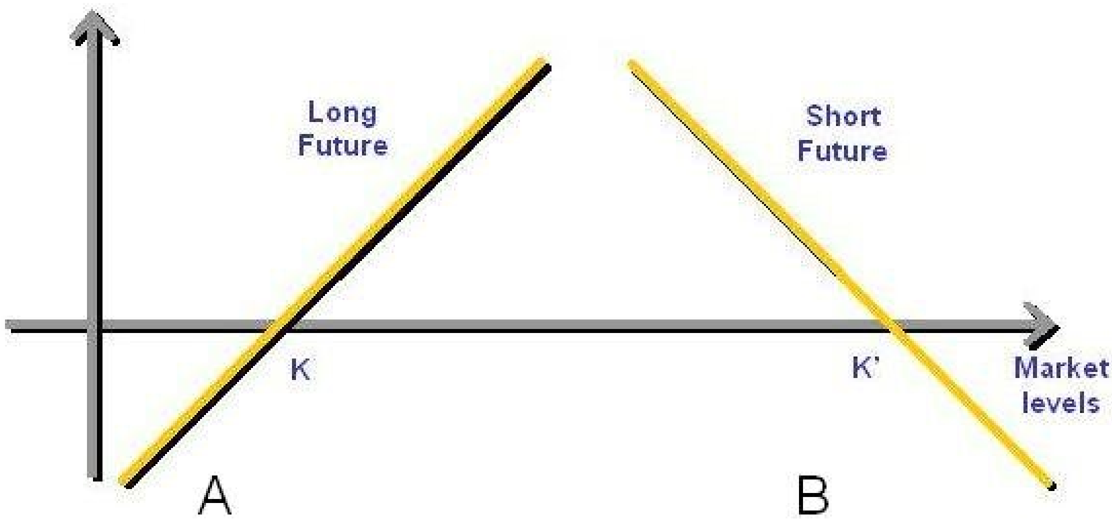


Figure 3.19: Payoff from forward contracts.

party assumes a *short position* and agrees to sell the asset on the same date for the same price. The specified price will be referred to as the delivery price [Hu 06].

The delivery price is chosen so that the value of the forward contract to both parties is zero. This means it costs nothing to take either a long or short position. A forward contract is settled at maturity. The holder of the short position delivers the asset to the holder of the long position in return for a cash amount equal to the delivery price.

The absence of an immediate payment represents an opportunity cost to the seller of the financial asset. The seller will thus have to be compensated for the loss of any opportunity to profit from another sale in the future. The forward price is, in general, higher than the spot price.

The value of the forward contract at delivery equals the value of the underlying less the delivery price paid. This can be stated as

$$V(T) = \phi\,[S(T) - K] \tag{3.10}$$

where $S(T)$ is the price of the underlying at maturity T and K is the delivery price that was fixed when the deal was entered into originally. Also,

$$\phi = \begin{cases} 1 & \forall \quad \text{longs} \\ -1 & \forall \quad \text{shorts} \end{cases}$$

Equation (3.10) also defines the payoff from a position in a forward contract. This is illustrated graphically in Fig. 3.19.

The forward price for a certain contract is defined as the delivery price which would make the contract have zero value. It follows that the forward price and delivery price are equal at the time, t_0, the contract is entered into, i.e.

$$V(t_0) = 0 = \phi\,[S(t_0) - K] \tag{3.11}$$

3.6.4 The Futures Contract

A futures contract is by definition exactly the same as a forward contract. However, a futures contract is transacted in the arena of a futures exchange. Transactions must be made in prescribed increments (i.e., whole numbers of futures contracts covering a designated "size" per contract), where the price-setting capability applies to a limited number of prospective settlement dates – all features of a futures contract is thus standardised.

Buyers and sellers of futures come together in one central place - the electronic matching system of the exchange. Transactions take place at the best bids and offers provided by the exchange members. Standardisation assists with the liquidity and price competitiveness of exchange derivatives, since all market activity is focused on a limited range of quoted contracts with clearly visible prices [Sk 03]. Furthermore, most recognised exchanges try to ensure the availability of prices for each of the listed contracts, through official and unofficial liquidity providers.

Futures transactions tend to be used primarily as price-setting mechanisms rather than as a means of transferring property. That is, when using futures contracts, buyers and sellers typically offset their original positions prior to the delivery date specified by the contract, and then they secure the desired currency via a spot market transaction.

The differences between a forward and futures contract are summarised in Table 3.5.

3.6.5 Cash Flow Differences

Cash flow obligations are very different for forward contracts and futures contracts. With a forward contract, a price is established on the trade date; but cash changes hands only on the expiration date (or settlement date after the expiration date), when, as agreed, the buyer pays the seller and takes possession of the property. With a futures contract, the change in value of the futures is passed between the two parties to the trade following movements of the futures price each day, making use of the clearinghouse as an intermediary. When the futures price rises, the buyer (who holds the long position) "earns" the change in value of the contract, and the seller (the short-position holder) loses. Opposite adjustments are made when the futures price declines. This daily cash adjustment thus collects from the loser and pays to the winner each day, with no extension of credit whatsoever. The daily dollar value that changes hands is called the "variation settlement."

	Forward	**Future**
Contract size	Customised	Standardised
Delivery date	Customised	Standardised
Participants	"Big" players and corporates. Speculation not encouraged	"Smaller" players. Qualified speculation
Liquidity and pricing	Low volumes and inefficient pricing	High liquidity and efficient pricing
Security deposit	Compensating bank balances or credit lines	Small security deposit required
Credit	Client's credit important	No credit issues
Documentation	All documentation like ISDAs to be in place	No documentation; Rules of exchange prevail
Clearing operation	Bank's back office	Clearing house.
Regulation	Self-regulation; Central Bank	Regulated Exchange; Central Bank
Liquidation	Mostly by actual delivery	Mostly by offset and cash
Transaction costs	Bank's bid/ask spread	Negotiated fees
Margining	Sometimes	Daily MtM and margins.

Table 3.5: Differences between Forwards and Futures.

This cash-flow aspect of the futures contact is perhaps the most difficult conceptual hurdle, as well as the hardest operational feature, for a potential futures market user. This will be discussed in detail in the following sections.

3.7 Arbitrage-free Pricing of Forwards and Futures

3.7.1 What is Arbitrage?

In §3.6 we defined a forward and a futures contract. If we want to trade such a contract, we need a price. In §3.6.3 we stated that the forward price cannot be the same as the spot price due to opportunity costs. This means that if we can define and calculate the value of the opportunity cost, we should be able to determine a fair price for a forward contract.

In finance (or financial mathematics), the value of this "opportunity cost" is usually calculated by the no-arbitrage principle. This principle is mostly used to determine fair prices and hedging strategies of forward contracts and other derivatives.

What is arbitrage?

The *"Law of One Price" is the first commandment of quantitative finance.*

This law is an economic law that can be stated as: "In an efficient market all identical goods must have only one price." With globalisation this should also hold between different markets. The intuition for this law is that all sellers will flock to the highest prevailing price, and all buyers to the lowest current market price. In an efficient market the convergence on one price is instant. This law also underpins the statement that two portfolios that will produce exactly the same cash flows in the future must have the same value to start with.

The concept of risk-free arbitrage, or the opportunity to earn a certain profit with no capital investment, is based on this Law. An intuitive grasp of how the force of arbitrage maintains price efficiency is among the most important concepts for any financial engineer to possess [Fi 03].

Formally, theoreticians define an arbitrage as a trading strategy that requires the investment of no capital, cannot lose money, and has a positive probability of making money. Equivalently, an arbitrage opportunity exists if it is possible to make a gain that is guaranteed to be at least equal to the risk free rate of return, with a chance of making a greater gain. Less rigorously, an arbitrage opportunity is a "free lunch".

The notion of true arbitrage is profoundly important in financial engineering and theoretical finance. In theory, a market in equilibrium will offer no arbitrage opportunities. Much of the theory of asset valuation is based on the assumption that prices must be set in a consistent manner that affords no true arbitrage opportunities between them. This is called arbitrage-free pricing. An arbitrage condition is a relationship that must prevail between certain prices if they are to be arbitrage-free. Examples of arbitrage conditions are

- interest rate parity for forward exchange rates;
- put-call parity for European options;
- cash-and-carry arbitrage conditions for forward commodity prices.

If arbitrage opportunities do exists, arbitrageurs will eventually cause prices to adjust until arbitrage is no longer possible i.e., prices are arbitrage-free. This can be stated otherwise

> the existence of arbitrage opportunities induces trading and price adjustments until an economic equilibrium is reached, with no arbitrage opportunities left in the economy [JT 00].

Financial markets are said to be well-functioning if there are no arbitrage opportunities i.e., an efficient market will have little of no arbitrage opportunities. The less efficient a market the more and bigger the arbitrage opportunities.

Most financial engineering models are what are known as relative pricing models. They price instruments based on prices of other instruments quoted in the market — an instrument's price is determined relative to other prices quoted in the market in such a manner as to preclude any arbitrage opportunities.

Can all of this help us to determine a price of a forward contract? Yes, it can. We will explain it with the use of Box Diagrams. The use of Box Diagrams is one effective approach to understand and capitalize on arbitrage opportunities because Box Diagrams provide an important visualization of the arbitrage process. While the approach is applicable to a number of arbitrage situations, we will focus on currency forwards and futures [Fi 03].

3.7.2 Box Diagrams

If you ask almost anyone who works in foreign exchange why the spot price and the forward price differ, they will respond with these three words [We 06]

> "interest rate differentials"

What are they talking about? Let's see if we can clarify this remark.

Study Fig. 3.20. We call this the "Basic Box Diagram" that shows us schemati-

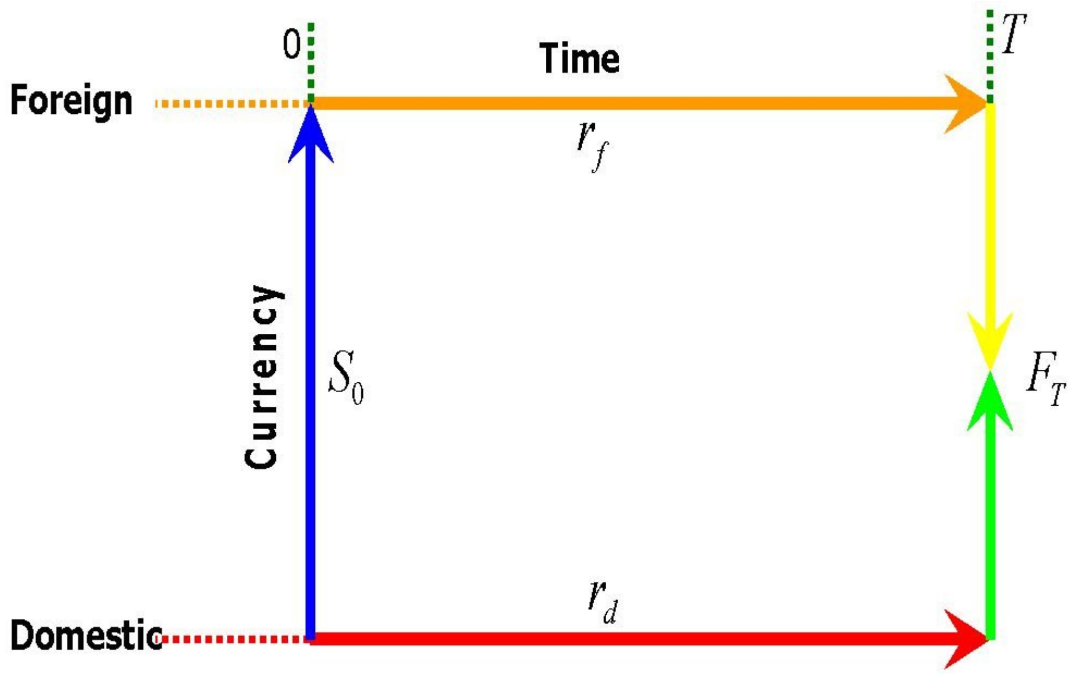


Figure 3.20: The basic Box Diagram.

cally what we want to achieve. We start at the spot rate S_0 and want to get to the forward rate F_T that is "one small step away from spot." Vertically, the Box Diagram moves between investments in a "domestic currency" and a "foreign currency." Horizontally, the Box dimension represents time, specifically the time between today and the expiration date of a currency futures contract for which you are attempting to establish an arbitrage-free price. This time period is shown as the variable$T$. The domestic risk-free investment/borrowing rate is represented by r_d and the foreign risk-free investment/borrowing rate by r_f. The spot currency exchange rate (today's rate, or at $T = 0$) is represented by S_0 and the futures currency exchange rate (priced today and deliverable at time T) is represented by F_T.

How does this all help us to establish an arbitrage-free price of the forward contract? Remember what the "law of one price" states about arbitrage opportunities. Let's look at an example where we have Ks1000. You need to decide how to invest this money today for a period of one year. However, you do not want to take risk but you have a choice: you can invest it in the local market in Shillings or you can invest it in the USA in Dollars! According to the Box Diagram you have two options:

1. Invest the Ks1000 for one year at the current Kenyan risk-free investment rate r_d (simple); or,

2. Convert the Ks1000 into another currency (US Dollars for this example) at the spot rate S_0 and invest the funds at the US risk-free investment rate r_f (simple) for the same period.

Fig. 3.21 shows the cash flows and what you will hold after one year. Let's assume the current spot USDKES exchange rate is 82, $r_d = 5\%$ and $r_f = 2\%$. At the end of one year you will have either Ks1050 or \$86.1. Due to the law of one price, the following must hold at T

$$F_T = \frac{1050}{12.3171} = 85.248\ USDZAR. \tag{3.12}$$

This spot-forward FX relationship is known as *Interest Rate Parity* or *Covered Interest Arbitrage* [We 06].

In the realm of no-arbitrage, after 1 year at T, my R1050 must be equal to \$12.3171. This will only be true if the exchange rate is 85.248 USDKES as calculated by equation 3.12. 85.248 is thus the rate that applies to the future date after 1 year — called the forward rate. Equation 3.12 can be formalised as follows by substituting the 1050 and 12.3171 with formulas that calculate interest

$$F_T = S_0 \frac{\left(1 + \frac{r_d}{m}\right)^{m\,T_d}}{\left(1 + \frac{r_f}{m}\right)^{m\,T_f}} \tag{3.13}$$

if the risk-free interest rates are given in compounded format (either NACA, NACS or NACQ). m is the number of compound period per annum.

We have to make a distinction in terms of the time parameter T. This is due to different day count conventions in different countries. This is a crucial point that are often overlooked by professionals in the market (see Fig. 1.15 and §3.2.1). T_d is thus not always equal to T_f — as Kenya uses "actual over 364", T_d will hardly ever be equal to T_f.

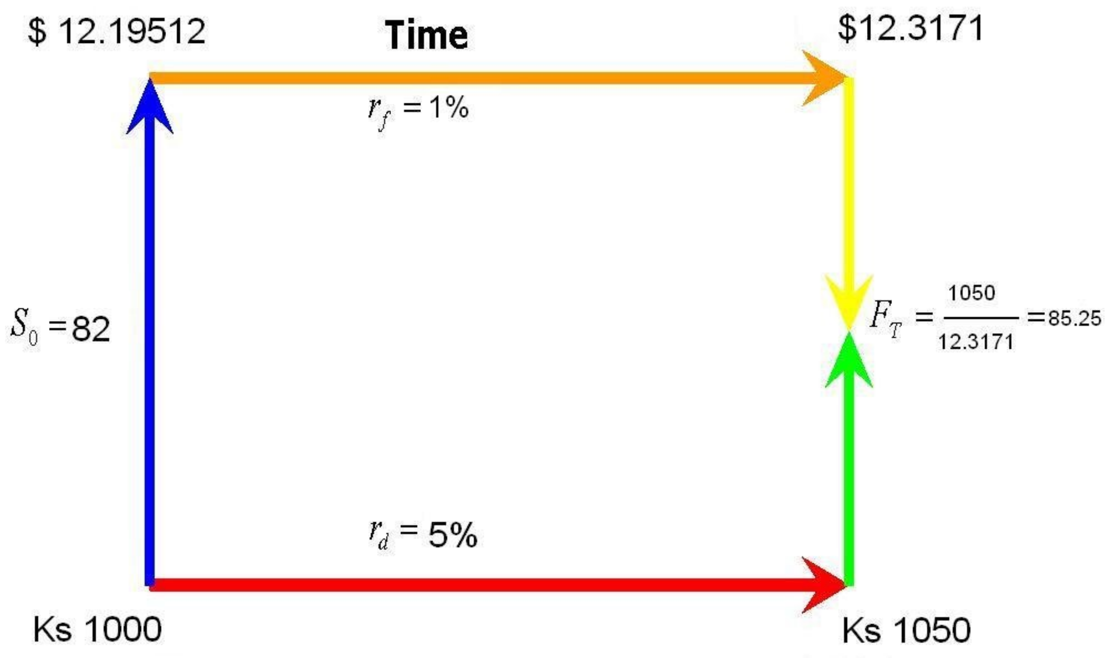


Figure 3.21: The cash flows for your investment.

If the interest rates are simple in format (on the yield — which will most probably be the case as most currency forwards are short term in nature with a maturity of 1 year or less), the formula in (3.13) changes as follows

$$F_T = S_0 \frac{1 + r_d T_d}{1 + r_f T_f} \tag{3.14}$$

Theory works better if one considers continuous rates. Then Eq. 3.14 becomes

$$F_T = S_0 e^{(r_d T_d - r_f T_f)} \tag{3.15}$$

This answers the question at the beginning of this section. Equation 3.15 explains the difference between the spot price and forward price as being due to "interest rate differentials". If $T_d = T_f = T$ this can simplify to

$$F_T = S_0 e^{(r_d - r_f)T}. \tag{3.16}$$

Equation 3.16 shows that, if $r_d - r_f$, $F_T = S_0$ or the spot price and the forward price are the same. Put another way, if the costs and benefits are the same in both countries, Spot = Forward.

3.7.3 Carry Cost and Forward Points

From Eqs. 3.14 and 3.15 we see that the forward price is the spot price adjusted by a certain amount. We term this the opportunity cost. However, Equation (3.14) shows that this is actually the net carry cost i.e.,

$$Netcarrycost = \frac{1 + r_d T_d}{1 + r_f T_f}. \tag{3.17}$$

This Equation states that we borrow money in the foreign market where we pay interest at r_f and we deposit the equivalent amount in the domestic market where we earn interest at r_d. We can now say that

$$\boxed{\textbf{Forward price = Spot price adjusted by the net carry cost}}$$

How does it work in the real world? If a client asks a bank for a forward price, the dealer does not quote the forward outright. Why not? Look at Eq 3.18. The forward is spot × a quantity. The spot moves around a lot meaning the forward will also move around a lot. The spot can move a couple of times per second — for the major currencies anyhow. It will be very hard for a dealer to quote a forward with the spot moving so quickly. However, if we define a quantity P such that

$$P = S_0 \left[\frac{1 + r_d T_d}{1 + r_f T_f} - 1 \right] \tag{3.18}$$

we can rewrite Eq. 3.14 to be

$$F_T = S_0 + P. \tag{3.19}$$

This quantity P is known as the forward points. Equation 3.19 can be rearranged to give

$$P = F_T - S_0. \tag{3.20}$$

The forward points is thus the difference between the forward price and spot price. Note: the forward points can be positive OR negative, depending on the interest rate differentials. Equation 3.14 is used to calculate F_T and one then uses Eq. 3.20 to obtain the forward points. Due to the different tenors quoted in the market, we obtain a forward points yield curve. This is shown in Fig. 3.22 for the USDZAR. We also show the forward curve for the Nigerian USDNGN on 21 February 2011 in Fig. 3.23.

From the no-arbitrage arguments mentioned above and Equations (3.14), (3.18) and (3.19) we learn that domestic and foreign interest rates play a crucial role in determining the forward price. Equation (3.14) actually shows us that a FX forward is similar to a pair of zero coupon bonds — a domestic deposit and foreign loan. Therefore, forward rates reflect interest rates in the two currencies. Equation (3.14) reflects forward interest parity. In order to price forwards effectively, we thus need to employ market related domestic and foreign yield curves.

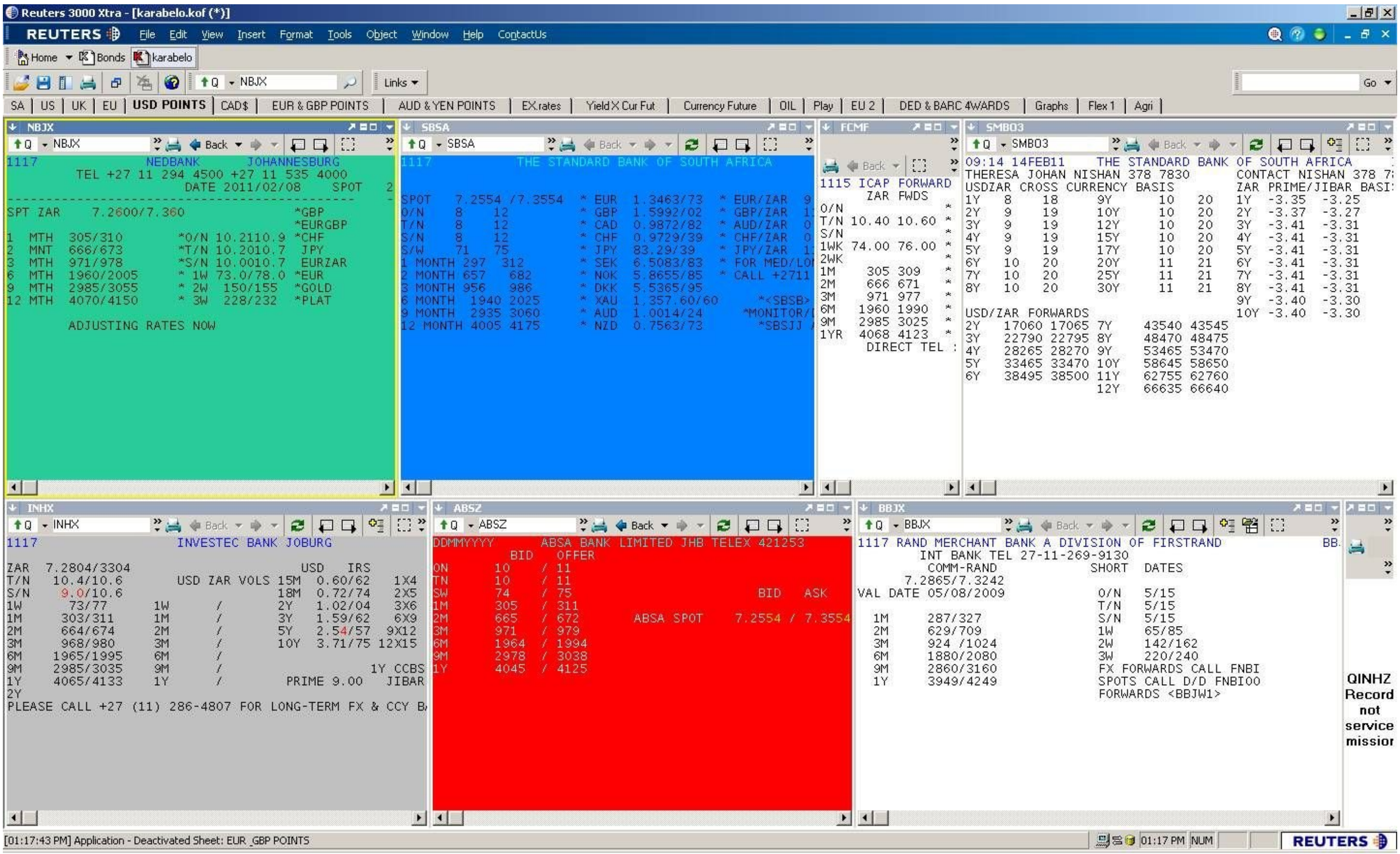
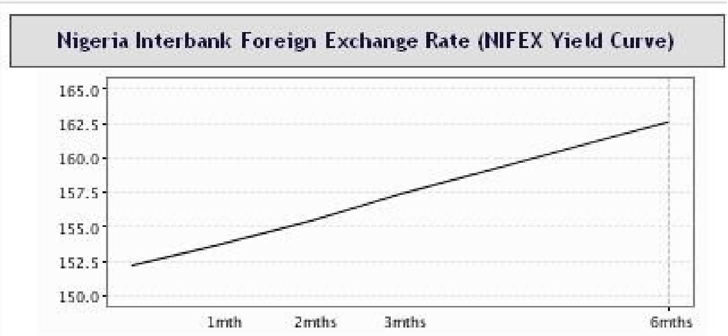


Figure 3.22: Different banks' quotes on USDZAR forward points on 14 Feb 2011.



Tenor	Swap	Forward Fixings	Date
Spot		153.5400 / 153.6400	21 Feb, 2011
30 Day	1.3635 / 1.3644	154.9035 / 155.0044	21 Feb, 2011
60 Day	2.9440 / 2.9459	156.4840 / 156.5859	21 Feb, 2011
90 Day	4.6925 / 4.6956	158.2325 / 158.3356	21 Feb, 2011
180 Day	9.9257 / 9.9322	163.4657 / 163.5722	21 Feb, 2011

Figure 3.23: Niara forward curve on 21 Feb 2011.

3.7.4 Trading Futures on Exchange

One aspect to consider, when dealing on an exchange, is fees. Brokers charge fees to clients and the exchange also charges a small fee. Let B be the all inclusive fee as a percentage. The following equations will hold for the possible trades a client can do

1. The client goes long:

$$F = S_0 + P + S_0\,B \tag{3.21}$$

1. The client goes short:

$$F = S_0 + P - S_0\,B \tag{3.22}$$

1. The client unwinds a long position:

$$F = S_0 + P - S_0\,B \tag{3.23}$$

1. The client unwinds a short position:

$$F = S_0 + P + S_0\,B \tag{3.24}$$

3.8 Non-Deliverable Forwards

In general, an OTC currency forward contract is a deliverable contract. That means that the buyer of the forward will take delivery of the underlying currency on the expiry settlement date.

On the other hand, a FX non-deliverable forward (NDF) is essentially an outright forward contract whereby, on the contracted settlement date, profit or loss is adjusted between the two counterparties basing on the difference between the contracted NDF rate and the prevailing spot FX rates on the original agreed notional amount. This means the client will not take physical delivery of the underlying currency. No principle amount is exchanged. Cash will change hands only. The cash is the notional/principle amount divided by the difference in the spot FX rate on T and the forward rate as quoted on the original deal date t_0. Hence NDFs are 'non-cash' products which are off-the-balance-sheet and as the principal sums do not move, possess much lower counterparty risks.

Every NDF has a fixing date and a settlement (delivery) date. The fixing date is the day and time whereby the comparison between the NDF rate and the prevailing spot rate is made. The settlement date is the day whereby the difference is paid or received. Depending on the currencies dealt, there are variations whereby for some currencies, the fixing date is one good business day before the settlement date and for other currencies, the fixing date is two good business days before the settlement date. Generally, the fixing of spot rate is based on a reference page on Reuters or Telerate with a fallback of calling four leading dealers in the relevant market for a quote.

Across the world, NDF markets have evolved as a result of restrictions in local forward markets i.e., regulatory issues (liquidity can also play a role). It allows hedging of currencies which would otherwise be 'unhedgeable' in the offshore market.

Let's look at an example to understand the cash flows. We take the example in the previous section. Let's assume today, the spot exchange rate is 7.25 ZAR/USD. A client phones the bank. The client wants to buy an NDF on R1 million notional. You quote the client a forward rate of 7.6 ZAR/USD. The NDF expires on 1 February 2007. We discuss three possible scenarios on the expiry date

- The prevailing USDZAR rate is equal to the NDF rate;
- The prevailing USDZAR rate is higher than the NDF rate;
- The prevailing USDZAR rate is lower than the NDF rate.

These scenarios are set out and discussed in Table 3.8.

The general formula for calculating the profit or loss is given by

$$PL = N\left(\frac{S_T}{F_T} - 1\right) \tag{3.25}$$

where PL is the profit or loss to the client and N is the notional amount.

3.9 Contango and Backwardation

These two unusual terms refer to the shape of the futures or forward price curve of a particular instrument. A contango is observed when the futures price is higher than the spot or cash price. Thus most commodities, equities, and other instruments exhibit a contango shape to their successive futures prices. This positive slope in price, as the time to expiry increases, reflects a lower income from an instrument than the return on cash, as we have noted above. Backwardation is the term used to describe a negatively sloping futures curve, that is, where it is more expensive to purchase a particular commodity in the underlying market now than to contract to purchase it later [Sk 03].

For most futures and forward markets, backwardation is rarely, if ever, observed since, according to our understanding of carry cost, it implies that the particular asset provides a greater income than cash for the period of the contract. It is usually only seen in some *currency markets*, and in the short-dated equity and bond futures

Scenario a.	Assuming that the prevailing USDZAR is exactly 7.6 on 1 February 2007. In this instance there is no difference between the NDF and prevailing rates, hence no payment is made by either parties and the NDF expires.
Scenario b.	Assuming that the prevailing USDZAR is 7.7 on expiry. In this instance, the ZAR has weakened and there is a difference of R0.10. Hence, Bank X will pay the difference to the client on the settlement date (5 February 2007) in Rand. This amount (profit to the client) is determined by the following method: On 21 Nov 2006, the client buys $$\frac{R1,000,000}{R/\$7.6} = \$131,578.95$$ from Bank X. On 1 Feb 2007, the client sells to Bank X $\$131,578.95 \times 7.7 = R1,013157.90$. The client thus makes a profit of R13,157.89.
Scenario c.	Assuming that the prevailing USDZAR is 7.55 on expiry. In this instance, the ZAR has strengthened and there is a difference of R0.05. Hence Bank X will receive the difference from the client on the settlement date (5 February 2007) in Rand. This amount (loss to the client) is determined by the following method: On 21 Nov 2006, the client buys $$\frac{R1,000,000}{R/\$7.6} = \$131,578.95$$ from Bank X. On 1 Feb 2007, the client sells to SBSA $\$131,578.95 \times 7.55 = R993,421.05$. The client thus makes a loss of R6,578.95.

Table 3.6: Explaining an NDF

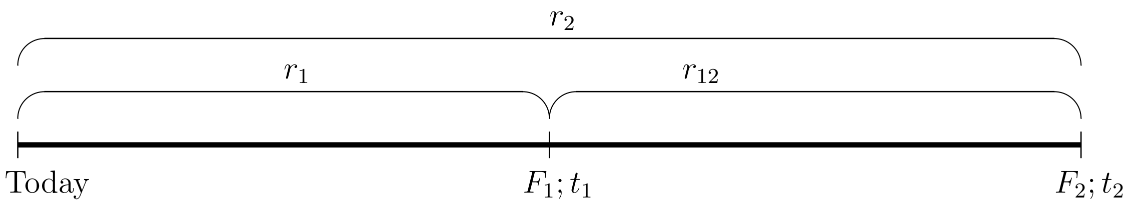


Figure 3.24: Marking-to-market a futures contract.

markets where the timing of dividends and coupons can have a distorting impact on futures prices. It can also be witnessed in some commodity markets where the successive futures contracts on a particular commodity relate to different harvests, or where normal arbitrage conditions do not apply.

3.10 Forward Prices versus Futures Prices

When interest rates are constant and the same for all maturities, the forward price for a contract with a certain delivery date is, theoretically, the same as the futures price for a contract with the same delivery date;

$$\text{Forward} = \text{Future}$$

In theory (if interest rates are constant) the value of a futures contract is given by one of Eqs. 3.14, 3.15 or 3.16 with the forward points defined in Eq. 3.18 (for simple interest).

However, when interest rates are unpredictable (as they are in the real world), forward and futures prices are in theory no longer the same. We explain this by considering a futures contract that is marked-to-market twice as shown in Figure 3.24. The value of a forward contract maturing at time t_2 is given by

$$F = Se^{r_2 t_2}.$$

using continuous interest rates. A futures contract is margined and its value at time t_1 is thus

$$F_1 = Se^{r_1 t_1}$$

and at time t_2 we have

$$\begin{aligned} F_2 &= F_1 e^{r_{12} t_{12}} \\ &= Se^{r_1 t_1 + r_{12} t_{12}}. \end{aligned}$$

Due to the unpredictability of interest rates, $r_1 \neq r_{12}$ and $F \neq F_2$. If interest rates were constant we'll have $r_2 = r_1 = r_{12}$ and thus $F = F_2$ and forwards and futures would be the same. This is the case for illiquid futures contracts. Market makers use Eqs. 3.15 and 3.16 to calculate the value of a futures contract.

Other factors to consider are taxes, transaction costs, scrip lending fees and the treatment of margins. In most circumstances, however, the differences between the prices of short dated futures and forward contracts are sufficiently small to be ignored.

In South Africa, the other factors mentioned do play a significant role. We will look at these in more detail later on.

3.11 Trading African Currency Derivatives

South Africa has a very active foreign exchange market. The OTC derivatives market is vibrant with many exotic structures trading daily. Currency futures can also be traded on the JSE's currency derivatives exchange. The clearing houses of the derivatives exchanges are most of the banks operating in South Africa.

Mauritius started a new exchange called GBOT recently[16]. It offers futures on Gold, Silver, EURUSD, GBPUSD, UASDJPY, USDMUR, USDZAR. It plans to extend the product list to include trading in Nigeria's Naira and the Kenyan and Ugandan shillings against the dollar. The State Bank of India, Mauritius's Banque des Mascareignes and Barclays Plc will be the clearing houses for the exchange. It is owned by Financial Technologies (India) Ltd.

Currency futures seems to be a growth market sector. On 2 February 2011 it was reported by the *Kenya Star* that the Central Bank of Nigeria stated it will begin foreign currency futures trading soon with auctions to be conducted as much as three months forward. There are moves to start a currency futures market in Kenya as well. On 31 January 2011, the Nigerian Central Bank issued this statement[17]

> "In support of the various initiatives led by the Authorised Dealers, over the years, in deepening the Nigerian inter-bank foreign exchange market, the CBN is releasing these guidelines on the risk management products Authorised Dealers are allowed to offer to their customers and the modalities under which the CBN intends to boost the trading liquidity in these products.
>
> The approved hedging products are FX Options, Forwards (Outright and Non-Deliverable), FX Swaps and Cross-Currency Interest Rate Swaps. Authorised Dealers are now allowed to offer European-styled FX call and put option contracts to their customers and in the inter-bank market. All hedge transactions with the customers must be backed by trade (visible and invisible) transactions. The CBN shall grant approvals for Authorised

[16] http://www.gbot.mu/

[17] http://www.cenbank.org/documents/FmdCirculars.asp — Guidelines for FX Derivatives and Modalities for CBN FX Forwards

> Dealers that qualify to engage in Options. Details on the approval process shall be released in due course."

All of the above is good news for Africa. It is becoming easier to trade African currencies. Africa is set for a landmark development which will stimulate further foreign trade and investments while ensuring good risk mitigation. This last point is extremely important — risk can only be mitigated if risk can be managed. Derivatives exchanges need good risk management practices which will filter through to the rest of the markets.

Chapter 4

Properties of Options Prices

Call Put (CME hand signals)

4.1 Introduction

The key to understanding options is the notion of a premium. Some derivatives are compared to insurance. Just as you pay an insurance company a premium in order to obtain protection on your car (if it is stolen) for instance, there are derivatives products that have a payoff contingent upon the occurrence of some event. This premium needs to be calculated. *Black & Scholes* developed a model for this. But, is such a model a true reflection of reality? Where does it come from? Options have been traded long before the publication of a model so why do we need a model? Isn't the price set by the market and isn't this market price "right" in the sense of the efficient market hypothesis?

A common misconception about option pricing models is that their typical use by option traders is to indicate the "right" price of an option. Options pricing models do not necessarily give the "right" price of an option. The "right" price is what someone is willing to pay for a particular option. An efficient market will give the best and truest prices for options. Many individuals trade warrants and some of them do not know what an option is nor do they have a pricing model.

In §1.4 we learnt that there are many benefits to having a model. One can almost state it is imperative to have such a model. The derivatives market would not have advanced to where it is today, was it not for the *Black & Scholes* model. In this chapter we get to know the formula better and we will understand the framework it is set in. We will concentrate on how the model can help us in our understanding of options and will touch on some practical implementation issues every trader needs to know when using the *Black & Scholes* formula.

4.2 The Two Types of Derivatives

There are two types of derivatives: linear derivatives and non-linear derivatives. A linear derivative is one whose payoff function is a linear function. For example, a futures contract has a linear payoff in that every one-tick movement translates *directly* into a specific currency value (KES for Kenya for instance). A non-linear derivative is one whose payoff changes with time and space.

Space in this instance is the location of the strike with respect to the actual spot value. An example is a plain vanilla option. As the option becomes progressively more in-the-money, the rate at which the position makes money increases. We will look at this later.

4.3 Option Basics

There are two types of options: calls and puts. A *call* option gives the holder the right, but not the obligation, to buy the underlying asset by a certain date for a certain price.

A *put* option gives the holder the right, but not the obligation, to sell the underlying asset by a certain date for a certain price.

The price in the contract is known as the *exercise* price or *strike* price. The date in the contract is known as the expiration, *expiry*, *exercise* or *maturity* date.

An option can not be obtained free of charge. There is a price attached to it called a premium. The premium is the price paid for an option. The buyer of an option pays a price for the right to make a choice — the choice to exercise or not.

Call and put options are defined in one of two ways: American or European. A European option can only be exercised at the maturity date of the option whereas an American option can be exercised at any time up and including the maturity date[1]. If

[1]The terms European and American used here bear no relation to geographic considerations. European options trade on American exchanges and American option trade on European exchanges.

the holder of the option decides to exercise the option, this option becomes a simple FX contract. The holder of the option will only exercise the option when the market is in his favour, otherwise the option contract expires worthless.

4.4 Exchange Traded Options

Options trade on many different exchanges around the world. The underlyings include stock, indices and futures. These contracts are standerdised by the exchanges and the options can be European of American in style.

In the USA, options on stock are very popular. Options on Microsoft, IBM and General Motors are very liquid. The two most popular option contracts on indices are the options on the S&P500 and S&P100 that trade on the *Chicago Board of Exchange* (CBOE). The S&P500 option is European and the S&P100 option American. These options are also settled in cash and not in the underlying shares.

Options on index futures are also very popular. The S&P500 short dated future trades nearly round the clock and options on this future are very liquid. In South Africa options on the liquid Alsi futures contract are also very popular.

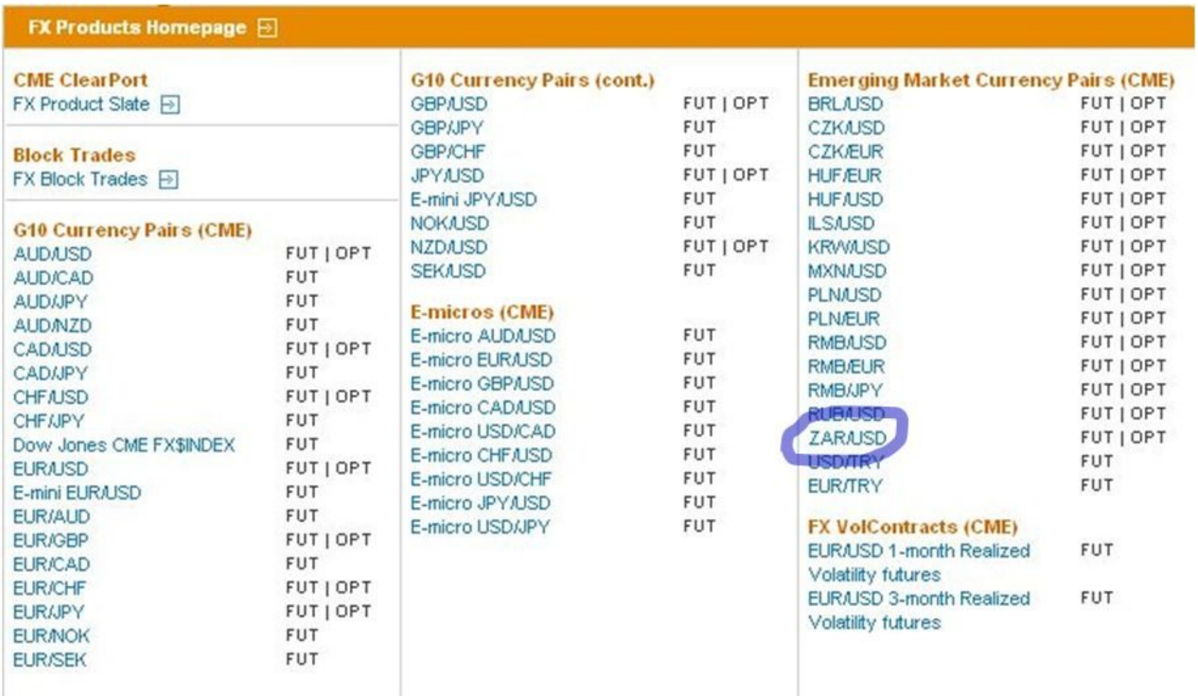
Options on currency futures are starting to become liquid. Fig. 4.1 shows currency options and futures traded on CME in the USA. The only African currency listed is USDZAR. PHLX U.S. dollar-settled South African Rand currency options also trade on Nasdaq. These options, similar to those on CME, are quoted in terms of U.S. dollars per unit of the underlying currency (South African Rand) and premium is paid and received in U.S. dollars. The contract size is Zar100,000.

4.5 Over-the-Counter Options

Not all options trade on an exchange. The *over-the-counter* (OTC) market, where financial institutions and corporations trade directly with one another, are very popular. In fact, the OTC market is much larger that the exchange traded market. The main advantage of an OTC option is that it can be tailored by a financial institution to meet the needs of a corporate client. It is also easy to incorporate non-standard features like an Asian feature into a contract.

Structured products are very popular. A Structured product is a portfolio of options where the strikes and expiry dates are set at certain levels and times to give particular payoff profiles sought by clients. We will expand on this in Chapter 8.

We will later define some other option called Asian options, Bermudan options and Parisian options.


FX Products Homepage

CME ClearPort
FX Product Slate

Block Trades
FX Block Trades

G10 Currency Pairs (CME)

AUD/USD	FUT | OPT
AUD/CAD	FUT
AUD/JPY	FUT
AUD/NZD	FUT
CAD/USD	FUT | OPT
CAD/JPY	FUT
CHF/USD	FUT | OPT
CHF/JPY	FUT
Dow Jones CME FX$INDEX	FUT
EUR/USD	FUT | OPT
E-mini EUR/USD	FUT
EUR/AUD	FUT
EUR/GBP	FUT | OPT
EUR/CAD	FUT
EUR/CHF	FUT | OPT
EUR/JPY	FUT | OPT
EUR/NOK	FUT
EUR/SEK	FUT

G10 Currency Pairs (cont.)

GBP/USD	FUT | OPT
GBP/JPY	FUT
GBP/CHF	FUT
JPY/USD	FUT | OPT
E-mini JPY/USD	FUT
NOK/USD	FUT
NZD/USD	FUT | OPT
SEK/USD	FUT

E-micros (CME)

E-micro AUD/USD	FUT
E-micro EUR/USD	FUT
E-micro GBP/USD	FUT
E-micro CAD/USD	FUT
E-micro USD/CAD	FUT
E-micro CHF/USD	FUT
E-micro USD/CHF	FUT
E-micro JPY/USD	FUT
E-micro USD/JPY	FUT

Emerging Market Currency Pairs (CME)

BRL/USD	FUT | OPT
CZK/USD	FUT | OPT
CZK/EUR	FUT | OPT
HUF/EUR	FUT | OPT
HUF/USD	FUT | OPT
ILS/USD	FUT | OPT
KRW/USD	FUT | OPT
MXN/USD	FUT | OPT
PLN/USD	FUT | OPT
PLN/EUR	FUT | OPT
RMB/USD	FUT | OPT
RMB/EUR	FUT | OPT
RMB/JPY	FUT | OPT
RUB/USD	FUT | OPT
ZAR/USD	FUT | OPT
USD/TRY	FUT
EUR/TRY	FUT

FX VolContracts (CME)

EUR/USD 1-month Realized Volatility futures	FUT
EUR/USD 3-month Realized Volatility futures	FUT

Figure 4.1: Currency options and futures traded on CME.

4.6 Why trade FX Options versus the Spot FX?

Currency options have gained acceptance as invaluable tools in managing foreign exchange risk. One of the primary benefits for trading FX Options versus Spot FX is that options provide investors with tremendous versatility including a wide range of strike prices and expiration months available for trading. Investors can implement single and multi-leg strategies, depending on their risk and reward tolerance. Investors can implement bullish, bearish and even neutral market forecasts with limited risk. FX Options also provide the ability to hedge against loss in value of an underlying asset. Options are attractive financial instruments to portfolio managers and corporate treasuries because of this flexibility.

Options can be a way for traders to limit their risk in a trade. For instance, if a trader believes the EURUSD will move upwards, he may purchase a call at a premium so that if the rate hits the option strike price he can exercise it. If the currency instead moves against the trader, all that is lost is the premium.

In general we note that options expanded the universe of tradable financial instruments. The consequence is that hedges can be tailored more precisely to the risk profile of the underlying and the risk can be managed more easily. Options allow an investor to construct different payoff profiles. You can mimic your actual exposure by trading in a portfolio of options. We will look at this in Chapter 8. All these benefits can be grouped together into six benefits [Sh 10]

Benefit One: The Ability to Leverage

Options provide both individuals and firms with the ability to leverage. In other words, options are a way to achieve payoffs that would usually be possible only at a much greater cost. Options can cause markets to become more competitive, creating an environment in which investors have the ability to hedge an assortment of risks that otherwise would be too large to sustain.

Benefit Two: Creating Market Efficiency

Options can bring about more efficiency in the underlying market itself. Option markets tend to produce information flow. Options enable investors to access and trade on information that otherwise might be unobtainable or very expensive. It is for instance difficult to short sell stock. This slows the process down in which adverse information is incorporated into stock prices and make markets less efficient. It is, however, easy to sell a future or put option.

Benefit Three: Cost Efficiency

Derivatives are cost efficient. Options can provide immense leveraging ability.An investor can create an option position that will imitate the underlying's position identically — but at a large cost saving.

Benefit Four: 24/7 Protection

Options provide relative immunity to potential catastrophic effects of gaps openings in the underlyings. Consider a stop-loss order put in place to prevent losses below a predetermined price set by the investor. This protection works during the day but what happens after market close. If the market gaps down on the opening the next day your stop-loss order might be triggered but at a price much lower than your stop-loss price. You could end up with a huge loss. Had you purchased a put option for downside protection you will also be protected against gap risk

Benefit Five: Flexibility

Options offer a variety of investment alternatives. You can hedge a myriad of risks under specific circumstances. We will look at structuring and constructing different payoff profiles later on in Chapter 8.

Benefit Six: Trading Additional Dimensions

Implementation of options opens up opportunities of additional asset classes to the investor that are embedded in options themselves. Options allow the investor not only to trade underlying movements, but to allow for the passage of time and the harnessing of volatility. The investor can take advantage of a stagnant or a range-bound market.

4.7 The Benefits of the *Black & Scholes* Model

A simple mathematical model is seldom a true reflection of reality. Options and futures were traded many years before any kind of valuation model was constructed. So the question that immediately springs to mind is: why do we need a model?

Options are part of the financial markets and thus for financial markets to function well, participants must thoroughly understand option pricing. *The task is to find the rational and fair price of an option.* Equity analysts spend a lot of time to talk to the management of a company, they analyse the financial statements and then calculate a *fair value* for that company. *Black & Scholes* took that one step further by using that information together with market information to calculate the *fair value* of an option on that company.

Although derivatives have been traded long before *Black & Scholes* published their formula, there are many benefits in having such a model. The true benefits of an option pricing model is that it provides an accurate "snap shot" of current market conditions. But, the market is dynamic; it changes continuously. A model is extremely important and useful for risk management purposes. With a model we can break the option's market price into each of the factors that comprise it. A model can be used to examine each factor separately and assess its individual contribution to the determination of the option's price. Furthermore, by forecasting each of the individual factors, one can forecast an option's price over a wide range of different scenarios. Models are also important when one wants to construct complex instruments with underlying optionality.

Here are some other benefits of having a model

- the concepts behind the *Black & Scholes* analysis provide the framework for thinking about option pricing;
- all the research in option pricing since the *Black & Scholes* analysis has been done either to extend it or to generalize it;
- by doing research many new insights into the behaviour of options have come to light;
- as a consequence of this research, more complex option structures have been developed — the so-called "exotics";
- quantitative risk management now possible;
- hedging of derivative structures and portfolios easier;

- the market has thus progressed and expanded.

An important reason for studying the Black-Scholes theory is that the financial world uses it as a standard. We will later see that all options structures (vanilla or complex) are priced in what is today known as the "*Black & Scholes world*". In fact, traders quote Black-Scholes volatility to each other, not the actual price of the options. Further, Black-Scholes prices still give very good approximations to the real prices of options.

4.8 Black, Scholes and Merton did not invent any Formula!

As we will see later on in this chapter, options traders use a pricing formula which they adapt by fudging and changing the tails and skewness by varying one parameter, the standard deviation of a Gaussian — called the volatility. This formula is popularly called the "Black-Scholes-Merton" formula owing to an attributed eponymous discovery (though changing the standard deviation parameter is in contradiction with it). Fischer Black, Myron Scholes and Robert Merton even were awarded the Nobel prize for it in 1997.

However, in 2008 Haug and Taleb stated the following: there is historical evidence that [HT 08]

1. Black, Scholes and Merton did not invent any formula, just found an argument to make a well known (and used) formula compatible with the economics establishment, by removing the "risk" parameter through "dynamic hedging" (see §1.3,

2. Option traders use (and evidently have used since 1902) heuristics and tricks more compatible with the previous versions of the formula of Louis Bachelier and Edward O. Thorp (that allow a broad choice of probability distributions) and removed the risk parameter by using put-call parity.

3. Option traders did not use formulas after 1973 but continued their bottom-up heuristics. The Bachelier-Thorp approach is more robust (among other things) to the high impact rare event. The paper draws on historical trading methods and 19th and early 20th century references ignored by the finance literature.

It is time to stop calling the formula by the wrong name.

Haug and Taleb further points out that, in a case of scientific puzzle, the exact formula called "Black-Sholes-Merton" was written down (and used) by Edward Thorp which, paradoxically, while being robust and realistic, has been considered unrigorous. This raises the following

1. The Black-Scholes-Merton was just a neoclassical finance argument, no more than a thought experiment,

2. There are no evidence of traders using their argument or their version of the formula.

It is high time to give credit where it belongs.

These two gentlemen go further

> "First, something seems to have been lost in translation: Black and Scholes (1973) and Merton (1973) actually never came up with a new option formula, but only an theoretical economic argument built on a new way of "deriving", rather rederiving, an already existing - and well known - formula. The argument, we will see, is extremely fragile to assumptions. The foundations of option hedging and pricing were already far more firmly laid down before them. The Black-Scholes-Merton argument, simply, is that an option can be hedged using a certain methodology called "dynamic hedging" and then turned into a risk-free instrument, as the portfolio would no longer be stochastic. Indeed what Black, Scholes and Merton did was 'marketing'; finding a way to make a well-known formula palatable to the economics establishment of the time, little else, and in fact distorting its essence."

These authors point out two myths

- Myth 1: People did not properly "price" options before the Black-Scholes-Merton theory.

- Myth 2: Option traders today use the Black-Scholes-Merton option "pricing formula".

It is true, these are myths. In §1.3 we mentioned people like *Sprenkle* (1961), *Ayres* (1963), *Boness* (1964), *Samuelson* (1965) and *Chen* (1970) who all produced valuation formulas similar in form to the *Black & Scholes* model. It is common sense that the *Black & Scholes* formula was known before 1973 when Black and Scholes published their paper [BS 73]. The *Boness* formula is identical to the *Black & Scholes* formula.

What is important though is *the way Black, Scholes* and *Merton* derived their formula based on continuous dynamic delta hedging or alternatively based on CAPM they were able to get independent of the expected rate of return. It is in other words not the formula itself that is considered the great discovery done by *Black, Scholes* and *Merton*, but how they derived it. This is among several others also pointed out by *Mark Rubinstein* when he wrote [Ru 06]

> "The real significance of the formula to the financial theory of investment lies not in itself, but rather in how it was derived. Ten years earlier the

> same formula had been derived by Case M. Sprenkle (1962) and A. James Boness (1964)."

However, *Haug* and *Taleb* wants *Edward Thorp* to have the honors for he, apparently, worked out the precise formula before Black-Scholes and Merton [Ha 07]. They conclude their paper by the following statement

> "This is why we call the equation Bachelier-Thorp. We were using it all along and gave it the wrong name, after the wrong method and with attribution to the wrong persons. It does not mean that dynamic hedging is out of the question; it is just not a central part of the pricing paradigm. It led to the writing down of a certain stochastic process that may have its uses, some day, should markets "spiral towards dynamic completeness". But not in the present."

Shortly after the publication of their paper, *Paul Wilmott* answered them on Wilmott Forum by saying

> "I will happily accept that the Black-Scholes formulae were around well before 1973...... In the first issue of our magazine (Wilmott magazine, September 2002) the cover story was about Ed Thorp and his discovery of the formulae and their use for making money (rather than for publication and a Nobel Prize!). Ed wrote a series of articles 'What I Knew and When I Knew it' to clarify his role in the discovery, including his argument for what is now called risk-neutral pricing. I particularly like the story of how Fischer Black asked Ed out to dinner to ask him how to value American options. By the side of his chair Ed had his briefcase in which there was an algorithm for valuation and optimal exercise but he decided not to share the information with Black since it was not in the interests of Ed's investors! Incorrect accreditation of discoveries is nothing new in mathematics, but usually there's a quid pro quo that *if you don't get your name attached to your discovery then at some stage you'll get your name attached to someone else's*!"

This controversy is similar to the *Newton* and *Leibniz* controversy over who invented *Calculus*[2].

Should we now discard the formula? Not at all. Wilmott summarised it very well on Wilmott Forum during April 2008

[2]Leibniz discovered "his" form of calculus in Paris between 1673 and 1676. By 1676, Leibniz realized that he was onto something "big"; he just didn't realize that Newton was on to the same big discovery because Newton was remaining somewhat tight lipped about his breakthroughs. In fact, it was actually the delayed publication of Newton's findings that caused the entire controversy. Leibniz published the first account of differential calculus in 1684 and then published the explanation of integral calculus in 1686.
Newton did not publish his findings until 1687. Yet evidence shows that Newton discovered his theories of fluxional calculus in 1665 and 1666. Evidence also shows that Newton was the first to

> "They say traders don't use Black-Scholes because traders use an implied volatility skew and smile that is inconsistent with the model. I think this is a red herring. Yes, sometimes traders use the model in ways not originally intended but they are still using a model that is far simpler than modern-day 'improvements.' One of the most fascinating things about the Black-Scholes model is how well it performs compared with many of these improvements. For example, the deterministic volatility model is an attempt by quants to make Black-Scholes consistent with the volatility smile. But the complexity of the calibration of this model, its sensitivity to initial data and ultimately its lack of stability make this far more dangerous in practice than the inconsistent 'trader approach' it tries to 'correct'!"

4.9 The *Black & Scholes* Environment

To obtain an understanding of what the *Black & Scholes* formula means, it is very important to know under what conditions the *Black & Scholes* formula hold. *Black & Scholes* (and researchers before them) understood very well that the market is complex. To be able to describe it mathematically and to enable them to obtain a useful model, they knew they had to simplify the market by making certain assumptions[3]. The following is a list of the more important assumptions *Black & Scholes* made in their analysis [MS 00]:

- *The underlying asset follows a lognormal random walk.* This was not a new assumption and was already proposed by Bachelier in 1900.
- *The efficient market hypothesis is assumed to be satisfied.* In other words, the markets are assumed to be liquid, have price-continuity, be fair and provide all players with equal access to available information. This implies that there are no transaction costs.

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establish the general method called the "theory of fluxions", was the first to state the fundamental theorem of calculus and was also the first to explore applications of both integration and differentiation in a single work. However, since Leibniz was the first to publish a dissertation on calculus, he was given the total credit for the discovery for a number of years.
Remember, many of the ideas about calculus were being discussed and studied in the years before Newton and Leibniz worked on them. Pierre de Fermat and Isaac Barrow made important discoveries. Some historians have taken this line of argument to the extreme and have sought to establish Barrow as the inventor of the calculus and to represent the labours of Newton and Leibniz merely as a translation of Barrow's work into algebraic form.
http://en.wikipedia.org/wiki/Leibniz_and_Newton_calculus_controversy

[3]What inevitably happens after such a "simple" model has been proposed and understood well, is that people start to relax certain assumptions to move closer to a more realistic model.

- *We live in a risk-neutral world i.e., investors require no compensation for risk.* The expected return on all securities is thus the riskfree interest rate with the consequence that there are no arbitrage opportunities. This was one of *Black & Scholes* ' insights and is known as risk-neutral valuation.
- The stock's *volatility is known* and does not change over the life of the option. In statistical talk we say the means and variances of the distribution or process is "stationary".
- The *short-term interest never changes.*
- *Short selling of securities with full use of the proceeds is permitted.*
- *There are no dividends.*
- *Delta hedging is done continuously.* This is impossible in a realistic market but makes their analysis possible.

The above describes the so-called *Black & Scholes* environment within which they did their analysis.

In general we can say that *Black & Scholes* assumed that the financial market is a system that is in equilibrium. With equilibrium we mean that, if there are no outside or exogenous influences, then the system is at rest - everything thus balances out; supply equals demand. Any distortion or perturbation is thus quickly handled by the market players so as to restore the equilibrium situation. More so, the systems reacts to the perturbation by reverting to equilibrium in a *linear fashion.* The system reacts immediately because it wishes to be at equilibrium and abhors being out of balance [Pe 91]. An option's price is thus the value obtained under this equilibrium situation.

These assumptions are very restrictive, as a matter of fact *Black* went on to say that "Since these assumptions are mostly false, we know the formula must be wrong" [Bl 88]. But, he might not be far from the truth when he further stated that "But we may not be able to find any other formula that gives better results in a wide range of circumstances."

Is the assumption of a lognormal random walk an accurate one? This is the easiest one to test in the real world. Figigure 4.2 shows the distribution of a few assets compared to that of the standard normal distribution — we include USDZAR and EURUSD. We sometimes have to use our imagination but the dynamics of some assets are fairly close to that of the normal distribution. In Fig. 4.3 we do the same but for some African currencies. Look at how leptokurtic the African currencies are[4]. The biggest problem stems from the tails. We'll return to that later.

[4] A pure leptokurtic distribution has a higher peak than the normal distribution and has heavier tails.

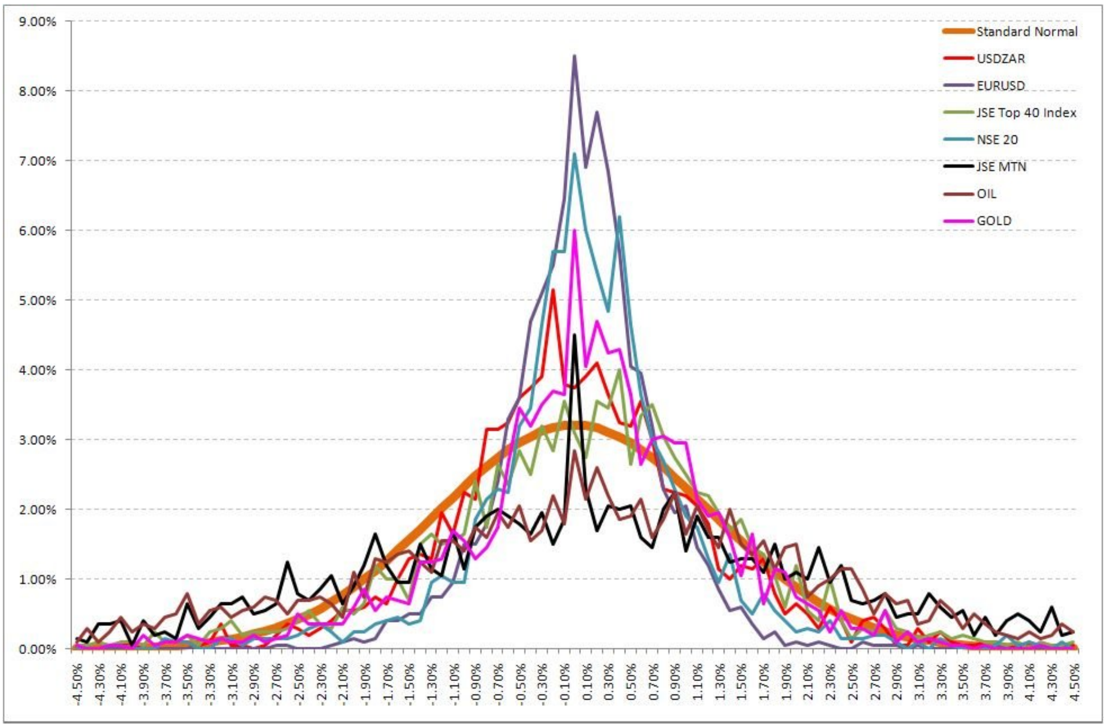


Figure 4.2: Distributions of a few assets classes compared to the standard normal distribution.

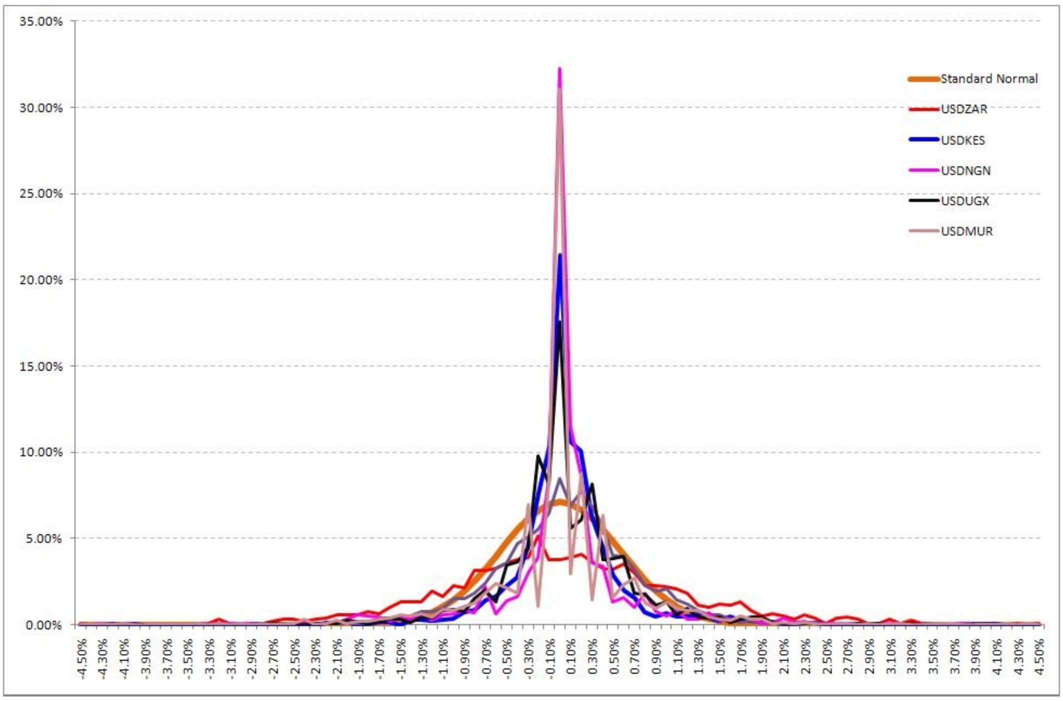


Figure 4.3: Distributions of a few African currencies compared to the standard normal distribution.

4.10 The *Black & Scholes* Analysis

We now know that the basic form of the option pricing formula was already known in the late sixties. These formulas, however, had two or more parameters that were hard to estimate. By using market data traders did estimate these. They would, for instance, calibrate the pricing formula every morning by using historical option prices.

4.10.1 The Option Payoff

So what was the problem back in the 1960's? Analytically, the payoff function of a European call and put is given by

$$\begin{aligned} V(T) = C(T) &= \max[0, S_T - K] \qquad (4.1) \\ V(T) = P(T) &= \max[0, K - S_T] \end{aligned}$$

with S_T and K the price of the underlying asset at expiry and the strike price respectively. To know what an option is worth one only needs to know the stock price at expiry. Easy.....? Well, we all know, to forecast stock prices is very difficult if not impossible.

4.10.2 The *Black & Scholes* Hedging Strategy: a Replicating Portfolio

Black & Scholes subscribed to the idea that stock prices were described by geometric Brownian motion. Their breakthrough came when they realized that, *when a hedged position for an option is in equilibrium, the expected return on such a hedge must be equal to the return on a riskless asset.* This insight led them to construct a riskless hedge for an option by forming a portfolio containing the underlying stock and European call options — this is also known as delta-hedging.

The reason why a riskless portfolio can be set up is because the stock price and derivative security price are both affected by the *same underlying source of uncertainty.* This means that in a short period of time the two are perfectly correlated. The portfolio is set up such that any gain (loss) from the stock position always offsets the loss (gain) from the derivative security position so that the overall value of the portfolio, at the end of the short period of time, is known with certainty. Remember that the portfolio remains *riskless for an infinitesimally short period of time only.* For the return on the hedge portfolio to remain riskless, the portfolio must continuously be adjusted in the appropriate manner as the asset price changes over time.

Black & Scholes further argued that the return on the portfolio must be the riskfree rate of interest if arbitrage opportunities are to be avoided. They showed that, *of the three securities: an option, the underlying stock and a riskless money market security,* *any two could be used to exactly replicate the third by a trading strategy.* The money market security must mature on the same date as the option and its face value should be equal to the striking price of the option.

In their original paper [BS 73] *Black & Scholes* showed that they could replicate the money market security. They stated that if one was short one derivative of value V and long an amount x of the underlying stock S, the portfolio is worth Π such that

$$\Pi = -V + xS. \tag{4.2}$$

This replicating portfolio has two restraints:

- *Equivalence of value*:
  the value of the replicating portfolio must equal the value of the call at all points in time until maturity.

- *Self-financing requirement*:
  This replicating portfolio must be self-financing, which means you neither consume from it nor add money to it beyond an initial loan or deposit Π [Le 00, HK 00].

Now, the change $\Delta\Pi$ in the portfolio during a short period of time Δt is given by

$$\Delta\Pi = -\Delta V + x\Delta S.$$

If we have a completely riskfree change $\Delta\Pi$ in the portfolio value Π then it must be the same as the growth we would get if we put the equivalent amount of cash in a riskfree interest-bearing account. This means that

$$\Delta\Pi = r\Pi\Delta t$$

where r is the riskless interest rate[5].

This replication of options by an appropriate portfolio of stock can be illuminated by the following example [CR 85]:

Example

Consider the following levered hedge:

1. Write 3 calls at C each (K =R50),

2. Buy two shares at R50 each,

3. Borrow R40 at 25% to be paid back at maturity.

Consider also the return from this hedge at maturity:

[5]The return on the portfolio is equal to the return on a riskless asset.

	value at current date	$S^* = 25$	$S^* = 100$
Write 3 calls	$3C$	0	-150
Buy 2 shares	-100	50	200
Borrow	40	-50	-50
Total	x	0	0

From this we see that, regardless of the outcome of S^*, the hedge exactly breaks even at expiration. The cash flows are zero at expiration so, if there are to be no arbitrage opportunities (and no risk), the current cash flow should also be zero. Therefore, $x = 0$ and thus $C = 20$ if there are no risks involved. ♣

This example shows that we can duplicate a pure position in a call if we buy shares and borrow in the right proportion against them. This can also be stated as follows:

- by assumption of the capital asset pricing model (CAPM), **any portfolio with a zero market risk (the so-called β-value) must have an expected return equal to the riskfree rate of interest**. We have thus established an equilibrium condition between the expected return on an option, the expected return on the stock and the riskfree rate [Me 90]. This means that option prices can be valued as if the underlying stock price increases at the riskfree rate!

Mathematically this riskless hedging and return arguments imply a parabolic differential equation[6] relating the value of the option to the value of the underlying asset. In using stochastic calculus *Black & Scholes* derived the relevant differential equation and subsequently the solution as well[7] (remember, many differential equations do not have solutions).

4.11 The Seminal Formula

Black & Scholes assumed that the underlying asset follows a random walk (Brownian Motion). They stated that its dynamics are governed by the following equation

$$\frac{dS}{S} = \mu dt + \sigma dW \tag{4.3}$$

where they assumed that trading takes place continuously in time. Here, μ is the expected return earned by an investor in a short period of time — the instantaneous

[6]A differential equation is a mathematical description of some phenomenon, system or process. This mathematical model can be used to analyse and describe the phenomenon. Once it is solved, we can describe the future behaviour of the phenomenon.

[7]For a clarification on the methods used by *Black & Scholes* to obtain the pricing formula see *Rosu* and *Stroock* [RS 01].

expected return. It is annualized and expressed as a proportion. The higher the risk the higher the return. μ is also dependent on the level of interest rates in the economy — the higher the interest, the higher the return. σ is the instantaneous standard deviation of return — in the market this is called the implied volatility. The volatility of a stock is exactly equal to the standard deviation of the continuously compounded return provided by the stock in one year. dW is a Wiener process [Hu 06, Na 88].

Black & Scholes had a breakthrough when they realised that they can transform the differential equation into the equation describing the heat transfer through a medium like a wall or glass. This is a well known equation in physics and was solved by physicists at the beginning of the nineteenth century[8]. This means that share prices dissipates through time in a similar manner that heat diffuses through a certain medium!

This insight and the assumptions described in the previous sections led them to solve the differential equation and derive their pricing formula. We outline their methodology in Appendix B where we show that the solution to the differential equation in Eq. B.1 is[9]

$$V(S,t) = \phi\left(Se^{-d\tau}N(\phi x) - Ke^{-r\tau}N(\phi y)\right) \tag{4.4}$$

where

$$\begin{aligned} x &= \left[\ln\frac{S}{K} + \left(r - d + \frac{1}{2}\sigma^2\right)\tau\right]\frac{1}{\sigma\sqrt{\tau}} \\ y &= x - \sigma\sqrt{\tau}. \end{aligned}$$

where ϕ is a binary parameter defined as

$$\phi = \begin{cases} 1 & \text{for a call option} \\ -1 & \text{for a put option.} \end{cases} \tag{4.5}$$

Here, S is the current spot stock price, K is the strike price, d is the dividend yield, σ^2 is the variance rate of the return on the stock prices, $\tau = T - t$ is the time to maturity and $N(x)$ is the cumulative standard normal distribution function[10]. Note that Eq. 4.4 in general format is; it holds for both calls and puts.

Equation (4.4) is not the original formula as published by *Black & Scholes* . They considered stock that do not pay dividends, i.e. $d = 0$. This formula is called the

[8] Heat or diffusion equations goes back to the beginning of the 19th century. They were successfully used to model smoke particles in air, flow of heat from one part of an object to another, chemical reactions, dispersions of populations and dispersions of pollutants in a running stream.

[9] *Clark* gives an extensive review of the derivation of the *Black & Scholes* equation [Cl 11]

[10] Note that

$$N(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-a^2/2}da$$

but it can be determined numerically.

modified Black & Scholes equation adapted by *Merton* in 1973 to include a continuous dividend yield d [Me 73]. He did this by correctly assuming that an option holder does not receive any cash flows paid by the underlying instrument. This fact should be reflected in a lower call price or a higher put price. The *Merton* model provides a solution by subtracting the present value of the continuous cash dividend flow from the price of the underlying instrument. The original equation had $d = 0$.

The beauty of this formula lies in the fact that one does not need to estimate market expectation or risk preferences. This was a revolutionary improvement over its predecessors. There are 3 parameters that needs to be estimated: the riskfree interest rate; the dividend yield and the variance or volatility. Note that the volatility needed in the *Black & Scholes* formula is the volatility of the underlying security that will be observed in the future time interval τ - *volatility thus needs to be predicted* [MS 00]. *Black & Scholes*, however, assumed that the variance is known and that it is constant.

4.12 The Intuitive *Black & Scholes*

The solution to the *Black & Scholes* differential equation can be interpreted in two ways:

4.12.1 As a Replicating Portfolio - Delta Hedging

Let's consider a call and we let $d = 0$. Now compare Eqs. (4.2) and (4.4)

$$\begin{aligned} V &= xS - \Pi \\ V &= N(x)S - Ke^{-r\tau}N(y). \end{aligned}$$

We immediately see that

$$\begin{aligned} x &= N(x) \equiv \text{ number of shares bought; called the Delta} \\ \Pi &= KN(y)e^{-r\tau} \equiv \text{ amount borrowed to pay for shares.} \end{aligned}$$

This shows that a European call option is a leveraged position in an asset plus an insurance policy. The leveraged transaction entails borrowing the present value of the strike K and purchasing one unit of the asset which sells for S [Ch 94]. Stated differently, the first term in (4.4) is the amount invested in the underlying asset and the second is the amount borrowed to pay for it.

This leads to the concept of delta-hedging. If we purchase an option we can trade in the cash instrument (called "trading spot" or "trading the cash") to hedge the option. By buying the option we pay the premium upfront. As the underlying's share price changes with time, the Delta will show us that we have to buy the cash at the lows and sell at the highs — we thus make money by delta hedging. In theory, if we hedge continuously we should make exactly what we paid for the option. On the other hand, if we sell the option, we earn the premium. The Delta will then show us to buy the cash at the highs and sell it at the lows - we thus loose money by delta hedging. By doing this continuously we should loose exactly what we earned with the premium. We will return to these arguments later in Chapter 5.

4.12.2 As a Present Value of Risk-neutral Expectation

The solution to the *Black & Scholes* differential equation can be interpreted in a risk-neutral expectations manner (we describe the call): If we exercise the call at maturity we will receive the stock but will have to pay the strike price K. This exchange will not take place if the call does not finish in-the-money. $N(x)$ is an "adjusted probability": it is the probability of the call being in-the-money adjusted for the depth in-the-money. The first term $S(t)N(x)$ can thus be seen as the weighted present value of receiving the stock (this is the stock's potential value and our potential benefit) if and only if $S_T > K$. The second term $-Ke^{-r\tau}N(y)$ is the weighted present value of paying the strike price (this is our potential cost or loss). $N(y)$ is the probability of the call being in-the-money.

In summary we can say

- $V \equiv$ difference between what you would earn from the stock at T and what you would pay for it (the strike price K) at T weighted with a probability.

4.13 A Currency Option Model

Garman and *Kohlhagen* provided a formula for the valuation of foreign currency options [GK 83]. They followed the *Black & Scholes* lines of thought but set their riskless hedge portfolio up by investing in foreign bonds, domestic bonds and the option. They had some more assumptions though

- It is easy to convert the domestic currency into the foreign currency;
- We can invest in foreign bonds without any restrictions.

Their analysis led them to the following *Black & Scholes* -type equation

$$V(S, K, \tau, \sigma, r_d, r_f, \phi) = \phi\left(Se^{-r_f\tau}N(\phi x) - Ke^{-r_d\tau}N(\phi y)\right) \tag{4.6}$$

where

$$\begin{aligned} x &= \left[\ln\frac{S}{K} + \left(r_d - r_f + \frac{1}{2}\sigma^2\right)\tau\right]\frac{1}{\sigma\sqrt{\tau}} \\ y &= x - \sigma\sqrt{\tau}. \end{aligned}$$

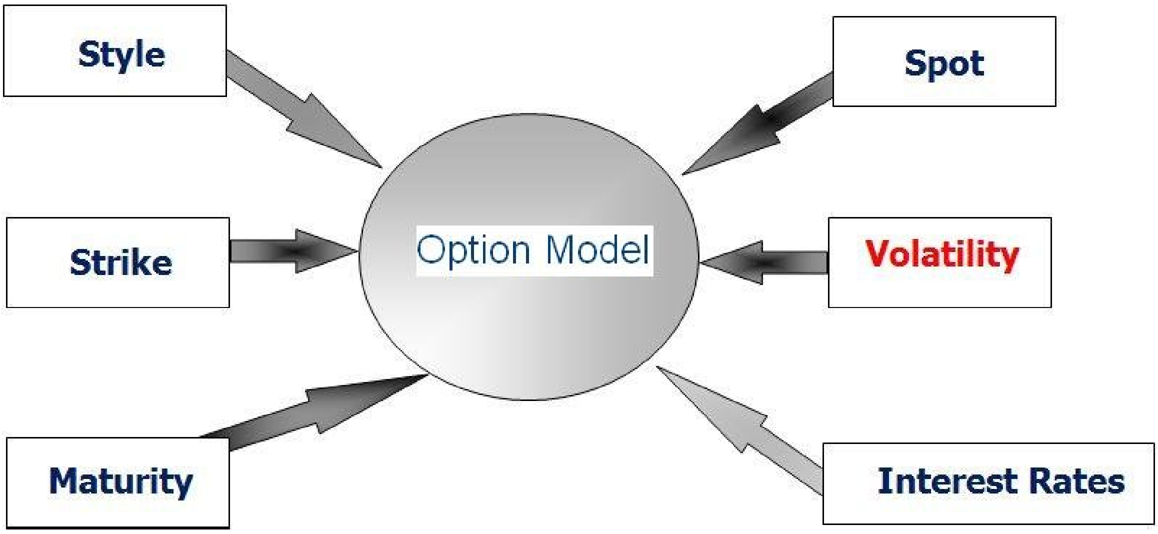


Figure 4.4: Information necessary to price an option.

with ϕ defined in Eq. 4.5. Note that S is the current spot exchange rate, K is the strike price, r_f is the foreign interest rate, r_d is the domestic interest rate σ^2 is the variance rate of the return on the exchange rate, $\tau = T - t$ is the time to maturity and $N(x)$ is the cumulative standard normal distribution function. We note that Eq. 4.6 is exactly the *Black & Scholes* equation given in Eq. 4.4 where we have substituted $d = r_f$ and $r = r_d$. To price an option we thus need six quantities as depicted in Fig. 4.4

Note that $V(S, K, t, \sigma, r_d, r_f, \phi)$ will sometimes be shortened to $V(S, \tau)$ or just $V(S)$. V denotes the value of a Call or Put. In future we sometimes also have

$$\begin{aligned} C(S, K, \tau, \sigma, r_d, r_f) &= V(S, K, t, \sigma, r_d, r_f, +1) \Rightarrow \text{Call} \\ P(S, K, \tau, \sigma, r_d, r_f) &= V(S, K, t, \sigma, r_d, r_f, -1) \Rightarrow \text{Put} \end{aligned} \tag{4.7}$$

4.14 Options on Forwards and Futures

In 1976 *Fischer Black* presented a model for pricing commodity options and options on forward contracts [Bl 76]. In §3.6.3 we discussed the concept of a forward contract. Equation 3.16 defined a forward contract's value to be

$$F_T = F = Se^{(r_d - r_f)T} \tag{4.8}$$

with S the spot currency exchange rate at the start of the forward contract. *Black* showed that a futures contract can be treated in the same way as a security providing a continuous dividend yield equal to the riskfree interest r. This means that we can use Eq. 4.4 where we have $r = d$ and we use the forward price F as the price of the underlying instead of the cash price S. Turning to currencies and the *Garman* and *Kohlhagen* model we put $r_d = r_f$.

The easiest way in obtaining *Black's* formula is to invert Eq. 4.8 to obtain S and substitute this into Eq. 4.6 to give

$$V(F,t) = \phi e^{-r_d \tau} \left[F N(\phi x) - K N(\phi y) \right] \tag{4.9}$$

if $T_d = T_f$.

4.15 Settlement Adjustments

Remember from §1.11 that FX spot transactions generally settle in two business days. There are thus four dates of importance for option contracts: today, spot, expiry and delivery [Cl 11]. The delivery date is usually set to the expiry spot, i.e., so that delivery bears the same spot settlement relationship to expiry. This means that if

spot = today + 2 (T+2), then delivery = expiry + 2 (T+2)

as well. In the *Black & Scholes* equation given in Eq. 4.4 we defined the time to expiry by stating $\tau = T - t$ where t is the trade date and T is the expiry date. Now that we have four dates, which ones are the correct ones to use when we price options? On a time line we have

$$T_{today} \to T_{spot} \longrightarrow T_{exp} \to T_{es}$$

and we usually have the delivery time $T_{del} = T_{es}$.

To understand why this is important, we refer back to the actual cash flows. The premium will only be received on T_{spot} although we enter into the contract at T_{today}. The premium should thus reflect the premium today at T_{today}. Also, if the option is in the money at expiry, the profit will flow on T_{del} only. Now it becomes tricky because the volatility applicable is the volatility over the period $T_{today} \leq t \leq T_{exp}$ because that is the real terms of the agreement. To price the option correctly, we price it from cash flow to cash flow thus from T_{spot} to T_{del}. We then discount the premium back from T_{del} to T_{today} and then forward value to T_{spot}.

This changes the *Black & Scholes* formula somewhat to

$$V(S,t) = S e^{-r_f (T_{es} - T_{spot})} N(\phi x) - K e^{-r_d (T_{es} - T_{spot})} N(\phi y) \tag{4.10}$$

where

$$\begin{aligned} x &= \left[\ln \frac{S}{K} + (r_d - r_f)(T_{es} - T_{spot}) + \frac{1}{2}\sigma^2 (T_{exp} - T_{today}) \right] \frac{1}{\sigma \sqrt{T_{exp} - T_{today}}} \\ y &= x - \sigma \sqrt{T_{exp} - T_{today}}. \end{aligned}$$

If $\tau = T_{es} - T_{spot} = T_{exp} - T_{today}$ Eq. 4.10 is exactly the same as Eq. 4.6.

If one extract the relevant interest rates from zero coupon yield curves, note the following: the interest rates should be the forward interest rates that hold from T_{spot} to T_{es}. We discussed forward rates in §3.2.7. Graphically we depict it in Fig. 4.5.

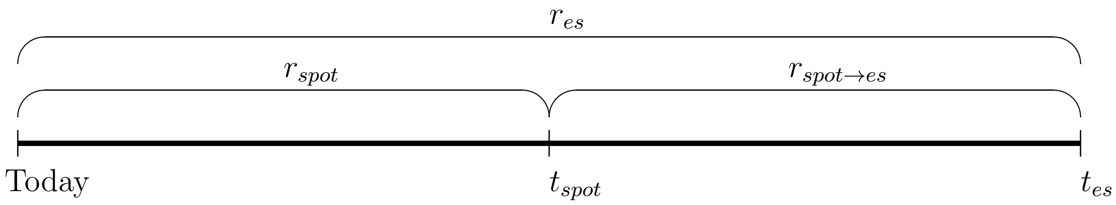


Figure 4.5: The correct interest rates for a currency option.

4.16 Option Pricing in Excel

An option pricing formula as given in Eqs. 4.4 and 4.6 can easily be implemented in Excel. We show this in Fig. 4.6. A better route though is to program it in VBA. We give some pseudo code in Fig. 4.7.

Let's calculate some option values by varying the input currency rate and plot this together with the payoff given in Eq. 4.2. The call graph is shown in Fig. 4.8.

4.17 Option Sensitivities

In the previous section we showed how to construct an option pricer in Excel. If we look at Eq. 4.6 we see there are six input parameters. They are

- the current FX rate;
- the strike price;
- the time to expiration;
- the local riskfree interest rate;
- the foreign riskfree interest rate;
- the volatility.

Let's see what happens to the premium if we increase the input parameters like the volatility or strike for instance. The outcomes are summarised in Fig. 4.9. We see the same results from our Excel calculator.

	A	B	C	D
1	DESCRIPTION			
2	Equity price	100		
3	Strike	100		
4	Current date	=NOW()		
5	Maturity date	=B4+181		
6	Interest Rate (NACA)	0.082		
7	Volatility	0.25		
8	Dividend Yield (NACA)	0.025		
9	Call/Put	p		
10				
11	T	=(INT(B5)-INT(B4))/365		
12	r CCR	=LN(1+B6)		
13	d CCR	=LN(1+B8)		
14	X	=(LN(B2/B3)+(B12-B13+B7*B7/2)*B11)/B7/SQRT(B11)	N(x)	=NORMSDIST(B14*B17)
15	Y	=B14-B7*SQRT(B11)	N(y)	=NORMSDIST(B17*B15)
16				
17	Phi	=IF(B9="c",1,-1)		
18	Value	=B17*(B2*EXP(-B13*B11)*D14-B3*EXP(-B12*B11)*D15)		
19	Delta	=B17*D14*EXP(-B13*B11)		
20				

Figure 4.6: Option pricing in Excel.

```
If (t < 0) Then
   V = 0
ElseIf (t = 0) Then
  V = IV ' IV = intrinsic value
Else
  x1 = (Log(S / K) + (rd - rf + sigma * sigma / 2) * t) _
         / (sigma * Sqr(t))
  x2 = x1 - sigma * Sqr(t)
  V  = phi * (S e^(rf * t) * Application.NormSDist(phi * x1) _
              - K * e^(rd * t) * Application.NormSDist(phi * x2))
End If
```

Figure 4.7: VBA pseudo-code showing the Black & Scholes implementation.

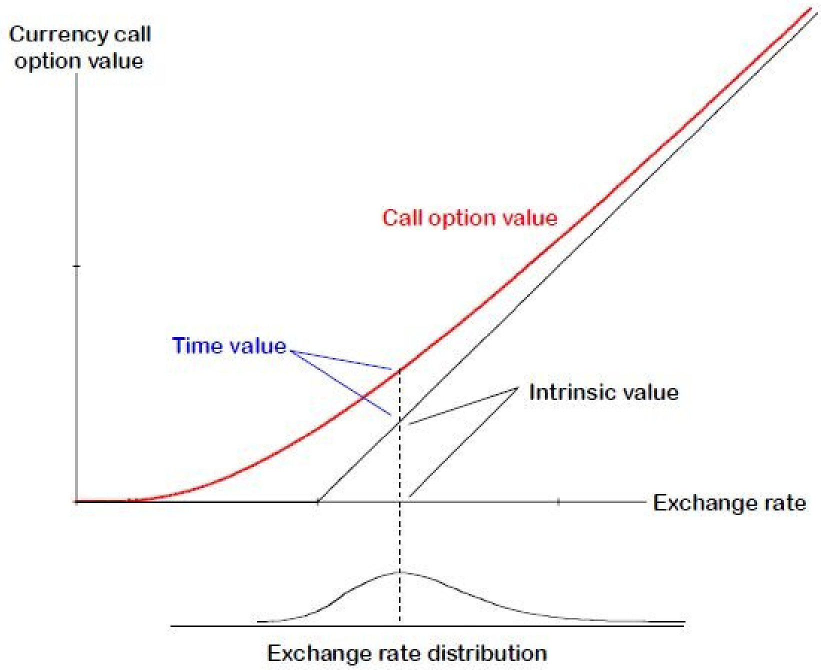


Figure 4.8: Call option and payoff function.

	Call	Put
Spot FX rate	↑	↓
Strike price	↓	↑
Domestic interest rate	↑	↓
Foreign interest rate	↓	↑
Volatility	↑	↑
Time to expiration	↑	↑

Figure 4.9: Option sensitivities by increasing the input parameters.

Chapter 5

The Mechanics of Option Prices

5.1 Introduction

In order to understand the mechanics of a derivative, it is important to understand what factors influence it, how these factors influence it and why they influence a derivative's behaviour. As traders we also need to understand what investors are looking for and what are their risk preferences in order to suggest the right strategy. One also has to understand how to construct a certain strategy and with which instruments. We will discuss the mechanics of options in this chapter.

5.2 Payoff Profiles

For most options and option structures, the important issue for investors is what the possible payoff profile looks like. So, what is a payoff profile?

Let's look at an example: suppose that an investor buys a share at Ks100. If the share price increases to Ks110, he can sell the share and realise a profit of Ks10. On the other hand he would incur a loss if it decreases. The exact same argument holds for an FX contract. If a Kenyan investor bought \$1000, by converting Kenyan Shilling at a rate of USDKES 80, it will cost him Ks80,000. If the exchange rate goes to 90, and he converts the Dollars back into Shillings, he will receive Ks90,000 realising a profit of Ks10,000. If the rate went to 70 he would have realised a loss of Ks10,000.

This profit and loss scenario can graphically be represented such as shown in Figure (5.1a). One can also sell the currency (short it by selling the Dollars) which would lead to the mirror image as shown in Figure (5.1b). The same would hold if you buy or sell a share[1], a forward contract or a futures contract.

[1]Short selling of shares: one can only do this if you can borrow the script. There is a cost attached to this in the region of 1% of the value. A big risk of short selling is the possible dividends that have to be paid. You have to reimburse the lending institution.

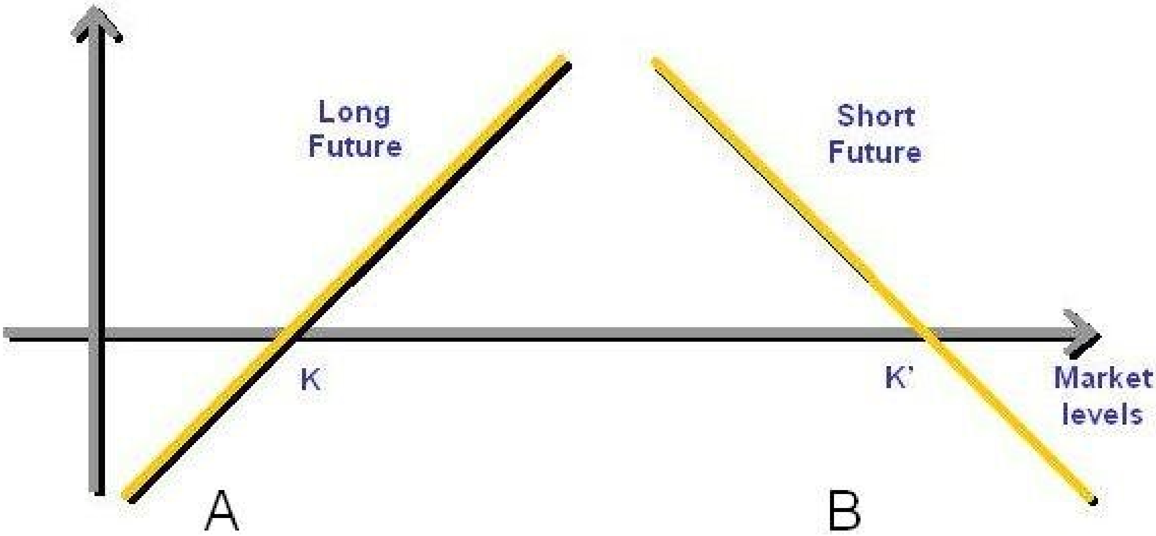


Figure 5.1: (a) Long position in future or share. (b) Short position in future or share.

Options also have payoff profiles. This is shown in Figure (5.2). This is just what one gets by supplying Eq. (4.2) with numbers where

$$V(T) = \phi \max[0, S - K]. \tag{5.1}$$

ϕ is defined in Eq. (4.4). $V(T)$ is also called the *intrinsic value* of an option.

A payoff profile shows the investor the payoff that would be received if the underlying is at its current level when the option expires. It is a simple graphical representation of the risk associated with the strategy: a long future has unlimited profit potential, but such a diagram also shows the potential losses.

It is also easy to work with payoff profiles because they are *additive* meaning that we can add or subtract them from one another. We'll later return to the payoff profiles of some more complex options and structures.

5.3 Building Blocks

The significance of these payoff profiles comes to light when one sees options as part of the "universe of basic building blocks" with which to construct more complex financial instruments. There are a few financial instruments in this universe: a zero-coupon-bond is probably the simplest. NCDs, BA's, TB's, term deposits and loans are all zero-coupon-bonds. Digital options and vanilla calls and puts also forms part of this universe.

I define these basic building blocks or instruments (loosely) in the same way that physicists define the basic particles of our universe. Physics now recognizes some 38

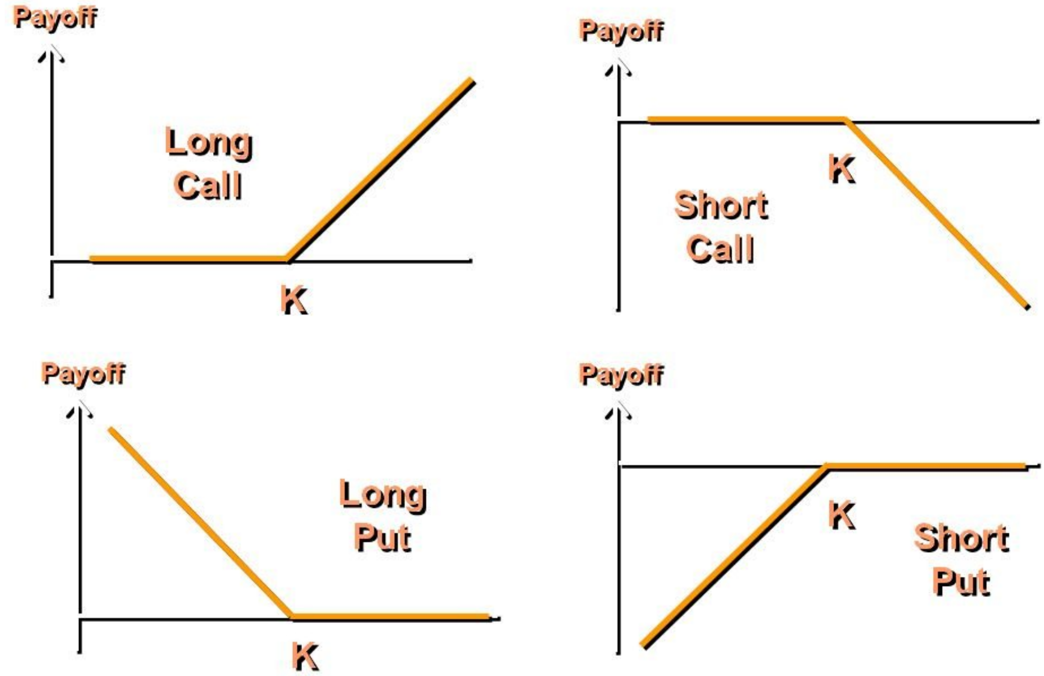


Figure 5.2: Payoffs from positions in European options.

different kinds of basic particles which one needs to build the hundreds of subatomic particles of matter that we know of - these, plus a little energy to bind them together in certain groups. These particles cannot be divided into smaller particles, they form the base of everything in our universe[2]

A coupon bond can be decomposed into a series of zero-coupon-bonds. Bond traders use a yield curve to price this series and then use this information to arbitrage bonds if there are anomalies. Even a long future can be decomposed into a long call and short put position. Can a zero-coupon-bond or vanilla option be decomposed further? No.

5.4 Put-Call-Parity

In the previous chapter we looked at the formulas available for trading European options. Can we say anything about a relationship between puts and calls?

Put-Call-Parity is a very important relationship that is distribution-free. It relationship that exists between the prices of European put and call options where both have the same underlier, strike price and expiry date. We derive it as follows: Consider a portfolio A which comprise a call option with maturity date T and a discount

[2]There are two families of basic particles: Leptons and Quarks. An electron is, for instance, a Lepton. Quarks form groups of two or three to form Baryons and Mesons. Protons and neutrons are Baryons while pions and kaons are Mesons.

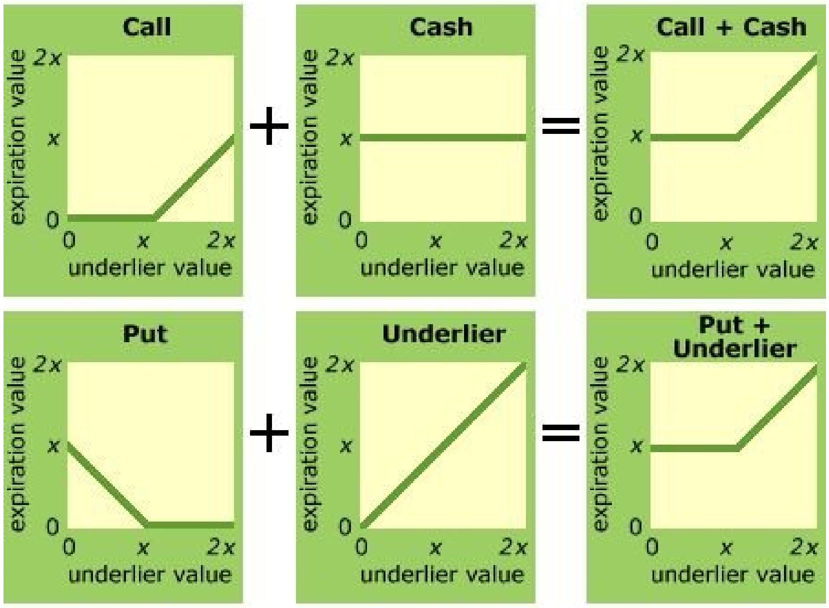


Figure 5.3: Put-Call-Parity.

bond that pays X at this date — this is similar to our replicating portfolio we discussed in Sections 4.10.2 and 4.12.1. Consider also a portfolio B with a put option and one share. Let's denote the underlying asset's price by S.

The values of these portfolios at maturity are shown in the next table[3] and are graphically depicted in Fig. 5.3. Here we show that that we can add the profiles for options and shares and options and bonds. Adding different instruments to generate another instrument is called a strategy.

$$\begin{array}{ll} \text{Portfolio A:} & \max[0, S_T - X] + K = \max[X, S_T] \\ \text{Portfolio B:} & \max[0, X - S_T] + S_T = \max[X, S_T]. \end{array}$$

We see both portfolios have the same value at maturity irrespective of the value of the underlier at expiration [BB 98]. This means both should have the same initial value at t, otherwise arbitrage would be profitable. From this we deduct that a call plus a bond is really a put plus the stock such that we have the following price equality

$$C + PV(X) = P + S$$

Written differently we have

$$C + Ke^{-r\tau} = P + Se^{-d\tau}. \tag{5.2}$$

[3]Note the following indentities:
$\max[x, y] = -\min[-x, -y] = y + \max[0, x - y] = y - \min[0, -(x - y)]$
$\min[x, y] = -\max[-x, -y] = y + \min[0, x - y] = b - \max[0, -(x - y)]$

Note, we used the same notation as in Eq. 4.4 where the strike price $K = X$ and we included a dividend yield d.

This is a fundamental arbitrage relationship which forces call and put prices to be tied to their underlying market and to each other. Note that it is not based on any option pricing model. It was derived purely using arbitrage arguments. Put-call parity offers a simple test of option pricing models. Any option pricing model that produces put and call prices that do not satisfy put-call parity must be rejected as unsound. Such a model will suggest trading opportunities where none exist. Put-call-parity is used to create synthetic securities. We will return to this later in Chapter 8.

From Eq. 5.2 and the *Garman* and *Kohlhagen* model given in Eq. 4.6 we obtain Put-Call-Parity for currency options to be

$$C + Ke^{-r_d\tau} = P + Se^{-r_f\tau}. \tag{5.3}$$

From Eq. 4.9 we deduce Put-Call-Parity for a currency option on a forward or futures contract is given by

$$\begin{aligned} C + Ke^{-r_d\tau} &= P + Fe^{-r_d\tau} \qquad (5.4) \\ \Rightarrow C - P &= (F - K)\, e^{-r_d\tau}. \end{aligned}$$

5.5 Option Dynamics and Risk Managements

If you have traded a few options but are relatively new to trading them, you are probably battling to understand why some of your trades aren't profitable. You start to realise that trying to predict what will happen to the price of a single option or a position involving multiple options as the market changes can be a difficult undertaking. You experience the forces of the market and see that an option price does not always appear to move in conjunction with the price of the underlying asset or share. If you want to trade in any financial instrument, it is important to understand what factors contribute to the movement in the price of that instrument, and what effect they have.

Option prices are influenced by six quantities or variables:

- the current FX rate;
- the strike price;
- the time to expiration;
- the local riskfree interest rate;
- the foreign riskfree interest rate;

- the volatility.

If you want to manage the risk associated with an option you need to understand the dynamics of option values in relation to these quantities.

Futures traders are almost exclusively interested in the direction of the market. Option traders, on the other hand, must also take note of how fast the market is moving or will move. If both futures and options traders take a long market position and the market does move higher, the futures trader is assured of a profit while the option trader may show a loss. To maintain a profit margin the option trader must analyze manage the risk associated with an option at once. He needs to understand the dynamics of option values in relation to these six quantities.

Because this is a difficult task we resort to theoretical models. The goal of theoretical (and numerical) evaluation of option prices is to analyze an option based on current market conditions as well as expectations about future conditions. This evaluation then *assists the trader in making an intelligent decision on an option* [Na 88]. Such an analysis can be done; all that is needed is information that characterizes the probability distribution of future FX rates and interest rates!

Here we consider what happens to options prices when one of these quantities changes while the others remain fixed/constant. We draw the relevant graphs for puts and calls and deduct from there the option's behaviour. The following have to be remembered: the FX rate, strike and time to expiry are known quantities. The riskfree interest rates and the volatility are mostly unknown. These have to be estimated. We will return to these later.

Please note that we use the same notation as for the currency option model in Eq. 4.6.

5.5.1 Asymptotics

If proper risk management is done, we need to understand what happens in the limits or when the six mentioned quantities either becomes very big or very small. Asymptotically we have [CR 85] where we denote the value of call by C and that of a put by P

$$\begin{aligned} S \to 0 &\;\Rightarrow\; C \to 0 \\ S \to \infty &\;\Rightarrow\; C \to \infty \end{aligned}$$

$$\begin{aligned} S \to 0 &\;\Rightarrow\; P \to K \\ S \to \infty &\;\Rightarrow\; P \to 0 \end{aligned}$$

$$\begin{aligned} K \to 0 &\;\Rightarrow\; C \to S \\ K \to \infty &\;\Rightarrow\; C \to 0 \end{aligned}$$

$$
\begin{aligned}
K \to 0 &\;\Rightarrow\; P \to 0 \\
K \to \infty &\;\Rightarrow\; P \to -\infty
\end{aligned}
$$

$$
\begin{aligned}
S < K : \tau \to 0 &\;\Rightarrow\; C \to 0 \\
S > K : \tau \to 0 &\;\Rightarrow\; C \to S - K \\
\tau \to \infty &\;\Rightarrow\; C \to S
\end{aligned}
$$

$$
\begin{aligned}
S < K : \tau \to 0 &\;\Rightarrow\; P \to K - S \\
S > K : \tau \to 0 &\;\Rightarrow\; C \to 0 \\
\tau \to \infty &\;\Rightarrow\; P \to K
\end{aligned}
$$

$$
\begin{aligned}
S < Ke^{-r_d\tau} : \sigma \to 0 &\;\Rightarrow\; C \to 0 \\
S > Ke^{-r_d\tau} : \sigma \to 0 &\;\Rightarrow\; C \to S - Ke^{-r_d\tau} \\
\sigma \to \infty &\;\Rightarrow\; C \to S
\end{aligned}
$$

$$
\begin{aligned}
S < Ke^{-r_d\tau} : \sigma \to 0 &\;\Rightarrow\; P \to Ke^{-r_d\tau} - S \\
S > Ke^{-r_d\tau} : \sigma \to 0 &\;\Rightarrow\; P \to 0 \\
\sigma \to \infty &\;\Rightarrow\; P \to K
\end{aligned}
$$

5.5.2 The Greeks

When we talk about these six variables in relation to option pricing, we call them the Greeks. You might have heard terms such as Delta or Vega and you immediately thought option trading is too difficult or risky.

However, what you will learn in this lesson is that learning things the 'Greek' way is like knowing the baby steps towards potential gains. While many traders focus on spot prices and trends, options pricing and its unpredictability seems to be a bigger problem. For one, the value of options is so uncertain that sometimes trends and factors provide no help at all. If you know about technical analysis of shares, try some of those analyses on option values. You will quickly realise that momentum or stochastics are of no use at all.

Further to the above, we pile on the fact that the Greeks cannot simply be looked up in your everyday option tables nor will you see them on screen where you see the option bids and offers. They need to be calculated which means you will need access to a computer or electronic calculator that calculates them for you.

The Delta

The delta is a measure of the ratio of option contracts to the underlying asset in order to establish a neutral hedge. We can also state that the Δ is a measure of how fast an option's value changes with respect to changes in the price of the underlying asset. From (4.6) we have

$$\Delta = \frac{\partial V}{\partial S} = \phi e^{-r_f \tau} N(\phi x). \tag{5.5}$$

From this we see that if the option is far out-of-the-money $\Delta \simeq 0$, however if the option is deep in-the-money we have $\Delta \simeq \phi$. Δ is thus the probability that the option will end in-the-money.

If you do not have a *Black & Scholes* calculator giving the Delta, you can easily calculate the numerical Delta as follows

$$\Delta_{num} = (V(S + 0.0001) - V(S)) \times 10000.$$

Just add one pip to the price. Here we multiply by 10,000 because of the pip size.

The Premium-Included Delta

If a Nairobi based FX trader wants to hedge his USDKES book in Shillings, he will use the Delta given in Eq. 5.5. However, if a trader has a USDKES book, but the trader sits in New York, his profits and losses will be computed in Dollars and what he really aim at is hedging the option values converted into Dollars. Hence, if V is the option value in Shillings and S is the USDKES spot exchange rate, V/S is the option value converted into Dollars. What the New York trader wants to hedge is

$$\frac{\partial \frac{V}{S}}{\partial S}.$$

This is called the *premium-included Delta* and is given by [Ca 10]

$$\Delta_{pi} = \phi \frac{K}{S} e^{-r_d \tau} N(\phi y). \tag{5.6}$$

Elasticity

The elasticity (denoted by Λ) is the elasticity of an option and shows the percentage change in its value that will accompany a small percentage change in the underlying asset price such that

$$\begin{aligned} \Lambda_c = \frac{\partial C}{\partial S}\frac{S}{C} &= \frac{S}{C}\Delta_C = e^{-r_f \tau}\frac{S}{C}N(x) \ > 1 \\ \Lambda_p = \frac{\partial P}{\partial S}\frac{S}{P} &= \frac{S}{P}\Delta_P = e^{-r_f \tau}\frac{S}{P}N(y) \ < 0. \end{aligned}$$

The elasticity increases when the FX rate decreases. Λ also increases as time to expiration decreases. A call will thus be more sensitive to FX rate movements 'in percentage terms', the shorter the time remaining to expiration [Ko 03].

Gamma

The Γ is the rate at which an option gains or loses deltas as the underlying asset's price move up or down. It is a measure of how fast an option is changing its market characteristics and is thus a useful indication of the risk associated with a position. We have

$$\Gamma = \frac{\partial^2 V}{\partial S^2} = \frac{\partial \Delta}{\partial S} = e^{-r_f \tau} \frac{N'(x)}{S\sigma\sqrt{\tau}} \tag{5.7}$$

where $N'(x)$ is the standard normal probability density function and the cumulative normal's derivative given by

$$N'(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right). \tag{5.8}$$

If the option is far out-of-the-money or far in-the-money, $\Gamma \simeq 0$. If Γ is small, Δ changes slowly and adjustments to keep the portfolio Δ-neutral need only be made relatively infrequently. If Γ is large, however, changes should be made frequently because the Δ is then highly sensitive to the price of the underlying asset. This happens when the FX rate is close to the strike price with very little time to expiry — such as in the morning of the option expires in the afternoon.

Speed

This quantity measures how fast the Γ is changing. It is given by

$$\frac{\partial^3 V}{\partial S^3} = \frac{\partial \Gamma}{\partial S} = -e^{r_f \tau} \frac{N'(x)}{S^2\sigma\sqrt{\tau}} \left(\frac{x}{\sigma\sqrt{\tau}} + 1\right) \tag{5.9}$$

with $N'(x)$ define in Eq. 5.8.

Theta

Both puts and calls lose value as maturity approaches. The Θ is the "time decay factor" and measures the rate at which an option loses its value as time passes such that

$$\Theta = \frac{\partial V}{\partial \tau} = -e^{-r_f \tau} \frac{SN'(x)\sigma}{2\sqrt{\tau}} + \phi\left[r_f S e^{-r_f \tau} N(\phi x) - \phi r_d K e^{-r_d \tau} N(\phi y)\right] \tag{5.10}$$

The size of the Γ correlates to the size of the Θ position where a large positive Γ goes hand in hand with a large negative Θ. A large negative Γ correlates with a large positive Θ. This means that every option position is a trade-off between market movement and time decay. Thus if Γ is large market movement will help the trader but time decay will hurt him and vice versa.

Some market participants calculates the numerical time decay defined as

$$\text{Time Decay} = V(t+i) - V(t) \tag{5.11}$$

where i is a day count parameter. If we put $i = 1$, $V(t+1)$ means the value of the option tomorrow keeping all the other parameters the same. The time decay is thus just the value of the option tomorrow minus the value today. The time decay over a weekend can be obtained by putting $i = 3$.

Charm

Charm is the change of Δ with time

$$\frac{\partial^2 V}{\partial S \partial \tau} = \frac{\Delta}{\partial \tau} = \phi e^{-r_f \tau}\left[N'(x)\frac{2(r_d - r_f)\tau - y\sigma\sqrt{\tau}}{2\tau\sigma\sqrt{\tau}} - r_f N(\phi x)\right] \tag{5.12}$$

Color

Color is the change of Γ with time

$$\frac{\partial^3 V}{\partial S^2 \partial \tau} = \frac{\Gamma}{\partial \tau} = -e^{-r_f \tau}\frac{N'(x)}{2S\tau\sigma\sqrt{\tau}}\left[2r_f\tau + 1 + \frac{2(r_d - r_f)\tau - y\sigma\sqrt{\tau}}{2\tau\sigma\sqrt{\tau}}x\right] \tag{5.13}$$

Vega

The Vega measures the change in the option's price as volatility changes such that

$$Vega = \frac{\partial V}{\partial \sigma} = Se^{-r_f \tau}\sqrt{\tau}N'(x) \tag{5.14}$$

"At-the-money" options are the most sensitive to volatility changes. Some market participants calculates the numerical Vega defined as

$$\text{Vega Numerical} = V(\sigma_{up}) - V(\sigma)$$

where $\sigma_{up} = \sigma + 1\%$ — the current volatility adjusted upwards by 1%.

Volga and Vanna

The Volga measures the speed of the change of the Vega

$$Volga = \frac{\partial^2 V}{\partial \sigma^2} = Se^{-r_f \tau}\sqrt{\tau}N'(x)\frac{xy}{\sigma}. \tag{5.15}$$

The Volga is also called the Vomma or Volgamma — due to the definition of the Γ. The Vanna measures how the Vega changes if the spot price changes

$$Vanna = \frac{\partial^2 V}{\partial \sigma \partial S} = \frac{\partial Vega}{\partial S} = -e^{r_f \tau}N'(x)\frac{y}{\sigma}. \tag{5.16}$$

Rho

The ρ measures the change of an option's price to changes in interest rates such that

$$\rho_d = \frac{\partial V}{\partial r_d} = \phi K \tau e^{-r_d \tau} N(\phi y) \tag{5.17}$$

$$\rho_f = \frac{\partial V}{\partial r_f} = -\phi S \tau e^{-r_f \tau} N(\phi x). \tag{5.18}$$

An increase in interest rates will decrease the value of an option by increasing carrying costs. This effect is, however, outweighed by considerations of volatility, time to expiration and the price of the underlying asset.

Dual Delta and Gamma

These quantities measure the changes in option prices as the strike changes

$$Dual\ \Delta = \Delta_K = \ = \frac{\partial V}{\partial K} = -\phi e^{-r_d \tau} N(phiy) \tag{5.19}$$

$$Dual\ \Gamma = \Gamma_K = \frac{\partial^2 V}{\partial K^2} = e^{-r_d \tau} \frac{N'(y)}{K \sigma \sqrt{\tau}}. \tag{5.20}$$

5.5.3 The Gamma and Theta Relationship

From the *Black & Scholes* differential equation in (B.1) we have

$$\frac{1}{2}\sigma^2 S^2 \Gamma + (r_d - r_f) S \Delta + \Theta = (r_d - r_f) V.$$

For a delta-neutral position we have have

$$\frac{1}{2}\sigma^2 S^2 \Gamma + \Theta = (r_d - r_f) V.$$

5.5.4 Intrinsic and Time Value

The intrinsic value of an option is the defined as the maximum of zero and the value it would have if it were exercised immediately. It is thus the gain (if any) if the option were exercised now. European options, in theory, do not have an intrinsic value until the expiry date. It is however still meaningful to know this for European options. We define the intrinsic value to be

$$\mathrm{IV} = \phi \max\left[0, (S - K)\right] \tag{5.21}$$

which is the same as the payoff function given in Eq. 4.2.

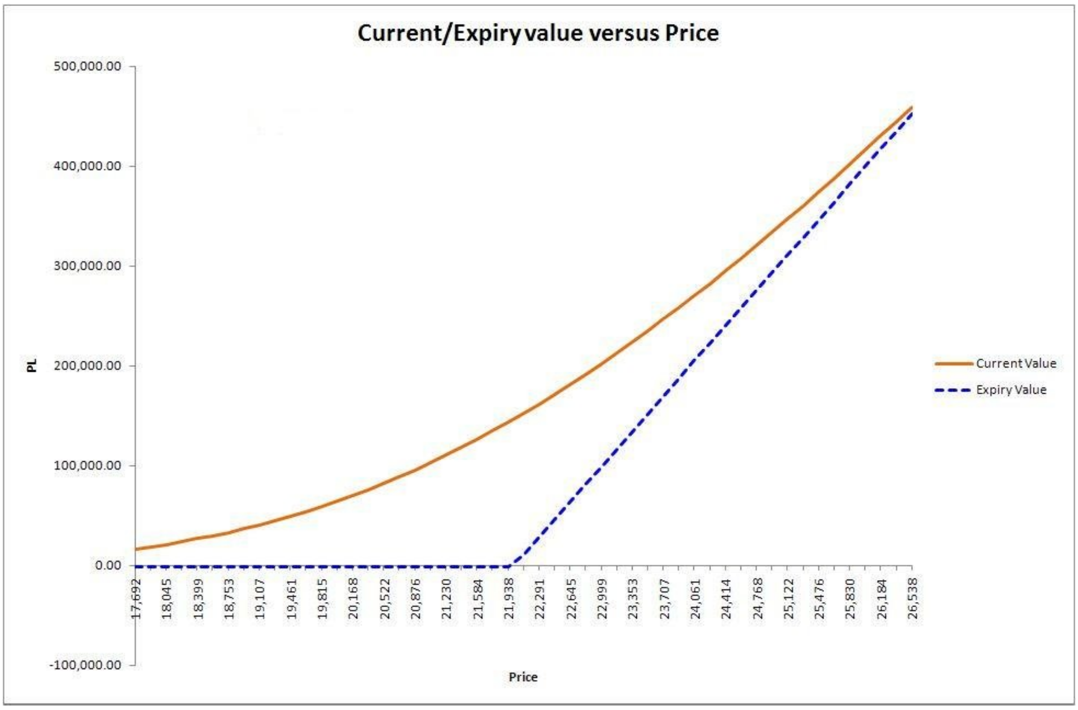


Figure 5.4: A call option and its payoff profile.

In Figure (5.4) we show the value of a call together with its payoff profile (intrinsic value). What happens if time passes by? The value of an option can be thought of as the sum of its intrinsic value and time value or the time value is difference between the option value and the intrinsic value

$$\text{Time Value} = V(S,T) - \text{IV} \ \geq 0. \tag{5.22}$$

The uncertainty surrounding the future value of the underlying asset means that the option value is generally different from the intrinsic value. There are two factors affecting time value: time remaining until expiration and how close to the money is the strike price.

Option sellers (writers) collect time value premium when they establish a position — time value that is paid by option buyers. The option seller benefits from the passage of time while the buyer gets ravaged by it.

If we look at the graph of time value versus time, we see that options loose value as time progresses — we say "time value decays". This is shown in Figures (5.5). One important dynamic of time value decay is that the rate is not constant. The decay is small when one is far from expiry but it accelerates the closer one gets to expiry - the rate of time value decay increases. This means that the amount of time premium disappearing from the option's price per day gets greater with each passing day which is clearly seen in Fig. 5.5. From this we see that TV $\rightarrow$ IntrinsicValue as $t \rightarrow 0$ which is shown in Fig. 5.6.

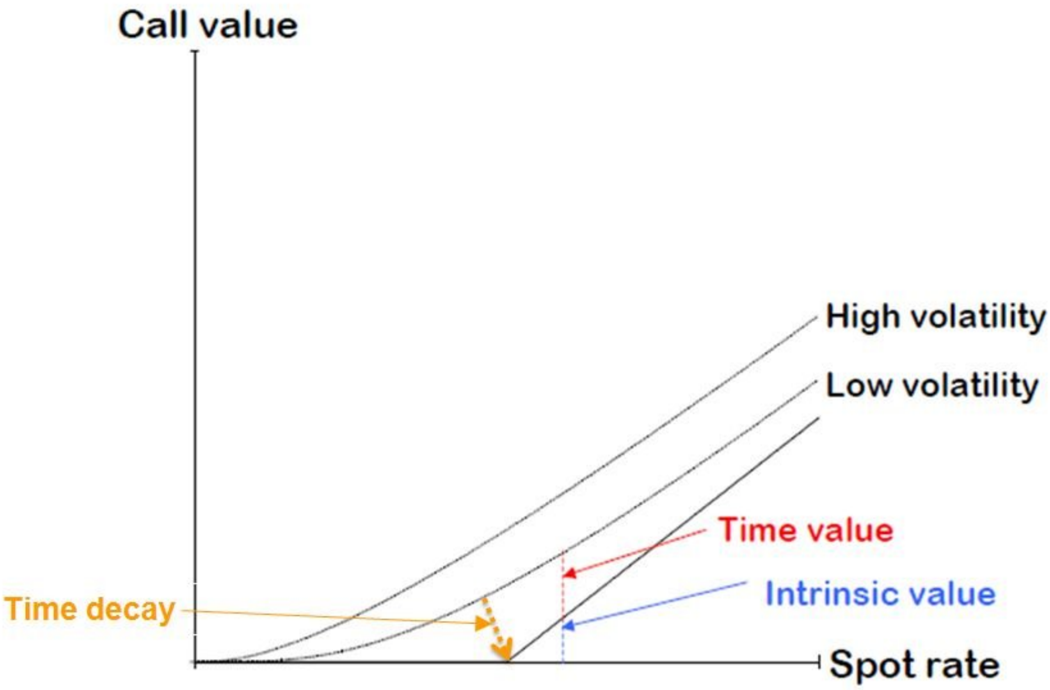


Figure 5.5: Time decay of an option.

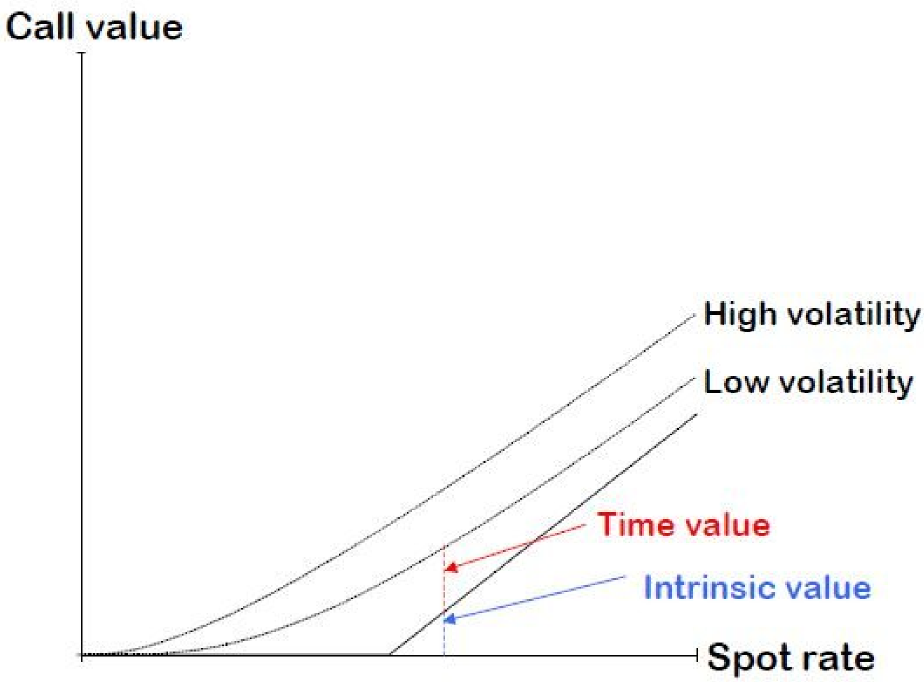


Figure 5.6: Time decay and intrinsic value of an option.

The time value is also a factor used in the decision of whether to exercise an American option, whether to hold on to it or to sell it. People who buy warrants should remember this. If the option is out-the-money, it is in general not worth your while in keeping it if there are less than 3 months to expiry.

Time value decay is always on the side of the option seller. Consider this: Markets only "trend" 1/3 of the time; they move sideways the other 2/3's. Studies have also shown that options expire worthless over 80% of the time. Being in the camp that wins 80% of the time is certainly a good place to be — there are risks though! Why are warrants so extremely successful across the globe?

5.6 Some more Basic Concepts

Note the following concepts

- Option at-the-money: The option is at-the-money when the strike and the forward rate are the same. The Delta = 50%.
- Option in-the-money: The option is in-the-money when
  - The outright is above the strike price for a call option.
  - The outright is below the strike price for a put option.

  The Delta will be between 50-100%.
- Option out-the-money: The option is out-the-money when
  - The outright is below the strike price for a call option.
  - The outright is above the strike price for a put option.

  The Delta will be between 0-49%.

5.7 Useful Relationships

5.7.1 Identities

The following identities are very useful when we do differentiation in calculating the theoretical Greeks for *Black & Scholes* -type formulas [Wy 06]

$$\begin{aligned}
\frac{\partial x}{\partial \sigma} &= -\frac{y}{\sigma} \\
\frac{\partial y}{\partial \sigma} &= \frac{x}{\sigma} \\
\frac{\partial x}{\partial r_d} = \frac{\partial y}{\partial r_d} &= \frac{\sqrt{\tau}}{\sigma} \\
\frac{\partial x}{\partial r_f} = \frac{\partial y}{\partial r_f} &= -\frac{\sqrt{\tau}}{\sigma} \\
Se^{-r_f\tau}N'(x) &= Ke^{-r_d\tau}N'(y)
\end{aligned} \tag{5.23}$$

Some identities also hold for the cumulative normal distribution function

$$\begin{aligned}
\frac{\partial N(\phi x)}{\partial S} &= \frac{\phi N'(\phi x)}{S\sigma\sqrt{\tau}} \\
\frac{\partial N(\phi y)}{\partial S} &= -\frac{\phi N'(\phi y)}{S\sigma\sqrt{\tau}} \\
\frac{\partial N'(x)}{\partial x} &= -xN'(x).
\end{aligned} \tag{5.24}$$

5.7.2 Symmetries

In Eq. 5.3 we wrote down the Put-Call-Parity relationship. There is also a Put-Call value symmetry for puts and calls with different strikes such that (where we use the same notation as set out in Eq. 4.8)

$$\begin{aligned}
C(S,K,\tau,\sigma,r_d,r_f) &= \frac{K}{Se^{(r_d-r_f)\tau}}P\left(S, \frac{\left(Se^{(r_d-r_f)\tau}\right)^2}{K}, \tau, \sigma, r_d, r_f\right) \\
\Rightarrow C(S,K,\tau,\sigma,r_d,r_f) &= \frac{K}{F}P\left(S, \frac{F^2}{K}, \tau, \sigma, r_d, r_f\right).
\end{aligned} \tag{5.25}$$

The symmetry for puts and calls on a forward or futures contract is quite simple

$$C(F,K,\tau,\sigma,r_d,r_f) = P(F,K,\tau,\sigma,r_d,r_f). \tag{5.26}$$

Another useful symmetry between puts and calls is given by [Ha 07]

$$C(S,K,\tau,\sigma,r_d,r_f) = P(-S,-K,\tau,-\sigma,r_d,r_f). \tag{5.27}$$

Let's say we wish to measure the value of the underlying in a different unit. That will effect the option value. The new option price can be calculated if we use the following state space transformation (also known as *space homogeneity*) [Ha 07, Wy 06]

$$a \times V(S, K, \tau, \sigma, r_d, r_f, \phi) = V(a \times S, a \times K, \tau, \sigma, r_d, r_f, phi)\ \forall\ a > 0 \tag{5.28}$$

where a is some constant.

From Eqs. 5.27 and 5.28 we deduce the follwing

$$\begin{aligned} C(S, K, \tau, \sigma, r_d, r_f) &= -P(S, K, \tau, -\sigma, r_d, r_f) \\ P(S, K, \tau, \sigma, r_d, r_f) &= -C(S, K, \tau, -\sigma, r_d, r_f). \end{aligned} \tag{5.29}$$

This is also know as "put-call-supersymmetry".

The symmetries mentioned here simplify coding and implementation of option pricing calculators.

5.7.3 Put-Call-Delta Parity

If we differentiate the put-call-parity relationship in Eq. 5.3 with respect to S we get

$$\Delta_C = \Delta_P + e^{-r_f \tau}. \tag{5.30}$$

For an option on a future we get from Eq. 5.5

$$\Delta_C = \Delta_P + e^{-r_d \tau}. \tag{5.31}$$

Now, look at the *space homogeneity* relationship in Eq. 5.28, and we differentiate both sides with respect to a and we then set $a = 1$, we get [Wy 06]

$$V = S\Delta + K\Delta_K \tag{5.32}$$

where Δ is the ordinary Delta and Δ_K is the dual delta defined in Eq. 5.20. This again can help in simplifying coding and option calculators. They also help in double checking or verifying the Greek numbers. These homogeneity methods can easily be extended to other more complex options.

5.7.4 Symmetries for Currency Options

By combining the Rho Greeks given in Eqs. 5.18 and 5.18 we obtain the *rates symmetry*

$$\begin{aligned} \frac{\partial V}{\partial r_d} + \frac{\partial V}{\partial r_f} &= -\tau V \\ \Rightarrow \rho_d + \rho_f &= -\tau V. \end{aligned} \tag{5.33}$$

We also have *foreign-domestic-symmetry* given by

$$\frac{1}{S}V(S,K,\tau,\sigma,r_d,r_f,\phi) = KV\left(\frac{1}{S},\frac{1}{K},\tau,\sigma,r_d,r_f,-\phi\right) \tag{5.34}$$

or

$$\frac{1}{S}C(S,K,\tau,\sigma,r_d,r_f) = KP\left(\frac{1}{S},\frac{1}{K},\tau,\sigma,r_d,r_f\right). \tag{5.35}$$

This equality is one of the faces of put-call-symmetry. The reason is that the value of an option can be computed both in a domestic as well as foreign scenario.

Chapter 6

American Options

American options are not common in the foreign exchange market. However, understanding them helps any trader. Pricing options using using numerical procedures like trees or lattices are very useful and much more generally applicable than the *Black & Scholes* formula — they can value a much broader range of options.

6.1 Introduction

A European option can only be exercised at expiry while an American option can be exercised at any point in time. An American option thus have an extra degree of freedom. Intuitively one should think that an American option should be more expensive than a European one due to this beneficial right to exercise. But, with this right also comes a headache: when should you exercise? If one can determine the point of optimal exercise, one should also be able to calculate the price of an American option. In this chapter we focus on American options and their pricing. In the last part we introduce tree models. These are very important because such models are used to price many exotic options.

6.2 Pricing American Options

Lots of papers have been written by many researchers who investigated the early exercise of American options. This has stimulated research into their pricing. There is as yet no closed-form solution to this problem. Many numerical techniques have been proposed: lattices, Monte Carlo simulation and finite difference techniques. There is always a trade off — speed versus accuracy.

The latest research point us to an optimal early exercise boundary. An exercise boundary is a time path of critical stock prices at which early exercise occurs. The optimal exercise boundary of an American option is not known ex ante and must be determined as part of the solution to the valuation problem. This leads to an analytic approximation. By doing this one can show that an American option is the sum of a European option and a term reflecting the premium one pays for the right to exercise an option early.

Methods to price American options are:

- Binomial tree (we discuss this later on)
- Optimised binomial trees
- Binomial trees with Richardson extrapolation
- Trinomial tree
- Method of lines
- Finite differences
- Barone-Adesi an Whaley (BAW) quadratic approximation
- Randomisation methods
- Analytic approximation: early exercise boundary

6.3 Optimal Early Exercies

Let $V(S,t)$ be the value of an American option and let $P(S,t)$ be the payoff for early exercise. It is possibly time-dependent and then we have the no-arbitrage constraint

$$V(S,t) \geq P(S,t) = \max[0, \phi(S-K)]. \tag{6.1}$$

that must hold everywhere. At expiry we have $V(S,T) = P(S,T)$.

It can now be shown, in the absence of dividends, that the value of an American call is the same as that of an European call[1]. The intuition behind this is that if we exercise early, we loose the insurance offered by the call and the interest earned on the strike K without gaining anything[2] (stock does not pay dividends). Dividends are the determining factor.

When dividends are expected, it can be optimal to exercise an American call. It is sometimes optimal to exercise an American call immediately prior to an ex-dividend date. This is due to the fact that the dividend will cause the share price to jump down making the option less attractive. It can also be shown that in most circumstances,

[1] *McMurray* and *Yadav*, however, found through empirical studies of the London market, that the early exercise premium for calls is sometime economically significant even if there are no expected dividends [MY 94].

[2] Think about the arbitrage free portfolio: you are long a call but to hedge one must sell the stock. When one sells the stock, one can earn the interest by investing that in the money market.

the only time that needs to be considered for the early exercise of an American call is the final ex-dividend date. Furthermore, if

$$D_n \leq K \left(1 + e^{-r(T-t_n)}\right) \tag{6.2}$$

and

$$D_i \leq K \left(1 + e^{-r(t_{i+1}-t_i)}\right) \tag{6.3}$$

where D_i is the i-th dividend with ex-dividend date t_i and $i = 1, 2, \ldots, n$, we can be certain that early exercise is never optimal.

Puts are different creatures all together. It can be optimal to exercise a put on a non-dividend paying stock early. A put option should always be exercised early if it is sufficiently deeply in-the-money. The intuition is that we loose the insurance but we can invest the strike proceeds at the riskfree rate[3].

Dividends, on the other hand, will tend to reduce the likely hood of early exercise. In general, the early exercise of a put becomes more attractive as S decreases, as r increases, as σ and transaction costs decrease. The probability of early exercise for a put should depend directly on the extend to which the option is in-the-money, directly on the time to expiry and negatively on the magnitude of the dividend on the last LDR date. The most likely time to exercise an American put is immediately after the last LDR date. Because there are certain circumstances when it is desirable to exercise an American put early, it must always be more valuable than the corresponding European put. But, since an American put is sometimes worth its intrinsic value, it follows that a European put must sometimes be worth less than its intrinsic value. This is shown in Figure (6.1).

6.4 The Early Exercise Boundary

In the previous Section showed that dividends are, in general, the deciding factor in deciding to exercise early. But, isn't there a more analytic way to calculate when exercise is optimal? This can be done by calculating the early exercise boundary. But, the early exercise boundary for the American put approaches the strike price at expiry with infinite velocity. This causes difficulties in developing efficient and accurate numerical procedures and consequently trading strategies, during the volatile period near expiry.

Huang, *Subrahmanyam* and *Yu* extended earlier work and provided a framework that is based on a recursive computation of the early exercise boundary. They showed that once the early exercise boundary is estimated, option values and hedge parameters can be obtained analytically. They concluded that the expression for the price

[3]Again think about the riskless portfolio: you are long a put and also long the stock. If the option is deep in the money you can exercise, sell the stock at K and invest the proceeds in the money market.

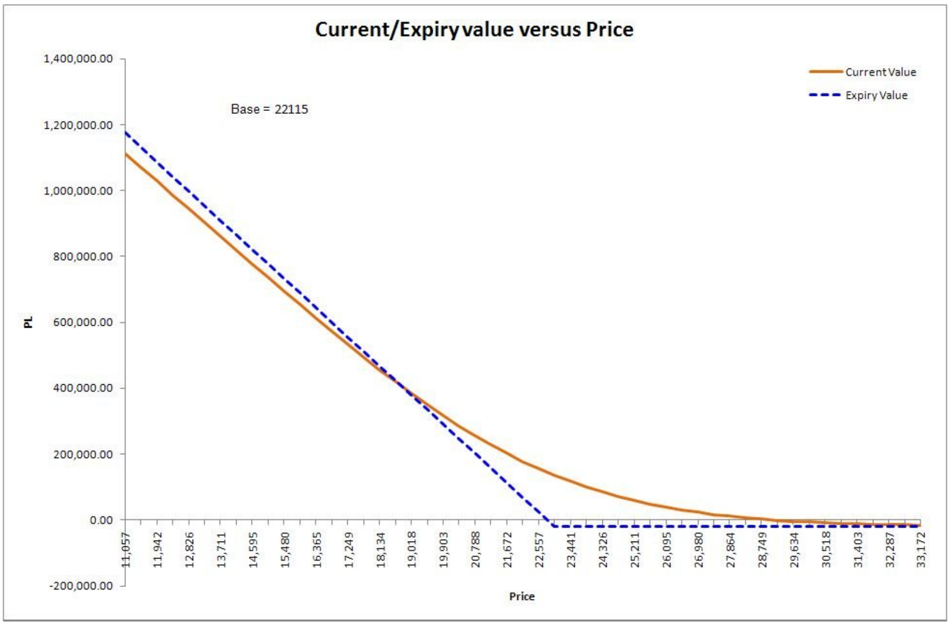


Figure 6.1: The value of an European put can be less than its intrinsic value.

of an American put is given by [HS 96]

$$P_0 = p_0 + \int_0^T \left[rKe^{-rt} N\left(-d_2(S_0, B_t, t)\right) - \alpha S_0 e^{-\alpha t} N\left(-d_1(S_0, B_t, t)\right)\right] \, dt \tag{6.4}$$

where $\alpha = r - b$, and b is the proportional cost of carry[4]. Further, p_0 is the price of a European put and

$$\begin{aligned} d_1(x, y, t) &= \frac{\ln(x/y) + (b + \sigma^2/2)t}{\sigma\sqrt{t}} \\ d_2(x, y, t) &= d_1 = \sigma\sqrt{t}. \end{aligned}$$

The time t optimal point on the early exercise boundary B_t satisfies the following equation

$$\begin{aligned} K - B_t &= rKe^{-r\tau_1} N\left(-d_2(B_t, K, \tau_1)\right) - \alpha B_t e^{-\alpha\tau_1} N\left(-d_1(B_t, K, \tau_1)\right) \\ &+ \int_t^T \left[rKe^{-r\tau_2} N\left(-d_2(B_t, B_s, \tau_2)\right) - \alpha B_t e^{-\alpha\tau_2} N\left(-d_1(B_t, B_s, \tau_2)\right)\right] ds \end{aligned} \tag{6.5}$$

where $\tau_1 = T - t$ and $\tau_2 = s - t$. Note that the boundary in Eq. (6.6) is independent of the underlying security price S. They also showed that the Greeks are equal to their European counterparts plus a term that depends on the early exercise boundary. The above framework can be implemented numerically quite efficiently.

[4]Actually, $b = r - d$ and thus $\alpha = d$.

The last two terms in Eq. (6.4) represent the early exercise premium. More specifically, the second term is the present value of the benefits from prematurely exercising the American put through the interest gained by receiving the exercise proceeds early. The last term captures the cost associated with the early exercise decision through the loss of insurance value, i.e., the cost of taking a short position on the underlying stock following exercising the put early.

In practise, American options are seldom exercised and traders would rather sell them. Studies have shown that traders have lost great deals of money by not exercising options at their optimal point [GS 85, MY 94].

In South Africa most OTC options and derivative structures are European in nature. Look at all the warrants listed, all the calls are American but, all the puts are European! Why? Because even the big international option writers are weary of American puts.

6.5 American versus European Options

Merton showed in 1973 that, in the absence of any dividend payments, an American call will never be exercised prior to expiration [Me 73]. In this context an American call is equivalent to a European call and the *Black & Scholes* formula can be used to price it. The intuition is simple: in the absence of dividends, the option is worth more 'alive' than 'dead' because of its time value. Killing the option would mean pocketing its intrinsic value while losing its speculative value. The investor is better off in selling the option rather than killing it. Remember, this does not hold for American puts.

Applying similar arguments to American options on a futures contract, one conclude that it is never optimal to exercise them. This holds for both puts and call. Here, as well, the investor is better off selling the option rather than exercising it. The *Black & Scholes* equation can thus be used in valuing them.

Traders in the agricultural futures market sometimes behave strangely. They do sometimes exercise American options on agricultural futures early. Why? This might be due to delivery concerns regarding the underlying commodity.

Most exchange traded options on futures are American in nature. Due to the previous arguments they are priced using the *Black & Scholes* formula. Now, note the following: if it is never optimal to exercise an option, the time value should always be positive meaning the option value can never be below the intrinsic value. From 5.22 we know that

$$\text{Time Value} \geq 0. \tag{6.6}$$

However, options that far in-the-money behave strangely. Using the *Black & Scholes* formula we do sometimes get a negative time value meaning the option value curve crosses the intrinsic curve. This is an abberation and solely due to the *Black & Scholes* model that prices these option incorrectly. If you use the *Black & Scholes* model impose the following limits

$$V(S,\tau) = \phi \max[0,(S-K)] \;\; iff \;\; V(S,\tau) < \phi \max[0,(S-K)]\,. \qquad (6.7)$$

6.6 Binomial Trees

Lattice methods use discrete-time and discrete-state approximations to *Black & Scholes* -type differential equations[5] to compute derivative prices. These methods are iterative in nature and easy to explain and implement.

The binomial tree is one type of lattice we can use to value an option. We use the same assumptions that *Black & Scholes* used — we thus live in a *Black & Scholes* world. We also use the risk-neutral valuation of *Cox* and *Ross* [CR 85]. One of the assumptions *Black & Scholes* made was that the underlying asset is traded on a continuous basis and that delta hedging is done on a continuous basis — returns are normally distributed (see §4.9). The discrete version of the normal distribution is the binomial distribution. In 1976 *Cox*, *Ross* and *Rubinstein* realised this and constructed a tree based method to value derivatives. Stock returns are assumed to be governed by a discrete probability measure; in this case the binomial distribution. The binomial model breaks down the time to expiration into potentially a very large number of time intervals, or steps. Using probability theory, a tree of stock prices is produced working forward from the present to expiration. This is graphically shown in Fig. 6.2. The consequence of this methodology is that, at expiry of an option, we have a discrete set of possible stock prices. We can now use these prices to value any derivative security. We are doing what Louis Bachelier stated more than 100 years ago. In his famous dissertation he mentioned that "you have to 'model' spot price movements' before you 'model' option values [We 06]. He realised that the unknown spot price in the future is just a scaled version of the current spot price.

We will later mention the tri-nomial tree which is an optimisation or extention of the binomial tree.

6.6.1 The Stock Price Tree

We start by dividing the time to maturity, τ, into N periods of equal length where $\Delta t = \tau/N$.

We start by making a very restrictive assumption. Let's start with a stock price S and we assume that after a small time interval Δt the price can take two values only: it can either move up to a new price of uS or down to dS (where u and d are numbers such that $u > d$). We also assume that the transition probability of moving to uS is p and to dS, $(1-p)$.

[5] Like the one we have in Eq. B.1.

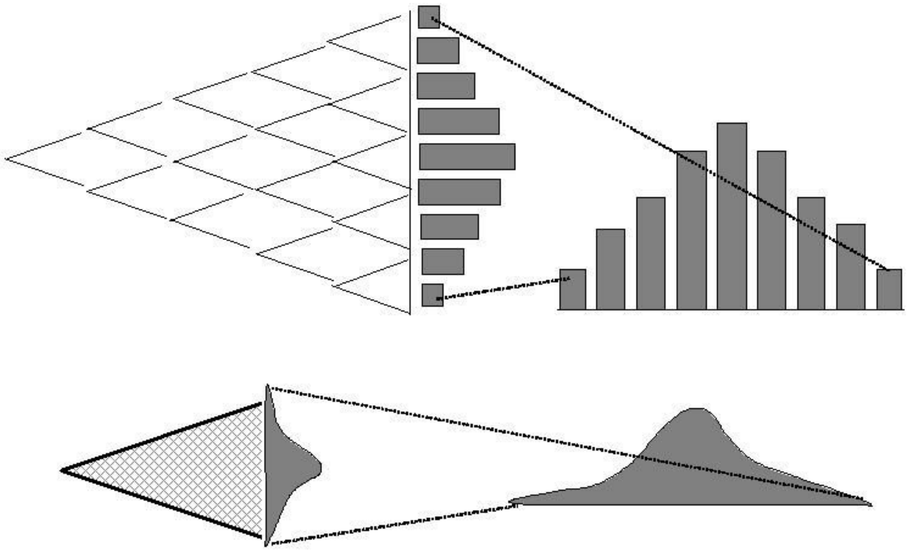


Figure 6.2: The binomial distribution is the discrete version of the normal distribution.

Extending this, we also restrict the new price uS such that it can only move up to a price uuS or down to a price udS. The same holds for the new price dS which can also only move up to a price duS or down to ddS. Such a two step tree is shown in Fig. 6.3.

The parameters u, d and p cannot be chosen arbitrarily; *they must give correct values for the mean and variance of the change in S during Δt* as determined by the binomial distribution. The parameters are obtained by using the risk-neutral argument which states that the expected return on all securities must be the riskfree interest rate. Now, let the stock price be governed by a Wiener process such that

$$\Delta S = rS\Delta t + \sigma S\epsilon\sqrt{\Delta t} \tag{6.8}$$

where ϵ is a number from a standard normal distribution. Here the return is given by rS and the standard deviation by $\sigma S\Delta t$ [Hu 06]. Note that Eq. 6.8 is the discrete version of Eq. 4.3.

In a risk-neutral world $\sigma = 0$ and $\Delta S = rS\Delta t$ which is a differential equation with solution $Se^{r\Delta t}$. This means that at the end of Δt the price of the stock has increased to an amount $Se^{r\Delta t}$; the stock price has thus earned interest at the risk-free rate r. From statistical arguments the expected value of S after Δt is

$$E[S(\Delta t)] = S[pu + (1-p)d].$$

Thus

$$e^{r\Delta t} = pu + (1-p)d. \tag{6.9}$$

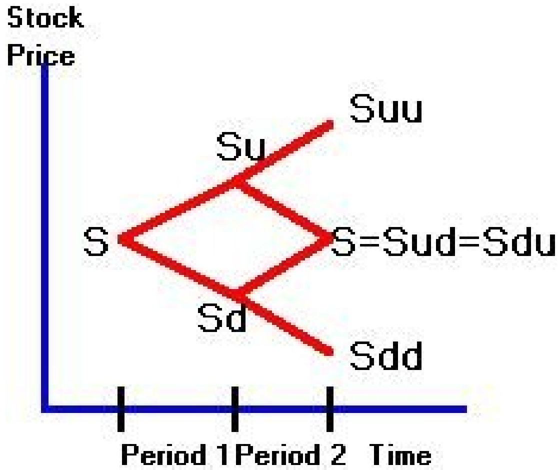


Figure 6.3: A two step/period binomial tree.

Provided Δt is small the variance of the change of the stock price during Δt is $\sigma^2 S^2 \Delta t$. The variance of S is also given by $\mathrm{Var}(S) = E[S^2] - (E[S])^2$ such that

$$\sigma^2 \Delta t = pu^2 + (1-p)d^2 - [pu + 1 - p)d]^2. \tag{6.10}$$

Equations (6.9) and (6.10) can be used to obtain the transition probabilities u, d and p; but we still need another equation. We currently have 2 equation in 3 unknowns. We need three equations in three unknowns to solve all 3 variables. The third condition is usually taken as

$$u = \frac{1}{d}. \tag{6.11}$$

The last three equations are solved and give

$$\begin{aligned} p &= \frac{a-d}{u-d} \\ u &= \frac{1}{2a}\left[a^2 + b^2 + 1 + \sqrt{(a^2+b^2+1)^2 - 4a^2}\right] \simeq e^{\sigma\sqrt{\Delta t}} \\ d &= \frac{1}{u} \end{aligned} \tag{6.12}$$

where

$$\begin{aligned} a &= e^{r\Delta t} \\ b^2 &= a^2(e^{\sigma^2 \Delta t} - 1). \end{aligned} \tag{6.13}$$

The stock price is now considered at times t (that is now), $t + \Delta t, t + 2\Delta t, \ldots, t + i\Delta t, \ldots, t + N\Delta t$. After the first period the stock price of uS can also move up or down with a certain probability. The stock price thus evolves according to a lattice or tree such as that shown in Fig. (6.4). Here we have $S = 100$, $K = 95$, $\tau = 0.5$, $r = 8\%$, $\sigma = 30\%$ and $N = 5$.

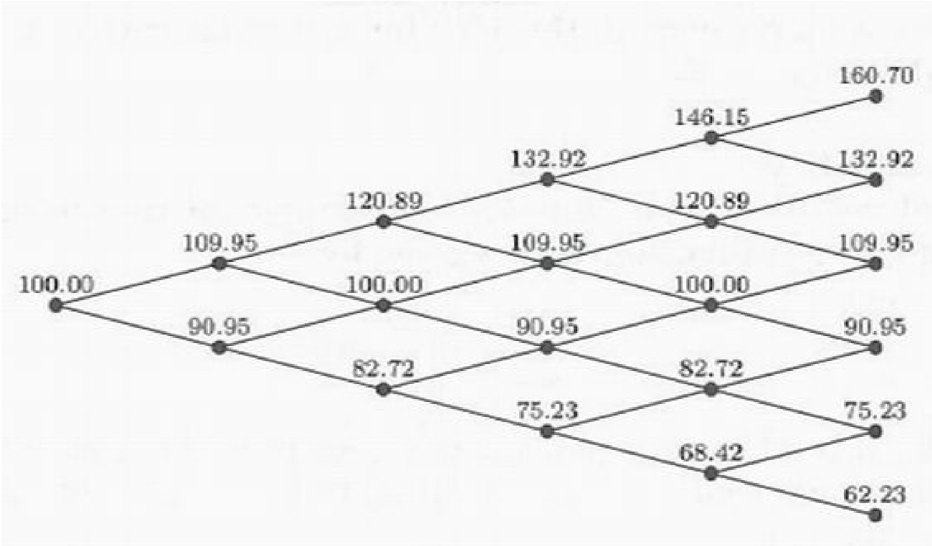


Figure 6.4: Five step tree of a nondividend-paying stock [Ha 07].

The tree consists of nodes which we can index as (i, j) where $i = 0, 1, 2, \ldots, N$ and $j = 0, 1, 2, \ldots, i$. The nodes on the lattice correspond to stock prices

$$S_{i,j} = Su^j d^{i-j} \tag{6.14}$$

which can be deduced directly from the binomial distribution. Note that the tree recombines in the sense that an up movement followed by a down movement leads to the same asset price as a down movement followed by an up movement. This means we need

$$ud = 1.$$

Once the tree of stock prices has been determined from t to expiry T, we can value options on the stock. We show a 5 step tree in Fig. (6.5).

6.6.2 Valuing Options on Currencies

For currencies, S is the FX rate. The derivative security is evaluated by starting at the time T and working backwards through the tree — this is called dynamic programming. We assume that the value of the underlying is known at T for all of the $N+1$ nodes. At the end of the tree - at expiration of the option - all the terminal option prices for each of the final possible stock prices are known — they simply equal their intrinsic values.

To see how this work we start with the simplest situation in which there is only one period left till expiration. Let C be the current value of a call on the currency with a current value of S. Also let C_u be the price of the call if the FX rate goes to uS and C_d if it goes to dS. Now, at expiry, we know that $C_u = \max[0, uS - K]$ and

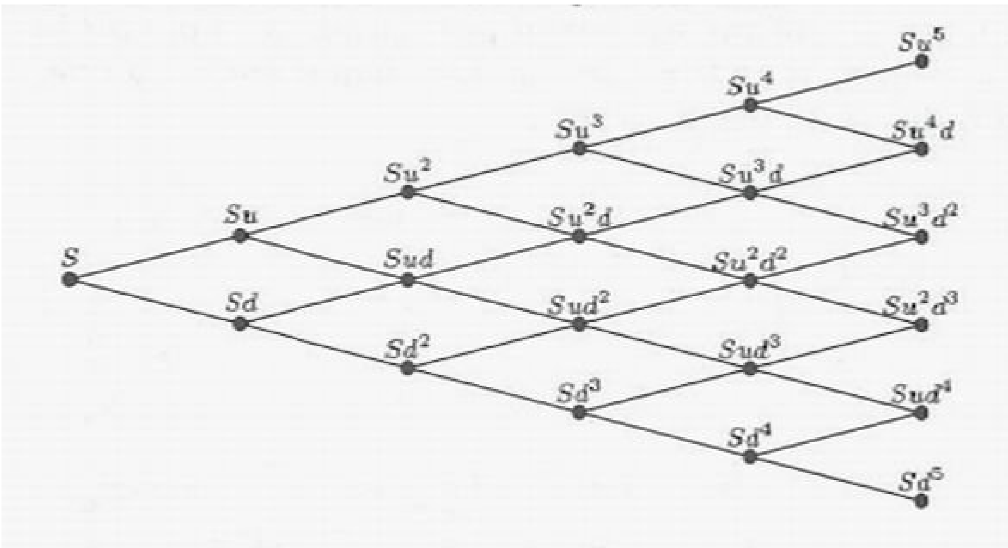


Figure 6.5: Five step tree [Ha 07].

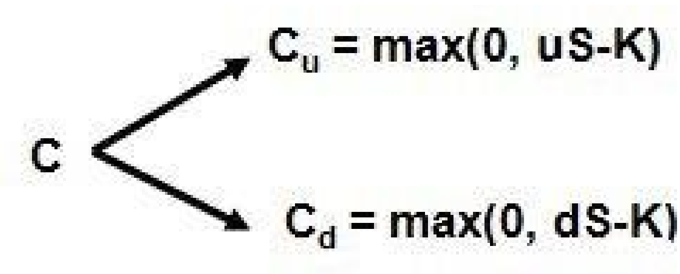


Figure 6.6: Terminal option values in a one step tree.

$C_d = \max[0, dS - K]$ with K the strike FX rate — stated differently, we know the option prices at time T (the expiry time). This is shown in Fig. 6.6.

Next, the option prices at each step of the tree are calculated working backward from expiration to the present. Because a risk-neutral world is being assumed, the value at each node at time $T - \Delta t$ can be calculated as the expected value at time T, discounted at rate r_d for a time period Δt [Hu 06]. This can also be seen if we set up a riskless hedge similar to *Black & Scholes* as described in §4.10.2. If you are trying to create a hedged portfolio Π consisting of a long a call and shorting a number x of the underlying currency, you should have

$$\Pi = +C - xe^{r_f \Delta t} S.$$

Why do we have $xe^{r_f \Delta t}$ in this equation? One has to consider all cash flows associated with the underlying asset. If the asset is a share, there are dividends to consider. A currency is similar to a bond, we always earn interest on it. In this case the foreign riskfree interest rate r_f. Now, what is the definition of a hedge?

One would be successfully hedged if the outcome is the same whether the spot price S goes up or down.

This implies that at expiration we should have

$$C_u - xe^{r_f \Delta t} uS = C_d - xe^{r_f \Delta t} dS$$

where x is the number of shares to trade in order to obtain a perfect hedge — see §4.10.2. Rearranging this we have [We 06]

$$x = e^{-r_f \Delta t} \frac{C_u - C_d}{uS - dS}. \tag{6.15}$$

If we look at the structure of Eq. 6.15 and we remember the definition of the Δ:

Δ = the change in option value with respect to changes in the price of the underlying asset

Equation 5.5 is the continuous version of this definition while Eq. 6.15 is the discrete or numerical version. We can thus state that $x = \Delta$. This is very important where we state in general:

Δ gives the number of underlying stock to trade in order to hedge the option

This is what we described in §4.12.1 where we discussed Delta hedging.

Also remember that this replicating portfolio must be self-financing, which means you neither consume from it nor add money to it beyond an initial loan or deposit which should be the same as the value of the portfolio. We thus have the loan is worth Π. This loan is a loan in the domestic currency. Further, taking on the *Black & Scholes* argument that if one is able to construct a riskless hedge, this portfolio ought to earn the domestic riskfree interest rate we should have

$$\begin{aligned} +C - \Delta e^{r_f \Delta t} S &= \Pi \\ C_u - \Delta e^{r_f \Delta t} uS = C_d - \Delta e^{r_f \Delta t} dS &= \Pi e^{r_d \Delta t}. \end{aligned}$$

This means that portfolio is worth Π today but it is worth $\Pi e r_d \Delta t$ at expiry. Solving these two equations for C and remembering the definition for p given in Eq. 6.13 we get

$$C = e^{-r_d \Delta t}[pC_u + (1-p)C_d]. \tag{6.16}$$

Please note that, due to the domestic and foreign interest rates in the equations above, our quantity a in Eq. 6.14 is now defined as

$$a = e^{(r_d - r_f)\Delta t}. \tag{6.17}$$

For currencies we use p, u and d as defined in 6.13.

The procedure discussed sofar was for a one step process. This is now generalised such that the option prices at each step are used to derive the option prices at the previous step of the tree by the process of backward induction as described above. We walk backwards through the tree starting at expiry. If we have a N-period (or step) tree, we first determine the value of the options at expiry such that at time T

$$C_{N,j} = \max[0, Su^j d^{i-j} - K] \quad (6.18)$$
$$P_{N,j} = \max[0, K - Su^j d^{i-j}]. \quad (6.19)$$

To obtain the correct put and call value we discount this back, with the probabilities p and $(1-p)$, over each Δt until we obtain $C_{0,0}$ and $P_{0,0}$ which are the correct call and put values respectively. For European options we then have in general

$$\begin{aligned} C_{i,j} &= e^{-r_d \Delta t}[pC_{i+1,j+1} + (1-p)C_{i+1,j}] \\ P_{i,j} &= e^{-r_d \Delta t}[pP_{i+1,j+1} + (1-p)P_{i+1,j}]. \end{aligned} \quad (6.20)$$

What makes the tree method so powerful is the fact that we know the option value at each node. We can thus add several conditions to alter the option price at a specific node. For instance, to price American options we stop at each node and find out whether it is optimal to exercise or note. This is done by comparing the option value to the intrinsic value. If the intrinsic value is higher, we should exercise and the option value should just be the intrinsic value. Extending Eq. 6.20 to include American option we have

$$\begin{aligned} C_{i,j} &= \max\left[Su^j d^{i-j} - K, e^{-r_d \Delta t}[pC_{i+1,j+1} + (1-p)C_{i+1,j}]\right] \\ P_{i,j} &= \max\left[K - Su^j d^{i-j}, e^{-r_d \Delta t}[pP_{i+1,j+1} + (1-p)P_{i+1,j}]\right]. \end{aligned} \quad (6.21)$$

The correct option premiums are given by $C_{0,0}$ and $P_{0,0}$.

In Fig. 6.7 we show a three step tree to value an American put on the Euro. We have the FX rate as $S = 1.05$, the strike is $K = 1.10$ with interest rates $r_d = 3.\%$ and $r_f = 5.5\%$.

6.6.3 Advantages of the Binomial Tree

The big advantage the binomial model has over the Black-Scholes model is that it can be generalised easily and one can impose a variety of conditions for which the Black-Scholes model cannot be applied. Binomial trees are extensively used to accurately price American options. This is because with the binomial model it's possible to check at every point in an option's life (i.e. at every step of the binomial tree) for the possibility of early exercise. The binomial model is also easy to adapt to value exotic options or options with no closed form solutions. Examples of such options are partial American options (mainly used in share incentive schemes), Asian options and Barrier options. This model becomes very useful when we have to incorporate the volatility smile into the pricing of options.

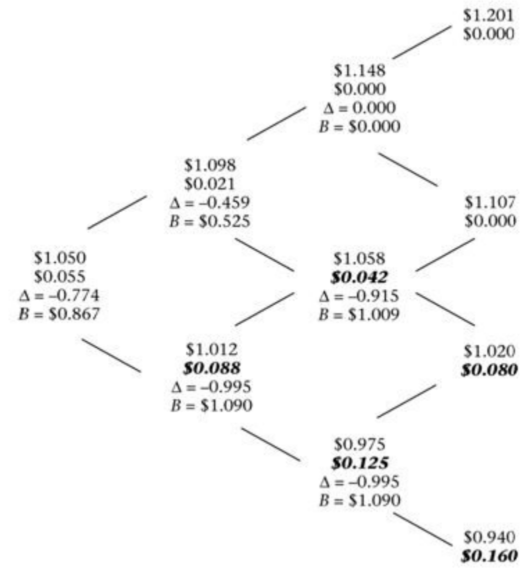


Figure 6.7: A three step tree to value an American put on the EURUSD.

6.6.4 Risk Parameters

The risk parameters are obtained numerically. To do that at time $t = 0$ we add two steps to the tree, thus $N' = N + 2$. Now $i = 0, 1, 2, \ldots, N'$ and the **correct call and put values are given by** $C_{2,1}$ **and** $P_{2,1}$ respectively. This corresponds to time $t = 0$. Thus for calls

$$
\begin{aligned}
\Delta(t=0) &= \frac{C_{2,2} - C_{2,0}}{S_{2,2} - S_{2,0}} \\
\Theta(t=0) &= \frac{C_{4,2} - C_{0,0}}{4\Delta t} \\
\Gamma(t=0) &= \frac{1}{h}\left[\frac{C_{2,2} - C_{2,1}}{S_{2,2} - S_{0,0}} - \frac{C_{2,1} - C_{2,0}}{S_{0,0} - S_{2,2}}\right] \\
h &= \frac{1}{2}(S_{2,2} - S_{2,0}).
\end{aligned}
$$

To evaluate VEGA we need to to calculate C and P at $(\sigma - \epsilon)$ and $(\sigma + \epsilon)$ where ϵ is usually 1%. The slope of the line between these two points gives the VEGA. The same procedure holds for RHO. Our example in Fig. 6.7 listed the Delta as well.

6.7 Convergence of the Binomial Model

For European options, the binomial model converges to the Black-Scholes formula as the number of binomial calculation steps increases, i.e. Δt tends to zero. In fact the Black-Scholes model for European options is really a special case of the binomial model where the number of binomial steps is infinite. In other words, the binomial

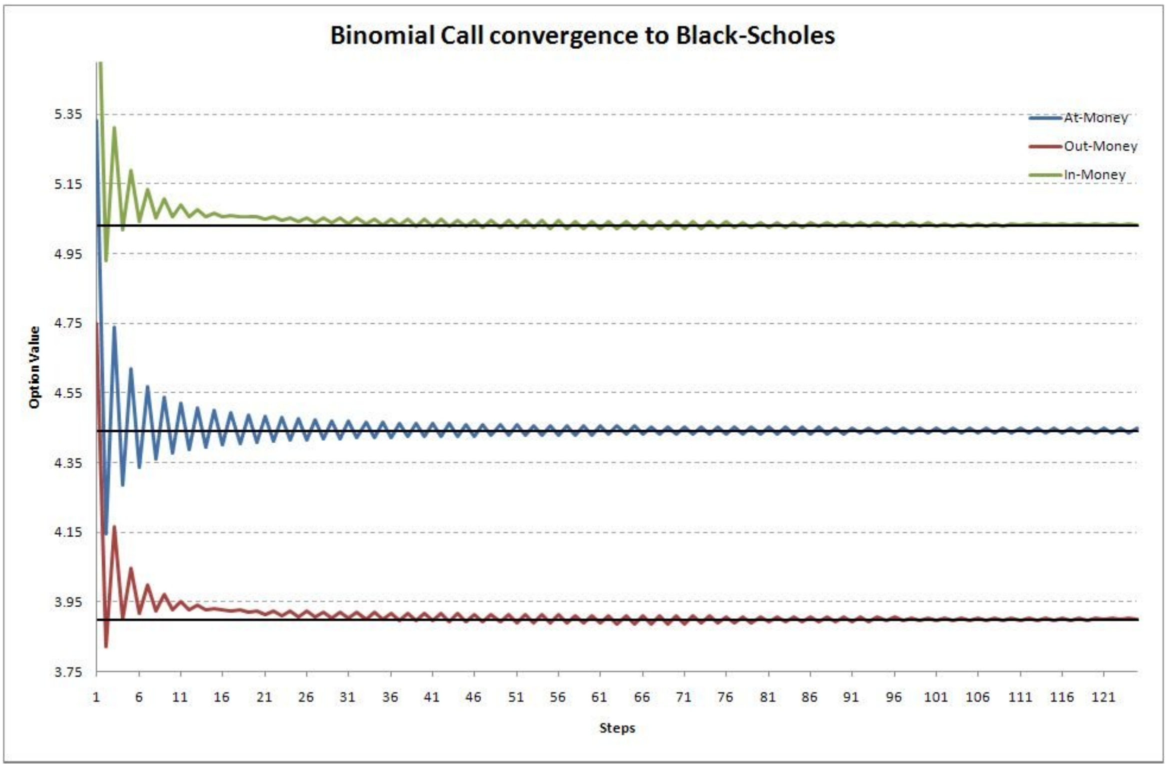


Figure 6.8: Wave-like pattern of the binomial model. We show the patterns for at-, in-, and out-the-money options.

model provides discrete approximations to the continuous process underlying the Black-Scholes model.

Beware of the convergence. It converges in a wave-like pattern as the number of steps increases. True convergence is only obtained for a large number of steps — typically $N = 5000$. This makes the binomial model slow to run. In Fig. 6.8 we show the convergence of a European USDKES call option where we have $S = 82$, $\sigma = 5\%$, $r_d = 5\%$, $r_f = 2\%$ and $T = 1$. We look at strikes $K_1 = 82$, $K_1 = 83$ and $K_2 = 81$. Note the structure of the convergence: for in- and out-the-money options, it converges from the above while for at-the-money convergence is symmetrical around the *Black & Scholes* value. This shows that we have good approximations with $N = 150$, however, the true *Black & Scholes* value is obtained with N equal to a couple of thousand. Use a first order Richardson expansion to to get quicker convergence. We do this by letting

$$V(S,K) = 2V(S,K,\tau,\sigma,r_d,r_f,N,\phi) - V(S,K,\tau,\sigma,r_d,r_f,N1,\phi) \tag{6.22}$$

where N is the number of steps in tree and $N1 = \int(N/2)$. We thus do the valuation twice and you might think that this will increase the time of execution. However, N can be reduced so much to $N = 500$ meaning it is much quicker.

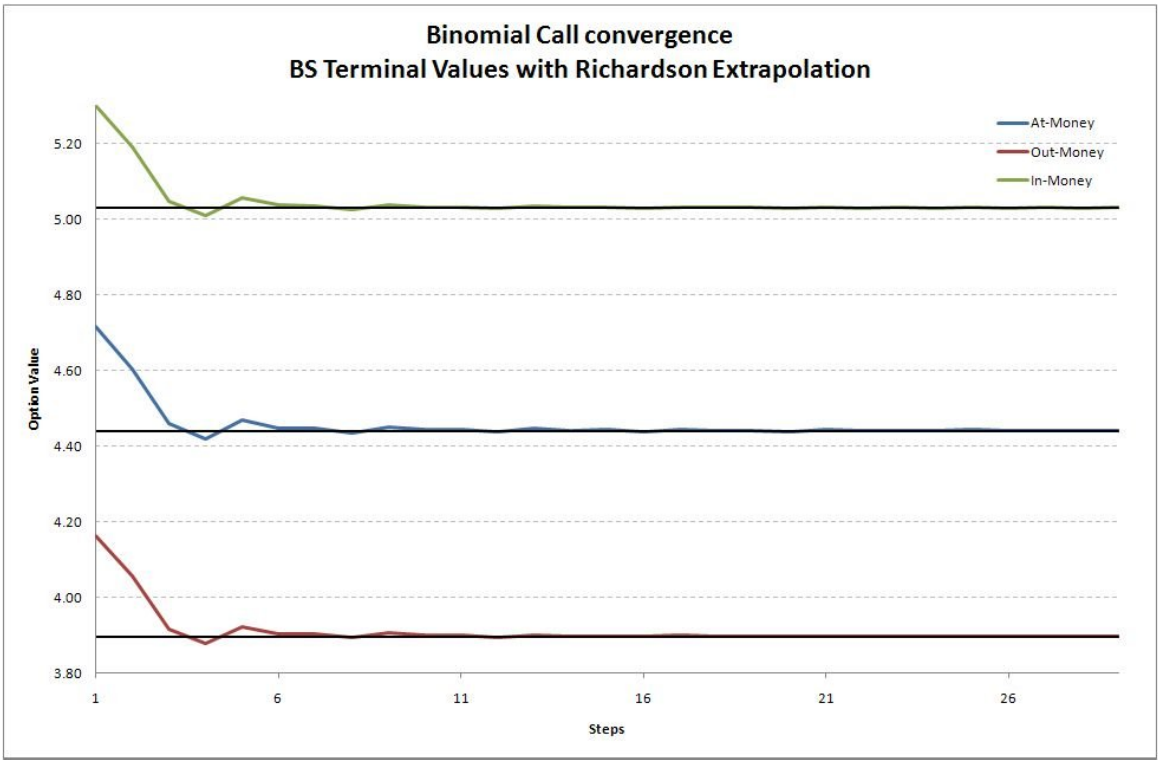


Figure 6.9: Optimising the binomial model.

Another trick, to get much quicker convergence, is to use the *Black & Scholes* formula in obtaining the expiration values. We then have from Eq. 6.19

$$V_{N,j} = V(Su^j d^{i-j}, K, \Delta t, \sigma, r_d, r_f, \phi) \tag{6.23}$$

The faster convergence using these tricks is shown in Fig. 6.9. Now, combine these two tricks and see what happens!

6.8 The Trinomial Tree

The trinomial tree procedure is an extension of the binomial option pricing model and is conceptually similar. It can also be shown that the approach is equivalent to the explicit finite difference method for option pricing. It was developed by *Phelim Boyle* in 1986 [Bo 86, Hu 06].

As one can guess by its name, the trinomial method is similar to the binomial lattice in that the stock price is modeled by a tree, but instead of two possible paths per node, the trinomial tree has three; an up, down and stable path. From any node this methodology assumes that the asset price can only go up to uS, down to dS and stay the same after a small time period Δt where, similar to the binomial model we have

$$\begin{aligned} u &= e^{\sigma\sqrt{2\Delta t}} \\ d &= e^{-\sigma\sqrt{2\Delta t}}. \end{aligned} \tag{6.24}$$

We also restrict the tree to be recombining similar to the binomial tree. We thus need $ud = 1$. The transition probabilities are given by

$$p_u = \left(\frac{e^{\frac{1}{2}(r_d - r_f)\Delta t} - e^{-\sigma\sqrt{\frac{\Delta t}{2}}}}{e^{\sigma\sqrt{\frac{\Delta t}{2}}} - e^{-\sigma\sqrt{\frac{\Delta t}{2}}}} \right)^2 \tag{6.25}$$

$$p_d = \left(\frac{e^{\sigma\sqrt{\frac{\Delta t}{2}}} - e^{\frac{1}{2}(r_d - r_f)\Delta t}}{e^{\sigma\sqrt{\frac{\Delta t}{2}}} - e^{-\sigma\sqrt{\frac{\Delta t}{2}}}} \right)^2 \tag{6.26}$$

$$p_m = 1 - p_u - p_d. \tag{6.27}$$

The tree is constructed by letting

$$S_{i,j} = S u^{\max[0,j-i]} d^{\max[0,i-j]}$$

where we note that i is the step counter going forward in time whilst j is the counter for every possible node at each time step i. We show a sample trinomial tree in Fig. 6.10. To price options we start with the terminal values given in Eq. 6.19 and work our way back through to the tree similar to what we did with a binomial tree. The option price is determined at every node such that

$$V_{i,j} = e^{-r_d\Delta t}[p_u V_{i+1,j+1} + p_m V_{i+i,j} + p + dV_{i+1,j-1}] \tag{6.28}$$

We show the convergence properties in 6.11. Note that the convergence of the trinomial tree is much smoother than that of the binomial. Convergence can further be enhanced by using Richardson extrapolation. Using the *Black & Scholes* values as terminal values do not work for the trinomial tree.

6.9 Options on Currency Futures on Safex/YieldX

The *Black & Scholes* model takes into consideration the fact that an option trader has financing costs — you either have to borrow money to buy the underlying or you deposit money if you have to sell the underlying to hedge yourself.

Safex margins futures and options. Because the margin requirements are so small relative to the value of the trade, in theory, we can disregard it. In theory we can thus buy and sell futures without any funding costs. Since the financing cost for a futures contract is zero, the option should not include this premium and its value should be

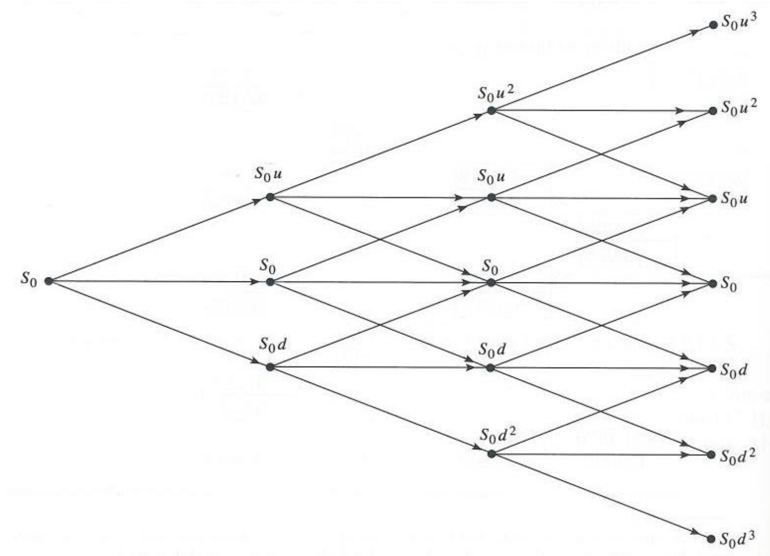


Figure 6.10: The trinomial tree [Hu 06].

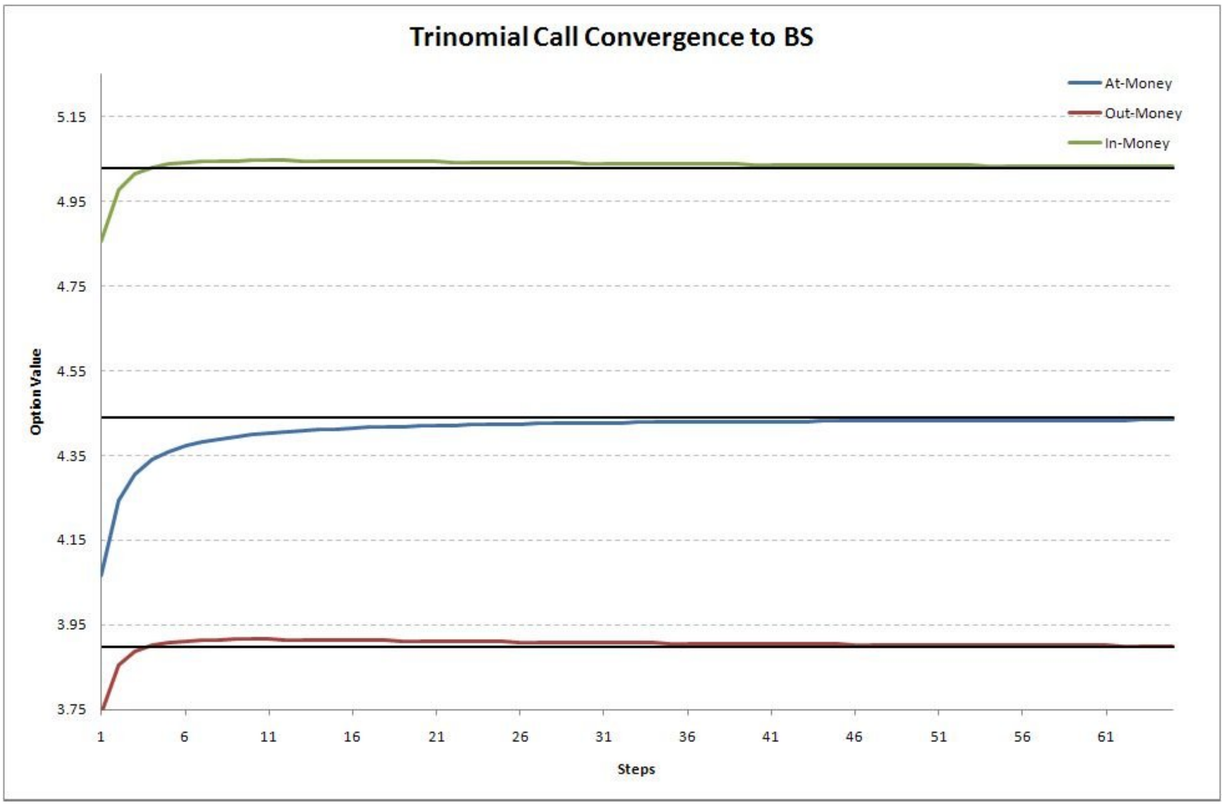


Figure 6.11: Convergence of the trinomial tree. We show the patterns for at-, in-, and out-the-money options.

lower. In practise we put $r_d = r_f = 0$ in Eq. (4.9) if we want to price a Safex option. We can then rewrite the formula as

$$V(F,t) = FN(\phi x) - KN(\phi y) \tag{6.29}$$

with F now the futures price. We see from this that there is no time value of money factor anywhere and $\rho = 0$. This means that we actually pay the premium at expiry — technically it is paid incrementally on a daily basis. To be strictly correct one also has to take interest due to the margin into account. This is not done in practise. They also impose the boundary condition as given in Eq. 6.7.

And finally, to get the right price per contract for an option on a currency, one has to multiply the option's value by the correct multiplier. The currencies that can be traded on the Yield-X platform is given in Table 6.9 together with their respective multipliers.

Currency Code	**Explanation**	**Multipliers**
USDZAR	US Dollar Rand	1,000
USDZAR	USD Maxi contract	100,000
EURZAR	Euro Rand	1,000
GBPZAR	British Pound Rand	1,000
AUDZAR	Aussie Dollar Rand	1,000
CHFZAR	Swiss Franc Rand	1,000
CADZAR	Canadian Dollar Rand	1,000
ZARJPY	Japanese Yen Rand	100,000
ZARCNY	Chinese Yuan Rand	10,000

Table 6.1: Tradable Currency futures and options in South Africa

Chapter 7

Volatility

If you ask option traders what they do, the say, "We trade 'vol'." They really trade options but they think about the volatility value embedded in those options. But what volatility are they trading because one of the stranger notions associated with volatility is that are several types of volatility? Let's unwrap volatility.

7.1 Introduction

In Chapter 5, §5.5, we mentioned that there are three uncertain parameters in the *Black & Scholes* formula: the dividends, the riskfree interest and the volatility. These have to be estimated. The most important of the three is the volatility. *Black & Scholes* highlighted this through their model. They, however, knew that volatility changes over time. As far back as 1976, *Fischer Black* wrote

> "Suppose we use the standard deviation ... of possible future returns on a stock ... as a measure of volatility. Is it reasonable to take that volatility as constant over time? I think not."

Figure 7.1 shows a graph of the annual volatility for the USDZAR using daily data. It is clear that volatility is not constant. Volatility is, however, statistically persistent. This means that volatility trends: if it is volatile today, then it should continue to be volatile. In Fig. 7.2 we show the logarithmic returns of the USDZAR and in Fig. 7.3 we show the same for the Kenyan Shilling. We clearly see clustering in both graphs.

But, what is this "volatility"? As a concept, volatility seems to be simple and intuitive. Even so, volatility is both the boon and bane of all traders — you can't live with it and you can't really trade without it. Without volatility, no trader can make money!

Most of us usually think of "choppy" markets and wide price swings when the topic of volatility arises. These basic concepts are accurate, but they also lack nuance. Volatility measures variability, or dispersion about a central tendency — it is simply

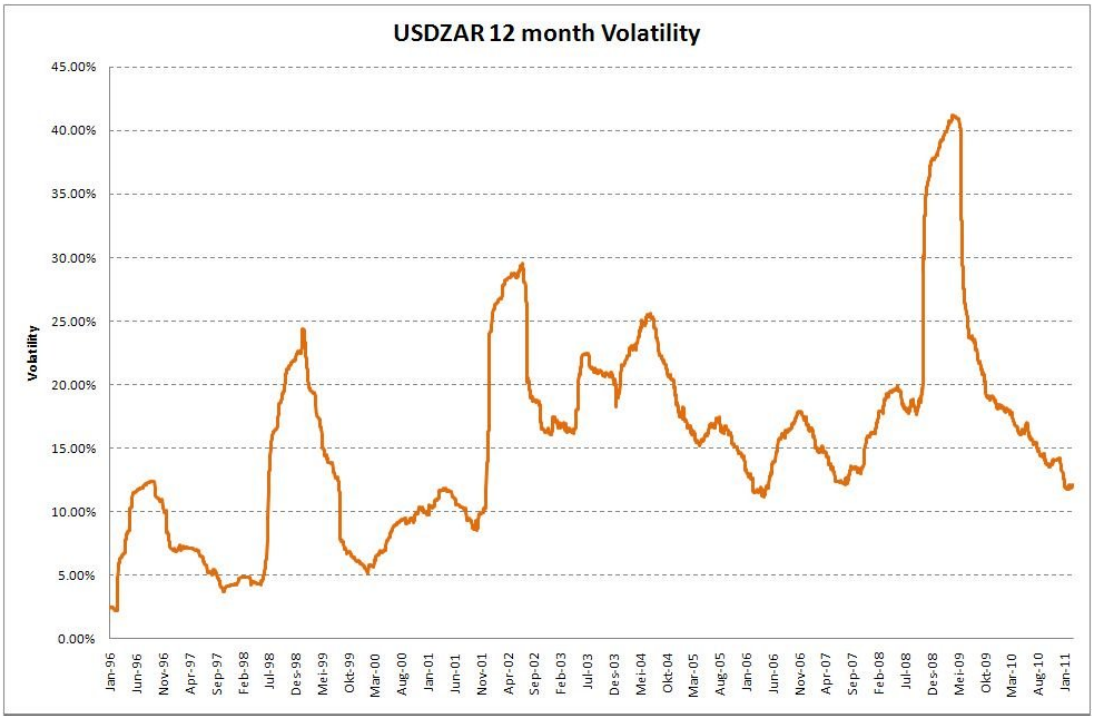


Figure 7.1: Annual volatility of the USDZAR since June 1995 using daily data.

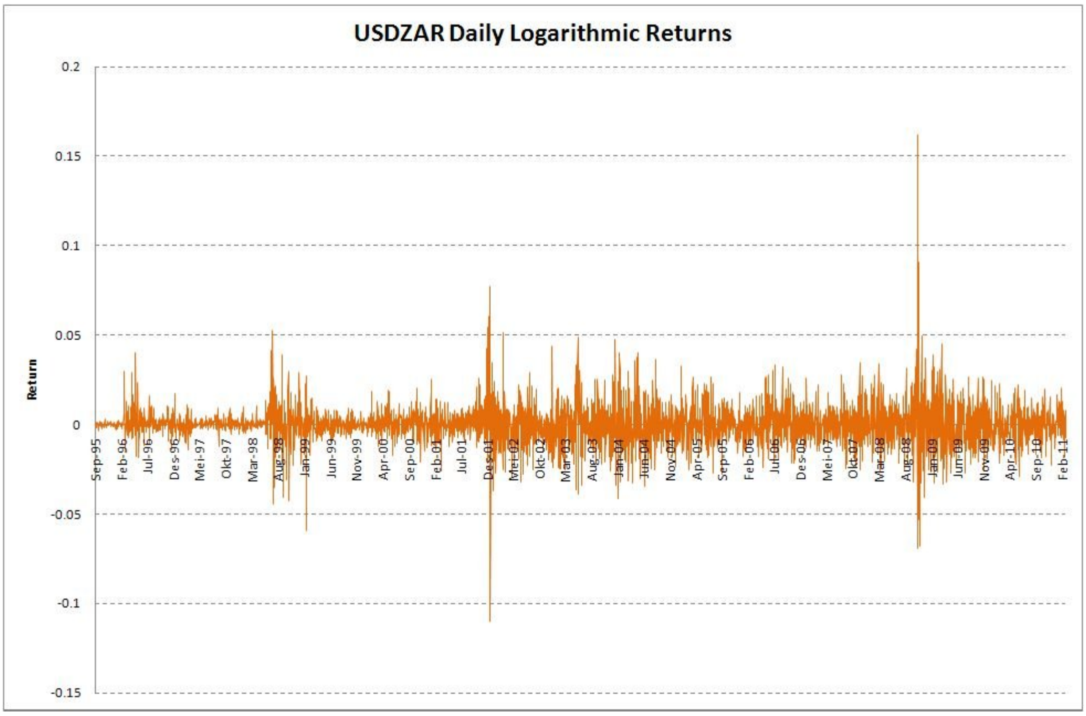


Figure 7.2: Logarithmic returns for USDZAR since June 1995.

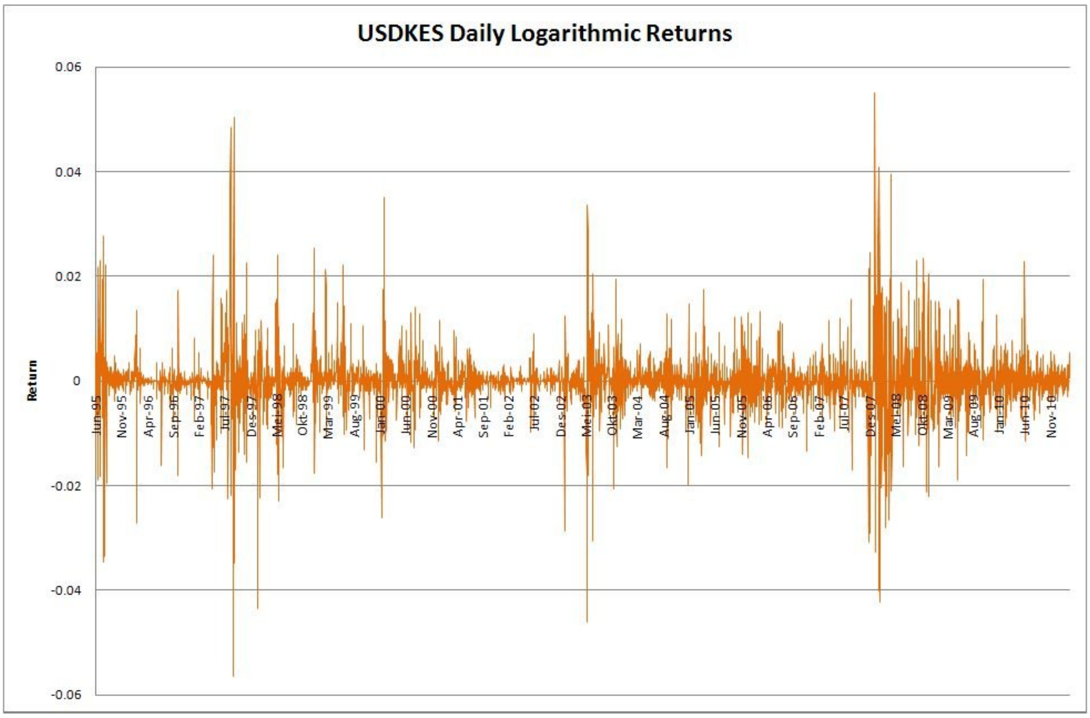


Figure 7.3: Logarithmic returns for USDKES since June 1995.

a measure of the degree of price movement in a stock, futures contract or any other market. Volatility also has many subtleties that make it challenging to analyze and implement. The following questions immediately comes to mind: is volatility a simple intuitive concept or is it complex in nature, what causes volatility, how do we estimate volatility and can it be managed?

What's necessary for traders is to be able to bridge the gap between the simple concepts mentioned above and the sometimes confusing mathematics often used to define and describe volatility. By understanding certain volatility measures, any trader — options or otherwise — can learn to make practical use of volatility analysis and volatility-based strategies. We'll explore these volatility calculations and discuss how to use them.

In this Chapter we will try to shed some light on these and many more questions. We'll explore these volatility calculations and discuss how to use them. We will concentrate on the practical estimation of volatility and how different measures thereof can help the trader and risk manager to understand and handle changing volatility.

7.2 Volatility Dynamics

The *Black & Scholes* option pricing model assumes that volatility is constant. However, a very interesting working paper by David Shimko reported very high negative correlations during the period 1987-1989 between changes in implied volatilities on

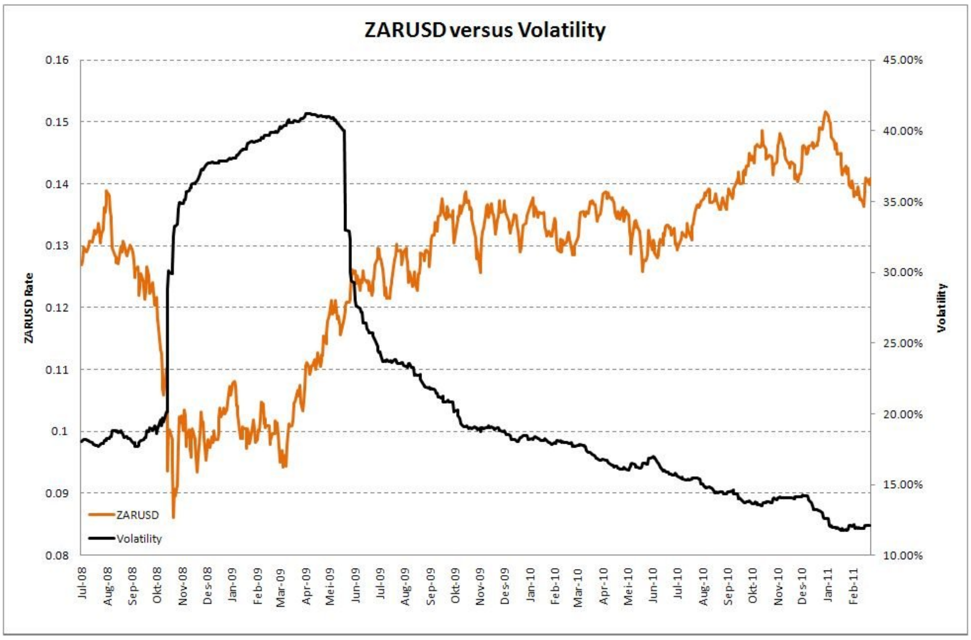


Figure 7.4: ZARUSD FX rate versus the 12 months historical volatility.

S&P 100 index options and the concurrent return of the index — a correlation which should be zero according to the Black-Scholes formula [Sh 93].

It is now an accepted fact, that when asset prices go up (down) volatility usually goes down (up); there is an inverse relationship between volatility and the underlying asset price. We show this in Fig. 7.4 where we plot the ZARUSD exchange rate together with the 12 month historical volatility[1]. The *Black & Scholes* option model with constant volatility will therefore produce option prices that do not match those traded in the market.

This deficiency of the *Black & Scholes* model arises because *Black & Scholes* assumed a constant volatility in deriving their option pricing formula as we discussed in §4.9. It is also argued that the source of this Black-Scholes deficiency can be attributed to the fact that the distribution of asset price levels at expiry is not lognormal [DFW 98].

Another feature of volatility is that it is mean reverting. We define mean reversion as follows

- An asset model is mean reverting if asset prices tend to fall (rise) after hitting a maximum (minimum).

Please note that it can be perfectly consistent with an efficient and arbitrage free market to have some processes in an asset model where all market participants do indeed know that a high or low point has been reached. In Fig. 7.5 we plot the 3

[1]Note that we plot the ZARUSD or inverse of USDZAR to get the full effect.

month rolling historical volatility for USDZAR and for USDKES. Note the long term mean which the volatility reverts to.

7.3 Volatility Defined

Volatility is a measure of instability [Sh 10]. A volatile substance will tend to change its form easily — for example, gasoline is volatile. This means that gasoline gives off fumes even at low temperatures and it is thus more likely to be ignited.

In the financial markets, volatility refers to the likelihood that securities will change in value over time. We then define volatility as the variation of an asset's returns.

Volatility indicates the range of a return's movement. Large values of volatility mean that returns fluctuate in a wide range — simply put, the price will fluctuate considerably over time. Fundamental assumptions used in assets models are that returns follow a normal distribution with a zero mean and that underlying prices are lognormally distributed. Thus, in a return's distribution, volatility is the *deviation of returns from their mean.*

If we assume the mean of returns is zero, then 10% Volatility represents that in one year returns will be within [-0.1; +0.1] with 68.3% probability (1 standard deviation); within [-0.2; +0.2], with 95.4% probability (2 standard deviations), and within [-0.3; +0.3], with 99.7% probability (3 standard deviations) — according to a normal distribution.

Volatility is a fundamental market force, and it must include risk. Traders can take advantage of market volatility and prosper from it. Remember, volatility is not merely an indicator that shows the market is likely to drop!

7.4 The Variance Rate of Return

In their paper in 1973, *Black & Scholes* mentioned the parameter σ^2 which they said was the "variance rate of the return" on the stock prices. *Black & Scholes* took this as a known parameter that is constant through the life of the option (see Section 4.11). *Black & Scholes* knew this parameter was not constant. Did they really know what this parameter was?

In a paper prior to their seminal one, *Black & Scholes* gave more insight into the variance rate of return [BS 72]. There they stated that they estimated the instantaneous variance from the historical series of daily stock prices. They thus defined volatility as the amount of variability in the returns of the underlying asset. *Black & Scholes* determined what is today known as the historical volatility and used that as a proxy for the expected volatility in the future. In that paper they tested several implications of their model empirically by using a sample of 2 039 calls and 3 052 straddles traded on the New York stock exchange between 1966 and 1969.

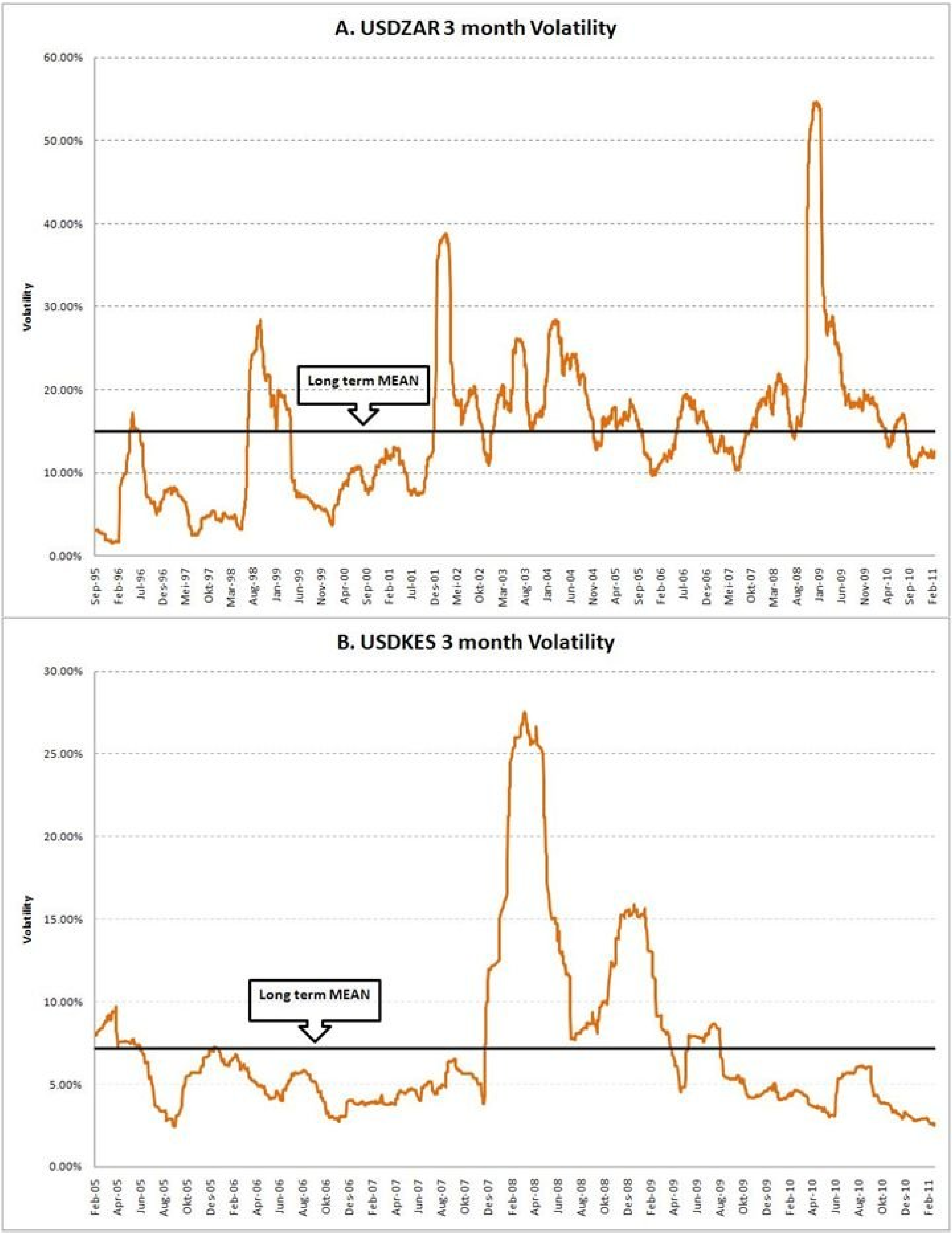


Figure 7.5: A. USDZAR 3 month volatility with long term mean. B. USDKES 3 month volatility with long term mean.

In analyzing their results they noted that the variance actually employed by the market is too narrow and that the historical estimates of the variance include an attenuation bias, i.e., the spread of the estimates is greater than the spread of the true variance. This would imply that for securities with a relatively high variance, the market prices would imply an underestimate in the variance, while using historical price series would overestimate the variance and the resulting *Black & Scholes* model price would thus be too high; the converse is true for relative low variance securities. Was this the first observation of a volatility skew? In further tests *Black & Scholes* found that their model performed very well when the true variance rate of the stock was known.

7.5 Empirical Research

Since *Black & Scholes* published their formula, a lot of empirical research has been undertaken to compare the *Black & Scholes* model price of options to market prices [Ro 87]. Much of this research focused on the volatility because this is the most important parameter that needs to be estimated. People started to realize that historical and implied volatility changes through time. They also observed the implied variance rate declines as the exercise price increases [MM 79]. People thus observed a volatility skew. This then started the quest to understand volatility better and the search for better estimates thereof.

7.6 Importance of Volatility

We know that volatility has been viewed as natural for a long time, certainly predating any sophistication in capital markets. At the beginning of the 21st century, almost every financial decision there is to make is interesting because of volatility [Ne 97]. Volatility holds a certain status in the finance industry and this is enhanced when prominent people talk about it. Every person in the markets takes note when Ben Bernanke, the governor of the Federal Reserve Bank in the USA, speaks. He often refers to the volatility in the financial markets. Volatility was a huge topic of discussion during the financial crises at the end 2008. Look at Fig. 7.5. Look at the spike in volatility at the end of 2008!

Since 1973 the array of available and actively traded products have expanded enormously. Some derivative instruments are extremely complex to value and understand. But, for complex or simple derivative instruments one thing is central: if it contains optionality its valuation model requires at least one volatility parameter. Volatility is thus important to a trader as well as to a risk manager who needs to understand the risks associated with these instruments.

People have also generalized the *Black & Scholes* model where the volatility is taken as a stochastic parameter [Le 00]. Popular models are the Heston model [He 93] and the well-known SABR model by *Hagan et al* [HK 02]. But, *this does not solve the problem of estimation* since, for these models, the user has to specify a set of parameters to define a time-varying stochastic volatility process. This can be difficult.

Any person concerned with the financial markets knows the importance of volatility. Whether you are an equities broker, derivatives trader or risk manager, events like that on 11 September 2001 influence the markets, volatility and thus your life!

7.7 Different Types of Volatility

There are different types of volatility:

- Implied volatility
- Historical volatility
- Noncentered Volatility
- Realized/Actual Volatility
- Extreme-Value Estimators

In Sec. 4.10 we saw that the volatility parameter used in the *Black & Scholes* equation should be the future volatility i.e., the volatility from the transaction date to the expiry date. This is not known and volatility thus needs to be *estimated or predicted*.

The question still remains: what volatility should be used? Also, traders might think that because volatility is a market "price" it is set by the market like the price of a share. However, how do you as an option buyer knows that this is the correct volatility? How did the writer of the option estimate the volatility in the first place especially if it is an OTC option? There are many variations in the method of measurement [Ta 97]. In this Chapter we will look at these questions but we will keep it simple and we will not wonder into the complex field of the ARCH[2] or regression models.

7.8 Trading or Nontrading Days

To estimate the volatility of a stock price empirically, the stock price is usually observed at fixed intervals of time. These intervals can be days, weeks or months[3]. Before any calculation can be done, however, a question one needs to answer is whether

[2] Auto Regressive Conditional Heteroskedasticity.

[3] One has to be consistent; if the frequency of observation is every Thursday at midnight, the returns all need to correspond to such a period

the volatility of an exchange-traded instrument is the same when the exchange is open as when it is closed.

Some people argue that information arrives even when an exchange is closed and this should influence the price. A lot of empirical studies have been done and researchers found that volatility is far larger when the exchange is open than when it is closed [Hu 06]. The consequence of this is that if daily data are used to measure volatility, the results suggest that days when the exchange is closed should be ignored.

7.9 Estimation of Volatility

7.9.1 Historical Volatility

Let S_i be the stock price at the end of the ith interval and let

$$u_i = \ln \frac{S_i}{S_{i-1}} \tag{7.1}$$

be the log relative prices[4] for $i = 1, 2, \ldots, n$ with n the number of returns in the historical sample [Hu 06]. The historical volatility estimate is then given by

$$\sigma = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (u_i - \overline{u})^2} \tag{7.2}$$

where $\overline{u}$ is the mean defined by

$$\overline{u} = \frac{1}{n} \sum_{j=1}^{n} u_j .$$

σ in Equation (7.2) gives the estimated volatility per interval and is a measure of the previous fluctuations in share price.

Note: in statistical terms the volatility or variance of an asset over time is known as the ‘second moment of the normal distribution.’

Scaling Volatility

To annualize the volatility, we need to scale this estimate with an annualization factor h which is the number of intervals per annum such that[5]

$$\sigma_{an} = \sigma * \sqrt{h}.$$

[4]Remember, the assumption is that stock prices are lognormally distributed. Their behaviour are described by geometric Brownian motion.

[5]*Diebhold*, *Hickman*, *Inoue* and *Schuermann* caution in using a scaling of $\sqrt{h}$. They call this a “first generation rule of thumb” that can be improved [DH 96].

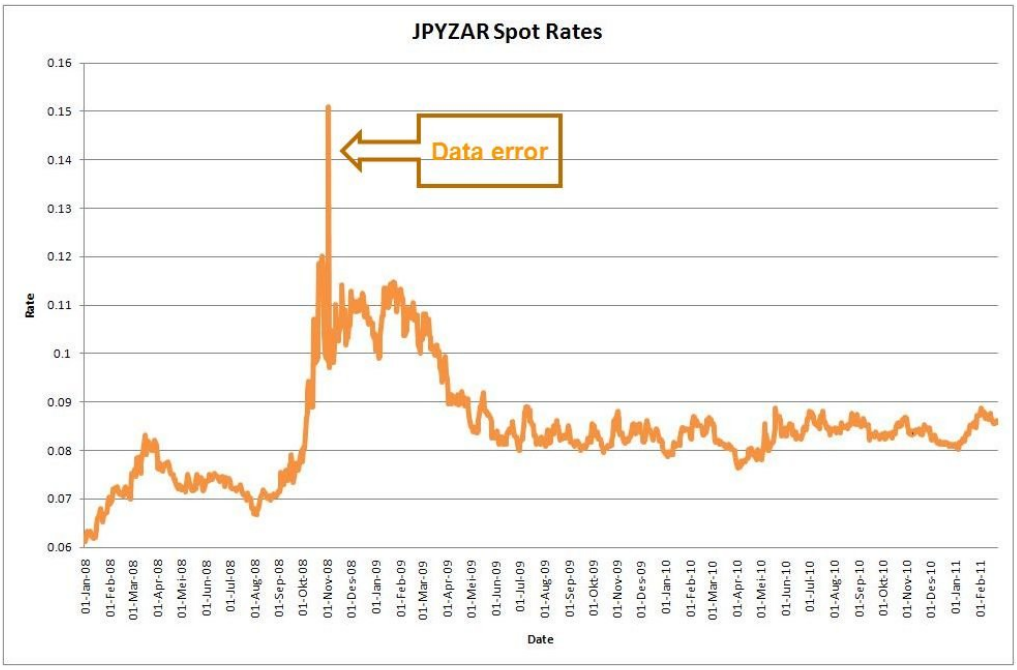


Figure 7.6: ZARJPY historical data from Reuters.

If daily data is used the interval is one trading day and we use $h \approx 252$, if the interval is a week, $h = 52$ and $h = 12$ for monthly data[6].

The Standard Deviation

Equation (7.2) is just the standard deviation[7] of the sampled series u_i. The *Black & Scholes* view was discussed in §7.4. Table 7.1 provides an example of historical volatility calculation. It shows daily prices of USDKES in succession for 1 month (November 2010) i.e., 21 trading days. This yields 20 returns. The average for the log relatives u_j is 0.000062 and the standard deviation is 0.0032419. This gives an annualized historical volatility of 5.1463%.

Beware of erroneous data received from data vendors. A single wrong data point can wreck havoc with volatility calculations. In Fig. 7.6 we plot the ZARJPY from data as extracted from Reuters 3000. Reuters has the exchange rate on 3 November 2008 as 0.150988 where it should actually be 0.0981195. The 3 month volatility of this time series from 1 September 2008 till 30 November 2008 is 30% while the correct volatility is 20%!

---

[6]There are approximately 252 trading days per annum
[7]In Microsoft Excel one can use the STDEV() function

	Date	**Price**	u_j	u_j^2
1	01-Nov-10	80.6000		
2	02-Nov-10	80.2500	-0.0043519	0.0000189
3	03-Nov-10	80.4000	0.0018674	0.0000035
4	04-Nov-10	80.0000	-0.0049875	0.0000249
5	05-Nov-10	80.0500	0.0006248	0.0000004
6	08-Nov-10	80.5000	0.0056057	0.0000314
7	09-Nov-10	80.2000	-0.0037337	0.0000139
8	10-Nov-10	80.5000	0.0037337	0.0000139
9	11-Nov-10	80.4000	-0.0012430	0.0000015
10	12-Nov-10	80.2500	-0.0018674	0.0000035
11	15-Nov-10	80.4000	0.0018674	0.0000035
12	16-Nov-10	80.2500	-0.0018674	0.0000035
13	17-Nov-10	80.0000	-0.0031201	0.0000097
14	18-Nov-10	80.1000	0.0012492	0.0000016
15	19-Nov-10	80.0000	-0.0012492	0.0000016
16	22-Nov-10	80.0000	0	0
17	23-Nov-10	79.9500	-0.0006252	0.0000004
18	24-Nov-10	80.2000	0.0031221	0.0000097
19	26-Nov-10	80.7000	0.0062151	0.0000386
20	29-Nov-10	80.9500	0.0030931	0.0000096
21	30-Nov-10	80.7000	-0.0030931	0.0000096
		Mean	0.0000620	
		StDev	0.0032419	
		Annual Vol σ	5.146288%	
			σ'	0.0032425
			Annual σ'	5.147278%

Table 7.1: Historical volatility for USDKES during November 2010.

7.9.2 Noncentered Volatility

In Table 7.1 we see that the mean return $\overline{u}$ is close to zero (especially for daily data). This is in general the case. So, what would happen if we discard it? As a bonus to very busy option traders, it means there is one parameter less to estimate. Equation (7.2) is then turned into the (nonweighted) noncentered volatility σ' that can be expressed as

$$\sigma' = \sqrt{\frac{1}{n}\sum_{j=1}^{n} u_j^2} \tag{7.3}$$

Another reason for ditching the mean return is that it makes the estimation of volatility closer to what would affect a trader's P/L. To illustrate this suppose we use a week's worth of returns to calculate the volatility and let's assume that each day was down 2%. If the mean were subtracted from each daily return, the volatility for the week would be zero. That certainly does not accord with a trader's expectation going into that week, and, the volatility the trader experienced during the week would be much higher than zero. The third reason is that zero-mean volatilities are better at forecasting future volatilities. In Table 7.1 we listed u_j^2 and calculated σ' to be 5.1473% for USDKES during November 2010. This is a bit higher than the historical volatility estimate.

Figlewski gives another strong reason for using σ' instead of σ. He reasons that, since volatility is measured in terms of deviations from the mean return, an inaccurate estimate of the mean will reduce the accuracy of the volatility calculation [Fi 94]. This is especially true for short time series like 1 to 3 months (which are the time frames used by most option traders to estimate volatilities). He shows that one can have a standard deviation around the true mean of up to 85%. In other words, roughly one third of the time, the trader's volatility estimate will be calculated from a sample mean that is more than 85 percentage points above or below the correct value on an annualized basis.

7.9.3 Realized/Actual Volatility

This is the historical volatility calculated looking "backward" when an option expires using Eq. 7.2. As an example, let's say a trader wants to write an option today, at t, that expires in 3 months time at time T. To estimate the volatility he/she might calculate the historical volatility for the past 3 months. If similar options are traded in the market he/she might calculate the implied volatility. The actual volatility will, however, only be known at expiry T. Once the 3 months have passed, one can calculate the realized volatility between t and T because the actual price path is known.

Through empirical research people found that implied volatility is not a very accurate predictor of the actual future historical or realised volatility. Historical volatility was also found to be an unreliable predictor, suggesting that in the stock volatility business, history does not necessarily repeat itself.

Does this mean that the market is always wrong? No. It simply means it is very difficult to predict the future price volatility of a stock. However, it also means this difficulty leads to *more trading opportunities* and *more market inefficiencies* to trade against.

7.9.4 Implied Volatility

Suppose we use the Black-Scholes model to infer the volatility used by option traders to price the option. We search for the volatility that makes the model give us an option price that corresponds to the market price. The model will then be said to 'imply a volatility', giving rise to the concept of the implied volatility.

We illustrate the basic idea with an example. On 21 February 2011 the USDKES FX rate closed the day at 80.00. Say you wanted to buy an at-the-money option that expires on 13 June 2011. You phone a bank and they quote you a price of Ks3.10 per option[8] or 9.997%. From the current local yield curve the riskfree interest rate is 12.05% and the foreign interest is 1%. One can then substitute all of these into Equation (4.6) and numerically solve this to back out the volatility σ. For this example one gets an implied volatility of 40.2731%. In Appendix D I give a useful method to calculate the implied volatility[9].

Remember, the volatility parameter used in the *Black & Scholes* equation in 4.6 is the future volatility. This volatility that the trader uses as an input to the *Black & Scholes* formula, is called the implied volatility. Implied volatility is interpreted to be the market's assessment of the average volatility over the remaining life of the option. This means that, if we move forward in time to the 13th June 2011 we can calculate the historical or realised volatility looking back till 21 February 2011. If the trader was a prophet, his implied quoted volatility on 21 Feb should have been exactly the same as the realised volatility calculated on 13 June. Always remember this: *Bruno Dupire*[10] once stated

> "Implied volatility is the wrong number to put into the wrong formula to obtain the correct price."

A high implied volatility indicates that the market anticipates the exchange rate to be volatile or keep moving significantly. A low implied volatility shows that the market expects FX rates to change moderately. Can we put some numbers to these vague terms 'moderately' and 'significantly'? In all *Black & Scholes* -type formulas we always have the following term: $\sigma\sqrt{\tau}$. This is consequence of the assumption of

[8] When trading on price traders actually quote the percentage of the option premium to the underlying's price.

[9] In Excel use the Goal Seek procedure.

[10] Guru on local volatility models.

Brownian motion. Stated differently: standard deviation increases proportionately to the square root of time[11].

If the underlying FX rate's implied volatility is at 25%, what does it translate to in terms of how much the rate may move in one day, one week or even one month? If there are 252 trading days per year, multiply 25% by 1/252 to arrive at one standard deviation for one day which is approximately 1.575%. Therefore, two-thirds of the time, the underlying will move up or down 1.6% on an average trading day[12]. To calculate the possible trading range (with a probability of 68.3%) of the underlying over the next 5 day, we multiply 25% by the square root of 5 divided by 252:

$$\text{Trading Range} = 25\%\sqrt{\frac{5}{252}} \simeq 3.5\%.$$

7.10 Extreme-Value Estimators

The volatility formula in Equation (7.2) is called the close-close (CC) estimator. Another class of estimator (the high-low (HL)) uses intra-interval highs and lows to characterize the distribution. These estimators are more efficient because they use additional information about movements throughout the interval that snapshots at the end of an interval cannot hope to summarize. The practical importance of this improved efficiency is that five to seven times fewer observations are necessary in order to obtain the same statistical precision as the CC estimator. Since models like *Black & Scholes* are based in continuous time, it is natural to want a volatility measure more broadly based on the price continuum. The pioneering work was done by *Parkinson* and *Garman* and *Klass*. We will later on show how these measures can help in the trading environment.

7.10.1 Parkinson Estimator

The form of the *Parkinson* estimator is [Pa 80]

$$\sigma_p = \sqrt{\frac{1}{n}\frac{1}{4\ln 2}\sum_{i=1}^{n}\left(\ln\frac{H_i}{L_i}\right)^2} \tag{7.4}$$

where H_i is the i-th interval *high* and L_i is the interval *low*. If daily data is used these will be the intra-day *high* and *low*. n is again the number of data points in the sample. σ_p is a general average over n intervals for the intra-interval volatility.

[11] This is due to Einstein who calculated way back in 1905 that a particle that exhibits a random motion are displaced proportionally with the square root of time.

[12] Remember that 1 standard deviation encompass 68.3% of all outcomes from a sample that is normally distributed.

Date	**Open**	**High**	**Low**	**Close**	$\ln \frac{S_H}{S_L}$	$\ln \frac{S_C}{S_O}$
01-Nov	80.7	80.8	80.4	80.6	0.00496	-0.00124
02-Nov	80.6	80.75	80.2	80.25	0.00683	-0.00435
03-Nov	80.46	80.7	80.25	80.4	0.00559	-0.00075
04-Nov	80.4	80.6	79.95	80	0.00810	-0.00499
05-Nov	80.05	80.25	79.9	80.05	0.00437	-
08-Nov	80.05	80.65	80.05	80.5	0.00747	0.00561
09-Nov	80.5	80.65	80.15	80.2	0.00622	-0.00373
10-Nov	80.65	80.75	80.3	80.5	0.00559	-0.00186
11-Nov	80.5	80.51	80.2	80.4	0.00386	-0.00124
12-Nov	80.46	80.6	80.25	80.25	0.00435	-0.00261
15-Nov	80.4	80.7	80.35	80.4	0.00435	-
16-Nov	80.4	80.5	80.1	80.25	0.00498	-0.00187
17-Nov	80.25	80.45	79.9	80	0.00686	-0.00312
18-Nov	80.15	80.2	79.8	80.1	0.00500	-0.00062
19-Nov	80.1	80.15	79.75	80	0.00500	-0.00125
22-Nov	80	80.15	79.85	80	0.00375	-
23-Nov	79.9	80.4	79.9	79.95	0.00624	0.00063
24-Nov	79.95	80.4	79.95	80.2	0.00561	0.00312
26-Nov	80.3	80.8	80.1	80.6	0.00870	0.00373
29-Nov	80.75	80.95	80.5	80.7	0.00557	-0.00062
30-Nov	80.85	81.1	80.65	80.95	0.00556	0.00124
				Parkinson	**5.54259%**	
					Garman-Klass	**5.99174%**

Table 7.2: Extreme value volatility estimators.

7.10.2 Garman-Klass Estimator

Garman and *Klass* improved the efficiency by using even more information [GK 80]. They also used the intra-interval *open* O_i and *close* C_i prices to obtain

$$\sigma_{gk} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left[\frac{1}{2}\left(\ln\frac{H_i}{L_i}\right)^2 - (2\ln 2 - 1)\left(\ln\frac{C_i}{O_i}\right)^2\right]} \tag{7.5}$$

To include overnight volatility one can substitute the opening price O_i with the previous interval's closing price C_{i-1}.

Table 7.2 list the values of the *Parkinson* and *Garman* and *Klas* estimators for the same data used in Table 7.1. We see that the intraday volatility is lower than the historical volatilities.

7.11 Higher Moments of the Normal Distribution

The concept of the “moment” in mathematics developed from the concept of the “moment” in physics. The moments articulate the nature of the distribution. Any distribution can be characterised by a number of features, such as the mean, the variance, the skewness and the kurtosis [Sh 10]. The mean is known as the first moment and we mentioned the variance which is the second moment. Let’s look at the next two.

7.12 Skewness

Skew is the degree of the asymmetry of a return’ s distribution around its mean — it is a measure of the shape of the distribution. The commonly pictured “bell-shaped” or normal distribution is symmetric and has zero skewness. In positive skewness (a lognormal distribution has positive skewness), a large probability of a small loss is offset by a small probability of a large gain (”long shot”). In negative skewness, a small probability of a large loss is offset by a large probability of a small gain.

Applied to options trading it can be suggested that skewness indicates, as well as Implied Volatility, how cheap or expensive options are. For example, a negative skew coefficient means that the density of a return’s distribution is asymmetric with a sharper fall on the right tail. Thus, put options are more expensive than call options.

Options models suggest that returns are normally distributed and underlying prices are lognormally distributed. Thus, the skew of a return’ s distribution is zero. Moving an up-and-down movement in theoretical models has the same probability.

Note: in statistical terms the skew or shape of the distribution is known as the ‘third moment of the normal distribution.’ Option traders seek to make money by accurately predicting when markets will rally or drop. But often the market goes against them and they lose money. They confront sizable negative returns, a situation they may not have anticipated at all [Sh 10].

This is the skew. It insinuates the ever-present possibility of large negative returns. This is actually negative skew. Skew is the contour, the unevenness, in a distribution, the dent in the bell curve. A negative skew suggests that the prospect of achieving negative returns is superior to that of achieving large positive returns. Of course, a distribution can possess positive skew as well. We show the skewness of USDZAR, USDKES and EURUSD in Fig. 7.7.

Note: A skew demonstrates the relationship between the movement of an underlying asset and its volatility.

7.13 Kurtosis

Kurtosis is the relative peakedness or flatness of a returns' distribution compared to the normal distribution (a normal distribution has a zero kurtosis). Kurtosis risk in statistics and decision theory denotes the fact that observations are spread in a wider fashion than the normal distribution entails. In other words, fewer observations cluster near the average, and more observations populate the extremes either far above or far below the average compared to the bell curve shape of the normal distribution.

A distribution is said to be leptokurtic if its tails are fatter than those of a corresponding normal distribution. It is said to be platykurtic if its tails are thinner than those of the normal distribution. Market returns for stocks tend to be slightly leptokurtic. This means that dramatic market moves occur with greater frequency than is predicted by the normal distribution.

Because kurtosis characterizes the flatness of returns, it can be applied to an option model. Consequently, a kurtosis greater than zero means fatter tails and the model underprices both out-of-the-money and in-the-money calls and puts. A kurtosis less than zero means thinner tails; and as a result, options are overpriced.

Note: in statistical terms the kurtosis or varying variance or volatility of volatility is known as the 'fourth moment of the normal distribution.' This moment is something that all traders can relate to. It signifies volatility of volatility of an underlying asset. What happens to the distribution of the curve when volatility changes? What occurs to the extreme downside skew if volatility changes? A changing volatility can cause the tails of the normal distribution to become "fatter" or "skinnier" than otherwise predicted, thereby increasing the potential risk [Sh 10].

We show this by returning to the distribution of a few currencies. We compare these to the standard normal distribution function in Fig. 7.7. Skewness and kurtosis are clearly visible.

7.14 The Volatility Measure?

Though there are no survey results to support the following claim, there is reason to believe that if 10 different quantitative analysts were asked to supply the volatility of a security, at least nine different answers would be offered [Ne 97]. Does this means that quants are careless hacks[13] practising careless, ad hoc methods? Or, is the volatility measure defined haphazardly?

Neither one is true. The disparity comes largely from the multitude of ways one could sample the data in calculating the standard deviation. Some traders with a long memory may argue that a longer sample, like a year, is necessary because more observations lead to better statistical accuracy. Other traders may be victims of some form of market amnesia and may favour a relative short sample period,

[13] And traders shout all together...YES!

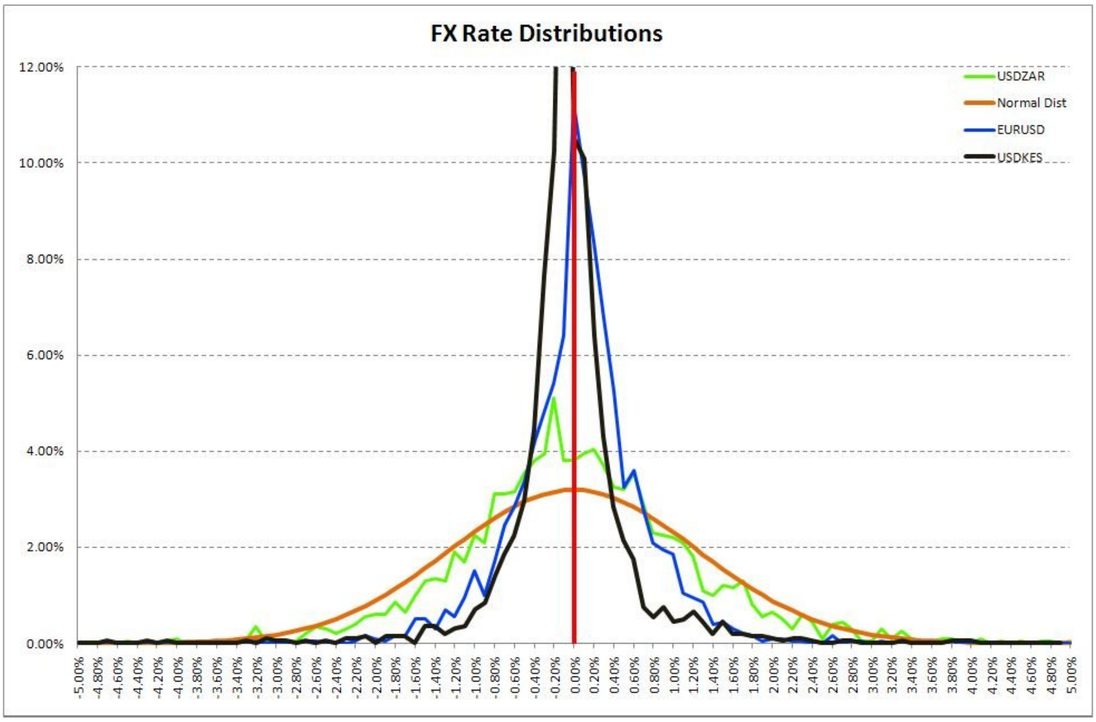


Figure 7.7: Skewness and kurtosis in the FX markets.

like a month, under the assumptions that the recent past is more indicative of the current environment. Market shocks (like the influence the attacks on the World Trade Centers on 11 September 2001 had on the market) influence these traders' view of the market and they give this data a high weighting in their volatility analysis. Others argue that one needs a sample based on the length of the time to maturity i.e., if the option expires in four months time, one needs to have a sample of four months of data to calculate the volatility.

Another source of disparity comes from the return (as defined in Equation (7.1)) calculation interval. Should the time subscript i be measuring weeks, days or even higher frequency intraday periods? Yet another way to characterize the width of the return distribution is to use the intraday highs and lows. We have described this in Section 7.10.

Still others may contend that the historical volatilities are unimportant because the market has already gleaned all the information from past movement and has incorporated the information into option prices. If the market is efficient at using all available information in forming the option price, and if the price model is correct, then the implied volatility is the best guess possible for ensuing volatility.

With so many methodological variations available, it seems impossible to specify the "right" approach to estimation. Indeed, there are *no universal truths*. There are, however, conditional truths and guidelines that will be discussed in the following sections.

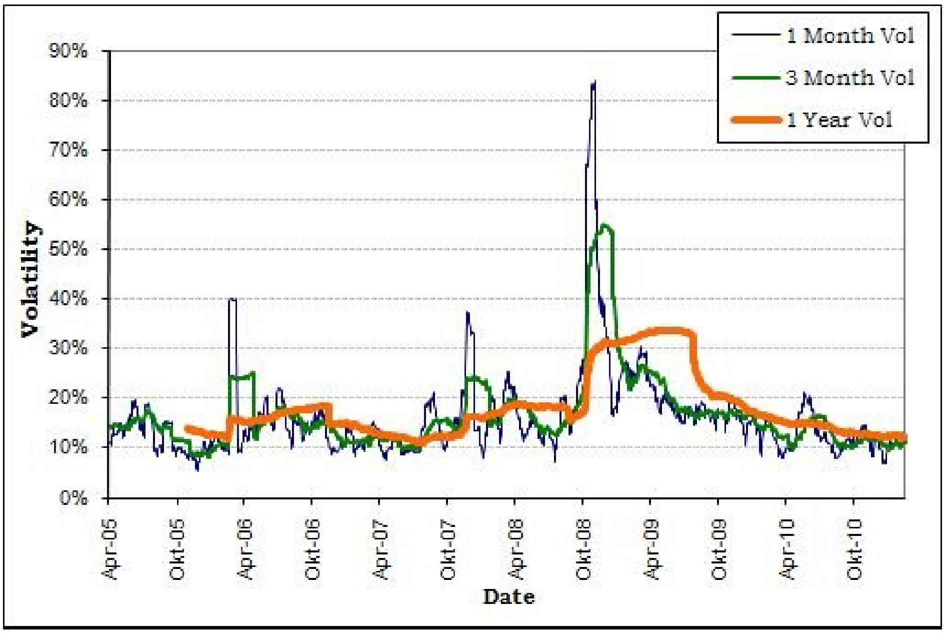


Figure 7.8: Windows of volatility.

7.15 Sample Size

In Section 7.14 we pointed out that the sample size in calculating the historical volatility is a contentious issue. We noted there are no fixed rules but we discuss two ways to get a better feel for what sample size might be more appropriate.

7.15.1 Volatility Plots

A useful tool is a plot of volatilities with different window-lengths (intervals). For instance, moving windows of one-month volatilities could be plotted with moving windows of three-month volatilities superimposed. See Figure 7.8 where we plotted the 1 month, 3 month and 12 month historical volatilities for USDZAR from 31 January 2005.

This can show how stable volatilities have been throughout time. It can also jog memories of specific events that may have caused spikes in volatility like the 9/11 attacks or the financial crises at the end of 2008. If the trader does not believe that a similar event will happen over the life of the option, the best volatility estimate will exclude that point.

Another property this tool can reveal is the effect of outliers on the volatility estimation. Certainly an event like 19 October 1987, shows up as a large volatility plateau on a plot like this. In Figure 7.8 one can clearly see the crash of September 1997, a few months later the Asian crises during 1998 and the 11 September 2001 World Trade Center attacks and the 2008 financial crisis. Not only is there a big jump when the point enters the sample, but also a precipitous drop as soon as it leaves the sample. The drop-off is bothersome. Does the market perceive risk so differently from the one day to the next? Probably not[14]. To overcome this one can use a time-weighted volatility.

7.15.2 Introducing Filtering

One way to handle the drop-off eluded to in the previous section is to filter the data. The historical volatility as calculated by Eq. 7.2 weighs each data point evenly — actually giving each a weight of one. In the markets this is hardly the case. Due to activity in the markets or information events/days, some days might be more important than others.

Filtering is a simple method whereby we can take this fact into account and give data points uneven weightings. Here we present a simplified version of the *Kalman* filter[15]: the exponential decay.

It gives the trader the flexibility to assign more importance to recent events in proportion to their distance away from the present. Note, however, that the trader should not take the measurement for gospel. He might have information about some political situation or some structural changes that's going to take place, that need to have an effect on the weighting.

The symbol λ is used for the decay factor where $0 \geq \lambda < 1$. To lengthen the memory let λ be closer to one. In using the noncentered volatility defined in Equation 7.3 we obtain the filtered volatility as

$$\sigma'' = \sqrt{\frac{1}{\Omega}\sum_{j=1}^{n} \lambda^{n-(j-1)} u_j^2} \tag{7.6}$$

where

$$\Omega = \sum_{j=1}^{n} \lambda^j = \lambda + \lambda^2 + \lambda^3 + \ldots + \lambda^n \approx \frac{1}{1-\lambda}$$

For the last approximation to hold, n needs to be very large (higher than 1000 observations as a general rule).

The closer λ is to 1, the longer the memory included in the calculation where $\lim_{\lambda\to 1} \sigma'' = \sigma'$. Table 7.3 list the values of the filtered volatility for the USDZAR with $\lambda = 0.95$. We calculate the filtered volatility to be 16.0698% (σ'' scaled by $\sqrt{252}$) while the noncentered volatility σ' is 16.7452%. *Van Vuuren*, *Botha* and *Styger* mention that, in general, emerging markets are assigned a $\lambda = 0.94$. They analyzed the South African market and give guidelines in estimating λ on a daily basis which

[14]Implied volatilities are not likely to behave that way

[15]For an introduction see http://www.computing.edu.au/ robin/thesis/node15.html.

Date	**FX Rate**	u_i	u_i^2	λ^j	$\lambda^j u_j^2$	j
01-11-2010	80.6					
02-11-2010	80.25	-0.004351888	0.00001894	0.34056	0.00000645	21
03-11-2010	80.4	0.001867414	0.00000349	0.35849	0.00000125	20
04-11-2010	80	-0.004987542	0.00002488	0.37735	0.00000939	19
05-11-2010	80.05	0.000624805	0.00000039	0.39721	0.00000016	18
08-11-2010	80.5	0.005605745	0.00003142	0.41812	0.00001314	17
09-11-2010	80.2	-0.00373367	0.00001394	0.44013	0.00000614	16
10-11-2010	80.5	0.00373367	0.00001394	0.46329	0.00000646	15
11-11-2010	80.4	-0.001243008	0.00000155	0.48767	0.00000075	14
12-11-2010	80.25	-0.001867414	0.00000349	0.51334	0.00000179	13
15-11-2010	80.4	0.001867414	0.00000349	0.54036	0.00000188	12
16-11-2010	80.25	-0.001867414	0.00000349	0.56880	0.00000198	11
17-11-2010	80	-0.003120127	0.00000974	0.59874	0.00000583	10
18-11-2010	80.1	0.001249219	0.00000156	0.63025	0.00000098	9
19-11-2010	80	-0.001249219	0.00000156	0.66342	0.00000104	8
22-11-2010	80	0	-	0.69834	-	7
23-11-2010	79.95	-0.000625195	0.00000039	0.73509	0.00000029	6
24-11-2010	80.2	0.003122076	0.00000975	0.77378	0.00000754	5
25-11-2010	80.6	0.004975135	0.00002475	0.81451	0.00002016	4
26-11-2010	80.7	0.001239926	0.00000154	0.85738	0.00000132	3
29-11-2010	80.95	0.003093105	0.00000957	0.90250	0.00000863	2
30-11-2010	80.7	-0.003093105	.00000957	0.95000	0.00000909	1
			SUM	12.52933	0.00010427	
				σ''	4.579392%	
				σ'	4.742434%	

Table 7.3: Filtered volatility for USDZAR with $\lambda = 0.95$.

they found to lead to more accurate volatility and hence Value-at-Risk values [VB 00].

7.16 The Volatility Skew

The Black and Scholes model assumes that volatility is constant. This is at odds with what happens in the market where traders know that the formula misprices deep in-the-money and deep out-the-money options. The mispricing is rectified when options (on the same underlying with the same expiry date) with different strike prices trade at different volatilities — traders say volatilities are skewed when options of a given asset trade at increasing or decreasing levels of implied volatility as you move through

Underlying Asset	Time Period of Analysis
S&P 500 Futures	25/03/1986 - 24/12/1996
FTSE Futures	02/01/1985 - 20/12/1996
Nikkei Dow Futures	25/09/1990 - 16/12/1996
DAX Futures	02/01/1992 - 20/12/1996
Bund Futures	20/04/1989 - 21/11/1996
BTP Futures	11/10/1991 - 21/11/1996
Gilt Futures	13/03/1986 - 22/11/1996
US T-Bond Futures	02/01/1985 - 15/11/1996
Deutsche Mark /US Dollar	03/01/1985 - 09/12/1996
British Pound / US Dollar	25/02/1985 - 09/12/1996
Japanese Yen / US Dollar	05/03/1986 - 09/12/1996
Swiss Franc / US Dollar	25/02/1985 - 09/12/1996
Euro Dollar	27/06/1985 - 16/12/1996
Euro Sterling	05/11/1987 - 18/12/1996
Euro D-mark	11/03/1990 - 16/12/1996
Euro Swiss Franc	15/10/1992 - 16/12/1996

Figure 7.9: The markets Tompkins studied.

the strikes. The empirical relation between implied volatilities and exercise prices is known as the "volatility skew".

The volatility skew can be represented graphically in 2 dimensions (strike versus volatility). The volatility skew illustrates that implied volatility is higher as put options go deeper in the money. This leads to the formation of a curve sloping downward to the right. Sometimes, out-the-money call options also trade at higher volatilities than their at-the-money counterparts. The empirical relation then has the shape of a smile, hence the term "volatility smile". This happens most often in the currency markets.

7.16.1 Universality of the Skew

The skew is a universal phenomenon. It is is seen in most markets around the globe. One of the best and comprehensive studies to confirm this was done by *Tompkins* in 2001 [To 01]. He looked at 16 different options markets on financial futures comprising four asset classes: equities, foreign exchange, bonds and forward rate agreements (FRA's). He compared the relative smile patterns or shapes across markets for options with the same time to expiration. His data set comprised more than 10 years of option prices spanning 1986 to 1996. The markets he examined are shown in Fig. 7.9 together with the time ranges for his data sets. He mentioned that

"In total, the number of option prices examined for all sixteen markets was

> 1,862,473. Given that we also had the underlying futures prices for the same dates (and at the same time) as the options, we were able to assure that both time series were consistent to each other. From this analysis, we were able to clean both series and assure our analysis was minimally impacted by errors in data."

Tompkins concluded that regularities in implied volatility surfaces exist and are similar for the same asset classes even for different exchanges. A further result is that the shapes of the implied volatility surfaces are fairly stable over time. We show his results in Fig. 7.10 for currencies and and 7.11 for stock indices. South Africa's stock indices and currencies exhibit similar shapes as those determined in this study. Note the 'smiles' in all markets. The *Tompkins* study looked at option with an expiry of 5 days to 90 days. From this we ascertain that all markets' skews tend to a smile for very short dated options.

7.16.2 Why do we observe a Skew?

In 1972 Black and Scholes mentioned in a paper "the historical estimates of the variance include an attenuation bias - the spread of the estimated variance is larger than the true variance" [BS 72]. This would imply that for securities with a relatively high variance (read volatility), the market prices would imply an underestimate in the variance, while using historical price series would overestimate the variance and the resulting model option price would be too high; the converse is true for relative low variance securities. Black and Scholes further showed that the model performed very well, empirically, if they use the right variance. In 1979, *Macbeth* and *Mervile* extended this empirical research of Black and Scholes and also showed that the skew existed [MM 79]. In this paper, *Macbeth* and *Merville* reported that the *Black & Scholes* model undervalues in-the-money and overvalues out-the-money options. At that point in time the skew wasn't pronounced but the market crash of October 1987 changed all of that.

If one looks at option prices before and after October 1987, one will see a distinct break. Option prices began to reflect an "option risk premium" — a crash premium that comes from the experiences traders had in October 1987. After the crash the demand for protection rose and that lifted the prices for puts; especially out-the-money puts. To afford protection, investors would sell out-the-money calls. There is thus an over supply of right hand sided calls and demand for left hand sided puts — alas the skew. A skew represents the market's bias toward calls or puts.

The skew tells us there exists multiple implied volatilities for a single underlying asset. This should be somewhat disconcerting. How can the market be telling us that there is more than one volatility for the asset? The real phenomenon underlying volatility skews is that either [Ma 95]

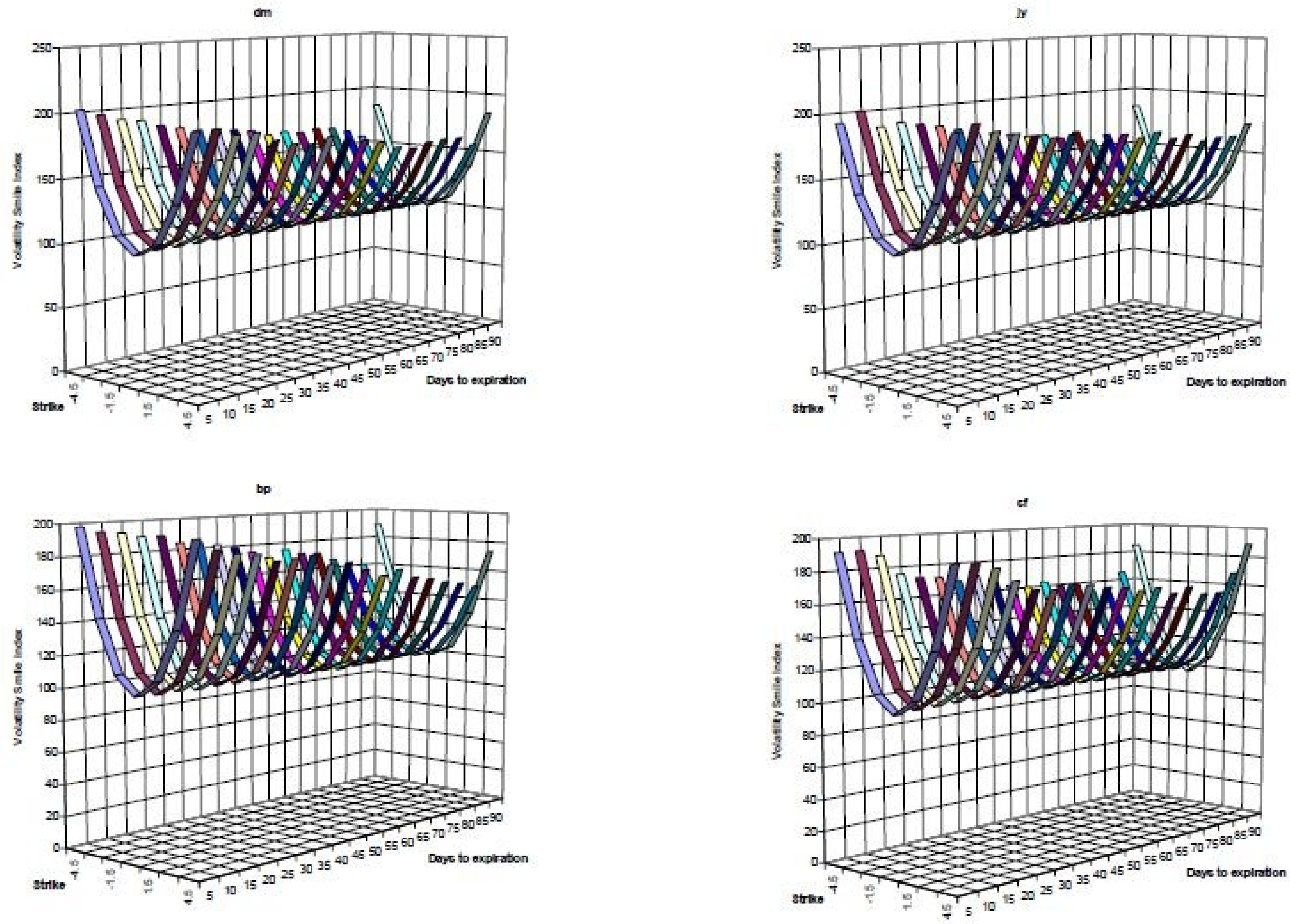


Figure 7.10: Volatility surfaces for currencies.

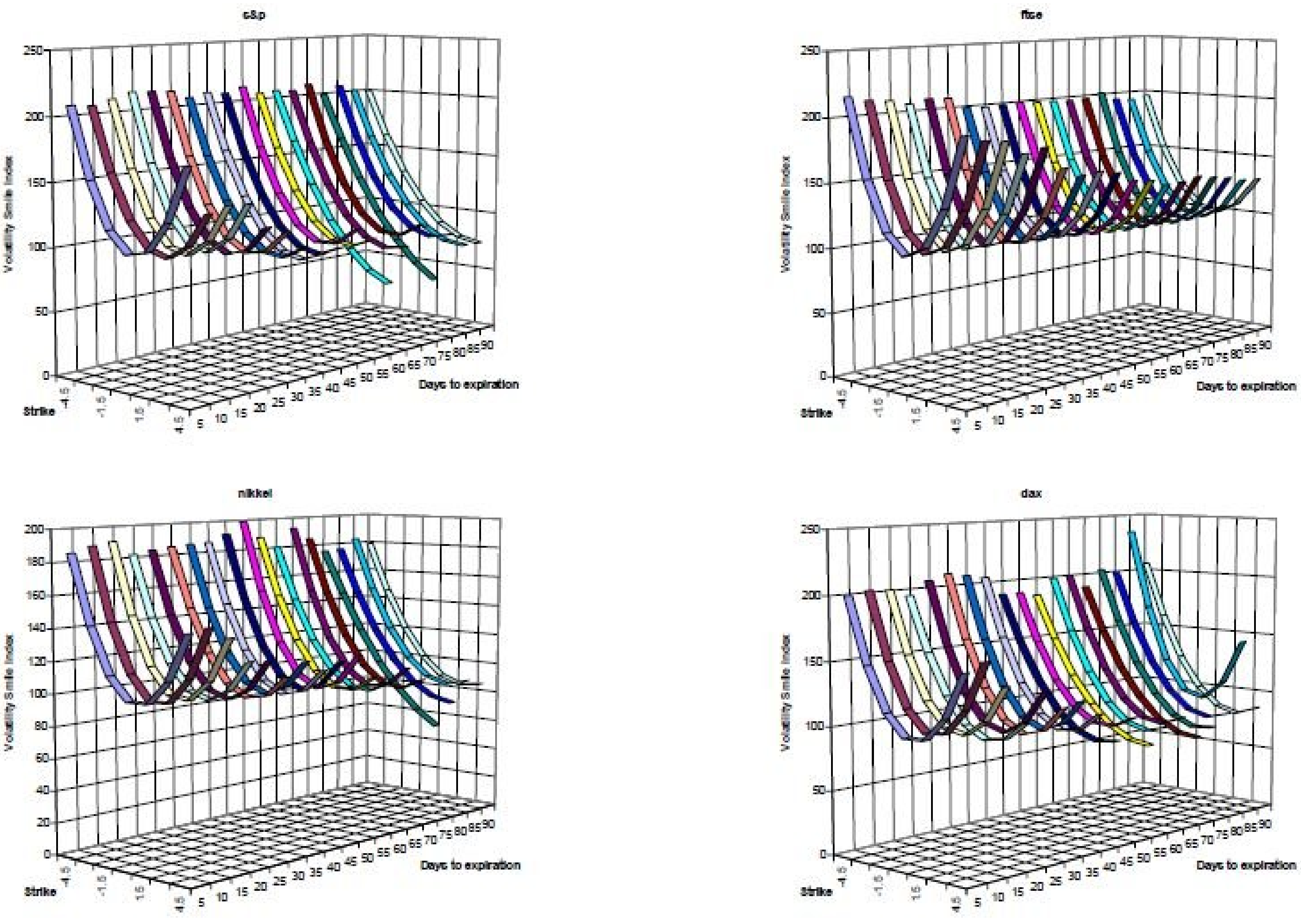


Figure 7.11: Volatility surfaces for stock indices.

- market imperfections systematically prevent prices from taking their true *Black & Scholes* values or
- the underlying asset price process differs from the lognormal diffusion process assumed by the *Black & Scholes* model[16].

These two points show us there is something wrong with the Black-Scholes model, which is that it fails to consider all of the factors that enter into the pricing of an option. It accounts for the stock price, the exercise price, the time to expiration, the dividends, and the risk-free rate. The implied volatility is more or less a *catch-all term*, capturing whatever variables are missing, as well as the possibility that the model is improperly specified or blatantly wrong. The volatility skew is thus the market's way of getting around Black and Scholes's simplifying assumptions about how the market behaves.

7.16.3 Shapes of the skew

There are three distinct shapes

- **Supply Skew**: The supply skew is defined by higher implied volatility for lower strikes and lower volatility for higher strikes. Supply refers to the natural hedging activity for the major players in the market who have a supply of something they need to hedge. Stock Index and interest rate markets have supply skews. The natural hedge for stock owners is to buy puts in order to protect the value of their "supply" of stock and sell calls to offset the puts' prices — collars. This natural action in the marketplace determines the structure of the skew. This type of skew is also known as a 'smirk'.
- **Demand Skew**: Demand skews have higher implied volatility at higher strikes and lower implied volatility at lower strikes. The natural hedger in demand markets is the end user. The "collar" for a demand hedger is to buy calls and sell puts. The grain markets and energy markets are good examples of demand skews. This type of skew is also known as a 'hockey stick'.
- **Smile Skew**: The third type of skew is called a smile skew. The smile skew is illustrated by higher implied volatility as you move away from the at-the-money strike. The at-the-money strike would have the lowest implied volatility while the strikes moving up and down would have progressively higher implied volatility. The smile skew is generally only observed in the currency markets. The natural hedger has to hedge currency moves in either direction depending of whether they have accounts payable or receivable in the foreign currency.

We show these shapes in Figs. 7.12 and 7.13. Note that the EURUSD smile is

[16]Stochastic volatility models closer to the truth.

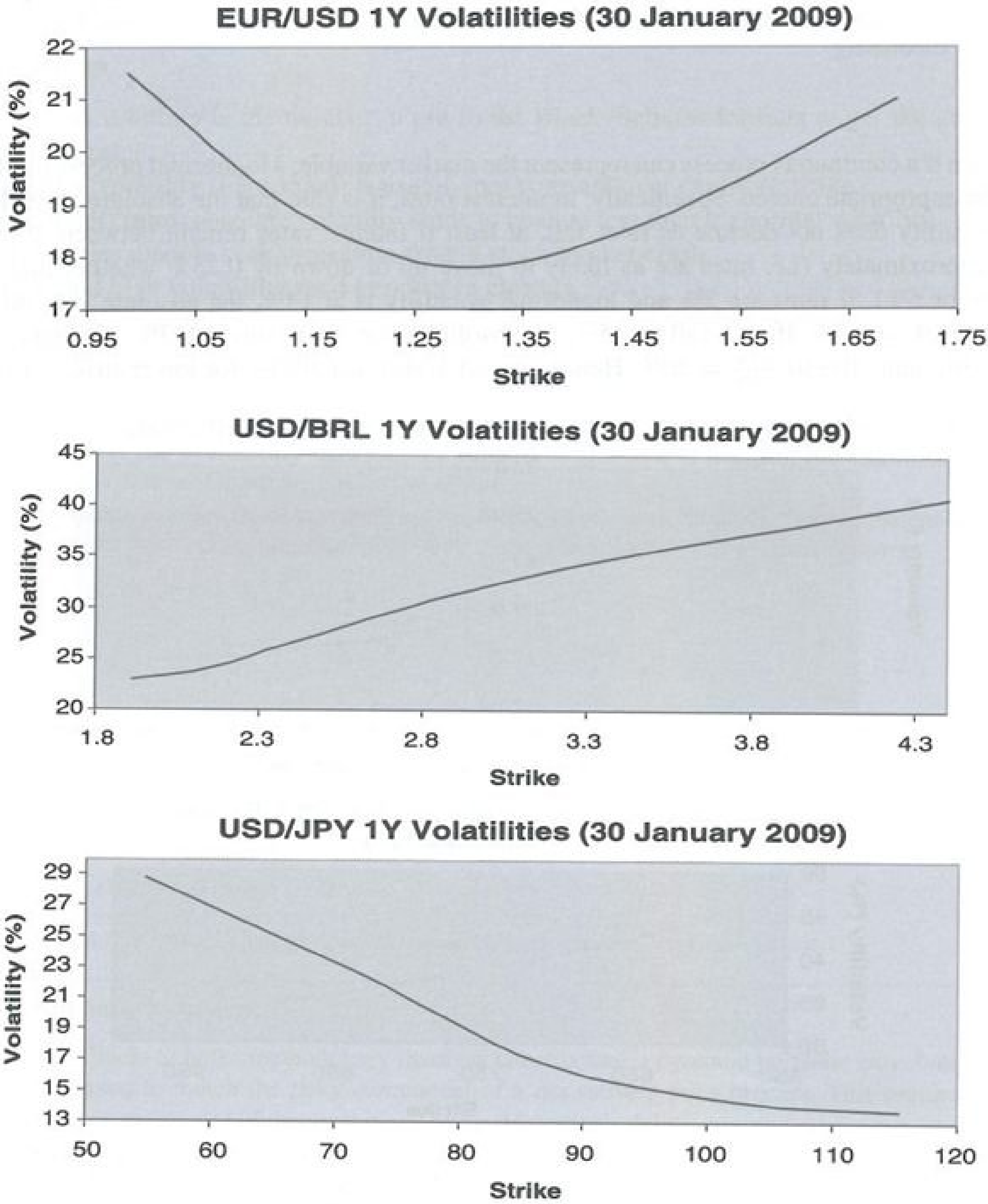


Figure 7.12: Different shape currency skews.

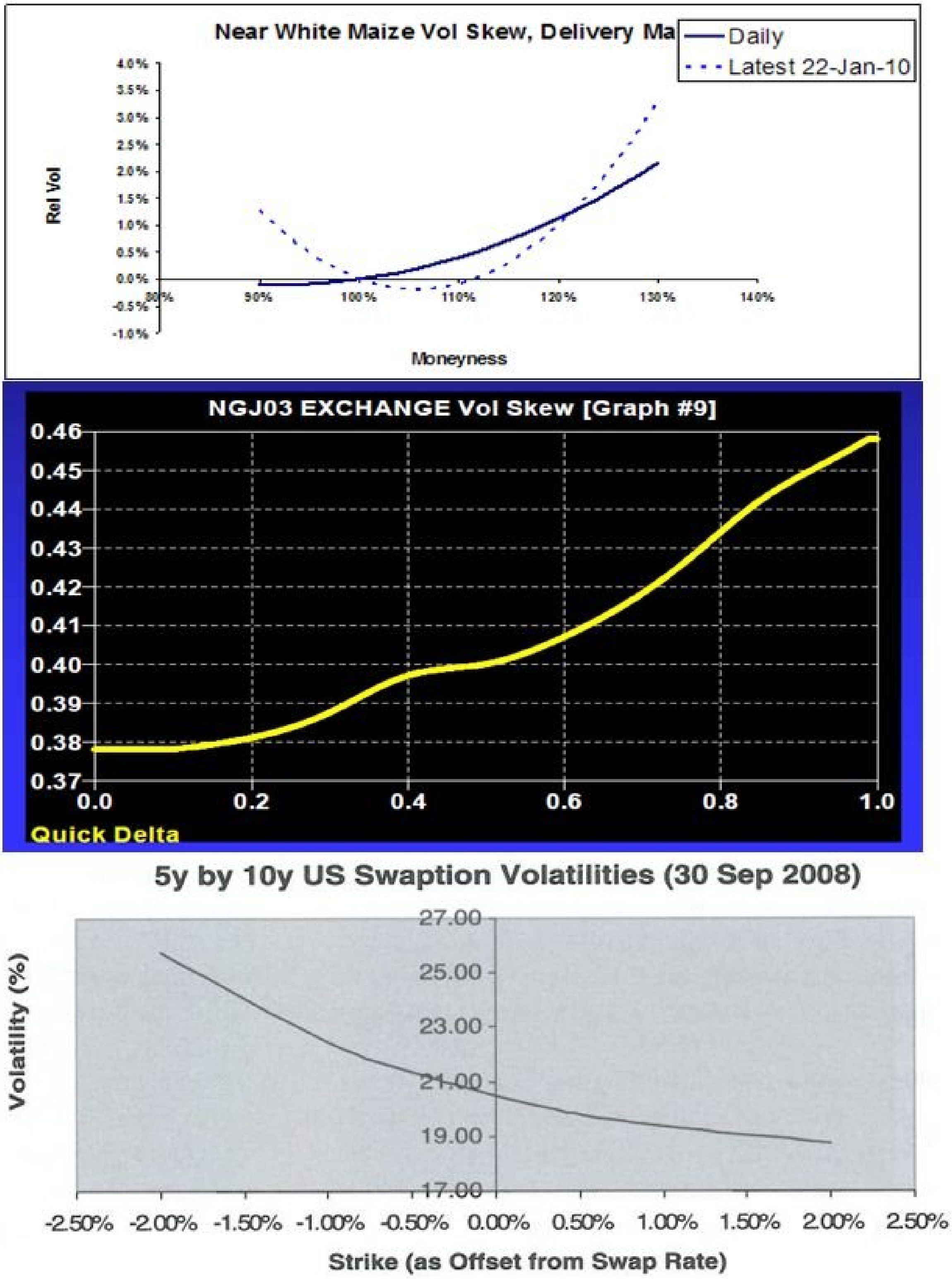


Figure 7.13: Skews for some other instruments: white maize, natural gas, swaptions and the S&P 500 index.

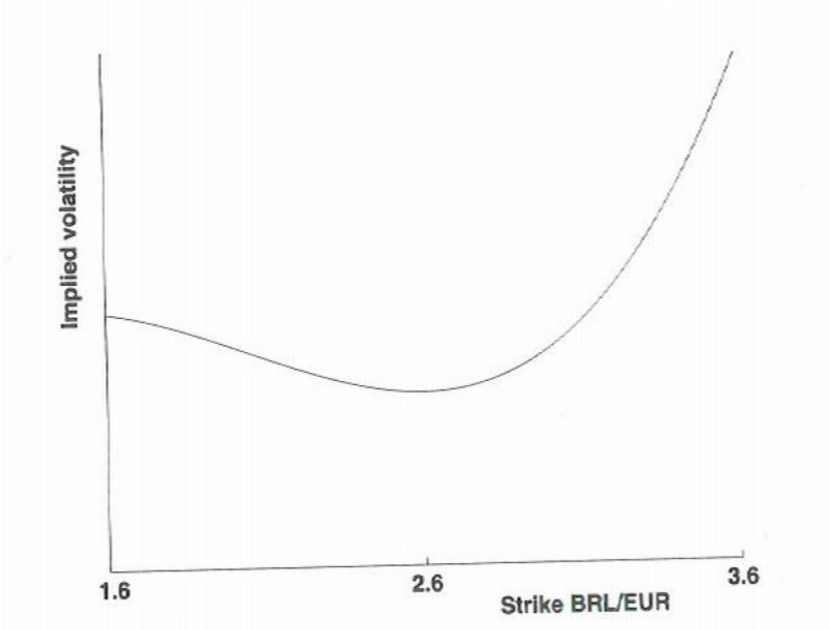


Figure 7.14: The smile for the BRLEUR which is not symmetrical.

symmetric around the ATM. Both currencies are perceived to be stable with similar risks. However, due to the currency being a pair, the risk is that either currency of the exchange rate can collapse. This means that the smile can be skewed to one side due to the country risk or stability of a particular exchange rate, e.g. the markets perceive the Kenyan country risk as being higher than the USD or Euro country risk. Look at the shape of the smile of the Brazilian Real and Euro in Fig. 7.14 [DW 08]. It is not symmetrical around the ATM. This was also shown in Fug. 7.12. This shows that while indices across the globe mostly have similar shaped skews, the skews for different currency pairs can be vastly different.

7.16.4 Delta Hedging and the Skew

Another view on the skew is the fact that if the markets go down they tend to become more volatile. Equity markets crash downward but hardly ever 'crash' or gap upward. However, the currency market with its smile do 'crash' upward and downward. This alone does not explain the skew as realised volatility is the same regardless of any strike price. The existence of the skew is apparently telling us that this increase in volatility has a bigger impact on lower strike options than on higher strike options.

The reason behind this becomes apparent when thinking in terms of realised gamma losses as a result of rebalancing the delta of an option in order to be delta hedged [DW 08]. In a downward spiraling market the gamma on lower strike option increases, which combined with a higher realised volatility causes the option seller to rebalance the portfolio more frequently, resulting in higher losses for the option seller.

Naturally the option seller of lower strike options wants to get compensated for this and charges the option buyer by assigning a higher implied volatility to lower strikes. This principle applies regardless of the of the in- or out-of-the-moneyness of an option. Whether it is a lower strike in-the-money call or a lower strike out-the-money put, makes no difference from a skew perspective. Indeed if there were a difference there would be an arbitrage opportunity.

7.16.5 The Term Structure of Volatility

Another aspect of volatility that is observed in the market is that at-the-money options with different expiries trade at different volatilities. The at-the-money volatilities for different expiry dates are usually decreasing in time meaning shorter dated options trade at higher volatilities to longer dated ones. However, since the 2008 financial crisis we see the term structure tend to slope upward more often. This provides another method for traders to gauge cheap or expensive options. A downward sloping term structure is natural in the market because short dated downside options need to be delta hedged more often resulting in higher losses — downside short dated options have higher Gammas than long dated ones.

The term structure of volatility arises partly because implied volatility in short options changes much faster than for longer options and partly due to the assumed mean reversion of volatility. The effect of changes in volatility on the option price is also less the shorter the option.

It is well-known that volatility is mean reverting; when volatility is high (low) the volatility term structure is downward (upward) sloping. We therefore postulate the following functional form for the ATM volatility term structure

$$\sigma_{atm}(\tau) = \frac{\theta}{\tau^{\lambda}}. \tag{7.7}$$

Here we have

- τ is the months to expiry,
- λ controls the overall slope of the ATM term structure; $\lambda > 0$ implies a downward sloping ATM volatility term structure, whilst a $\lambda < 0$ implies an upward sloping ATM volatility term structure, and
- θ controls the short term ATM curvature.

We show the current term structure for USDZAR in Fig. 7.16.5)

7.16.6 What is a Volatility Surface?

Combining the ATM term structure of volatility and the skew per expiry date, will render a 3 dimensional graph (time to expiry versus strike versus volatility). This is known as the volatility surface.

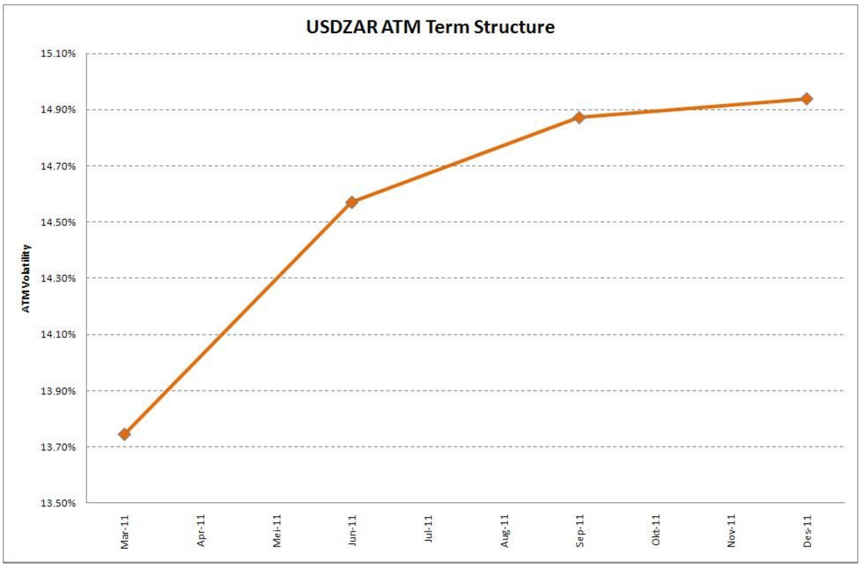


Figure 7.15: The USDZAR market fitted at-the-money volatility term structure during February 2011.

7.16.7 Skews in South Africa

Index Futures

In Fig. 7.16 we show the February 2011 volatility surface for the ALSI (index futures) contracts traded on Safex. The index option market is liquid enough such that Safex can generate these skews from traded data[17]. The methodology was developed and implemented by *Kotzé* and *Joseph* [KJ 09]. *Kotzé* and *Joseph* fits the data with the following functional form

$$\sigma_{model}(\beta_0,\, \beta_1,\, \beta_2) = \beta_0 + \beta_1\, K + \beta_2\, K^2. \tag{7.8}$$

In this equation we have

- K is the strike price in moneyness format (Strike/Spot),
- β_0 is the constant volatility (shift or trend) parameter, $\beta_0 > 0$. Note that
$$\sigma \underset{K\to 0}{\longrightarrow} \beta_0,$$
- β_1 is the correlation (slope) term. This parameter accounts for the negative correlation between the underlying index and volatility. The no-spread-arbitrage condition requires that $-1 < \beta_1 < 0$ and,
- β_2 is the volatility of volatility ('vol of vol' or curvature/convexity) parameter. The no-calendar-spread arbitrage convexity condition requires that $\beta_2 > 0$.

Note that Eq. (7.8) is also linear in the wings as $K \to \pm\infty$.

[17] http://www.jse.co.za

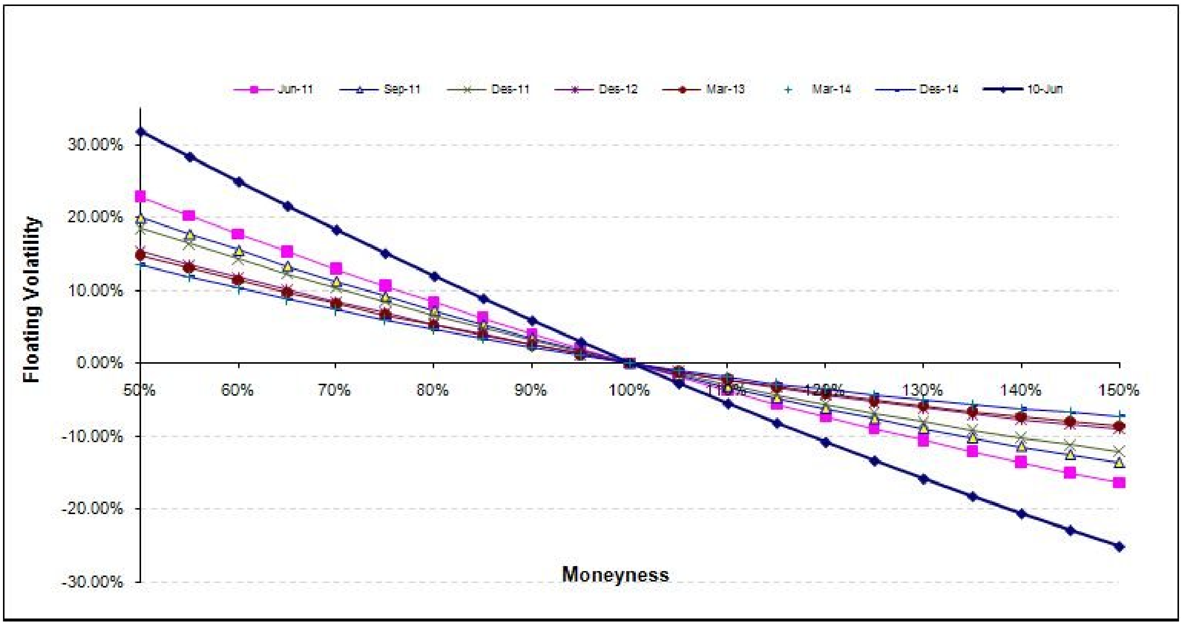


Figure 7.16: ALSI volatility surface during February 2011.

Currency Futures

In Table (7.4) we give the February 2011 volatility surface for the USDZAR contracts traded on Yield-X. We show the skew on a relative or floating basis where the ATM strikes are given as 100% and the ATM floating volatilities are given as 0%. The first column shows the percentage of moneyness[18] and the second column shows the relative volatility i.e., what number has to be added or subtracted from the ATM volatility to give the real volatility.

The current skews are plotted in Figure 7.17. Yield-X supplies the skews on a monthly basis. The currency option market in South Africa is new and the liquidity not great. Yield-X does not generate these itself. They are supplied by a London based company called 'Super Derivatives[19].' There are other companies who supply skew data to market players. Another one active in the South African market is called 'Markit[20].'

So what happens if the strike price lies between two strikes given in the skew? Then we have to interpolate. We do straight line linear interpolation and this is explained next.

[18]Moneyness shows how far the strike is from the ATM strike.
[19]https://www.superderivatives.com/
[20]http://www.markit.com/en/?

Moneyness	Mar-11	Jun-11	Sep-11	Des-11
70%	3.515%	8.263%	5.376%	3.556%
75%	4.873%	5.327%	3.542%	1.929%
80%	6.745%	2.634%	1.618%	0.535%
85%	7.460%	0.561%	0.062%	-0.439%
90%	5.655%	-0.561%	-0.687%	-0.815%
95%	2.431%	-0.704%	-0.628%	-0.618%
100%	0.000%	0.000%	0.000%	0.000%
105%	0.082%	1.401%	0.982%	0.900%
110%	2.404%	3.266%	2.215%	1.996%
115%	6.193%	5.340%	3.625%	3.215%
120%	10.678%	7.372%	5.137%	4.485%
125%	15.087%	9.106%	6.678%	5.731%
130%	18.660%	10.298%	8.173%	6.887%

Table 7.4: The official floating Yield-X volatility skew for USDZAR during February 2011

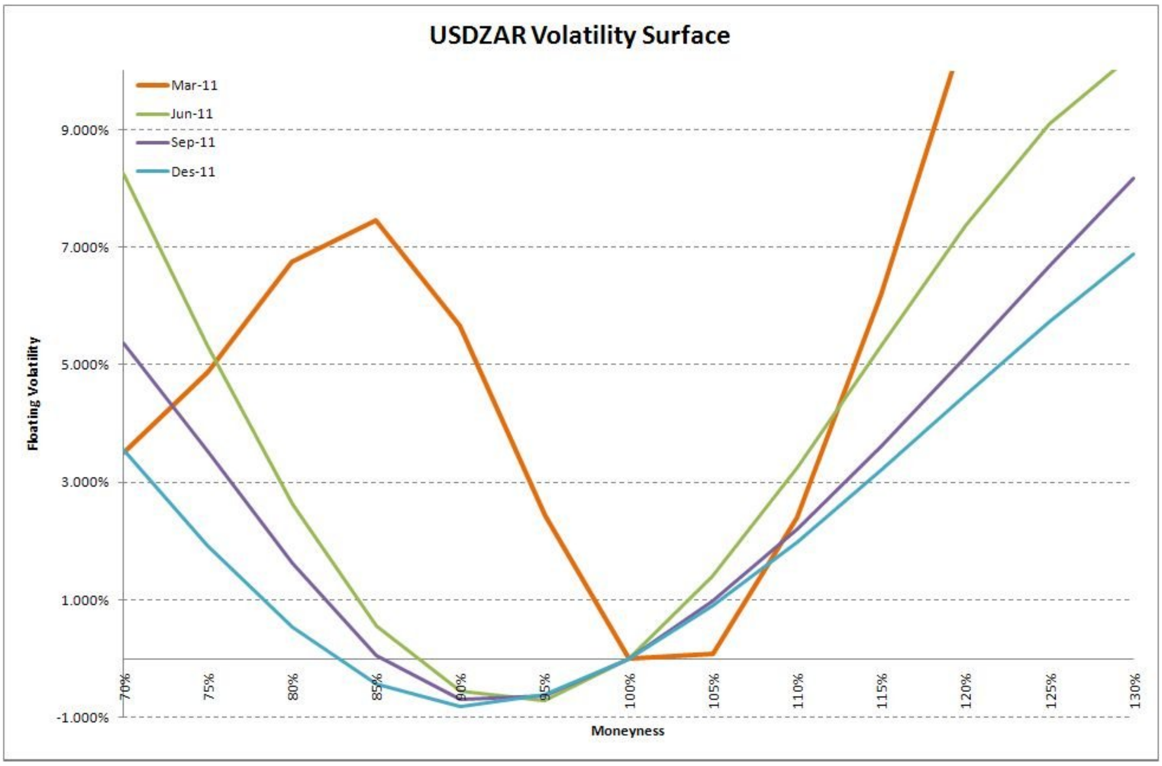


Figure 7.17: USDZAR volatility surface during February 2011.

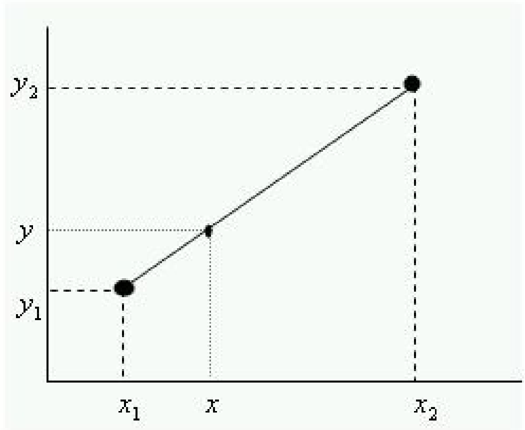


Figure 7.18: Linear interpolation between 2 points.

7.17 Linear Interpolation

In the mathematical subfield of numerical analysis, interpolation is a method of constructing new data points within the range of a discrete set of known data points. In engineering and science one often has a number of data points, as obtained by sampling or experimentation, and tries to construct a function which closely fits those data points. This is called curve fitting or regression analysis. Interpolation is a specific case of curve fitting, in which the function must go exactly through the data points[21].

There are many methods of interpolation like polynomial, spline and linear. However, linear interpolation is one of the simplest interpolation methods and used widely in the markets. In Figure (7.18) we plot a general xy-cartesian plane. Also drawn are two points P_1 and P_2 with coordinates (x_1, y_1) and (x_2, y_2) respectively. Let's assume these coordinates are known. The formula describing the line between P_1 and P_2 is obtained from plane analytic geometry and is given by

$$y = mx + c. \tag{7.9}$$

It can be shown that

$$c = y_1 - mx_1$$

and

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

with m the slope of the line and c the y-axis intercept. By using Eq. (7.8) we can calculate the coordinates of any point that lies on the straight line between P_1 and P_2.

[21] http://en.wikipedia.org/wiki/Interpolation

If the volatility surface is given as a set of discrete points like that shown in Table 7.4 and we need a volatility that is not explicitly given, we need to interpolate to obtain it. The simplest and easiest way is to interpolate the data as given: Here the strike prices would be on the x-axis and the volatilities on the y-axis. Supply any strike and determine the corresponding volatility. We can also use linear interpolation across time to obtain a new skew between two known skews. Is this correct though?

Interpolation across Strikes

As can be seen by looking at the skew graphs, we ascertain that the volatility is not linear in the strike. Market practitioners use the logarithmic strike which is linear in the variance. Thus, instead of using the strike price K, use $\ln(K)$ on the x-axis and σ^2 on the y-axis.

Interpolation across Time

Using naive linear interpolation with regard to time can lead to unrealistic forward volatility dynamics or simply negative forward volatilities. Even interpolation on the variance can lead to negative forward volatilities. A scheme that works well is the *flat forward interpolation* methodology.

Let's assume at time points t_i we have corresponding implied volatilities denoted by σ_i where $i = 1, 2, 3, \ldots, N$. We now want to obtain a volatility for a time point t where $t \neq t_i$. The flat forward volatility interpolator is given by [Cl 11]

$$\sigma(t) = \begin{cases} \sigma_1 & t < t_1, \\ \sqrt{\frac{1}{t}\left[\sigma_i^2 t_i + \sigma_{i,i+1}^2 (t - t_i)\right]} & t_i \leq t < t_{i+1} \text{ for } i < N, \\ \sigma_N, & t \geq t_N \end{cases} \tag{7.10}$$

where

$$\sigma_{i,i+1}^2 = \frac{\sigma_{i+1}^2 t_{i+1} - \sigma_i^2 t_i}{t_{i+1} - t_i}. \tag{7.11}$$

7.18 Sticky Volatilities

Sticky strike and sticky delta are trader phrases to describe the behaviour of volatility when the price moves. It can be shown that both the sticky Delta and sticky strike rules produce arbitrage opportunities should the volatility surface move as predicted by them. This is the reason why they are mainly regarded as quoting mechanisms and not expressions of actual behaviours of volatility surfaces [Ca 10].

Delta	Mar-11	Jun-11	Sep-11	Des-11
Δ_{25}^{p}	1.4873%	-0.7198%	-0.0999%	-0.3640%
Δ_{30}^{p}	1.1899%	-0.5758%	-0.0799%	-0.2912%
Δ_{35}^{p}	0.8924%	-0.4319%	-0.0600%	-0.2184%
Δ_{40}^{p}	0.5949%	-0.2879%	-0.0400%	-0.1456%
Δ	0.0000%	0.0000%	0.0000%	0.0000%
Δ_{40}^{c}	0.0579%	0.7247%	1.3071%	3.0039%
Δ_{35}^{c}	0.0672%	1.0569%	1.6176%	3.0032%
Δ_{30}^{c}	0.0764%	1.3891%	1.9281%	3.0026%
Δ_{25}^{c}	0.0857%	1.7213%	2.2386%	3.0020%

Table 7.5: The official Yield-X volatility skew for USDZAR for February 2011 using Deltas instead of moneyness

7.18.1 Sticky Delta

In the sticky delta model (also known as a relative or floating skew), the implied volatility depends on the moneyness only (spot divided by strike - S/K). The ATM implied volatility does not change as the underlying spot changes. This entails that the smile floats with moneyness (or Delta) as spot is shifted such that the delta of options are preserved. This means that when the underlying asset's price moves and the Delta of an option changes accordingly, a different implied volatility has to be used in the *Black & Scholes* model. In this model, moneyness is plotted against relative volatility (difference in volatility from the ATM volatility). The sticky-delta rule quantifies the intuition that the current level of at-the-money volatility — the volatility of the most liquid options — should remain unchanged as spot changes. We listed such a skew in Table 7.4. We show the same skew in Table 7.5 where we map the volatilities against the put Deltas on the left of the ATM and call Deltas on the right of the ATM.

7.18.2 Sticky Strike

In a sticky strike model ("absolute skew"), the implied volatility of each option is constant as the spot changes or the volatility of a given strike is unaffected by a change in price. Another way to put this is that skew is kept fixed at strikes as the spot is shifted. This means the volatility is independent of the spot, it depends on the strike only. A sticky strike skew plots volatility against actual strikes. In Table 7.6 we give the February USDZAR skew for options traded on Yield-X. This is the same skew as that listed in Table 7.4. Intuitively, "sticky strike" is a poor man's attempt to preserve the Black-Scholes model. It allows each option an independent existence, and doesn't worry about whether the collective options market view of the spot is consistent.

7.18.3 Which is better: sticky strike or sticky delta?

There is no conclusion yet although market players seem to prefer the sticky strike model. *Rubinstein* and *Jackwerth* in 1997 compared several models and found that sticky-strike best predicts future smiles [JR 96]. However, *Derman* found in 1999 that market conditions should set the tone [De 99].

Sticky Delta

He found that if the markets are trending, where the market is undergoing significant changes in levels without big changes in realized volatility, sticky delta rules [De 99]. Then, in the absence of a change in risk premium or an increased probability of jumps, the realised volatility will be the dominant input to the estimation of the implied volatility of (high-Gamma) at-the-money options. As the underlying moves to new levels, it is sensible to re-mark the current at-the-money implied volatility to the value of the previous at-the-money volatility. The at-the-money volatility "stick" to the ATM spot level.

Sticky Strike

Sticky strike rules if the market trades in a range (jumps are unlikely) without a significant change in the current realised volatility. As markets are range bound most of the time, sticky strike is the most common rule used.

Daglish, *Hull* and *Suo* concluded in 2006 that all versions of the sticky strike rule are inconsistent with any type of volatility smile or volatility skew [DHS 06]. They state that "If a trader prices options using different implied volatilities and the volatilities are independent of the asset price, there must be arbitrage opportunities." They further found that the relative sticky delta rule can be at least approximately consistent with the no-arbitrage condition.

Strike	14-Mar-11	13-Jun-11	19-Sep-11	19-Des-11
6.7500	16.5626	13.8135	14.1262	14.1205
6.8000	16.1105	13.8181	14.1295	14.1259
6.8500	15.6748	13.8394	14.1455	14.1409
6.9000	15.2617	13.8770	14.1736	14.1653
6.9500	14.8770	13.9308	14.2133	14.1985
7.0000	14.5270	14.0002	14.2639	14.2403
7.0500	14.2177	14.0850	14.3248	14.2902
7.1000	13.9552	14.1849	14.3954	14.3480
7.1500	13.7455	14.2994	14.4751	14.4131
7.2000	13.5934	14.4282	14.5633	14.4854
7.2500	13.4980	14.5710	14.6593	14.5643
7.3000	13.4574	14.7274	14.7626	14.6495
7.3500	13.4693	14.8968	14.8725	14.7407
7.4000	13.5317	15.0785	14.9885	14.8375
7.4500	13.6424	15.2718	15.1104	14.9395
7.5000	13.7994	15.4761	15.2379	15.0464
7.5500	14.0005	15.6908	15.3709	15.1579
7.6000	14.2436	15.9151	15.5092	15.2739
7.6500	14.5266	16.1483	15.6526	15.3942
7.7000	14.8474	16.3899	15.8009	15.5187
7.7500	15.2040	16.6391	15.9539	15.6471
7.8000	15.5940	16.8954	16.1114	15.7793
7.8500	16.0156	17.1579	16.2733	15.9151
7.9000	16.4664	17.4261	16.4393	16.0544
7.9500	16.9446	17.6993	16.6094	16.1969
8.0000	17.4478	17.9769	16.7832	16.3424

Table 7.6: The official sticky strike Yield-X volatility skew for USDZAR during February 2011

Chapter 8

Option Strategies

In §2.7 we discussed the various hedging instruments available to investors who want to hedge a certain liability in the foreign exchange market. Further, in §2.8 we gave some guidelines for corporates in emerging markets who want to hedge FX exposures. We also mentioned a study showing that many corporation do use FX options in their hedging activities.

However, before you buy or sell options you need a strategy, and before you choose an options strategy, you need to understand how you want options to work in your portfolio. A particular strategy is successful only if it performs in a way that helps you meet your investment goals. If you hope to increase the income you receive from the underlying asset, for example, you'll choose a different strategy from an investor who wants to hedge a position in the underlying asset.

Generally, an *Option Strategy* involves the simultaneous purchase and/or sale of different option contracts (known as an Option Combination), and possibly an underlying position. Options strategies can favour movements in the underlying that are bullish, bearish or neutral. In the case of neutral strategies, they can be further classified into those that are bullish on volatility and those that are bearish on volatility. The option positions used can be long and/or short positions in calls and/or puts at various strikes.

Option strategies can complement portfolios in many different ways. So it's worth taking the time to identify a goal that suits your view on the markets or your end gaol. Once you've chosen a goal, you'll have narrowed the range of strategies to use. As with any type of investment, only some of the strategies will be appropriate for your objective.

In this chapter we will discuss some of the basic option strategies and where they are applicable.

8.1 Introduction

We have now learnt much about single options. The next question is: can we put options with different strikes or different maturities together? If you are long a certain currency pair, and you want to hedge against market moves, is your only option to buy a put or are there other ways in which to use not just one, but a few options? If we have a certain market view, are there any time proven option strategies that can help you?

The answer to all these questions is yes. Options are very versatile and used in combination with the underlying instruments give powerful strategies that can help you to hedge a portfolio, get synthetic exposure to the market or gear a position. However, in order to make a sensible decision as to which strategy is the right one, you'll first need to make a careful assessment of your current financial and psychological situation, your market outlook and your investment goals. Your strategy selection should then focus on only those positions that are consistent with all of these benchmarks.

We have already discussed put-call-parity. This can be seen as a strategy to convert a call into a put and a put into a call.

8.2 Basic Strategies

8.2.1 Buying Calls

Buying just a call can be a strategy. Who should consider buying calls:

1. An investor who is bullish on a particular asset.
2. An investor who would like to take advantage of what purchasing options offers: leverage with a limited risk.
3. An investor who anticipates a rise in the value of a particular asset but does not want to commit all of the capital needed to purchase the asset.

Buying a call is a simple and popular option strategy. Although buying calls may not be suitable for everyone, it allows the investor the opportunity to profit from an upward move in the underlying stock while having very little capital at risk compared to the amount needed to own the stock.

Call buyers need to balance their particular risk and reward tolerances and recognize the differences when choosing strike prices. Before deciding on the purchase of a particular strike price, the individual should have sincere expectations on the underlying equity. For those who are bullish on a particular stock, buying calls is a viable alternative strategy to use. The owner of a call has the opportunity to profit from a rise in the stock with very little capital at stake compared to the amount

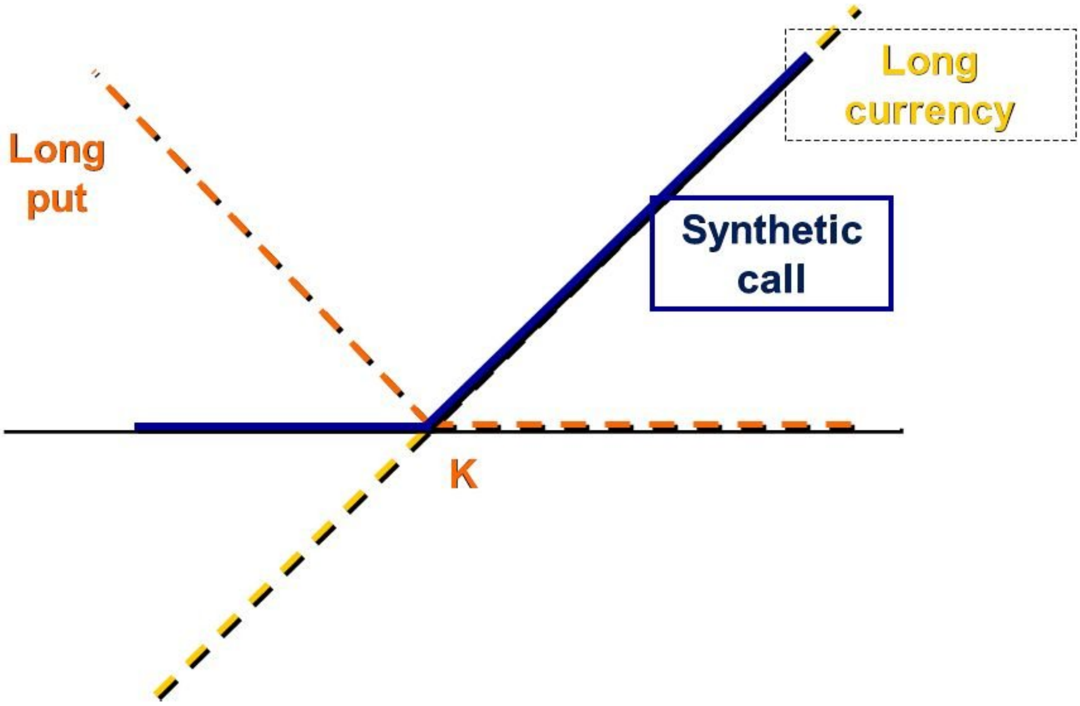


Figure 8.1: Put strategy.

necessary to buy the underlying security. Choosing the right strike price can be as important as choosing the right stock.

8.2.2 Buying Puts

Buying a put can also be a strategy. Puts are most often used for hedging a long position in the underlying. The primary motivation of an investor who buys a naked put is to realize financial reward from a decrease in price of the underlying security. This investor is generally more interested in the dollar amount of his initial investment and the leveraged financial reward that long puts can offer than in the number of contracts purchased.

Experience and precision are key in selecting the right option (expiration and/or strike price) for the most profitable result. In general, the more out-of-the-money the put purchased is the more bearish the strategy, as bigger decreases in the underlying stock price are required for the option to reach the break-even point.

In Fig. 8.1 we show the resultant payoff profile for a strategy consisting out of a long put together with a long position in the underlying.

8.2.3 Bull and Bear Spreads

A bull call spread is defined as the purchase of a call and the simultaneous sale of another call on the same underlying asset with a higher strike and the same expiration (the second strike is called the cap). A bull put spread is defined as the selling of a put and the simultaneous purchase of another put on the same underlying equity with a lower strike and the same expiration. In applying these strategies, the investor hopes that the stock price will appreciate.

Here the investor takes advantage of selling time premium as an efficient means to effect his strategy. Using options as the tools to enact the specific strategies while at the same time quantifying and limiting risk shows the value that options can add to a portfolio. For the neutral to mildly bullish investor, bull spreads afford lower breakevens and lower risk and that's music to an investor's ears.

The reverse of a bull spread is a bear spread. Here the investor hopes that the stock price will decline. We show all four types of spreads in Fig. 8.2.

For example a company wants to buy 1 Million EUR. At maturity T

1. If $S_T < K_1$, it will not exercise the option. The overall loss will be the option's premium. But instead the company can buy EUR at a lower spot in the market.
2. If $K_1 < S_T < K_2$, it will exercise the option and buy EUR at strike K_1.
3. If $S_T > K_2$, it will buy the 1 Million EUR at a rate $K_2 - K_1$ below S_T.

Let's look at an example [Wy 06]: A company wants to hedge receivables from an export transaction in USD due in 12 months time. It expects a stronger EUR/weaker USD. The company wishes to be able to buy EUR at a lower spot rate if the EUR weakens on the one hand, but on the other hand be protected against a stronger EUR. The Vanilla Call is too expensive, but the company does not expect a large upward movement of the EUR.

In this case a possible form of protection that the company can use is to buy a Call Spread as for example listed in Table 8.1. If the company's market expectation is correct, it can buy EUR at maturity at the strike of 1.1400. If the EURUSD exchange rate is below the strike at maturity the option expires worthless. However, the company would benefit from a lower spot when buying EUR. If the EURUSD exchange rate is above the short strike of 1.1800 at maturity, the company can buy the EUR amount 400 pips below the spot. Its risk is that the spot at maturity is very high.

The EUR seller can buy a EUR Put Spread in a similar fashion.

8.2.4 Risk Reversal

A risk reversal is also known as a zero-cost-collar. This strategy is implemented if the investor wants to hedge his international cash flows. Since buying a call requires

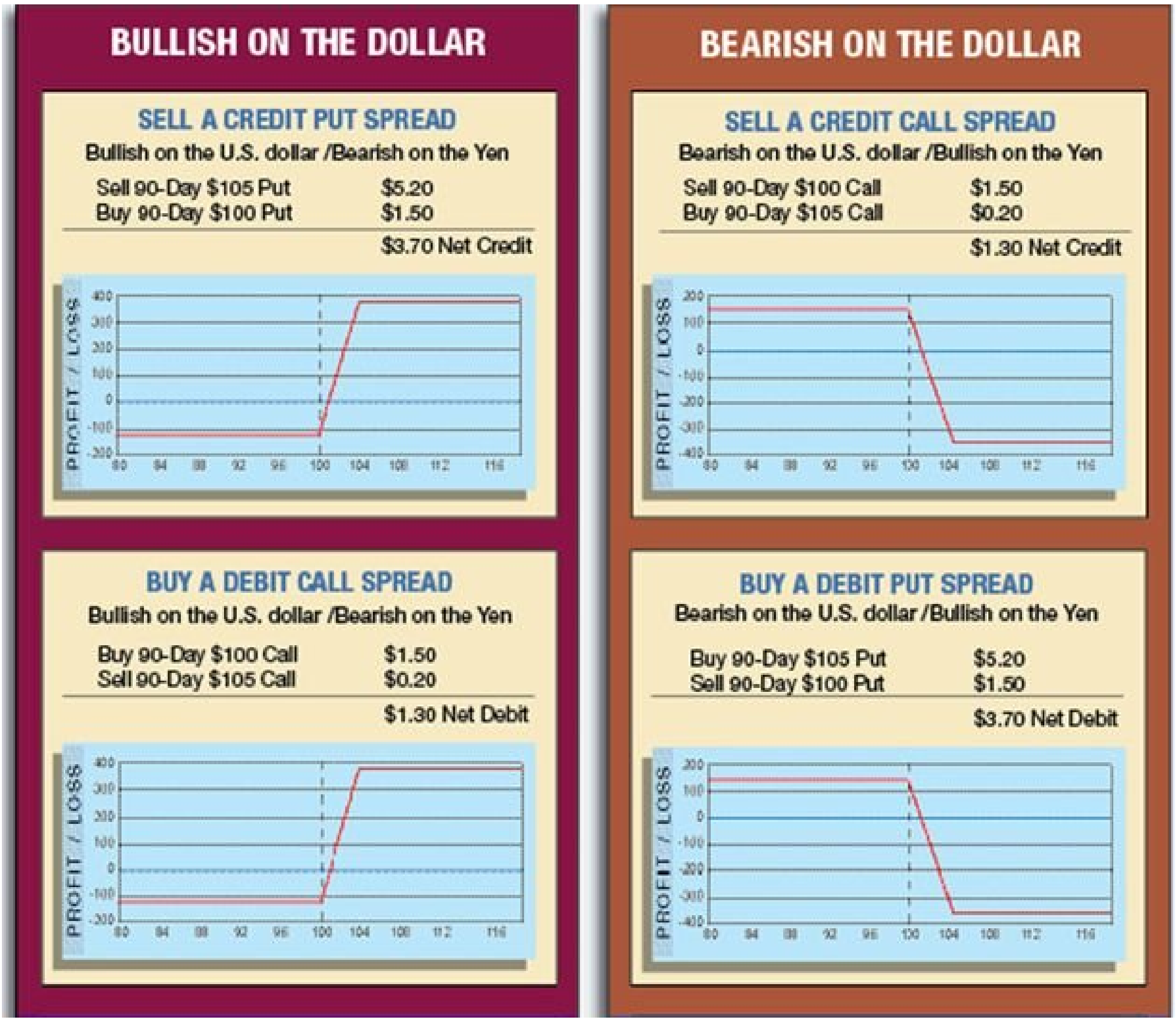


Figure 8.2: Bull and Bear spreads.

Spot reference	1.1500 EUR-USD
Company buys	EUR call USD put with lower strike
Company sells	EUR call USD put with higher strike
Maturity	1 year
Notional of both the Call option	EUR 1,000,000
Strike of the long Call option	1.1400 EUR-USD
Strike of the short Call option	1.1800 EUR-USD
Premium	EUR 14,500.00
Premium of the long EUR call only	EUR 40,000.00

Table 8.1: Hedging with a bull call spread.

a premium, the buyer can sell another options to finance the call's premium. The investor buys a call option and short a put option. The strike of the call is determined by ensuring that the call premium is the same as the put premium — total cash flow thus zero.

This strategy entitles the holder to buy an agreed amount of currency (say KES) on a specified date (maturity) at a pre-defined rate (long strike) assuming the exchange rate is above the long strike at maturity [Wy 06]. However, if the FX rate is below the strike of the short put at maturity, the holder is obliged to buy the amount of KES at the short strike. Therefore, buying a risk reversal provides full protection against rising KES.

The portfolio is thus fully hedged, but in return for the lower risk, the investor gives up some of the upside potential — he caps his upside returns. Due to the zero-cost nature of these strategies, they are very popular — especially when the market turns bearish. In Fig. 8.3 we show the general risk reversal where we are short the put and long the call only. Secondly, in Fig. 8.4 we show the situation where we are long the underlying currency and then we buy the put and short the call.

The information content of risk reversals is very useful to any market participant. Firstly, they are a representation of the market's expectations on the exchange-rate direction. Filtered properly, risk reversals can generate profitable overbought and oversold signals. Risk reversals are quoted in terms of the difference in volatility between the two options. While in theory these options should have the same implied volatility, in practice they often differ in the market. A positive number indicates that calls are preferred to puts and that the market is expecting a move up in the

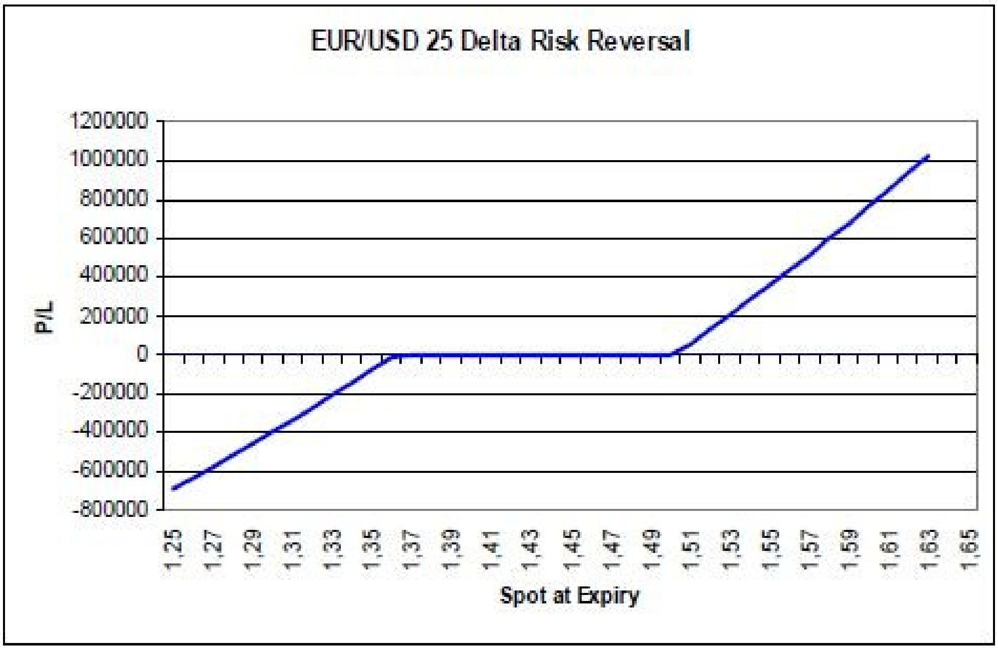


Figure 8.3: The general risk reversal.

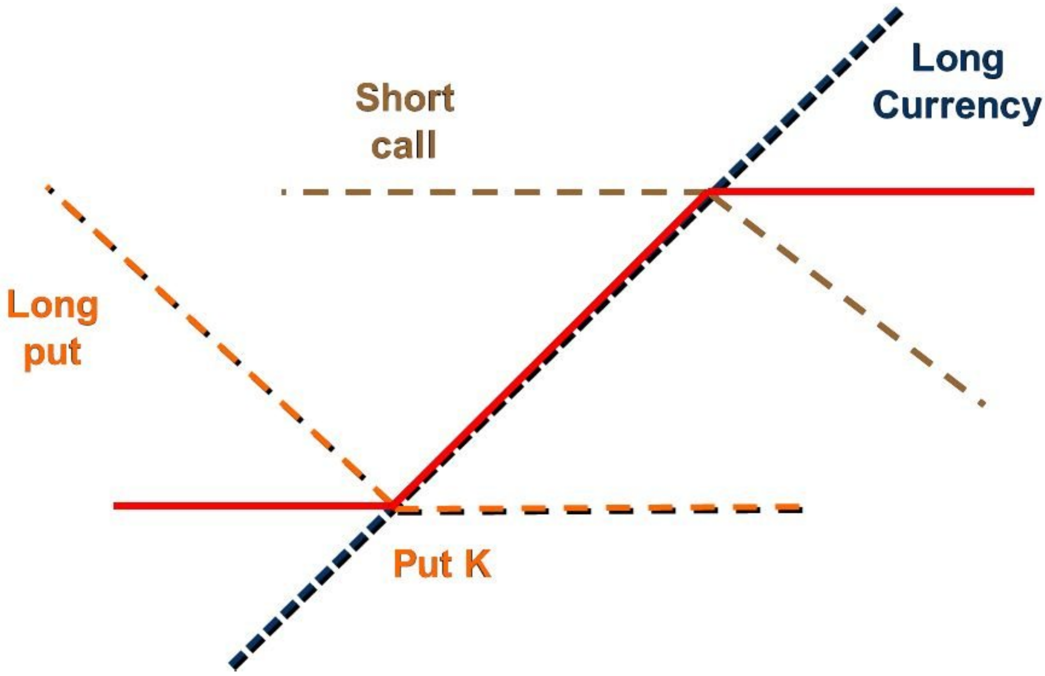


Figure 8.4: The zero-cost spread or collar.

underlying currency. Likewise, a negative number indicates that puts are preferred to calls and that the market is expecting a move down in the underlying currency.

Secondly, they convey the most information when they are at relatively extreme values. These extreme values are commonly defined as one standard deviation beyond the averages of positive and negative values. Therefore, we are looking at values one standard deviation below the average of negative risk-reversal figures, and values one standard deviation above the average of positive risk-reversal figures. When risk reversals are at these extreme values, they give off contrarian signals; when the entire market is positioned for a rise in a given currency, it makes it that much harder for the currency to rally, and that much easier for it to fall on negative news or events. A large positive risk-reversal number implies an overbought situation, while a large negative risk-reversal number implies an oversold situation. The buy or sell signals produced by risk reversals are not perfect, but they can convey additional information used to make trading decisions.

For example, a company wants to sell 1 Million USD. At maturity T:

1. If $S_T < K_1$, it will be obliged to sell USD at K_1. Compared to the market spot the loss can be large. However, compared to the outright forward rate at inception of the trade, K_1 is usually only marginally worse.

2. If $K_1 < S_T < K_2$, it will not exercise the call option. The company can trade at the prevailing spot level.

3. If $S_T > K_2$, it will exercise the option and sell USD at strike K_2.

Let's look at an example [Wy 06]: A company wants to hedge receivables from an export transaction in USD due in 12 months time. It expects a stronger EUR/weaker USD. The company wishes to be fully protected against a stronger EUR. But it finds that the corresponding plain vanilla EUR call is too expensive and would prefer a zero cost strategy by financing the call with the sale of a put. In this case a possible form of protection that the company can use is to buy a Risk Reversal as for example indicated in Table 8.2. If the company's market expectation is correct, it can buy EUR at maturity at the strike of 1.2250. The risk is when EUR-USD exchange rate is below the strike of 1.0775 at maturity, the company is obliged to buy 1 Mio EUR at the rate of 1.0775. The strike at 1.2250 is the guaranteed worst case, which can be used as a budget rate.

8.3 Neutral Strategies

These are shown in Fig. 8.5.

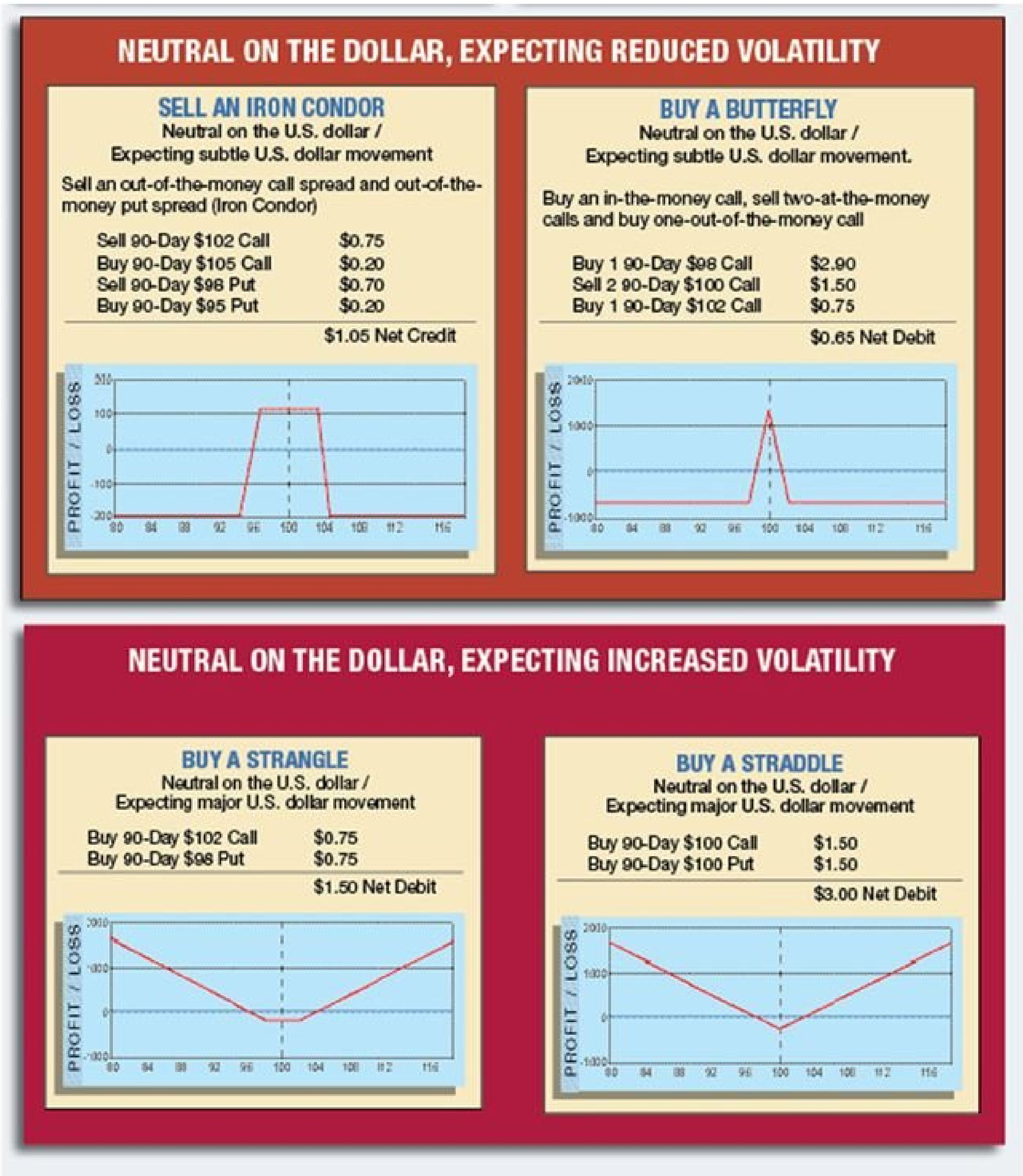


Figure 8.5: Neutral strategies.

Spot reference	1.1500 EUR-USD
Company buys	EUR call USD put with higher strike
Company sells	EUR put USD call with lower strike
Maturity	1 year
Notional of both the Call option	EUR 1,000,000
Strike of the long Call option	1.2250 EUR-USD
Strike of the short Put option	1.0775 EUR-USD
Premium	EUR 0.00

Table 8.2: Example of a risk reversal.

8.3.1 Straddles

A *bottom straddle* involves buying a call and put with the same strike and expiration date. This is a volatility play because the investor hopes that the stock price will have large moves. The direction does not matter - either way he makes money. If the asset price is close to the strike at expiry, he looses his initial premium. They are quite expensive though.

A *top straddle* is the opposite. Here the investor sells the call and put. This strategy is a high risk one. Why? The investor hopes for a sideways move in the FX rate.

For example, a company has bought a Straddle with a nominal of 1 Million EUR. At maturity T:

1. If $S_T < K$, it would sell 1 Mio EUR at strike K.
2. If $S_T > K$, it would buy 1 Mio EUR at strike K.

Let's look at an example [Wy 06]: A company wants to benefit from believing that the EUR-USD exchange rate will move far from a specified strike (Straddle's strike). In this case a possible product to use is a Straddle as for example listed in Table 8.3. If the spot rate is above the strike at maturity, the company can buy 1 Mio EUR at the strike of 1.1500. If the spot rate is below the strike at maturity, the company can sell 1 Mio EUR at the strike of 1.1500. The break even points are 1.0726 for the put and 1.2274 for the call. If the spot is between the break even points at maturity, then the company will make an overall loss.

Spot reference	1.1500 EUR-USD
Company buys	EUR call USD put
Company buys	EUR put USD call
Maturity	1 year
Notional of both the options	EUR 1,000,000
Strike of both options	1.1500 EUR-USD
Premium	EUR 77,500.00

Table 8.3: Example of a straddle.

8.3.2 Strips and Straps

The same as straddles but you either buy two puts or two calls. This is a low risk directional play.

8.3.3 Strangles

This strategy is also called a *bottom vertical combination*. The investor buys a put and call with different strike prices — both out-the-money. The investor hopes for very large price movements in either direction. They are also very expensive but cheaper than straddles.

For example, a company has bought a Strangle with a nominal of 1 Million KES. At maturity T

1. If $S_T < K_1$, it would sell 1 Mio KES at strike K_1.
2. If $K_1 < S_T < K_2$, it would not exercise either of the two options. The overall loss will be the option's premium.
3. If $S_T > K_2$, it would buy 1 Mio KES at strike K_2.

Let's look at an example [Wy 06]: A company wants to benefit from believing that the EUR-USD exchange rate will move far from two specified strikes (Strangle's strikes). In this case a possible product to use is a Strangle as for example listed in Table 8.4. If the spot rate is above the call strike at maturity, the company can buy 1 Mio EUR at the strike of 1.2000. If the spot rate is below the put strike at maturity, the company can sell 1 Mio EUR at the strike of 1.1000. However, the risk is that,

Spot reference	1.1500 EUR-USD
Company buys	EUR call USD put
Company buys	EUR put USD call
Maturity	1 year
Notional of both the options	EUR 1,000,000
Put Strike	1.1000 EUR-USD
Call Strike	1.2000 EUR-USD
Premium	EUR 40,000.00

Table 8.4: Example of a strangle.

if the spot rate is between the put strike and the call strike at maturity, the option expires worthless. The break even points are 1.0600 for the put and 1.2400 for the call. If the spot is between these points at maturity, then the company makes an overall loss.

8.3.4 Butterfly Spreads

Here you buy a call with a relatively low strike and you buy a call with a relatively high strike. You also sell two calls with a strike halfway between the two outer strikes. The mid strike is generally close to the current FX rate. It is actually a combination of a long strangle and a short straddle.

An investor would enter into such a strategy if he believes that large rate movements are unlikely - a low volatility regime. If the FX rate does move adversely in either direction, his down side is protected. The constituent options of a butterfly is shown in Fig. 8.6 (see http://en.wikipedia.org/wiki/Butterfly_%28options%29).

Advantages [Wy 06]

- Limited protection against market movement or increasing volatility
- Maximum loss is the premium paid
- Cheaper than the straddle.

Disadvantages

- Limited profit

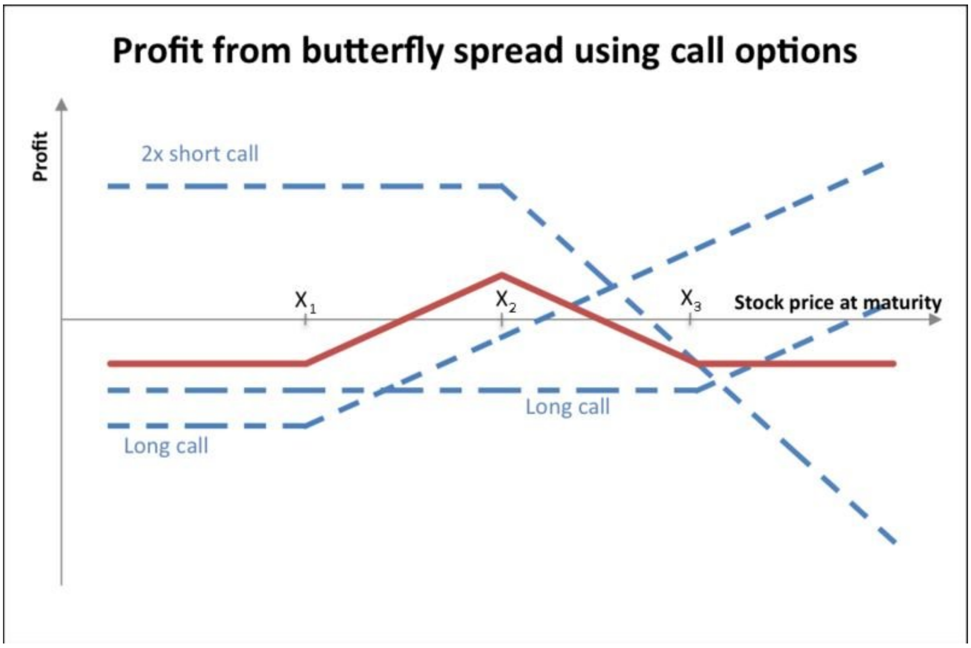


Figure 8.6: A butterfly broken into its constituent options.

- Not suitable for hedge-accounting as it should be clear if the client wants to sell or buy KES.

For example, a company has bought a long Butterfly with a nominal of 1 Million EUR and strikes $K_1 < K_2 < K_3$. At maturity T:

1. If $S_T < K_1$, it would sell 1 Mio EUR at a rate $K_2 - K_1$ higher than the market.
2. If $K_1 < S_T < K_2$, it would sell 1 Mio EUR at strike K_2.
3. If $K_2 < S_T < K_3$, it would buy 1 Mio EUR at strike K_2.
4. If $S_T > K_3$, it would buy 1 Mio EUR at a rate $K_3 - K_2$ less than the market.

Let's look at an example [Wy 06]: A company wants to benefit from believing that the EURUSD exchange rate will remain volatile from a specified strike (the middle strike K_2). In this case a possible product to use is a long Butterfly as for example listed in Table 8.5. If the spot rate is between the lower and the middle strike at maturity, the company can sell 1 Mio EUR at the strike of 1.1500. If the spot rate is between the middle and the higher strike at maturity, the company can buy 1 Mio EUR at the strike of 1.1500. If the spot rate is above the higher strike at maturity, the company will buy EUR 100 points below the spot. If the spot rate is below the lower strike at maturity, the company will sell EUR 100 points above the spot.

Spot reference	1.1500 EUR-USD
Maturity	1 year
Notional of both the options	EUR 1,000,000
Lower strike K_1	1.1400 EUR-USD
Middle strike K_2	1.1500 EUR-USD
Upper strike K_3	1.1600 EUR-USD
Premium	EUR 30,000.00

Table 8.5: Example of a butterfly spread.

8.3.5 The Iron Condor

It is the same as a butterfly but, where the middle two positions have different strike prices. Broken down it consists out of a bull put spread and a bear call spread with the same expiration. The number of call spreads will be equal to the number of put spreads.

The position is so named due to the shape of the profit/loss graph, which loosely resembles a large-bodied bird, such as a condor. In keeping with this analogy, traders often refer to the inner options collectively as the "body" and the outer options as the "wings". The word iron in the name of this position indicates that, like an iron butterfly, this position is played across the current spot price of the underlying instrument having one vertical spread below and one vertical spread above the current spot price. This distinguishes the position from a plain Condor position, which would be played with all strikes above, or below the current spot price of the underlying instrument. A Call Condor would be played with all call contracts and a Put Condor would be played with all put contracts[1].

8.3.6 Fence or Seagull

A long Seagull Call strategy is a combination of a long call, a short call and a short put — it is also known as a fence. It is similar to a Risk Reversal. So it entitles its holder to purchase an agreed amount of a currency (say KES) on a specified date (maturity) at a pre-determined long call strike if the exchange rate at maturity is between the long call strike and the short call strike (see below for more information). If the

[1] http://en.wikipedia.org/wiki/Iron_condor

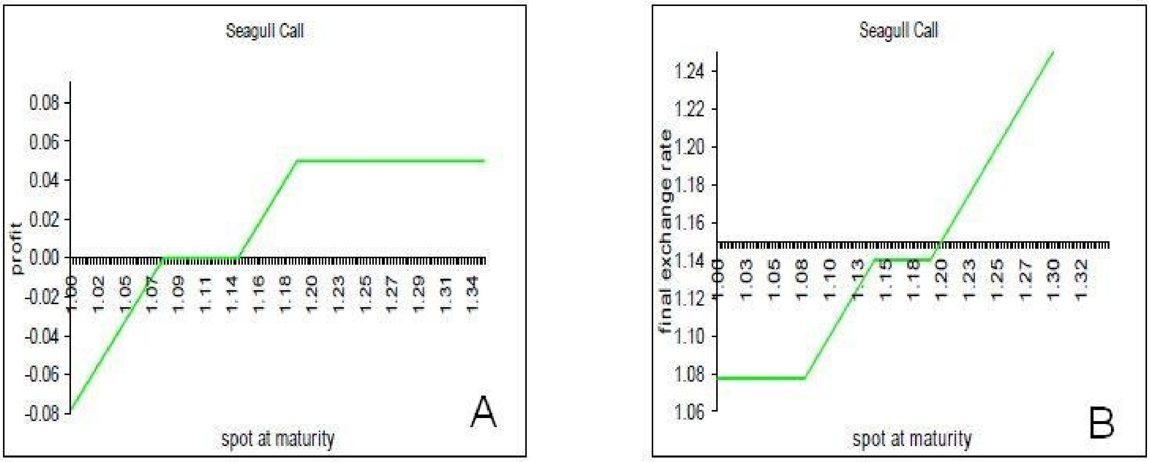


Figure 8.7: A: The Seagull call. B: The flying seagull

exchange rate is below the short put strike at maturity, the holder must buy this amount in KES at the short put strike. Buying a Seagull Call strategy provides good protection against a rising KES.

Advantages

- Good protection against stronger EUR/weaker USD
- Better strikes than in a risk reversal
- Zero cost product

Disadvantages

- Maximum loss depending on spot rate at maturity and can be arbitrarily large.

The protection against a rising KES is limited to the interval from the long call strike and the short call strike. The biggest risk is a large upward movement of KES. Figure 8.7 shows the payoff and final exchange rate diagram of a Seagull. Rotating the payoff clockwise by about 45 degrees shows the shape of a flying seagull [Wy 06].

For example, a company wants to sell 1 Million USD and buy EUR. At maturity T:

1. If $S_T < K_1$, the company must sell 1 Mio USD at rate K_1.
2. If $K_1 < S_T < K_2$, all involved options expire worthless and the company can sell USD in the spot market.

Spot reference	1.1500 EUR-USD
Maturity	1year
Notional	USD 1,000,000
Company buys	EUR call USD put strike 1.1400
Company sells	EUR call USD put strike 1.1900
Company sells	EUR put USD call strike 1.0775
Premium	USD 0.00

Table 8.6: Example of a seagull call.

3. If $K_2 < S_T < K_3$, the company would buy EUR at strike K_2.
4. If $S_T > K_3$, the company would sell 1 Mio USD at a rate $K_2 - K_1$ less than the market.

Let's look at an example [Wy 06]: A company wants to hedge receivables from an export transaction in USD due in 12 months time. It expects a stronger EUR/weaker USD but not a large upward movement of the EUR. The company wishes to be protected against a stronger EUR and finds that the corresponding plain vanilla is too expensive and would prefer a zero cost strategy and is willing to limit protection on the upside. In this case a possible form of protection that the company can use is to buy a Seagull Call as for example presented in Table 8.6. If the company's market expectation is correct, it can buy EUR at maturity at the strike of 1.1400. If the EUR-USD exchange rate will be above the short call strike of 1.1900 at maturity, the company will sell USD at 500 points less than the spot. However the risk is that, if the EURUSD exchange rate is below the strike of 1.0775 at maturity, it will have to sell 1 Mio USD at the strike of 1.0775.

8.4 Trading Strategies and Market Conditions

In the following sections we will summarise some strategies that can be followed under certain market conditions [2].

[2]See http://www.commodityworld.com/options_strategies.htm

8.4.1 Key Points to Remember

- All options eventually lose ALL of their "time value."
- All "Out-Of-The-Money" options expire worthless.
- Markets only "trend" 1/3 of the time; they move sideways the other 2/3.

8.4.2 BULLISH Market Strategies

Option Spread Strategy	**Description**	**Reason to use**	**When to use**
Buy a Call	Strongest bullish option position.	Loss limited to premium paid.	Undervalued option with volatility increasing.
Sell a Put	Neutral bullish option position.	Profit limited to premium received.	High volatility, bullish trending market.
Buy Vertical Bull Call Spread	Buy Call & sell Call of higher strike price.	Loss limited to debit.	Small debit, bullish market.
Sell Vertical Bear Put Spread	Sell Put & buy Put of lower strike price.	Loss limited to strike price difference less premium received.	Large credit, bullish market.

8.4.3 BEARISH Market Strategies

Option Spread Strategy	**Description**	**Reason to use**	**When to use**
Buy a Put	Strongest bearish option position.	Loss limited to premium paid.	Undervalued option with increasing volatility.
Sell a Call	Neutral bearish option position.	Profit limited to premium received.	Option overvalued, market flat to bearish.
Buy Vertical Bear Put Spread	Buy at the money Put & sell out of the money Put.	Loss limited to debit.	Small debit, bearish market.
Sell Vertical Bull Call Spread	Sell Call & buy Call of higher strike price.	Loss limited to strike price difference minus credit.	Large credit, bearish market.

8.4.4 NEUTRAL Market Strategies

Option Spread Strategy	**Description**	**Reason to use**	**When to use**
Strangle	Sell out of the money Put & Call.	Maximum use of time value decay.	Trading range market with volatility peaking.
Arbitrage	Buy & sell similar options simultaneously.	Profit certain if done at credit.	Any time credit received.
Calendar	Sell near month, buy far month, same strike price.	Near month time value decays faster.	Small debit, trading range market.
Butterfly	Buy at the money Call (Put) & sell 2 out of the money Calls(Puts) & buy out of the money Call (Put).	Profit certain if done at credit.	Any time credit received.
Guts	Sell in the money Put & Call.	Receive large premium.	Options have time premium & market in trading range.
Box	Sell Calls & Puts same strike price.	Profit certain if done at credit.	Any time credit received.
Ratio Call	Buy Call & sell Calls of higher strike price.	Neutral, slightly bullish.	Large credit & difference between strike price of option bought & sold.
Conversion	Buy futures & buy at the money Put & sell out of the money Call.	Profit certain if done at credit.	Any time credit received.

8.4.5 Special Market Situations

Option Spread Strategy	**Description**	**Reason to use**	**When to use**
Straddle Purchase	Buy Put & Call.	Options will lose time value premium quickly.	Options undervalued & market likely to make a big move.
Covered Call	Buy future & sell Call.	Collect premium on Calls sold.	Neutral to slightly bullish.
Covered Put	Sell future & sell Put.	Collect premium of Puts sold.	Neutral to slightly bearish.
Synthetic futures position.	Buy Call (Put) & Sell Put (Call).	Neutral, slightly trending market.	Receive credit, option sold far out of the money.

Appendix A

Create an Effective Risk Management Policy

GTNEWS 02 Sep 2008

In this Q&A, Joseph Maurer, executive board member of HiFX and former vice president and treasurer of Levi Strauss, discusses FX risk management policy, how risks are defined, risk tolerances, accountabilities and performance measurement.

Joseph Maurer is an executive board member of HiFX and former vice president and treasurer, Levi Strauss. During his tenure as treasurer of Levi Strauss, he led more than US$22bn of public and private debt financings. His responsibilities as treasurer also encompassed the management of employee benefit funds, insurable risk management, and real estate. Maurer is an engineering graduate of Michigan State University and holds an MBA from Stanford University.

Q (gtnews): As former treasurer of Levi Strauss & Co., what were your responsibilities in terms of FX risk management?

A (Joseph Maurer, former VP and treasurer, Levi Strauss): I was the treasurer of the company and, before that, treasurer of its international group. Throughout my tenure of nearly 30 years at the company I was fully responsible for the management of all financial risks, including foreign exchange. In that role, I developed the company's risk management policies, strategies, and executed their implementation. During that time, FX management became increasingly sophisticated and evolved into a European treasury centre staffed with nine professionals charged with dynamically managing the company's currency risks. I was fully accountable for the results of those efforts and the management of that team.

Q (gtnews): The nine professionals you mention - what were their primary roles and how was interaction between those team members facilitated?

A (Maurer): The organisation was a classic risk management design that one would commonly find in a group that was charged with adding value and where there was significant latitude for decision making. The group was led by an assistant treasurer and reported to me. Two FX traders who were supported by an analyst executed the transactions. The controller who had three people reporting to him performed the control and settlement function. All of the reporting and settlements were performed by this control section. The most important function - that of risk control officer - was performed by an expert in risk assessment. He had extensive academic training and was well versed in the specialised mathematics of risk management. It is this position that performed the stress testing, simulations, Monte Carlo analysis and continuous mark-to-market of all positions. His reporting relationship, for control purposes, bypassed the treasury organisation and went directly to the chief financial officer. While he was resident in the treasury organisation, it was an important control point to have his reporting relationship outside of treasury, giving an added layer of protection and separation.

Q (gtnews): How do you define a company's currency risk management policy?

A (Maurer): A company's currency risk management policy is the foundation upon which its currency risk management programme is built. Developing and implementing it is the single most important thing a company must do in managing FX risk. It defines what the company expects to achieve, how risks are defined, controls over processes and risks, risk tolerances, accountabilities and performance measurement. Without a comprehensive and well thought out policy, there is the potential for chaos.

Q (gtnews): Do you create a risk management policy that remains in place over a period of time, or is the policy ever-evolving, as market conditions change?

A (Maurer): I believe that to be effective, a risk management policy needs to continually evolve to keep pace with changing conditions within the firm as well as the markets. The fundamental elements of the policy remain consistent over time, but the details must reflect the development of new risk management tools as well the firm's economic environment. Q (gtnews): What are the main objectives of a risk management policy?

A (Maurer): The main policy objectives are to put in place an auditable framework for the management of currency risks. It sets the ground rules for when exposures will be recognised and who is responsible for that recognition. It defines who will manage the risks and the latitude given to the manager. It sets the parameters for measuring the performance and effectiveness of the manager and the risk levels that are acceptable. It also defines oversight responsibilities and the control processes over the operation.

Q (gtnews): Generally, what kind of latitude are FX managers given?

A (Maurer): It depends a great deal on the firm's approach to currency risk management. For example, if a firm chooses a fairly passive approach of identifying exposures and hedging them with no intention of altering those positions in response to market movements, then the FX managers would have very little latitude. The other end of the spectrum is the approach that is dynamic and attempts to add value by allowing the FX managers to essentially trade their positions, in which case they would have significant latitude and require sophisticated risk management and measurement expertise.

Q (gtnews): What kind of risks need to be managed with FX?

A (Maurer): Foreign exchange risks arise in the ordinary conduct of businesses that operate across country borders. The first line of defence against FX risk is through the natural processes of how the business is financed, its currency of invoicing and currency of product sourcing and finally, extracting the value of earnings. FX markets are most useful in mitigating the residual risks, that is the risks that otherwise cannot be offset through the normal processes available to the business.

Q (gtnews): How are the risk management policies you describe risk tolerant?

A (Maurer): There are many possible approaches to currency risk management. The simplest approach would be one that aggressively seeks to identify every currency risk exposure and then fully hedge it as soon as it is identified. Any approach other than that has embedded in it some level of risk and therefore a level of risk tolerance. The risk management policy defines exactly how the risk is defined and the tolerance for that risk.

Q (gtnews): What kinds of tools and analysis does one use to measure and assess risk?

A (Maurer): The tools and analytical techniques depend largely on the size and complexity of the basket of risks being measured and the sophistication attendant in the risk management programme. The selection is also driven by the desires of the managing board of the company. Typically, the tools include Value at Risk (VaR), scenario analysis, stress testing and Monte Carlo analysis.

Q (gtnews): What kinds of controls are there in the risk management process?

A (Maurer): There are several types of controls involved in the risk management process. There are the typical accounting and payment controls, but in addition there are the ones unique to currency management. If the policy delineates certain risk parameters, they must be monitored to assure compliance. It would typically be controls on open positions (or per cent hedged) and controls driven off stress testing. Another control parameter frequently used is VaR.

Q (gtnews): How is VaR used?

A (Maurer): VaR is used to assess an overall level of risk using a standardised statistical measure, which is widely accepted. Unfortunately, I do not think it is well understood by senior managers who are not risk management professionals. As a result, I believe it can at times give these senior managers a potentially false sense of security. I much prefer scenario analysis that calculates potential outcomes under a variety of conditions.

Q (gtnews): How does one measure performance?

A (Maurer): The performance being measured depends on the objectives of the currency management policy. Performance measures are then built to ascertain compliance with policy. When evaluating financial performance, benchmarks must be used. To be effective, the benchmarks need to be achievable and set objectively at the time the magnitudes of the currency exposures are identified.

Q (gtnews): What kind of benchmarks do you use?

A (Maurer): We used the simplest of benchmarks. At the time an exposure was identified, it was assigned a benchmark foreign exchange rate. That rate was the rate currently available in the market for the settlement date of the transaction. That met the test of being easily verified and understood and was a rate that was achievable at the time. Treasury was charged with delivering value added based on that rate.

Appendix B

Solution to the BS Differential Equation

The differential equation obtained by *Black & Scholes* is[1]

$$\frac{1}{2}\sigma^2 S^2 \frac{\partial^2}{\partial S^2} V(S,t) + \frac{\partial}{\partial t} V(S,t) + (r-d) S \frac{\partial}{\partial S} V(S,t) = rV(S,t) \tag{B.1}$$

where $V(S,t)$ is the option value, S is the current spot stock price, d is the dividend yield and σ^2 is the variance rate of the return on the stock prices. To obtain Eq. (B.1) no assumption about the specific kind of option has been made. This partial differential equation is valid for both calls and puts. The chosen type will be obtained by selecting the appropriate boundary conditions. For a call this is given in Eq. (4.2).

Black & Scholes solved the partial differential equation (PDE) given in (B.1) for a stock that does not pay a dividend. We here follow their methodology in solving the call option and put $d = 0$. The boundary condition for a call is given in Eq. (4.2). Subsequently researchers have developed other methods to solve this PDE and other methods to solve the option pricing problem without having to solve the PDE directly e.g., using probability theory [Hu 06, HK 00, Cl 11].

Fischer Black was a physicist and in mentioning the problem to his colleagues they realised that this rather “complex-looking” PDE could be transformed into a form that looks simpler and was known. They made the following ansatz [BS 73]

$$V(S,t) = e^{-r\tau} y(u,v) \tag{B.2}$$

where

$$\begin{aligned} u(S,t) &= \frac{2}{\sigma^2}\left(r - \frac{\sigma^2}{2}\right)\left[\ln\left(\frac{S}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)\tau\right] \\ v(S,t) &= \frac{2}{\sigma^2}\left(r - \frac{\sigma^2}{2}\right)^2 \tau. \end{aligned}$$

[1] I here give Merton’s general equation that he derived in 1973 by including a continuous constant dividend yield d [Me 73]. The original equation had $d = 0$.

This substitution transforms (rotates) the *Black & Scholes* PDE to

$$\frac{\partial y}{\partial v} = \frac{\partial^2 y}{\partial u^2} \tag{B.3}$$

and the boundary condition becomes

$$y(u,0) = \begin{cases} 0, & u < 0 \\ K\left[\exp\left(\dfrac{u\sigma^2/2}{r-\sigma^2/2}\right) - 1\right], & u \geq 0. \end{cases}$$

Equation (B.3) is the PDE for heat transfer through a medium; well-known to the physics community [Ha 87, Bo 83]. This equation has been studied thoroughly by physicists and the solution (using these boundary conditions) is obtained by using Fourier series (see [Ch 63] or any good book on this subject).

In our notation the solution is given by

$$y(u,v) = \frac{1}{2\pi}\int_{-u/\sqrt{2v}}^{\infty} K\left(\exp\left[\frac{(u+q\sqrt{2v})\sigma^2/2}{r-\sigma^2/2}\right] - 1\right) e^{-q^2/2}dq. \tag{B.4}$$

Substituting (B.4) into (B.2), and simplifying we find

$$V(S,t) = S(t)N(x) - Ke^{-r\tau}N(y) \tag{B.5}$$

where $N(x)$ is the Normal/Gaussian distribution function given by

$$N(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-a^2/2}da$$

and

$$\begin{aligned} x &= \frac{1}{\sigma\sqrt{\tau}}\left[\ln\left(\frac{S}{K}\right) + (r + \frac{1}{2}\sigma^2)\tau\right] \\ y &= \frac{1}{\sigma\sqrt{\tau}}\left[\ln\left(\frac{S}{K}\right) + (r - \frac{1}{2}\sigma^2)\tau\right] = x - \sigma\sqrt{\tau}. \end{aligned}$$

Equation (B.5) is the well-known *Black & Scholes* solution for the value of a call option. The put value can be obtained by either imposing the put boundary conditions onto (B.1) and solving or from the *put-call-parity* relation $C + Ke^{-r\tau} = P + S$ such that

$$V(S,t) = Ke^{-r\tau}N(-y) - SN(-x). \tag{B.6}$$

Note that the expected return on the stock does not appear in the solutions and the solutions are risk-neutral.

Appendix C

Probabilistic Solution

Question: in knowing that the underlying asset has a lognormal distribution, what is the expected value of a call option at expiration [Ga 86]?

To answer this we let C^* be the call at expiration. The expected value of the call is then given by

$$E[C^*] = E[\max(0, S^* - K)]$$

with S^* the value of the stock at expiration, K is the striking price and E is the expectation operator[1]. Now let $S^* = SR$ where R is the return on the underlying asset which is lognormally distributed; hence

$$E[C^*] = E[\max(0, SR - K)].$$

If $L(R)$ is the lognormal density function, we have

$$E[C^*] = \int_{K/S}^{\infty} (SR - K)L(R)dR. \tag{C.1}$$

NOTE: $L(R)$ is only defined for $R \in [0, \infty]$ but we only integrate from K/S because $S^* > K \Rightarrow R > K/S$.

If R is lognormally distributed, $\ln R$ is normally distributed. Now, if we let $\ln R = X$, we obtain from (C.1)

$$E[C^*] = \int_{\ln K/S}^{\infty} (Se^X - K)N(X)dX \tag{C.2}$$

[1] Let $\{X_i\}$, $i = 1, 2, \ldots, N$, be a discrete set of random variables with probability density function $f(X)$. Then

$$E[X] = \sum_{i=0}^{N} X_i f(X_i).$$

If $\{X\}$ are continuous we have

$$E[X] = \int_{-\infty}^{\infty} X f(X)dX.$$

where $N(X)$ is the normal density function. If we further assume that the stock prices are governed by a Wiener process we have

$$\ln R = \ln \frac{S^*}{S} = \mu\tau + \sigma\epsilon\sqrt{\tau}$$

where μ is the mean and σ the volatility of the stock prices and ϵ is a random variable from a standard normal distribution. Using this assumption and (C.2) we have

$$E[C^*] = \int_{[\ln(K/S)-\mu\tau]/\sigma\tau}^{\infty} (Se^{\mu\tau+\sigma\epsilon\sqrt{\tau}} - K)N(\epsilon)d\epsilon \tag{C.3}$$

where $\ln R > \ln(K/S)$ if the option is exercised.

Also, in a risk-neutral world with continuous risk-free rates of interest r the expected return on any asset should just be the risk-free rate i.e.,

$$E(R) = e^{r\tau} = e^{d\tau}e^{\mu\tau+\sigma^2\tau/2} \Rightarrow \mu = \ln\frac{r}{d} - \frac{1}{2}\sigma^2 \tag{C.4}$$

over period τ. Here d is an extra riskless payoff return obtained during the period τ (a dividend for instance).

Taking the above into account, we can put

$$C = E[C^*]e^{-r\tau} \tag{C.5}$$

i.e., the call value C at t is equal to the expected call value C^* at T discounted by the risk-free rate of interest. So, substituting (C.4) into (C.3) and using (C.5), we obtain

$$C = Se^{-d\tau}\int_a^{\infty} e^{-\frac{1}{2}\sigma^2\tau+\sigma\epsilon\sqrt{\tau}}N(\epsilon)d\epsilon - Ke^{-r\tau}\int_a^{\infty} N(\epsilon)d\epsilon \tag{C.6}$$

where

$$a = \frac{1}{\sigma\sqrt{\tau}}\left[\ln\frac{K}{S} + (d - r + \frac{1}{2}\sigma^2)\tau\right].$$

In calculating the integrals[2] we obtain

$$C = Se^{-d\tau}N(x) - Ke^{-r\tau}N(y) \tag{C.7}$$

where

$$\begin{aligned} x &= \ln\frac{S}{K} + (r - d + \frac{1}{2}\sigma^2)\tau \\ y &= x - \sigma\sqrt{\tau}. \end{aligned}$$

Equation (C.7) is Black's model which can be used for options on stock that pays a known dividend yield d, currency options and options on futures. If $d = 0$, (C.7) reduces to the ordinary Black & Scholes option pricing formula.

[2]Use the general substitution $y = -\sqrt{c}(x - b/c)$, with $c = 1$ and $b = \sigma\sqrt{\tau}$, to transform the first and $y = -x$ to transform the second integral into more appropriate forms.

Appendix D

Calculating the Implied Volatility

If one wants to obtain the implied volatility used in the Black-Scholes equation, one can start by implementing the "brute force" technique. Here one uses a numerical routine to solve Equation (3.2) for σ. This can be done quite easily by using a Newton-Raphson technique [PF 86]. In Excel one can use the *Goal Seek* procedure.

Corrado and *Miller*, however, derived an accurate formula to compute implied volatility [Ne 97]. They refer to it as the improved quadratic formula where

$$\sigma\sqrt{T} = \frac{\sqrt{2\pi}}{S+X}\left[V - \frac{S-X}{2} + \sqrt{\left(V - \frac{S-X}{2}\right)^2 - \frac{(S-X)^2}{\pi}}\right].$$

Here $X = Ke^{-rT}$ is the discounted strike price, S is the stock price and T is the time to expiry. It is accurate over a wide range of strike prices.

Appendix E

Black-Scholes or Black Holes?

> The Nobel laureates Fischer Black, Myron Scholes and Robert Merton revolutionised financial economics with the publication of their option valuation formula in 1973. The model, however, was devised for an elementary, ideal and frictionless world. Understanding the framework and underlying simplifying assumptions behind their formulation will help to minimize the risks in trading and managing derivative securities[1].

The Brady Commission that investigated the 1987 crash on Wall Street put part of the blame on the trading strategies derived from the Black-Scholes (BS) analysis. In 1994 George Soros reiterated that, if there is an overwhelming amount of delta hedging in the same direction, the theoretical equilibrium will be disturbed leading to discontinuous price movements - the premise of the BS model will break down. A well-known example is the downfall of Long Term Capital Management (LTCM) in 1998. In a recent letter to shareholders Warren Buffett warned that derivatives were "financial weapons of mass destruction" and derivatives trading sat on a "time bomb".

Statements like these contain some truths. It is due to many practitioners seeing the formula as a black hole of gravitating mathematical forces that overwhelms them. They perceive the model as flawed but, contrary to belief, the flaws lie in misconceptions and its practical application. Many traders, fund and risk managers use the outputs clinically in their analysis. This "black box" approach is wrong. It is imperative that every derivatives practitioner understands the universe in which the BS formula was devised. In this note we discuss and illuminate the black hole surrounding this enigmatic formula.

Mathematical modelling of financial markets can be traced back to Louis Bachelier's 1900 dissertation on speculation in the Paris markets. Financial economics, however, only came of age in 1973 with the publication of the preference-free option pricing formula by Fischer Black, Myron Scholes and Robert Merton. Their model

[1] This article appeared in the South African Financial Markets Journal [Ko 03]

Figure E.1: Saggitarius A*: supermassive black hole at the Milky Way's center.

established the everyday use of mathematical models as essential tools in the world of finance, both in the classroom and on the trading floor.

One fact we constantly have to remember is that the BS model is just a model, an abstraction of reality. Modelling, however, is not merely a collection of techniques but an art in blending the relevant aspects of a problem and its unforeseen consequences with a descriptive, yet tractable, mathematical methodology. Building and analysing models are important. This is illustrated by the following benefits that resulted from the BS formula.

- the concepts behind the formula provided the framework for thinking about option valuation and dynamics;
- such research led to an expanded universe of financial instruments encompassing more complex option structures;
- quantitative risk management, stress testing and scenario analysis became possible;
- hedging of derivatives were easier.

You might feel that quantitative analysts confuse everyone with their mathematical jargon and, shouldn't the market determine the prices of traded securities? That is correct and hence we talk about option valuation or estimation. Pricing generally refers to where the market price is rather than where it should be.

One of the first lessons in option valuation is that the model should only be used to guide us toward the correct value, giving somebody a BS calculator doesn't make him a trader. The model is a means of thinking about the problem we're tackling but it does not necessarily give us the answer. It is a filter that lets us turn our perceptions about volatility into Rand values or hedge ratios. Traders use the model as a standardized communication tool for productive trading conversations.

How should the BS model then be used? A good start is to comprehend its limitations and applicability to the case at hand. By considering practicalities like transaction costs, liquidity and frequency of hedging, we will be guided on how to overlay these problems onto the actual bare-bones price of the model and decide whether that's a price we can live with. Trading with a model is definitely not the simple and clinical procedure many people imagine.

Let's now look at the BS world, meaning the environment they created which underpins their formula. They assumed:

- the underlying stock price follows a continuous random walk - stock prices diffuse through time linearly proportional to the spot price i.e. constant volatility. The price in the future is unpredictable but will most likely be close to some mean or expected value,

- the efficient market hypothesis holds - markets are liquid, have price-continuity, are fair, are complete and all players have equal access to information,

- investors live in a risk-neutral world - they require no compensation for taking risk. There are no arbitrage opportunities and the expected return is the risk-free interest rate. This was perhaps their most important insight leading them to construct a self-financing riskless hedge; in a portfolio of three securities - an option, the underlying stock and a riskless money market security - any two could be used to exactly replicate the third by a trading strategy,

- delta hedging is done continuously - for the return on the hedge portfolio to remain riskless, the portfolio must continuously be adjusted as the asset price changes over time.

In the rest of this note we will look at the two most common violations of the BS environment namely imperfect volatility forecasts and dynamic hedging.

We first look at dynamic hedging. In the BS environment, the key to understanding derivatives is the notion of a premium. The premium of an option is given by the model with the volatility as a vital input, but the premium and the traded price of an option are rarely the same. This is explained by the concept of delta-hedging. The delta tells a trader to buy or sell the underlying stock in order to hedge the market risk of an option. In theory, if delta-hedging is done dynamically, trading profits (losses) should equal premium outflows (income).

The premium is thus equivalent to the cost of hedging the option. However, dynamic hedging in real markets is not a risk-free proposition. Some of the additional risks are changes in volatility, changes in interest rates, changes in dividends, trading costs and liquidity. Risk is reduced if we know what our real hedge strategy and portfolio is and what it is going to cost. This is not realistic but it does mean that the true value of the option, in general, is substantially different to the BS value.

Secondly we look at the concept of volatility. The problem arises because we should use the future volatility in the model, which is not known and needs to be estimated. Many practitioners use the implied volatility in managing their books. Implied volatility is seen as the market's collective forecast of volatility to be realised over the life of the option. Studies comparing implied volatilities with actual realised volatilities generally find little agreement between the two. As far back as 1972, BS noted that the implied volatility employed by the market is too narrow and that the historical estimates of volatility include an attenuation bias.

Today, this is called the volatility skew or term structure of volatility. The skew is a consequence of the market's view that options with different strikes and different expiries, have different risks and should be valued as such. It only rose to prominence after the crash of October 1987 reflecting a risk or "crash" premium.

Using implied volatility is complicated because it might include the effect of mark-ups and it is impossible to derive any volatility expectation from a single option contract — a problem in illiquid markets. Because of these problems some traders use implied volatility for valuations but their own forecasts (utilising Garch analysis for instance) for hedging.

This attribution belies the BS picture where it is assumed that there is only one underlying stock and hence one evolutionary process. The skew and different volatilities used for the same trade show that the real underlying process differs from the random walk assumed by BS.

To overcome this, researchers look at different processes like the parabolic, Levy or jump diffusion processes. Due to its complexity this has not caught on but models where the volatility is assumed to follow a stochastic process are being used successfully. However, many skilled traders still use BS and they account for this "discrepancy" by utilising the volatility skew effectively, that is, volatility is used to reflect all the un-tradable risks in the market.

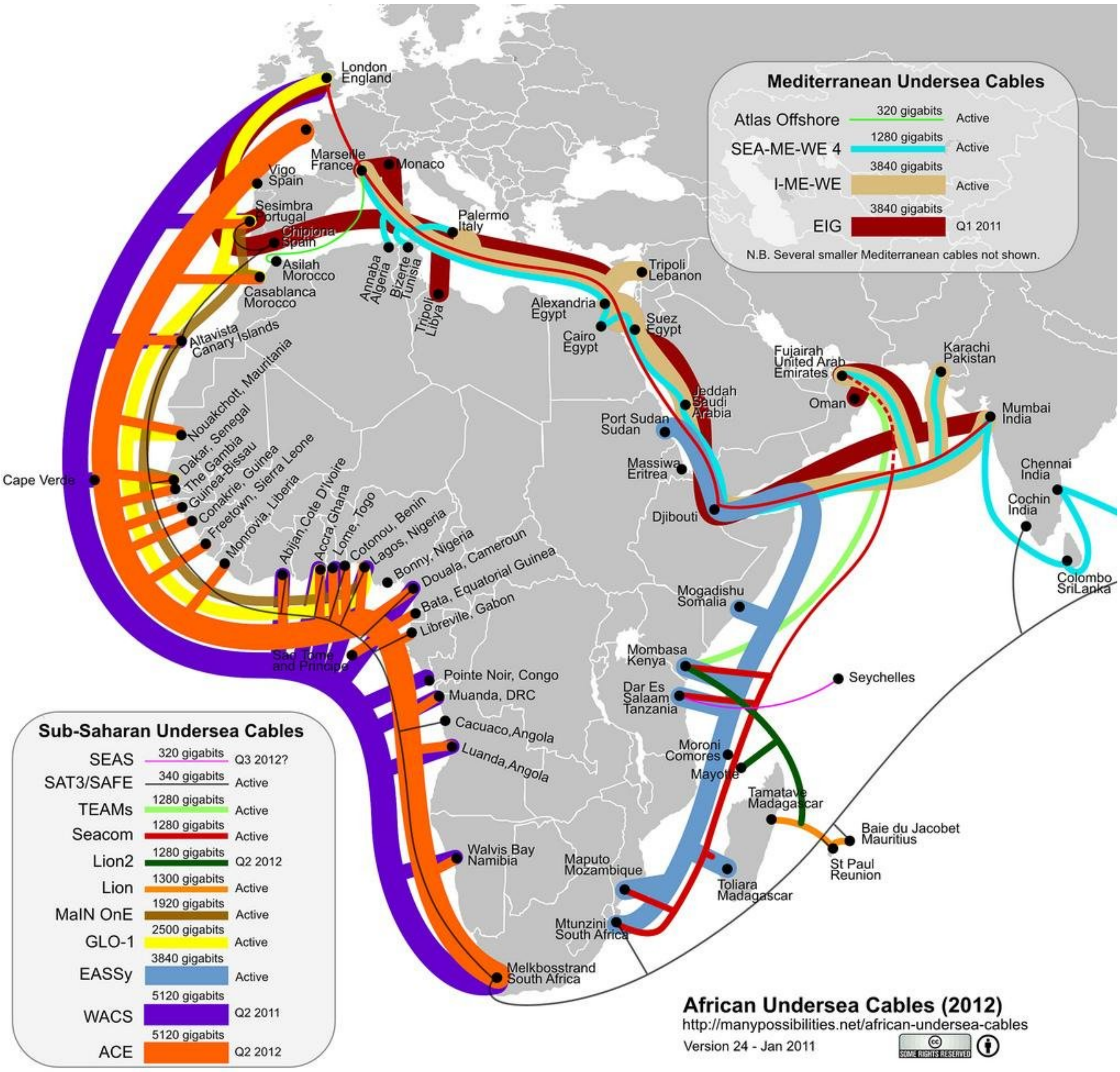
From the abovementioned arguments we see that the BS assumptions are demonstrably wrong for real world markets. Still the model is used by everyone working in derivatives. It is used confidently in situations for which it was not designed for, usually successfully and the model is remarkably robust.

The black hole is not that dark and practitioners who understand the BS environment use a "bag of tricks" to map this simplified world to the real one. They use stress testing, scenario analysis and procedures like Monte Carlo simulations where they can calculate option values and hedge ratios discretely in time. They also learn to use fudges like shadow delta and shadow gamma.

Intelligent traders iterate between imagination and model use in a way that belies easy categorization and testing. There is an old saying, "It isn't the size of the wand; it's the wizard who waves it". This is precisely the reason why the BS model has survived all these years. It does not try to be fancy, it lets the operator himself be fancy at will!

Appendix F

Connecting Africa



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