Working title: EEG Paper

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Abstract

Keywords:

1. Introduction

2. Preliminaries

Electroencephalogram:

The spontaneous electrical signals generated in the brain contain various information about the brain activity. These signals can be systematically captured using electroencephalography (EEG), a non-invasive technique that records voltage fluctuations on the scalp produced by the collective activity of neurons. The electrical activity measured by EEG originates primarily from postsynaptic potentials in pyramidal neurons, which are large, vertically aligned cells in the cerebral cortex. The potential of a single neuron is minuscule (on the order of microvolts), so detectable EEG signals require the summation of activity from millions of similarly oriented and synchronously active pyramidal neurons.

Due to the brain's non-linear anatomical structure, the tissue, cerebrospinal fluid, and bone structure vary across the region, so not all neurons contribute similarly to the EEG. As a result, EEG predominantly captures activity from superficial cortical regions rather than deeper brain structures (e.g., thalamus or basal ganglia). These potentials are extremely weak, typically ranging from 10 to 100 microvolts, necessitating high-gain amplifiers and noise reduction techniques. However, EEG is capable of capturing abnormal electrical discharge, such as epileptiform spikes or slow waves indicative of encephalopathy. Its millisecond-level temporal resolution enables real-time monitoring of dynamic brain processes, making it an invaluable tool for diagnosing epilepsy.

Fast Fourier Transform:

This computationally efficient algorithm is used to compute the Discrete Fourier transform (DFT), which decomposes a finite sequence of time-domain signal samples into its constituent frequency components. For instance, consider a discrete signal $x[n]$ of length N, the DFT is defined as:

$$X[k] = \sum_{n=0}^{N-1} x[n].e^{-i\frac{2\pi}{N}kn}, \quad \text{for } k = 0, 1, \ldots, N-1$$

This holds the computational complexity of $O(N^2)$, which blows up for higher values of N. FFT solves this problem by recursively decomposing the components into smaller sub-problems exploiting symmetry and periodicity. After applying the FFT, the signal is partitioned into frequency bins with resolution $\Delta f = \frac{f_s}{N}$, where f_s is the sampling rate. This way, the EEG signal is quantified as oscillatory activity in canonical bands(delta: 0.5-4 Hz, theta: 4-8 Hz, alpha: 8-13 Hz, beta: 13-30 Hz, gamma: 30-100 Hz).

Power Spectral Analysis:

The power spectrum (or Power Spectral Density) captures the power distribution of a continuous signal in the frequency components present in the signal. For signals consisting of voltages(such as the EEG signal), the power spectrum is simply the squared value of the voltages itself per frequency and hence the units are in terms of $\mu V^2\text{Hz}^{-1}$. The summation or integration of the spectral components yields the total power or variance. For a discrete-time EEG signal $x[n]$ of length N, the PSD is classically estimated using the periodogram, defined as the squared magnitude of the Discrete Fourier Transform (DFT):

$$\text{PSD}[k] = \frac{1}{N\,f_s} \left| \sum_{n=0}^{N-1} x[n]\, e^{-i\frac{2\pi}{N}kn} \right|^2, \quad \text{for } k = 0, 1, \ldots, N-1$$

where f_s is the sampling rate, and $PSD[k]$ represents power at the frequency, $f = \frac{kf_s}{N}$(in Hz). The simple PSD (without any windowing and segment averaging) suffers from high variance and bias due to spectral leakage. PSD estimation is particularly useful for identifying abnormal rhythmic activity, and this remains integral to EEG due to its computational simplicity and interoperability.

Welch Transform:

This method is an upgraded version of a signal's standard Power Spectral Density calculations, improving the robustness and limiting the spectral leakage and high variance. For an EEG signal $x[n]$ of length N it proceeds by,

1. Segmentation: Dividing the data into M segments of some length L with an overlap of D samples, $x_m[n]$.
2. Windowing: Applying a window function $w[n]$ (e.g. Hamming, Hanning) to each segment to suppress edge discontinuities, $x_{w,m}[n] = x_m[n].w[n]$.
3. Computing periodogram: Calculating the modified periodogram for each windowed segment,

$$\mathrm{PSD}[k] = \frac{1}{L.U.f_s} \left| \sum_{n=0}^{L-1} x_{w,m}[n]\, e^{-i\frac{2\pi}{L}kn} \right|^2, \quad \text{where, } U = \frac{1}{L}\sum_{n=0}^{L-1} |w[n]|^2$$

4. Averaging: Over all M periodograms to obtain the final PSD welch estimate as,

$$\mathrm{PSD}_{\mathrm{Welch}}[k] = \frac{1}{M}\sum_{m=0}^{M-1} \mathrm{PSD}[k]$$

This method is particularly advantageous for analyzing non-stationary neural signals with inherent noise (e.g. artifacts, line noise). It smoothens out transient disturbances and enhances the visibility of oscillatory components by averaging over overlapping epochs.

Convoluted Neural Network:

A deep learning-based method in machine learning facilitates the automatic identification and analysis of relevant data features without human intervention. This method has an advantage over other methods in avoiding over-fitting by a weight-sharing feature and is capable of scalability in implementation for large networks. The CNN consists mainly of two sections:

a) Convolution layer or feature extraction and b) A fully connected layer or classifier. The classification section can be replaced with other classifier methods like Linear Regression (LR) or Support Vector Machine (SVM) supervised learning models.

a) Convolution layer (Feature extraction):
This consists of casting layers of filtration on the data matrix and sub-sampling of feature maps to determine whether certain features are available in the data matrix.
i) Data processing: For an image, the data matrix will be the values of the pixels arranged in the dimension of the image. Considering it is initially in RGB format and changed to grayscale images before translating them to NumPy array data matrix reduces the dimensionality.
ii)Applying a filter or kernel: It can be considered a matrix with elements representing the kernel weight. These values can be randomly initialized, or the kernel can be kept constant, such as using Laplace Kernel throughout the convolution.

$$\text{Laplace kernel}_{5\times5} = \begin{bmatrix} 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & -2 & -1 & 0 \\ -1 & -2 & 16 & -2 & -1 \\ 0 & -1 & -2 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 \end{bmatrix}$$

Convoluting using a kernel involves "sliding" the kernel horizontally and vertically over the image data matrix depending on the stride value that defines the step size. Except for stride value 1, other strides skip some pixels from convoluting, thus reducing the output size. The kernel is applied to the pixels, and it applies dot product element-wise to the data matrix. The corresponding values are summed up to reach a single scalar value. This sliding is done concurrently until it is no longer allowed. The output matrix of scalars represents the feature map. Sometimes, adding extra pixels (mostly zeros) around the initial data matrix helps determine the border size information.
iii)Pooling layer: The generated feature map is then passed on to a pooling layer subroutine. This process sub-samples the initial feature maps' size and shrinks to even smaller feature maps. Like the convolution step, a pool kernel and a stride value are again initiated for this process. Considering we apply a max pool kernel of size 16×16 with stride value 1, we pool the maximum

values from each window while covering the image data matrix horizontally and vertically.
iv)Flattening: Each feature map thus generated is flattened or resized into a vector space data matrix.

One can avoid the fully connected layer method of CNN as a classifier based on the need. Contrarily, we can use:
a)Sigmoid function-based Linear Regression model: This is a supervised learning model for classifying objects. After splitting the flattened vectors of max pooled feature maps into train and test datasets, a linear model can be generated such as:

$$z = X_{\text{train}} \cdot w + b$$

where X_{train} is the matrix of input features, w represents the weights, and b is the bias term. This computed value, z, is then passed through a sigmoid function:

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

This maps the linear model into the range (0,1) and yields a probabilistic output for classification.
or b)Support Vector Machine model: This is also a supervised learning model for classifying objects. In this approach, we use a Support Vector Machine with a polynomial kernel of some degree n, defined by:

$$K(x, x') = (\gamma\langle x, x'\rangle + r)^n$$

Here, $\langle x, x'\rangle$ represents the dot product of the input vectors, γ is the scaling parameter, and r is a coefficient. This kernel function transforms the original feature space into a higher-dimensional space, enabling the SVM to capture complex, non-linear relationships between features.
While training the models, we concurrently update the weights and biases to align the model with the solution space. We can further change the hyperparameters (learning rate, iterations) to achieve different outputs in the result.
v) Loss function: Binary cross-entropy loss (BCE), also known as log loss, is a critical component of training binary classification models (e.g., sigmoid-based models). It quantifies the dissimilarity between predicted probabilities and true binary labels (0 or 1). Considering the output $\sigma(z)$ after linear regression as y_{pred}, or say predicted probability, we can write the Binary Cross

Entropy (BCE) loss function as:

$$\text{BCE} = -\frac{1}{N}\sum_{i=1}^{N}\left[y_{\text{true}}\log(y_{\text{ pred}}) + (1 - y_{\text{true}})\log(1 - y_{\text{pred}})\right]$$

Here, N is the total number of samples, and y_{true} represents the true label. During training phase, the classification loss is minimized in order to optimize the parameters of the classifier.
vi) Confusion matrix after testing: After evaluating the model, this matrix provides information about how accurate the predictions are compared to reality in classification problems. It tabulates into:

	Predicted Class 1	Predicted Class 0
Actual Class 1	True Positives(TP)	False Negatives (FN)
Actual Class 0	False Positives(FP)	True Negatives(TN)

vi)Accuracy: It measures the overall correctness of the model when making the predictions.

$$\text{Accuracy} = \frac{\text{TP+TN}}{\text{TP+TN+FP+FN}}$$

Some other parameters, such as the: Precision, Recall, F1-Score, Sensitivity, Specificity is used to evaluate the robustness of the model against the data.

3. Methodology

The data used for all the analyses is retrieved from the Clinic and Polyclinic of Epileptology, University Hospital of Bonn, Germany, collected using a standardized electrode placement scheme (Plant a figure). The data set consists of individual sets (A, B, C, D, and E) of at least five healthy and five epileptic patients. Each set has 100 single-channel time series segments captured at 173.61 Hz for 23.6 seconds of the time window frame. Extracranial data sets A and B are from healthy individuals kumawith eyes open and closed condition, respectively. Intra-cranial C and D sets are taken from the Hippocampal and Epileptogenic brain regions of epileptic patients. Set E is of patients experiencing a seizure (ictal) condition.

The data is standardized to make it more robust and tolerant to other external factors (demographics, noise, etc.). For visualization purposes, each dataset is transformed from a time domain to a frequency domain using the direct Fast Fourier Transform (FFT) method. Welch's Power Spectral Density (PSD) method is used for further analysis. This involves multiplying a window function to each segmented data of constant duration and averaging over all periodograms generated after applying FFT operation on all the window segments. This method allowed for quantification of oscillatory activity in canonical frequency bands (delta: 0.5-4 Hz, theta: 4-8 Hz, alpha: 8-13 Hz, beta: 13-30 Hz, gamma: 30-100 Hz) while minimizing noise and non-stationary artifacts which are inherent in an EEG recording. The power spectral plots generated by Fast Fourier transformed magnitudes and from Welch's PSD showed some frequent peculiar peaks for several datasets for some particular range of frequencies. As a matter of interest, the EEG time series is again reconstructed for the full, and those particular range values from Welch's frequency domain PSD plots and Fast Fourier transformed magnitude power spectral plots.

For phase space analysis, the electrode potential values are considered as the x-axis, plotted against their time derivatives as the y-axis for the original and reconstructed EEG time series. Following the generation of detailed phase plots, these images are systematically fed into a Convoluted Neural Network (CNN) model. This model is designed to extract and analyze complex spatial and temporal patterns inherent in the phase space representations. By leveraging its convolutional layers, CNN efficiently identifies key features within the high-dimensional data, facilitating robust downstream classification tasks.

The CNN model is constructed to perform binary classification between healthy and epileptic patient states using phase space images derived from both the original and reconstructed EEG signals. The model employs a Laplace kernel during convolution, which enhances its ability to capture critical edge and texture features within the images. Furthermore, a variable pooling strategy is implemented, with pooling sizes ranging from 2 x 2 to 16 x 16, allowing for multi-scale analysis that adapts to the diversity of signal characteristics. After flattening the max pooled datasets, they were fed into two different classification models (Linear Regression (sigmoid) and Support Vector Machine) for classification. Notably, the architecture of the fully connected neural network omits hidden layers, culminating in a single output node that directly conveys the binary classification result.

The results obtained from various qualitative analyses reveal frequent peaks in two peculiar ranges in the gamma section with a subsequent weightage for a potential classification of healthy and epileptic patients, which are further discussed in the following section, along with more statistical data interpretations.

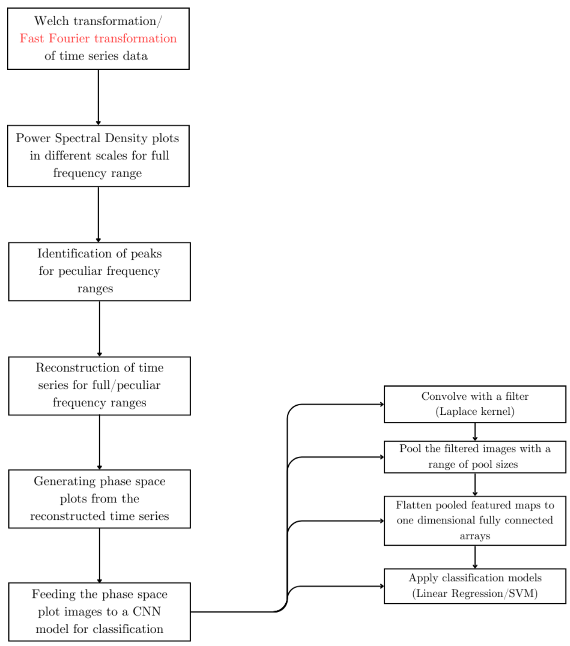

Figure 1: Caption

4. Results and Discussion

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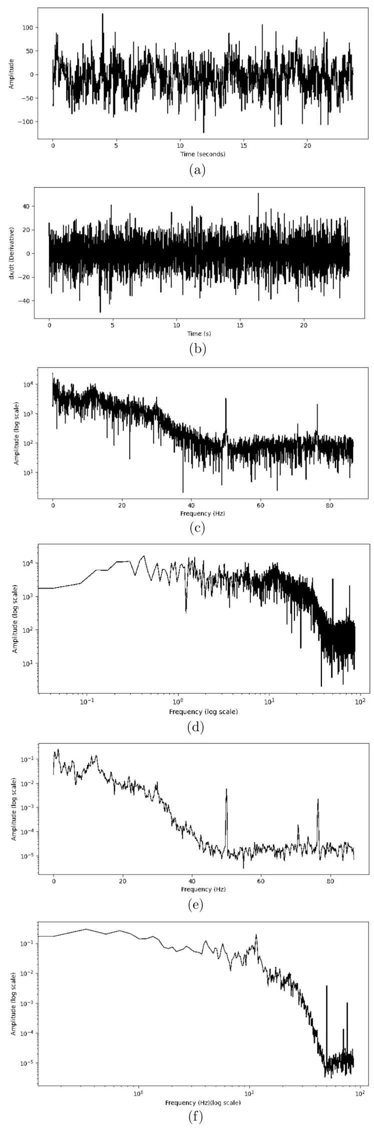
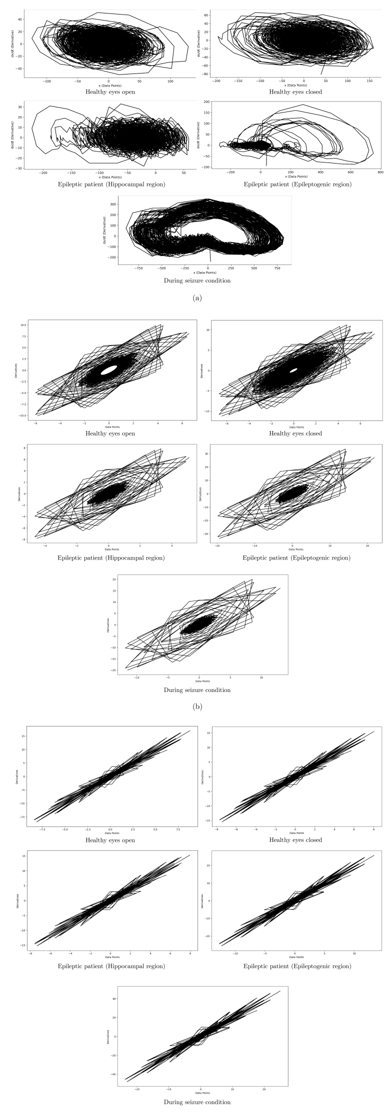
5. Conclusion

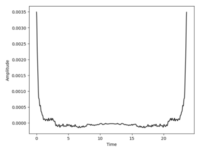
References



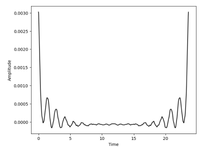
(a) Example EEG signal from UoB dataset.



(b) Example RPS imgaes of EEG signals from UoB dataset.



Healthy eyes open



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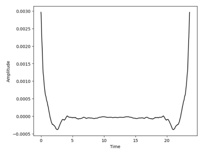
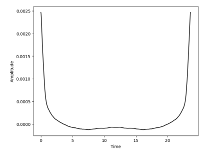
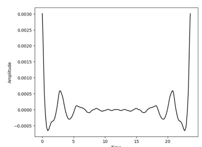


Epileptic patient (Hippocampal region)

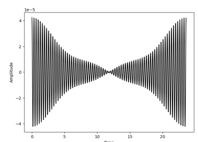


Epileptic patient (Epileptogenic region)

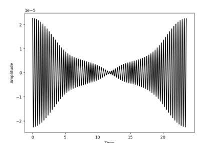


During seizure condition

(a)

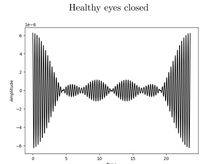


Healthy eyes open

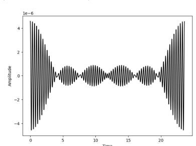


Healthy eyes closed

Epileptic patient (Hippocampal region)

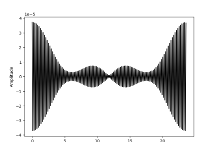


Epileptic patient (Epileptogenic region)

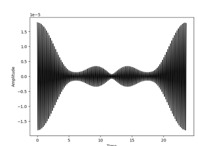


During seizure condition

(b)



Healthy eyes open



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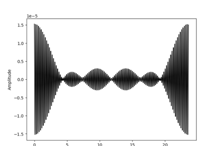
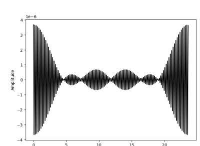
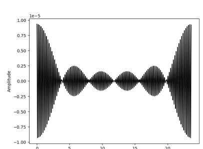


Epileptic patient (Hippocampal region)



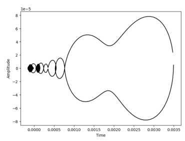
Epileptic patient (Epileptogenic region)



During seizure condition

(c)

Figure 3: Caption



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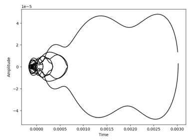

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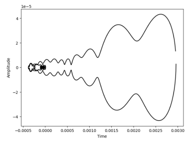
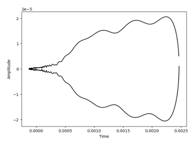
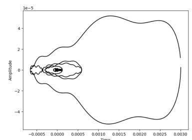


Epileptic patient (Hippocampal region)

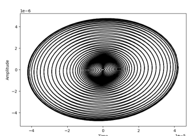


Epileptic patient (Epileptogenic region)

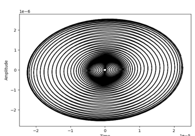


During seizure condition

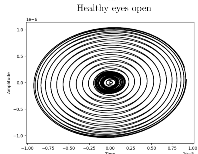
(a)



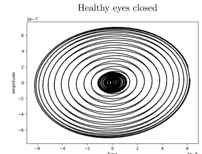
Healthy eyes open



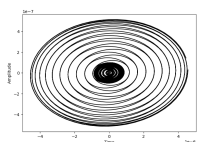
Healthy eyes closed



Epileptic patient (Hippocampal region)

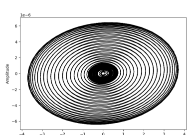


Epileptic patient (Epileptogenic region)

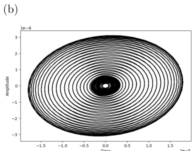


During seizure condition

(b)



Healthy eyes open



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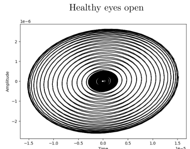
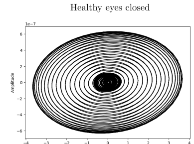
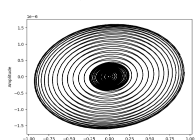


Epileptic patient (Hippocampal region)



Epileptic patient (Epileptogenic region)



During seizure condition

(c)

Figure 4: Caption

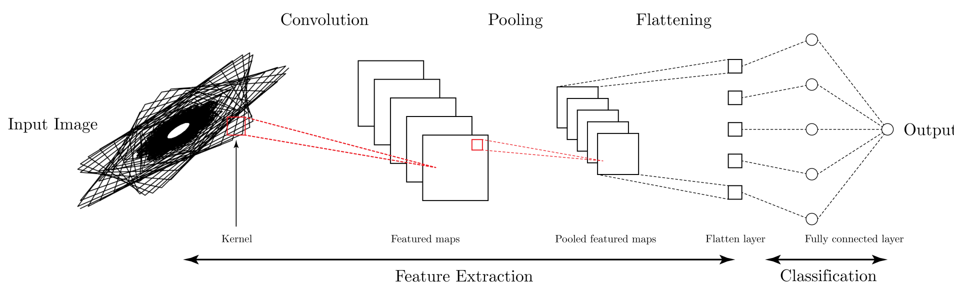


Figure 5: Caption