Sheet 1:

a.i) $T = \begin{pmatrix} R(\theta) & t \\ 0 & 1 \end{pmatrix}$, $R(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$, $t = \begin{pmatrix} x \\ y \end{pmatrix}$

a) $x_i = (x_i, y_i, \theta_i)^T$

$T_i = \begin{pmatrix} R_\theta(\theta_i) & t_i \\ 0 & 1 \end{pmatrix}$ $\quad l = (l_x, l_y)$

$$l_g = T \times P_l = T_i \begin{pmatrix} l \\ 1 \end{pmatrix}$$

$$= \begin{bmatrix} R(\theta_i) & t_i \\ 0 & 1 \end{bmatrix} \begin{bmatrix} l \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} R(\theta_i)\, l + t_i \\ 1 \end{bmatrix}$$

$$R(\theta_i)\, l = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} l_x \\ l_y \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta\, l_x - \sin\theta\, l_y \\ \sin\theta\, l_x + \cos\theta\, l_y \end{bmatrix}$$

$$l_g = (x_i + \cos\theta l_x - \sin\theta l_y,\ y_i + \sin\theta l_x + \cos\theta l_y)$$

b) $T_1 P_1 = P_{1q}$

$P_1 = T_1^{-1} P_{1q}$

$$= \begin{bmatrix} R(\theta_1)^T & -R(\theta_1)^T t_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1_q \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$1 =$ $\cos^{-1}\big(\cos\theta_1 (1_{xq} - x_1) + \sin\theta_1 (1_{yq} - y_1),$

$\sin\theta_1 (x_1 - 1_{xq}) + \cos(1_{yq} - y_1)\big)$

c) $T_1 T_{12} = T_2$

$T_{12} = T_1^{-1} T_{02} = \begin{bmatrix} R(\theta_1)^T & -R(\theta_1)^T t_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R(\theta_2) & t_2 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} R(-\theta_1) & -R(\theta_1) t_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R(\theta_2) & t_2 \\ 0 & 1 \end{bmatrix}$

$T_{12} = \begin{bmatrix} R(\theta_2 - \theta_1) & t_2 R(-\theta_1) - R(-\theta_1) t_1 \\ 0 & 1 \end{bmatrix}$

d) $T_2 P_{l'} = P_{lg}$

$$P_{l'} = T_2^{-1} P_{lg}$$

$$= \begin{bmatrix} R(\theta_2-\theta_1)^T & -R(\theta_2-\theta_1)^T R(-\theta_1)(t_2-t_1) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} l_g \\ 1 \end{bmatrix}$$

(l_g from camera frame)

$$P_{l'} = \begin{bmatrix} R(\theta_1-\theta_2) l_g + R(\theta_1-\theta_2) R(\theta_1)(t_1-t_2) \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} l' \\ 1 \end{bmatrix}$$

$$l' = (l_x, l_y)$$

Note - $T = \begin{bmatrix} R(\theta) & t \\ 0 & 1 \end{bmatrix}$, $R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$,

$P = \begin{pmatrix} l \\ 1 \end{pmatrix}$ $\quad t = \begin{bmatrix} x \\ y \end{bmatrix}$

can be rewritten as

$$T = \begin{bmatrix} \cos\theta & -\sin\theta & x \\ \sin\theta & \cos\theta & y \\ 0 & 0 & 1 \end{bmatrix} \quad P = \begin{bmatrix} l_x \\ l_y \\ 1 \end{bmatrix}$$