Introduction to Multi-Objective Optimization

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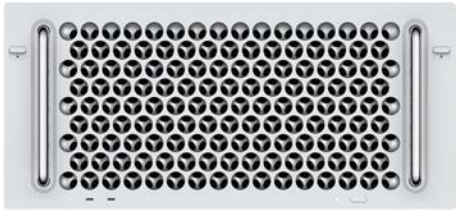
Roadmap

- What is Optimization?
- What is Multi-Objective Optimization?
- The History of Multi-Objective Optimization.
- Common Methods.
- Applications of Multi-Objective Optimization.

Optimization

- Life is about making decisions.
- We always attempt to make the “best” decision within a specified set of possible decisions.
- Consider buying a computer?



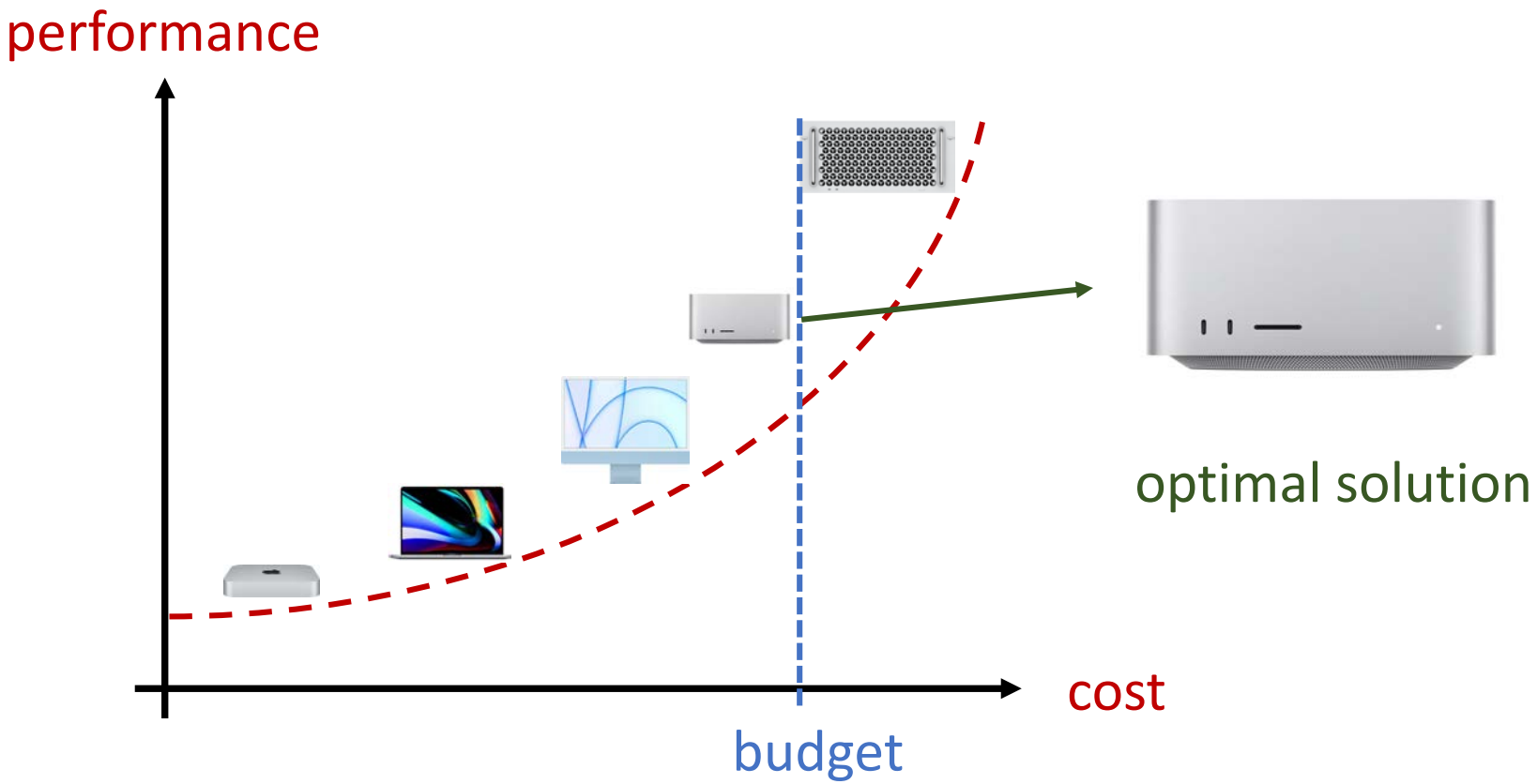



Optimization

- Consider buying a computer?

Objective: Max performance

Subject to: Budget-feasible



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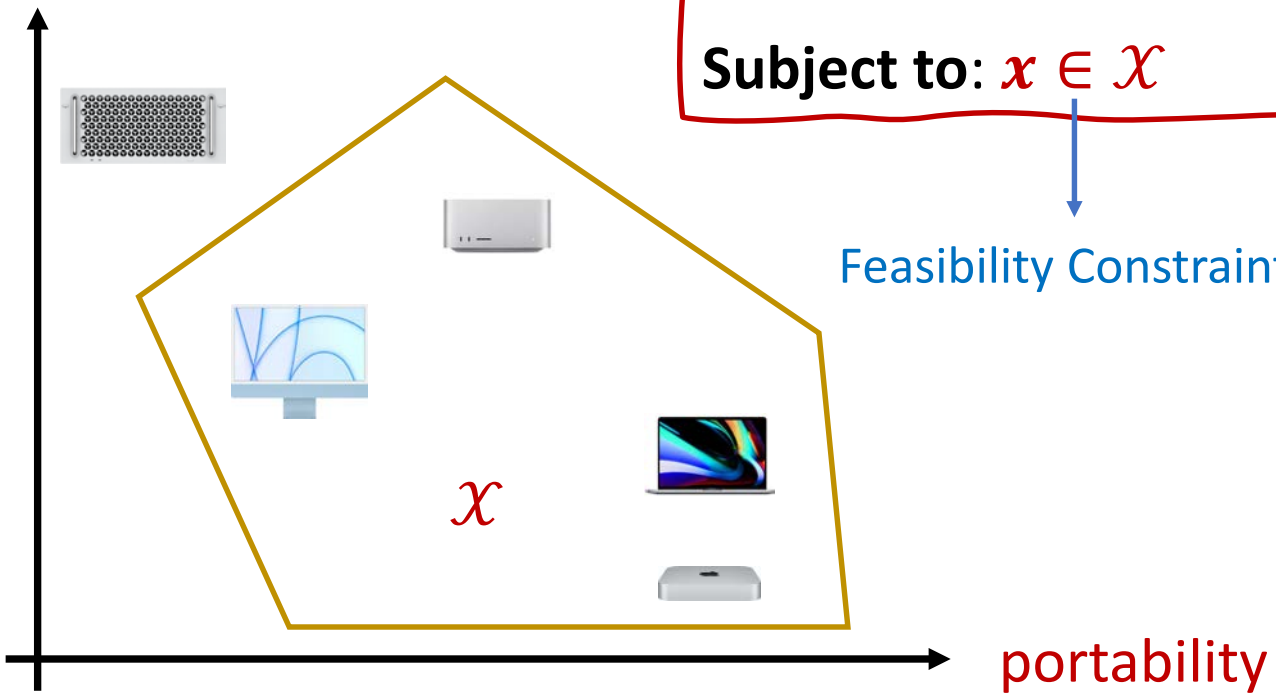
Multi-Objective Optimization

Multiple Objectives

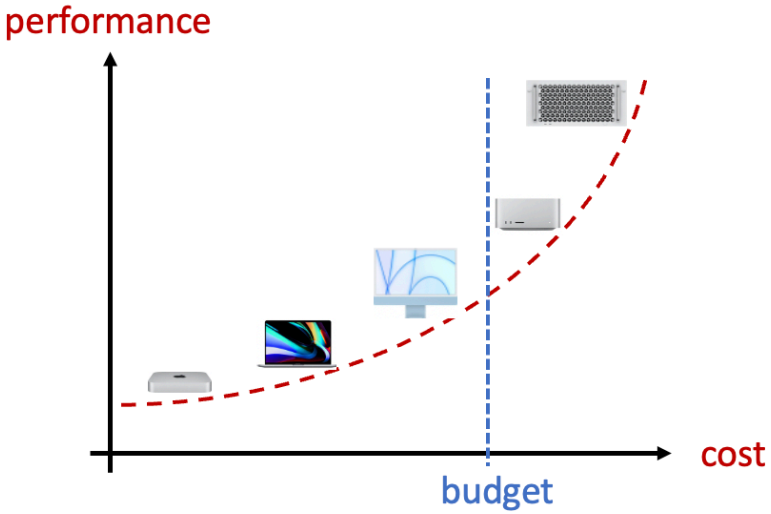
Objective: Max $f_1(\boldsymbol{x}), \cdots, f_k(\boldsymbol{x})$

Subject to: $\boldsymbol{x} \in \mathcal{X}$

Feasibility Constraints



Multi-Objective Optimization



Objective 1: Max performance

Subject to: Budget-feasible

- In the single-objective optimization problem, the superiority of a solution over other solutions is easily determined by comparing their objective function values.
- *How about multi-objective optimization problem?*

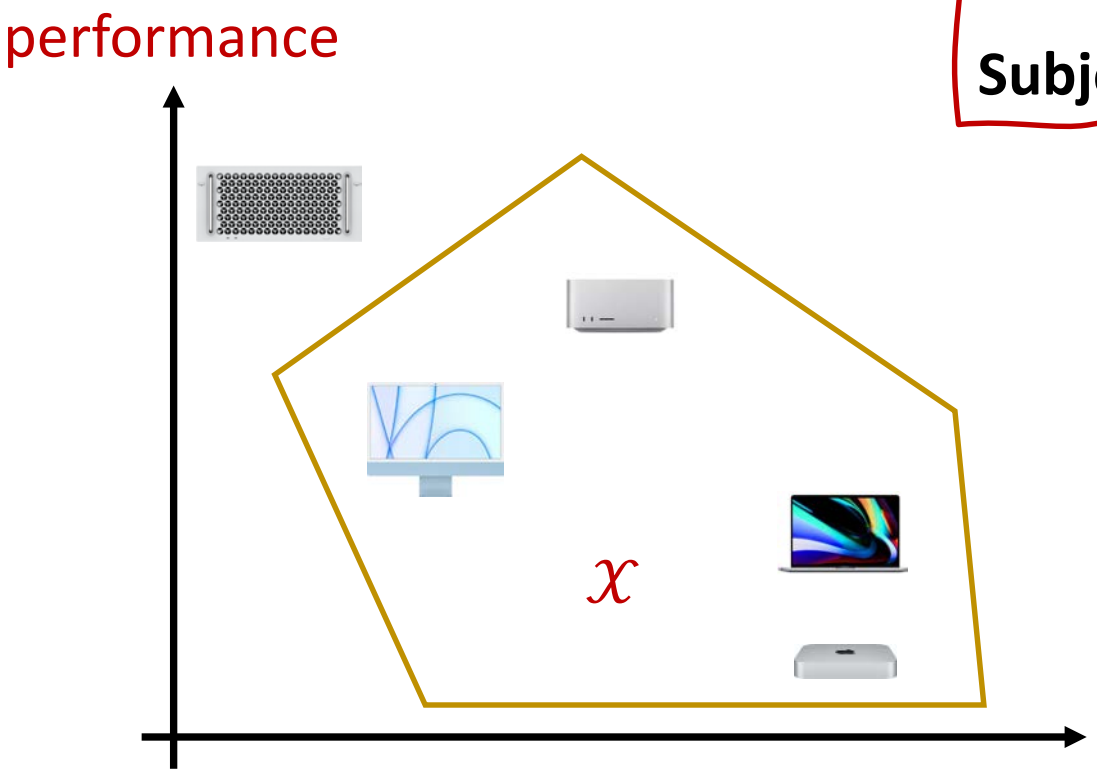
Multi-Objective Optimization

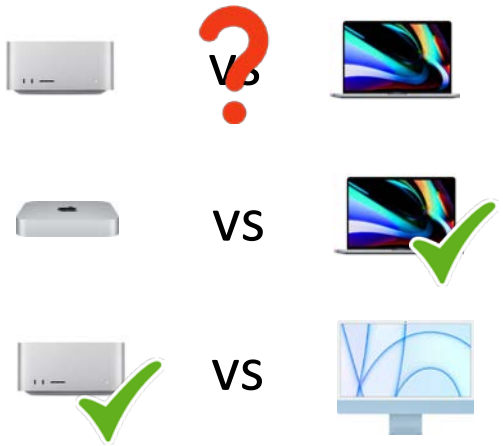
- Consider buying a computer?

Objective 1: Max performance

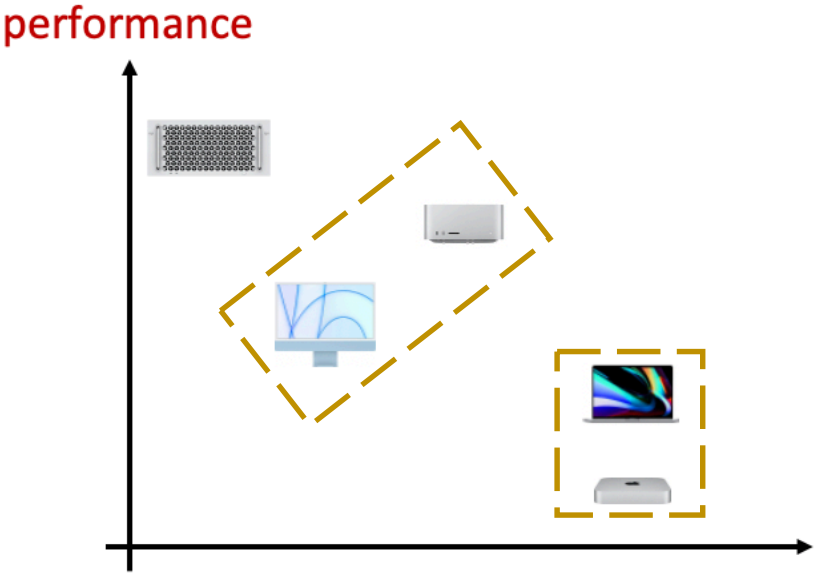
Objective 2: Max portability

Subject to: Budget-feasible





Multi-Objective Optimization



Objective 1: Max performance

Objective 2: Max portability

Subject to: Budget-feasible

- In multi-objective optimization problem, the goodness of a solution is determined by the **dominance**:

  Solution x_1 **(Pareto) dominates** Solution x_2 if
  (1) x_1 is no worse than x_2 in all objectives, and
  (2) x_1 is strictly better than x_2 in at least one objective.

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Multicriteria Decision Making

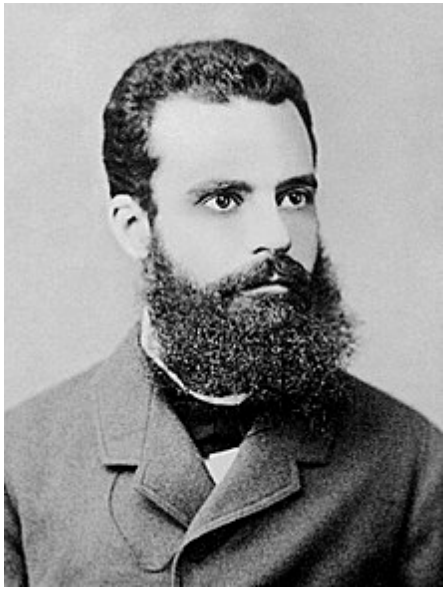
- In 1881, King's College (London) and later Oxford Economics *Prof. F.Y. Edgeworth* is the first to define an optimum for multicriteria economic decision making.
- He does so for the multiutility problem within the context of two consumers, P and π:

"It is required to find a point (x, y) such that in whatever direction we take an infinitely small step, P and π do not increase together but that, while one increases, the other decreases."

Reference:

- Edgeworth, F.Y., Mathematical Psychics, P. Keagan, London, England, 1881.

Vilfredo Pareto

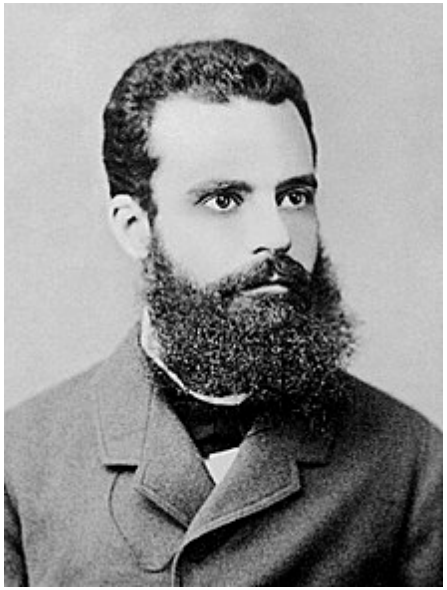


- Born in Paris in 1848 and graduated from the University of Turin in 1870 with a degree in Civil Engineering.

Thesis: "The Fundamental Principles of Equilibrium in Solid Bodies"

- While working in Florence as a Civil Engineer from 1870-1893, Pareto takes up the study of philosophy and politics and is one of *the first to analyze economic problems with mathematical tools*.

Vilfredo Pareto



- In 1893, Pareto becomes the Chair of Political Economy at the University of Lausanne in Switzerland, where he creates his two most famous theories:
  - ✓ Circulation of the Elites
  - ✓ *The Pareto Optimum*

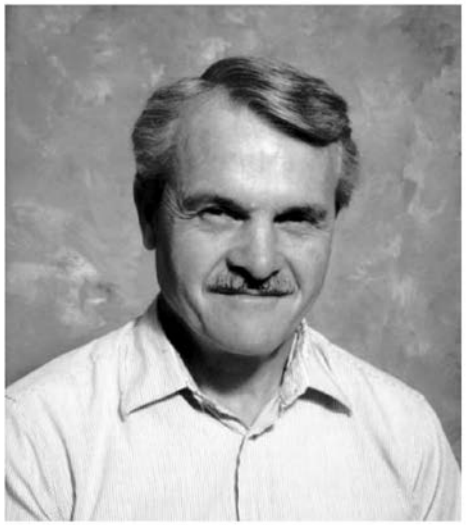
> *"The optimum allocation of the resources of a society is not attained so long as it is possible to make at least one individual better off in his own estimation while keeping others as well off as before in their own estimation."*

Reference:

- Pareto, V., Manuale di Economia Politica, Societa Editrice Libraria, Milano, Italy, 1906. *Translated into English by A.S. Schwier as Manual of Political Economy, Macmillan, New York, 1971*.

Extension to Engineering

- After the translation of Pareto's Manual of Political Economy into English, *Prof. Wolfram Stadler* of San Francisco State University begins to *apply the notion of Pareto Optimality to the fields of engineering and science* in the middle 1970's.
- The applications of multi-objective optimization in engineering design grew over the following decades.



Reference:

- Stadler, W., "A Survey of Multicriteria Optimization, or the Vector Maximum Problem," Journal of Optimization Theory and Applications, 1979.
- Stadler, W. "Applications of Multicriteria Optimization in Engineering and the Sciences (A Survey)," Multiple Criteria Decision Making –Past Decade and Future Trends, ed. M. Zeleny, JAI Press, Greenwich, Connecticut, 1984.
- Ralph E. Steuer, "Multicriteria Optimization -Theory, Computation and Application", 1985

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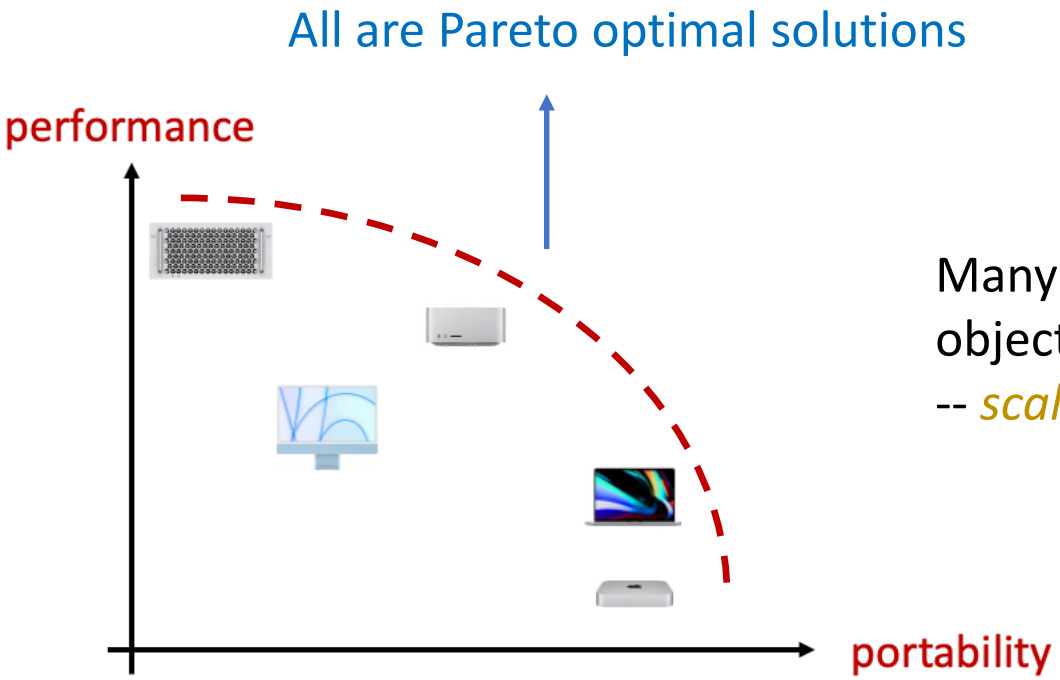
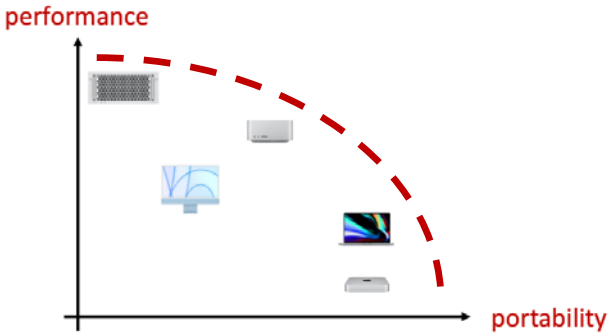
Solutions

- Multiple Pareto optimal solutions.
- There are various ways to define "solving a multi-objective optimization problem".



Many methods convert multiple objectives into a single-objective -- *scalarized problem*.

Solutions



- When *decision making* is emphasized, the objective of solving a multi-objective optimization problem is referred to supporting a *decision maker* in finding the most preferred Pareto optimal solution according to his/her subjective preferences.
  - ✓ **No preference methods**: no decision maker is available, but a neutral compromise solution is found without preferences.
  - ✓ **Priori methods**: preferences are first asked from the decision maker and then a solution best satisfying them is found.
  - ✓ **Posteriori methods**: a representative set of Pareto optimal solutions is first found and then the decision maker chooses one.
  - ✓ **Interactive methods**: the decision maker is allowed to iteratively search for the most preferred solution.

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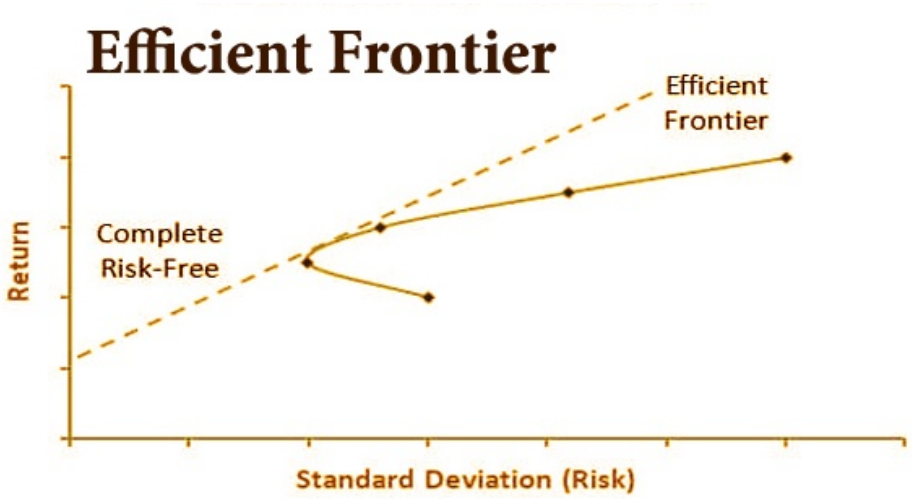
Finance

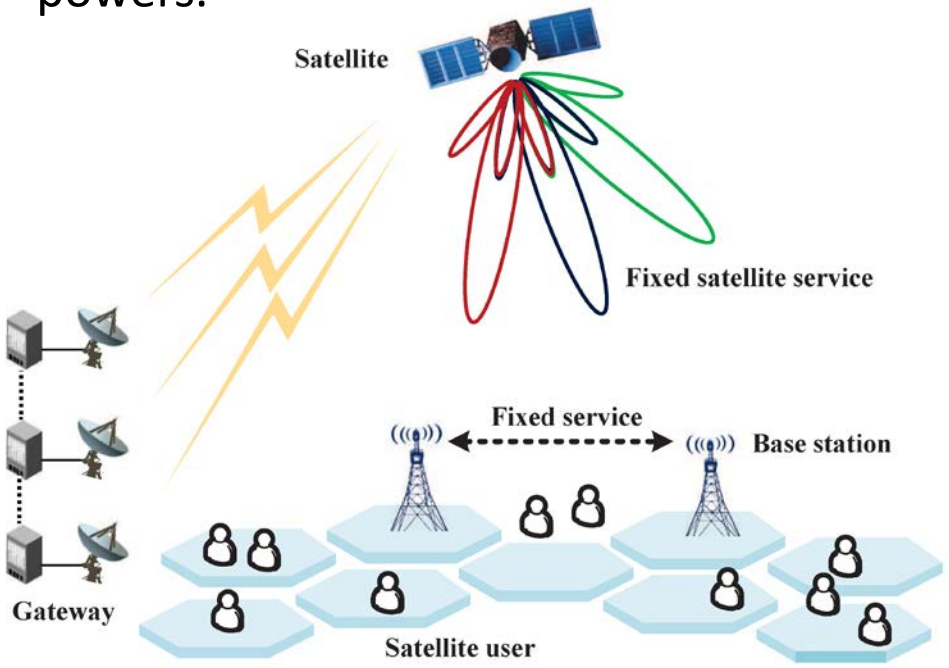
- In finance, a common problem is to choose a portfolio when there are two conflicting objectives:
  - the desire to have the expected value of portfolio returns be as high as possible, and
  - the desire to have risk, often measured by the standard deviation of portfolio returns, be as low as possible.



- *Efficient Frontier:* the best combinations of risk and expected return that are available.

Radio Resource Management

- The purpose of radio resource management is to satisfy the data rates that are requested by the users of a cellular network.
- The main resources are time intervals, frequency blocks, and transmit powers.



Each user has its own objective function.

These objectives are conflicting since the frequency resources are very scarce.

Inspection of Infrastructure

- Typically, planning inspection has been viewed as a single-objective optimization problem, where one aims to minimize the energy or time spent in inspecting an entire target structure.
- For complex, real-world structures, however, covering 100% of an inspection target is not feasible, and generating an inspection plan may be better viewed as a multi-objective optimization problem, where one aims to both maximize inspection coverage and minimize time and costs.

Thank you!