COMP1002
Computational Thinking and Problem Solving

Lecture 9
Problem Solving III

Lecture 9

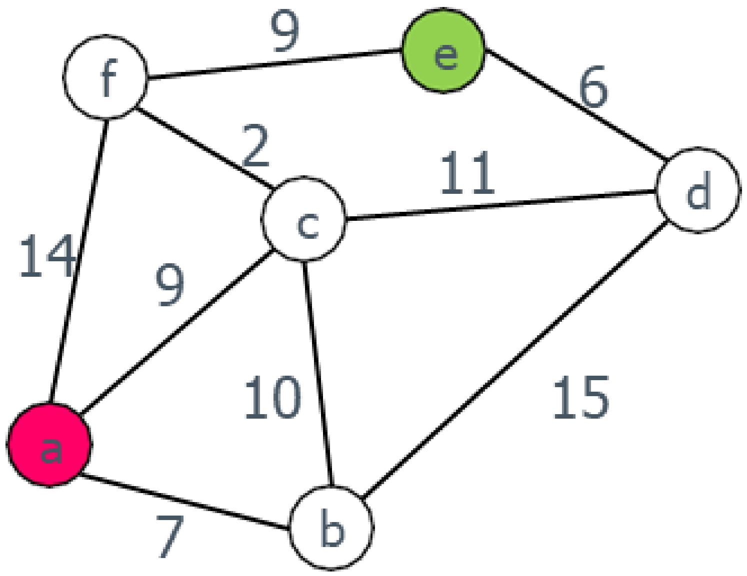
- Graph
  - Modelling
  - Implementation
    - Adjacency List
    - Adjacency Matrix
- A Glimpse of the Shortest Path Algorithm
- Final Tips on Problem Solving

Graph

- From the last lecture, we know *graphs* are so common for data modeling
  - How can we represent a graph in computers?
  - There are two parts
    - Nodes: can be represented as a list or a set
    - Edges: can be represented as a list or a set
  - For each edge, we would need a pair or a tuple expressing the two nodes
  - Some graphs need more information on the edges
    - Weight of an edge
    - Direction of an edge

Graph

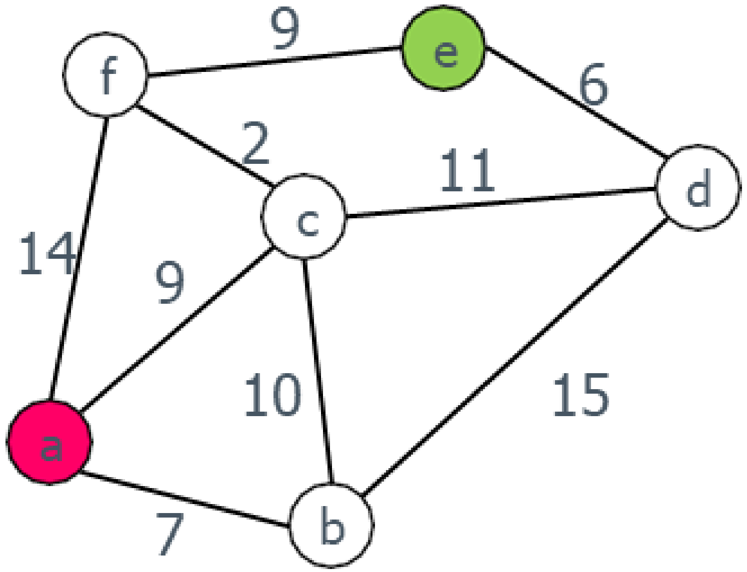
- For example
  - Nodes
    - {a,b,c,d,e,f} (6 nodes)
  - Edges
    - {ab, ac, af, bc, bd, cd, cf, de, ef} (9 edges)
  - With weights on edges:
    - {ab=7, ac=9, af=14, bc=10, bd=15, cd=11, cf=2, de=6, ef=9}
  - This graph is undirected (no direction on edges)



Graph

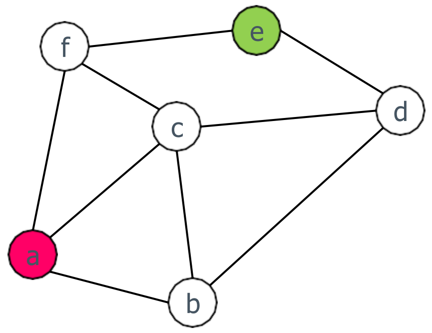
- We need to build the data model for the graph before we can use the computer to process it
  - It is easy to represent the nodes, but perhaps harder with the edges
  - We may represent nodes and edges separately as two different groups
  - It is more natural to represent edges as linked to nodes. There are two common representations:
    - Adjacency list
    - Adjacency matrix
  - Note that an edge in an undirected graph is equivalent to a pair of edges in a directed graph

Graph Representation



- Graph Representation
  - Simply G = (V, E)
    - Set of nodes
      - V = {a,b,c,d,e,f}
    - Set of edges
      - E = {ab, ac, af, bc, bd, cd, cf, de, ef}
      - E = {(a,b), (a,c), (a,f), (b,c), (b,d), (c,d), (c,f), (d,e), (e,f)}
    - Edges storing the weights
      - E = {ab=7, ac=9, af=14, bc=10, bd=15, cd=11, cf=2, de=6, ef=9}
      - E = {(a,b,7), (a,c,9), (a,f,14), (b,c,10), (b,d,15), (c,d,11), (c,f,2), (d,e,6), (e,f,9)}
    - Should we try to store nodes and edges together instead of separately?

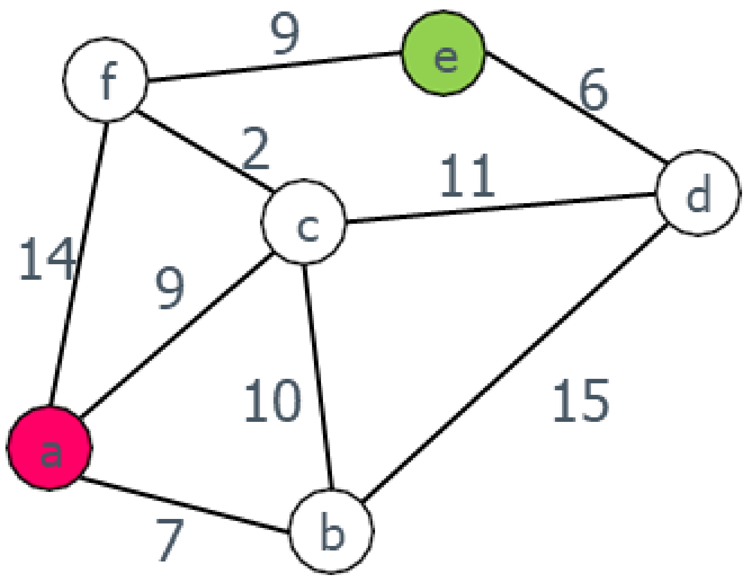
Adjacency List



- For each node, put together the edges for each node
- An adjacency list is a list for each node, showing the neighbours of that node, i.e., edges
- Example
  - Nodes, a set
    - {a,b,c,d,e,f}
  - Edges in 6 adjacency lists: D(node), each being a set

    D(a) = {b,c,f}
    D(b) = {a,c,d}
    D(c) = {a,b,d,f}
    D(d) = {b,c,e}
    D(e) = {d,f}
    D(f) = {a,c,e}

Adjacency List



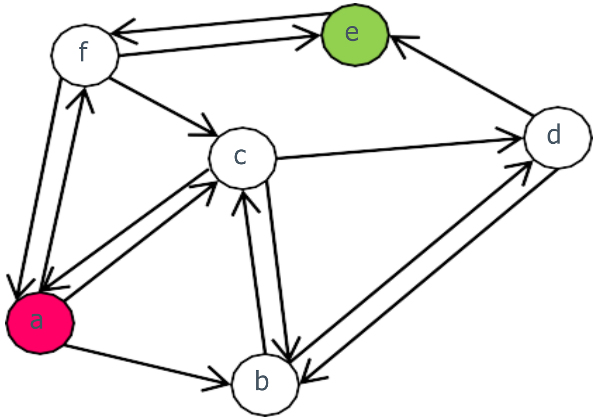
- For edges with weights, an adjacency list is a list for each node, showing the edges and the weights
- Example
  - Nodes, a set
    - {a,b,c,d,e,f}
  - Edges in adjacency lists: D(node), each being a set of tuples

    D(a) = {(b,7),(c,9),(f,14)}
    D(b) = {(a,7),(c,10),(d,15)}
    D(c) = {(a,9),(b,10),(d,11),(f,2)}
    D(d) = {(b,15),(c,11),(e,6)}
    D(e) = {(d,6),(f,9)}
    D(f) = {(a,14),(c,2),(e,9)}

Adjacency List

- For a directed graph (graph with direction on edges), an adjacency list is a list for each node, showing the next reachable node and perhaps also the weights
- Example
  - Nodes, a set
    - {a,b,c,d,e,f}
  - Edges in adjacency lists: D(node), each being a set

    D(a) = {b,c,f}
    D(b) = {c,d}
    D(c) = {a,b,d}
    D(d) = {b,e}
    D(e) = {f}
    D(f) = {a,c,e}



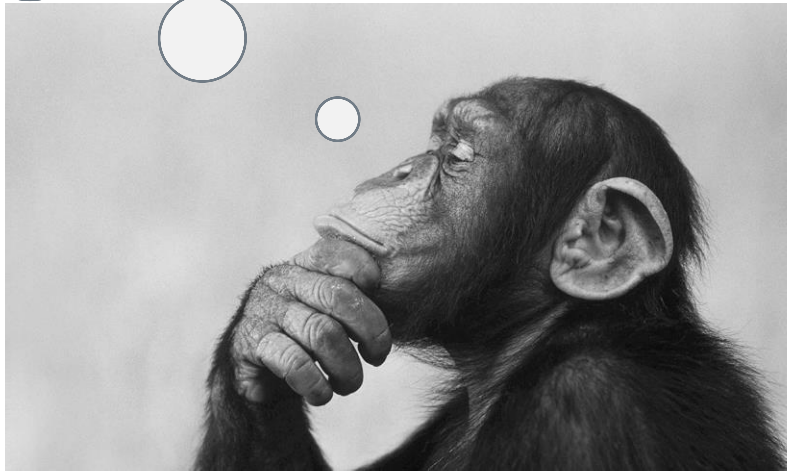
Adjacency List

- Modeling in Python
  - Nodes can be represented as a list
  - Edges can be represented as a list of lists
  - Example
    - Nodes as a list

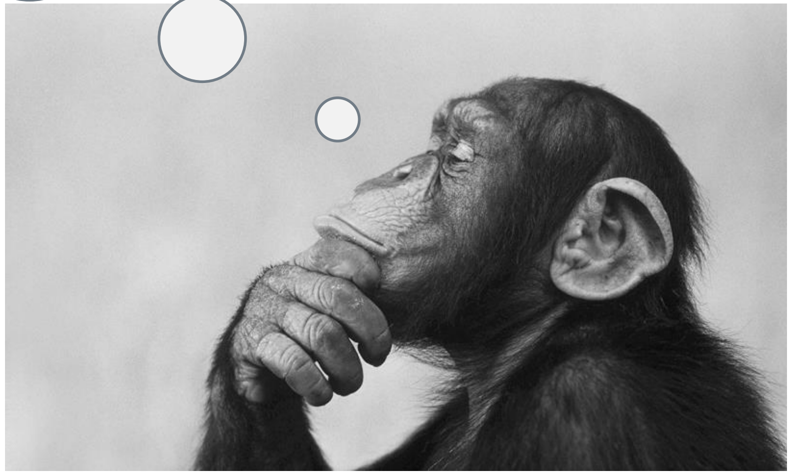
      N = [”a”, ”b”, ”c”, ”d”, ”e”, ”f”]
    - Edges as a list of lists

```
D = [  [”b”, ”c”, ”f”],
       [”a”, ”c”, ”d”],
       [”a”, ”b”, ”d”, ”f”],
       [”b”, ”c”, ”e”],
       [”d”, ”f”],
       [”a”, ”c”, ”e”]  ]
```

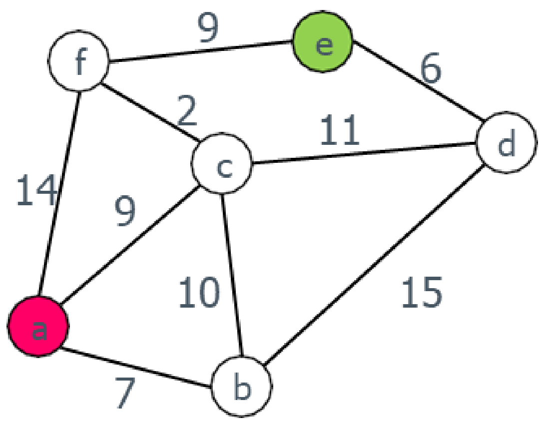
Can the edges be modeled as a list of sets?



Any other way(s) in Python to model the nodes and edges?



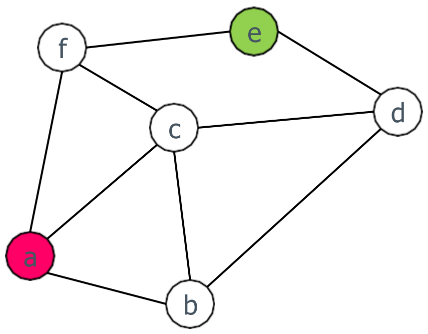
Adjacency List



- Modeling in Python
  - Nodes can be represented as a list
  - Edges with weights can be represented as a list of lists of tuples
  - Example
    - Nodes as a list
      - N = [”a”, ”b”, ”c”, ”d”, ”e”, ”f”]
    - Edges as a list of lists of tuples
      - D = [[(”b”,7),(”c”,9),(”f”,14)],
        [(”a”,7),(”c”,10),(”d”,15)],
        [(”a”,9),(”b”,10),(”d”,11),(”f”,2)],
        [(”b”,15),(”c”,11),(”e”,6)], [(”d”,6),(”f”,9)],
        [(”a”,14),(”c”,2),(”e”,9)]]

Adjacency Matrix

- For each pair of nodes, maintain a matrix to show the neighborhood between the pair
  - An adjacency matrix has each node in a row (source) and in a column (destination)
  - 1 / True means an edge
  - 0 / False means no edge
- Example
  - Matrix D[6,6] for a graph with 6 nodes



	a	b	c	d	e	f
a	0	1	1	0	0	1
b	1	0	1	1	0	0
c	1	1	0	1	0	1
d	0	1	1	0	1	0
e	0	0	0	1	0	1
f	1	0	1	0	1	0

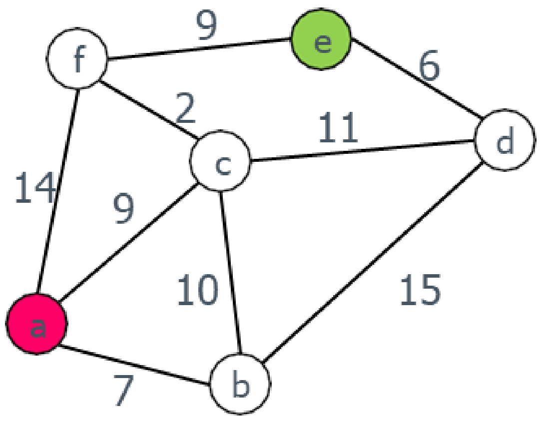
Adjacency Matrix

› An adjacency matrix for a graph with weight have matrix elements showing neighborhood and weight

› This is often called a distance matrix when the weight is the distance

› Example

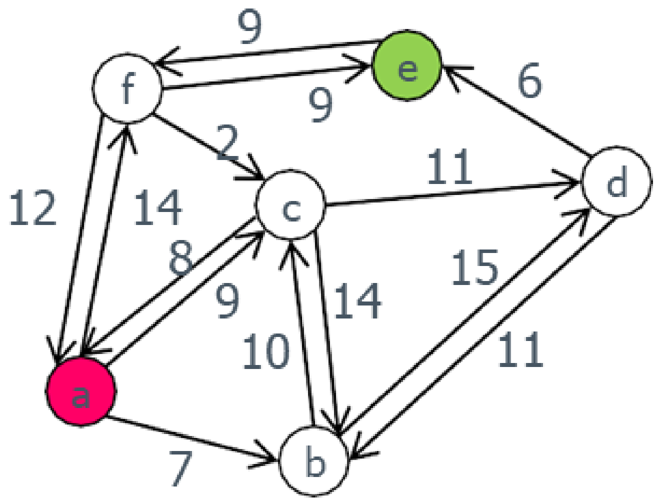
– Matrix D[6,6] for a graph with 6 nodes



	a	b	c	d	e	f
a	0	7	9	∞	∞	14
b	7	0	10	15	∞	∞
c	9	10	0	11	∞	2
d	∞	15	11	0	6	∞
e	∞	∞	∞	6	0	9
f	14	∞	2	∞	9	0

Adjacency Matrix

- A distance matrix for a directed graph with weight have matrix elements showing the weight/distance from node in row i to node in column j
- Example
  - Matrix D[6,6] for a graph with 6 nodes



	a	b	c	d	e	f
a	0	7	9	∞	∞	14
b	∞	0	10	15	∞	∞
c	8	14	0	11	∞	∞
d	∞	11	∞	0	6	∞
e	∞	∞	∞	∞	0	9
f	12	∞	2	∞	9	0

Adjacency Matrix

› The 2-D matrix is normally represented as a list of lists as a logical 2-D array

– Example

› Nodes as a list:

N = [”a”, ”b”, ”c”, ”d”, ”e”, ”f”]

› Matrix as a list of lists:

```
D = [  [0, 1, 1, 0, 0, 1],
       [1, 0, 1, 1, 0, 0],
       [1, 1, 0, 1, 0, 1],
       [0, 1, 1, 0, 1, 0],
       [0, 0, 0, 1, 0, 1],
       [1, 0, 1, 0, 1, 0]  ]
```

Adjacency Matrix

- The 2-D matrix can also be represented as a dictionary of dictionary
- Example
  - Nodes as a list
    - N = [”a”, ”b”, ”c”, ”d”, ”e”, ”f”]
  - Matrix as a dictionary of dictionary

```
D = {  ”a”: {”a”:0,”b”:7,”c”:9,”d”:inf,”e”:inf,”f”:14},
       ”b”: {”a”:inf,”b”:0,”c”:10,”d”:15,”e”:inf,”f”:inf},
       ”c”: {”a”:8,”b”:14,”c”:0,”d”:11,”e”:inf,”f”:inf},
       ”d”: {”a”:inf,”b”:11,”c”:inf,”d”:0,”e”:6,”f”:inf},
       ”e”: {”a”:inf,”b”:inf,”c”:inf,”d”:inf,”e”:0,”f”:9},
       ”f”: {”a”:12,”b”:inf,”c”:2,”d”:inf,”e”:9,”f”:0}  }
```

  - Here, inf = float(”infinity”) in Python, meaning infinity (∞).

Shortest Path

› A highly common application on a graph is to find the shortest path from one node to another

– The starting node is often called *source node* in graph theory

– The target node is often called *destination node*

– The graphs may contain weights, or without

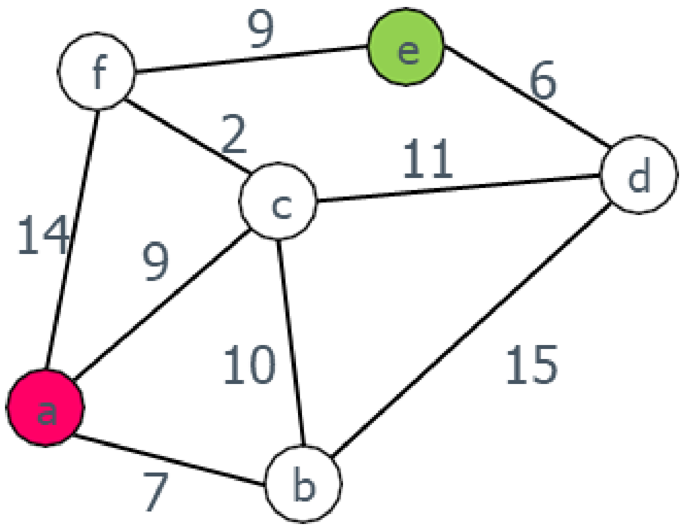
› We call them weighted graphs and unweighted graphs respectively

– The graphs may contain edges with or without directions

› We call these two types directed graphs and undirected graphs respectively

Shortest Path

- Can you find the shortest path from a to e?
  - Algorithm 1: Layman approach to find all possible paths first

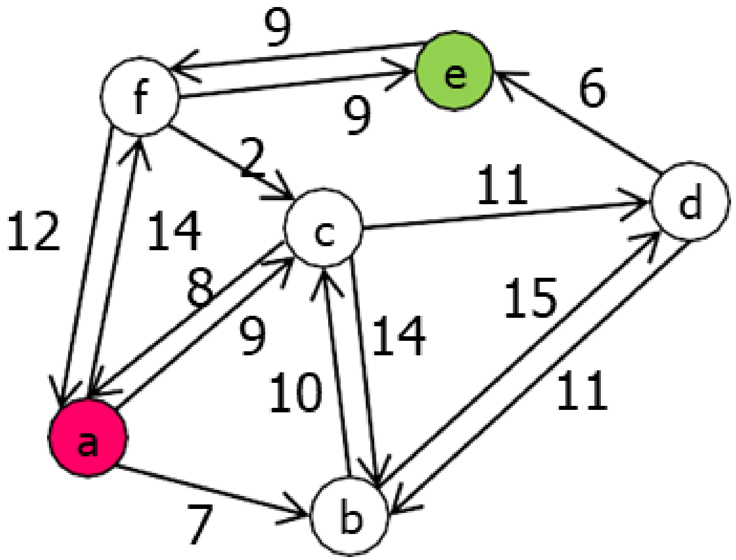


a → f → e	23
a → c → f → e	20
a → c → d → e	26
a → b → c → d → e	34
a → b → d → e	28
a → c → b → d → e	40
Any more?	
a → b → c → f → e	28
a → b → d → c → f → e	44
Yet more?	
a → f → c → d → e	33
a → f → c → b → d → e	47

  - Mission impossible for a large graph!

Shortest Path

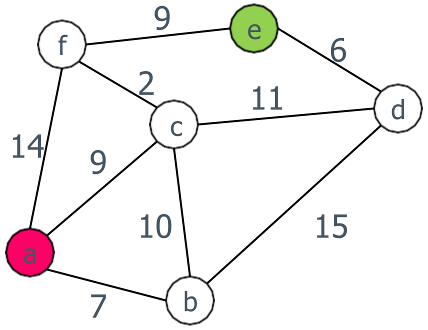
› Can you find the shortest path from a to e?
  - This is a directed graph
  - There are fewer possible paths than previous one



a → f → e	23
a → c → d → e	26
a → b → c → d → e	34
a → b → d → e	28
a → c → b → d → e	44
a → f → c → d → e	33
a → f → c → b → d → e	51

  - Still mission impossible for a large graph!

Improvement

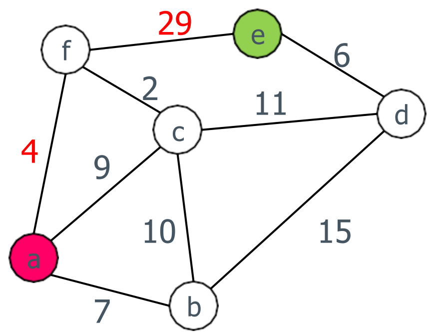


- There are so many possible paths
- Algorithm 2: search for paths starting with fewer steps

a → f → e	23
a → b → d → e	28
a → c → d → e	26
a → c → f → e	20
Stop here?	
a → b → c → d → e	34
a → b → c → f → e	28
a → c → b → d → e	40
a → f → c → d → e	33
Stop here?	
a → b → d → c → f → e	44
a → f → c → b → d → e	47

Improvement

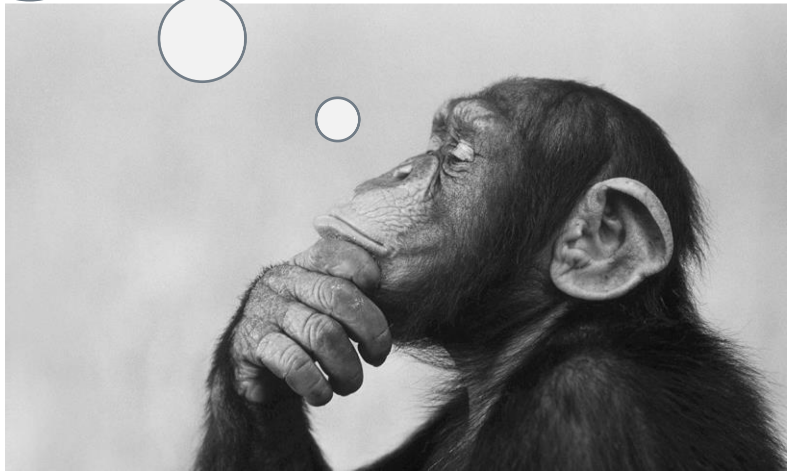
› Consider this graph



› Search for paths starting with fewer steps

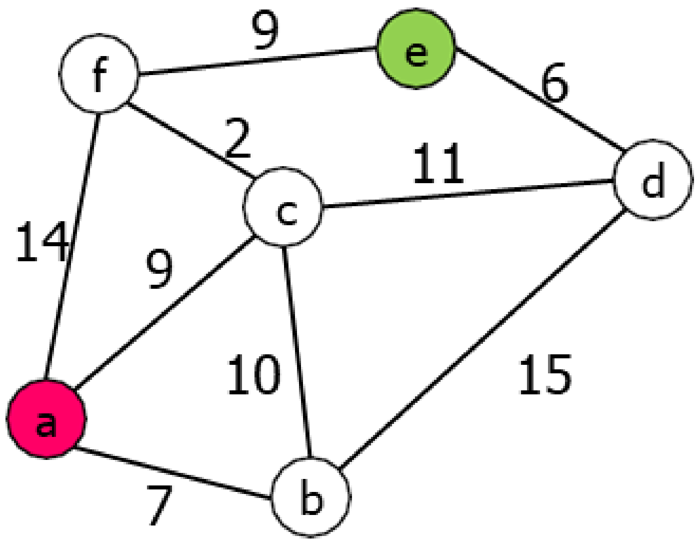
▪ a → f → e	▪ 33
▪ a → b → d → e	▪ 28
▪ a → c → d → e	▪ 26
▪ a → c → f → e	▪ 40
▪ Stop here?	
▪ a → b → c → d → e	▪ 34
▪ a → b → c → f → e	▪ 48
▪ a → c → b → d → e	▪ 40
▪ a → f → c → d → e	▪ 23
▪ Stop here?	
▪ a → b → d → c → f → e	▪ 64
▪ a → f → c → b → d → e	▪ 37

We may miss the correct answer if not trying out all possible paths...



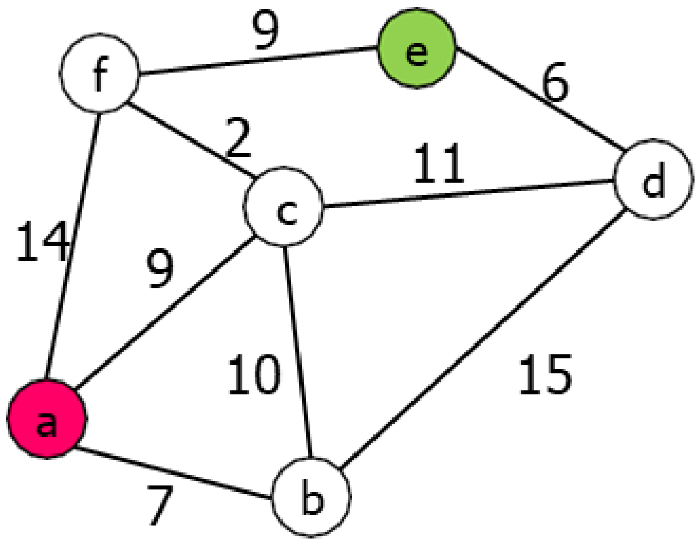
Improvement

- We need a more clever and systematic approach!
- Possible approach
  - Look at edges and use them to improve on existing paths
  - An edge is said to lead to improvement, if passing through it would lead to a better path
  - Example
    - Going from a to c directly, the distance is 9.
    - Going from a to f directly, the distance is 14.
    - Going from a to f via c, the distance improves to 11 < 14.



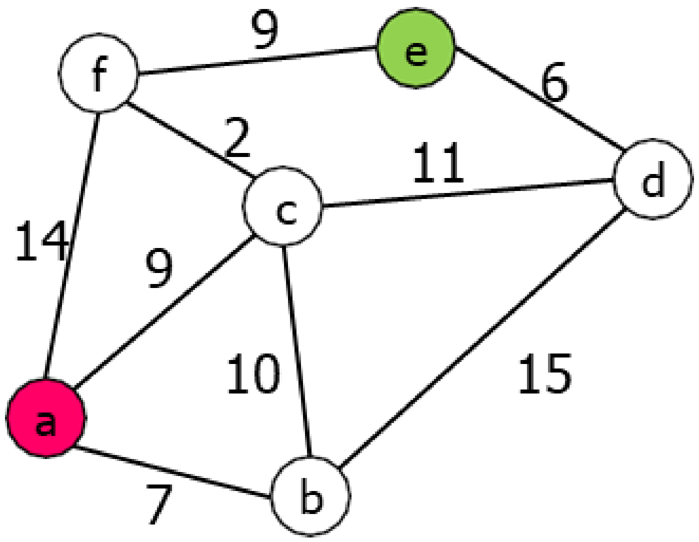
Improvement

- A sketch of the idea
  - Starting from a, find the shortest distance to its neighbouring node, e.g., node b
    - Save the path to other neighbour and record the distance(s)
  - Then, use this path to go to b's neighbour
    - Compare the distance of the saved path with the distance when passing through b
      - To go to c, should we take a -> b -> c?
  - Explore from the newly done node, i.e., c, to its neighbours and continue the process until the shortest paths to all nodes are found



Further Improvement

- The shortest path problem will be carefully studied in algorithm courses
  - Further Reading - Dijkstra's algorithm
    - https://en.wikipedia.org/wiki/Dijkstra%27s_algorithm



Final Tips on Problem Solving

- Data abstraction
  - Use data structures to model the data and their relationship
    - In our example, use graph
  - Use proper tools in the programming language to represent the model
    - List, set and dictionaries
- Procedure abstraction
  - Try stupid solutions first, as long as it is logical
    - In our graph example, find all possible paths
  - Refine your solution based on your understanding of the problem and try to find out patterns/structures of the problem
    - A problem can be broken down in to smaller ones. Sometimes the smaller problem has a similar pattern as the parent one
      - In our graph example, find the shortest path to near nodes first

Final Tips on Problem Solving

- There is no magic to problem solving
  - The holy grail is to
    - Practice, practice, and practice
- There is no magic to programming
  - The holy grail is to
    - Practice, practice, practice, and yet more practice
    - Do not aim at producing the shortest or cleverest program unless you are an expert (how? Practice, practice, and practice!)

Summary

- Graph
  - Modelling
  - Implementation
    - Adjacency List
    - Adjacency Matrix
- A Glimpse of the Shortest Path Algorithm
- Final Tips on Problem Solving