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Q3) a) <u>Probability Mass Function:</u>

Binomial Function:

$P(X = x) = C(n, x) * p^x * (1 - p)^{(n - x)}$

But because we need to have r-1 successes in the first z-1 trials, and then 1 success on the zth trial

$P(X=k; r, p) = C((z - 1), (r - 1)) * p^{(r - 1)} * (1 - p)^{((z - 1) - (r - 1))} * p$

Which is equal to:

$P(X=k; r, p) = C((z - 1), (r - 1)) * p^{(r)} * (1 - p)^{(z - r)}$

Let $z = r + k$

Thus,

$P(X=k; r, p) = C((r + k - 1), (k)) * p^{(r)} * (1 - p)^{(k)}$

<u>Mean (E(X)):</u>

$E(X) =$

$$\sum_{K=0}^{\infty} K * C((r + k - 1), (k)) * p^{(r)} * (1 - p)^{(k)}$$

∞

$$\sum_{K=1} (r + k - 1)!/((r - 1)!(k - 1)!) * p^{r} * (1 - p)^{k}$$

$$\sum_{K=1}^{\infty} r(1-p)/p * C((r + k - 1), (k - 1)) * p^{(r + 1)} * (1 - p)^{(k - 1)}$$

But, z = r + k

$$r(1 - p)/p * \sum_{z=0}^{\infty} C((r + 1 + z - 1), (z)) * p^{(r + 1)} * (1 - p)^{(z)}$$

= r(1 - p)/p

Thus, E(X) = r(1 - p)/p

Variance (Var(X)):

Given the formula for E(X),

E(X - EX^2) = EX^2 - (EX)^2
E(X(X - 1)) = E(X^2 - X) = EX^2 - EX
EX^2 = E(X(X - 1)) + EX
Thus, Var(X) = E(X(X - 1)) + EX - (EX)^2

E(X(X - 1)) =

$$\sum_{X=0}^{\infty} C((r + k - 1), (k)) * p^{(r)} * (1 - p)^{(k)} * k(k - 1)$$

But C((r + k - 1)!, (k!))* k(k - 1) =
r(r + 1) * C((r + k - 1), (k - 2))

E(X(X - 1)) =

$$\sum_{X=2}^{\infty} C((r + k - 1), (k)) * p^{(r)} * (1 - p)^{(k)} * k(k - 1)$$

Let y = k - 2
q = r + 2
Thus, r + k - 1 = q + y - 1

E(X(X - 1)) =

$$r(r + 1) * \sum_{y = 0}^{\infty} C((q + y - 1), (q)) * p^{(q - 2)} * (1 - p)^{(k)} * k(y + 2)$$

$$(r * (r + 1) * (1 - p)^2)/ p^2 \sum_{y = 0}^{\infty} C((q + y - 1), (q)) * p^{(q - 2)} * (1 - p)^{(k)} * k(y + 2)$$

Thus, Variance $Var(X) = E(X(X - 1)) + EX - (EX)^2$
$= (r * (r + 1) * (1 - p)^2)/ p^2 + (r * (1 - p) / p) - (r * (1 - p) / p)^2$
$= (r * (1 - p) / p) [(r + 1)(1- p)/p + 1 - r * (1 - p) / p]$
$= (r * (1 - p) / p^2)[r - rp + 1 - p + p - r + rp]$
$= r * (1 - p) / p^2$

Hence, $Var(X) = r * (1 - p) / p^2$

Q3) C)

Number of lives (r) = 9
Prob of being hit (p) = 1/20

$E(X) = r(1 - p)/p$
$= 9 * (19/20) / (1/20) = 9 * 0.95 / 0.05$
$= 171$

(or)

$E(X) = r/p$
$= 9/(1/20)$
$=180$

Thus, expected value is 171 or 180

Standard Deviation = $sqrt(Var(X)) = sqrt(r * (1 - p) / p^2)$

$= sqrt(9 * (19/20) / (1/20)^2)$
$= sqrt(3420)$
$= 58.480$

Q3) D)

P (X>=k; r, p) =

$$1- \sum_{k=0}^{k-1} C((r + k - 1), (k)) * p^{(r)} * (1 - p)^{(k)}$$

Here, r = 1
P = 0.05
K = 104

$$1- \sum_{k=0}^{103} C((104), (104)) * 0.05^{(1)} * (0.95)^{(104)}$$

=

$$1- \sum_{k=0}^{103} 0.05 * (0.95)^{(104)}$$

= 1 - (0.05 + 0.0475 + 0.045125 + 0.04286875 + 0.0407253125…)
= 0.0053 (approx.)

Thus, probability of Montgomery surviving if he has 1 life left is 0.0053

P (X>=k; r, p) =

$$1 - \sum_{k=0}^{k-1} C((r + k - 1), (k)) * p^{(r)} * (1 - p)^{(k)}$$

Here, r = 9
P = 0.05
K = 104

$$1 - \sum_{k=0}^{103} C((8+k), (k)) * 0.05^{(9)} * (0.95)^{(104)}$$

= 0.89 (approx.)
Thus, probability of Montgomery surviving if he has 9 life left is 0.89