COMP 1433: Introduction to Data Analytics &
COMP 1003: Statistical Tools and Applications

Lecture 3 – Statistics Basics for Data Analytics

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Discussion: Tears of Sally

- *Sally Clark* was the victim of a miscarriage of justice in 1999.
- She was found guilty of the murder of two infant sons, both died within the first few weeks of their births; and only the mother is present when the babies die.
- Here are the arguments from the defense:
  - 1 in 8,500 babies died from sudden infant death syndrome (SIDS), which causes the babies' death in the first few weeks of their births.
  - The probability that two babies both died from SIDS is 1 in 73 million (the square of 1/8500)
  - The probability that the mother kills the sons is $1 - \frac{1}{73M} \approx 1$, so the mother is the murder!
- *QUESTION: How can you fight back?*

Roadmap

- Expectation and Variance
- Sample Statistics
- Hypothesis Testing
- Statistics vs. Data Analytics
  - Naïve Bayes (cont.)
  - Probabilistic Language Models (cont.)

Roadmap

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Expectation of Random Variables

- The *expected value* for discrete random variable X
  - $E[X] \equiv \sum_k x_k P(X = x_k) = \sum_k x_k p_X(x_k)$
  - It represents the *weighted average sum* of X
  - Also called the *mean* of X
- **Example**. The *expected value* of *rolling a die* is:
  - $E(X) = 1 \cdot \frac{1}{6} + 2 \cdot \frac{1}{6} + \cdots + 6 \cdot \frac{1}{6} = \frac{1}{6} \cdot \sum_{k=1}^{6} k = \frac{7}{2}$
- **PROPERTIES**
  - $E[aX] = aE[X]$
  - $E[aX + b] = aE[X] + b$

Variance and Standard Deviation

- Let the mean of X be $\mu = E[X]$
- Then the *variance* of X is:
  - $Var(X) \equiv E[(X-\mu)^2] = \sum_k (x_k - \mu)^2 p_X(x_k)$
  - The weighted *square distance* from the *mean*.
- We have another form of *variance as:*
  - $Var(X) = E[X^2] - \mu^2$
- The *standard deviation* is $\sigma(X) = \sqrt{Var(x)}$
  - The weighted *distance* from the *mean*.
- The *variance* of *rolling the die* is:
  - $Var(X) = \sum_{k=1}^{6}[k^2 \cdot \frac{1}{6}] - \mu^2 = \frac{1}{6} \cdot \frac{6(6+1)(2\cdot 6+1)}{6} - \left(\frac{7}{2}\right)^2 = \frac{35}{12}$
  - The *standard deviation* $\sigma = \sqrt{\frac{35}{12}} \cong 1.70$

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What is *Sampling*?

- *Sampling* can consist:
  - *Gathering random data* from a *large* population, e.g.,
    - Measuring the *height of randomly selected adults*.
    - Measuring the *starting salary of random CS students*.
  - *Recording* the *results of experiments*, e.g.,
    - Measuring the *breaking strengths* of randomly selected bolts
    - Measuring the *lifetime* of randomly selected bulbs.
- **Assumptions**:
  - The population is *infinite* (or very *large*).
  - The observations are *independent*.
    - The experiment outcome does not affect other experiments.

What is *Sample Statistics*?

- A random sample from a population consists of:
  - *Independent*, *identically distributed* random variables,

$$X_1, X_2, \dots, X_n$$

- The values of X_i are called the *outcomes* of the experiment.
- A *statistic* is a *function* of $X_1, X_2, \dots, X_n$.
- Thus, a *statistic* itself is a *random variable*.

Important Statistics

- The *sample mean*:
  - $\bar{X} \equiv \frac{1}{n}(X_1 + X_2 + \cdots + X_n)$
- The sample variance and standard deviation:
  - $S^2 \equiv \frac{1}{n-1}\sum_{k=1}^{n}(X_k - \bar{X})^2$ and $S = \sqrt{\frac{1}{n-1}\sum_{k=1}^{n}(X_k - \bar{X})^2}$
- The *order statistic* in which the observation are *ordered in size*, e.g., *increasing order*
- The *sample median* is:
  - The *mid-value* of the *order statistic* (if n is an odd)
  - The *average* of the *two middle values* (if n is an even)
- The *sample range* is the *difference between the largest and smallest observations*.

Example: Important Statistics

- For the 8 observations:
  - *-0.737, 0.511, -0.083, 0.066, -0.562, -0.906, 0.358, 0.359*
- What is the *sample mean*:
  - $\bar{X} = \frac{1}{8}(-0.737 + 0.511 - 0.083 + 0.066 - 0.562 - 0.906 + 0.358 + 0.359) = -0.124$
- What is the *sample variance*:
  - $S^2 = \frac{1}{8-1}[(-0.737 + 0.124)^2 + (0.511 + 0.124)^2 + (-0.083 + 0.124)^2 + (0.066 + 0.124)^2 + (-0.562 + 0.124)^2 + (-0.906 + 0.124)^2 + (0.358 + 0.124)^2 + (0.359 + 0.124)^2] = 0.297$
- What is the *sample standard deviation*:
  - $S = \sqrt{0.297} = 0.545$

Example: Important Statistics

- For the 8 observations:
  - *-0.737, 0.511, -0.083, 0.066, -0.562, -0.906, 0.358, 0.359*
- What is the *order statistics*?
  - *-0.906, -0.737, -0.562, -0.083, 0.066, 0.358, 0.359, 0.511*.
- What is the *sample median*?
  - $\frac{-0.083+0.066}{2} = -0.0085$
- What is the *sample range*?
  - $0.511 - (-0.906) = 1.417$

Discussion: The Trap of Mean

- ***NEWS TITLE***: The *average income* of Company T's employees is *100K HKD/month*
- Is the news title *misleading*?
- Considering the following example:
  - *10% of the employees earn 910K*
  - *90% of the employees earn 10K*
- What is the *mean* of their monthly income?
- Is it able to reflect the overall situation?
- What is a better alternative?

Sample Mean vs. Population Mean

- Suppose the population mean and standard deviation are μ and σ.
- The sample mean is $\bar{X} = \frac{1}{n}(X_1 + X_2 + \cdots + X_n)$
    - It is also a *random variable*:
    - Expected value
        - $\mu_{\bar{X}} \equiv E[\bar{X}] = E\left[\frac{1}{n}(X_1 + X_2 + \cdots + X_n)\right] = \mu$
    - Variance
        - $\sigma_{\bar{X}}^2 \equiv Var(\bar{X}) = \frac{\sigma^2}{n} \rightarrow n \to +\infty$ and $\sigma_{\bar{X}}^2 \to 0$
- So, the expected value of *sample mean* is the *population mean* μ.

Law of Large Number

- *Chebyshev's Inequality*. For any random variables:
  - $P(|X-\mu| \geq \epsilon) \leq \frac{\sigma^2}{\epsilon^2}$ ($\mu = E[X]$ and $\sigma = \sqrt{Var(X)}$)
- Then $P(|\bar{X}-\mu| \leq \epsilon) \geq 1 - \frac{\sigma_{\bar{X}}^2}{\epsilon^2} = 1 - \frac{\sigma^2}{n\epsilon^2}$
- For $n \to +\infty$, $P(|\bar{X}-\mu| \leq \epsilon) = 1$ for any $\epsilon > 0$
- The *sample mean* approximates the *population mean* μ for very large n.

Markov Inequality and Chebyshev's Inequality

- *Chebyshev's Inequality*. For any random variables, we have:
  - $P(|X-\mu| \geq \epsilon) \leq \frac{\sigma^2}{\epsilon^2}$, where $\mu = E[X]$ and $\sigma = \sqrt{Var(X)}$
  - $P(|X-\mu| \geq \epsilon) = P\left((X-\mu)^2 \geq \epsilon^2\right) \leq \frac{\sigma^2}{\epsilon^2}$
- *Markov Inequality*. For discrete random variable $X \geq 0$ and $\epsilon > 0$:
  - $P(X \geq \epsilon) \leq \frac{E[X]}{\epsilon}$
    - $E[X] = \sum_{x\geq 0} xp(x) \geq \sum_{x \geq \epsilon} xp(x) \geq \epsilon \sum_{x\geq\epsilon} p(x) = \epsilon P(X \geq \epsilon)$
  - It also holds for continuous random variables!

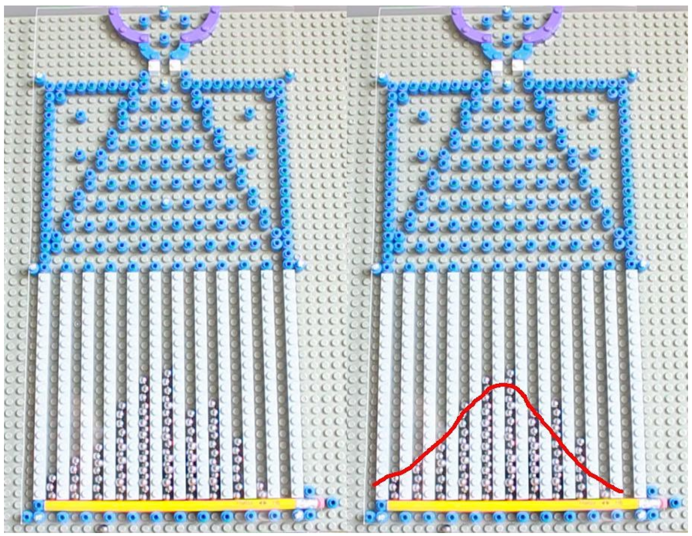
Law of Large Number

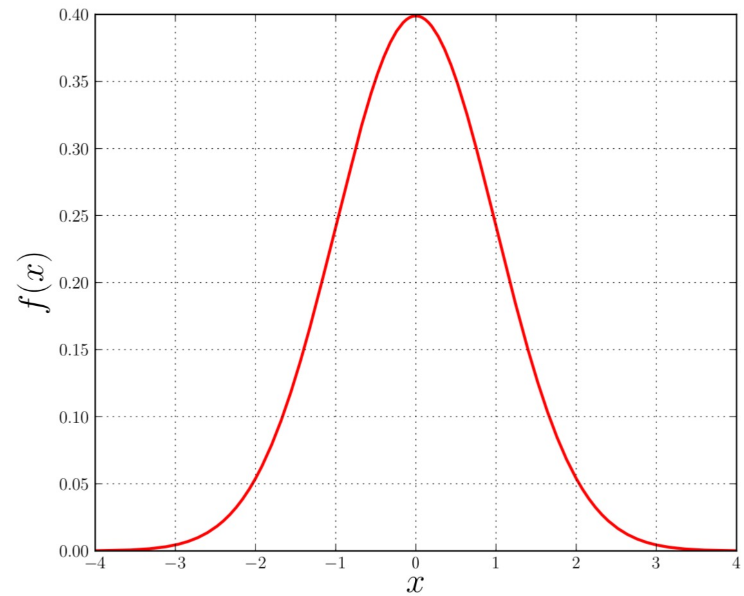
- *Chebyshev's Inequality*. For any random variables:
  - $P(|X-\mu| \geq \epsilon) \leq \frac{\sigma^2}{\epsilon^2}$ ($\mu = E[X]$ and $\sigma = \sqrt{Var(X)}$)
- Then $P(|\bar{X}-\mu| \leq \epsilon) \geq 1 - \frac{\sigma_{\bar{X}}^2}{\epsilon^2} = 1 - \frac{\sigma^2}{n\epsilon^2}$
- For $n \to +\infty$, $P(|\bar{X}-\mu| \leq \epsilon) = 1$ for any $\epsilon > 0$
- The *sample mean* approximates the *population mean* μ for very large n.

Central Limit Theorem

- Suppose the population mean and standard deviation are μ and σ.
- The sample mean is $\bar{X} = \frac{1}{n}(X_1 + X_2 + \cdots + X_n)$
  - $\mu_{\bar{X}} \equiv E[\bar{X}] = E\left[\frac{1}{n}(X_1 + X_2 + \cdots + X_n)\right] = \mu$
  - $\sigma_{\bar{X}}^2 \equiv Var(\bar{X}) = \frac{\sigma^2}{n}$
- **NOTE**. $\bar{X}$ is approximately *general normal* (or satisfies *normal distribution*) for very large n.
  - The proof requires advanced knowledge in Calculus.
  - This explains why normal distribution is so important!

General Normal





Galton Knocked Boards

$$\frac{P(x \leq X \leq x + \Delta x)}{\Delta x}$$

where Δx is very small

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$$

General and Standard Normal

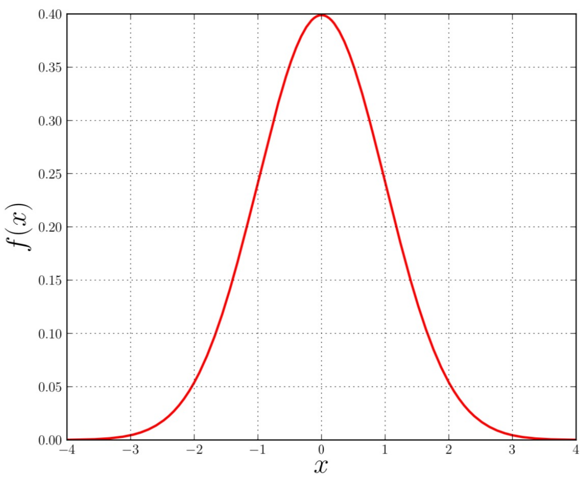
- For *continuous random variable* X, if for all $x \in R$
  - We have $\frac{P(x \le X \le x+\Delta x)}{\Delta x} \equiv f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$
  - where Δx is a very small value

Probability Density Function

- We say X is *general normal* or $X \sim N(\mu, \sigma^2)$

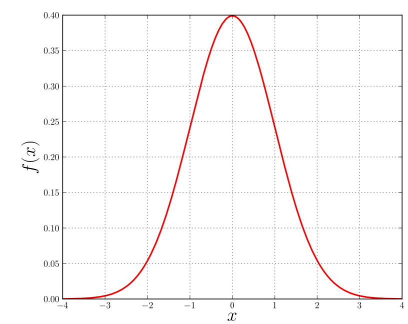
- **PROPERTIES**.
  - $E[X] = \mu$ and $Var(X) = \sigma^2$.

- When $\mu = 0$ and $\sigma = 1$
  - $f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$
  - X is *standard normal* or $X \sim N(0,1)$



General and Standard Normal

- For *continuous random variable* X, if for all $x \in R$
  - We have $\frac{P(x \le X \le x+\Delta x)}{\Delta x} = f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$
  - where Δx is a very small value
- We say X is *general normal* or $X \sim N(\mu, \sigma^2)$
- **PROPERTIES**.
  - $\frac{X-\mu}{\sigma} \sim N(0,1)$
  - ***BECAUSE***:
    - $f\left(\frac{x-\mu}{\sigma}\right) = \frac{P\left(x \le \frac{X-\mu}{\sigma} \le x+\Delta x\right)}{\Delta x} = \frac{P(x\sigma+\mu \le X \le (x+\Delta x)\sigma+\mu)}{\Delta x\sigma}\sigma = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$

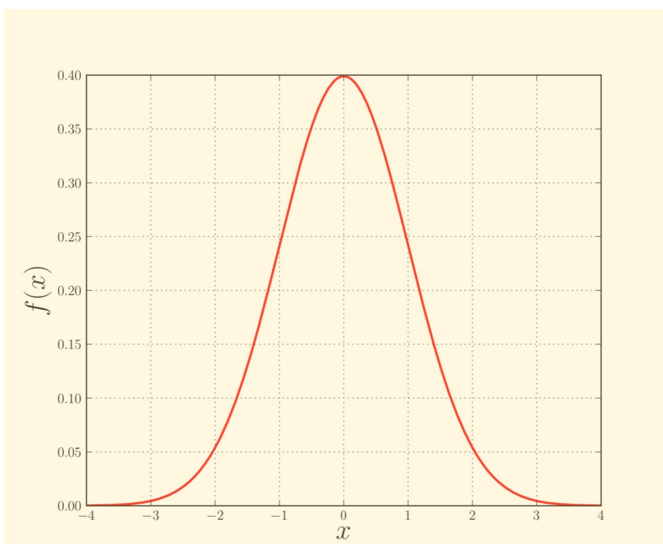


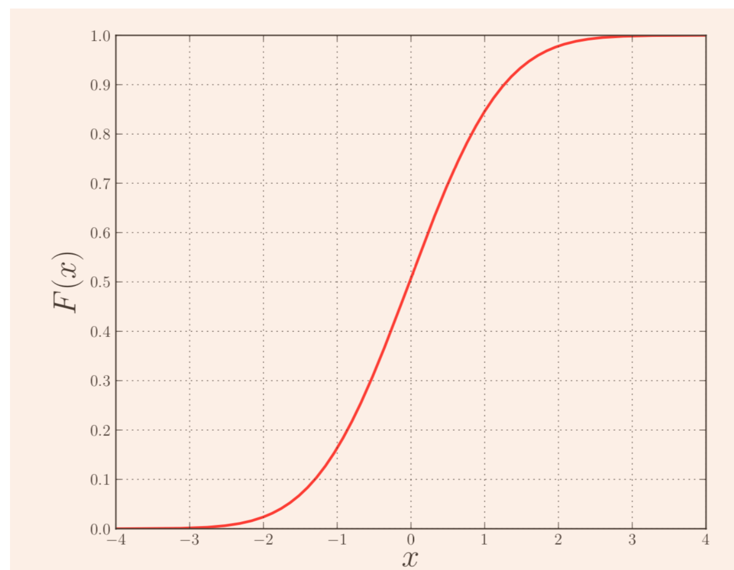
Central Limit Theorem

- Suppose the population mean and standard deviation are μ and σ.
- The sample mean is $\bar{X} = \frac{1}{n}(X_1 + X_2 + \cdots + X_n)$
    - $\mu_{\bar{X}} \equiv E[\bar{X}] = E\left[\frac{1}{n}(X_1 + X_2 + \cdots + X_n)\right] = \mu$
    - $\sigma_{\bar{X}}^2 \equiv Var(\bar{X}) = \frac{\sigma^2}{n}$
- **NOTE**. $\bar{X}$ is approximately *general normal* (or satisfies *normal distribution*) for very large n.
    - $\frac{\bar{X}-\mu_{\bar{X}}}{\sigma_{\bar{X}}} = \frac{\bar{X}-\mu}{\frac{\sigma}{\sqrt{n}}}$ is approximately *standard normal* for very large n.

Standard Normal

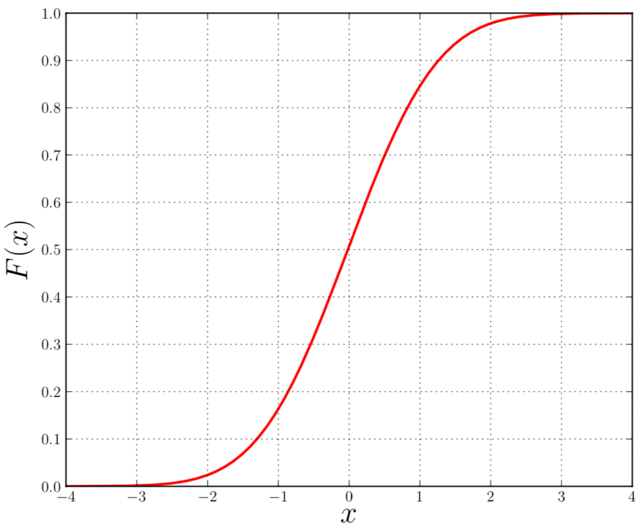
- For *continuous random variable* X, if for all $x \in R$
  - $f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$, say $X \sim N(0,1)$
  - The *cumulative distribution function*
    - $F(x) \equiv P(X \leq x) \equiv \Phi(x)$





Standard Normal

- For *continuous random variable* X, if for all $x \in R$
  - $f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$, say $X \sim N(0,1)$
  - The *cumulative distribution function*
    - $F(x) \equiv P(X \leq x) \equiv \Phi(x)$
    - $P(-x \leq X \leq x) = 1 - 2\Phi(-x)$ $(x > 0)$



z	$\Phi(z)$	z	$\Phi(z)$
0.0	.5000	-1.2	.1151
-0.1	.4602	-1.4	.0808
-0.2	.4207	-1.6	.0548
-0.3	.3821	-1.8	.0359
-0.4	.3446	**-2.0**	**.0228**
-0.5	.3085	-2.2	.0139
-0.6	.2743	-2.4	.0082
-0.7	.2420	-2.6	.0047
-0.8	.2119	-2.8	.0026
-0.9	.1841	-3.0	.0013
-1.0	.1587	-3.2	.0007

QUESTION*. How about positive* z ?

Roadmap

- Expectation and Variance
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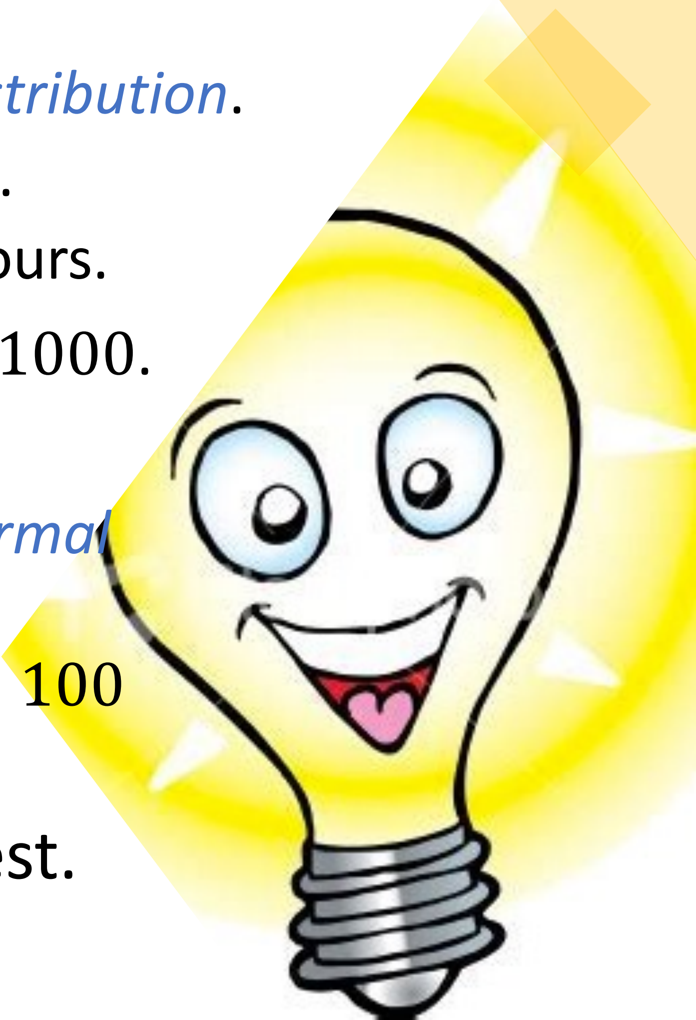
- Statistics vs. Data Analytics
  - Naïve Bayes (cont.)
  - Probabilistic Language Models (cont.)

Hypothesis Testing

- How to *test* whether a *hypothesis* is *true or false*?
- First, we gather *data* (i.e., *samples*).
- Second, we hypothesize that a random variable *has a given mean*.
- Third, we decide to *accept* or *reject* the hypothesis based on the data we collect.

Example: Hypothesis Tests

- We want to order a large shipment of light bulbs.
- The manufacturer *claims* that:
  - The lifetime of bulbs has a *normal distribution*.
  - The *mean* lifetime is $\mu = 1000$ hours.
  - The *standard deviation* is $\sigma = 100$ hours.
- We want to *test the hypothesis* that $\mu = 1000$.
- We assume that:
  - The lifetime of bulbs has indeed a *normal distribution*.
  - The *standard deviation* is indeed $\sigma = 100$ hours.
- We sample 25 light bulbs for the test.



Example: Hypothesis Tests

- The manufacturer *claims* that:
  - $\mu = 1000$ hours.
  - $\sigma = 100$ hours.
- We assume that:
  - Indeed, $\sigma = 100$ hours.
- We sample 25 light bulbs for the test.
  - The *sample* average lifetime is $\bar{X} = 960$
  - Do we accept the hypothesis that $\mu = 1000$?
- We know $P(\bar{X} \leq 960) = \Phi\left(\frac{960-1000}{\frac{100}{\sqrt{25}}}\right) = \Phi(-2.0) = 2.28\%$

One-sided probability

Would you accept and pay for the shipment!

Example: Hypothesis Tests

- The manufacturer *claims* that:
  - $\mu = 1000$ hours.
  - $\sigma = 100$ hours.
- We assume that:
  - Indeed, $\sigma = 100$ hours.
- We sample 25 light bulbs for the test.
  - The *sample* average lifetime is $\bar{X} = 1040$
  - Do we accept the hypothesis that $\mu = 1000$?
- We know $P(\bar{X} \geq 1040) = \Phi\left(\frac{1040-1000}{\frac{100}{\sqrt{25}}}\right)$

$$= \Phi(-2.0) = 2.28\%$$

One-sided probability

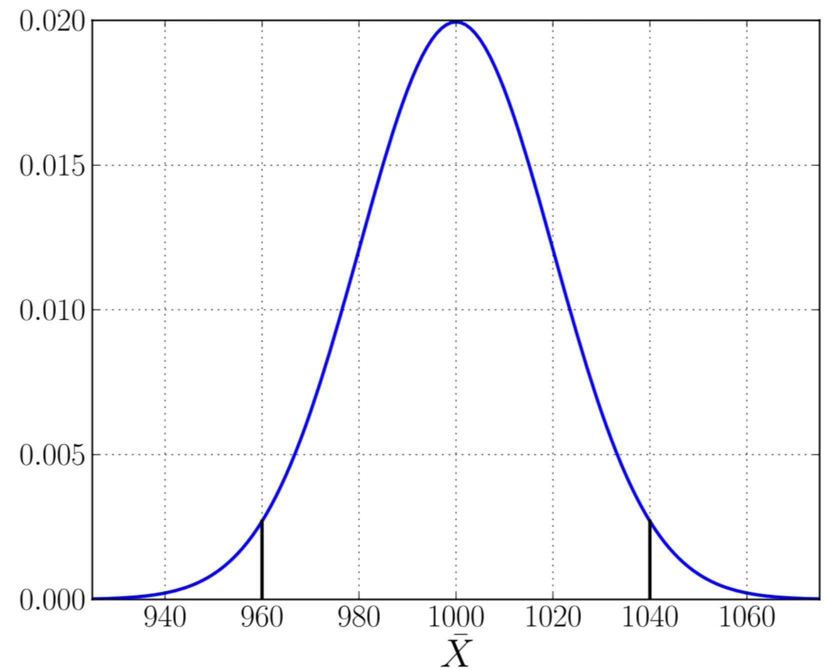
Would you accept and pay for the shipment!

Example (cont.)

- The manufacturer *claims* that:
  - $\mu = 1000$ hours.
  - $\sigma = 100$ hours.
- We assume that:
  - Indeed, $\sigma = 100$ hours.
  - We *accept* that $\mu = 1000$ if

$$960 \le \bar{X} \le 1040$$

- We sample 25 light bulbs for the test
- The probability that we *accept*:
  - $P(|\bar{X} - 1000| \le 40) = 1 - 2\Phi(\frac{960-1000}{100/\sqrt{25}})$
    $= 1 - 2\Phi(-2.0) \cong 95\%$
- And we *reject* with 5% probability



Example 1 (two-sided)

- **Given** a sample size 9 from a normal population with $\sigma = 0.2$, has sample mean $\bar{X} = 4.88$
- **Claim**: The population mean is $\mu = 5.00$ (***Null Hypothesis*** H_0)
- Since $Z \equiv \frac{\bar{X}-\mu}{\sigma/\sqrt{n}}$ is standard normal, the ***p-value*** is
  - $P(|\bar{X} - \mu| \geq 0.12) = P\left(|Z| \geq \frac{0.12}{\frac{0.2}{\sqrt{9}}}\right) = 2\Phi(-1.8) \cong 7.18\%$
  - We *reject* the hypothesis if $P(|\bar{X} - \mu| \geq 0.12)$ is *rather small*, say if $P(|\bar{X} - \mu| \geq 0.12) < 10\%$ (*level of significance* 10%).
  - We *reject* the hypothesis if $P(|\bar{X} - \mu| \geq 0.12) < 5\%$ (*level of significance* 5%) --- *We are more tolerant* .

Example 2 (one-sided)

- **Given** a sample size 64 from a normal population with $\sigma = 0.234$, has sample mean $\bar{X} = 4.847$
- We will test the *Null Hypothesis* $H_0: \mu \leq 4.8$
- We will *reject* H_0 if $P(\bar{X} \geq 4.847)$ is small, e.g., $P(\bar{X} \geq 4.847) < 5\%$
- Then the p-value:
  - $P(\bar{X} - \mu \geq 0.847) = P\left(\frac{\bar{X}-\mu}{\sigma} \geq \frac{4.847-4.8}{\frac{0.234}{8}}\right) = P\left(\frac{\bar{X}-\mu}{\sigma} \geq 1.6\right) = \Phi(-1.6) = 5.48\%$
- We (barely) accept H_0 at the level of significance 5%
- We reject H_0 at the level of significance 10%

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Descriptive vs. Inferential Statistics

- **Descriptive:** e.g., Median; describes data you have but can't be generalized beyond that
- **Inferential:** e.g., hypothesis testing; enables inferences about the population beyond our data samples
  - These are the techniques we'll leverage for many Machine Learning techniques

Examples of Business Questions

- **Simple (descriptive) Statistics**
  - E.g., “Who are the most profitable customers?”
- **Hypothesis Testing (answering yes/no with samples)**
  - E.g., “Is there a difference in value to the company of these customers?”
- **Segmentation/Classification**
  - E.g., What are the common characteristics of these customers?
- **Prediction**
  - E.g., Will this new customer become a profitable customer? If so, how profitable?

Statistics vs. Data Analytics

- Most business questions are causal: what would happen if? (e.g., I show this ad)
- But it’s easier to ask correlational questions, (what happened in this past when I showed this ad).
- Generalize the statistics of observations to predict in the future scenarios.

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Put them in in the log space

Instead of this:

This:
$$c_{NB} = \operatorname*{argmax}_{c_j \in C} P(c_j) \prod_{i \in positions} P(x_i \mid c_j)$$

Parameters to be estimated from the data

$$c_{\mathrm{NB}} = \operatorname*{argmax}_{c_j \in C} \left[\log P(c_j) + \sum_{i \in \mathrm{positions}} \log P(x_i | c_j) \right]$$

This is ok since log doesn't change the ranking of the classes (class with highest prob still has highest log prob)

Model is now just max of sum of weights: a *linear* function of the inputs

So naive bayes is a *linear classifier*

Learning the Multinomial Naive Bayes Model

- First attempt: maximum likelihood estimates
  - simply use the frequencies in the data

$$\hat{P}(c_j) = \frac{doccount(C = c_j)}{N_{doc}}$$

$$\hat{P}(w_i \mid c_j) = \frac{count(w_i, c_j)}{\sum_{w \in V} count(w, c_j)}$$

V is the vocabulary maintaining all the words used for classification.

Parameter estimation

$$\hat{P}(w_i \mid c_j) = \frac{count(w_i, c_j)}{\sum_{w \in V} count(w, c_j)}$$

fraction of times word w_i appears among all words in documents with class c_j

- Create mega-document for class *j* by concatenating all docs with the class label
  - Use frequency of *w* in mega-document

Problem with Maximum Likelihood

- What if we have seen no training documents with the word ***fantastic*** and classified in the topic **positive** (***thumbs-up)***?

$$\hat{P}(\text{"fantastic"} \mid \text{positive}) = \frac{count(\text{"fantastic"}, \text{positive})}{\sum_{w \in V} count(w, \text{positive})} = 0$$

- Zero probabilities cannot be conditioned away, no matter the other evidence!

$$c_{MAP} = \text{argmax}_c \, \hat{P}(c) \prod_i \hat{P}(x_i \mid c)$$

Laplace (add-1) Smoothing for Naive Bayes (empirically, it works!)

$$\hat{P}(w_i \mid c) = \frac{count(w_i, c) + 1}{\sum_{w \in V} \left(count(w, c) + 1 \right)}$$

$$= \frac{count(w_i, c) + 1}{\left(\sum_{w \in V} count(w, c) \right) + |V|}$$

Unknown words

- What about unknown words
  - that appear in our test data
  - but not in our training data or vocab
- We *ignore* them
  - Remove them from the test document!
  - Pretend they weren't there!
  - Don't include any probability for them at all.
- Why don't we build an unknown word model?
  - It doesn't help: knowing which class has more unknown words is not generally a useful thing to know!

Stop words

- Some systems ignore another class of words:
- **Stop words**: very frequent words like *the* and *a*.
  - Sort the whole vocabulary by frequency in the training, call the top 10 or 50 words the *stopword list*.
  - Now we remove all stop words from the training and test sets as if they were never there.
- But in some specific text classification applications, removing stop words don't help, so it's more common to **NOT** use stopword lists and use all the words in Naive Bayes (case-by-case).

Multinomial Naïve Bayes: Learning

- From training corpus, extract *Vocabulary*

- Calculate $P(c_j)$ terms
  - For each c_j in C do

    $docs_j \leftarrow$ all docs with class $=c_j$

$$P(c_j) \leftarrow \frac{|docs_j|}{|\text{total \# documents}|}$$

- Calculate $P(w_k \mid c_j)$ terms
  - $Text_j \leftarrow$ single doc containing all $docs_j$
  - For each word w_k in *Vocabulary*

    $n_k \leftarrow$ # of occurrences of w_k in $Text_j$

$$P(w_k \mid c_j) \leftarrow \frac{n_k + \alpha}{n + \alpha |Vocabulary|}$$

e.g., $\alpha = 1$ (add-1)

Let's do a worked sentiment example!

	Cat	Documents
Training	-	just plain boring
	-	entirely predictable and lacks energy
	-	no surprises and very few laughs
	+	very powerful
	+	the most fun film of the summer
Test	?	predictable with no fun

A worked sentiment example

	Cat	Documents
Training	-	just plain boring
	-	entirely predictable and lacks energy
	-	no surprises and very few laughs
	+	very powerful
	+	the most fun film of the summer
Test	?	predictable ~~with~~ no fun

Prior from training:

P(-) = 3/5
P(+) = 2/5

Drop "with"

Likelihoods from training:

$$P(\text{“predictable”}|-) = \frac{1+1}{14+20} \qquad P(\text{“predictable”}|+) = \frac{0+1}{9+20}$$

$$P(\text{“no”}|-) = \frac{1+1}{14+20} \qquad P(\text{“no”}|+) = \frac{0+1}{9+20}$$

$$P(\text{“fun”}|-) = \frac{0+1}{14+20} \qquad P(\text{“fun”}|+) = \frac{1+1}{9+20}$$

Scoring the test set:

$$\boxed{P(-)P(S|-) = \frac{3}{5} \times \frac{2 \times 2 \times 1}{34^3} = 6.1 \times 10^{-5}}$$

$$P(+)P(S|+) = \frac{2}{5} \times \frac{1 \times 1 \times 2}{29^3} = 3.2 \times 10^{-5}$$

Roadmap

- Expectation and Variance
- Sample Statistics
- Hypothesis Testing
- Statistics vs. Data Analytics
    - Naïve Bayes (cont.)
    - Probabilistic Language Models (cont.)

Estimating N-gram Probabilities

$$P(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

Count

Example: <s> I am Sam </s>

<s> Sam I am </s>

<s> I do not like green eggs and ham </s>

$P(\texttt{I} \mid \texttt{<s>}) = \frac{2}{3} = .67 \quad P(\texttt{Sam} \mid \texttt{<s>}) = \frac{1}{3} = .33 \quad P(\texttt{am} \mid \texttt{I}) = \frac{2}{3} = .67$

$P(\texttt{</s>} \mid \texttt{Sam}) = \frac{1}{2} = 0.5 \quad P(\texttt{Sam} \mid \texttt{am}) = \frac{1}{2} = .5 \quad P(\texttt{do} \mid \texttt{I}) = \frac{1}{3} = .33$

More Examples

- can you tell me about any good cantonese restaurants close by
- mid priced thai food is what i'm looking for
- tell me about chez panisse
- can you give me a listing of the kinds of food that are available
- i'm looking for a good place to eat breakfast
- when is caffe venezia open during the day

Raw bigram counts

Out of 9,222 sentences

	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
lunch	2	0	0	0	0	1	0	0
spend	1	0	1	0	0	0	0	0

Raw Bigram Probabilities

Normalize by unigrams:

i	want	to	eat	chinese	food	lunch	spend
2533	927	2417	746	158	1093	341	278

Results

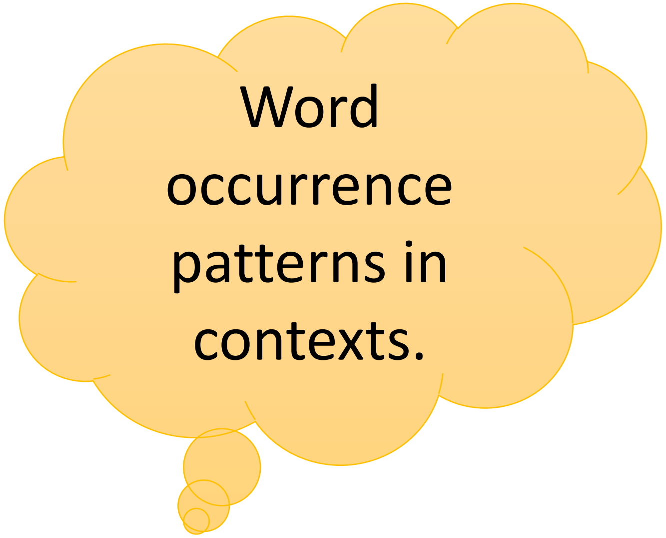
	i	want	to	eat	chinese	food	lunch	spend
i	0.002	0.33	0	0.0036	0	0	0	0.00079
want	0.0022	0	0.66	0.0011	0.0065	0.0065	0.0054	0.0011
to	0.00083	0	0.0017	0.28	0.00083	0	0.0025	0.087
eat	0	0	0.0027	0	0.021	0.0027	0.056	0
chinese	0.0063	0	0	0	0	0.52	0.0063	0
food	0.014	0	0.014	0	0.00092	0.0037	0	0
lunch	0.0059	0	0	0	0	0.0029	0	0
spend	0.0036	0	0.0036	0	0	0	0	0

Bigram Estimates of Sentence Probabilities

$$P(< s > \ I \, want \, english \, food \ </s >) = P(I| < s >) \cdot P(want|I) \cdot P(english|want) \cdot P(food|english) \cdot P(</s > |food) = 0.000031$$

What did we learn?

- $P(english|want) = 0.0011$
- $P(chinese|want) = 0.0065$
- $P(to|want) = 0.66$
- $P(eat \mid to) = 0.28$
- $P(food \mid to) = 0$
- $P(want \mid spend) = 0$
- $P\,(i \mid <s>) = .25$



A slide to take away

- What is ***expectation*** and ***variance***?
- What are the ***statistics*** to represent the data ?
- How to connect expectation with the sample mean with the ***law of large number***?
- How to describe the distribution of sample means with the ***central limit theorem***?
- What is the ***p-value*** and when to ***accept/reject a hypothesis***?
- How to estimate the likelihoods and priors to ***train a Naïve Bayes classifier***?
- How to estimate **N-gram probabilities**?