Mini project 1

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OBJECTIVE: This project demonstrates how numerical errors accumulate during summation, inspired by the Vancouver Stock Exchange's 1982-1983 issue, where rounding errors caused the index to drop by about 25 points per month, leading to a 50% loss before correction in November 1983 [1].

Key mathematical concepts used are *round-off errors* and *truncation errors*. *Round-off errors* occur when numbers are rounded to a certain number of digits, while *truncation errors* occur when approximations are used. Both contribute to deviations from true values in numerical computations, and these errors grow over time due to the limited precision of computers.

Understanding how *floating-point arithmetic* handles these errors is important because this type of arithmetic is widely used in computing and finance, where even small discrepancies lead to major problems, as in fields like high-frequency trading (HFT), where quantitative models exploit small discrepancies between markets.

SUMMARY OF PROCEDURE: I summed 10,000 random numbers using three methods: full precision, rounding to three decimal places, and truncating (chopping) to three decimal places. Results were recorded at intervals of 1,000 numbers to observe how each method's sum evolved over time and were saved to arrays. This method allows for easy comparison of cumulative errors across methods. Results were saved in a CSV file.

RESULTS AND DISCUSSION: Numerical errors from rounding and chopping impacted the total sum differently. The *full precision sum* closely matches the true total, with minimal error. The *rounded sum* deviates slightly, but the difference remains small and grows gradually. However, the *chopped sum* shows the largest deviations, particularly as n increases. This is because truncation consistently underestimates values.

The accumulation of absolute errors as more terms are added results in the observed differences. By 10,000 terms, the chopped sum is about 5 units lower than the full precision sum. As error propagation theory states:

$$|E_{\text{sum}}| = |E_1| + |E_2| + \cdots + |E_n|$$

This leads to the growing discrepancy in the chopped sum. A similar issue occurred in the Vancouver Stock Exchange between 1982-1983, where truncating the index to three decimal places after each update introduced small errors. With around 3,000 updates daily, these errors compounded over time, causing the index to drop by 25 points per month. This is much like the cumulative error in my summation process.

Number of Terms (n)	*Full Precision Sum*	*Rounded Sum*	*Chopped Sum*
1000	510.881	510.880	510.380
2000	1016.879	1016.878	1015.880
3000	1523.549	1523.541	1522.049
4000	2027.866	2027.858	2025.866
5000	2517.092	2517.066	2514.589
6000	3016.872	3016.859	3013.862
7000	3513.914	3513.912	3510.384
8000	4004.523	4004.512	4000.502
9000	4506.411	4506.397	4501.888
10000	4989.929	4989.920	4984.897

Table 1: Comparison of sum results using full precision, rounding, and chopping methods.

Conclusively, this project shows the importance of precision in numerical computations. Chopping introduces larger cumulative errors over time, while rounding keeps the sums closer to the true value. As seen in the Vancouver Stock Exchange case, small numerical discrepancies compounded and led to significant deviations from the true value.

REFERENCES

[1] Wikipedia: Vancouver Stock Exchange,
https://en.wikipedia.org/wiki/Vancouver_Stock_Exchange