# Mini project 1 : Vancouver Stock Exchange drop

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OBJECTIVE: This project shows how numerical errors accumulate during summation, inspired by the Vancouver Stock Exchange's 1982-1983 issue, where truncation errors caused the index to drop by about 25 points per month, leading to a 50% loss before correction in November 1983 [1].

To demonstrate this, I used *rounding to three decimal places* and *chopping to three decimal places*. *Rounding errors* occur when numbers are rounded to a certain number of digits, as shown in equation 2, while *chopping errors* happen when digits beyond a certain point are simply discarded, as implemented in equation 3. Both contribute to deviations in numerical computations, accumulating over time due to limited computer precision. We compare these methods to full precision summation.

SUMMARY OF PROCEDURE: To observe the effect of numerical errors, I performed the summation of 10,000 uniformly distributed random numbers within the interval [0, 1), denoted as $\{X_i\}$, using three distinct methods.

Using full precision:

$$S_n^{(F)} = \sum_{i=1}^{n} X_i \tag{1}$$

Using rounding to three decimal places:

$$S_n^{(R)} = \sum_{i=1}^{n} \text{round}(X_i, 3) \tag{2}$$

Using chopping to three decimal places:

$$S_n^{(C)} = \sum_{i=1}^{n} \left\lfloor X_i \times 10^3 \right\rfloor / 10^3 \tag{3}$$

The results were recorded at intervals of 1,000 terms (i.e., $n = 1000, 2000, \ldots, 10000$) for comparison. The definitions in Equations 1, 2, and 3 show the methods applied.

RESULTS AND DISCUSSION: Numerical errors from rounding and chopping to 3 decimal places impacted the total sum differently. The *full precision sum* closely matches the true total, with minimal error. The *rounded sum* deviates slightly, but the difference remains small and grows gradually. However, the *chopped sum* shows the largest deviations, particularly as $n$ increases. This is because chopping consistently underestimates values.

The accumulation of absolute errors as more terms are added results in the observed differences. By 10,000 terms, the chopped sum is 5 units lower than the full precision sum as seen in Table 1. As error propagation theory [2] states:

$$|E_{\text{sum}}| = |E_1| + |E_2| + \cdots + |E_n| \tag{4}$$

This leads to the growing discrepancy in the chopped sum detailed in 3. A similar issue occurred in the Vancouver Stock Exchange between 1982-1983, where chopping the index to three decimal places after each update introduced small errors. With around 3,000 updates daily, these errors compounded over time, causing the index to drop by 25 points per month. This is much like the cumulative error in my summation process (see Table 1).

Conclusively, this project shows the importance of precision in numerical computations. Chopping to 3 decimal places introduces larger cumulative errors over time, while rounding keeps the sums closer to the true value. As seen in the Vancouver Stock Exchange case, small numerical discrepancies compounded and led to significant deviations from the true value. Additional resources are available on GitHub [3].

| Number of Terms (n) | Full Precision Sum | Rounded Sum | Chopped Sum |
|:---:|:---:|:---:|:---:|
| 1000 | 510.881 | 510.880 | 510.380 |
| 2000 | 1016.879 | 1016.878 | 1015.880 |
| 3000 | 1523.549 | 1523.541 | 1522.049 |
| 4000 | 2027.866 | 2027.858 | 2025.866 |
| 5000 | 2517.092 | 2517.066 | 2514.589 |
| 6000 | 3016.872 | 3016.859 | 3013.862 |
| 7000 | 3513.914 | 3513.912 | 3510.384 |
| 8000 | 4004.523 | 4004.512 | 4000.502 |
| 9000 | 4506.411 | 4506.397 | 4501.888 |
| 10000 | 4989.929 | 4989.920 | 4984.897 |

Table 1: Comparison of sum results from full precision, detailed in Equation 1, rounding detailed in Equation 2, and chopping, detailed in Equation 3.

## REFERENCES

[1] Wikipedia: Vancouver Stock Exchange, `https://en.wikipedia.org/wiki/Vancouver_Stock_Exchange`, accessed 10th October 2024.

[2] Lorenzo, J. (n.d.). *Propagation of Uncertainties, Part 2*. Louisiana State University. `https://www.geol.lsu.edu/jlorenzo/geophysics/uncertainties/Uncertaintiespart2.html`, accessed 10th October 2024.

[3] J. Raj. *Numerical Analysis Projects*. GitHub repository, `https://github.com/jyoutir/numerical-analysis-projects/tree/main/miniproject_1`. Accessed: 17th October 2024.