Mini project 1

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OBJECTIVE: This project shows how numerical errors accumulate during summation, inspired by the Vancouver Stock Exchange's 1982-1983 issue, where rounding errors caused the index to drop by about 25 points per month, leading to a 50% loss before correction in November 1983 [1].
To demonstrate this, I used *round-off errors* and *truncation errors*. *Round-off errors* occur when numbers are rounded to a certain number of digits, while *truncation errors* happen when digits beyond a certain point are simply discarded. Both contribute to deviations in numerical computations, growing over time because of limited computer precision. We compare these methods to full precision.

SUMMARY OF PROCEDURE: To observe the effect of numerical errors, I summed 10,000 random numbers $\{X_i\}$ using three different methods:

Using full precision:

$$S_n^{(F)} = \sum_{i=1}^{n} X_i \tag{1}$$

Using rounding to three decimal places:

$$S_n^{(R)} = \sum_{i=1}^{n} \text{round}(X_i, 3) \tag{2}$$

Using truncation (chopping) to three decimal places:

$$S_n^{(C)} = \sum_{i=1}^{n} \text{chop}(X_i, 3) \tag{3}$$

where

$$\text{chop}(X_i, 3) = \lfloor X_i \times 10^3 \rfloor / 10^3 \tag{4}$$

The results were recorded at intervals of 1,000 terms (i.e., $n = 1000, 2000, \ldots, 10000$) for comparison. The definitions in Equations 1, 2, and 3 show the methods applied, with truncation detailed in Equation 4.

RESULTS AND DISCUSSION: Numerical errors from rounding and chopping impacted the total sum differently. The *full precision sum* closely matches the true total, with minimal error. The *rounded sum* deviates slightly, but the difference remains small and grows gradually. However, the *chopped sum* shows the largest deviations, particularly as n increases. This is because truncation consistently underestimates values.

The accumulation of absolute errors as more terms are added results in the observed differences. By 10,000 terms, the chopped sum is about 5 units lower than the full precision sum. As error propagation theory [2] states:

$$|E_{\text{sum}}| = |E_1| + |E_2| + \cdots + |E_n|$$

This leads to the growing discrepancy in the chopped sum. A similar issue occurred in the Vancouver Stock Exchange between 1982-1983, where truncating the index to three decimal places after each update introduced small errors. With around 3,000 updates daily, these errors compounded over time, causing the index to drop by 25 points per month. This is much like the cumulative error in my summation process (see Table 1).

Conclusively, this project shows the importance of precision in numerical computations. Chopping introduces larger cumulative errors over time, while rounding keeps the sums closer to the true value. As seen in the Vancouver Stock Exchange case, small numerical discrepancies compounded and led to significant deviations from the true value.

Number of Terms (n)	*Full Precision Sum*	*Rounded Sum*	*Chopped Sum*
1000	510.881	510.880	510.380
2000	1016.879	1016.878	1015.880
3000	1523.549	1523.541	1522.049
4000	2027.866	2027.858	2025.866
5000	2517.092	2517.066	2514.589
6000	3016.872	3016.859	3013.862
7000	3513.914	3513.912	3510.384
8000	4004.523	4004.512	4000.502
9000	4506.411	4506.397	4501.888
10000	4989.929	4989.920	4984.897

Table 1: Comparison of sum results from full precision, rounding, and chopping.

References

[1] Wikipedia: Vancouver Stock Exchange,
`https://en.wikipedia.org/wiki/Vancouver_Stock_Exchange`

[2] Lorenzo, J. (n.d.). *Propagation of Uncertainties, Part 2*. Louisiana State University.
`https://www.geol.lsu.edu/jlorenzo/geophysics/uncertainties/Uncertaintiespart2.html`