Portfolio VI - Estimating Stationary Distribution

Introduction

In this report, I estimate the limiting stationary distribution of a traffic light system with three states: Green (G), Yellow (Y), and Red (R). The light system is modelled as a discrete-time Markov chain, with all states being recurrent, meaning the system always returns to each state after some time. The transition probabilities between the states are defined as follows, without violating real traffic light behavior:

$$P = \begin{pmatrix} 0.1 & 0.9 & 0.0 \\ 0.0 & 0.2 & 0.8 \\ 0.7 & 0.0 & 0.3 \end{pmatrix}$$

- $G \to R$: set to 0, as direct green-to-red transitions are not possible.
- $Y \to G$: set to 0, as yellow does not transition directly back to green.
- $R \to Y$: set to 0, as red does not go directly to yellow.

To approximate the limiting stationary distribution, which represents the long-term proportion of time the model spends in each state, we simulate the Markov chain over $10,000$ time steps. To quantify the uncertainty in the estimates, I use a block averaging technique, dividing the simulation into 100 blocks of 100 steps each. This method allows us to estimate the variance of the state probabilities and construct 95% confidence intervals..

Estimating the Stationary Distribution

My model yielded the following estimates for the stationary distribution, along with their 95% confidence intervals calculated using block averaging (see **Table 1**):

State	**Mean Probability**	**Standard Error**	**95% Confidence Interval**
G	0.2955	0.0020	[0.2916, 0.2994]
Y	0.3305	0.0025	[0.3255, 0.3355]
R	0.3740	0.0031	[0.3680, 0.3800]

Table 1: Estimated stationary distribution and confidence intervals

The estimates suggest that, in the long run, the model spends approximately 29.55% of the time in the Green state, 33.05% in Yellow, and 37.40% in Red. The visual representation of these estimates, including the 95% confidence intervals, can be seen in **Figure 1**.

In producing this plot, I learned:

- How to estimate the limiting stationary distribution of a Markov chain using block averaging.
- Importance of transition probabilities in finding long-term state proportions in Markov chains.
- Thought careful behind building a realistic traffic light model, accounting for feasible state transitions only.

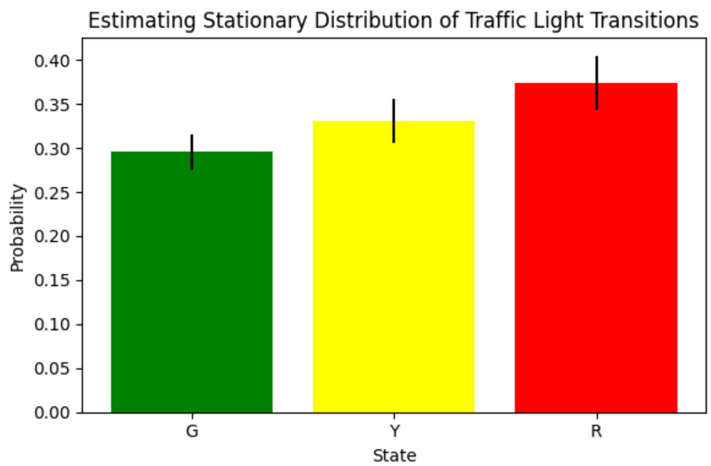

Figure 1: Estimated stationary distribution of traffic light transitions with 95% confidence intervals.