Portfolio VIII - Event-Based Simulation

Introduction

In a twist of national importance with the recent elections, the FBI has provided a trillion-dollar funding boost to ensure enough polling stations are set up to avoid any undercover agents getting stuck in long lines. With this generous support they have sub-contracted top mathematicians globally, thus my role is to use event-based simulation to model queue dynamics at polling stations. The goal is to ensure sufficient voting booths are available to minimize voter wait times for regular people and undercover agents. Using an **M/M/1 queue model**, I simulate voter arrivals and service times as exponential random variables to observe queue lengths over time.

Methodology

The polling station is modeled as an M/M/1 queue with finite capacity K, using event-based simulation. Key characteristics include:

- **Arrival Rate** ($\lambda = 20$ voters/hour): Arrivals follow a Poisson process.
- **Service Rate** ($\mu = 30$ voters/hour): Service times are exponentially distributed.
- **Queue Capacity** ($K = 10$ voters): Maximum queue length.
- **Single Server**: One voting booth.

The simulation advances in discrete time steps ($\Delta t = 1$ minute). At each step:

- A voter arrives based on the arrival probability P_{arrival}.
- A voter is serviced based on the service probability P_{service}.

To accurately represent the continuous Poisson arrival and exponential service processes in discrete time, the per-step probabilities are calculated using the following formulas:

Arrival Probability:

$$P_{\text{arrival}} = 1 - e^{-\lambda \Delta t}$$

Service Probability:

$$P_{\text{service}} = 1 - e^{-\mu \Delta t}$$

At each time step, two independent uniform random variables $U_1, U_2 \in [0, 1)$ are generated to determine events:

Arrival Event:

- If $U_1 < P_{\text{arrival}}$ and the queue length is less than K, a voter joins the queue.

Service Event:

- If the queue length is greater than zero and $U_2 < P_{\text{service}}$, a voter is serviced.

Results

The simulation reported:

Total Voters Processed: 149

Average Queue Length: 0.7

Maximum Queue Length: 5 (below capacity $K = 10$)

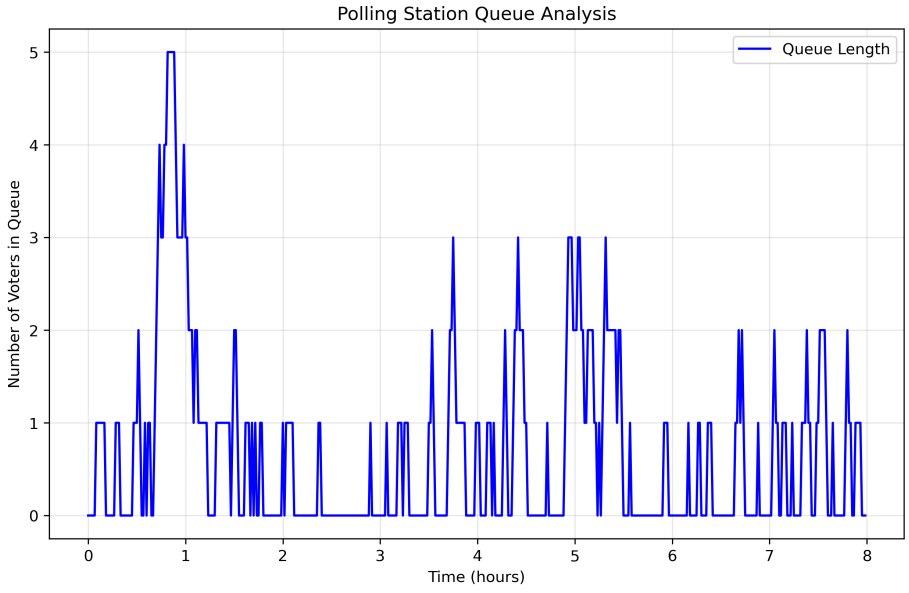

Figure 1: Queue Length Over Time

Figure 1 shows queue dynamics, with queue length well within capacity, indicating efficient queues that manage voters.

Since the queue length remained below capacity throughout the day. This suggests that the FBI can be confident there are enough polling booths to minimize voter wait times and ensure their agents will not get stuck in the queues.

Limitations

This simulation assumes a constant arrival rate and a single polling booth throughout the day. However, in reality:

- **Varying Arrival Times**: Voter arrivals may fluctuate during the day (e.g., peak times in the morning or evening). Future model will consider variable arrival rates.
- **Multiple Polling Booths**: Real polling stations often have more than one booth, which would reduce wait times and alter queue dynamics.