Portfolio II - Expectation

Figures 1, 2, and 3 show distribution of sample means for an exponential distribution with rate parameter $\lambda = 1$. I generated samples of exponential random variables for different sample sizes $n = 5, 20, and 1000$. Each figure shows how sample means are distributed, and the true mean is shown by the red dashed line.

For each sample size n, I calculated the sample mean and the sample's 90% confidence interval (CI):

Sample Size (n)	**Mean**	**90% Confidence Interval**
5	2.0753	(1.3329, 2.8176)
20	0.8210	(0.5428, 1.0993)
1000	1.0213	(0.9725, 1.0700)

Table 1: Means and 90% Confidence Intervals for different sample sizes

In producing this plot I learned:

- Showcased that the random variables that are exponentially distributed obey the Central Limit Theorem.
- As the sample size increases, the confidence interval (CI) becomes narrower. For $n = 5$, the CI is wide showing smaller sample sizes lead to lower precision, even though the mean is relatively accurate.
- For $n = 1000$ the CI is narrow, showing larger sample sizes lead to accurate and precise sample means that are close to the true mean.
- Used sample variance $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$ to find error and then get 90% CIs by calculating lower and upper bounds.

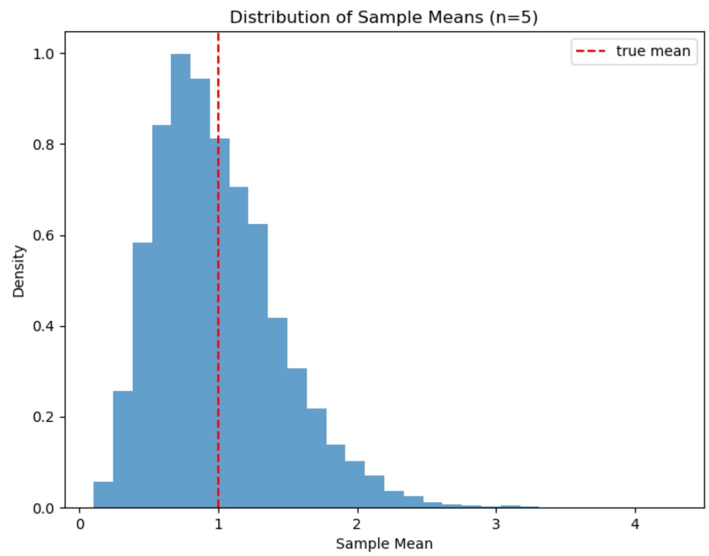

Figure 1: Distribution of sample means for an exponential distribution with $\lambda = 1$ and $n = 5$. The red dashed line represents the true mean.

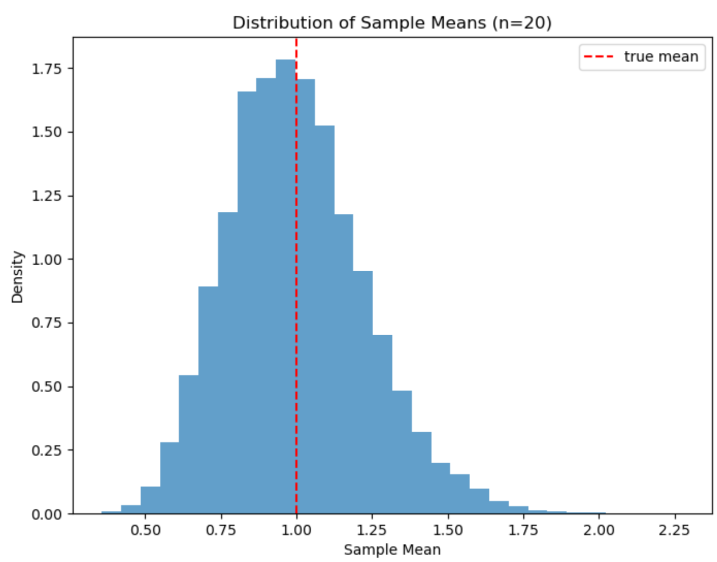

Figure 2: Distribution of sample means for an exponential distribution with $\lambda = 1$ and $n = 20$. The red dashed line represents the true mean.

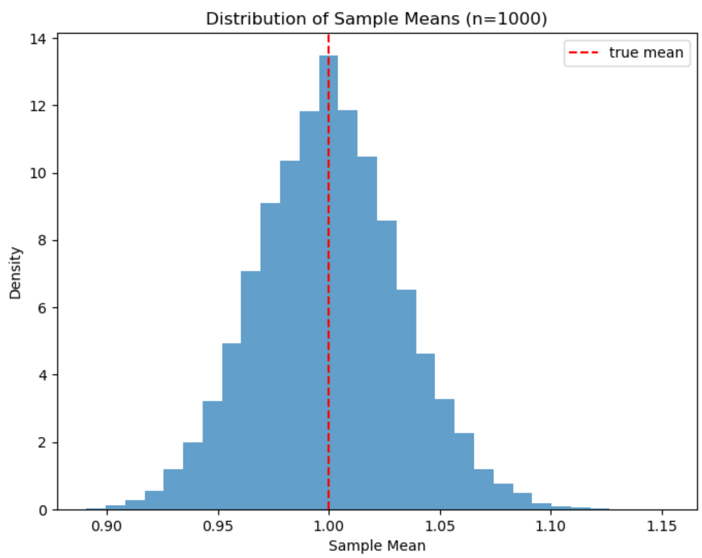

Figure 3: Distribution of sample means for an exponential distribution with $\lambda = 1$ and $n = 1000$. The red dashed line represents the true mean.