Portfolio IX - Inhomogeneous Arrival Rates in Event Simulation

Introduction

Building upon my previous work with the FBI on simulating queues at polling stations, this report addresses a significant limitation: the fluctuation of voter turnout throughout the day. By incorporating time-dependent (inhomogeneous) arrival rates, we recognise that voters don't arrive in a steady stream but tend to peak during meal times and breaks. This refinement aims to improve accuracy of our analysis of queue lengths and wait times, as well as determining if more polling stations are necessary.

Methodology

The polling station is modelled as an M/M/1 queue with finite capacity K using event-based simulation. Key characteristics include:

- **Base Arrival Rate** (λ_0): The constant background arrival rate.
- **Service Rate** (μ): Exponentially distributed service times.
- **Queue Capacity** (K): Maximum queue length.
- **Single Server**: One voting booth handling the queue.

To account for fluctuations in voter turnout during different times of the day, I introduce a time-varying arrival rate $\lambda(t)$ which is given as follows:

$$\lambda(t) = \lambda_0 + \sum_{i=1}^{N} A_i \cdot \exp\left(-\frac{(t - t_i)^2}{2\sigma_i^2}\right)$$

where A_i, t_i, and σ_i represent the amplitude, time, and width of each peak, respectively. The simulation advances in discrete time steps (Δt), and at each step:

- **Compute Probabilities:**

$$P_{\text{arrival}}(t) = \lambda(t)\Delta t, \quad P_{\text{service}} = \mu\Delta t$$

- **Determine Events:**
  - **Arrival Event:** If a uniform random variable $U_1 \in [0, 1)$ is less than $P_{\text{arrival}}(t)$ and $Q(t) < K$, increment the queue length. If $Q(t) = K$, increment the count of rejected voters.
  - **Service Event:** If a uniform random variable $U_2 \in [0, 1)$ is less than P_{service} and $Q(t) > 0$, decrement the queue length.

Results

The simulation was conducted over a 24-hour period with the following peak characteristics:

Peak	**Amplitude** A_i (voters/hour)	**Time** t_i (hours)	**Width** σ_i (hours)
Morning Peak	30	9	1.5
Lunch Peak	20	12	2
Afternoon Peak	25	16.5	1.5

Table 1: Defined Peaks for Time-Dependent Arrival Rate

Key Statistics

- **Base Arrival Rate**: 5 voters/hour
- **Total Voters Served**: 380
- **Rejected Voters**: 57
- **Average Queue Length**: 6.9 voters
- **Max Queue Length**: 20 voters (at capacity K)
- **Avg Arrival Rate**: 17.8 voters/hour

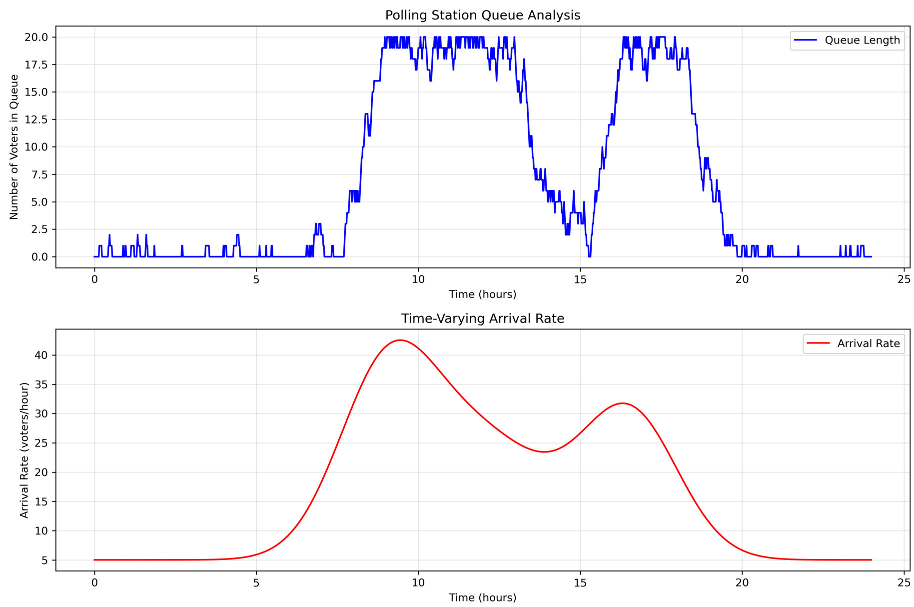

Figure 1: Top plot: Queue size over 24 hours, showing peak congestion during morning, lunch, and late afternoon. Bottom plot: Time-varying arrival rate with distinct peaks and declines.

Conclusion and Limitations

The simulation shows that one polling station is insufficient during peak times, leading to queues hitting capacity and 57 voters turned away. More polling stations are needed to avoid voter frustration. The FBI should work on increasing capacity for smoother voting.

A key limitation is the use of a constant service rate, which ignores factors like voter impatience or staff shifts that could slow service. Future models could use time-varying service rates for more accuracy.

What I Learned

- Incorporating time-varying arrival rates captures daily fluctuations better and provides more realistic queue predictions.
- Even simple simulations can reveal important system bottlenecks that aren't obvious with static homogeneous models.