Portfolio V - Hitting Probabilities for a Gambler

Introduction

This report models a gambler starting with an initial amount of money between \$1,000 and \$512,000, aiming to reach \$1,024,000 by playing a game which doubles their money on each win (50% chance). If the gambler loses, the total is halved. The process is represented by a Markov chain with **state 0** (ruin) and **state 11** (winning) as absorbing states. The transition matrix P is:

$$P = \begin{pmatrix} 1 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0.5 & 0 & 0.5 & 0 & \dots & 0 & 0 \\ 0 & 0.5 & 0 & 0.5 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 0.5 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 1 \end{pmatrix}$$

where:

$$P_{0,0} = 1, \quad P_{11,11} = 1, \quad P_{i,i-1} = 0.5, \quad P_{i,i+1} = 0.5 \quad \text{for } 1 \leq i \leq 10,$$

and all other entries are zero.

Hitting Probabilities

The hitting probabilities u_i for reaching the winning state from each starting state i were calculated using the fundamental matrix $N = (I - Q)^{-1}$, where Q represents transitions among non-absorbing states. The probabilities for reaching \$1,024,000 from each starting amount are shown in Figure 1.

As shown in Figure 1, as the starting amount increases, the probability of reaching the target rises. Starting from lower amounts leads to a higher chance of ruin, while starting from \$512,000 gives a near-certain chance of reaching \$1,024,000.

In producing this plot, I learned:

- How to calculate hitting probabilities in Markov chains.
- That starting position greatly affects the chance of reaching the target, and the odds of winning big while gambling are quite slim over multiple plays.
- The concept of hitting times in symmetric random walks.

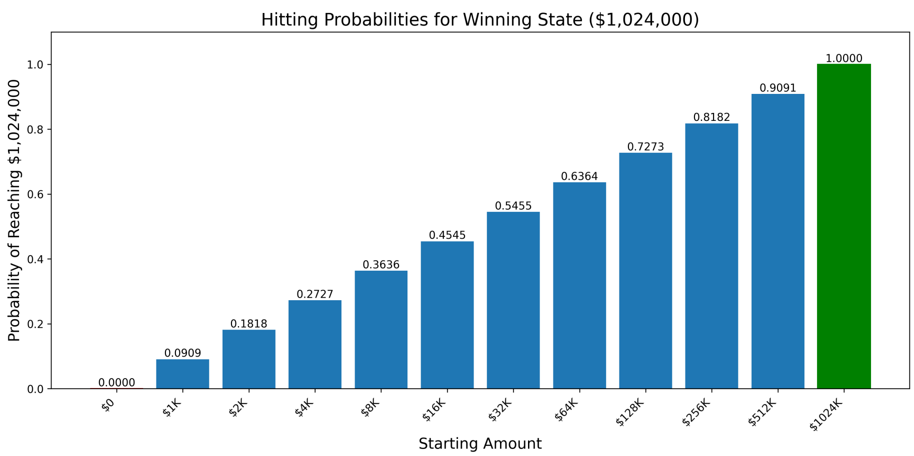

Figure 1: Hitting probabilities for reaching $1,024,000 from various starting amounts.