Portfolio VI - Estimating Stationary Distribution

Introduction

In this report, I estimate the limiting stationary distribution of a traffic light system with three states: Green (G), Yellow (Y), and Red (R). The light system is modelled as a discrete-time Markov chain, with all states being recurrent, meaning the system always returns to each state after some time. The transition probabilities between the states are defined as follows:

$$P = \begin{pmatrix} 0.8 & 0.1 & 0.1 \\ 0.2 & 0.7 & 0.1 \\ 0.1 & 0.2 & 0.7 \end{pmatrix}$$

To approximate the limiting stationary distribution, which represents the long-term proportion of time the model spends in each state, we simulate the Markov chain over $10,000$ time steps. To quantify the uncertainty in the estimates, I use a block averaging technique, dividing the simulation into 100 blocks of 100 steps each. This method allows us to estimate the variance of the state probabilities and construct 95% confidence intervals..

Estimating the Stationary Distribution

My model yielded the following estimates for the stationary distribution, along with their 95% confidence intervals calculated using block averaging (see **Table 1**):

State	Estimated Probability	95% Confidence Interval
G	0.4559	[0.4361, 0.4757]
Y	0.3098	[0.2938, 0.3258]
R	0.2343	[0.2186, 0.2500]

Table 1: Estimated stationary distribution and confidence intervals

The estimates suggest that, in the long run, the model spends approximately 45.59% of the time in the Green state, 30.98% in Yellow, and 23.43% in Red. The visual representation of these estimates, including the 95% confidence intervals, can be seen in **Figure 1**.

In producing this plot, I learned:

- How to estimate the limiting stationary distribution of a Markov chain using block averaging, specifically applied to model traffic light systems.

- Importance of transition probabilities in determining long-term state proportions in Markov chains.
- How block averaging helps in estimating the variance of the estimates to construct confidence intervals.

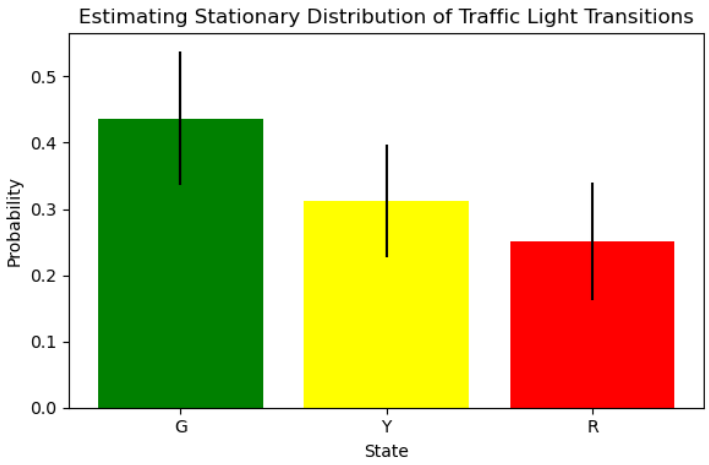

Figure 1: Estimated stationary distribution of traffic light transitions with 95% confidence intervals.