Portfolio VIII - Event-Based Simulation

Introduction

In a twist of national importance with the recent elections, the FBI has provided a trillion-dollar funding boost to ensure enough polling stations are set up to avoid any undercover agents getting stuck in long lines. With this generous support, they have sub-contracted top mathematicians globally. My role is to use event-based simulation to model queue dynamics at polling stations, aiming to ensure sufficient voting booths are available to minimise voter wait times for regular people and undercover agents. Using an **M/M/1 queue model**, I simulate voter arrivals and service times as exponential random variables to observe queue lengths over time.

Methodology

The polling station is modelled as an M/M/1 queue with finite capacity K. Key characteristics include:

- **Arrival Rate** ($\lambda = 20$ voters/hour): Arrivals follow a Poisson process.
- **Service Rate** ($\mu = 30$ voters/hour): Service times are exponentially distributed.
- **Queue Capacity** ($K = 10$ voters): Maximum queue length.
- **Single Server**: One voting booth.

The simulation advances in discrete time steps ($\Delta t = 1$ minute). At each step:

- A voter arrives based on the probability $P_{\text{arrival}} = 1 - e^{-\lambda \Delta t}$.
- A voter is serviced based on the probability $P_{\text{service}} = 1 - e^{-\mu \Delta t}$.

To accurately represent the continuous Poisson arrival and exponential service processes in discrete time, the probabilities are calculated using the formulas above. At each time step, two uniform random variables $U_1, U_2 \in [0, 1)$ are generated:

- **Arrival Event**: If $U_1 < P_{\text{arrival}}$ and the queue length is less than K, a voter joins the queue.

- **Service Event**: If the queue length is greater than zero and $U_2 < P_{\text{service}}$, a voter is serviced.

Results

The simulation reported:

- **Total Voters Processed**: 149
- **Average Queue Length**: 0.7 voters
- **Maximum Queue Length**: 5 (below capacity $K = 10$)

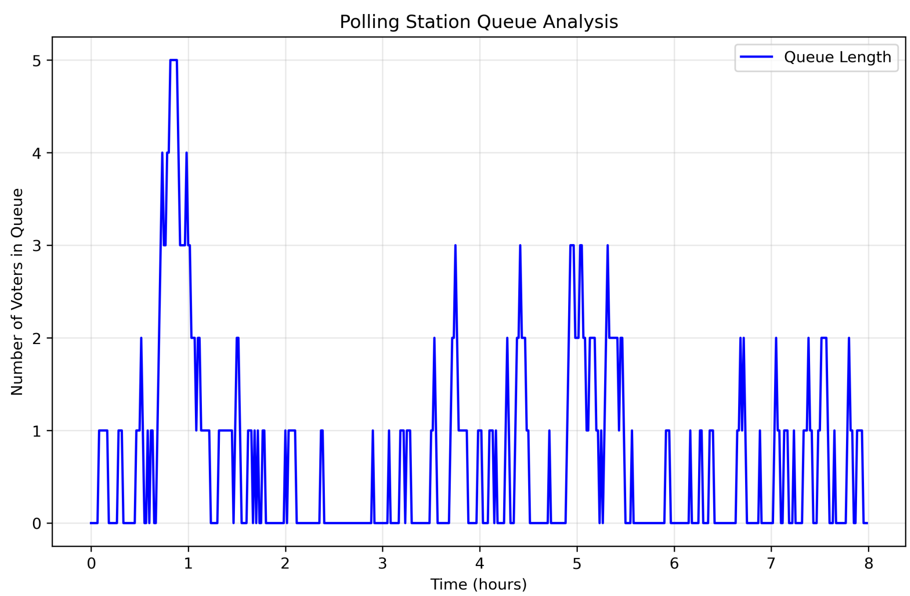

Figure 1: Queue Length Over Time

Figure 1 shows number of voters waiting in a queue throughout hours of the day.

Conclusion and Limitations

Figure 1 shows queue dynamics, with the queue length well within capacity, indicating efficient queues that manage voters. Since the queue length remained below capacity throughout the day, this suggests that the FBI can be confident there are enough polling booths to minimise voter wait times and ensure their agents will not get stuck in the queues.

This simulation assumes a constant arrival rate and a single polling booth throughout the day. However some limitations of this model are:

- **Varying Arrival Times**: Voter arrivals may fluctuate during the day (e.g., peak times in the morning or evening). Future models will consider variable arrival rates.

- **Multiple Polling Booths**: Real polling stations often have more than one booth, which would reduce wait times and alter queue dynamics.

What I Learned

- I gained insight into using object-oriented programming (OOP) for simulations. OOP made event handling simple and avoids spagetti code functions with many parameters - definitely a practice I'll use in future projects.

- I learned to break down seemingly complex events (like queue dynamics throughout the day) into smaller much simpler objects (voters, servers) to mathematically model it.