National Cheng Kung University   Department of Engineering Science

Instructor: Chi-Hua Yu

HW 6
Programming, Due 09:00, Wednesday, April 20th , 2022

注意事項：

1. Homework 的時間為公布題目後至下次上課前結束(上課當天 09:00)。

2. 請在規定的時段內完成作業，並用你的學號與 HW number 做一個檔案夾 (e.g., N96091350_HW3), 將你的全部 ipynb 檔放入檔案夾，壓縮後上傳至課程網站 (e.g., N96091350_HW3.zip)，超過期限後不予補交。

Homework Submission Procedure (請仔細閱讀)

1. You should submit your Jupyter notebook and Python script (`*.py`, in Jupyter, click File, Download as, Python (`*.py`)).
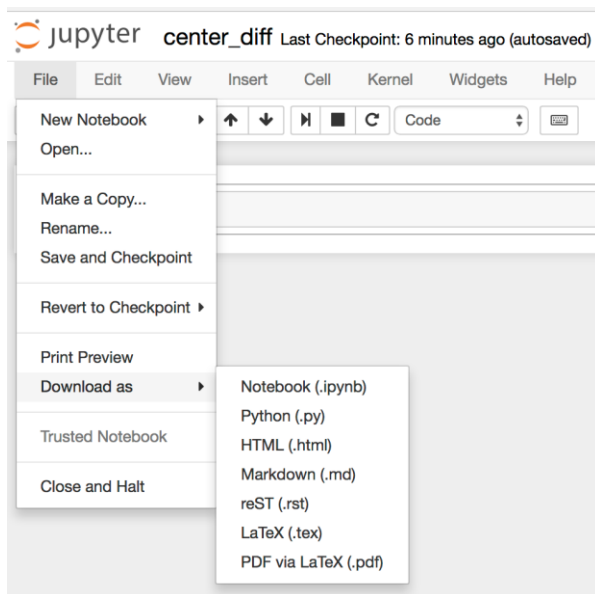
2. Name a folder using your student id and lab number (e.g., n96081494_HW1), put all the pdf and all the Jupyter notebooks and python scripts into the folder and zip the folder (e.g., n96081494_HW1.zip).

3. Submit your lab directly through the course website.

1. **(20%)** Name your pdf file HW6_student id (e.g., HW6_n96081494.pdf). Please state the details and derivations in a professional format and submit a pdf file to Moodle. Taking the Taylor series of f around $a = x_j$ and compute the series at $x = x_{j-2}$, x_{j-1}, x_{j+1}, x_{j+2}. Show that the resulting equations can be combined to form an approximation for $f'(x_j)$ which is $O(h^4)$.

2. **(40%)** Name your Jupyter notebook `central_difference.ipynb` and Python script `central_difference.py`. Consider the function $f(x) = \cos(x)$. We know that the derivative of $\cos(x)$ is $-\sin(x)$. Write a Python program to differentiate $f(x) = \cos(x)$ without using the function `np.diff`. Please write a function which name `my_central_diff` to complete the differentiation. Plot the difference and print the value of maximum error between the aforementioned numerical differentiation methods and their accuracy.

   Below is the running example:

   The wide of the blue line is 8

   The wide of the orange line is 4
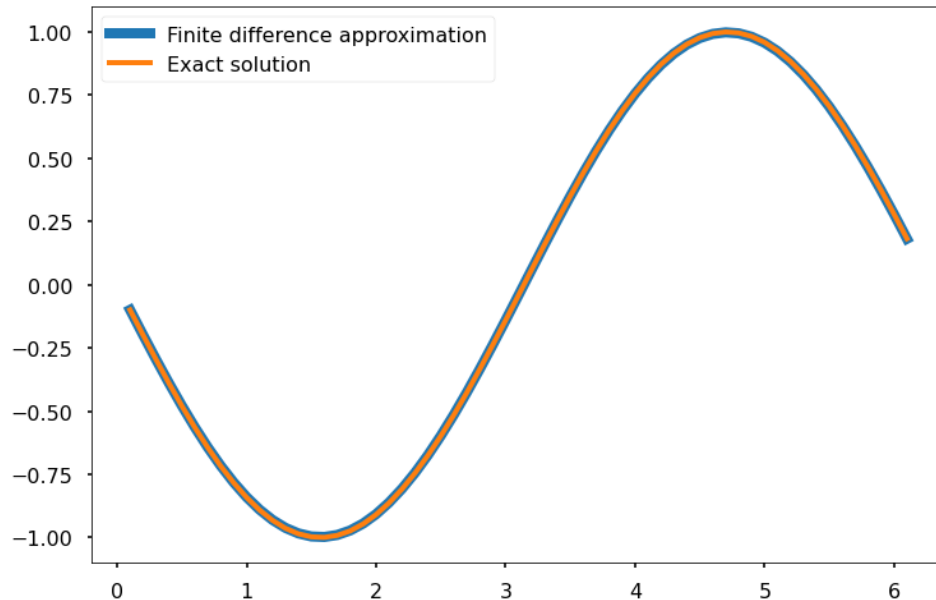
   The maximum error is 0.0016657056915946322

3. **(40%)** Name your Jupyter notebook `iterate_step_size.ipynb` and Python script `iterate_step_size.py`. Continue the previous question, compute the numerical derivative of $f(x) = \cos(x)$ using the central-difference formula for decreasing step size, h. Please plot the maximum error between the approximated derivative and the true derivative versus h.

Below is the running example:

The number of iterations is 15

The decreasing size of step is h/2