Numerical Method

National Cheng Kung University Department of Engineering Science

Instructor: Chi-Hua Yu

HW 9

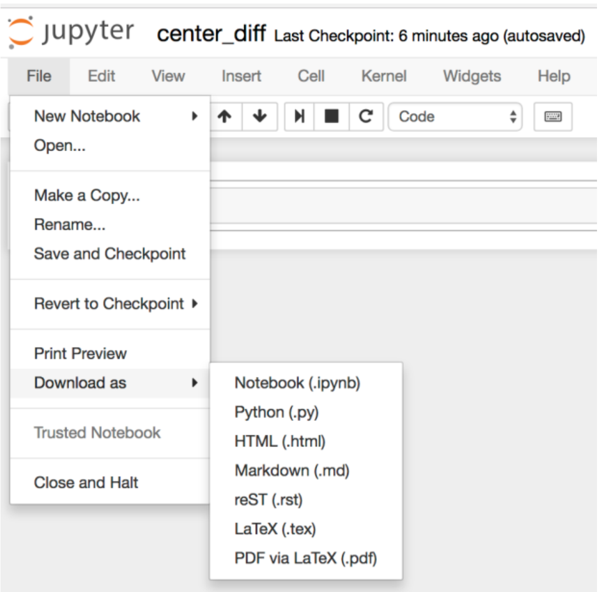
Programming, Due 09:00, Wednesday, June 1st , 2022

注意事項：

1. Homework 的時間為公布題目後至下次上課前結束(上課當天 09:00)。
2. 請在規定的時段內完成作業，並用你的學號與 HW number 做一個檔案夾 (e.g., N96091350_HW3), 將你的全部 ipynb 檔放入檔案夾，壓縮後上傳至課程網站 (e.g., N96091350_HW3.zip)，超過期限後不予補交。

<u>Homework Submission Procedure (請仔細閱讀)</u>

1. You should submit your Jupyter notebook and Python script (`*.py`, in Jupyter, click File, Download as, Python (`*.py`)).



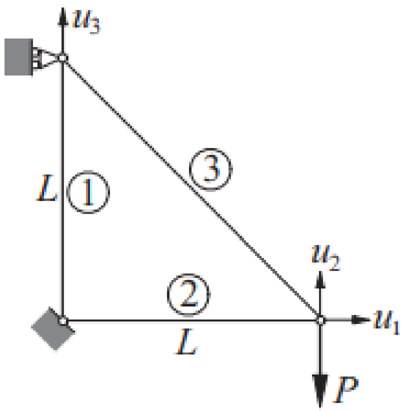
2. Name a folder using your student id and lab number (e.g., n96081494_HW1), put all the pdf and all the Jupyter notebooks and python scripts into the folder and zip the folder (e.g., n96081494_HW1.zip).

3. Submit your lab directly through the course website.

1. (50%) Name your Jupyter notebook `minimize_volume.ipynb` and Python script `minimize_volume.py`. The displacement formulation of the truss shown results in the following simultaneous equations for the joint displacements **u:**

$$\frac{E}{2\sqrt{2}L}\begin{bmatrix} 2\sqrt{2}A_2 + A_3 & -A_3 & A_3 \\ -A_3 & A_3 & -A_3 \\ A_3 & -A_3 & 2\sqrt{2}A_1 + A_3 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -P \\ 0 \end{bmatrix}$$

where E represents the modulus of elasticity of the material and A_i is the cross-sectional area of member i. Use Powell's method to minimize the structural volume (i.e., the weight) of the truss while keeping the displacement u_2 below a given value δ.



Below is the running example:

Penalty multiplier is 100.

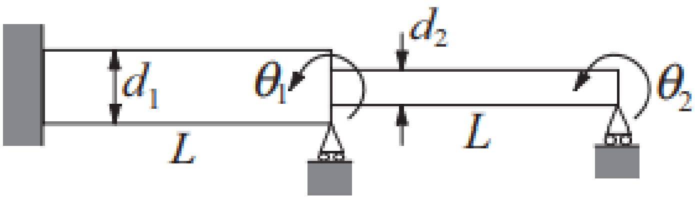
```
xStart = array([1.0, 1.0, 1.0])
x,numIter = powell(F, xStart)
print("x = ", x)
print("v = ", v)
print("Relative weight F = ", weight)
print("Number of cycles = ", numIter)

x =  [3.73870376 3.73870366 5.28732564]
v =  [-0.26747239 -1.06988953 -0.26747238]
Relative weight F =  14.95481504706307
Number of cycles =  10
```

2. (50%) Name your Jupyter notebook `minimize_diameters.ipynb` and Python script `minimize_diameters.py`. The fundamental circular frequency of the stepped shaft is required to be higher than ω_0 (a given value). Use the downhill simplex to determine the diameters d_1 and d_2 that minimize the volume of the material without violating the frequency constraint. The approximate value of the fundamental frequency can be computed by solving the eigenvalue problem (obtainable from the finite element approximation)

$$\begin{bmatrix} 4(d_1^4 + d_2^4) & 2d_2^4 \\ 2d_2^4 & 4d_2^4 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \frac{4\gamma L^4 \omega^2}{105E} \begin{bmatrix} 4(d_1^2 + d_2^2) & -3d_2^2 \\ -3d_2^2 & 4d_2^2 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$$

where

$$\begin{aligned} \gamma &= \text{ mass density of the material} \\ \omega &= \text{ circular frequency} \\ E &= \text{ modulus of elasticity} \\ \theta_1, \theta_2 &= \text{ rotations at the simple supports} \end{aligned}$$



Below is the running example:

Penalty multiplier is 10^6.

```
dStart = np.array([1.0, 1.0])
d = downhill(F, dStart, 0.1)
print("d = ", d)
print("eigenvalue = ", eVal)

d =  [1.07512696 0.79924677]
eigenvalue =  0.39999775723764786
```