Curve Sketching Unit Assignment
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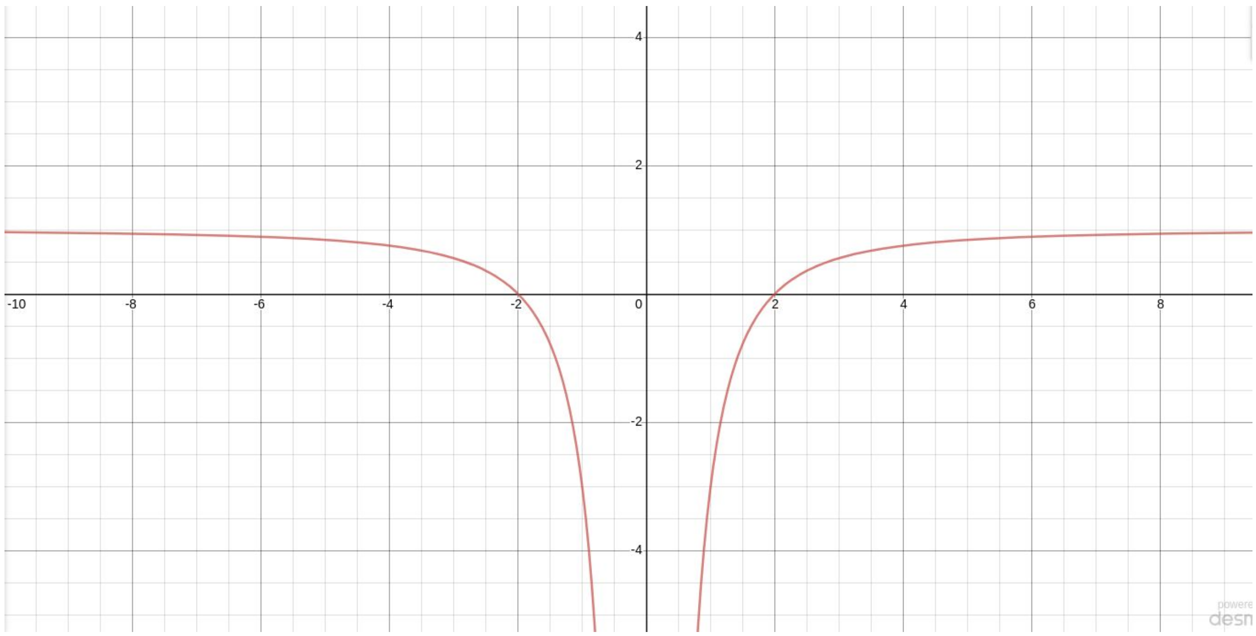
1. **Sketch the functions on graph paper with properly labeled axes and make sure to label the coordinates or equation of all important features, including the intercepts, asymptotes, critical points, and points of inflection.**

a) $y = \frac{x^2-4}{x^2}$



i) x-intercept

$y = 0 \;=>\; \frac{x^2-4}{x^2} = 0,\; x^2 - 4 = 0 \times x^2,\; x^2 - 4 = 0,\; x^2 = 4$

$x = \pm 2$

Thus, x-intercept would be $(2, 0),\; (0, 2)$.

ii) y-intercept

$x = 0 => y = \frac{x^2-4}{x^2},\; y = \frac{0-4}{0}$

Since $\frac{0}{0}$ can be interpreted as undefined, we can say that y-intercept does not exist.

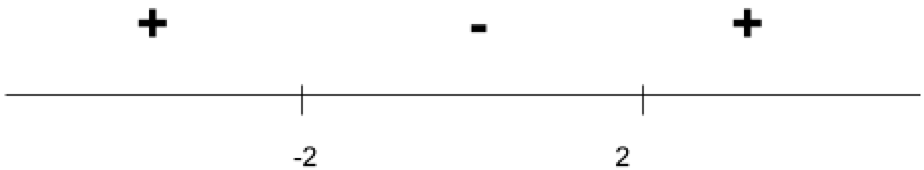
iii) Sign chart

To get a sign chart, we need to calculate at which x point we can get a positive y point.

$\frac{x^2-4}{x^2} > 0,\; x^2 - 4 > 0,\; x^2 > 4$

Thus, x could be $x >-2 \; or \; x \; > \; 2$, to satisfy the condition.

We can draw the sign chart as the following.



iv) End behavior

$lim_{x->\infty}\frac{x^2-4}{x^2} = lim_{x->\infty}(1 - \frac{4}{x^2}) = 1$

$lim_{x->-\infty}\frac{x^2-4}{x^2} = lim_{x->-\infty}(1 - \frac{4}{x^2}) = 1$

We can say that $\frac{1}{x^2}$ would go 0 since if we send x to infinite (or minus infinite), the variable would go to infinite (or minus infinite).

v) Critical points

$f(x) = \frac{x^2-4}{x^2}$

To use the quotient rule to get $\frac{dy}{dx}$ as the following.

$f'(x) = \frac{2x(x^2)-(x^2-4)2x}{x^4} = \frac{2x^3-2x^3+8x}{x^4} = \frac{8x}{x^4} = \frac{8}{x^3}$

To get the critical point, $f'(x) = 0$, however, if this is the case, x should be zero.
If x is zero, $f'(x)$ would be undefined. Thus, the critical point of a given equation does not exist.

vi) Asymptote

- Vertical asymptote
  - To get a vertical asymptote, solving the equation by letting $n(x) = 0$, where $n(x)$ is denominator of the function.
  - $f(x) = \frac{x^2-4}{x^2},\ n(x) = x^2$
  - $x^2 = 0$
  - $x = 0$
- Horizontal asymptote
  - To find a horizontal asymptote when the degree of the denominator and the degree of the numerator is the same, note the fact that the horizontal asymptote equals the leading coefficient of the numerator divided by the leading coefficient of the denominator.
  - $f(x) = \frac{x^2-4}{x^2},\ horizontal\ asymptote\ =\ 1$
- Slant asymptote
  - To find a slant asymptote, we acknowledge the fact that occurs when the polynomial in the numerator is a higher degree than the polynomial in the denominator.
  - However, the degree of the numerator and the degree of the denominator is the same. Thus, slant asymptote does not exist.

vii) Slope chart

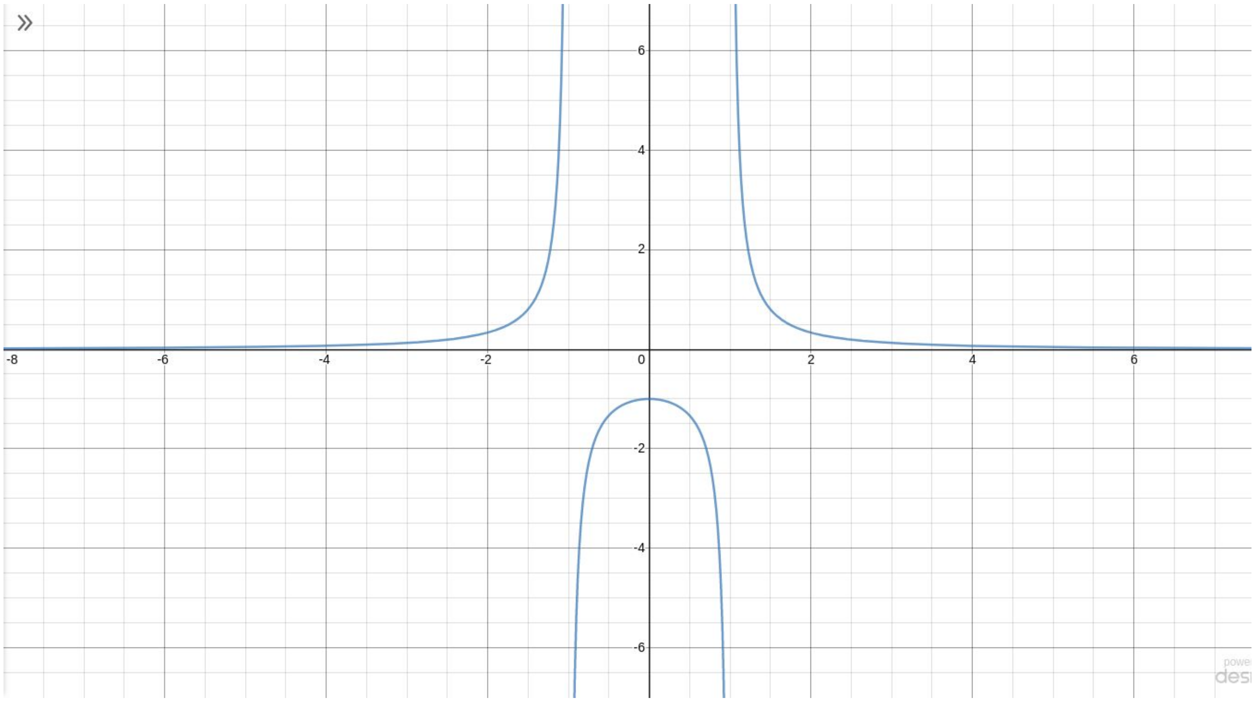
- $f'(x) = \frac{dy}{dx} = \frac{8}{x^3}$

x	f'(x)
-3	$-\frac{8}{27}$
-2	-1
-1	-8
0	undefined
1	8
2	1
3	$\frac{8}{27}$

ix) Concavity Chart

- To become concave upward, $f''(x) > 0$.
- $f'(x) = \frac{8}{x^3} = 8 \times \frac{1}{x^3}$
- In order for getting the second derivatives, use Quotient's Law to get the answer.
  - The Quotient's Law is the following. $\frac{1}{g(x)}' = -\frac{g'(x)}{g(x)^2}$
  - $f''(x) =- \frac{3x^2}{x^6} =- \frac{3}{x^4}$
  - $f''(x) =- \frac{3}{x^4} > 0$
  - The value cannot be solved since if $x = 0$, the formula is undefined.
- Thus, the formula is concave downward at all x points.

b) $y = \frac{1}{x^2-1}$

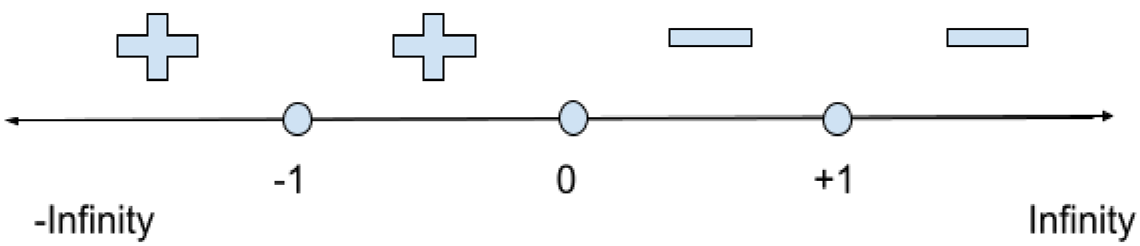


- x-intercept
  - To get the x-intercept, substitute y=0 into the formula.
  - $0 = \frac{1}{x^2-1}$
  - Since there is no answer, there is no x-intercept.
- y-intercept
  - To get the y-intercept, substitute x=0 into the formula.
  - $y =- 1$
  - Thus, y-intercept is $(0,-1)$.
- Critical point
  - In order for getting the first derivatives, use Quotient's Law to get the answer.
  - The Quotient's Law is the following. $\frac{1}{g(x)}' = -\frac{g'(x)}{g(x)^2}$
  - $f'(x) =- \frac{2x}{(x^2-1)^2} = 0$
  - $At\ x = 0,$ the first derivates would be zero, while $f(0) = \frac{1}{0-1} =- 1$.
  - Thus, the critical point would be $(0,-1)$.
- Asymptotes
  - For the vertical Asymptote, set the denominator equals zero.
  - $(x^2 - 1) = 0$
  - $x^2 = 1$
  - $x =\pm 1$
  - $lim_{x->-1}(\frac{1}{x^2-1}) = \frac{1}{0} = \infty$
  - $lim_{x->+1}(\frac{1}{x^2-1}) = \frac{1}{0} = \infty$
  - Thus, $x = 1\ or\ x =- 1$ is on the vertical asymptote.

- For the slant asymptote, if the degree of the numerator is bigger than the denominator, there is no horizontal asymptote.

- For the horizontal asymptote, If the degree of the denominator is bigger than the degree of the numerator, the horizontal asymptote is the x-axis $(y = 0)$.
- Thus, x-axis , $y = 0$, is the horizontal asymptote.

- End behaviour
  - If $x\ goes\ \infty$, $lim_{x->\infty}(\frac{1}{x^2-1}) = \frac{1}{\infty} = 0$.
  - Thus, the function goes to 0.
- Sign Chart
  - To get a sign chart, we need to calculate at which x point we can get a positive y point.
  - $\frac{1}{x^2-1} > 0,\ x^2 - 1 > 0,\ x^2 > 1$
  - Thus, x could be $x > 1\ or\ x > \ -1$, to satisfy the condition.
  - Interval of Increase: $(-\infty, -1) \cup (-1, 0)$
  - Interval of Decrease: $(0, 1) \cup (1, \infty)$
  - We can draw the sign chart of f'(x) as the following.



- For point of inflection, $f''(x) = 0$.
  - To get the second derivative, we need to get the first derivative.
  - $f'(x)$ could be calculated by using the Quotient's Law.
  - $f(x) = \frac{1}{x^2-1}$
  - $f'(x) = -\frac{2x}{(x^2-1)^2}$
  - $f''(x) = \frac{2(3x^2+1)}{(x^2-1)^3} = 0$
  - Since there is no answer, we can conclude that there is no inflection point.
- Slope Chart
  - $f'(x) = -\frac{2x}{(x^2-1)^2}$

x	f'(x)
-3	$\frac{7}{64}$
-2	$\frac{26}{27}$

-1	undefined
0	-2
1	undefined
2	$\frac{26}{27}$
3	$\frac{7}{64}$

- Concavity Chart

ix) Concavity Chart

- To become concave upward, $f''(x) > 0$.
- $f''(x) = \frac{2(3x^2+1)}{(x^2-1)^3} > 0$
- Concave upward on $(-\infty, -1) \cup (1, \infty)$
- Concave downward on $(-1, 1)$
- Thus, the formula is concave downward at all x points.
- We can draw the sign chart of $f''(x)$ as the following.