

Unit Assignment: Rational Functions and Inequalities

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Question 1.

1-1. What is a rational function?

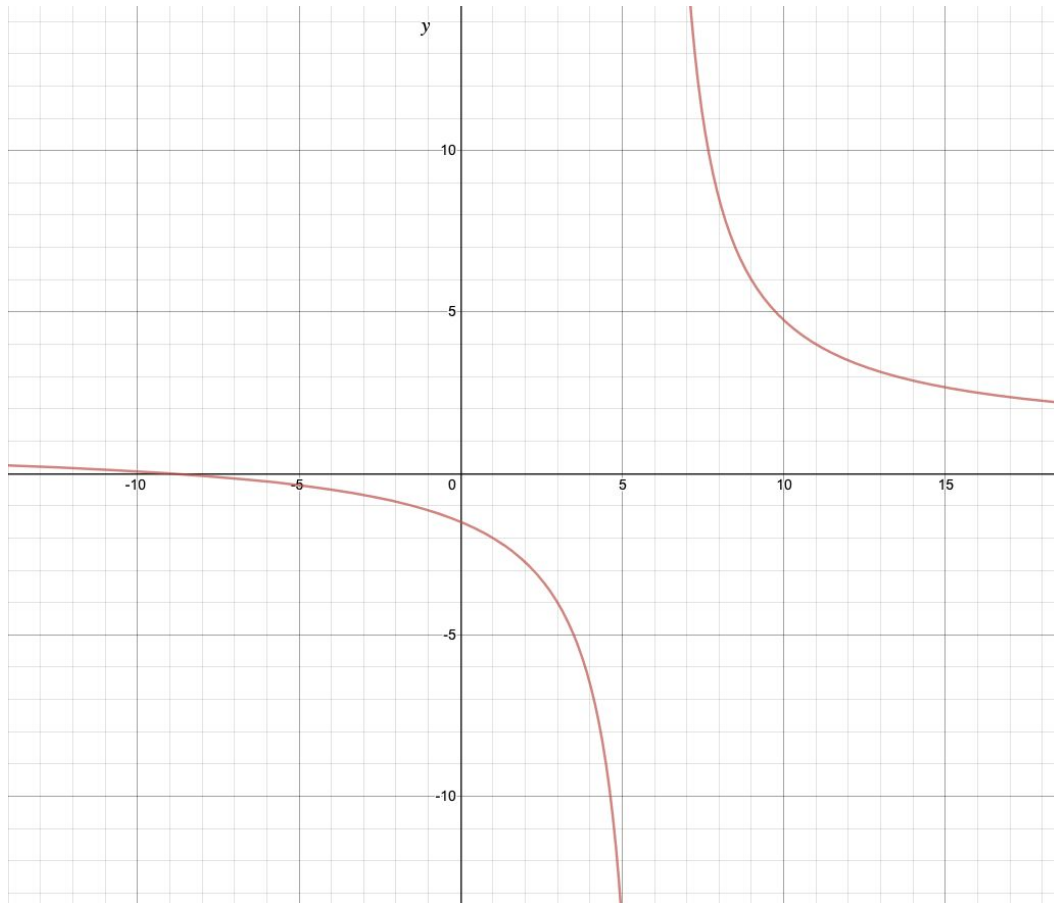
- A rational function has a ratio of two polynomial functions where its divider is not zero. The function has x-intercepts, y-intercepts, turning points, local maximum, and local minimum, and asymptotes.
- The domain of $f(x) = \frac{P(x)}{Q(x)}$ is all point of x where its denominator $Q(x)$ is not zero and $f(x)$, $g(x)$ are polynomials.

1-2. How is it different from a polynomial?

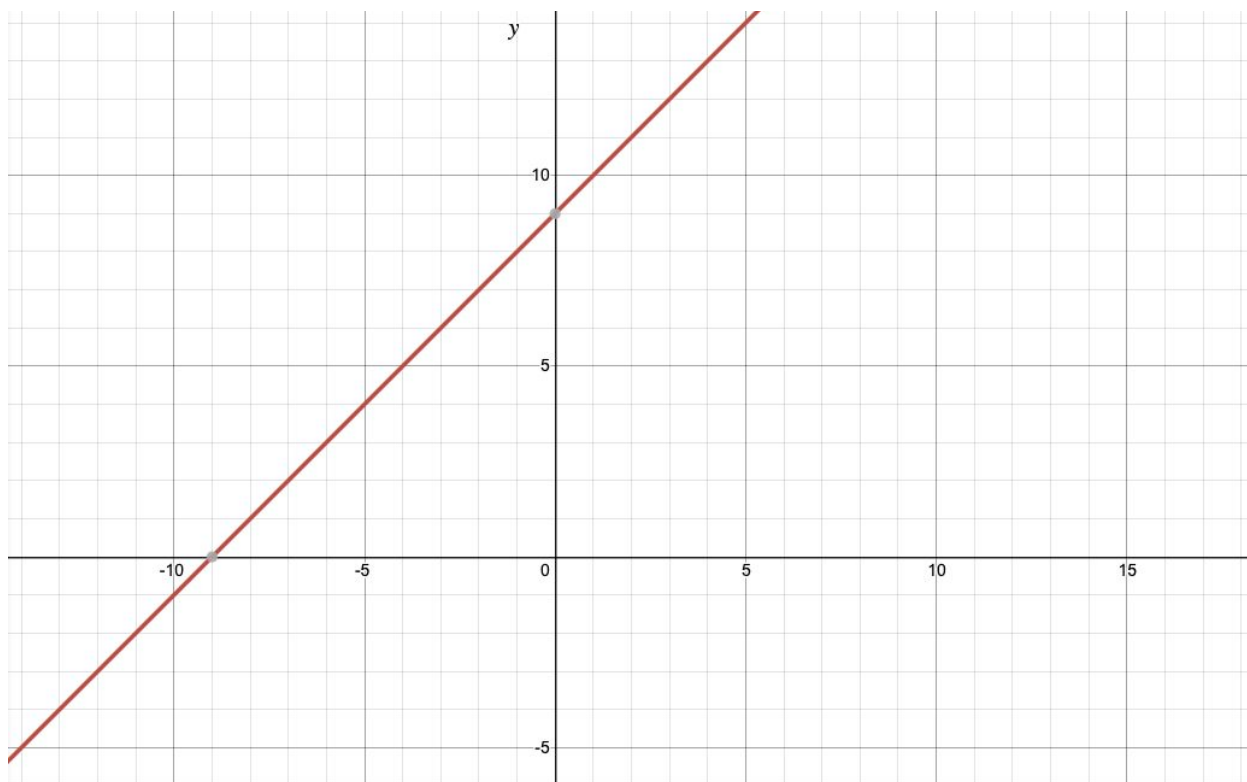
- A polynomial function is composed of one or multiple monomials while a rational function has a denominator, which is a polynomial function, to divide another polynomial function.

1-3. Provide a graph of each to demonstrate the difference.

- The graph of $f(x) = \frac{x+9}{x-6}$



- The graph of $f(x) = x + 9$



Question 2.

2-1. $f(x) = \frac{2x-1}{x+5}$

- x-intercept

$$0 = \frac{2x-1}{x+5}$$

$$0 = 2x - 1$$

$$1 = 2x$$

$$x = \frac{1}{2}$$

Therefore, the x-intercept is $\frac{1}{2}$

- y-intercept

$$f(0) = \frac{2x-1}{x+5}$$

$$f(0) = \frac{2(0)-1}{0+5}$$

$$f(0) = \frac{-1}{5}$$

Therefore, the y-intercept is $\frac{-1}{5}$

- holes

Factor: $\frac{2x-1}{x+5}$

There is no hole since no common factor viable.

- Vertical asymptotes

Factor: $\frac{2x-1}{x+5}$

Thus, $x = -5$

- Horizontal and Oblique asymptotes

The numerator and denominator have the same degree, thereby horizontal asymptotes are $y = \frac{a}{c}$ where a, c are both leading coefficient of the numerator and denominator.

$y = \frac{2}{1}, y = 2$

- End behaviors

i) when $x \rightarrow -5^-$

$$f(-5.001) = \frac{2*(-5.001)-1}{(-5.001)+5} = \frac{-11.002}{-0.001}$$

$y \rightarrow \infty$

ii) when $x \rightarrow -5^+$

$$f(-4.999) = \frac{2*(-4.999)-1}{(-4.999)+5} = \frac{-10.998}{0.0001}$$

$y \rightarrow -\infty$

iii) if $x \rightarrow \infty$

$$f(9999) = \frac{2*(9999)-1}{9999+5}$$

$x \rightarrow \infty$

iv) if $x \rightarrow -\infty$

$$f(-9999) = \frac{2*(-9999)-1}{(-9999)+5}$$

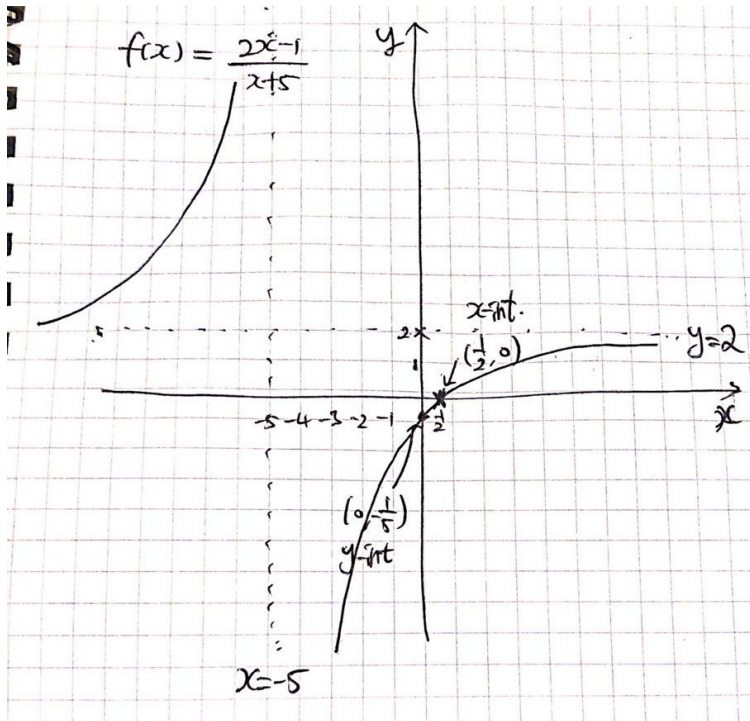
$x \rightarrow -\infty$

- Defining Intervals

$$f(x) = \frac{2x-1}{x+5}$$

	$x < -5$	$-5 < x < \frac{1}{2}$	$x > \frac{1}{2}$
$2x - 1$	-	-	-
$x + 5$	+	+	+
total	-	-	+

- Sketch by hand



2-2. $f(x) = \frac{x^2-9}{x}$

- x-intercept

$$0 = \frac{x^2-9}{x}$$

$$0 = x^2 - 9$$

$$x = 3 \text{ or } x = -3$$

- y-intercept

$$f(x) = \frac{0-9}{0}$$

It is not allowed to let the denominator equals zero. Thus, there is no y-intercept.

- Holes

Since the nominator and denominator do not have a common factor, there is no hole.

- Vertical Asymptotes

$$f(x) = \frac{x^2-9}{x}$$

$$x \neq 0$$

Thus, $x = 0$ is a vertical asymptote.

- Horizontal/Oblique Asymptotes

The numerator has a larger degree than denominator which means there are no horizontal asymptotes.

There is an oblique asymptote, which is $y = x$.

To denote the following equation, $y = ax + b + \frac{e}{cx+d}$ ($a \neq 0$) has an oblique asymptote which is $y = ax + b$.

$$y = x - \frac{9}{x}$$

- End behaviours

i) $x \rightarrow 0^-$

$$y = \frac{(-0.001)^2 - 9}{(-0.001)} = \frac{(0.000001) - 9}{-0.001} > 0$$

$$y \rightarrow \infty$$

ii) $x \rightarrow 0^+$

$$y = \frac{(0.001)^2 - 9}{(0.001)} = \frac{(0.000001) - 9}{0.001} < 0$$

$$y \rightarrow -\infty$$

iii) $x \rightarrow \infty$

$$f(999) = \frac{(999)^2 - 9}{999} > 0$$

$$y \rightarrow \infty$$

iv) $x \rightarrow -\infty$

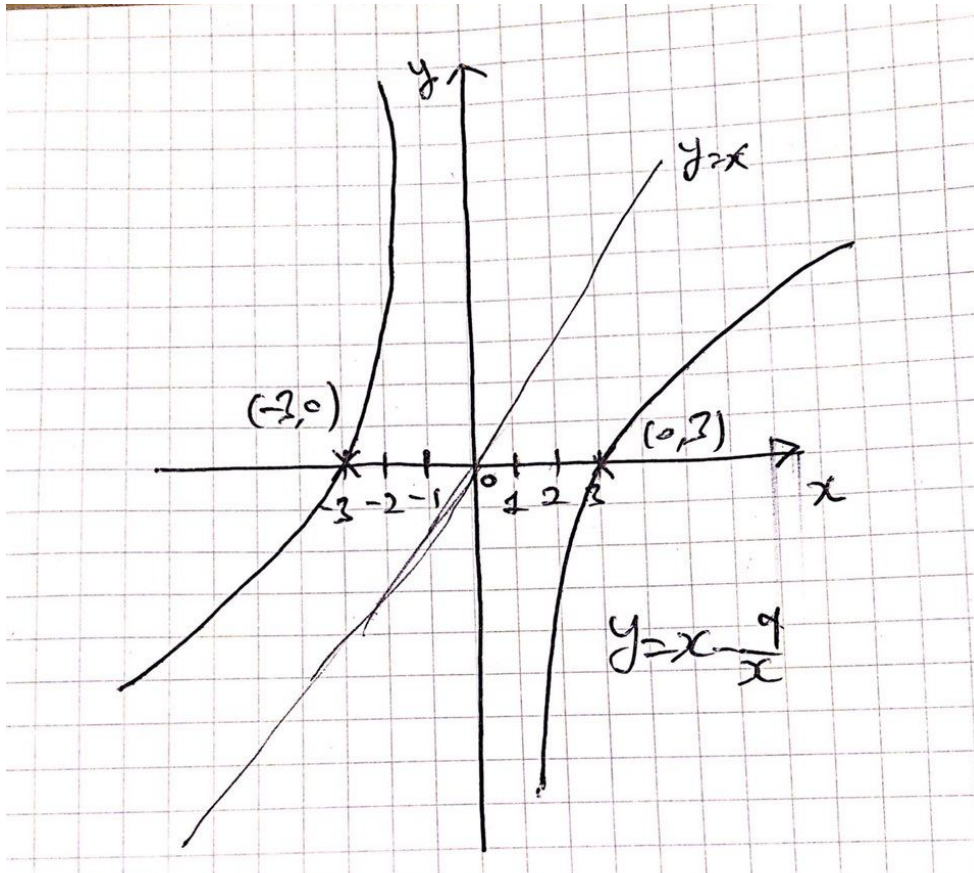
$$f(-999) = \frac{(-999)^2 - 9}{(-999)} < 0$$

$$y \rightarrow -\infty$$

- Defining Intervals

	$x < -3$	$0 < x < 3$	$x > 3$
x	-	+	+
$x^2 - 9$	+	-	+
	-	-	+

- Sketch by hand



2-3. $f(x) = \frac{x-3}{x^3-2x^2-5x+6}$

$$f(x) = \frac{(x-3)}{(x-1)(x+2)(x-3)}$$

- x-intercept

$$0 = \frac{1}{(x-1)(x+2)}$$

$0 * (x-1)(x+2) = 1$ which is undefined.

There is no x-intercept.

- y-intercept

$$f(x) = \frac{1}{(0-1)(0+2)} = -\frac{1}{2}$$

- Holes

$$\text{Factor } f(x) = \frac{(x-3)}{(x-1)(x+2)(x-3)}$$

$(x-3)$ is the common factor

$$x = 3$$

Substitute $x = 3$ to $f(x)$

$$\frac{(3-3)}{(3-1)(3+2)(3-3)} = \frac{1}{10}$$

Therefore hole is $(3, \frac{1}{10})$

- Vertical Asymptotes

$$(x-1)(x+2)(x-3) \neq 0$$

$$x \neq -2, 1, 3$$

However, $x = 3$ is a hole value

Therefore, the vertical asymptote is $-2, 1$

- Horizontal/Oblique Asymptotes

The denominator has a bigger degree than the numerator

The horizontal asymptote happens in $y = 0$

There is no slant asymptote.

- End behaviors

i) $x \rightarrow -2^-$

$$f(x) = \frac{(-2.001-3)}{(-2.001-1)(-2.001+2)(-2.001-3)} = \frac{1}{(-3.001)(-0.001)} > 0$$

$$f(x) \rightarrow \infty$$

ii) $x \rightarrow -2^+$

$$f(x) = \frac{(-1.999-3)}{(-1.999-1)(-1.999+2)(-1.999-3)} = \frac{1}{(-2.999)(0.0001)} < 0$$

$$f(x) \rightarrow -\infty$$

iii) $x \rightarrow 1^-$

$$f(x) = \frac{(0.999-3)}{(0.999-1)(0.999+2)(0.999-3)} = \frac{1}{(-0.001)(2.999)} < 0$$

$$f(x) \rightarrow -\infty$$

iv) $x \rightarrow 1^+$

$$f(x) = \frac{(1.001-3)}{(1.001-1)(1.001+2)(1.001-3)} = \frac{1}{(0.001)(3.001)} > 0$$

$$f(x) \rightarrow \infty$$

v) $x \rightarrow \infty$

$$f(999) = \frac{(999-3)}{(999-1)(999+2)(999-3)} > 0$$

$$f(x) \rightarrow \infty$$

vi) $x \rightarrow -\infty$

$$f(-999) = \frac{(-999-3)}{(-999-1)(-999+2)(-999-3)} > 0$$

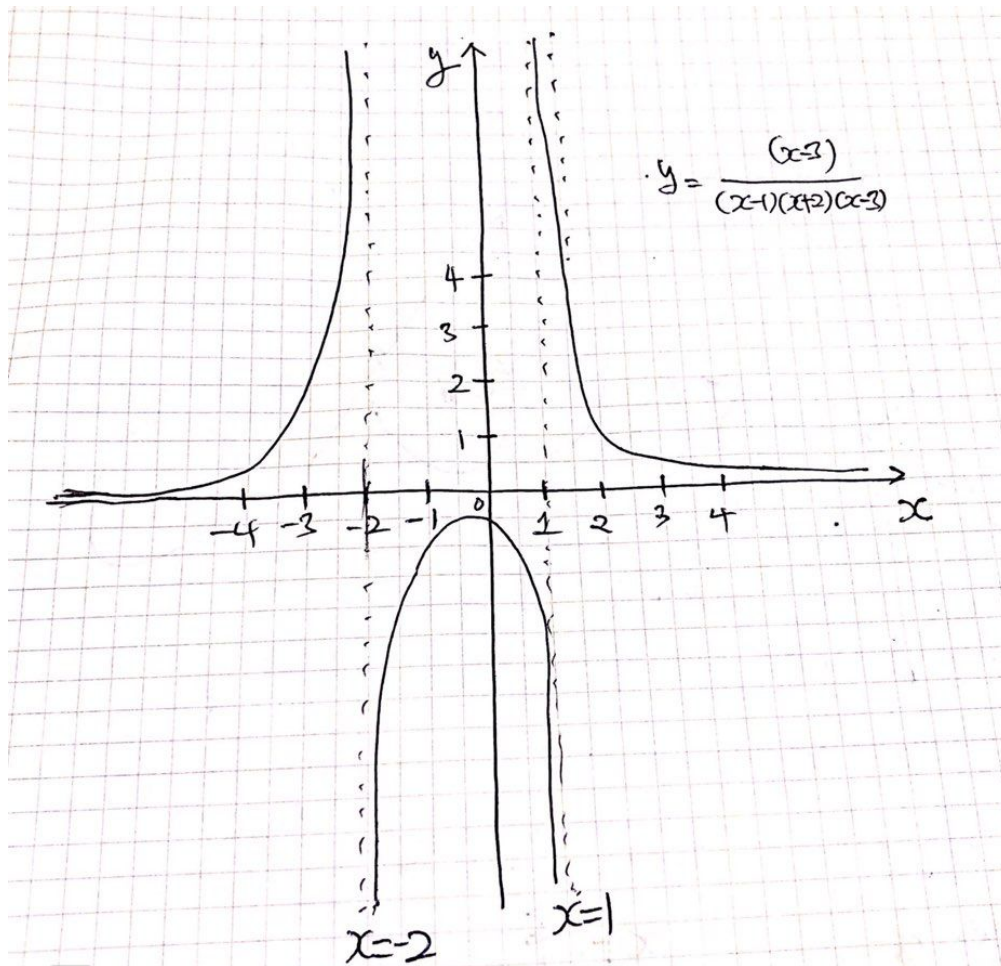
$$f(x) \rightarrow \infty$$

- Defining Intervals

	$x < -2$	$-2 < x < 1$	$x > 2$
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$x+2$	-	+	+
$x-1$	-	-	+
$x-3$	-	-	+
	-	+	+

- Sketch by hand



Question 3.

a. $\frac{16x^2-9}{x^2+4x-12} = 0$
 $\frac{(4x+3)(4x-3)}{(x-2)(x+6)} = 0$

To extract the value that makes the function undefined.

$$(x-2) \neq 0, (x+6) \neq 0$$

$$x \neq 2, x \neq -6$$

To get the value that makes the function zero.

$$(4x+3) = 0$$

$$\text{or } (4x-3) = 0$$

$$x = -\frac{3}{4} \text{ or } x = \frac{3}{4}$$

b. $x(2x-13) < -20$

$$2x^2 - 13x < -20$$

$$2x^2 - 13x + 20 < 0$$

$$(2x-5)(x-4) < 0$$

	$x < \frac{5}{2}$	$\frac{5}{2} < x < 4$	$x > 4$
$(2x-5)$	-	+	+
$(x-4)$	-	-	+
	+	-	+

Therefore, $\frac{5}{2} < x < 4$

c. $\frac{1}{r+3} > \frac{r+4}{r-2} + \frac{6}{r-2}$

$$\frac{1}{r+3} > \frac{r+10}{r-2}$$

$$(r-2) > (r+10)(r+3)$$

$$(r-2) > r^2 + 13r + 30$$

$$r^2 + 12r + 32 < 0$$

$$(r+4)(r+8) < 0$$

	$-8 < r$	$-8 < r < -4$	$r > -4$
$(r+4)$	-	-	+

$(r + 8)$	-	+	+
	+	-	+

The answer is $-8 < r < -4$

Question 4.

The following formula determines the minimum number of hours of studying required to attain a test score, x .

$$p(x) = \frac{0.31x}{100.5-x}$$

- a. How many hours of study are needed to score an 80?

$$p(80) = \frac{0.31(80)}{100.5-80}$$

$$p(80) = \text{about } 1.21$$

Therefore, 1.21 hours of study are needed to score an 80.

- b. What score can you achieve if you study for 6 hours?

$$6 = \frac{0.31x}{100.5-x}$$

$$6(100.5 - x) = 0.31x$$

$$603 - 6x = 0.31x$$

$$6.31x = 603$$

$$x = 95.56$$

Therefore, I would score 95.56 after 6 hours of study.

- c. How many hours of study are needed to score the mark you would like to obtain in this course?

I would like to obtain 100, thus would put 100 into the equation.

$$P(100) = \frac{0.31(100)}{100.5-100}$$

$$P = 62$$

Therefore, I should study for 62 hours to obtain 100.