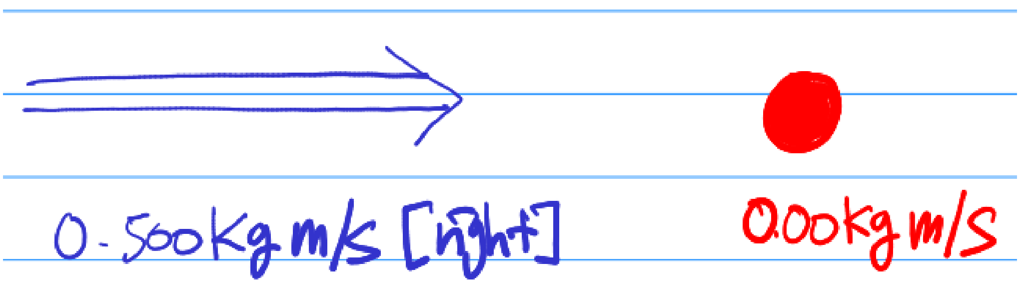
Assignment: Collisions

1. **Collisions in 2D**
   - Draw the diagram and show $\vec{p}_i$, the momentum before the collision.

     $P_{1i} = (0.500kg)(1.000m/s) = 0.500kg\ m/s$

     $P_{2i} = (1.500kg)(0.000m/s) = 0.000kg\ m/s$

     $P_i = (0.500kg\ m/s) + (0.000kg\ m/s) = 0.500kg\ m/s$

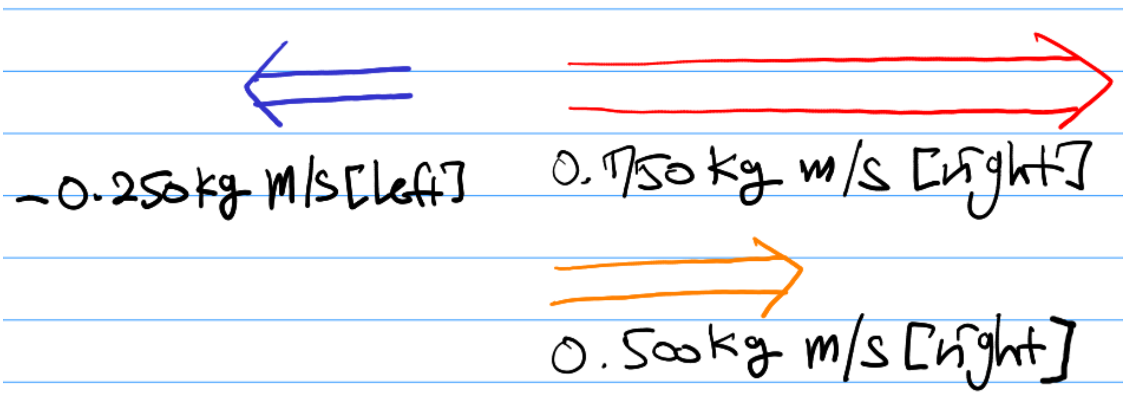


   - Underneath that diagram, draw another diagram that shows two vectors $\vec{P}_{1f}$ and $\vec{P}_{2f}$, the final momentum of ball and ball after the collision. Include a third vector that represents — the sum of the former two vectors.

     $P_{1f} = (0.500kg)(-0.500m/s) = \ -0.250kg\ m/s$

     $P_{2f} = (1.500kg)(0.500m/s) = \ 0.750kg\ m/s$

     $P_f = 0.750kgm/s - 0.250kgm/s = 0.500kg\ m/s$

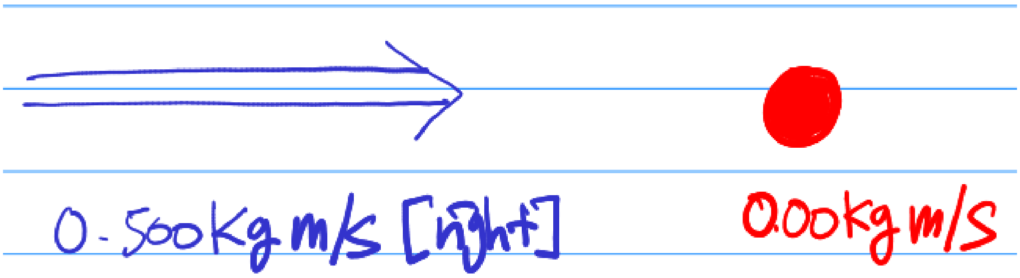


   - Using formula, calculate the total kinetic energy before and the total energy after the collision.

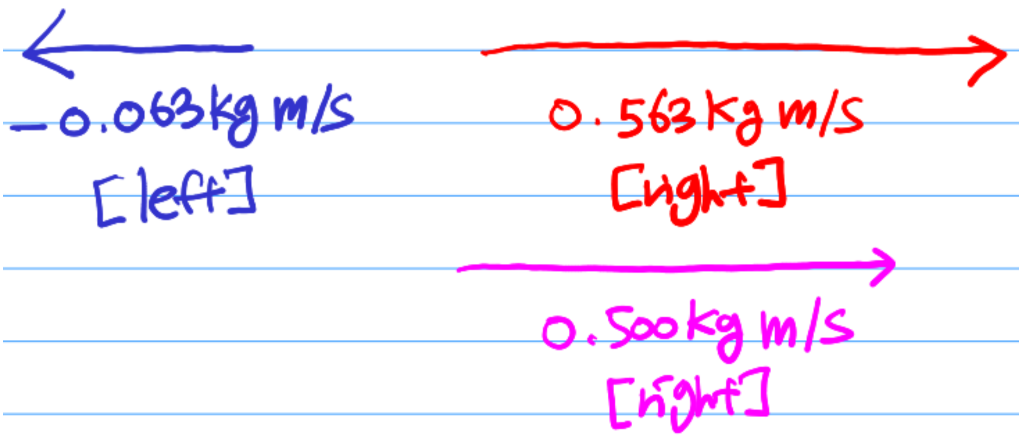
     $K_E\ before: \frac{1}{2} \times (0.5) \times (1.0)^2 + \frac{1}{2} \times (0.5) \times 0^2 = 0.25J$

     $K_E\ after: \frac{1}{2} \times (0.5) \times (-0.5)^2 + \frac{1}{2} \times (1.5) \times (0.5)^2 = 0.0625 + 0.1875 = 0.25J$

- Repeat Step 1~3 but first, change the Elasticity to 50%.
  $P_{1i} = (0.500kg) \times (1.000m/s) = 0.500kg\ m/s$
  $P_{2i} = (1.500kg) \times (0.000m/s) = 0.000kg\ m/s$
  $P_i = 0.500kg\ m/s\ +\ 0.000kg\ m/s\ = 0.500kg\ m/s$

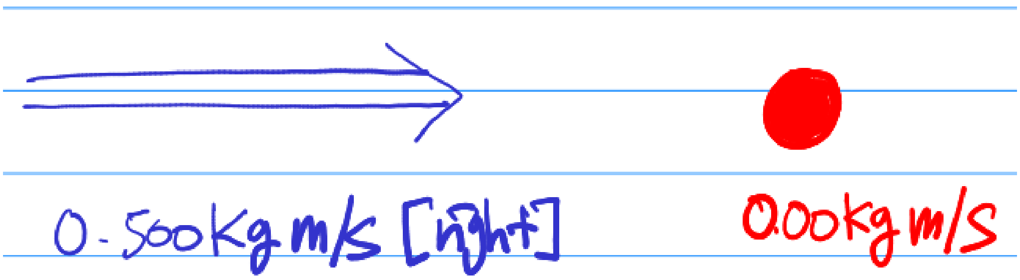


$P_{1f} = (0.500kg)(-\ 0.125m/s) =-\ 0.063kg\ m/s$
$P_{2f} = (1.500kg)(0.375m/s) = 0.563kg\ m/s$
$P_f = 0.563kg\ m/s\ -\ 0.063kg\ m/s = 0.500kg\ m/s$



$K_E\, before\ = \frac{1}{2}(0.5)(1.0)^2 + \frac{1}{2}(1.5)(0)^2 = 0.25\ +\ 0\ =\ 0.25J$
$K_E\, after\ = \frac{1}{2}(0.5)(-\ 0.125)^2 + \frac{1}{2}(1.5)(0.375)^2 = \frac{1}{256} + \frac{27}{256} \simeq 0.109J$

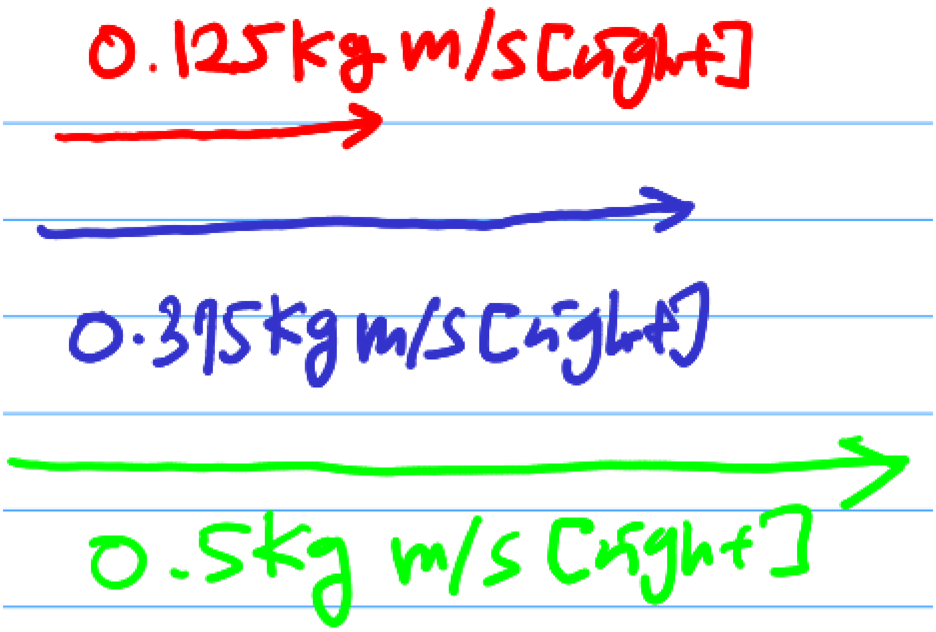
- Repeat Step 1~3 but first, change the elasticity to 0%.
  $P_{1f} = (0.500kg) \times (1.000m/s) = 0.500kg\ m/s$
  $P_{2f} = (1.500kg) \times (0.000m/s) = 0.000kg\ m/s$
  $P_f = 0.500kg\ m/s + 0.000kg\ m/s = 0.500kg\ m/s$



$P_{1f} = (0.500kg)(0.25m/s) = 0.125kg\ m/s$

$P_{2f} = (1.500kg)(0.25m/s) = 0.375kg\ m/s$

$P_f = 0.375kg\ m/s\ +\ 0.125kg\ m/s = 0.5kg\ m/s$



$K_E\ before = \frac{1}{2}(0.5)(1.0)^2 + \frac{1}{2}(1.5)(0)^2 = 0.25 + 0 = 0.25J$

$K_E\ after = \frac{1}{2}(0.5)(0.25)^2 + \frac{1}{2}(1.5)(0.25)^2 = 0.015625 + 0.046875 \simeq 0.0625J$

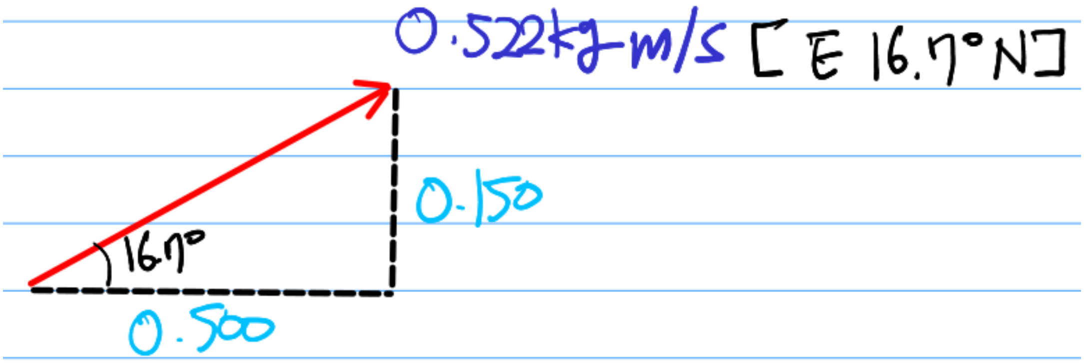
2. Collisions in 2D

- Run the simulation and draw a diagram that shows two vectors $\vec{p}_1$ and $\vec{p}_2$ the initial momentum of ball and ball before the collision. Add a third vector to that diagram $\vec{p}_i$ the total momentum before the collision.

$P_{1i} = \sqrt{(0.500)^2 + (0.150)^2} \simeq 0.522kgm/s$

$tan(\theta) = \frac{0.150}{0.500}$

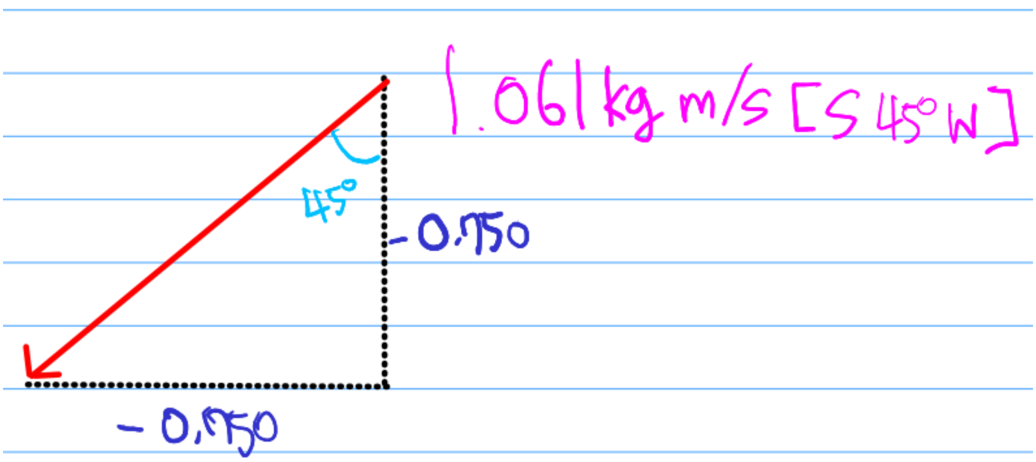
$\theta \simeq 16.7^o$



$P_{2i} = \sqrt{(-\ 0.750)^2 + (-\ 0.750)^2} \simeq 1.061 kgm/s$

$tan(\theta)\ = \frac{-0.750}{-0.750} = 1$

$\theta\ \simeq\ 45^o$

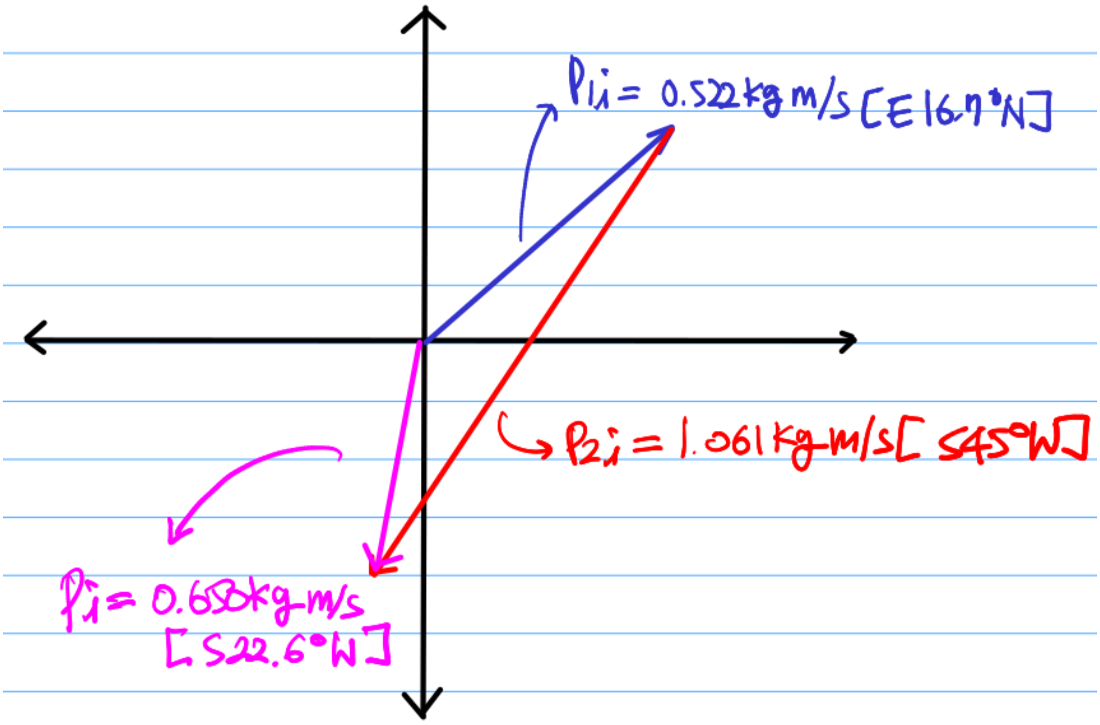


$P_i = [0.500\ +\ (-\ 0.750),\ 0.150\ -\ (0.750)]\ =\ [-\ 0.250, -\ 0.600]$

$tan(\theta)\ = \frac{-0.250}{-0.600}$

$\theta\ \simeq\ 22.6^o$

$\sqrt{(0.250)^2 + (-\ 0.600)^2} = 0.650 kgm/s$

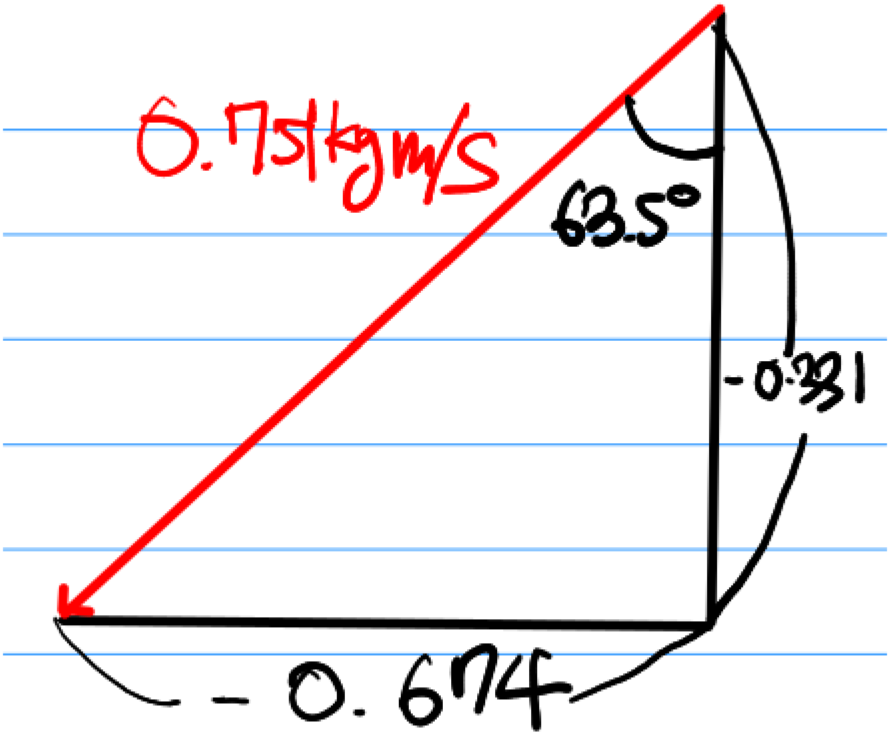


- Underneath that diagram, draw another diagram that shows two vectors: $\vec{p}_1$ and $\vec{p}_2$ — the final momentum of ball 1 and ball 2 after the collision. Include a third vector that represents $\vec{p}_f$ — the sum of the former two vectors.

$P_{1f} = \sqrt{(-0.674)^2 + (-0.331)^2} \simeq 0.751 kgm/s$

$tan\,\theta = \frac{-0.674}{-0.331}$

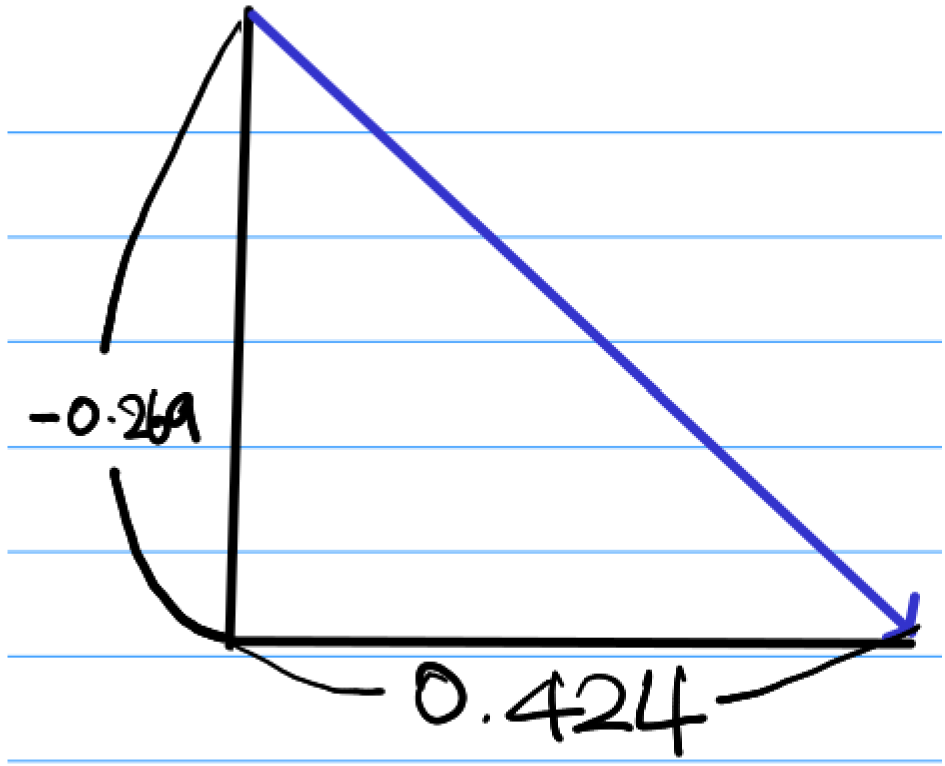
$\theta \simeq 63.8^o$



$$P_{2f} = \sqrt{(0.424)^2 + (-0.629)^2} \simeq 0.494 kgm/s$$

$$tan\,\theta = \frac{0.424}{-0.629}$$

$$\theta \simeq -57.6^o$$

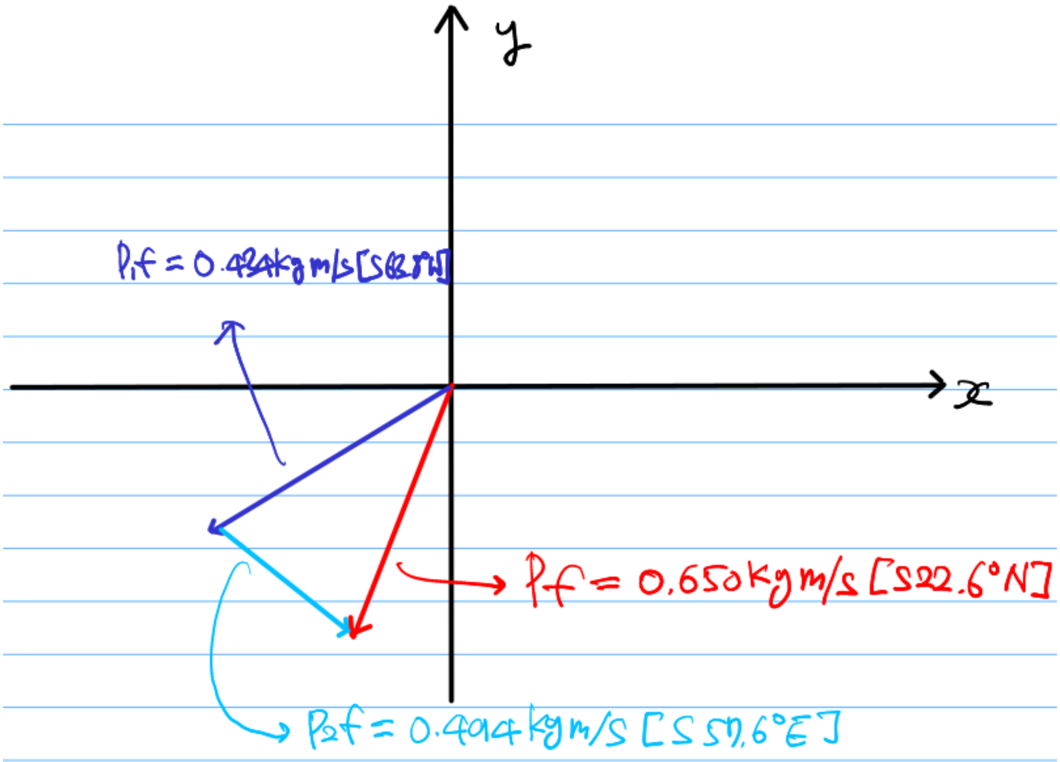


$P_f = [(-\ 0.674) + (0.424),\ (-\ 0.331) + (-\ 0.629)] = [-\ 0.250, -\ 0.600]$

$tan\,\theta\ = \frac{-0.250}{-0.600}$

$\theta\ \simeq\ 22.6^o$

$\sqrt{(-\ 0.250)^2 + (-\ 0.600)^2} \simeq 0.650 kgm/s$



Questions:

1. **In the first setup, you experienced an elastic collision; in the second, a non-elastic collision; and in the third, a completely inelastic collision. Make some general statements about momentum and kinetic energy conservation based on your findings.**
    - Elastic Collision: Elastic collision preserves both velocity and kinetic energy in a 1D collision.
        - Momentum: The initial momentum P_x was $0.500kgm/s$. After the collision, the momentum of Ball 1 changed to $-\ 0.250kgm/s$ and that of Ball 2 changed to $0.750kgm/s$. By summing them up, the final momentum became $-\ 0.250kgm/s\ +\ 0.750kgm/s\ =\ 0.500kgm/s$ which is the same as the initial momentum.
          $m_{1i}v_{1i} = m_{1f}v_{1f} + m_{2f}v_{2f}$
          $(0.500)(1.000)\ =\ (0.500)(-\ 0.500)\ +\ (1.500)(0.500)$
          $0.500\ =-\ 0.250\ +\ 0.750,\ Conserved$
        - Kinetic Energy: The initial velocity of the Ball 1 was $1.000m/s$ in a positive direction. After the collision, the Ball 2 makes its final velocity to $0.500m/s$ in a positive direction, which conserves the final energy of Ball 1. Since the kinetic energy reduced, the final velocity of Ball 1 became $0.500m/s$ in a negative direction.
          $\frac{1}{2}m_{1i}v_{1i} = \frac{1}{2}m_{1f}v_{1f} + \frac{1}{2}m_{2f}v_{2f}$
          $\frac{1}{2}(0.500)(1.000)^2 = \frac{1}{2}(0.500)(-\ 0.500)^2 + \frac{1}{2}(1.500)(0.500)^2$
          $0.250\ =-\ 0.125\ +\ 0.375$

$0.250 = 0.250,\ Conserved$

Thus, from the observations and calculations shown above, we can be confident that both the momentum and kinetic energy are conserved in an elastic collision.

- Non-elastic Collision: In a 1D collision, inelastic collision only conserves the momentum not the kinetic energy.
  - Momentum: The initial momentum P_x was $0.500kgm/s$. After the collision, the momentum of Ball 1 changed to $0.125kgm/s$ and that of Ball 2 changed to $0.375kgm/s$. By summing them up, the final momentum became $0.125kgm/s + 0.375kgm/s = 0.500kgm/s$ which is the same as the initial momentum.
    $m_{1i}v_{1i} = m_{1f}v_{1f} + m_{2f}v_{2f}$
    $(0.500)(1.000) = (0.500)(-0.125) + (1.500)(0.375)$
    $0.500 = -0.0625 + 0.5625,\ Conserved$
  - Kinetic Energy: The initial velocity of the Ball 1 was $1.000m/s$ in a positive direction. After the collision, the Ball 2 makes its final velocity to $0.375m/s$ in a positive direction, which does not conserve the final energy of Ball 1. Since the kinetic energy reduced, the final velocity of Ball 1 became $0.125m/s$ in a negative direction.
    $\frac{1}{2}m_{1i}v_{1i} = \frac{1}{2}m_{1f}v_{1f} + \frac{1}{2}m_{2f}v_{2f}$
    $\frac{1}{2}(0.500)(1.000)^2 = \frac{1}{2}(0.500)(-0.125)^2 + \frac{1}{2}(1.500)(0.375)^2$
    $0.250 = 0.00390625 + 0.10546875$
    $0.250 \neq 0.109375,\ Not\ Conserved$

    Thus, from the observations and calculations shown above, we can be confident that the momentum is conserved but the kinetic energy is not conserved in an inelastic collision. This means that there was a loss in the kinetic energy. It is noted that the energy could have been transformed into other kinds of energy including sounds which implies that the energy itself is gone to nowhere.

    $KE_{lost} = \frac{1}{2}m_{1i}v_{1i} = \frac{1}{2}m_{1f}v_{1f} + \frac{1}{2}m_{2f}v_{2f}$
    $KE_{lost} = 0.250 - 0.109375$
    $KE_{lost} = 0.140625J$

- Completely Inelastic collisions: In a 1D collision, completely inelastic collisions can only happen when the kinetic energy is perfectly lost while the momentum is conserved.
  - Momentum: The initial momentum P_x was $0.500kgm/s$. After the collision, the momentum of Ball 1 changed to $0.125kgm/s$ and that of Ball 2 changed to $0.375kgm/s$. By summing them up, the final

momentum became $0.125kgm/s + 0.375kgm/s = 0.500kgm/s$ which is the same as the initial momentum.

$m_{1i}v_{1i} = m_{1f}v_{1f} + m_{2f}v_{2f}$

$(0.500)(1.000) = (0.500)(0.25) + (1.500)(0.25)$

$0.500 = 0.125 + 0.375, \textit{ Conserved}$

- Kinetic Energy: The initial velocity of the Ball 1 was $1.000m/s$ in a positive direction. After the collision, the Ball 2 makes its final velocity to $0.25m/s$ in a positive direction, which does not conserve the final energy of Ball 1. Since the kinetic energy reduced, the final velocity of Ball 1 became $0.25m/s$ in a positive direction.

  $\frac{1}{2}m_{1i}v_{1i} = \frac{1}{2}m_{1f}v_{1f} + \frac{1}{2}m_{2f}v_{2f}$

  $\frac{1}{2}(0.500)(1.000)^2 = \frac{1}{2}(0.500)(0.25)^2 + \frac{1}{2}(1.500)(0.25)^2$

  $0.250 = 0.015625 + 0.046875$

  $0.250 \neq 0.0625, \textit{ Not Conserved}$

  Thus, from the observations and calculations shown above, we can be confident that the momentum is conserved but the kinetic energy is not conserved in a completely inelastic collision. This means that there was a loss in the kinetic energy. It is noted that the energy could have been transformed into other kinds of energy including sounds which implies that the energy itself is gone to nowhere.

  $KE_{lost} = \frac{1}{2}m_{1i}v_{1i} = \frac{1}{2}m_{1f}v_{1f} + \frac{1}{2}m_{2f}v_{2f}$

  $KE_{lost} = 0.250 - 0.00625$

  $KE_{lost} = 0.24375J$

2. **In the non-elastic collisions, the total kinetic energy was smaller after the collision. Does this violate the Law of Conservation of Energy? Explain.**

   No, the Law of Conservation of Energy is not broken. When compared to an elastic impact, the total kinetic energy was not preserved in the virtual lab experiment. This did not, however, imply that the collision broke the Law of Conservation of Energy. The collision forces may have taken or added kinetic energy in the inelastic collision, but the overall energy remained the same. This is just a shift in energy forms between the colliding items. Heat energy or stored energy can be transformed from the new type of energy. When the inelastic balls contact, another typical energy shift is the vibration of the balls or the sound energy.

3. **In steps 1 and 2 , you drew $\vec{p}_i$ and $\vec{p}_f$. Are these vectors equal? Both of these vectors are the third side of their respective triangles. Describe your triangles in terms of whether or not they are identical, similar, or congruent?**

The two vectors, $\vec{p}_i$ and $\vec{p}_f$, have equal magnitude and direction vectors, resulting in two similar triangles. For both vectors, the px component was $-\ 0.250kgm/s$, while the py component was$-\ 0.600kg\ m/s$. For elastic 2D collisions, this means that the momentum is preserved.

4. **Summarize the rules for conservation of momentum and kinetic energy in a collision.**

   The conservation of momentum and kinetic energy for each elastic and inelastic collision were calculated based on the results of the experiment. Because the forces between the colliding objects are conservative, the momentum and kinetic energy were all preserved in the elastic collision. When two elastic items contact, the objects compress for a brief while at the impact location before springing again. (OpenStaxCollege,n.d.) The objects temporarily hold elastic potential energy as a result of this action, but it is eventually transformed back to kinetic energy. As a result, momentum and energy are both preserved. (“Energy and Momentum in Collisions”, n.d.) The momentum, on the other hand, was conserved in the inelastic impact, but the kinetic energy was not. Because energy is a scalar number, it may be converted into a variety of forms, as previously stated. However, because momentum is a vector variable, it cannot be lost and is thus conserved even in inelastic collisions.

Works Cited

OpenStaxCollege. Inelastic Collisions in One Dimension. (n.d.). Retrieved June 20, 2021, from https://opentextbc.ca/physicstestbook2/chapter/inelastic-collisions-in-one-dimension/.

Energy and Momentum in Collisions. (n.d.). Retrieved June 20, 2021, from http://www.softschools.com/notes/ap_physics/energy_and_momentum_in_collisions/.