Unit Assignment: Trigonometry

MHF4U

Virtual High School

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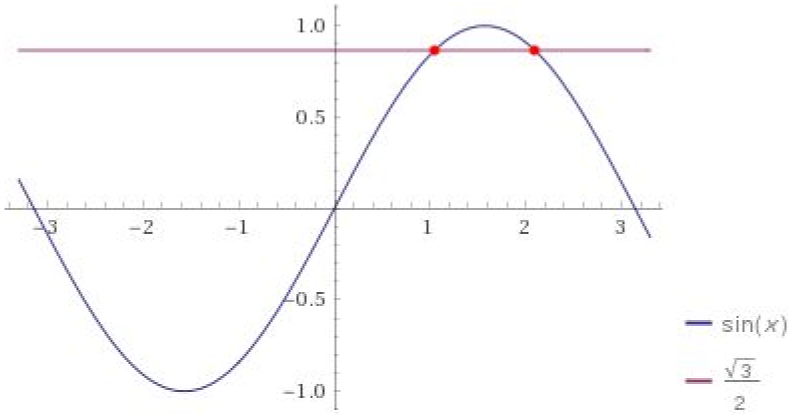
Question 1.

a. Find all solutions for $sin(x) = \frac{\sqrt{3}}{2}$

Graph



Radians: $x = \frac{\pi}{3} + 2\pi n,\ x = \frac{2\pi}{3} + 2\pi n$

- Take the inverse side of both sides of the equation to extract x from inside the sine.
  - $x = arcsin(\frac{\sqrt{3}}{2})$
- The exact value of $arcsin(\frac{\sqrt{3}}{2})\ is\ \frac{\pi}{3}$
  - $x = \frac{\pi}{3}$
- The sine function is positive in the first and second quadrants. To find the second solution, subtract the reference angle from π to find the solution in the second quadrant.
  - $x = \pi - \frac{\pi}{3}$
- Simplify $\pi - \frac{\pi}{3}$
  - $x = \frac{2\pi}{3}$
- Find the period of $sin(x)$
  - 2π
- The period of the $sin(x)$ function is 2π so values will repeat every 2π radians in both directions.
  - $x = \frac{\pi}{3} + 2\pi n$ (for any Integer n)
  - $x = \frac{2\pi}{3} + 2\pi n$ (for any Integer n)

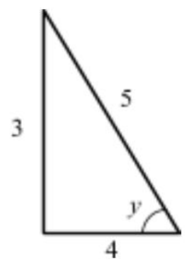
b. If $sin(x) = \frac{1}{3}$ and $sec(y) = \frac{5}{4}$, where $0 \le x \le \frac{\pi}{2}$ and $0 \le y \le \frac{\pi}{2}$, evaluate the expression $sin(x - y)$.

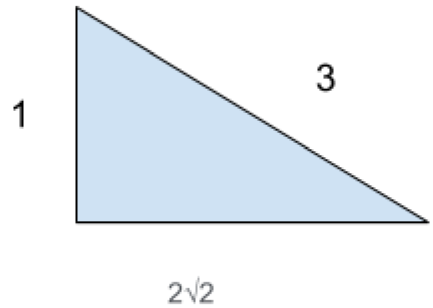
Step 1.

- $2\ sin(x - y)\ +\ 2\ sinycosx\ =\ 2\ sinxcosy$
- Now, divide by 2 which yield:
- $sin(x - y)\ +\ sinycosx\ =\ sinxcosy$
- Finally, solve for sin(x-y) and we get the desired identity.
- $sin(x - y)\ =\ sinxcosy\ -\ sinycosx$

Step 2.

- $sec(y) = \frac{5}{4}$
- $\frac{1}{cos(y)} = \frac{5}{4}$
- $5cos(y) \ = \ 4$
- $cos(y) \ = \ \frac{4}{5}$





Step 3.

- $sin(x-y) \ = \ sinxcosy \ - \ sinycosx$
- $sin(x-y) = \frac{1}{3} * \frac{4}{5} - \frac{3}{5} * cos(x)$
- $cos(x) = \frac{2\sqrt{2}}{3}$
- $sin(x-y) = \frac{1}{3} * \frac{4}{5} - \frac{3}{5} * \frac{2\sqrt{2}}{3}$
- $sin(x-y) = \frac{4}{15} - \frac{2\sqrt{2}}{5}$
- $sin(x-y) = \frac{4-6\sqrt{2}}{15}$

Answer: $sin(x-y) = \frac{4-6\sqrt{2}}{15}$

Question 2.

Solve for all values of x in the given intervals:

a) $2cos(x) + sin(2x) = 0$ for $0 \leq x \leq 2\pi$

- $2cos(x) + 2sin(x)cos(x) = 0$
- $2(cos(x) + sin(x)cos(x)) = 0$
- $2cos(x)(1 + sin(x)) = 0$

- Case 1.
  - $2cos(x) = 0$
  - $cos(x) = 0$
  - $x = \frac{\pi}{2}, \frac{3\pi}{2}$
- Case 2.
  - $1 + sin(x) = 0$
  - $sin(x) =- 1$

- $x = \frac{3\pi}{2}$

- Answer: $x = \frac{\pi}{2}, \frac{3\pi}{2}$

b) $2sin^2(x) = 1\, for\, x \in R$

- $sin^2(x) = \frac{1}{2}$
- $sin(x) = -\sqrt{\frac{1}{2}}, \sqrt{\frac{1}{2}}$

- Case 1. $sin(x) = -\sqrt{\frac{1}{2}}$
- $sin(x) = -\frac{\sqrt{2}}{2}$
- $arcsin(sin(x)) = arcsin(-\frac{\sqrt{2}}{2})$
- $x = -\frac{\pi}{4}$
- The sine function to be negative in the 3 and 4 quadrants.
- Subtract the reference angle from 2π to find a reference angle.
  - $x = 2\pi - (-\frac{\pi}{4})$
- Add the aforementioned reference angle to π to find the solution in the third quadrant.
  - $x = 2\pi + \frac{\pi}{4} + \pi$
- Simplify the expression.
  - $x = \frac{5\pi}{4}$
- Since the period of $sin(x)$ is 2π, we can write as the follows:
  - $x = \frac{5\pi}{4} + 2\pi n$
- Add 2π to every negative angle to get positive angles.
  - $-\frac{\pi}{4} + 2\pi$
  - $x = \frac{7\pi}{4}$
- The answer is
  - $x = \frac{5\pi}{4} + 2\pi n$ (for any Integer n)
  - $x = \frac{7\pi}{4} + 2\pi n$ (for any Integer n)

- Case 2. $sin(x) = \sqrt{\frac{1}{2}}$
- $sin(x) = \frac{\sqrt{2}}{2}$
- $arcsin(sin(x)) = arcsin(\frac{\sqrt{2}}{2})$
- $x = \frac{\pi}{4}$
- The sine function to be positive in the 1 and 2 quadrants.
- Subtract the reference angle from π in the second quadrant.
  - $x = \pi - \frac{\pi}{4}$
- Simplify $\pi - \frac{\pi}{4}$
  - $x = \frac{3\pi}{4}$
- Since the period of $sin(x)$ is 2π, which means that the values will repeat every 2π radians in both directions, thus we can write the function as the follows:

- $x = \frac{\pi}{4} + 2\pi n$ (for any Integer n)
- $x = \frac{3\pi}{4} + 2\pi n$ (for any Integer n)

- The answer is the following by consolidating the previous answers:
    - $x = \frac{\pi}{4} + \pi n$ (for any Integer n)
    - $x = \frac{3\pi}{4} + \pi n$ (for any Integer n)

c) $tan^2(x) - 3 = 0\, for\, x \in R$

- $tan^2(x) = 3$
- $tan(x) = \sqrt{3}, -\sqrt{3}$

- Case 1. $tan(x) = \sqrt{3}$
- $arctan(tan(x)) = arctan(\sqrt{3})$
- $arctan(tan(x)) = \frac{\pi}{3}$
- $x = \frac{\pi}{3}$
- Since the period of $tan(x)$ is π, the answer is the as follows:
    - $x = \frac{\pi}{3} + \pi n$

- Case 2. $tan(x) = -\sqrt{3}$
- $arctan(tan(x)) = arctan(-\sqrt{3})$
- $arctan(tan(x)) = -\frac{\pi}{3}$
- $x = -\frac{\pi}{3}$
- The tangent function to be negative in the 1 and 4 quadrants.
- In order to find second solution, subtract the reference angle from π to get a solution in the third quadrant.
    - $x = \pi - \frac{\pi}{3}$
    - $x = \frac{2\pi}{3}$
- The period of the $tan(x)$ function is π which means that values would repeat every π radians in both sides.
    - $x = \frac{2\pi}{3} + \pi n$ (for any Integer n)

- The answer is the following:
    - $x = \frac{\pi}{3} + \pi n$ (for any Integer n)
    - $x = \frac{2\pi}{3} + \pi n$ (for any Integer n)

Question 3.

Prove the following identities: (If it is a one step problem please state the formula used)

a) $sin(\frac{\pi}{2}+x) = cos(x)$

- We usually begin to work on the side of equality that seems to be more complicated. Thus, choose to work from the left side.
- Using the following formula to solve the equation.
  - sin(α±β)=sin(α)cos(β)±cos(α)sin(β)
  - angle a equals $\frac{\pi}{2}$ while angle b equals x
- $sin(\frac{\pi}{2})cos(x) + cos(\frac{\pi}{2})sin(x) = cos(x)$
- $1cos(x) + cos(\frac{\pi}{2})sin(x) = cos(x)$
- $1cos(x) + 0sin(x) = cos(x)$
- $cos(x) = cos(x)$, prove done.

b) $sin(x)cot(x) = cos(x)$

- Applying the following formula:
  - $cot(x) = \frac{cos(x)}{sin(x)}$
- $sin(x)\frac{cos(x)}{sin(x)} = cos(x)$
- $\frac{sin(x)cos(x)}{sin(x)} = cos(x)$
- Cancel the common factor of the left side.
- $cos(x) = cos(x)$, prove done.

c) $cot^2(x) + sec^2(x) = tan^2(x) + csc^2(x)$

- Manipulate the left side using the following identity:
  - $cot^2(x) =- 1 + csc^2(x)$
- $-1 + csc^2(x) + sec^2(x) = tan^2(x) + csc^2(x)$
- Manipulate the left side using the following identity:
  - $-1 + sec^2(x) = tan^2(x)$
- $tan^2(x) + csc^2(x) = tan^2(x) + csc^2(x)$, proof done.

d) $sin^2(x) - sin^2(y) = sin(x+y)sin(x-y)$

- We will choose to work on the right side to reach the left side.
- Using the sine of a sum formula:
  - **sin(α±β)=sin(α)cos(β)±cos(α)sin(β)**
  - We can write as, $sin(x)^2 - sin(y)^2 = (sin(x)cos(y) + cos(x)sin(y))sin(x-y)$
  - Thus, $sin(x)^2 - sin(y)^2 = (sin(x)cos(y) + cos(x)sin(y))(sin(x)cos(y) - cos(x)sin(y))$
- $sin(x)^2 - sin(y)^2 = (sin(x)cos(y))^2 - (cos(x)sin(y))^2$
- Using the following formula:
  - $cos^2(y) = 1 - sin(y)^2$
  - We can write as, $sin(x)^2 - sin(y)^2 = (sin(x)^2(1 - sin(y)^2)) - (cos(x)sin(y))^2$
  - Thus, $sin(x)^2 - sin(y)^2 = sin(x)^2 - sin(x)^2sin(y)^2 - (cos(x)sin(y))^2$
- Factoring by $sin(y)^2$

- $sin(x)^2 - sin(y)^2 = sin(y)^2(-cos(x)^2 - sin(x)^2) + sin(x)^2$

- Factoring by -1
  - $-sin(y)^2(cos(x)^2 + sin(x)^2)$
- Using the following formula:
  - $sin^2(x) + cos^2(x) = 1$
  - We can write as, $-sin(y)^2$
  - Thus, $sin(x)^2 - sin(y)^2 = sin(x)^2 - sin(y)^2$, prove done.

Question 4.
Describe how to use both an equivalent trigonometric identity and a diagram to demonstrate that two trigonometric ratios are equivalent.

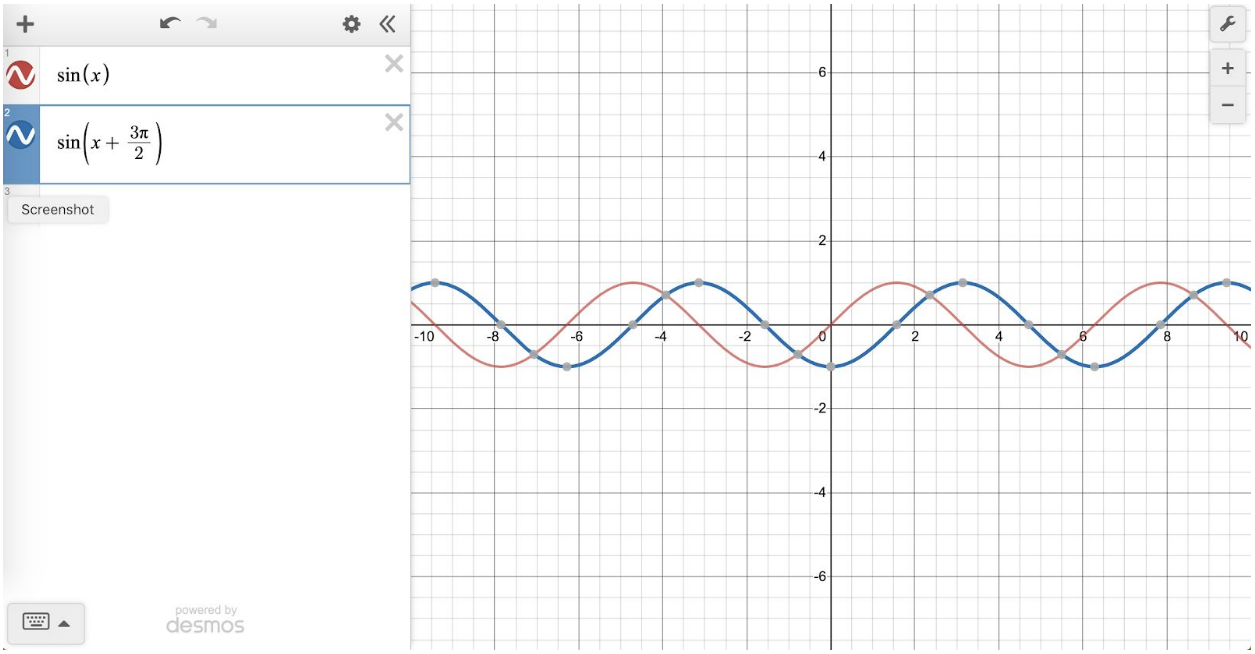
1. Use one of the following equivalent trigonometric expressions:

- $sin(\theta + \frac{3\pi}{2}) = -cos(\theta)$
- we will choose to work on the left side to reach the right side.
- Use the following formula:
  - $\mathbf{sin(\alpha \pm \beta) = sin(\alpha)cos(\beta) \pm cos(\alpha)sin(\beta)}$
  - where angle a equals θ, angle β equals $\frac{3\pi}{2}$
  - $sin(\theta)cos(\frac{3\pi}{2}) + cos(\theta)sin(\frac{3\pi}{2}) =- cos(\theta)$
- Since $sin(\frac{3\pi}{2}) = -1$
  - $sin(\theta)cos(\frac{3\pi}{2}) - cos(\theta) =- cos(\theta)$
- Since $cos(\frac{3\pi}{2}) = 0$
  - $0 * cos(\frac{3\pi}{2}) - cos(\theta) =- cos(\theta)$
- $-cos(\theta) =- cos(\theta)$, Proof done.

2. Using a diagram demonstrates how the related angle formulas are true. Create an example to illustrate your findings in part a) (choose a value for θ and solve both sides to prove that they are equal.)

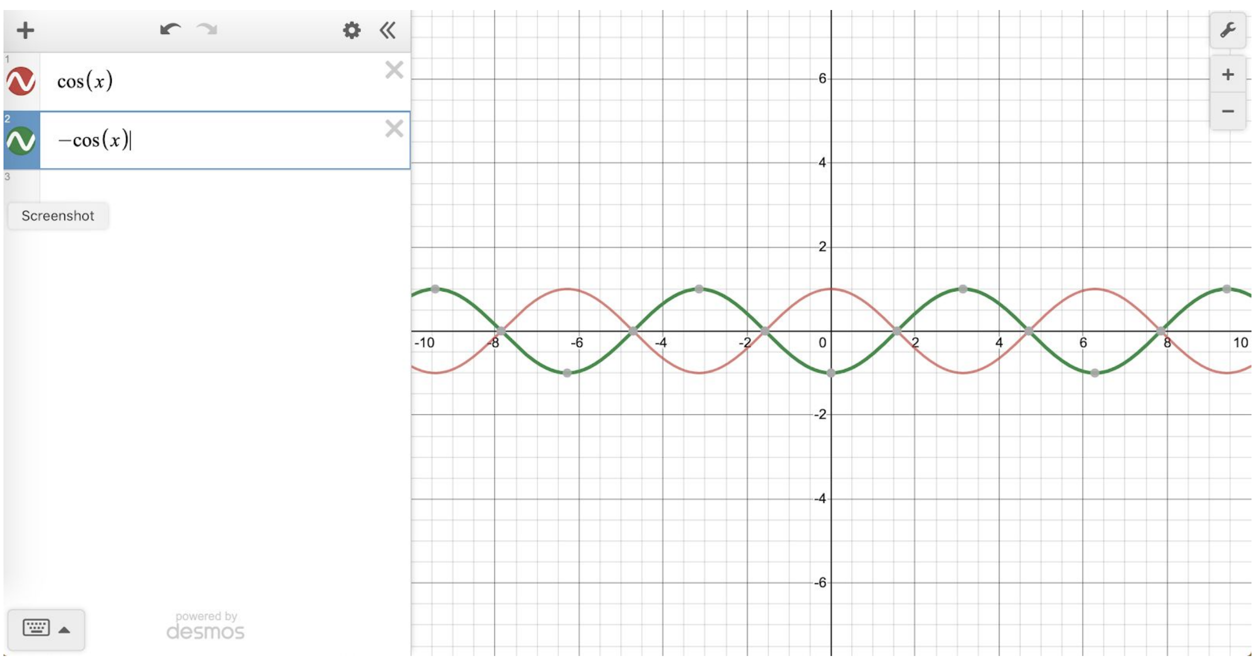
The graph of $sin(\theta)$ and $sin(\theta + \frac{3\pi}{2})$

- The graph is phase-shifted to the left by $\frac{3\pi}{2}$



The graph of $cos(x)$ and $-cos(x)$

- The graph is reflected by x-axis.



We can acknowledge that two graphs are overlapped by each other, which means that the two graphs are identical.

ex) The θ which is set as $\frac{7\pi}{12}$ to prove that two expressions are identical.

- $sin(\theta + \frac{3\pi}{2}) =- cos(\theta)$

- $sin(\frac{7\pi}{12} + \frac{3\pi}{2}) =- cos(\frac{7\pi}{12})$
- $sin(\frac{7\pi+18\pi}{12}) =- cos(\frac{7\pi}{12})$
- $sin(\frac{25\pi}{12}) =- cos(\frac{7\pi}{12})$
- $\frac{\sqrt{6}-\sqrt{2}}{4} = \frac{\sqrt{2-\sqrt{3}}}{2}$
- The left side 0.25881904 is equal to the right side 0.25881904 , which means that the given statement is always true. True