Underlying Concepts of Calculus Unit Assignment
MCV4U

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1. **Determine the following limits, if they exist!**

a) $\lim_{x \to 3}(3x^4 + 2x^2 - 5)$

- Since x goes close to 3, we can deem that x is almost 3. Thus substitute x with 3.

  $= (3 * 3^4 + 2 * 3^2 - 5) = 3^5 + 18 - 5 = 243 + 13 = 256$

- Thus, the limit exists while the answer is 256.

b) $\lim_{x \to 5}(\frac{4x}{x-5})$

- Since x goes close to 5, we can deem that x is almost 5. Thus substitute x with 5.

  $= (\frac{20}{0})$

- However, we can find that the denominator goes zero, which means that it is undefined.
- Thus, the limit does not exist.

c) $\lim_{x \to 4}(\frac{x^2 + x - 20}{8 - 2x})$

- To begin with, we need to subtract common factors between denominator and numerator.
- Thus, we need to factorize both of them.

  $= \lim_{x \to 4}(\frac{(x-4)(x+5)}{2(4-x)})$

  $= \lim_{x \to 4}(\frac{-(x+5)}{2})$

- Since x goes close to 4, we can deem that x is almost 4. Thus substitute x with 4.

  $= (\frac{-9}{2})$

- Thus, the limit exists while the answer is $-\frac{9}{2}$.

2. Evaluate the following limits (if they exist).

a) $\lim_{x \to 1} \frac{x^2 - 1}{x - 1}$

- To begin with, we need to subtract common factors between denominator and numerator.
- Thus, we need to factorize both of them.

  $\lim_{x \to 1} \frac{(x+1)(x-1)}{x-1}$

  $\lim_{x \to 1}(x+1)$

- Since x goes close to 1, we can deem that x is almost 1. Thus substitute x with 1.

  $= (2)$

b) $\lim_{x \to 0} \frac{\sqrt{x+16}-4}{x}$

- To begin with, we can appreciate that the value would become $\frac{0}{0}$, which is indeterminate. Thus, we need to avoid this by finding common factors to cancel between denominator and numerator.
- In order to do this, we need to multiply $\sqrt{x+16}+4$ to each denominator and numerator while keeping the same value as given above.
- Thus, the formula would be the following.

$$= \lim_{x \to 0} \frac{(x+16)-16}{x(\sqrt{x+16}+4)}$$

$$= \lim_{x \to 0} \frac{x}{x(\sqrt{x+16}+4)}$$

$$= \lim_{x \to 0} \frac{1}{(\sqrt{x+16}+4)}$$

$$= \frac{1}{8}$$

- Thus, the limit exists while the value is $\frac{1}{8}$.

c) $\lim_{x \to 64} \frac{\sqrt[3]{x}-4}{x-64}$

- We can use the following formula to solve the equation.

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$lim_{x->64} \frac{(x)^{\frac{1}{3}}-4}{(x^{\frac{1}{3}})^3-4^3}$$

$$= lim_{x->64} \frac{(x^{\frac{1}{3}}-4)}{(x^{\frac{1}{3}}-4)(x^{\frac{2}{3}}+16+4x^{\frac{1}{3}})}$$

$$= lim_{x->64} \frac{1}{x^{\frac{2}{3}}+16+4x^{\frac{1}{3}}}$$

$$= \frac{1}{(64)^{\frac{2}{3}}+16+4(64)^{\frac{1}{3}}}$$

$$= 48$$

- Thus, the limit exists while the value is 48.

3. From first principles (i.e. using the tangent slope method), find the slope of the following curves at the given value of x.

a) $f(x) = 2x^2 - 6x \; at \; x = 3$

$$f'(x) = lim_{h->0} \frac{f(x+h)-f(x)}{h} = lim_{h->0} \frac{2(x+h)^2-6(x+h)-(2x^2-6x)}{h} = lim_{h->0} \frac{4hx+2h^2-6h}{h}$$

$$= lim_{h->0}(4x + 2h - 6) = 4x - 6$$

$$f'(x)|_{x=-4} = 4(-4) - 6 =- 16 - 6 =- 22$$

Slope = -22

b) $f(x) = 3x^3 + 1\ at\ x = -4$

$f'(x) = lim_{h->0}\frac{f(x+h)-f(x)}{h} = lim_{h->0}\frac{3(x+h)^3+1-(3x^3+1)}{h} = lim_{h->0}\frac{9hx^2+9h^2x+3h^3}{h}$

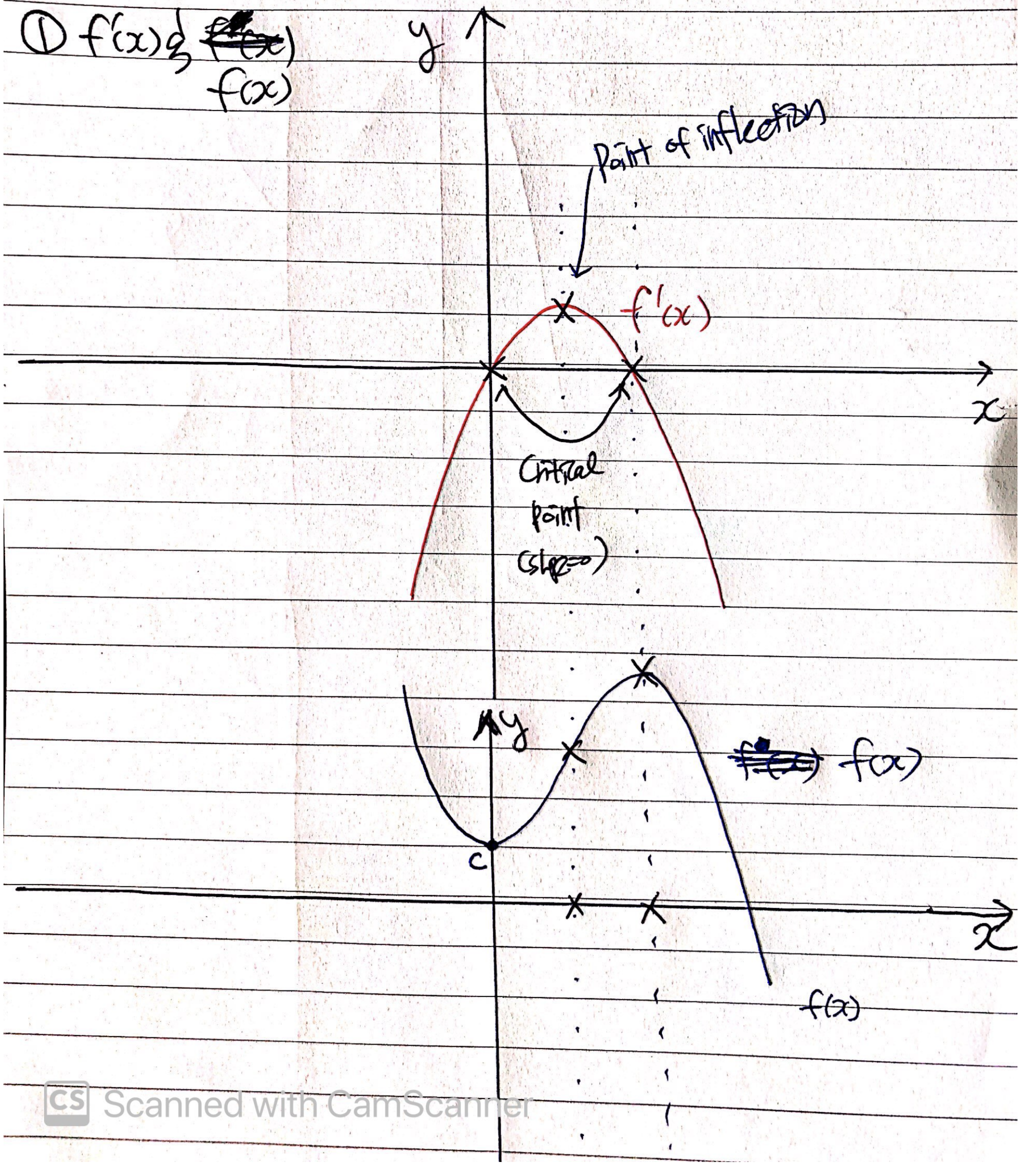
$= lim_{h->0}(9x^2 + 9hx + 3h^2) = 9x^2$

$f'(x)|_{x=-4} = 9(-4)^2 = 9 \times 16 = 144$

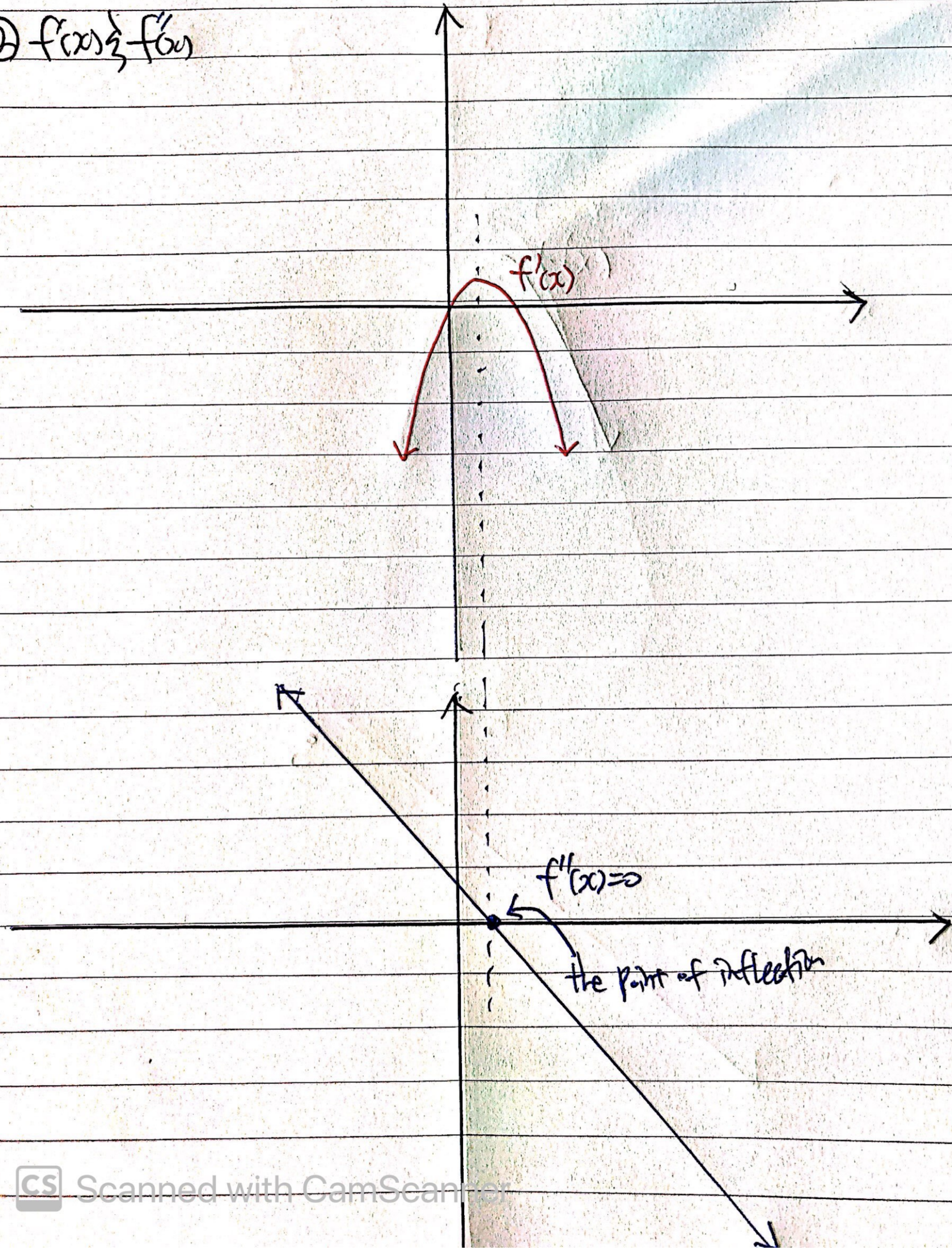
Slope = 144

4. Look at the graph below of the derivative f(x). From this, make a sketch of the original function f(x) and of the second derivative f''(x), and explain your reasoning.

① $f'(x)$ & ~~f(x)~~ $f(x)$



② $f'(x)$ & $f''(x)$



- Function $f'(x)$is a quadratic function which means that $f(x)$ is a cubic function and also means that $f''(x)$ is a linear function
- The vertices of the initial equation shall correspond to the x-intercepts of the first derivative. Since the value is changing from positive to negative, the left x-intercept of the first derivative must be the local limit of the initial function f(x).
- Since the first derivative's right intercept changes from negative to positive, it must be the local minimum.
- The x-intercepts of the second derivative would refer to the vertex of the first derivative. Since the vertex is the smallest, the second derivative's x-intercept must be changing from negative to positive. As a result, the second derivative function should be a positive-sloped linear function.