

# **Unit Assignment: Trigonometry**

MHF4U

Virtual High School

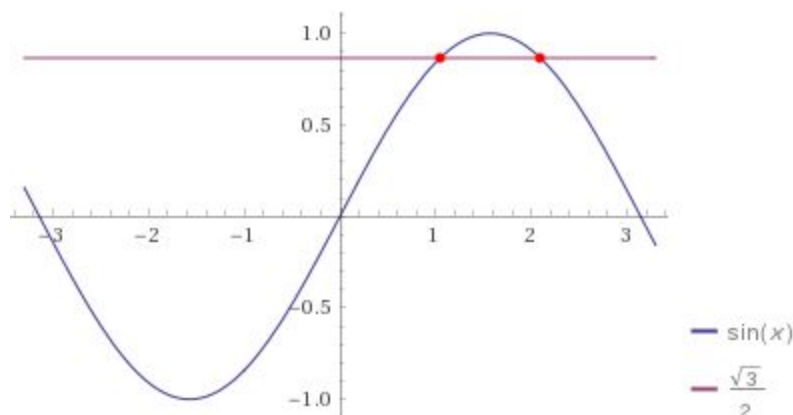
Jin Hyung Park

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### Question 1.

- a. Find all solutions for  $\sin(x) = \frac{\sqrt{3}}{2}$

#### Graph



Radians:  $x = \frac{\pi}{3} + 2\pi n$ ,  $x = \frac{2\pi}{3} + 2\pi n$

- Take the inverse side of both sides of the equation to extract  $x$  from inside the sine.
  - $x = \arcsin\left(\frac{\sqrt{3}}{2}\right)$
- The exact value of  $\arcsin\left(\frac{\sqrt{3}}{2}\right)$  is  $\frac{\pi}{3}$ 
  - $x = \frac{\pi}{3}$
- The sine function is positive in the first and second quadrants. To find the second solution, subtract the reference angle from  $\pi$  to find the solution in the second quadrant.
  - $x = \pi - \frac{\pi}{3}$
- Simplify  $\pi - \frac{\pi}{3}$ 
  - $x = \frac{2\pi}{3}$
- Find the period of  $\sin(x)$ 
  - $2\pi$
- The period of the  $\sin(x)$  function is  $2\pi$  so values will repeat every  $2\pi$  radians in both directions.
  - $x = \frac{\pi}{3} + 2\pi n$  (for any Integer  $n$ )
  - $x = \frac{2\pi}{3} + 2\pi n$  (for any Integer  $n$ )

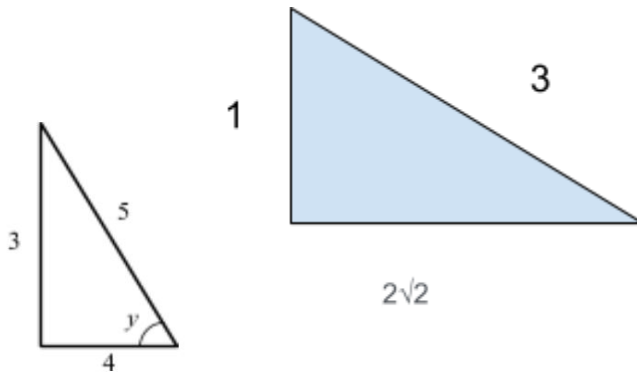
- b. If  $\sin(x) = \frac{1}{3}$  and  $\sec(y) = \frac{5}{4}$ , where  $0 \leq x \leq \frac{\pi}{2}$  and  $0 \leq y \leq \frac{\pi}{2}$ , evaluate the expression  $\sin(x - y)$ .

#### Step 1.

- $2 \sin(x - y) + 2 \sin y \cos x = 2 \sin x \cos y$
- Now, divide by 2 which yield:
- $\sin(x - y) + \sin y \cos x = \sin x \cos y$
- Finally, solve for  $\sin(x - y)$  and we get the desired identity.
- $\sin(x - y) = \sin x \cos y - \sin y \cos x$

### Step 2.

- $\sec(y) = \frac{5}{4}$
- $\frac{1}{\cos(y)} = \frac{5}{4}$
- $5\cos(y) = 4$
- $\cos(y) = \frac{4}{5}$



### Step 3.

- $\sin(x-y) = \sin x \cos y - \sin y \cos x$
- $\sin(x-y) = \frac{1}{3} * \frac{4}{5} - \frac{3}{5} * \cos(x)$
- $\cos(x) = \frac{2\sqrt{2}}{3}$
- $\sin(x-y) = \frac{1}{3} * \frac{4}{5} - \frac{3}{5} * \frac{2\sqrt{2}}{3}$
- $\sin(x-y) = \frac{4}{15} - \frac{2\sqrt{2}}{5}$
- $\sin(x-y) = \frac{4-6\sqrt{2}}{15}$

Answer:  $\sin(x-y) = \frac{4-6\sqrt{2}}{15}$

### Question 2.

Solve for all values of x in the given intervals:

a)  $2\cos(x) + \sin(2x) = 0$  for  $0 \leq x \leq 2\pi$

- $2\cos(x) + 2\sin(x)\cos(x) = 0$
- $2(\cos(x) + \sin(x)\cos(x)) = 0$
- $2\cos(x)(1 + \sin(x)) = 0$

- Case 1.

- $2\cos(x) = 0$
- $\cos(x) = 0$
- $x = \frac{\pi}{2}, \frac{3\pi}{2}$

- Case 2.

- $1 + \sin(x) = 0$
- $\sin(x) = -1$

- $x = \frac{3\pi}{2}$

- Answer:  $x = \frac{\pi}{2}, \frac{3\pi}{2}$

b)  $2\sin^2(x) = 1$  for  $x \in \mathbb{R}$

- $\sin^2(x) = \frac{1}{2}$

- $\sin(x) = -\sqrt{\frac{1}{2}}, \sqrt{\frac{1}{2}}$

- Case 1.  $\sin(x) = -\sqrt{\frac{1}{2}}$

- $\sin(x) = -\frac{\sqrt{2}}{2}$

- $\arcsin(\sin(x)) = \arcsin(-\frac{\sqrt{2}}{2})$

- $x = -\frac{\pi}{4}$

- The sine function to be negative in the 3 and 4 quadrants.

- Subtract the reference angle from  $2\pi$  to find a reference angle.

- $x = 2\pi - (-\frac{\pi}{4})$

- Add the aforementioned reference angle to  $\pi$  to find the solution in the third quadrant.

- $x = 2\pi + \frac{\pi}{4} + \pi$

- Simplify the expression.

- $x = \frac{5\pi}{4}$

- Since the period of  $\sin(x)$  is  $2\pi$ , we can write as the follows:

- $x = \frac{5\pi}{4} + 2\pi n$

- Add  $2\pi$  to every negative angle to get positive angles.

- $-\frac{\pi}{4} + 2\pi$

- $x = \frac{7\pi}{4}$

- The answer is

- $x = \frac{5\pi}{4} + 2\pi n$  (for any Integer  $n$ )

- $x = \frac{7\pi}{4} + 2\pi n$  (for any Integer  $n$ )

- Case 2.  $\sin(x) = \sqrt{\frac{1}{2}}$

- $\sin(x) = \frac{\sqrt{2}}{2}$

- $\arcsin(\sin(x)) = \arcsin(\frac{\sqrt{2}}{2})$

- $x = \frac{\pi}{4}$

- The sine function to be positive in the 1 and 2 quadrants.

- Subtract the reference angle from  $\pi$  in the second quadrant.

- $x = \pi - \frac{\pi}{4}$

- Simplify  $\pi - \frac{\pi}{4}$

- $x = \frac{3\pi}{4}$

- Since the period of  $\sin(x)$  is  $2\pi$ , which means that the values will repeat every  $2\pi$  radians in both directions, thus we can write the function as the follows:

- $x = \frac{\pi}{4} + 2\pi n$  (for any Integer  $n$ )
- $x = \frac{3\pi}{4} + 2\pi n$  (for any Integer  $n$ )

• The answer is the following by consolidating the previous answers:

- $x = \frac{\pi}{4} + \pi n$  (for any Integer  $n$ )
- $x = \frac{3\pi}{4} + \pi n$  (for any Integer  $n$ )

c)  $\tan^2(x) - 3 = 0$  for  $x \in R$

- $\tan^2(x) = 3$
- $\tan(x) = \sqrt{3}, -\sqrt{3}$

• Case 1.  $\tan(x) = \sqrt{3}$

•  $\arctan(\tan(x)) = \arctan(\sqrt{3})$

•  $\arctan(\tan(x)) = \frac{\pi}{3}$

•  $x = \frac{\pi}{3}$

• Since the period of  $\tan(x)$  is  $\pi$ , the answer is the as follows:

- $x = \frac{\pi}{3} + \pi n$

• Case 2.  $\tan(x) = -\sqrt{3}$

•  $\arctan(\tan(x)) = \arctan(-\sqrt{3})$

•  $\arctan(\tan(x)) = -\frac{\pi}{3}$

•  $x = -\frac{\pi}{3}$

• The tangent function to be negative in the 1 and 4 quadrants.

• In order to find second solution, subtract the reference angle from  $\pi$  to get a solution in the third quadrant.

- $x = \pi - \frac{\pi}{3}$
- $x = \frac{2\pi}{3}$

• The period of the  $\tan(x)$  function is  $\pi$  which means that values would repeat every  $\pi$  radians in both sides.

- $x = \frac{2\pi}{3} + \pi n$  (for any Integer  $n$ )

• The answer is the following:

- $x = \frac{\pi}{3} + \pi n$  (for any Integer  $n$ )
- $x = \frac{2\pi}{3} + \pi n$  (for any Integer  $n$ )

### Question 3.

Prove the following identities: (If it is a one step problem please state the formula used)

a)  $\sin(\frac{\pi}{2} + x) = \cos(x)$

- We usually begin to work on the side of equality that seems to be more complicated. Thus, choose to work from the left side.
- Using the following formula to solve the equation.
  - $\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$
  - angle  $a$  equals  $\frac{\pi}{2}$  while angle  $b$  equals  $x$
- $\sin(\frac{\pi}{2})\cos(x) + \cos(\frac{\pi}{2})\sin(x) = \cos(x)$
- $1\cos(x) + 0\sin(x) = \cos(x)$
- $1\cos(x) + 0\sin(x) = \cos(x)$
- $\cos(x) = \cos(x)$ , prove done.

b)  $\sin(x)\cot(x) = \cos(x)$

- Applying the following formula:
  - $\cot(x) = \frac{\cos(x)}{\sin(x)}$
- $\sin(x)\frac{\cos(x)}{\sin(x)} = \cos(x)$
- $\frac{\sin(x)\cos(x)}{\sin(x)} = \cos(x)$
- Cancel the common factor of the left side.
- $\cos(x) = \cos(x)$ , prove done.

c)  $\cot^2(x) + \sec^2(x) = \tan^2(x) + \csc^2(x)$

- Manipulate the left side using the following identity:
  - $\cot^2(x) = -1 + \csc^2(x)$
- $-1 + \csc^2(x) + \sec^2(x) = \tan^2(x) + \csc^2(x)$
- Manipulate the left side using the following identity:
  - $-1 + \sec^2(x) = \tan^2(x)$
- $\tan^2(x) + \csc^2(x) = \tan^2(x) + \csc^2(x)$ , proof done.

d)  $\sin^2(x) - \sin^2(y) = \sin(x+y)\sin(x-y)$

- We will choose to work on the right side to reach the left side.
- Using the sine of a sum formula:
  - **$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$**
  - We can write as,  $\sin(x)^2 - \sin(y)^2 = (\sin(x)\cos(y) + \cos(x)\sin(y))\sin(x-y)$
  - Thus,  $\sin(x)^2 - \sin(y)^2 = (\sin(x)\cos(y) + \cos(x)\sin(y))(\sin(x)\cos(y) - \cos(x)\sin(y))$
- $\sin(x)^2 - \sin(y)^2 = (\sin(x)\cos(y))^2 - (\cos(x)\sin(y))^2$
- Using the following formula:
  - $\cos^2(y) = 1 - \sin(y)^2$
  - We can write as,  $\sin(x)^2 - \sin(y)^2 = (\sin(x)^2(1 - \sin(y)^2)) - (\cos(x)\sin(y))^2$
  - Thus,  $\sin(x)^2 - \sin(y)^2 = \sin(x)^2 - \sin(x)^2\sin(y)^2 - (\cos(x)\sin(y))^2$
- Factoring by  $\sin(y)^2$

- $\sin(x)^2 - \sin(y)^2 = \sin(y)^2(-\cos(x)^2 - \sin(x)^2) + \sin(x)^2$
- Factoring by  $-1$ 
  - $-\sin(y)^2(\cos(x)^2 + \sin(x)^2)$
- Using the following formula:
  - $\sin^2(x) + \cos^2(x) = 1$
  - We can write as,  $-\sin(y)^2$
  - Thus,  $\sin(x)^2 - \sin(y)^2 = \sin(x)^2 - \sin(y)^2$ , prove done.

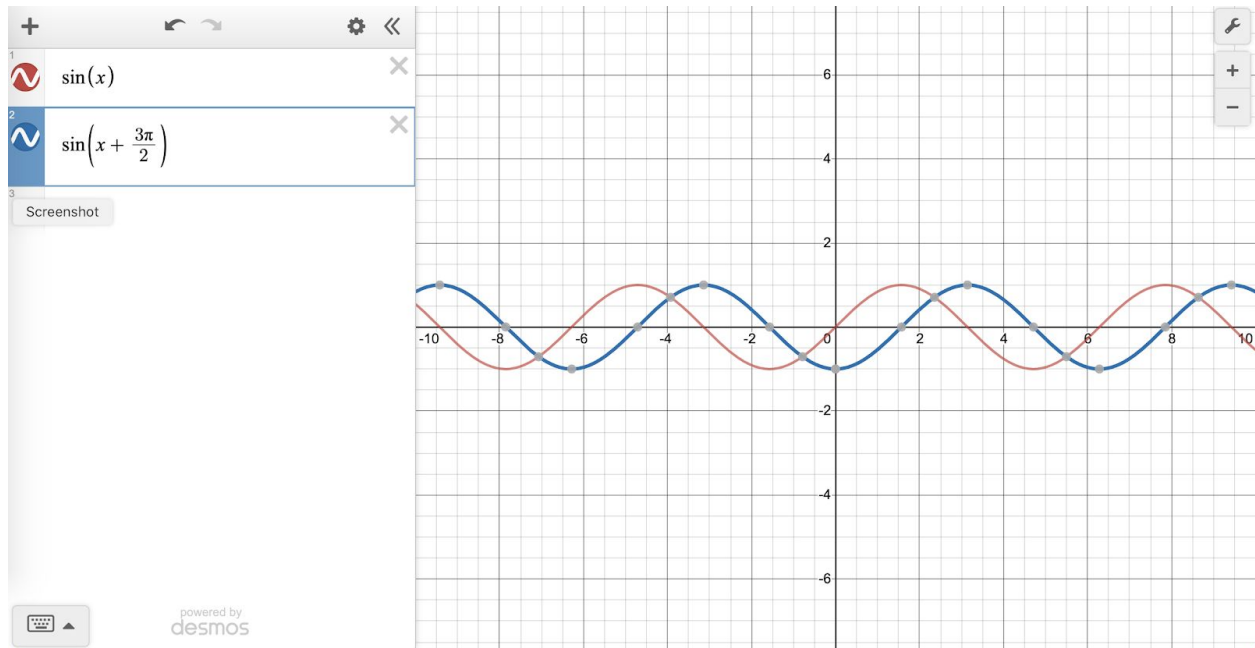
#### Question 4.

Describe how to use both an equivalent trigonometric identity and a diagram to demonstrate that two trigonometric ratios are equivalent.

1. Use one of the following equivalent trigonometric expressions:
  - $\sin(\theta + \frac{3\pi}{2}) = -\cos(\theta)$
  - we will choose to work on the left side to reach the right side.
  - Use the following formula:
    - $\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$
    - where angle  $\alpha$  equals  $\theta$ , angle  $\beta$  equals  $\frac{3\pi}{2}$
    - $\sin(\theta)\cos(\frac{3\pi}{2}) + \cos(\theta)\sin(\frac{3\pi}{2}) = -\cos(\theta)$
  - Since  $\sin(\frac{3\pi}{2}) = -1$ 
    - $\sin(\theta)\cos(\frac{3\pi}{2}) - \cos(\theta) = -\cos(\theta)$
  - Since  $\cos(\frac{3\pi}{2}) = 0$ 
    - $0 * \cos(\frac{3\pi}{2}) - \cos(\theta) = -\cos(\theta)$
  - $-\cos(\theta) = -\cos(\theta)$ , Proof done.
2. Using a diagram demonstrates how the related angle formulas are true. Create an example to illustrate your findings in part a) (choose a value for  $\theta$  and solve both sides to prove that they are equal.)

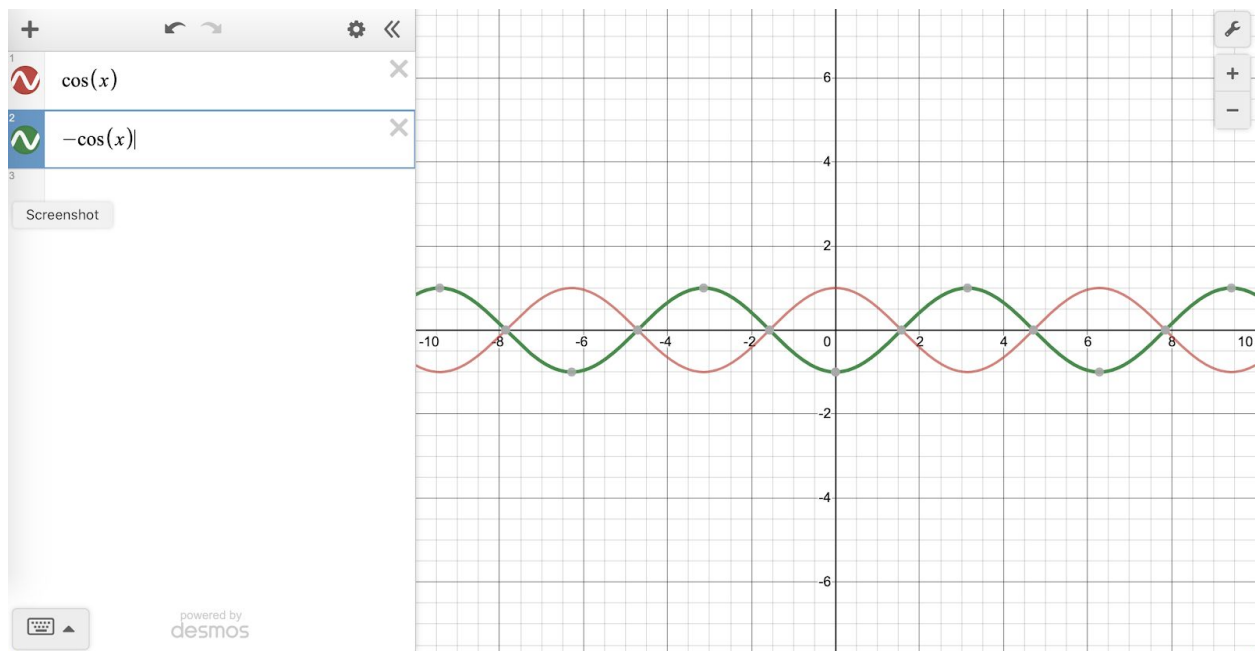
The graph of  $\sin(\theta)$  and  $\sin(\theta + \frac{3\pi}{2})$

- The graph is phase-shifted to the left by  $\frac{3\pi}{2}$



The graph of  $\cos(x)$  and  $-\cos(x)$

- The graph is reflected by x-axis.



We can acknowledge that two graphs are overlapped by each other, which means that the two graphs are identical.

ex) The  $\theta$  which is set as  $\frac{7\pi}{12}$  to prove that two expressions are identical.

- $\sin\left(\theta + \frac{3\pi}{2}\right) = -\cos(\theta)$



- $\sin(\frac{7\pi}{12} + \frac{3\pi}{2}) = -\cos(\frac{7\pi}{12})$

- $\sin(\frac{7\pi+18\pi}{12}) = -\cos(\frac{7\pi}{12})$

- $\sin(\frac{25\pi}{12}) = -\cos(\frac{7\pi}{12})$

- $\frac{\sqrt{6}-\sqrt{2}}{4} = \frac{\sqrt{2-\sqrt{3}}}{2}$

- The left side 0.25881904 is equal to the right side 0.25881904 , which means that the given statement is always true. True