

Unit Assignment: Exponential and Logarithmic Functions

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Question 1.

a) $\frac{3^2}{3^{-4}}$

• $\frac{(3^2)(3^4)}{1}$

• answer: 3^6

b) $\frac{2^5 \cdot 2^{-2}}{2^2} = \frac{2^5}{(2^2)(2^2)} = \frac{2^5}{2^4} = 2$

• answer: 2

c) $\frac{(x^2y)^4}{xy^3} = \frac{x^8y^4}{xy^3} = x^7y$

• answer: x^7y

d) $\frac{(a^5b^{-1}c)(a^{-2}bc^2)}{a^3b^2c^{-3}} = \frac{(a^5)(a^{-2})(b^{-1})(b)(c)(c^2)}{a^3b^2c^{-3}} = \frac{a^3b^0c^3}{a^3b^2c^{-3}}$

• answer: $\frac{c^6}{b^2}$

Question 2.

$f(x) = -(4)^x + 2$

• domain: $(-\infty, \infty)$

• range: $(-\infty, 2)$

• y-intercept: (0, 1)

- Originally, the y-intercept of the graph exists at (0, 1). However, after reflecting function on the x-axis and did a vertical translation up by 2 units, the y-intercept changes from (0, 1) to (0, -1) to (0, 1) respectively.

- Algebraically,

$$y = -(4)^0 + 2$$

$$y = -1 + 2$$

$$y = 1$$

Therefore, the y-intercept is (0, 1)

• Horizontal Asymptote: $y = 2$

- Originally, the parent function has a horizontal asymptote at $y = 0$. However, as the graph has a vertical transition up to 2 unit, the horizontal asymptote exists at $y = 2$.

Question 3.

When John was born, his parents invested \$7000 into a savings account that pays 3% interest, compounded monthly. How much will the investment be worth on John's 18th birthday?

- Let us set variable as the following:

- $P = \$7000$

- $r = 3\% = 0.03$ (assuming yearly nominal rate)

- $t = 18$ years

- $n = 12$ (monthly)

- We can use the following compounded interest formula:

- $A(t) = P(1 + \frac{r}{n})^{nt}$ which means to multiply monthly interests over current balance.

- $A(t) = 7000 * (1 + \frac{0.03}{12})^{12*18}$
 - $A(t) = 7000(1 + 0.0025)^{12*18}$
 - $A(t) = 7000 \times 1.0025^{12*18}$
 - $A(t) = 7000 \times 1.714851$
 - $A(t) = 12003.956118$, to round up is 12004

• Answer is \$12004

Question 4.

1) $f(x) = 10^x$

- let $f(x) = y$
- $y = 10^x$
- $\log_{10} y = x$

2) $f(x) = (\frac{1}{7})^x$

- let $f(x) = y$
- $y = (\frac{1}{7})^x$
- $\log_{\frac{1}{7}} y = \log_{\frac{1}{7}} (\frac{1}{7})^x$
- $\log_{\frac{1}{7}} y = x$

3) $f(x) = \log_5 x$

- let $f(x) = y$
- $\log_5 x = y \Leftrightarrow 5^y = x$
- $x = 5^y$

4) $f(x) = \log_{\frac{3}{5}} x$

- let $f(x) = y$
- $\log_{\frac{3}{5}} x = y \Leftrightarrow (\frac{3}{5})^y = x$
- $x = (\frac{3}{5})^y$

Question 5.

1) $\log_3 x - \log_3 8 = \log_3 4 + \log_3 1$

- $\log_3 (\frac{x}{8}) = \log_3 (4 * 1)$
- $\log_3 (\frac{x}{8}) = \log_3 4$
- $3^{\log_3 (\frac{x}{8})} = 3^{\log_3 4}$
- $(\frac{x}{8}) = 4$
- $x = 32$

2) $\log_3 (x + 2) - \log_3 (x + 28) = -3$

- $\log_3 (\frac{x+2}{x+28}) = -3$

- $3^{\log_3(\frac{x+2}{x+28})} = 3^{-3}$
- $\frac{x+2}{x+28} = \frac{1}{3^3}$
- $\frac{x+2}{x+28} = \frac{1}{27}$
- $27(x+2) = (x+28)$
- $27x + 54 = x + 28$
- $26x = -26$
- $x = -1$

Question 6.

- 1) $2^x = 6$
 - $\log_2 6 = x$
 - $x = 2.5850$ (after round)
 - Check:
 - $2^{2.5820} = 6.0002$
 - 6.0002 is similar to 6
- 2) $3^{x+2} = 2$
 - $\log_3 2 = x + 2$
 - $\log_3 2 - 2 = x$
 - $x = -1.3691$ (after round)
 - Check:
 - $3^{-1.3681+2} = 2$
 - 1.9999 is similar to 2
- 3) $7^{2x} = 52$
 - $\log_7 52 = 2x$
 - $\frac{\log_7 52}{2} = x$
 - $x = 1.0153$
 - Check:
 - $7^{2(1.0153)} = 52$
 - $7^{2.0356} = 52$
 - 52.0063 is similar to 52
- 4) $4^{3x-1} = 90$
 - $\log_4 90 = 3x - 1$
 - $\frac{\log_4 90 + 1}{3} = x$
 - $x = 1.4143$
 - Check:
 - $4^{3(1.4143)-1} = 90$
 - $4^{3.2459} = 90$
 - 89.9967 is similar to 90

Question 7.

In a particular factory, the ambient noise has a loudness of 100 dB. If prolonged exposure to a loudness of 85 dB can cause permanent damage, how much more intense is the factory than the damage threshold?

$$15 - > 10^{1.5}$$

<https://lms.virtualhighschool.com/d2l/le/content/61528/viewContent/562536/View>