Unit Assignment: Trigonometric Functions and Graphs

MHF4U

Virtual High School

2021.01.19

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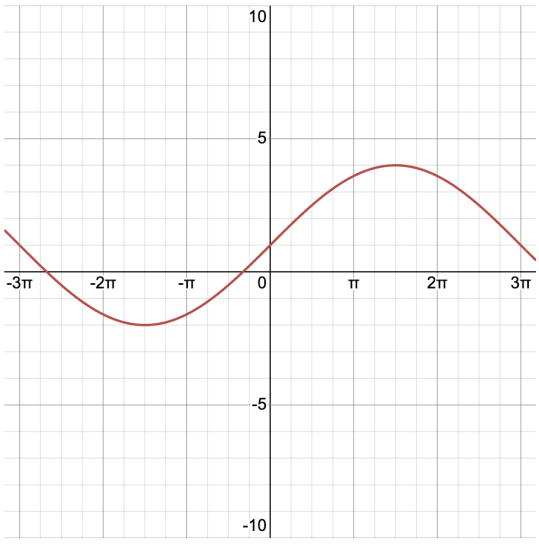
Question 1.

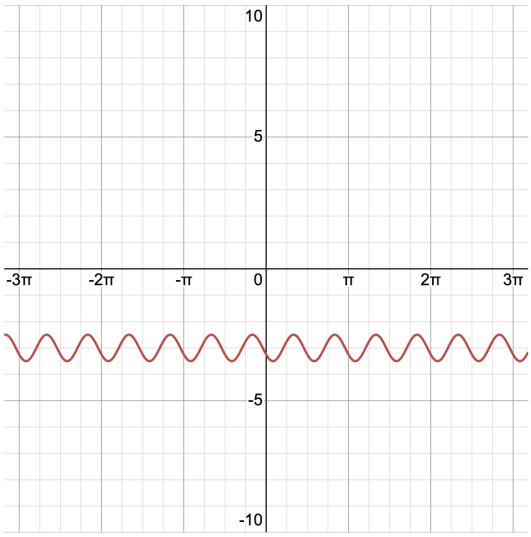
a) $f(x) = 3sin(\frac{1}{3}x) + 1$

- Use the form $asin(b(x - c)) + d$ to find the variables.
  - $a = 3, b = \frac{1}{3}, c = 0, d = 1$
- Domain: $(-\infty, \infty)$, $\{x|x \in R\}$
- Range: $[-2, 4]$, $\{y| -2 \leq y \leq 4\}$
  - Find the magnitude of the trig term of the function by taking the absolute value of the coefficient.
  - coefficient = 3
  - Find the lower bound of the range.
  - The lower bound of the range for sine is found by substituting the negative magnitude of the coefficient into the equation.
  - $y = -3 + 1$
  - $y = -2$
  - Find the upper bound of the range.
  - The upper bound of the range for sine is found by substituting the positive magnitude of the coefficient into the equation.
  - $y = 3 + 1$
  - $y = 4$
- Minimum and Maximum values:
  - local minimum: $(\frac{9\pi}{2} + 6\pi n, -2)$, when $n \ is \ any \ integer$
  - local maximum: $(\frac{3\pi}{2} + 6\pi n, 4)$, when $n \ is \ any \ integer$
- Period: $\frac{2\pi}{\frac{1}{3}} = 6\pi$
- Phase Shift: 0 to the right
- Amplitude: 3



b) $f(x) = -\frac{1}{2}cos(4x - \frac{\pi}{3}) - 3$

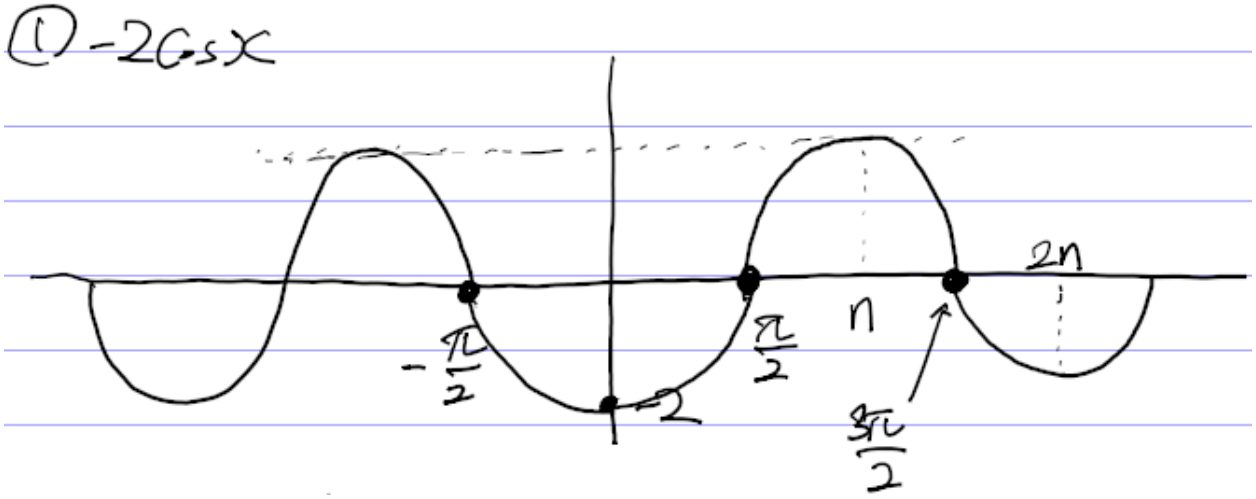
- Use the form $acos(b(x - c)) + d$ to find the variables.
  - $f(x) = -\frac{1}{2}cos(4(x - \frac{\pi}{12})) - 3$
  - $a = -\frac{1}{2}, b = 4, c = \frac{\pi}{12}, d = -3$
- Domain: $(-\infty, \infty)$, $\{x|x \in R\}$
- Range: $[-\frac{7}{2}, -\frac{5}{2}], \{y|\frac{-7}{2} \leq y \leq \frac{-5}{2}\}$
  - Find the magnitude of the trig term of the function by taking the absolute value of the coefficient
  - magnitude = 1
  - Find the lower bound of the range.
  - The lower bound of the range for cosine is found by substituting the negative magnitude of the coefficient into the equation.
  - $y = -1 - \frac{1}{2} - 3$
  - $y = -\frac{9}{2}$
  - Find the upper bound of the range.
  - The upper bound of the range for cosine is found by substituting the positive magnitude of the coefficient into the equation.
  - $y = 1 - \frac{1}{2} - 3$
  - $y = -\frac{5}{2}$
- Minimum and Maximum values:
  - local minimum: $(\frac{\pi}{12} + \frac{\pi}{2}n, -\frac{7}{2})$, when $n\ is\ any\ integer$
  - local maximum: $(\frac{\pi}{3} + \frac{\pi}{2}n, \frac{-5}{2})$, when $n\ is\ any\ integer$
- Period: $\frac{2\pi}{4} = \frac{\pi}{2}$
- Phase Shift: $\frac{\pi}{12}$ to the right
- Amplitude: $\frac{1}{2}$



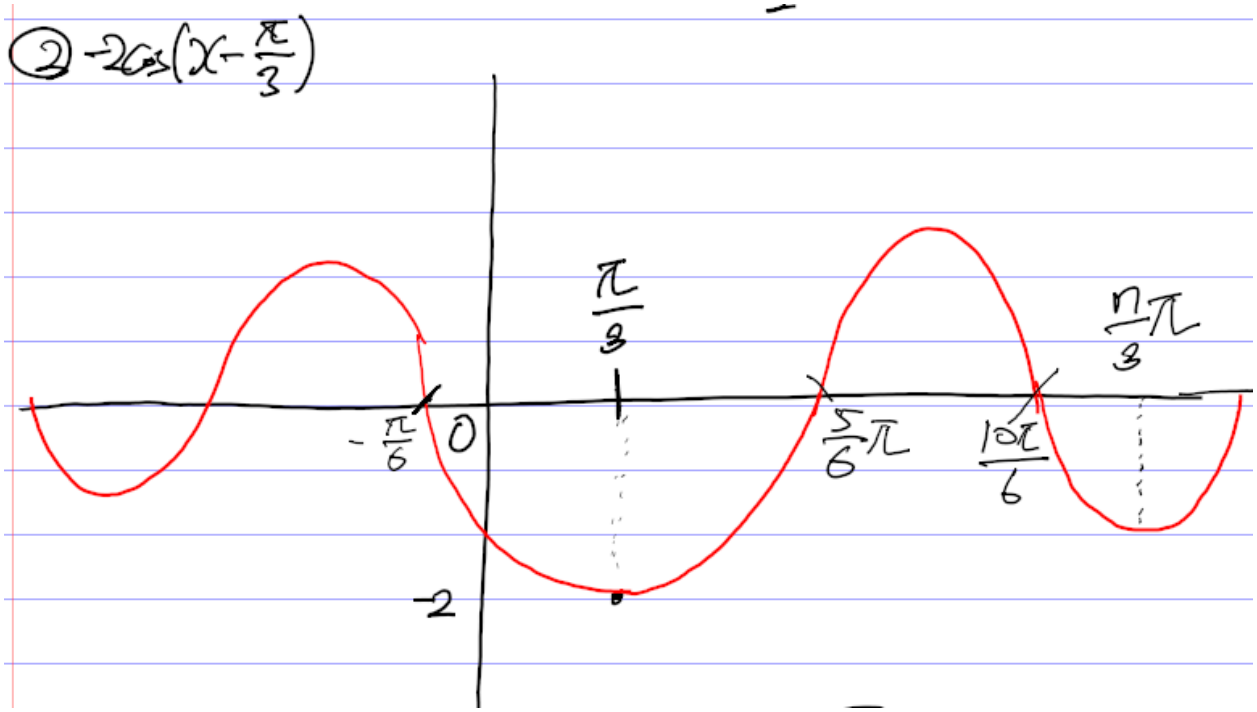
Question 2.

1) $f(x) = -2cos(3x - \frac{\pi}{3}) + \frac{1}{2}\ when\ 0 \leq x \leq 2\pi$

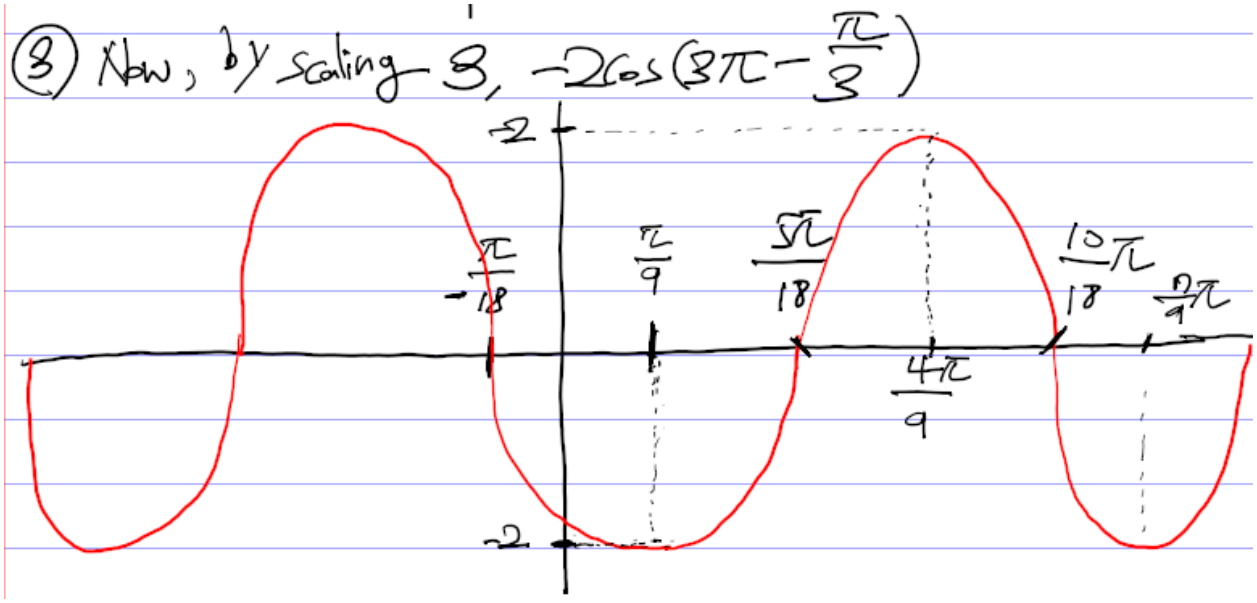
Now, we can first draw the simple $-2cosx$.



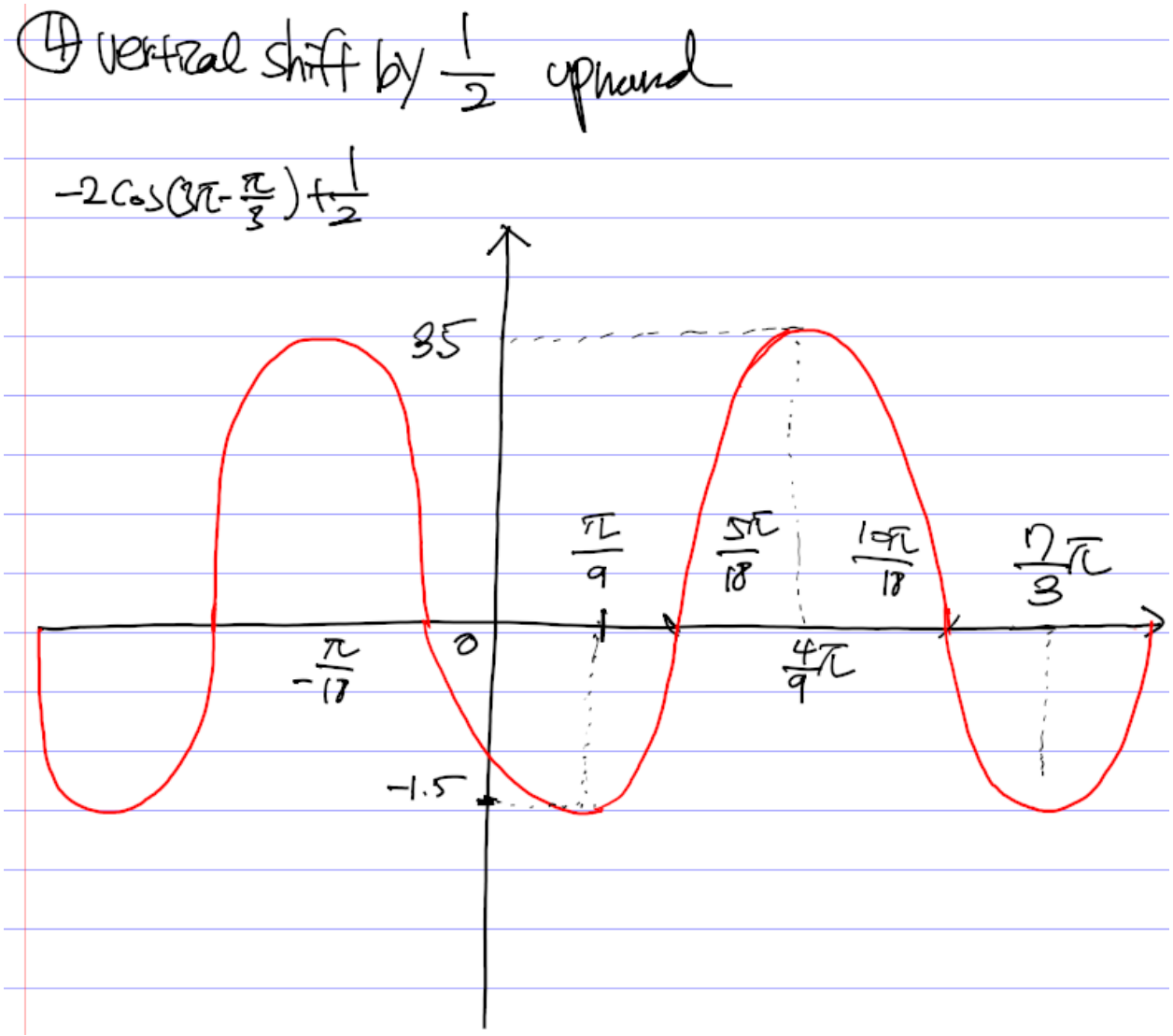
We can move the x-axis toward $\frac{\pi}{3}$.



Now, by scaling 3, we can achieve $-\ 2cos(3\pi\ -\frac{\pi}{3})$.

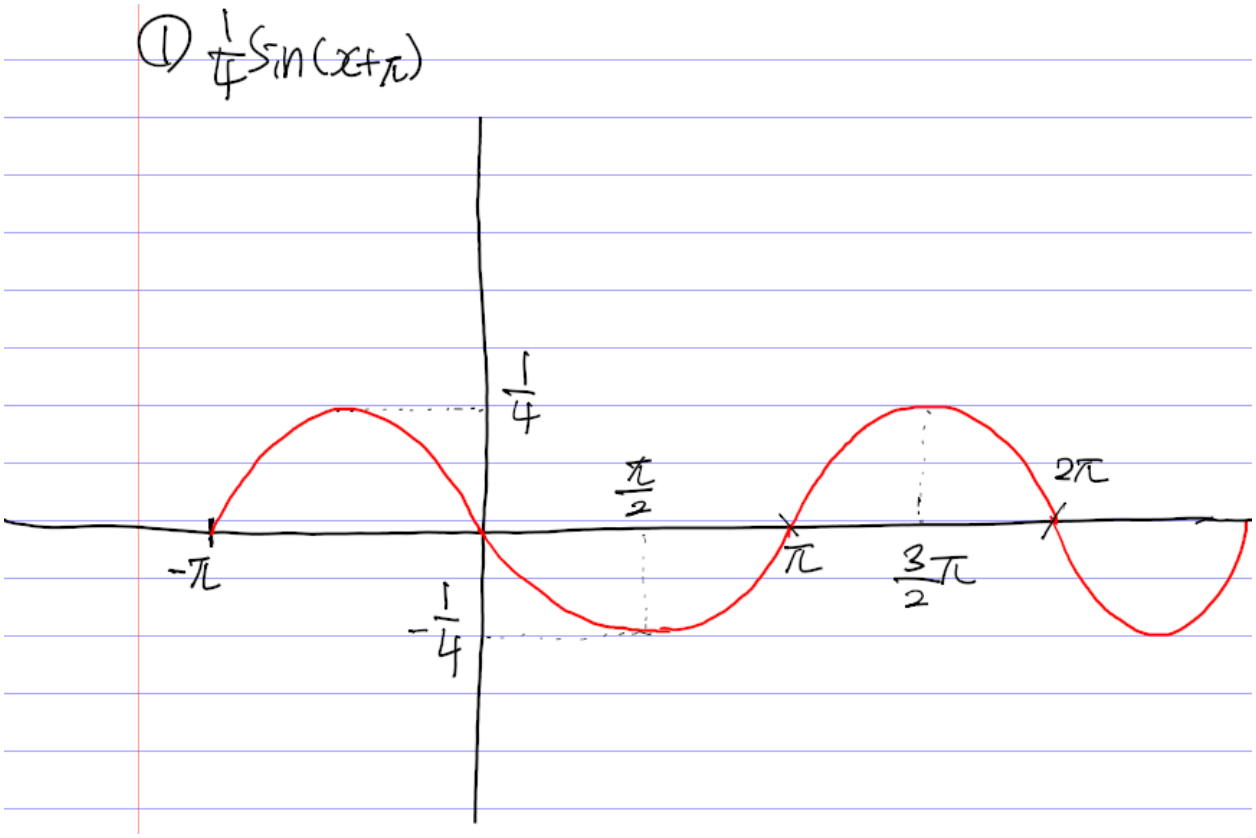


Finally, we can vertically shift by $\frac{1}{2}$upward.



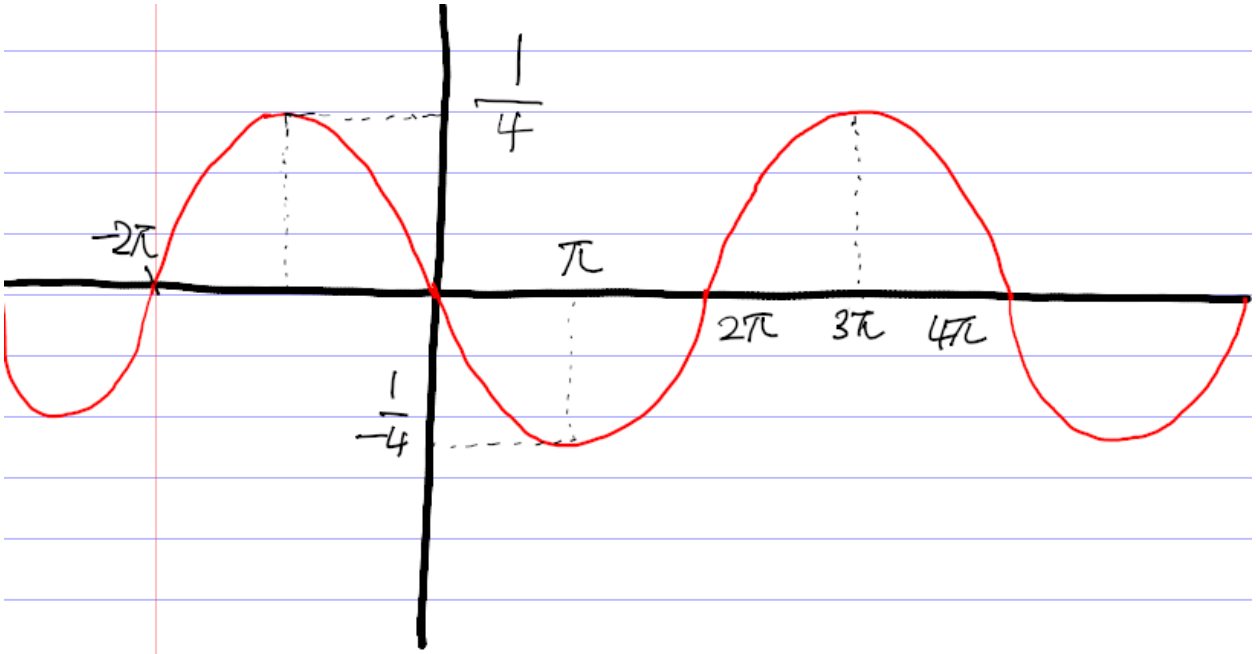
b) $f(x) = \frac{1}{4}sin(\frac{1}{2}x + \pi) - 3\ when\ 0 \leq x \leq 2\pi$

- To begin with, draw the basic function which is $\frac{1}{4}sin(x + \pi)$



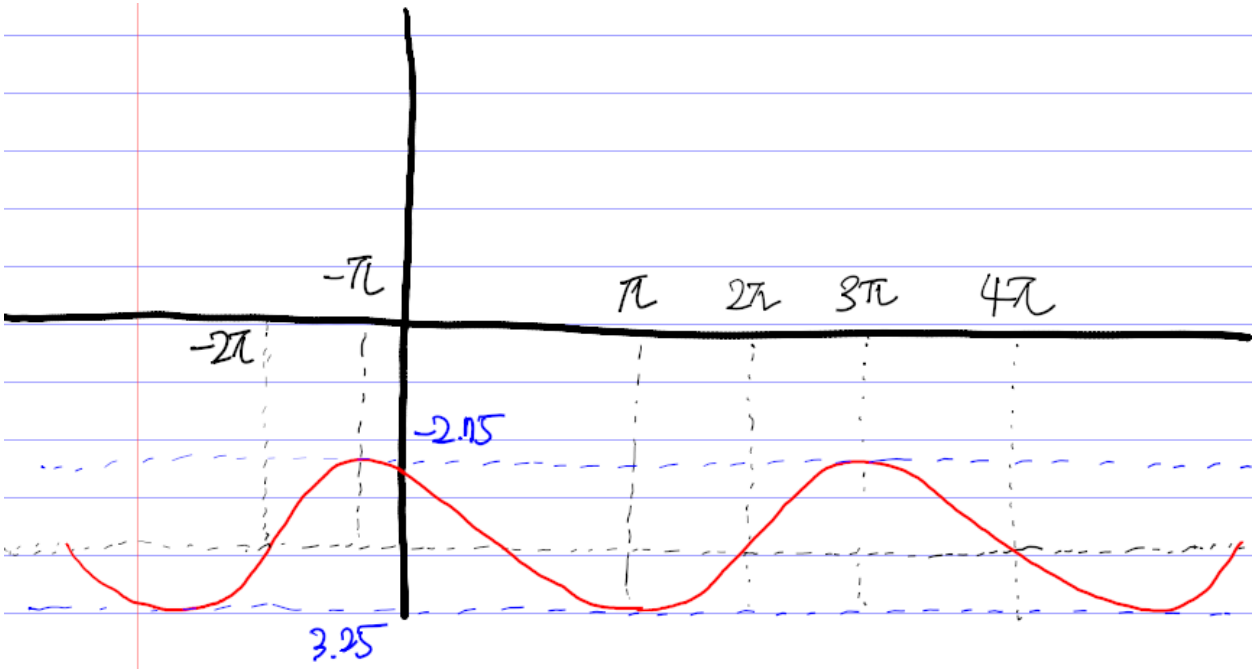
In addition, scale the graph of $\frac{1}{2}$.

The graph would be $f(x) = \frac{1}{4}sin(\frac{1}{2}x + \pi)$



Now, down vertical shift of 3 unit.

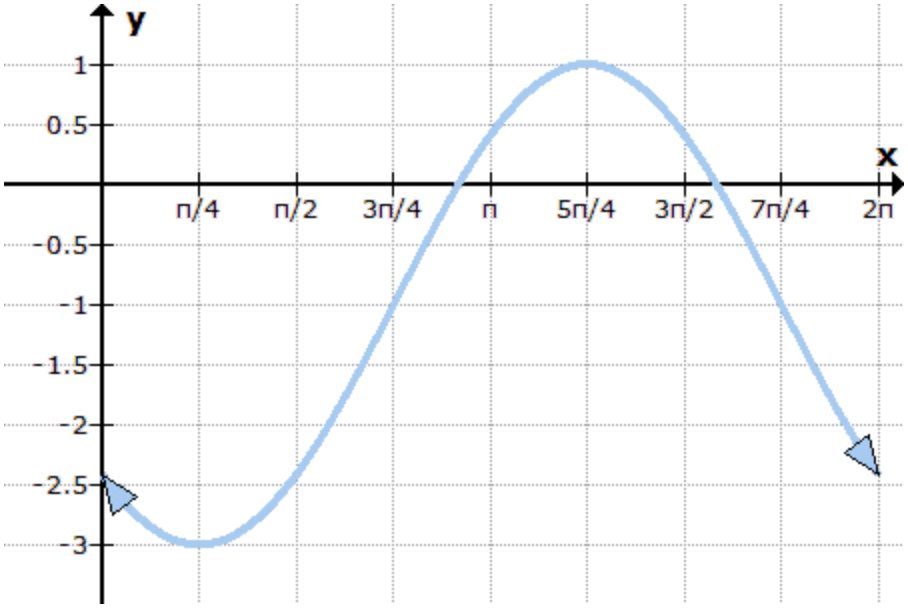
The graph would be $\frac{1}{4}sin(\frac{1}{2}x + \pi) - 3$.



Question 3.

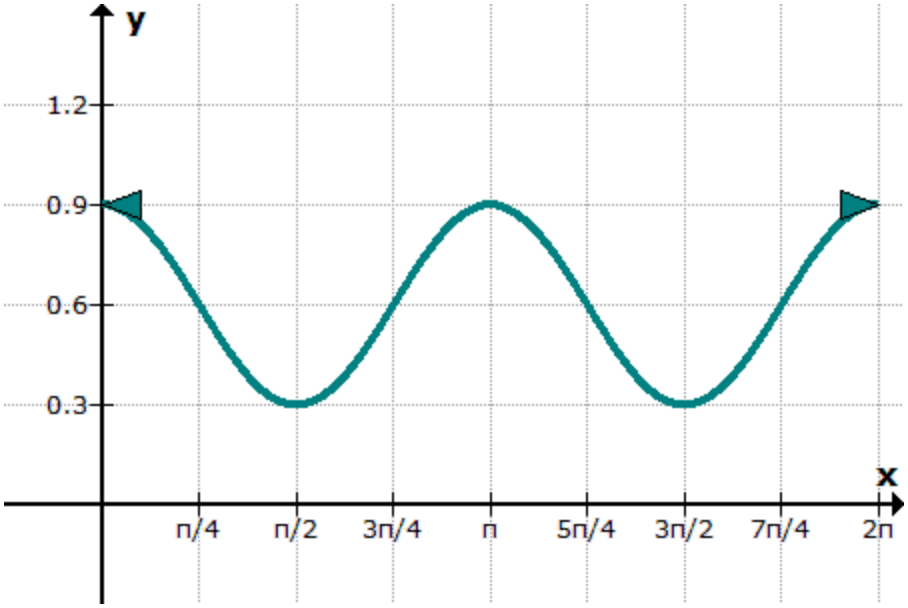
$f(x) = acos(k(x - d)) + c$ or $f(x) = asin(k(x - d)) + c$

1)



- Amplitude:
  - $\frac{maximum-minimum}{2} = \frac{1-(-3)}{2} = 2$
  - $a = 2$
- Axis of curve:
  - $y = -1$which means that the graph is vertically shifted down by 1
  - $c = -1$
- Period:
  - Half of period: $\frac{5\pi}{4} - \frac{4\pi}{4} = \pi$
  - Period: 2π
  - $k = 1$
- Phase shift:
  - The maximum which happens at first located at: $\frac{5\pi}{4}$
  - We could horizontally shift the graph right by $\frac{5\pi}{4}$
  - $d = \frac{5\pi}{4}$
- The graph is $f(x) = 2cos(x - \frac{5\pi}{4}) - 1$

2)



- Amplitude:
  - $\frac{minimum-maximum}{2} = \frac{0.9-0.3}{2} = 0.3$
  - $a = 0.3$
- Axis of curve:
  - $y = 0.6$which means that the graph is vertically shifted up by 0.6
  - $c = 0.6$
- Period:
  - $\pi - 0 = \pi$
  - Period = π,
  - $k = 2$
- Phase shift:
  - The maximum which happens first located at: 0
  - We could not horizontally shift the graph since the value is 0
  - $d = 0$
- The graph is $f(x) = 0.3cos(2x) + 0.6$

Question 4.

At its highest point, the second hand on a clock is 1.9 m above the ground. At its lowest point, the second hand is 1.6 m above the ground. The second hand starts at its highest point when the clock is started.

a) Let such function to be $h(t) = acos(k(t - d)) + c$

- Amplitude = $0.3/2 = 0.15\ m$
- $a = 0.15$
- Mid point of the function = $(1.9 + 1.6)/2 = 1.75$

- $c = 1.75$
- $period = 60s$
- $k = \frac{2\pi}{60} = \frac{\pi}{30}$
- The second hand starts at the maximum point
- $d = 0$
- $h(t) = 0.15[cos(\frac{\pi}{30}t)] + 1.75$

b)

- $h(10) = 1.75 + 0.15 * cos(\frac{\pi}{3}) = 1.825\, m$
- $h(35) = 1.75 + 0.15 * cos(\frac{35\pi}{30}) = 1.620\, m$