Basic Skills Review Assignment

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MHF4U

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Question 1. Rewrite the following relationships using function notation.

A. An airplane needs to travel 400 km. Determine a function for the speed of the airplane, with respect to time.

Let s represents speed in km/h. $s = speed\ in\ km/h$
Let t represents time in an hour. $t = time\ in\ hour$
Thus, $s(t) = 400/t$

B. An ice cream cone is left sitting in the hot sun. Sarah notices that the ice cream melts and loses half of its volume every 5 minutes. If the starting volume was 125 mL, determine a function for the volume, with respect to the number of hours it takes to travel.

Let v represents the volume in mL. $v = volume\ in\ mL$
Let t represents time in a minute. $t = time\ in\ a\ minute$
Thus, $v(t) = 125 * (1/2)^t/5$

C. Scott wants to calculate the distance from his house to each of his friends' houses. If he drives at 50 km/h, find a function for the distance, with respect to the number of hours it takes to travel.

Step 1. Let d represents the distance in km. $d = distance\ in\ km$
Step 2. Let t represents time in an hour. $t = time\ in\ an\ hour$
Step 3. Let n represents the number of friends to visit. $n = number\ of\ visiting\ home$
Thus, $d(t) = 50 * t * n$

Question 2. Find the inverses of each of the functions below algebraically.

(a) $p(r) = 2r^2 + 2r - 1$

Step 1. Inverse between p and r.

$r = 2p^2 + 2p - 1$

Step 2. To change to full square, p are grouped into common constants - 2.

$r = 2(p^2 + p) - 1$

Step 3. Changes a given expression to full square shape.

$r = 2[(p + 1/2)^2 - 1/4] - 1$

$r = 2(p + 1/2)^2 - 1/2 - 1$

$r = 2(p + 1/2)^2 - 3/2$

Step 4. Finish the calculation by moving variable p to the right while others to the left.

$r + 3/2 = 2(p + 1/2)^2$

$1/2(r + 3/2) = (p + 1/2)^2$

$r/2 + 3/4 = (p + 1/2)^2$

$(2r + 3)/4 = (p + 1/2)^2$

$\mp\sqrt{(2r + 3)/4} = (p + 1/2)$

$\mp\sqrt{(2r + 3)/4} - 1/2 = p$

Answer: $p^-1(r) = \mp\sqrt{(2r + 3)/4} - 1/2$

(b) $3y + 5x = 18$

Step 1. Move 5x to the right side for integrating the form.

$3y = -5x + 18$

Step 2. Inverse between x and y.

$3x = -5y + 18$

Step 3. Finish the calculation by arranging y to right side, another variable to left side.

$3x - 18 = -5y$

$(3x - 18)/5 = y$

$y = (-3x + 18)/5$

(c) $h(t) = -4.9(t + 3)^2 + 45.8$

Step 1. Inverse t and h.

$t = -4.9(h + 3)^2 + 45.8$

Step 2. Rearrange t and constant to the left side while h to the right side.

$t - 45.8 = -4.9(h + 3)^2$

Step 3. Finish the calculation.

$(t - 45.8)/ - 4.9 = (h + 3)^2$

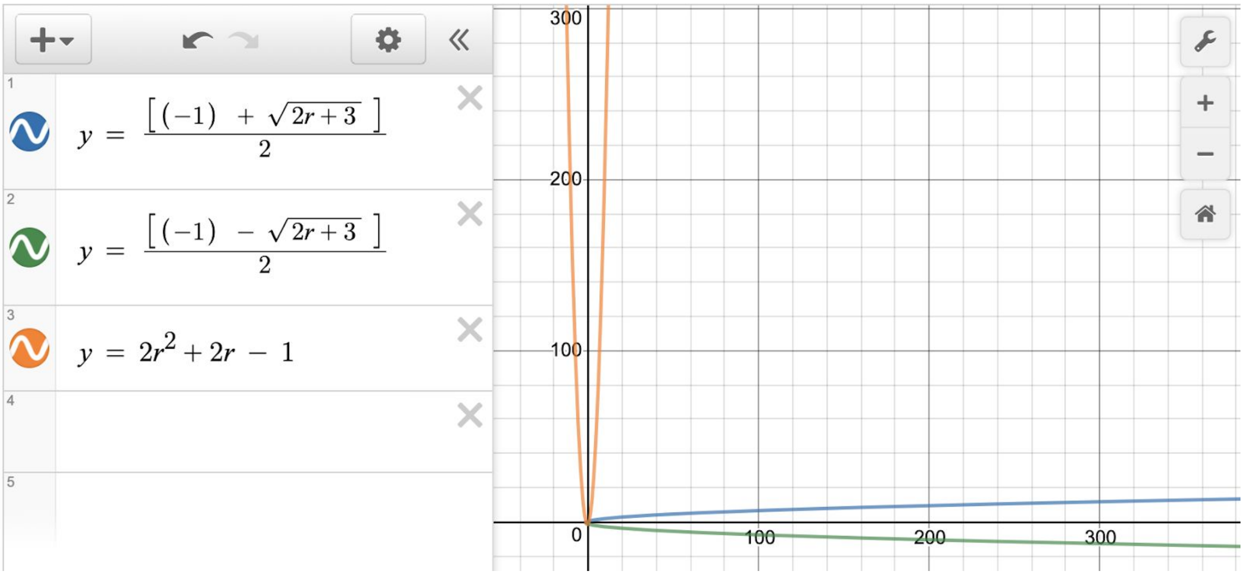
$\mp\sqrt{(t - 45.8)/ - 4.9} = h + 3$

$\mp\sqrt{(t - 45.8)/ - 4.9} - 3 = h$

$h^-1(t) = \mp\sqrt{(t - 45.8)/ - 4.9} - 3$

Question 3. With the aid of graphs, explain whether or not the inverses in question 2 are functions.

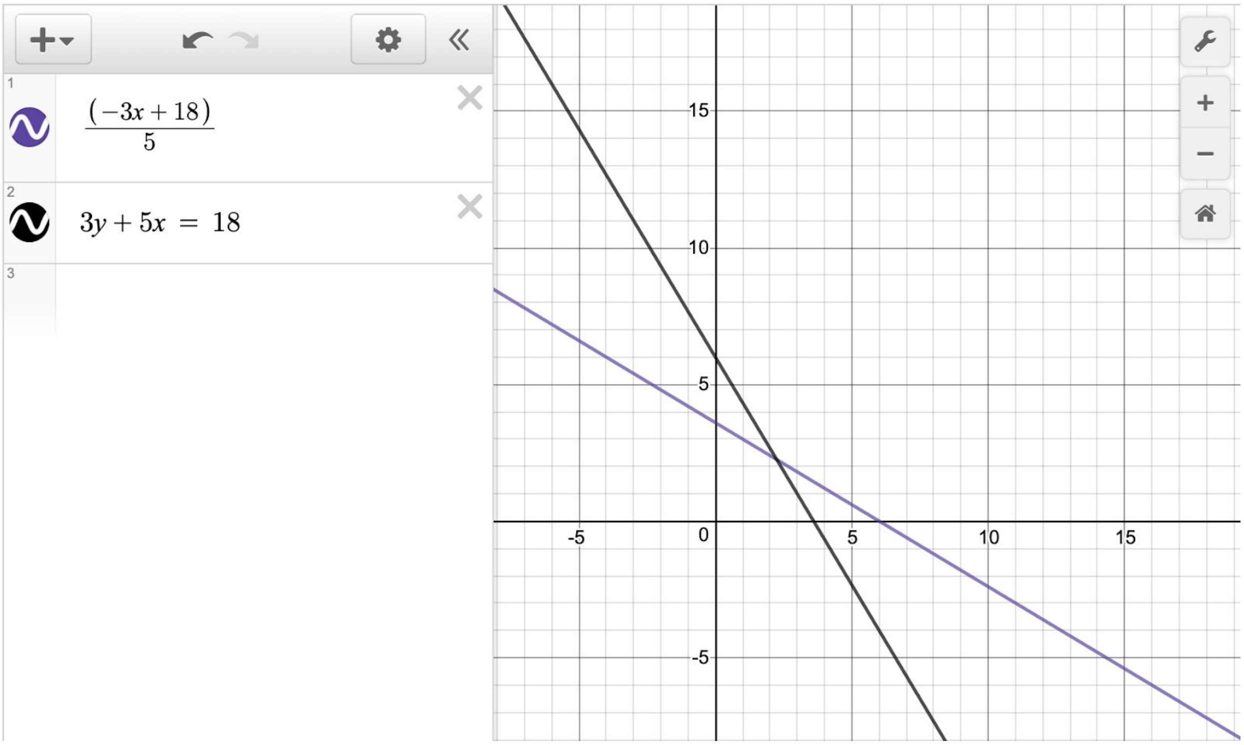
(a) $p^{-}1(r) = [(-1) \mp \sqrt{2r+3}]/2$ which is the inverse function of $p(r) = 2r^2 + 2r - 1$



The given inverse function is **NOT** a function since the graph returns two outputs per one input. This happens since the graph shows the inverse of the quadratic function. If the graph shows only one function that has a positive constant or negative constant, it would be a function.

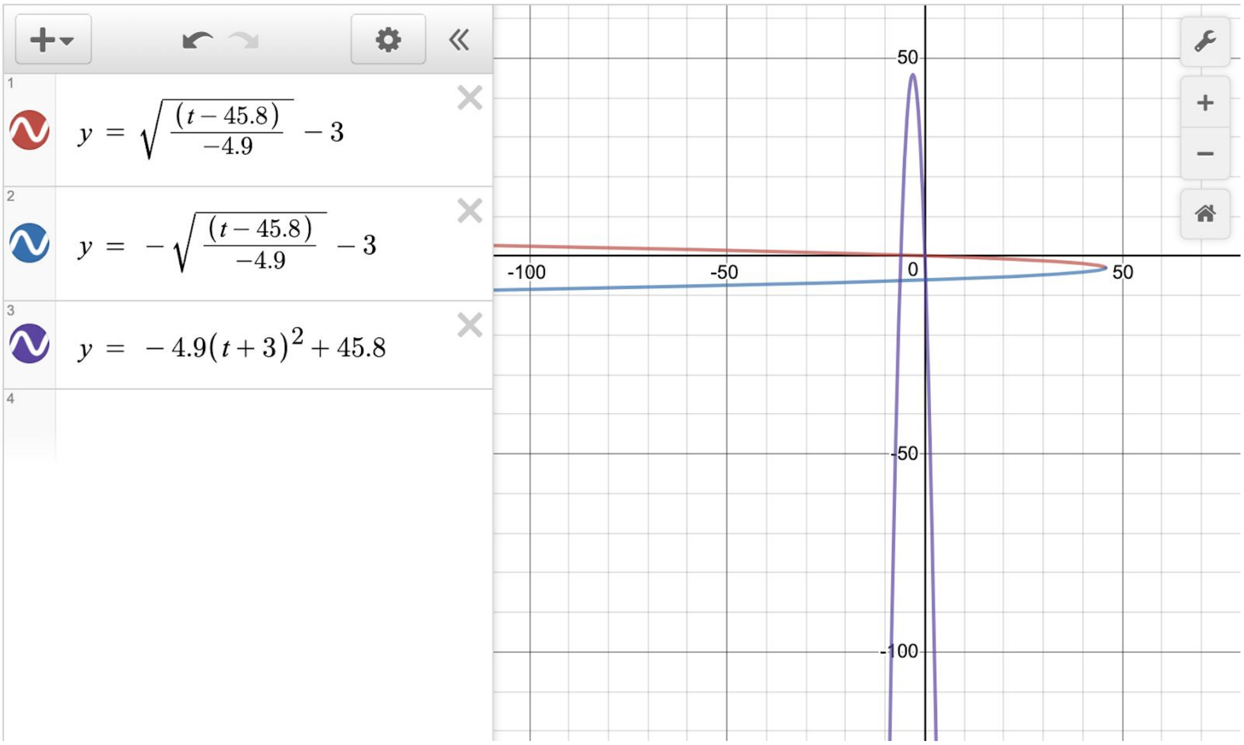
(b) $y{-}1 = \frac{-3x+18}{5}$ which is the inverse function of $3x + 5y = 18$

note) the above "-1" is not a constant but it means y-inverse.



The attached image shows the graph of $y{-}1 = \dfrac{-3x+18}{5}$ **which is a function**. There are two reasons to illustrate this. First is that it passes the vertical line test, which clearly states that the function returns only one output per one input. Secondly, when seeking the solution of two equations - original and inverse one - one can find that value x and value y is the same (9/4, 9/4). Therefore, the function $y{-}1 = \dfrac{-3x+18}{5}$ is a function.

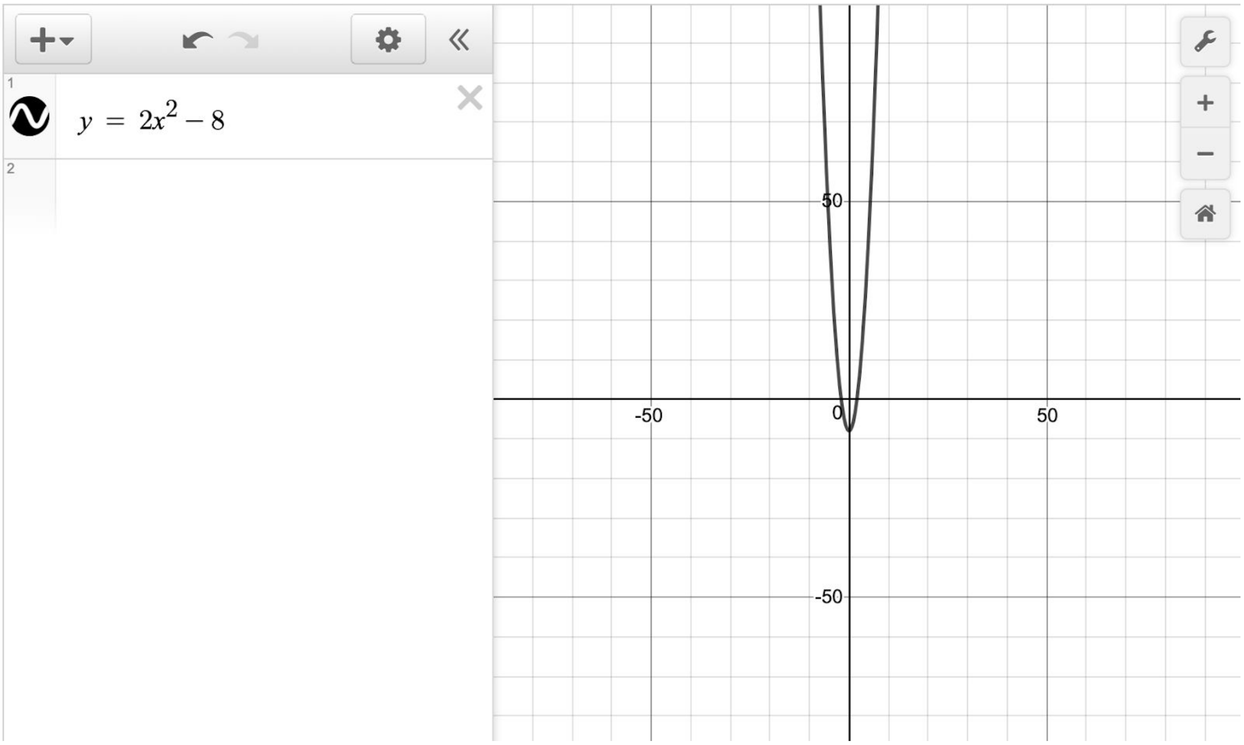
(c) $y = \mp\sqrt{\dfrac{t-45.8}{-4.9}}$ which is the reverse function of $h(t) = -4.9(t+3)^2 + 45.8$



$y = \mp\sqrt{\frac{t-45.8}{-4.9}}$ is **NOT** a function since <u>it gives two outputs per input</u>, which means that the graph does not pass the vertical line test. The graph would be recognized as the function is it has only one positive or negative constant but represents both cases. $y = \mp\sqrt{\frac{t-45.8}{-4.9}}$ is NOT a function since it gives two outputs per input, which means that the graph does not pass the vertical line test. The graph would be recognized as the function is it has only one positive or negative constant but represents both cases.

<u>Question 4. For each of the functions below, state the domain and range, the restrictions, the intervals of increasing and decreasing, the roots, y-intercepts, and vertices.</u>

(a) $f(x) = 2x^2 - 8$



a-1. Domain: {x|x∈R}
a-2. Range: {y|y≥−8,y∈R}
a-3. Restriction

- There is no restriction in variable x.
- $y \geq -8$

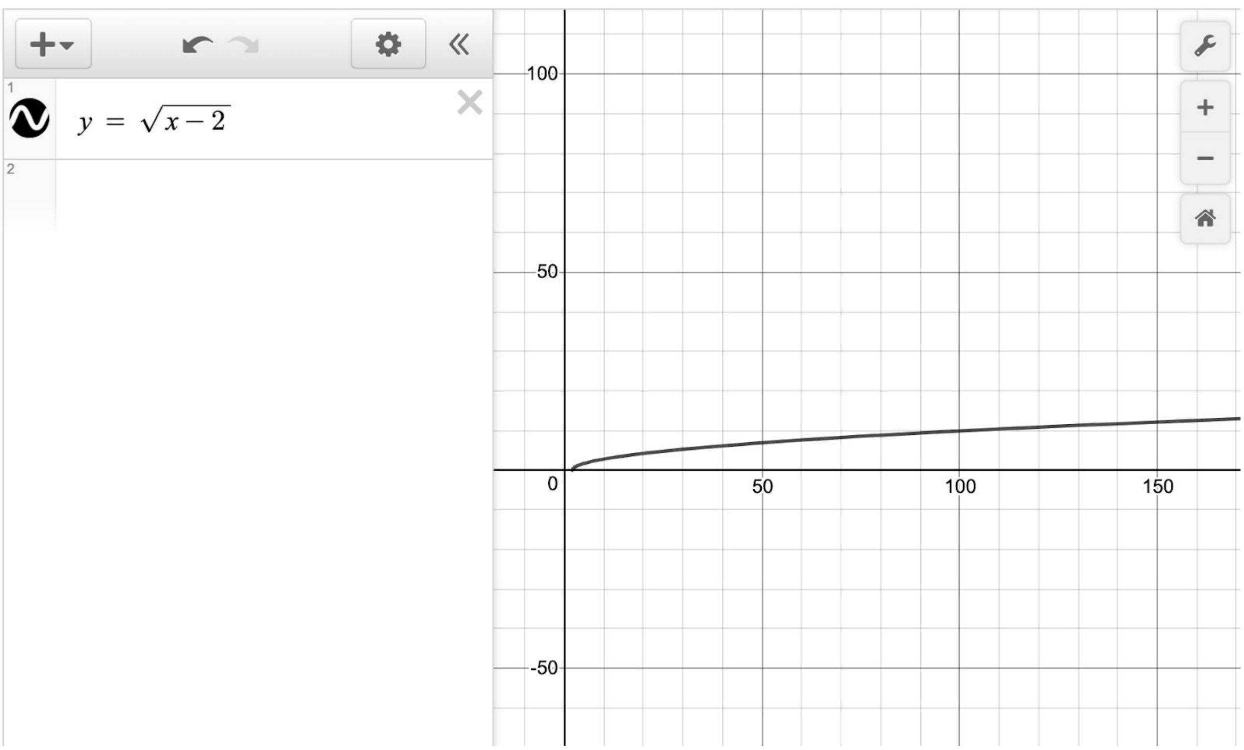
a-4. Intervals
- Increasing: (0,∞)
- Decreasing: (−∞,0)

A-5. root:∓ 2

- To solve, insert 0 to $f(x) = 2x^2 - 8$
- $0 = 2x^2 - 8$
- $0 = 2(x^2 - 4)$
- $0 = 2(x + 2)(x - 2)$
- $y = \mp 2$

- Y-intercept: $(0, -8)$
  - Insert 0 to f(x), $f(x) = 2x^2 - 8$
  - $f(0) = 2 * (0)^2 - 8$
  - $f(0) = 0 - 8$
  - $f(0) = -8$
  - $y = -8$
- Vertices: the vertices of f(x) is the minimum value of f(x) since it is the quadratic function, which is (0, -8).

(b) $f(x) = +\sqrt{x-2}$

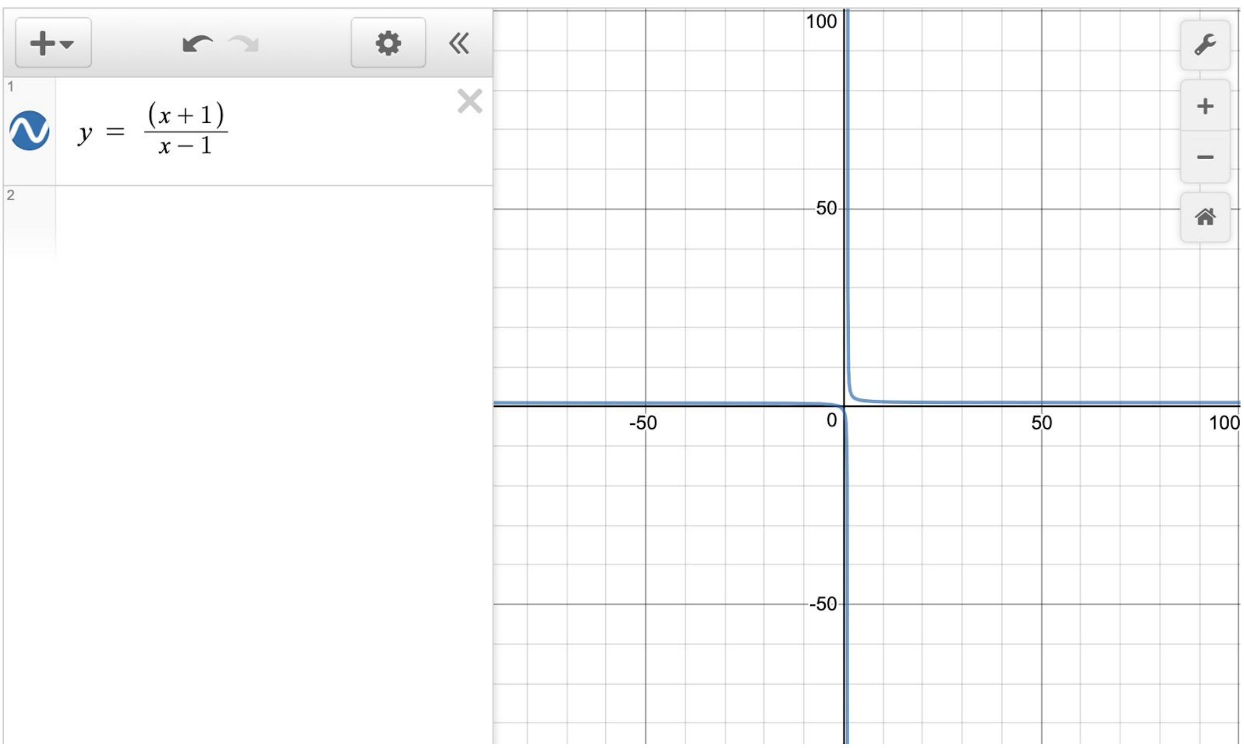


- b-1. Domain: $\{x|x \geq 2, x \in R\}$
- b-2. Range: $\{y|y \geq 0, y \in R\}$
- b-3. Restriction: $x \geq 2, y \geq 0$
- b-4. Intervals
  - Increasing: (2,∞)
  - Decreasing: no intervals
- b-5. Root: 2
  - $f(x) = \sqrt{x-2}$, insert 0 to f(x)

- $0 = +\sqrt{x-2}$
- $0 = x-2$
- $x = 2$

- b-6. Y-intercept: not exists
  - Since the number inside the sqrt cannot be minus, we can say it is undefined.
  - $f(x) = +\sqrt{x-2}$
  - $f(0) = +\sqrt{0-2}$
  - $f(0) = +\sqrt{-2}$
- b-7. Vertices: (2,0)
  - (2, 0) is the minimum value of the graph seen above.

(c) $f(x) = \dfrac{x+1}{x-1}$



- c-1. Domain: $\{x|x \neq 1, x \in R\}$
- c-2. Range: $\{y|y \neq 1, y \in R\}$
- c-3. Restriction: $x \neq 1$, $y \neq 1$
- c-4. Intervals
  - Increasing: none
  - Decreasing: (−∞,∞)

- c-5. Root: -1
    - $f(x) = \frac{x+1}{x-1}$ and insert 0 to f(x)$f(x) = \frac{x+1}{x-1}$ and insert 0 to f(x)
    - $0 = \frac{x+1}{x-1}$
    - $0 = x + 1$
    - $x = -1$
- c-6. Y-intercept: (0, -1)
    - $f(x) = \frac{x+1}{x-1}$
    - $f(0) = \frac{0+1}{0-1}$
    - $f(0) = 1/-1$
    - $f(0) = -1$
- c-7. Vertices: none
    - There is no vertices according to the graph.

<u>Question 5. The point (1,-2) is on the graph of f(x). Describe the following transformations on f(x), and determine the resulting point.</u>

- The notion of transformations of functions should be utilized in order for solving the problems. $y = \mp af(\mp b(x-h)) + \mp k$ function is used to determine horizontal/vertical stretches and compression, horizontal translation, and vertical translation.
    - a determines horizontal stretch or vertical compression.
    - The given function should pass a vertical line test.
    - b determines horizontal stretch or horizontal compression.
    - h determines horizontal stretch or horizontal compression.
    - k determines vertical stretch or vertical compression which determines y-intercept.

(a) $g(x) = 2f(x) + 3$

When applying the function into $y = \mp af(\mp b(x-h)) + \mp k$, the function can be interpreted to $y = 2 * f(1(x-0)) + 3$.

- 2 represents a, which means that the function is vertically stretched by the number of $+2$.
- 3 represents k which means that the graph is translated up by 3.
- $(1, -2)$ then transformed to $(1/1 + 0, 2 * (-2) + 3)$, which is $(1, -1)$.

(b) $g(x) = f(x+1) - 3$

When applying the function into $y = \mp af(\mp b(x-h)) + \mp k$, the function can be interpreted to $y = 1 * f((x-(-1))) - 3$.

- -1 represents h which determines the horizontal translation. The graph is translated to the left by 1 since -1 is a negative value.
- -3 represents k which determines y-intercept. The graph is vertically translated to the down by 3 since -3 is a negative value.
- $(1, -2)$ then transformed to $(1/1-1, 1*(-2)-3)$ that results in $(0, -5)$.

(c) $g(x) = -f(2x)$

When applying the function into $y = \mp af(\mp b(x-h)) + \mp k$, the function can be interpreted to $y = (-1)*f(2(x-0))+0$.

- -1 represents a which means that the graph is reflected on the x-axis.
- 2 represents b which means that the graph is compressed horizontally by 2.
- $(1, -2)$ would be transformed to $(1/2+0, (-1)*(-2)+0)$ with $(1/2, 2)$.

(d) $g(x) = -f(-x-1)+3$

When applying the function into $y = \mp af(\mp b(x-h)) + \mp k$, the function can be interpreted to $y = (-1)*f[-1*(x-(-1)]+3$.

- -1 represents a which means that the graph is reflected in the x-axis.
- -1 represents b which means that it is also reflected in the x-axis.
- -1 represents h which means that it is translated to the left by 1.
- +3 represents k which means that the graph is translated to the up by 3.
- (1, -2) would be transformed to $((1/-1)-1, (-1)*(-2)+3)$ which results (-2, 5).$((1/-1)-1, (-1)*(-2)+3)$ which results (-2, 5).

Question 6. Create a multimedia presentation to explain your reasoning behind one of your solutions to Question 3.

- I chose to record a short audio file to describe the solution of Question 3 - 1.
- Link: https://drive.google.com/file/d/1_JSNWuP7WwRvSp5N2MZg6wdT5k2jNcnV/view?usp=sharing
- Script: Hi, this is Jin who is taking the advanced functions in Virtual High School. I would like to talk about the function the reason behind inverse of p(r) = 2r^2 + 2r - 1 is not a function. Let us review the requisite for function. It requires to be a function to have only one output per one input. When it comes to the inverse function, p-1® = (-1) +- sqrt(2r+3) divided by 2 is not a function since it has two outputs per one input. The inverse of quadratic function has two rational functions - in a positive and a negative way. If the graph represents only half of the inverse graphs, it would be a function. However, since it shows the whole graph of positive and negative, it is not a function. Let me compare to another function which is 3y+5x = 18. It has the inverse function, which is y inverse equals -3x+18/5. Since it is the inverse function of the linear graph, it

has only one inverse graph. This means that it passes the vertical line test which suggests that the graph has only one output per one input. Thanks for listening.