Derivatives Unit Assignment
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1. **Find the derivatives of each of these functions.**

(a) $y = (x+7)^3(x-9)^4$

$= (x+7)^3(4(x-9)^3) + (x-9)^4(\frac{d}{dx}[(x+7)^3])$

$= (x+7)^3(4(x-9)^3) + 3(x-9)^4(x+7)^2$

$= (x+7)^2(x-9)^3(4(x+7) + 3(x-9))$

(b) $y = \frac{x^2-5}{x-5}$

$= \frac{(x-5)(2x)-(x^2-5)}{(x-5)^2}$

$= \frac{2(x-5)x-(x^2-5)}{(x-5)^2}$

$= \frac{x^2-10x+5}{(x-5)^2}$

(c) $y = (x^3+4)\sqrt{4x^2+2x-5}$

$= (x^3+4)\frac{d}{dx}[\sqrt{4x^2+2x-5}] + \sqrt{4x^2+2x-5}\frac{d}{dx}[x^3+4]$

$= (x^3+4)(\frac{1}{2}(4x^2+2x-5)^{-\frac{1}{2}}\frac{d}{dx}[4x^2+2x-5]) + (4x^2+2x-5)^{\frac{1}{2}} \bullet (3x^2)$

$= (8x+2)(x^3+4)\frac{1}{2(4x^2+2x-5)^{\frac{1}{2}}} + 3x^2(4x^2+2x-5)^{\frac{1}{2}}$

(d) $y = \frac{\sqrt[3]{2x^2+5}}{(x+3)^4}$

$= \frac{(x+3)^4\frac{d}{dx}(2x^2+5)^{\frac{1}{3}}-(2x^2+5)^{\frac{1}{3}}\frac{d}{dx}[(x+3)^4]}{(x+3)^8}$

$= \frac{(x+3)^4(\frac{1}{3}(2x^2+5)^{-\frac{2}{3}}\frac{d}{dx}[2x^2+5])-(2x^2+5)^{\frac{1}{3}}\frac{d}{dx}[(x+3)^4]}{(x+3)^8}$

$= \frac{\frac{(x+3)^4}{3(2x^2+5)^{\frac{2}{3}}}\frac{d}{dx}[2x^2+5]-(2x^2+5)^{\frac{1}{3}}\frac{d}{dx}[(x+3)^4]}{(x+3)^8}$

$= \frac{\frac{(x+3)^4}{3(2x^2+5)^{\frac{2}{3}}}(2(2x)+0)-(2x^2+5)^{\frac{1}{3}}\frac{d}{dx}[(x+3)^4]}{(x+3)^8}$

$= \frac{\frac{(x+3)^4}{3(2x^2+5)^{\frac{2}{3}}}(4x)-(2x^2+5)^{\frac{1}{3}}\frac{d}{dx}[(x+3)^4]}{(x+3)^8}$

$= \frac{\frac{4(x+3)^4x}{3(2x^2+5)^{\frac{2}{3}}}-(2x^2+5)^{\frac{1}{3}}\frac{d}{dx}[(x+3)^4]}{(x+3)^8}$

$= \frac{3(2x^2+5)^{\frac{2}{3}}(\frac{4(x+3)^4x}{3(2x^2+5)^{\frac{2}{3}}})+3(2x^2+5)^{\frac{2}{3}}(-(2x^2+5)^{\frac{1}{3}}\frac{d}{dx}[(x+3)^4])}{3(2x^2+5)^{\frac{2}{3}}(x+3)^8}$

$= \frac{(4(x+3)^4x)-3(2x^2+5)^{\frac{1}{3}+\frac{2}{3}}\frac{d}{dx}[(x+3)^4]}{3(2x^2+5)^{\frac{2}{3}}(x+3)^8}$

Differentiate the function.

$= \frac{4(x+3)^4x-3(2x^2+5)^1(4(x+3)^3)}{3(2x^2+5)^{\frac{2}{3}}(x+3)^8}$

Simplify the function.

$$= \frac{4(-5x^2+3x-15)}{3(2x^2+5)^{\frac{2}{3}}(x+3)^5}$$

2. Use the process of implicit differentiation to find $\frac{dy}{dx}$ given that: $5x^3y^4 + \frac{3y^2}{4x^3} = 5xy$. Apply implicit differentiation directly without first simplifying the equation. (10 marks)

$$5x^3y^4 + \frac{3y^2}{4x^3} = 5xy$$
$$\rightarrow [5x^3(4y^3\frac{dy}{dx}) + y^4(15x^2)] + \frac{4x^3(6y\frac{dy}{dx}) - 3y^2(12x^2)}{(4x^3)^2} = 5x(\frac{dy}{dx}) + 5y$$
$$\rightarrow 20x^3y^3\frac{dy}{dx} + 15x^2y^4 + \frac{24x^3y\frac{dy}{dx} - 36x^2y^2}{16x^6} - 5x(\frac{dy}{dx}) = 5y$$
$$\rightarrow 320x^9y^3\frac{dy}{dx} + 240x^8y^4 + (24x^3y\frac{dy}{dx} - 36x^2y^2) - 80x^7(\frac{dy}{dx}) = 80x^6y$$
$$\rightarrow \frac{dy}{dx}(320x^9y^3 + 24x^3y - 80x^7) = 80x^6y - 240x^8y^4 + 36x^2y^2$$
$$\rightarrow \frac{dy}{dx} = \frac{80x^6y - 240x^8y^4 + 36x^2y^2}{320x^9y^3 + 24x^3y - 80x^7}$$
$$\rightarrow \frac{dy}{dx} = \frac{4x^2(20x^4y - 60x^6y^4 + 9y^2)}{4x^2(80x^7y^3 + 6xy - 20x^5)}$$
$$\rightarrow \frac{dy}{dx} = \frac{20x^4y - 60x^6y^4 + 9y^2}{80x^7y^3 + 6xy - 20x^5}$$

3. Give an example of a situation in which composite differentiation might be used. Give examples of functions that might be applicable in your situation, and show how the relevant rates of change might be calculated. (8 marks)

The following is an example of a situation in which component derivatives can be used:
Suppose the spherical balloon expands and the radius increases by 6 centimeters per second. Let R represent the radius of the balloon in centimeters, t in seconds, and V in cubic centimeters in volume. Then we can know.

$$V = \frac{4}{3}\pi R^3, \ R = 6t$$
$$V = \frac{4}{3}\pi(6t)^3$$

To differentiate composite functions we have to use the chain rule.

$$\frac{dV}{dt} = \frac{dV}{dR} \bullet \frac{dR}{dt}$$
$$\frac{dv}{dR} = 4\pi R^2$$
$$\frac{dR}{dt} = 6$$
$$\frac{dv}{dt} = 4\pi R^2 \bullet 6 = 24\pi(6t)^2 = 864\pi t^2$$

4. The position of a point on a line is given by the equation $s(t) = t^3 - 6t^2 + 9t - 4$, where s is measured in metres and t in seconds.

(a) What is the velocity of the point after 2 seconds?

$$s'(t) = velocity = \frac{ds}{dt} = 3t^2 - 12t + 9$$
$$s'(2) = 3(2)^2 - 12(2) + 9 = 12 - 24 + 9 =- 3$$
$$\rightarrow \ -3m/s \ in \ opposite \ direction$$

(b) What is its acceleration after 4 seconds?

$s''(t) = acceleration = \frac{dv}{dt} = \frac{d^2s}{dt^2} = 6t - 12$

$s''(4) = 6(4) - 12 = 12$

Answer: $12m/s^2$

(c) Where is it when it first stops moving?

$s'(t) = 0,\ 3t^2 - 12t + 9 = 0$

$3(t^2 - 4t + 3) = 0$

$3(t - 1)(t - 3) = 0$

$t = 1,\ 3$

$s(1) = 1 - 6 + 9 - 4 = 0$

(d) How far has it travelled when its acceleration is 0?

$s''(t) = 6t - 12 = 0$

$t = 2,\ s(2) = 2^3 - 6(2)^2 + 9(2) - 4 = 8 - 24 + 18 - 4 = 4 - 6 =- 2$

$s(0) =- 4$

Distance = -2-(-4)=2, The answer is $2m$ in the opposite direction.

(e) After 2 second, is it moving toward or away from the origin?

$s(2) = (2)^3 - 6(2)^2 + 9(2) - 4 = 8 - 24 + 18 - 4 =- 16 + 14 =- 2,\ - 2m$

$s''(2) = 6(2) - 12 = 0$

It is the negative side of the origin from 2 meters away in the opposite direction. However, it is worth noting that the point does not move in two seconds.