SPH4U Energy and Momentum
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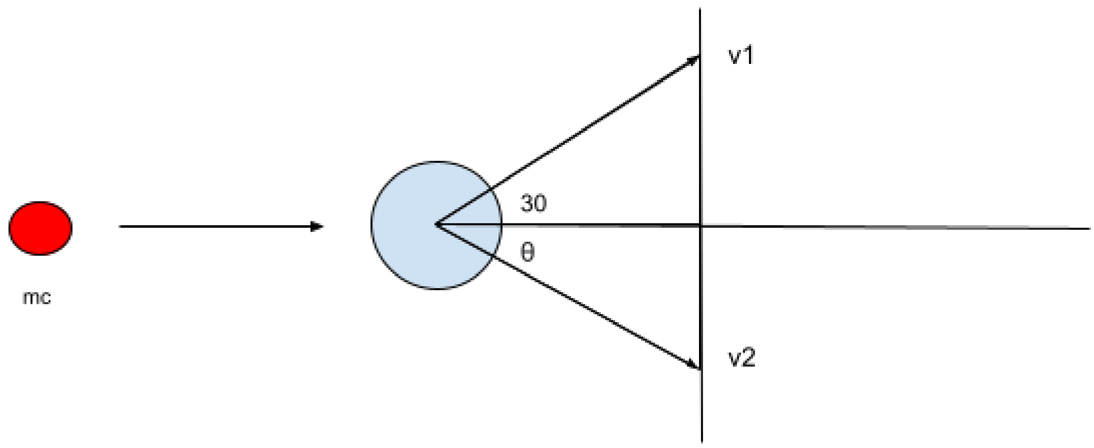
1. **Calculate the final speed of a 1500 kg roller coaster if it has an initial speed of 16 m/s but reduces its elevation by 8.6 m by coasting down a frictionless incline. (5 marks)**

   To begin with, we should use the idea of mechanical energy conservation.
   $P_{initial} + K_{initial} = P_{final} + K_{final}$
   $mgh = \frac{1}{2}mv^2$ $(potential\ energy\ with\ regards\ to\ gravity = kinetic\ energy)$
   $1500(kg) \times 9.8m/s \times 8.6(m) + \frac{1}{2} \times 1500 \times (16)^2 = 0 + \frac{1}{2} \times 1500 \times v^2$
   $v^2 = 9.8 \times 8.6 \times 2 + (16)^2$
   $v^2 = 424.56$
   $v = 20.6$
   $v = 2.1 \times 10^1 m/s$

   Thus, the final speed of a 1500 kg rollercoaster with initial speed of 16m/s is $2.1 \times 10^1 m/s.$

2. **A 0.50 kg cue ball makes a glancing blow to a stationary 0.50 kg billiard ball. After the collision, the cue ball deflects with a speed of 1.2 m/s at an angle of 30.0o from its original path. Calculate the original speed of the cue ball if the billiard ball ends up travelling at 1.6 m/s. (7 marks)**

   Let v as the initial velocity of cue ball, while θ as the angle between the path of the billiard ball and cue ball. To begin with, consider the direction perpendicular to the original direction of the cue ball, which is y direction. According to the conservation of momentum principle, we can write the following.



   $0 = 0.5 \times 1.2 \times sin30^o - 0.5 \times 1.6 \times sin\,\theta$
   $sin\,\theta = \frac{3}{8}$

$cos\,\theta\ = \sqrt{1 - \frac{9}{64}} = \frac{\sqrt{55}}{8}$

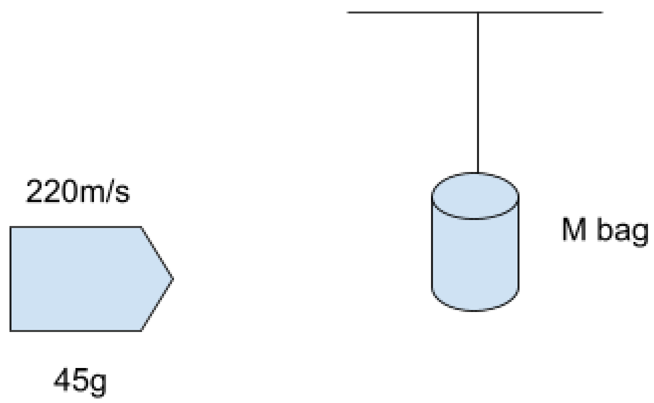
Let us apply the conservation of the momentum principle to the original direction of the cue ball, which is x direction.

$0.5\ \times\ v\ =\ 0.5\ \times\ 1.2cos30^{o}\ +\ 0.5\ \times\ 1.6\ \times\ cos\,\theta$

$v\ =\ 0.6\ \times\ \sqrt{3}\ +\ 0.4\ \times\ \frac{\sqrt{55}}{8}\ \approx\ 2.5m/s$

Thus, the original ball of the cue ball is $2.5\ \times\ 10^{0}m/s$.

3. **A bullet of mass 45 g is fired at a speed of 220 m/s into a 5.0 kg sandbag hanging from a string from the ceiling. The sandbag absorbs the bullet and begins to swing. To what maximum vertical height will it rise? (8 marks)**

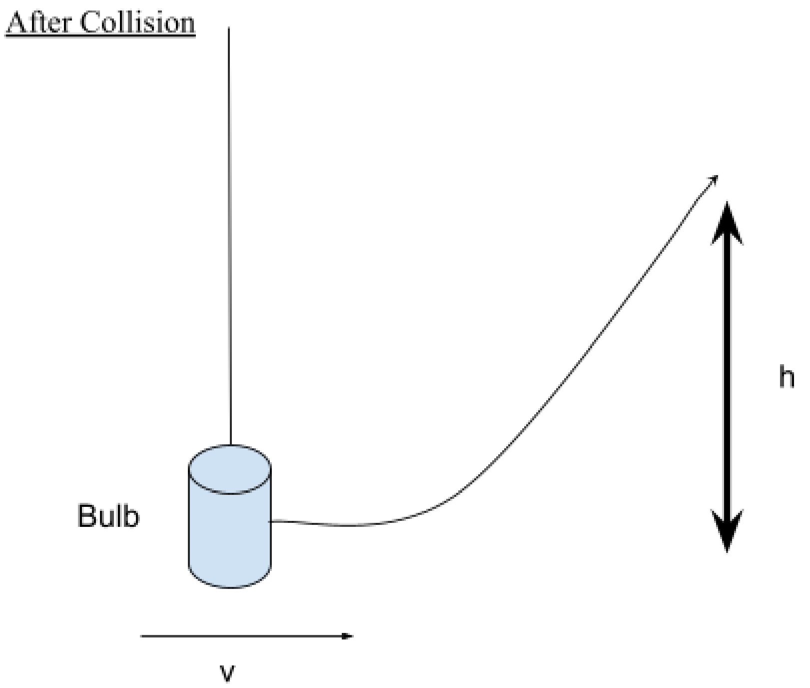


Before Collision

Let us v mean the velocity of a bullet while h is the height of the light bulb after getting collided with a mass.

Let us calculate the initial momentum.

$Initial\ Momentum\ =\ (\frac{45}{1000}\ \times\ 220)kg/m\ seconds.$



Let us calculate the final momentum.

$Final\ Momentum\ =\ (\frac{45}{1000}\ +\ 5)v$

Using the concept of momentum conservation and rewriting the equation as follows.

$P_{initial}\ = P_{final}$

$(\frac{45+5000}{1000})v\ =\ \frac{45}{1000}\ \times\ 220$

$v\ =\ (\frac{45\times200}{5045})m/s\ \approx\ 1.96m/s$

Since the kinetic energy is equal to the gravity potential energy, we can render the following involved.

$mgh\ = \frac{1}{2}mv^2$

$gh\ = \frac{1}{2}v^2$

$h\ = \frac{v^2}{2g}\ = \frac{(1.96)^2}{2\times9.8}m\ \approx\ 0.196m$

Thus, the maximum vertical height to be risen is $2.0\ \times\ 10^{-1}m$.