Unit Assignment: Exponential and Logarithmic Functions

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Question 1.

a) $\frac{3^2}{3^{-4}}$

- $\frac{(3^2)(3^4)}{1}$
- answer: 3^6

b) $\frac{2^5 * 2^{-2}}{2^2} = \frac{2^5}{(2^2)(2^2)} = \frac{2^5}{2^4} = 2$

- answer: 2

c) $\frac{(x^2y)^4}{xy^3} = \frac{x^8y^4}{xy^3} = x^7y$

- answer: x^7y

d) $\frac{(a^5b^{-1}c)(a^{-2}bc^2)}{a^3b^2c^{-3}} = \frac{(a^5)(a^{-2})(b^{-1})(b)(c)(c^2)}{a^3b^2c^{-3}} = \frac{a^3b^0c^3}{a^3b^2c^{-3}}$

- answer: $\frac{c^6}{b^2}$

Question 2.

$f(x) = \ -(4)^x + 2$

- domain: $(-\infty, \infty)$
- range: $(-\infty, 2)$
- y-intercept: $(0,\ 1)$
    - Originally, the y-intercept of the graph exists at $(0,\ 1)$. However, after reflecting function on the x-axis and did a vertical translation up by 2 units, the y-intercept changes from $(0,\ 1)$ to $(0,\ -1)$ to $(0,\ 1)$ respectively.
    - Algebraically,
      $y =-(4)^0 + 2$
      $y =-1 + 2$
      $y = 1$
      Therefore, the y-intercept is $(0, 1)$
- Horizontal Asymptote: $y = 2$
    - Originally, the parent function has a horizontal asymptote at $y = 0$. However, as the graph has a vertical transition up to 2 unit, the horizontal asymptote exists at $y = 2$.

Question 3.

When John was born, his parents invested $7000 into a savings account that pays 3% interest, compounded monthly. How much will the investment be worth on John's 18th birthday?

- Let us set variable as the following:
    - $P\ = \$7000$
    - $r = 3\% = 0.03\ (assuming\ yearly\ nominal\ rate)$
    - $t = 18\ years$
    - $n = 12\ (monthly)$
- We can use the following compounded interest formula:
    - $A(t) = P(1 + \frac{r}{n})^{nt}$ which means to multiply monthly interests over current balance.

- $A(t) = 7000 * (1 + \frac{0.03}{12})^{12*18}$
    - $A(t) = 7000(1 + 0.0025)^{12*18}$
    - $A(t) = 7000 \times 1.0025^{12*18}$
    - $A(t) = 7000 \times 1.714851$
    - $A(t) = 12003.956118,\ to\ round\ up\ is\ 12004$
- Answer is $\$12004$

Question 4.

1) $f(x) = 10^x$
- $let\ f(x) = y$
- $y = 10^x$
- $log_{10}y = x$

2) $f(x) = (\frac{1}{7})^x$
- $let\ f(x) = y$
- $y = (\frac{1}{7})^x$
- $log_{\frac{1}{7}}y = log_{\frac{1}{7}}(\frac{1}{7})^x$
- $log_{\frac{1}{7}}y = x$

3) $f(x) = log_5 x$
- $let\ f(x) = y$
- $log_5 x = y \Leftrightarrow 5^y = x$
- $x = 5^y$

4) $f(x) = log_{\frac{3}{5}} x$
- $let\ f(x) = y$
- $log_{\frac{3}{5}} x = y \Leftrightarrow \frac{3}{5}^y = x$
- $x = \frac{3}{5}^y$

Question 5.

1) $log_3 x - log_3 8 = log_3 4 + log_3 1$
- $log_3(\frac{x}{8}) = log_3(4 * 1)$
- $log_3(\frac{x}{8}) = log_3 4$
- $3^{log_3(\frac{x}{8})} = 3^{log_3 4}$
- $(\frac{x}{8}) = 4$
- $x = 32$

2) $log_3(x + 2) - log_3(x + 28) =- 3$
- $log_3(\frac{x+2}{x+28}) =- 3$

- $3^{log_3(\frac{x+2}{x+28})} = 3^{-3}$
- $\frac{x+2}{x+28} = \frac{1}{3^3}$
- $\frac{x+2}{x+28} = \frac{1}{27}$
- $27(x+2) = (x+28)$
- $27x + 54 = x + 28$
- $26x =- 26$
- $x =- 1$

Question 6.

1) $2^x = 6$

- $log_2 6 = x$
- $x = 2.5850\ (after\ round)$
- Check:
  - $2^{2.5820} = 6.0002$
  - 6.0002 is similar to 6

2) $3^{x+2} = 2$

- $log_3 2 = x + 2$
- $log_3 2 - 2 = x$
- $x =- 1.3691\ (after\ round)$
- Check:
  - $3^{-1.3681+2} = 2$
  - 1.9999 is similar to 2

3) $7^{2x} = 52$

- $log_7 52 = 2x$
- $\frac{log_7 52}{2} = x$
- $x = 1.0153$
- Check:
  - $7^{2(1.0153)} = 52$
  - $7^{2.0356} = 52$
  - 52.0063 is similar to 52

4) $4^{3x-1} = 90$

- $log_4 90 = 3x - 1$
- $\frac{log_4 90+1}{3} = x$
- $x = 1.4143$
- Check:
  - $4^{3(1.4143)-1} = 90$
  - $4^{3.2459} = 90$
  - 89.9967 is similar to 90

Question 7.
In a particular factory, the ambient noise has a loudness of 100 dB. If prolonged exposure to a loudness of 85 dB can cause permanent damage, how much more intense is the factory than the damage threshold?

- $L_1 - L_2 = 10log(\frac{I_1}{I_2})$
- $100dB - 85dB = 10log(\frac{I_1}{I_2})$
- $15dB = 10log(\frac{I_1}{I_2})$
- $15 = 10log(\frac{I_1}{I_2})$
- $1.5 = log(\frac{I_1}{I_2})$
- $\frac{I_1}{I_2} = 10^{1.5}$
- $\frac{I_1}{I_2} = 31.622$
- Answer: The loudness of the factory is 31.622 times more intense than the damage threshold.