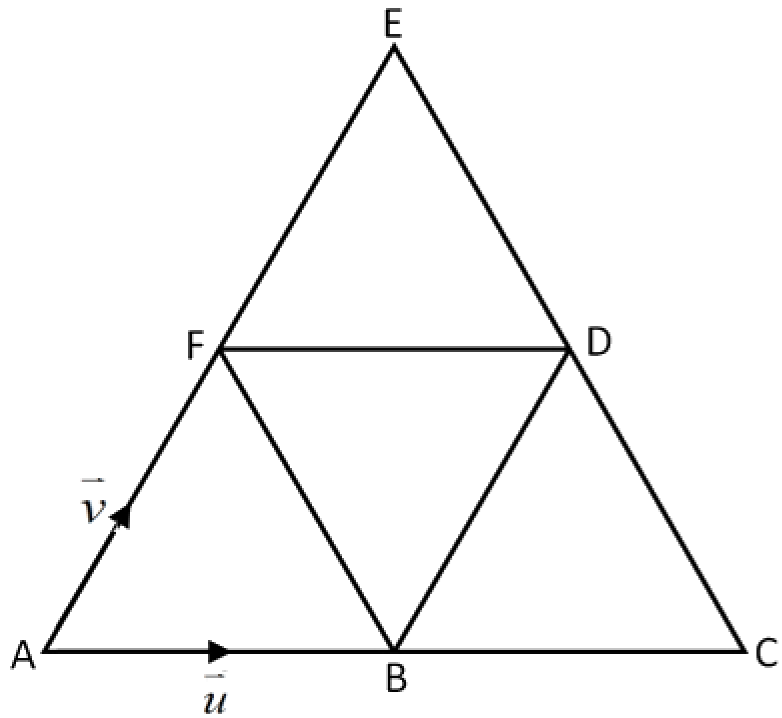
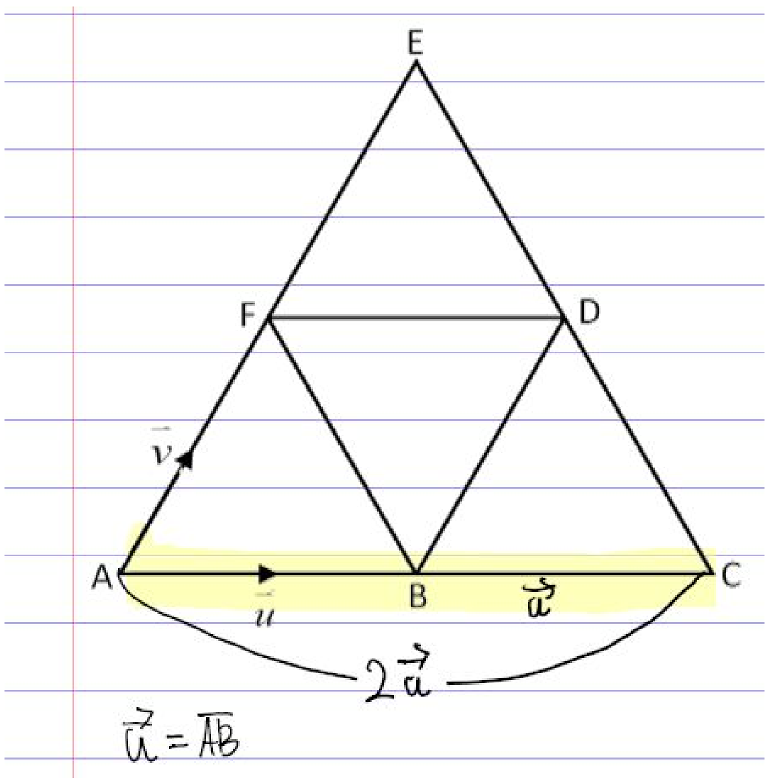
MCV4U Vectors Unit Assignment
Jin Hyung Park

1. **State whether each quantity is a vector or scalar.**
    1. Speed
    - Because a scalar has only magnitude with no direction, speed is a scalar. $speed = \frac{distance}{time}$
    2. Velocity
    - Considering that velocity can be specified with magnitude in a designated direction, we can think that $velocity = \frac{vec(d)}{t}$ has both magnitude and direction.
    3. Weight
    - Force is a product of mass, which is a scalar, and acceleration, which is a vector. When considering the formula that weight can be calculated by multiplying mass and acceleration, we can think that weight is a vector product taking both direction and magnitude.
    4. Mass
    - Mass itself is only a scalar quantity with only presenting magnitude. Mass does not change no matter where you are living and moving.
    5. Area
    - The area element is correctly defined only if its magnitude and direction are given. When it comes to the area of a parallelogram, we can appreciate the orientation of the area element while given as $vec(s) = vec(a) * vec(b)$ (where $vec(a)\ and\ vec(b)$ are its two sides). Thus, the area element is a vector quantity.

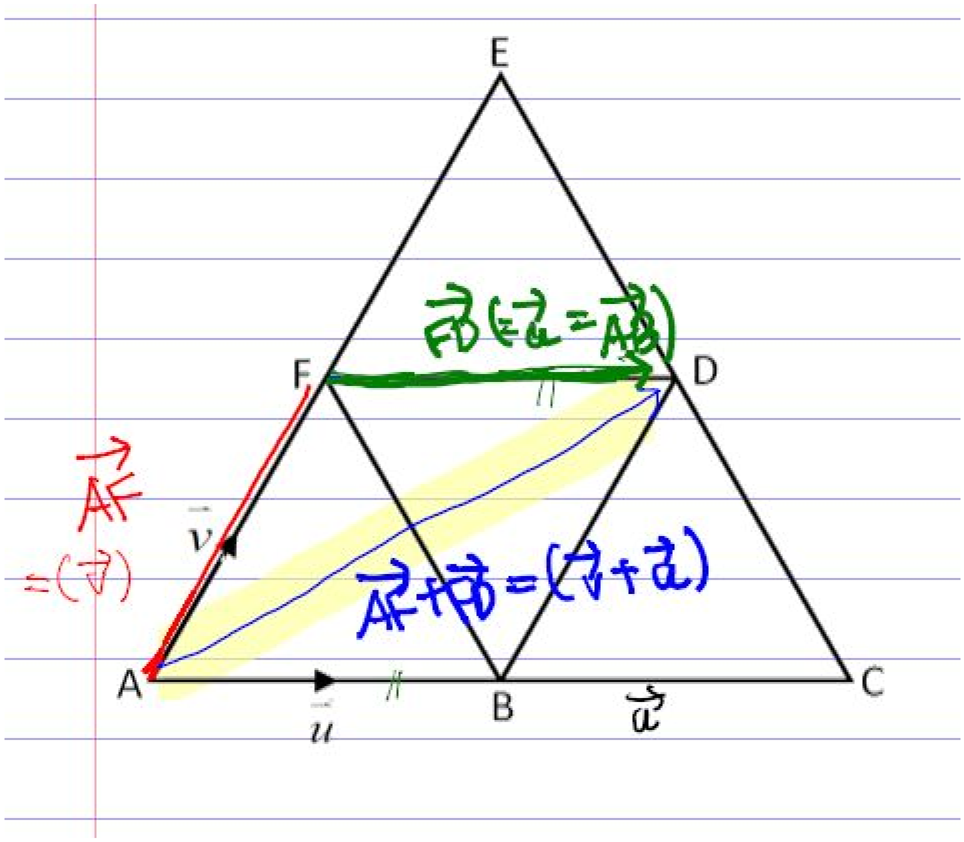
2. In the diagram, ACE is an equilateral triangle. B, D, and F are the midpoints of AC, CE, and EA. $\vec{AB} = \vec{u},\ \vec{AF} = \vec{v}$ **. Write the following vectors in terms of** $\vec{u}$ **and** $\vec{v}$ **.**



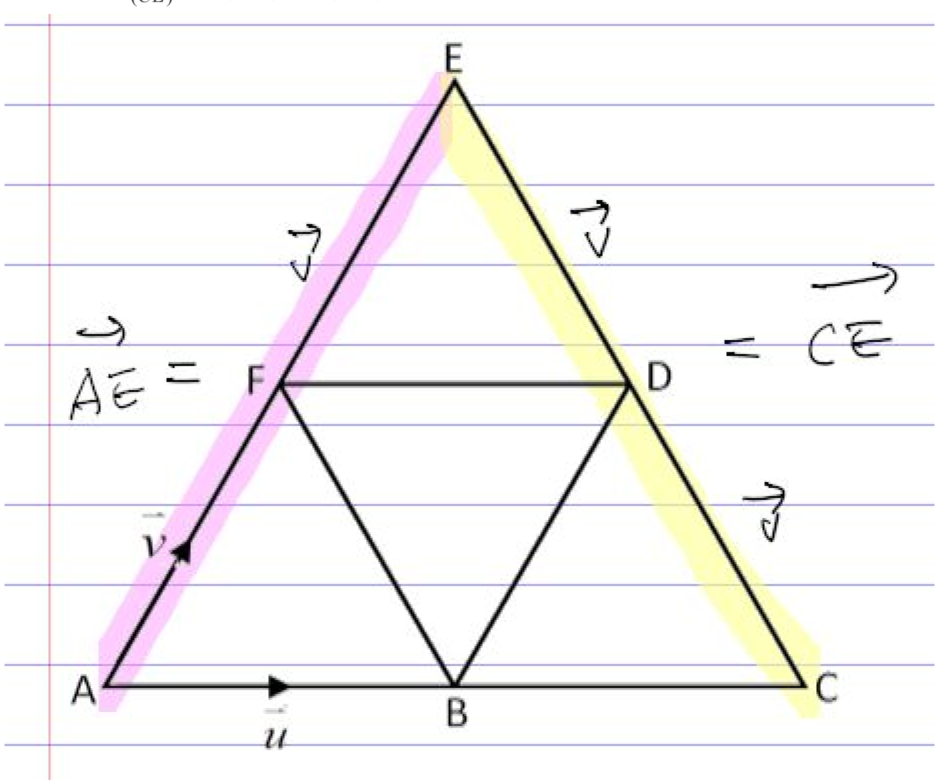
a. $\vec{AC} = 2\vec{AB} = 2\vec{u}$



b. $\vec(AD)=\vec(AF)+\vec(FD)=\vec(v)+\vec(u)$



c. $\rightarrow_{(CE)} = 2(\rightarrow_{v}) - 2(\rightarrow_{u})$



d. $\vec(EB) = ((\vec(EF) + \vec(FA)) + \vec(AB) = -\vec(v) - \vec(v) + \vec(u)$

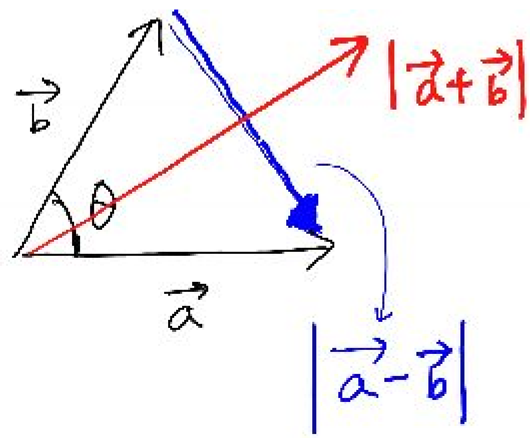
Give an example of a vector that is equal to:

e. $2\vec(v) = 2\overline{AF} = \overline{AE}$

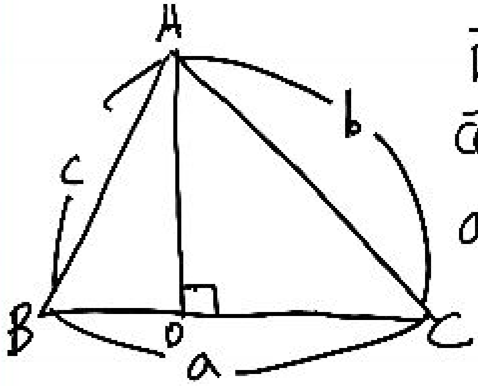
f. $\vec(u) - \vec(v) = \vec(\overline{FD}) + \vec(DC) = \vec(FC)$

3. Draw diagrams to show two vectors, vec(a) and vec(b), and the two vectors (vec(a)+vec(b)) and (vec(a)-vec(b)).

To begin with, let me define some elements that are required to solve the problems.



i) $|\vec{a}-\vec{b}|$ — 2nd law of cosine.



$\overline{BD} = c \cdot \cos B$

$\overline{CD} = b \cdot \cos C$

$a = c\cos B + b\cos C$

$$a = c\cos B + b\cos C \xrightarrow{\times a} a^2 = ac\cos B + ab\cos C$$

$$b = a\cos C + c\cos A \xrightarrow{\times b} b^2 = ab\cos C + bc\cos A$$

$$c = b\cos A + a\cos B \xrightarrow{\times c} c^2 = bc\cos A + ac\cos B$$

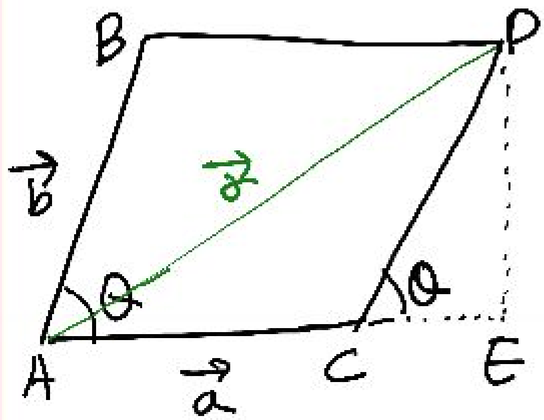
$$(a^2 - b^2) = (ac\cos B + \cancel{ab\cos C}) - (\cancel{ab\cos C} + bc\cos A)$$

$$= ac\cos B + bc\cos A$$

$$(a^2 - b^2 + c^2) = (ac\cos B + \cancel{bc\cos A}) + (\cancel{bc\cos A} + ac\cos B)$$

thus, $b^2 = a^2 + c^2 - 2ac\cos B$ ■

b. $|\vec{a}+\vec{b}|$



$\overline{CE} = \vec{b}\cdot\cos\theta$, $\overline{AE} = \vec{a}+(\vec{b}\cos\theta)$, $\overline{ED}=\vec{b}\sin\theta$

$\overrightarrow{AD} = \vec{\gamma} = (\vec{a}+\vec{b}\cos\theta)^2+(\vec{b}\sin\theta)^2$

According to pythagoras's law,

$= (\vec{a}^2+2\vec{a}\vec{b}\cos\theta+\vec{b}\cos\theta^2)+(\vec{b}\sin\theta)^2$

$= (\vec{a}^2+2\vec{a}\vec{b}\cos\theta)+\vec{b}^2(\cos\theta^2+\sin\theta^2)$

$(\because \cos\theta^2+\sin\theta^2=1)$

$= \vec{a}^2+2\vec{a}\vec{b}\cos\theta+\vec{b}$

a)

Let θ be the angle between $\vec{(a)}$ and $\vec{(b)}$ as shown in the attached figure. Thus, the range of θ is to be $0 \le \theta \le \frac{\pi}{2}$.

$|\overrightarrow{a} + \overrightarrow{b}| = \sqrt{|\overrightarrow{a}|^2 + |\overrightarrow{b}|^2 + 2|\overrightarrow{a}||\overrightarrow{b}|\cos\theta}$

$|\overrightarrow{a} - \overrightarrow{b}| = \sqrt{|\overrightarrow{a}|^2 + |\overrightarrow{b}|^2 - 2|\overrightarrow{a}||\overrightarrow{b}|\cos\theta}$

Considering the given condition that is $|\overrightarrow{a} + \overrightarrow{b}| > |\overrightarrow{a} - \overrightarrow{b}|$,

$$\sqrt{|\vec{a}|^2+|\vec{b}|^2+2|\vec{a}||\vec{b}|\cos\theta} > \sqrt{|\vec{a}|^2+|\vec{b}|^2-2|\vec{a}||\vec{b}|\cos\theta}$$

$$\implies \cos\theta > -\cos\theta$$

$$\implies 2\cos\theta > 1 \implies \cos\theta > \frac{1}{2} \implies \theta \in \left[0, \frac{\pi}{3}\right)$$

Thus, the answer is $0 \le \theta < \frac{\pi}{3}$.

b)

Considering the given condition that is $|\vec{a}+\vec{b}| < |\vec{a}-\vec{b}|$,

$$\sqrt{|\vec{a}|^2+|\vec{b}|^2+2|\vec{a}||\vec{b}|\cos\theta} < \sqrt{|\vec{a}|^2+|\vec{b}|^2-2|\vec{a}||\vec{b}|\cos\theta}$$

$$\implies \cos\theta < -\cos\theta$$

$$\implies 2\cos\theta < 1 \implies \cos\theta < \frac{1}{2} \implies \theta \in \left(\frac{\pi}{3}, \frac{\pi}{2}\right]$$

Thus, the answer is $\frac{\pi}{3} < \theta < \pi$.

c)

Considering the given condition that is $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$,

$$\sqrt{|\vec{a}|^2+|\vec{b}|^2+2|\vec{a}||\vec{b}|\cos\theta} = \sqrt{|\vec{a}|^2+|\vec{b}|^2+2|\vec{a}||\vec{b}|\cos\theta}$$

$$\implies \cos\theta = -\cos\theta$$

$$\implies \cos\theta = 0 \implies \theta = \frac{\pi}{2}$$

Thus, the answer is $\theta = \frac{\pi}{2}$.

d)

Considering the given condition that is $|\vec{a}+\vec{b}| = |\vec{a}|+|\vec{b}|$,

$$\sqrt{|\vec{a}|^2+|\vec{b}|^2+2|\vec{a}||\vec{b}|\cos\theta} = |\vec{a}|+|\vec{b}|$$

$$\implies |\vec{a}|^2+|\vec{b}|^2+2|\vec{a}||\vec{b}|\cos\theta = |\vec{a}|^2+|\vec{b}|^2+2|\vec{a}||\vec{b}|$$

$$\implies \cos\theta = 1 \implies \theta = 0$$

Thus, the answer is $\theta = 0$.

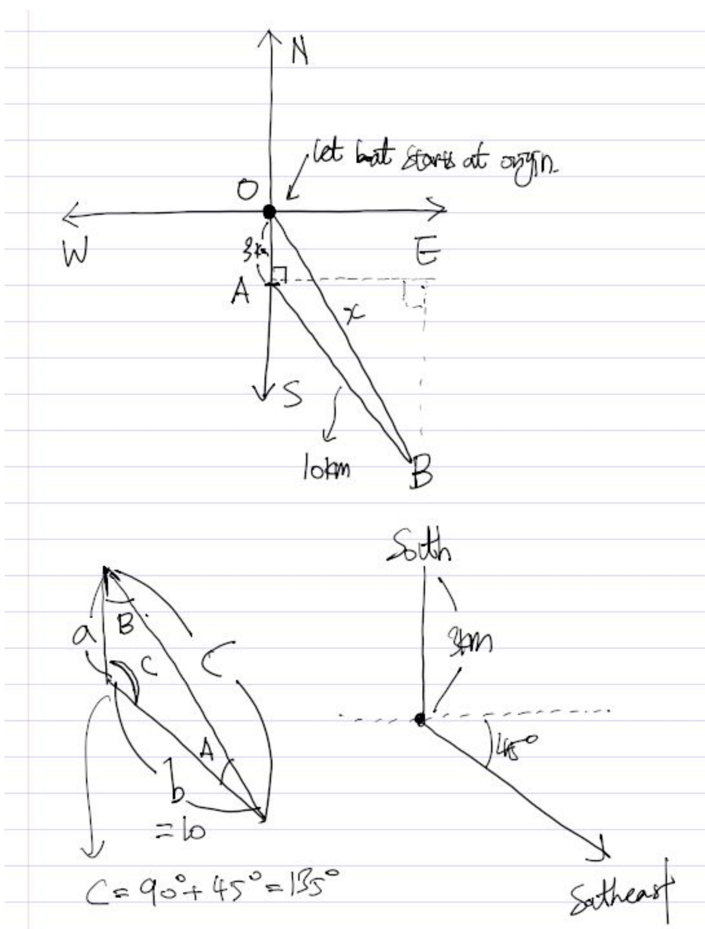
e)

$|\vec{a} + \vec{b}| = \sqrt{|\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos\theta}$ would be minimum when $\theta = 180$ *degree*.

Since $cos(180) = -1$,

$|\vec{a} + \vec{b}| = \sqrt{|\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|} = ||a| - |b||$

4. A boat sails 3 km South, then 10 km Southeast. Use trigonometry to find the boat's distance and bearing from its starting point.



Using cosine theorem,

$x^2 = (a^2 + b^2) - 2abcos(c) = (10)^2 + (3)^2 - 2 * 10 * 3 * cos(90 + 45)$

$= 100 + 9 - 60 * cos(135) = 109 - 60(-cos45) = 109 + \frac{60}{\sqrt{2}}$

$x = 12.30km = boat's\ distance$

Using Lemi's theorem,

$\frac{c}{sinc} = \frac{b}{sinb} = \frac{a}{sina}$

$\frac{12.30}{sin(90+45)} = \frac{10}{sinb}$, $sinb = \frac{10}{12.30\sqrt{2}}$

$b = sin^{-1}[\frac{10}{12.30\sqrt{2}}]$

$b = sin^{-1}(0.57) = 34.75$

$180 - 34.75 = 145.25 = bearing\ from\ starting\ point$

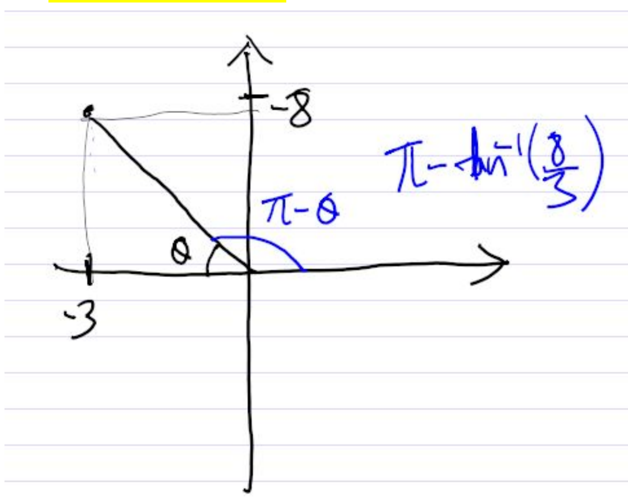
5. Convert the following vectors;

a) 75m/s on a bearing of 295 degrees to Cartesian form

- Since $\gamma = 75m/s$ while $\theta = 155\ degrees$, we can get [x, y] points by using trigonometry formulas.
- $x = \gamma cos\theta = 75cos(155) = 75 * (-0.487161) =- 36.5370$
- $y = \gamma sin\theta = 75sin(155) = 75 * (-0.873311) =- 65.4983$
- Thus, the answer is $(-36.5370,\ -65.4983)$.

b) [-3, 8] to direction/magnitude form

- $magnitude = \sqrt{(-3)^2 + (8)^2 = 9 + 64} = \sqrt{73} = 8.544$
- $tan\theta = \frac{8}{3}$
- $\theta = tan^{-1}(\frac{8}{3}) \approx 69^o$
- $direction = 69^o$



6. Express as a single vector.

a. $(\vec{PS}) + (\vec{SR}) = (\vec{PR})$

b. $(\vec{EF}) - (\vec{DF})$

$(\vec{EF}) + (\vec{FD}) = (\vec{ED})$

$$\boxed{\vec{(EF)} + \vec{(-FD)}) = \vec{(ED)}}$$

c. $\vec{(MP)} - \vec{(QR)} + \vec{(NM)} + \vec{(RP)}$

$= \vec{(MP)} + \vec{(RQ)} + \vec{(NM)} + \vec{(PR)}$

$= \vec{(NM)} + \vec{(MP)} + \vec{(PR)} + \vec{(RQ)})$

$= \vec{(NP)} + \vec{(PR)} + \vec{(RQ)}$

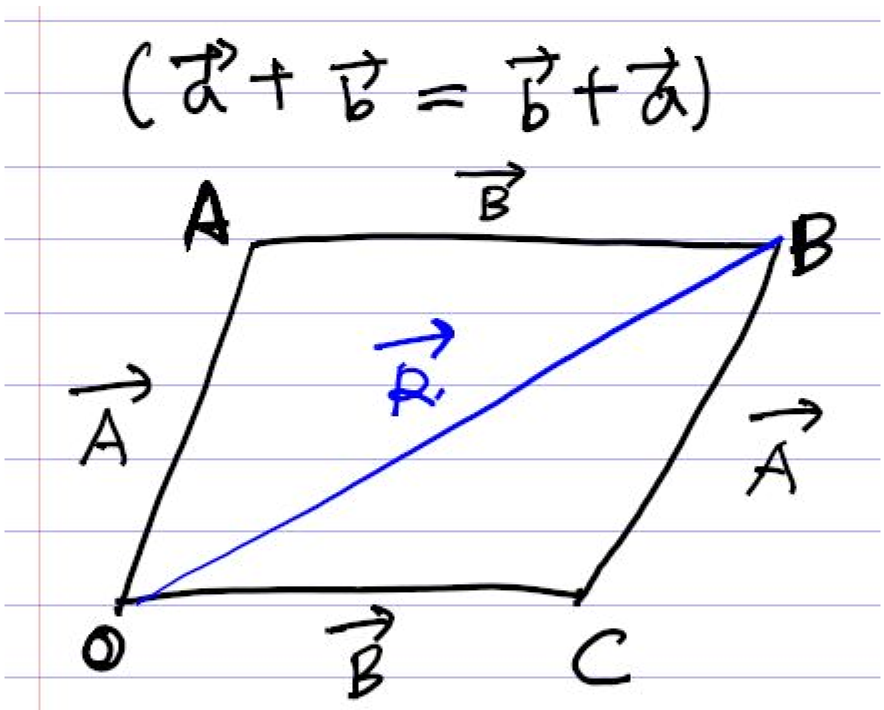
$= \vec{(NR)} + \vec{(RQ)}$

$\boxed{= \vec{(NQ)}}$

7. Demonstrate using diagrams

a) that vector addition is commutative: $\vec{(a)} + \vec{(b)} = \vec{(b)} + \vec{(a)}$

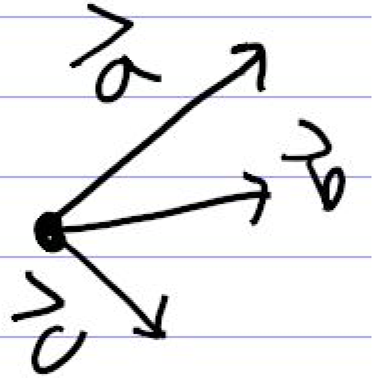
Constructing a parallelogram as following using two vectors as the adjacent sides.



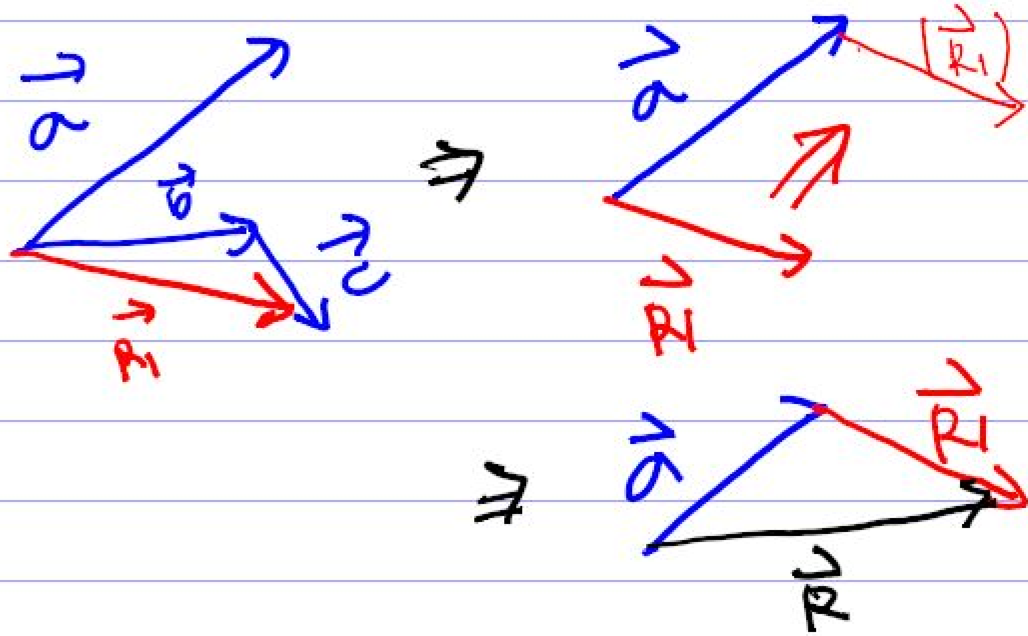
For the sake of convenience in typing, I will omit vector indication to each side below.
Using triangle law of vector addition in ΔOAB , $A + B = R$
and also using triangle law of vector addition in ΔOBC , $B + A = R$
Thus, $A + B = B + A$

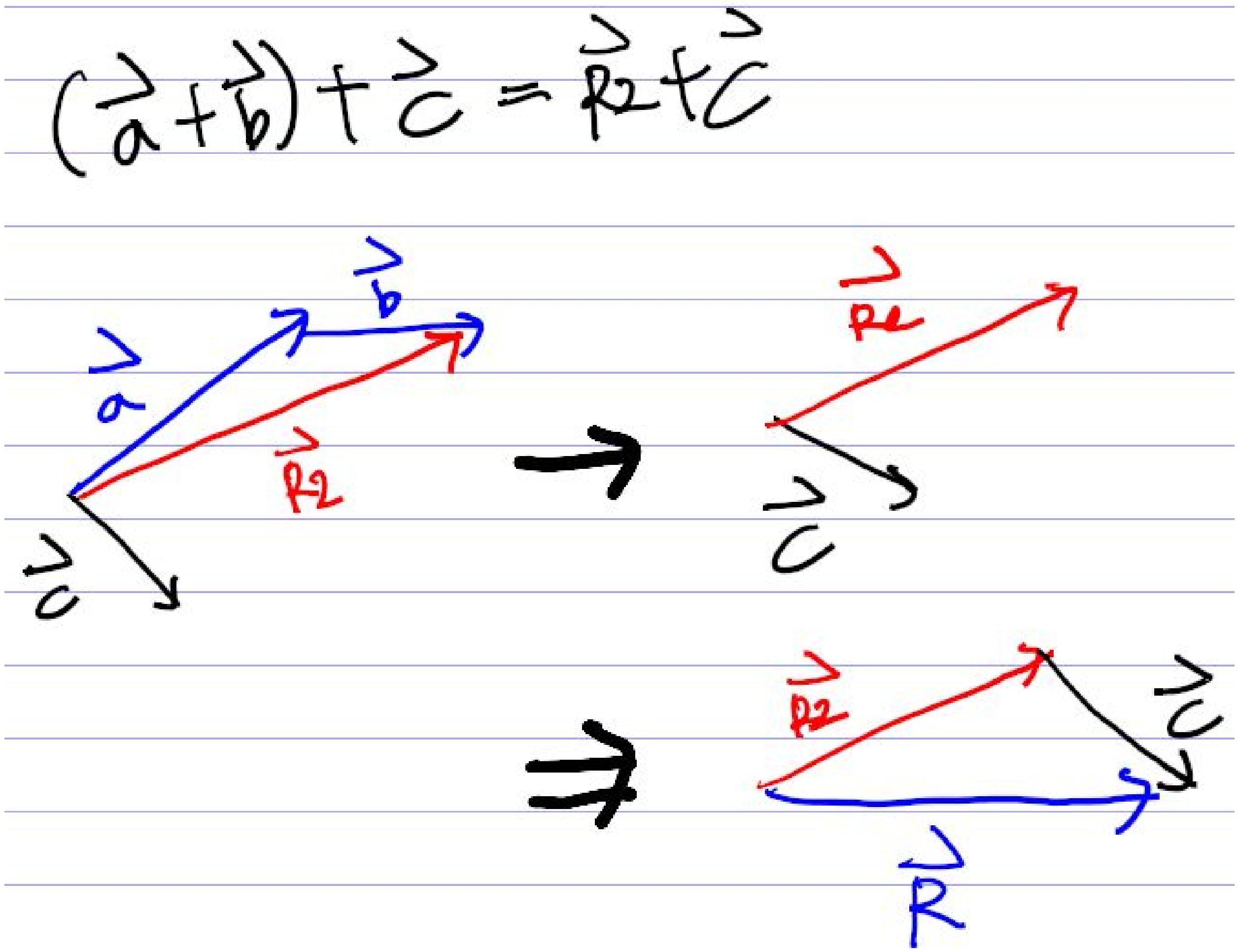
b) that vector addition is associative: $\vec{(a)} + (\vec{(b)} + \vec{(c)}) = (\vec{(a)} + \vec{(b)}) + \vec{(c)})$

Let $\rightarrow_a$, $\rightarrow_b$, $\rightarrow_c$ as the following.



$$\vec{a}+(\vec{b}+\vec{c}) = \vec{a}+\vec{R_1}$$





Thus, the resultant $\vec{R}$ is the same for $\vec{a} + (\vec{b} + \vec{c})$ and $(\vec{a} + \vec{b}) + \vec{c}$, which means that the vector addition is associative.

8. Research an example of the use of vectors, and explain how the mathematics is used, for example in engineering, computer animation, gaming, 3-D printing or GPS technology

The use of vectors is ubiquitous in artificial intelligence. Vectors are used to store internal representations of artificial intelligence models including deep learning networks and linear classifiers. To build an ML model, what we need to do first is to vectorize data, such as word vectorization. For example, the famous ML model in natural language processing is Word2Vec.

To briefly introduce how the model works, one of the things we can think of is changing text into numbers into vectors. The simplest way to change a word to a vector is to number the word and change it to a vector with only 1 element corresponding to the number and 0 for the rest. For example, there are a total of five words, and let's say that the word "puppy" is ranked twice. "Puppy" is then expressed as a five-dimensional vector with an only one-second element and all the rest zero. This way of changing words into vectors is called one-hot encoding. If there are N-words, each word is represented by an N-dimensional vector with only one element.

The model that changes words into vectors is called a word embedding model. word2vec is a representative model among word embedding models, using vector ideas.