MCV4U Unit 2 Assignment
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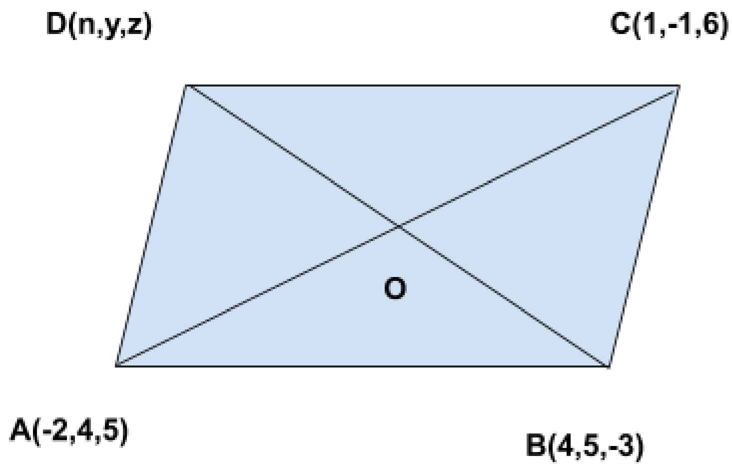
1. **Write an example of each of the following (assuming it is in 3-dimensional space).**
- A point lying on the x-axis.
  Example: $(2, 0, 0)$
- A point lying on the yz plane.
  Example: $(0, 2, 3)$
- A point lying on both the xy and xz planes.
  A point can lie on both xy planes and yz planes if and only if the point lies on the x-axis.
  Example: $(1, 0, 0)$
- A point lying on all three planes.
  Example: $(2, 3, 4)$
- A point lying on none of the three planes, but equidistant from the xz and yz planes.
  Example: $(0, 0, 0)$

2. **Triangle ABC has vertices A(4, 7, 7), B(1, 6, 5), and C(–2, 9, 8). What kind of triangle is ΔABC?**
- To begin with, getting the distance between two points having coordinates $(x1, y1, z1),\ (x2, y2, z2)$ are $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$.
- We could calculate the edge lengths of the given triangle as the following.
  - $AB = \sqrt{(1-4)^2 + (6-7)^2 + (5-7)^2} = \sqrt{9+1+4} = \sqrt{14}$
  - $AC = \sqrt{(-2-4)^2 + (9-7)^2 + (8-7)^2} = \sqrt{36+4+1} = \sqrt{41}$
  - $BC = \sqrt{(-2-1)^2 + (9-6)^2 + (8-5)^2} = \sqrt{9+9+9} = \sqrt{27}$
- We can get whether the given triangle ABC is a right angle triangle or not using the Pythagorean theorem.
  - $AC^2 = AB^2 + BC^2$
  - $\sqrt{41}^2 = \sqrt{14}^2 + \sqrt{27}^2$
  - $41 = 14 + 27$
  - Hence, the given triangle ABC is a right-ended triangle.

3. **The points (–2, 4, 5), (4, 5, –3), and (1, –1, 6) are three of four vertices of parallelogram ABCD. Explain why there are three possibilities for the location of the fourth vertex, and find the three points.**

There are three possibilities for the 4th vertex because either of the three vertices (A, B, or C) can be the 4th vertex.

<u>First Case.</u>

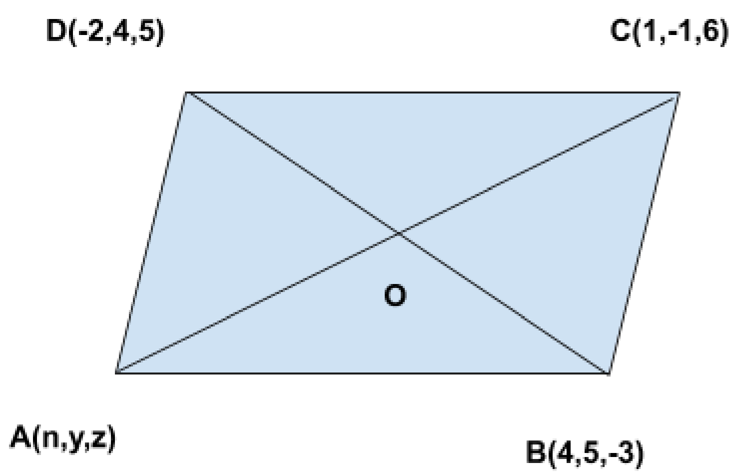


We will find the case where the 4th vertex is at the first vertex of the parallelogram. And then, I would like to calculate the known values of the points and use them to determine the value of Point O, the mid-point of the two points, which is the value of the summing each of unknown value of 4th vertex to that of opposite point.

With Point D and Point B, we can get the midpoint as follows. $O \equiv (\frac{n+4}{2}, \frac{y+5}{2}, \frac{z-3}{2})$
With Point A and Point C, we can get the determined value of midpoint as follows.
$O \equiv (\frac{-2+1}{2}, \frac{4-1}{2}, \frac{5+6}{2})$
We can calculate the values of undetermined midpoint like the following.
$\frac{n+4}{2} =- \frac{1}{2},\ \frac{y+5}{2} = \frac{3}{2},\ \frac{z-3}{2} = \frac{11}{2}\ => \ n =- 5,\ y =- 2,\ z = 14$

Thus, Point D is $(-5,\ 2,\ 14)$.

<u>Second Case.</u>

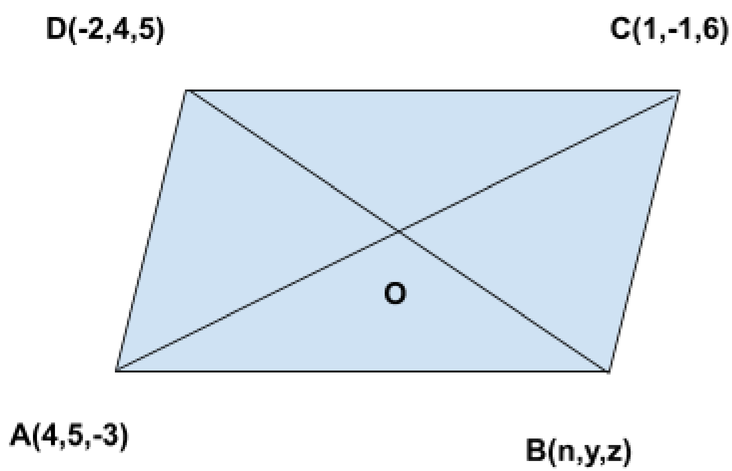


We will find the case where the 4th vertex is on the left lower vertex of the parallelogram. And then, I would like to calculate the known values of the points and use them to determine the value of Point O, the mid-point of the two points, which is the value of the summing each of unknown value of 4th vertex to that of opposite point.

With Point A and Point C, we can get the midpoint as follows. $O \equiv (\frac{n+1}{2}, \frac{y-1}{2}, \frac{z+6}{2})$
With Point A and Point C, we can get the determined value of midpoint as follows.
$O \equiv (\frac{4+2}{2}, \frac{5+4}{2}, \frac{-3+5}{2})$
We can calculate the values of undetermined midpoint like the following.
$\frac{n+1}{2} = 1,\ \frac{y-1}{2} = \frac{9}{2},\ \frac{z+6}{2} = 1\ \Rightarrow\ n = 1,\ y = 10,\ z = -4$

Thus, Point D is $(1,\ 10,\ -4)$.

Third Case.

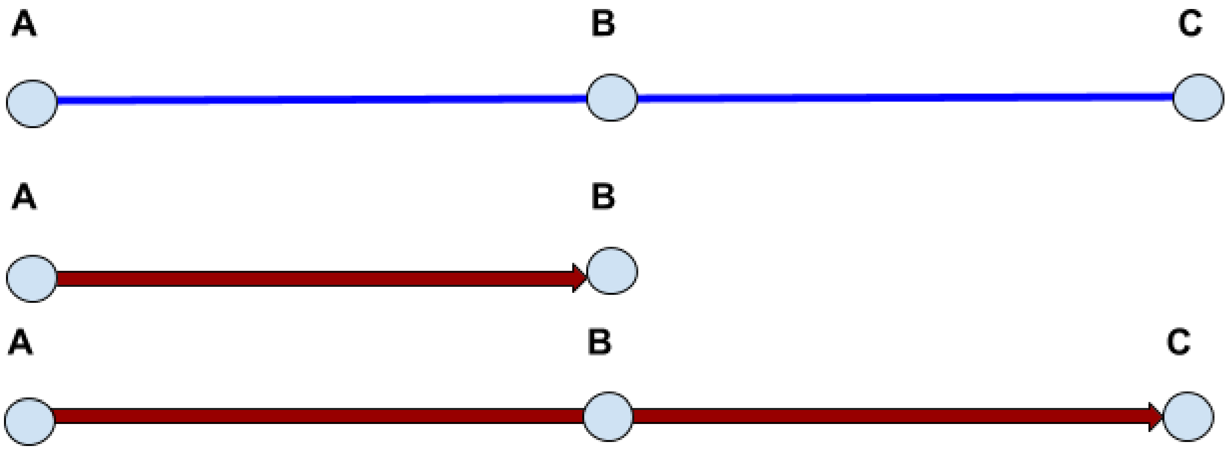


We will find the case where the 4th vertex is on the right lower vertex of the parallelogram. And then, I would like to calculate the known values of the points and use them to determine the value of Point O, the mid-point of the two points, which is the value of the summing each of unknown value of 4th vertex to that of opposite point.

With Point A and Point C, we can get the midpoint as follows. $O \equiv (\frac{n-2}{2}, \frac{y+4}{2}, \frac{z+5}{2})$
With Point A and Point C, we can get the determined value of midpoint as follows.
$O \equiv (\frac{5}{2}, 2, \frac{3}{2})$
We can calculate the values of undetermined midpoint like the following.
$\frac{n-2}{2} = \frac{5}{2},\ \frac{y+4}{2} = 2,\ \frac{z+5}{2} = \frac{3}{2}\ \Rightarrow\ n = 7,\ y = 0,\ z = -2$

Thus, Point D is $(7,\ 0,\ -2)$.

4. **The points A(–2, –1, z), B(2, 4, 3), and C(10, y, –1) are collinear. Find the values of y and z.**
   To understand the concept of collinear values, let us draw two possible vectors,
$\overline{AB}$ *and* $\overline{AC}$.

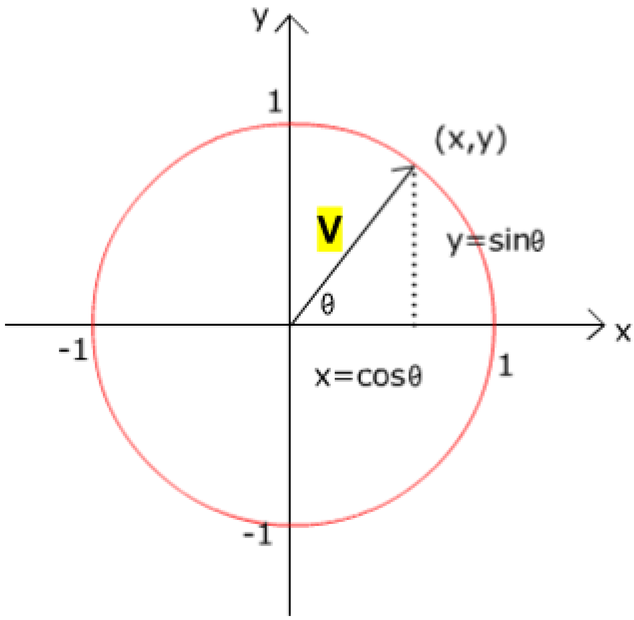


It is clearly appreciatable that the aforementioned vectors can be scaled up and down in length to shrink to another vector. In addition, these vectors are only inclined to change according to the sign of the scaling factor. For example, $2 \rightarrow_i + 4\rightarrow_j$ is a multiple of $1\rightarrow_i + 2\rightarrow_j$ by a factor of 2, while $-2\rightarrow_i - 4\rightarrow_j$ is only available for scaled down by a factor of 2.

$\overline{AB} = (2, 4, 3) - (-2, -1, z) = (4, 5, 3 - z)$
$\overline{AC} = (10, y, -1) - (-2, -1, z) = (12, y + 1, -1 - z)$
Using scaling logic, let the following.
$\overline{AB} = k\overline{AC} \Rightarrow (4, 5, 3 - z) = k(12, y + 1, -1 - z) = (12k, ky + k, -k - kz)$
Comparing the respective given elements.
$12k = 4,\ ky + k = 5,\ \ -k - zk = 3 - z$
$element\ X\ = k = \frac{1}{3}$
$element\ Y\ = \frac{1}{3}(y + 1) = 5 \Rightarrow y = 14$
$element\ Z = \frac{1}{3}(-1 - z) = 3 - z \Rightarrow z = 5$

Thus, the answer would be $y = 14,\ z = 5$

5. Explain the meaning of direction angles and their relation to direction vectors.
   Direction angle means the angle between a vector and the positive x-axis.



In the picture shown above, vector $\vec{v}$ has a direction angle θ while $\vec{v} = |\vec{v}|\,cos\theta i + |\vec{v}|\,sin\theta j$ where $|\vec{v}|$ is the magnitude of vector $\vec{v}$.

- What are the direction angles of the vector [–5, 1, 8]?