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Basic Skills Review Assignment

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September 15, 2018
MFH4U: Advanced Functions 12
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Part 1: Communication Problems

The first part of the assignment consists of 5 questions that require precise communication of their solutions. They will require a good understanding of the material presented in this review, but your mark will be based on the presentation and communication of your answers. Use the rubric provided to ensure you are meeting all of the expectations.

1. Rewrite the following relationships using function notation.

   a. An airplane needs to travel 400 km. Determine a function for the speed of the airplane, with the respect to time.

   Let s represent speed in km/h : s = *speed in km/h*

   Let t represent time in hour : t = *time in hour*

   $s(t) = \frac{400}{t}$

   b. An ice cream cone is left sitting in the hot sun. Sarah notices that the ice cream melts and loses half of its volume every 5 minutes. If the starting volume was 125 mL, determine a function for the volume, with respect to the number of hours it takes to travel.

   Let v represent volume in mL : v = *volume in mL*

   Let t represent time in minute : t = *time in minute*

   $v(t) = 125 \cdot (\frac{1}{2})^{\frac{t}{5}}$

   Scott wants to calculate the distance from his house to each of his friends' houses. If he drives at 50 km/h, find a function for the distance, with respect to the number of hours it takes to travel.

   Let d represent distance in km : d = *distance in km*

   Let time represent time in hour : t = *time in hour*

   $d(t) = 50t$

2. Find the inverses of each of the functions below algebraically.

- In order to find the inverses of each of the functions:
    - Step 1: f(x) is replaced with y to make the process easier.
    - Step 2: The switched equation is solved for y.
    - Step 3: y is replaced with $f^{-1}(x)$

a) $p(r) = 2r^2 + 2r - 1$

$p(r) = 2r^2 + r - 1$

$r = 2p^2 + p - 1$ ← inverse r and p, r=p

$r = 2(p^2 + p + \frac{1}{4} - \frac{1}{4}) - 1$

$r = 2(p^2 + p + \frac{1}{4}) - 1 - \frac{1}{4} \cdot 2$

$r = 2(p^2 + p + \frac{1}{4}) - 1 - \frac{1}{2}$

$r = 2(p + \frac{1}{2})^2 - \frac{3}{2}$

$r + \frac{3}{2} = 2(p + \frac{1}{2})^2$

$\frac{r}{2} + \frac{3}{4} = (p + \frac{1}{2})^2$

$\frac{2r+3}{4} = (p + \frac{1}{2})^2$

$\pm\sqrt{\frac{2r+3}{4}} = p + \frac{1}{2}$

$\pm\sqrt{\frac{2r+3}{4}} - \frac{1}{2} = p$

$\frac{\pm\sqrt{2r+3}}{2} - \frac{1}{2} = p$

$\frac{-1 \pm \sqrt{2r+3}}{2} = p$

$p^{-1}(r) = \frac{-1 \pm \sqrt{2r+3}}{2}$

b) $3y + 5x = 18$

$3y + 5x = 18$

$3y = -5x + 18$

$3x = -5y + 18$ ← inverse x and y, x=y

$3x - 18 = -5y$

$\frac{3x-18}{-5} = y$

$y^{-1} = \frac{-3x+18}{5}$

c) $h(t) = -4.9(t+3)^2 + 45.8$

$h(t) = -4.9(t+3)^2 + 45.8$

$t = -4.9(h+3)^2 + 45.8$ ← inverse t and h, t=h

$t - 45.8 = -4.9(h+3)^2$

$\frac{t-45.8}{-4.9} = (h+3)^2$

$\pm\sqrt{\frac{t-45.8}{-4.9}} = h+3$

$\pm\sqrt{\frac{t-45.8}{-4.9}} - 3 = h$

$h^{-1}(t) = \pm\sqrt{\frac{t-45.8}{-4.9}} - 3$

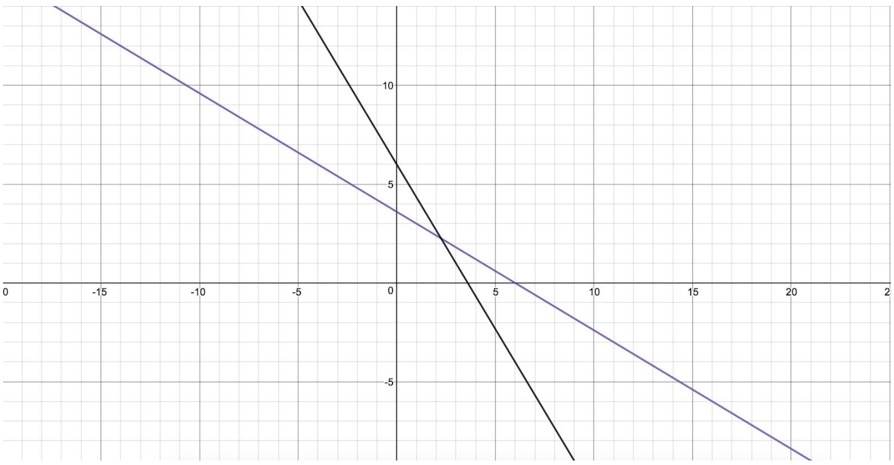
3. With the aid of graphs, explain whether or not the inverses in questions 2 are functions.

a) $p^{-1}(r) = \frac{(-1)\pm\sqrt{2r+3}}{3}$ is the inverse function of $p(r) = 2r^2 + 2r - 1$

[Figure 1. Green: $p(r) = 2r^2 + 2r - 1$, Red: $\frac{(-1)+\sqrt{2r+3}}{3}$, Green: $\frac{(-1)-\sqrt{2r+3}}{3}$]

In terms of function, for every input, there must be only one output. It is not a function because it does not pass the vertical line test. [Figure 1] shows that there are two outputs for one input. This graph represents the graph of inverse of quadratic function. If the graph represents only the half of the inverse graph, it would be a function. However, since it shows the whole graph of $p^{-1}(r) = \frac{(-1)+\sqrt{2r+3}}{3}$ and $p^{-1}(r) = \frac{(-1)-\sqrt{2r+3}}{3}$, it is not a function. Therefore, $p^{-1}(r) = \frac{\pm\sqrt{2r+3}-1}{3}$ is not a function.

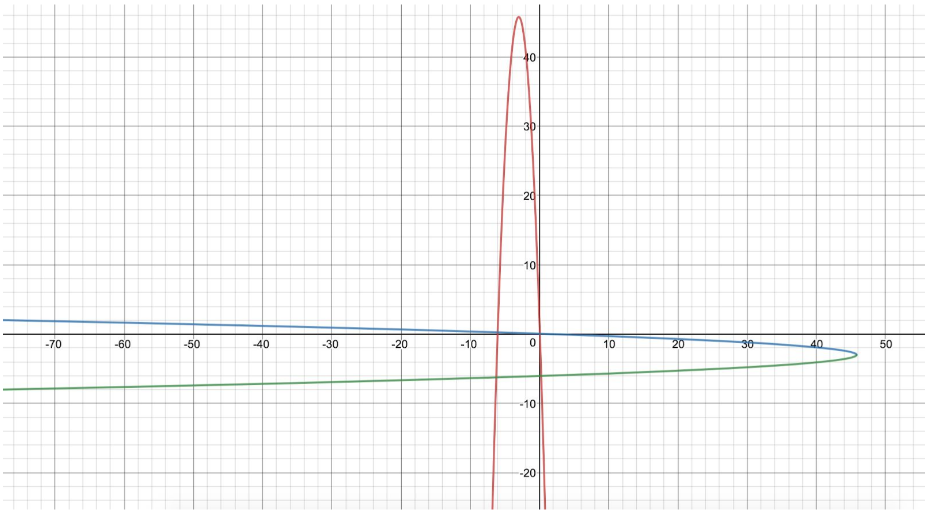
b) $y^{-1} = \frac{-3x+18}{5}$ is the inverse function of $3y + 5x = 18$



[Figure 2. Graph of $y^{-1} = \frac{-3x+18}{5}$]

This is the graph of $y^{-1} = \frac{-3x+18}{5}$. It is a function because it passes the vertical line test. In order to pass the vertical line test, for every input, there must be only one output. Moreover, this inverse function is reflected on y=x as an axis of symmetry. Therefore, $y^{-1} = \frac{-3x+18}{5}$ is a function.

c) $h^{-1}(t) = \pm\sqrt{\frac{t-45.8}{-4.9}} - 3$ is the inverse function of $h(t) = -4.9(t+3)^2 + 45.8$

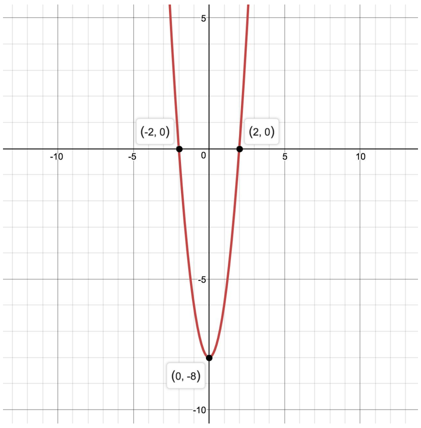


[Figure 3. Graph of $h^{-1}(t) = \pm\sqrt{\frac{t-45.8}{-4.9}}$]

$h^{-1}(t) = \pm\sqrt{\frac{t-45.8}{-4.9}}$ is not a function. First, this does not pass a vertical line test. This graph gives two outputs in one input. However, in terms of function, for every input, there must be only one output. It would be a function if the graph only shows half of the graph. Since the graph represents the both symbols, it is not a function.

4. For each of the functions below, state the domain and range, the restrictions, the intervals of increasing and decreasing, the roots, y-intercepts, and vertices.

a) $f(x) = 2x^2 - 8$



[Figure 4. Graph of $f(x) = 2x^2 - 8$]

- Domain
  - $\{x \mid x \in \mathbb{R}\}$
- Range
  - $\{y \mid y \geq -8, y \in \mathbb{R}\}$
- Restriction
  - No restriction for x
  - $y \geq -8$
- Intervals of increasing
  - $(0, \infty)$
- Intervals of decreasing
  - $(-\infty, 0)$
- Root : ± 2
  - $f(x) = 2x^2 - 8$

    Insert 0 to $f(x)$

    $0 = 2x^2 - 8$

    $0 = 2(x^2 - 4)$

    $0 = 2(x + 2)(x - 2)$

    $0 = (x + 2)(x - 2)$

    $x = \ \pm 2$
- Y-intercept: (0, -8)
  - $f(x) = 2x^2 - 8$

    $f(0) = 2x^2 - 8$

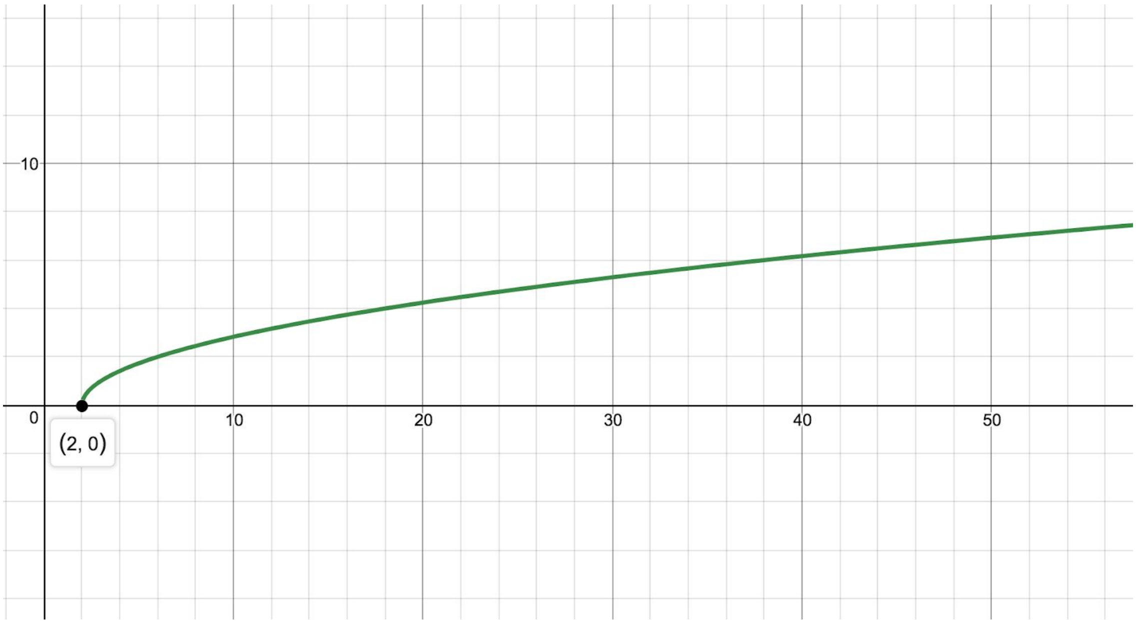
    $f(0) = 2(0)^2 - 8$

    $f(0) = 0 - 8$

    $f(0) = -8$

    $y = \ -8$
- Vertices
  - Since the vertices is the minimum value of f(x), it is (0, -8).

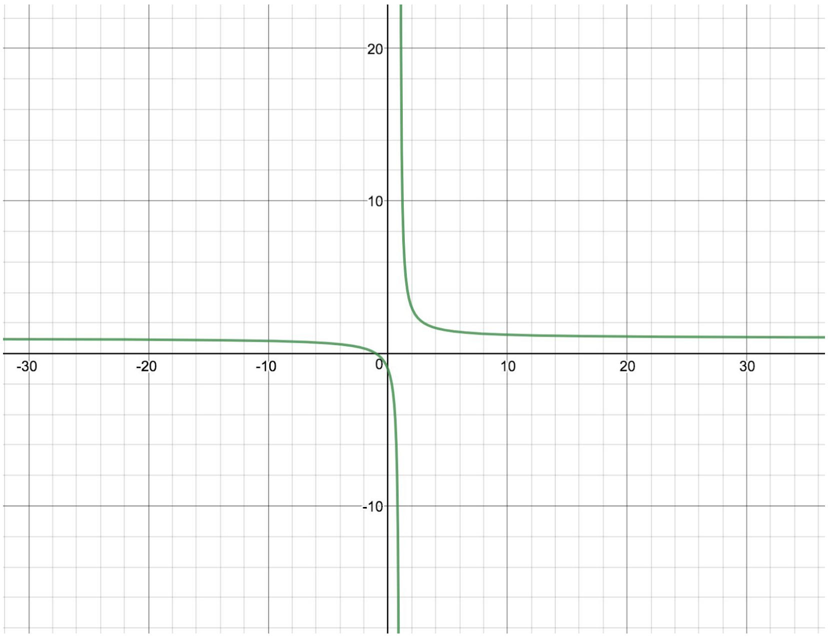
b) $f(x) = +\sqrt{x-2}$



[Figure 5. Graph of $f(x) = +\sqrt{x-2}$]

- Domain
  - $\{x \mid x \geq 2,\ x \in \mathbb{R}\}$
- Range
  - $\{y \mid y \geq 0, y \in \mathbb{R}\}$
- Restriction
  - $x \geq 2$
  - $y \geq 0$
- Intervals of increasing
  - $(2, \infty)$
- Intervals of decreasing
  - There is no intervals of decreasing in this function.
- Root: 2
  - $f(x) = +\sqrt{x-2}$
    Insert 0 to $f(x)$
    $0 = +\sqrt{x-2}$
    $0 = x - 2$
    $x = 2$
- Y-intercept: Undefined
  - $f(x) = +\sqrt{x-2}$
    $f(0) = +\sqrt{0-2}$
    $f(0) = +\sqrt{-2}$
    $f(0) = undefined$
- Vertices
  - According to [Figure 5], (2,0) will be the minimum value of f(x).

c) $f(x) = \frac{x+1}{x-1}$



[Figure 6. Graph of $f(x) = \frac{x+1}{x-1}$]

- Domain
  - $\{x \mid x \neq 1,\ x \in \mathbb{R}\}$
- Range
  - $\{y \mid y \neq 1,\ y \in \mathbb{R}\}$
- Restriction
  - $x \neq 1$
  - $y \neq 1$
- Intervals of increasing
  - None
- Intervals of decreasing
  - $(-\infty, \infty)$
- Root: -1
  - $f(x) = \frac{x+1}{x-1}$

    Insert 0 to $f(x)$

    $0 = \frac{x+1}{x-1}$

    $0 = x + 1$

    $x = -1$
- Y-intercept: (0, -1)
  - $f(x) = \frac{x+1}{x-1}$

    $f(0) = \frac{0+1}{0-1}$

    $f(0) = \frac{1}{-1}$

    $f(0) = -1$
- Vertices
  - According to [Figure 6], there is no vertices.

5. The point $(1,-2)$ is on the graph of $f(x)$. Describe the following transformations on $f(x)$, and determine the resulting point.

- In order to solve the transformation of functions, $y = af\{k(x-d)\} + c$ should be used to determine horizontal and vertical stretches or compressions, a horizontal translation, and a vertical translation.
  - First, a determines the vertical stretch/compressions and a reflection in the x-axis.
  - Second, k determines the horizontal stretch/compressions and a reflection in the y-axis.
  - Third, d determines a horizontal translation.
  - Fourth, c determines the y-intercept which is a vertical translation.

a) $g(x) = 2f(x) + 3$

When this functions is applied to $y = af\{k(x-d)\} + c$, $g(x) = 2f(x) + 3$ will change to $y = 2 \cdot f\{1(x-0)\} + 3$. 2 represents *a* and 3 represents *c*. 2 determines if the graph of this function has a vertical stretch or compression and a reflection on the x-axis. Since $|a| = |2|$ is bigger than 1, it is vertically stretched by the factor of $|2|$. Also, since 2 is bigger than 0, it is not reflected in the x-axis. *c* determines the vertical translation. Since *c* is bigger than 0, 3 > 0, the graph is translated up by 3. Therefore, $(1,-2)$ is transformed to $(\frac{1}{1} + 0,\ 2 \cdot (-2) + 3)$ which results (1, -1).

b) $g(x) = f(x+1) - 3$

When this functions is applied to $y = af\{k(x-d)\} + c$, $g(x) = f(x+1) - 3$ will change to $y = 1 \cdot f[\{x-(-1)\}] - 3$. -1 represents *d* and -3 represents *c*. The value of *d* which is -1 determines if there is a horizontal translation. Since -1 is smaller than 0, the graph is translated to the left by 1. Also, the value of *c* which is -3 determines the y-intercept. Since -3 is smaller than 0, the graph is translated down by 3. Therefore, $(1,-2)$ is transformed to $(\frac{1}{1} - 1,\ 1 \cdot (-2) - 3)$ which results $(0,-5)$.

c) $g(x) =-f(2x)$

When this functions is applied to $y = af\{k(x-d)\} + c$, $g(x) =-f(2x)$ will change to $y = (-1) \cdot f\{2(x-0)\} + 0$. -1 represents *a* and 2 represents *k*. -1 determines if the graph of this function has a vertical stretch or compression and a reflection on the x-axis. Since $|a| = |-1|$ is smaller than 1, it is reflected on the x-axis. Also, 2 which is the value of *k* determines the horizontal stretch or compression. Since $|2|$ is bigger than 1, the graph is compressed horizontally by the factor of $\frac{1}{|2|}$. Therefore, $(1,-2)$ is transformed to $(\frac{1}{2} + 0,\ (-1) \cdot (-2) + 0)$ which results $(\frac{1}{2}, 2)$.

d) $g(x) =-f(-x-1) + 3$

When this functions is applied to $y = af\{k(x-d)\} + c$, $g(x) =- f(-x-1) + 3$ will change to $y = (-1) \cdot f[-1\{x-(-1)\}] + 3$. $a =- 1$, $k =- 1$, $d =- 1$, $c = 3$. First, since *a* determine the vertical stretch or compression, according do $-1 < 0$, the graph is reflected in the x-axis. Second, *k* determines the horizontal stretch or compressions, according to the value of -1, it is also reflected in the y-axis. Third, *d* determines the horizontal translation, $-1 < 0$ shows that it is translated to the left by 1. Lastly, since the value of *c* determines the vertical translation, $3 > 0$ shows that the graph is translated up by the factor of 3. Therefore, $(1,-2)$ is transformed to ($\frac{1}{-1} - 1$, $(-1) \cdot (-2) + 3)$ which results $(-2, 5)$.

Part 2: Communication Presentation

The second part of this assignment requires you to create a multimedia presentation of the concepts involved in the topic specified by Question 6 (below). You may choose one of the following to do:

- *Record a short video* ☒
- *Record a short audio file* ☑
  - *Teaching or explaining the problem and its solution. Think of it like a radio show where viewers may or may not have mathematical background. How can you help all of the viewers to understand?*
- *Create a slideshow* ☒

6. Create a multimedia presentation to explain and justify your process steps to one of your solutions in Question 4.

- Link to the audio file
  - https://www.youtube.com/watch?v=tMWQicKYwzo

Script:

Hi, my name is Nawon Kwak, currently taking the advanced functions course in grade 12. Today, I am going to talk about the function $f(x) = 2x^2 - 8$ and its domain and range, restrictions, intervals of increasing and decreasing, roots, and y-intercept. This is a quadratic function. The standard form of this function is $f(x) = ax^2 + bx + c$. This function will show a parabola shape on the graph. The value a is the coefficient of x^2. If the value of a is negative, the parabola shape will open downward. However, when the value of a is positive, the parabola shape will open upward. In this case, since the value of a is 2, the graph will open upward.

The domain of $f(x) = 2x^2 - 8$ is $\{x \mid x \in \mathbb{R}\}$. The value of c determines the vertical translation of the graph. Since $-8 = c$, the graph is translated down by 8. Therefore, the range of this graph will be $\{y \mid y \geq -8, y \in \mathbb{R}\}$. Since it is a parabola-shaped graph, there is no restriction for x. However, since the vertical translation of this graph is -8, there is a restriction of y which is $y \geq -8$. The intervals of increasing would be $(0, \infty)$ and intervals of decreasing would be $(-\infty, 0)$. So far, I covered the domain, range, restriction, intervals of increasing, and intervals of decreasing. In terms of its roots, it is another term for x-intercept. In the graph of quadratic function, there are two roots. First, you need to substitute 0 into y. In this situation, 0 will equal to $2x^2 - 8$. When you solve for x, there will be two x values which are +2 and -2. Therefore, the roots of this function would be ± 2. In addition, to solve for the y-intercept, the equation would be $f(0) = 2(0)^2 - 8$. When you solve for the value of y-intercept, it is (0, -8). Lastly, since the vertice means the minimum value of the function, the value of vertices would be (0, -8).

Thank you for listening.