# Mid-Unit Assignment: Real-World Periodic Functions

**MHF4U**

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**Report**

1. **Periodic Functions**

   **1.1 Analog Clock**

   If we look at the classroom or home, there is at least one clock that helps us to notice the time. The first known clock with a water-powered escapement mechanism, which transformed rotational energy into periodic motions, dates from the 3rd century BC in ancient Greece; in the 10th century, Chinese engineers developed clocks with mercury-powered escapement mechanisms, followed by Arabic engineers. The clock has been the rudimentary device for human civilization from the past. (Wikipedia Contributors) Although the digital clock has been the trend in recent days, we can see many analog clocks even in the most digitized device, such as smart watches. Apple Watch provides the analog clock screen so that many users watch an analog clock through a digital screen every day. Thus, although the pendulum clock has disappeared recently, the appearance of analog clocks is still widely used.

   **1.2 Why the Analog Clock is Periodic**

   When it comes to analog clocks to say the time, the clock hands spin in circles and point to numbers ranging from 1 to 12. Since the hands return to their original location after an hour for the minute hand and 12 hours for the hour hand, the rotation of the hands reflects a periodic motion. This means that the analog clock's minute hand can be interpreted as a periodic function and trigonometric functions. I am going to create a trigonometric cosine equation that shows the height of the minute-hand of the clock (time in minutes) in this report.

2. **Calculations**

   **2.1 Data Table**

   At 2 p.m., I photographed the minute hand of the clock for an hour. The clock began at 2 p.m. and finished at 3 p.m., allowing the hand precisely one hour to fly. The shot is taken every five minutes, and Table 1 shows the angle at which the minute hand travels every five minutes.

| Time | Picture | Angle |
| --- | --- | --- |
| 14:00 |  | The angle hand travels in one minute: $\frac{2\pi}{60} = \frac{\pi}{30}$ |
| 14:05 |  | The angle hand travels in one minute: $\frac{2\pi}{60} = \frac{\pi}{30}$<br><br>The angle to travel from the initial (14:00) : $\frac{\pi}{30} \bullet 5 = \frac{5\pi}{30} = \frac{\pi}{6}$ |
| 14:10 |  | The angle hand travels in one minute: $\frac{2\pi}{60} = \frac{\pi}{30}$<br><br>The angle to travel from the initial (14:00) : $\frac{\pi}{30} \bullet 10 = \frac{10\pi}{30} = \frac{\pi}{3}$ |

| | | |
|---|---|---|
| 14:15 |  | The angle hand travels in one minute: $\frac{2\pi}{60} = \frac{\pi}{30}$<br><br>The angle to travel from the initial (14:00) : $\frac{\pi}{30} \bullet 15 = \frac{15\pi}{30} = \frac{\pi}{2}$ |
| 14:20 |  | The angle hand travels in one minute: $\frac{2\pi}{60} = \frac{\pi}{30}$<br><br>The angle to travel from the initial (14:00) : $\frac{\pi}{30} \bullet 20 = \frac{20\pi}{30} = \frac{2\pi}{3}$ |
| 14:25 |  | The angle hand travels in one minute: $\frac{2\pi}{60} = \frac{\pi}{30}$<br><br>The angle to travel from the initial (14:00) : $\frac{\pi}{30} \bullet 25 = \frac{25\pi}{30} = \frac{5\pi}{6}$ |

| | | |
|---|---|---|
| 14:30 |  | The angle hand travels in one minute: $\frac{2\pi}{60} = \frac{\pi}{30}$<br><br>The angle to travel from the initial (14:00) :<br>$\frac{\pi}{30} \bullet 30 = \frac{30\pi}{30} = \pi$ |
| 14:35 |  | The angle hand travels in one minute: $\frac{2\pi}{60} = \frac{\pi}{30}$<br><br>The angle to travel from the initial (14:00) :<br>$\frac{\pi}{30} \bullet 35 = \frac{35\pi}{30} = \frac{7\pi}{6}$ |
| 14:40 |  | The angle hand travels in one minute: $\frac{2\pi}{60} = \frac{\pi}{30}$<br><br>The angle to travel from the initial (14:00) :<br>$\frac{\pi}{30} \bullet 40 = \frac{40\pi}{30} = \frac{4\pi}{3}$ |

| | | |
|---|---|---|
| 14:45 |  | The angle hand travels in one minute: $\frac{2\pi}{60} = \frac{\pi}{30}$<br><br>The angle to travel from the initial (14:00) : $\frac{\pi}{30} \bullet 45 = \frac{45\pi}{30} = \frac{3\pi}{2}$ |
| 14:50 |  | The angle hand travels in one minute: $\frac{2\pi}{60} = \frac{\pi}{30}$<br><br>The angle to travel from the initial (14:00) : $\frac{\pi}{30} \bullet 50 = \frac{50\pi}{30} = \frac{5\pi}{3}$ |
| 14:55 |  | The angle hand travels in one minute: $\frac{2\pi}{60} = \frac{\pi}{30}$<br><br>The angle to travel from the initial (14:00) : $\frac{\pi}{30} \bullet 55 = \frac{55\pi}{30} = \frac{11\pi}{6}$ |
| 15:00 |  | The angle hand travels in one minute: $\frac{2\pi}{60} = \frac{\pi}{30}$<br><br>The angle to travel from the initial (14:00) : $\frac{\pi}{30} \bullet 60 = \frac{60\pi}{30} = 2\pi$ |

### 2.2 Determining Significant Values

- The base point is the top of the table, which is 1.5 m high that was measured with a tapeline from the floor of my home to the top of the table. Since the bottom section of the clock is just above the table, where the minimum of the minute-hand can be determined automatically, the base point was selected as the top of the table.
- The diameter of the clock was about 15cm, which is 0.15 m.
  - The initial y-value for the minute-hand is about 1.65m (adding 0.15m to 3.5m) above the ground, because the initial position (x=0) of the minute-hand of the clock was pointing at number 12.
  - The maximum value of the graph would be 1.65m and the minimum value of the graph would be 1.5m. The respective value would be the minute-hand's highest and lowest of the clock.
  - The amplitude of the graph would be 0.075m since the amplitude equals the absolute value of the difference between the maximum and minimum value of the graph which is divided by 2 ( $\frac{1.65-1.5}{2}$ )
  - The period is 60 minutes since it is the time that minute-hand completes a cycle and the head came back to the initial position. For the period of the trigonometric function, we can represent as $\frac{2\pi}{|k|}$. Since the period is 60 minutes, we can write that $\frac{2\pi}{|k|} = 60$. We can conclude that $period = \frac{\pi}{30}$.
  - Let us calculate the vertical translation of the clock using the fact that the maximum is at 1.65m above the ground, the minimum 1.5m above the ground while the amplitude is 0.075m. The parent trigonometric function does not have vertical translation since their maximum and minimum is 1 and -1.
    - $maximum = 1$
    - $minimum =- 1$
    - $amplitude = 1$
    - $maximum - amplitude = minimum + amplitude = 0$
  - For the minute-hand,
    - $maximum - amplitude = 1.65 - 0.075 = 1.575$
    - $minimum + amplitude = 1.5 + 0.075 = 1.575$
  - It means that the minute-hand graph of the equation will render a vertical translation up by 1.575m.
  - Remind that the cosine function of the form.
    - $f(x) \ = \ acos(k(x - d)) \ + \ c$

- Thus, the equation will be $f(x) = 0.075cos(\frac{\pi}{30}x) + 1.575$

- The cosine equation can be converted into the sine equation.
  - Using the known fact the following:
    - $cos(x) = sin(\frac{\pi}{2} - x)$
  - Thus, the equation would be $f(x) = 0.075sin(\frac{\pi}{30}(15 - x)) + 1.575$
- The equations that have been drawn from the analog clock are all trigonometric functions and periodic functions.
  - $cos(x) = sin(\frac{\pi}{2} - x)$
  - $f(x) = 0.075sin(\frac{\pi}{30}(15 - x)) + 1.575$

**2.4 Determining the y-values at the Given Points**

- Let us determine the y-values at the points $x = \frac{period}{3}$ and $x = \frac{5 \bullet period}{6}$ with cosine function.
  - $f(x) = 0.075cos(\frac{\pi}{30}x) + 1.575$
- $x = \frac{period}{3} = \frac{\pi}{90}\ (20\ minutes)$
  - $f(x) = 0.075cos(\frac{\pi}{30}(\frac{\pi}{90})) + 1.575$
  - $f(x) = 0.075cos(\frac{\pi^2}{2700}) + 1.575$
  - $0.075cos(\frac{\pi^2}{2700}) = 0.075 \cdot 0.99999331 = 0.07499949$
  - $f(x) = 0.07499949 + 1.575 = 1.64999949$
  - At around 20 minutes, the answer would be 1.64999949
    - (20, 1.64999949)
- $x = \frac{5 \bullet period}{3} = \frac{5}{3} \bullet \frac{\pi}{30} = \frac{5\pi}{90}\ (50\ minutes)$
  - $f(x) = 0.075cos(\frac{\pi}{30}(\frac{5\pi}{90})) + 1.575$
  - $f(x) = 0.075cos(\frac{5\pi^2}{2700}) + 1.575$
  - $0.075cos(\frac{5\pi^2}{2700}) = 0.075cos(\frac{\pi^2}{540}) = 0.075 \cdot 0.99983297$
    $= 0.07498747$
  - $f(x) = 0.07498747 + 1.575$
  - $f(x) = 1.64998747$
  - At around 50 minutes, the answer would be 1.64998747
    - (50, 1.64998747)

3. **Verification**

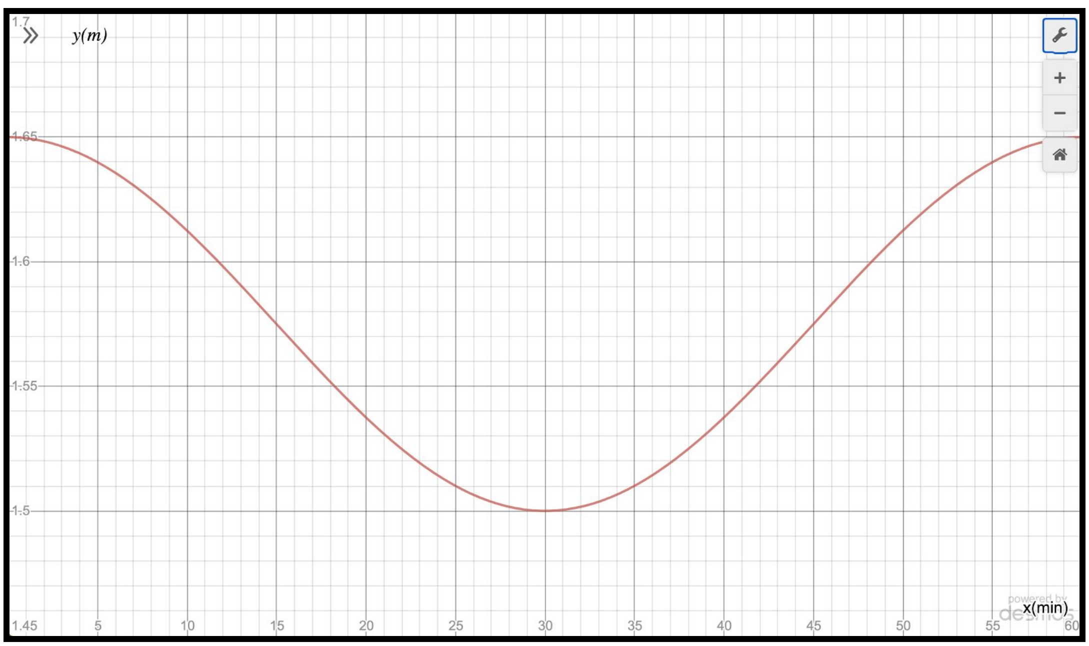
**3.1 Ways to Verify the Functions**

- The formulated function can be tested in a variety of ways. The first method involves entering numbers or points from the equation and determining if the answers are correct. When the two inputs, which are minutes, were substituted in the function, the acceptable height of the minute-hand appeared, much as in the previous two cases. It's a perfect way to ensure that the formulated function is right if the responses are matched to the case.
- The other way is to graph the function using technology. The graph itself, as well as the points on the graph, must be checked after the functions are drawn. The maximum and minimum values, original y-value, time, and amplitude had to be calculated for my task. One of the most useful sites is Mathway, which is the service that provides the feature of drawing function graphs but it requires users to subscribe to their service. Another drawing service that provides it for free is Desmos.com.
- As we did the first method in the previous part, we are going to draw the function by technology in this part.

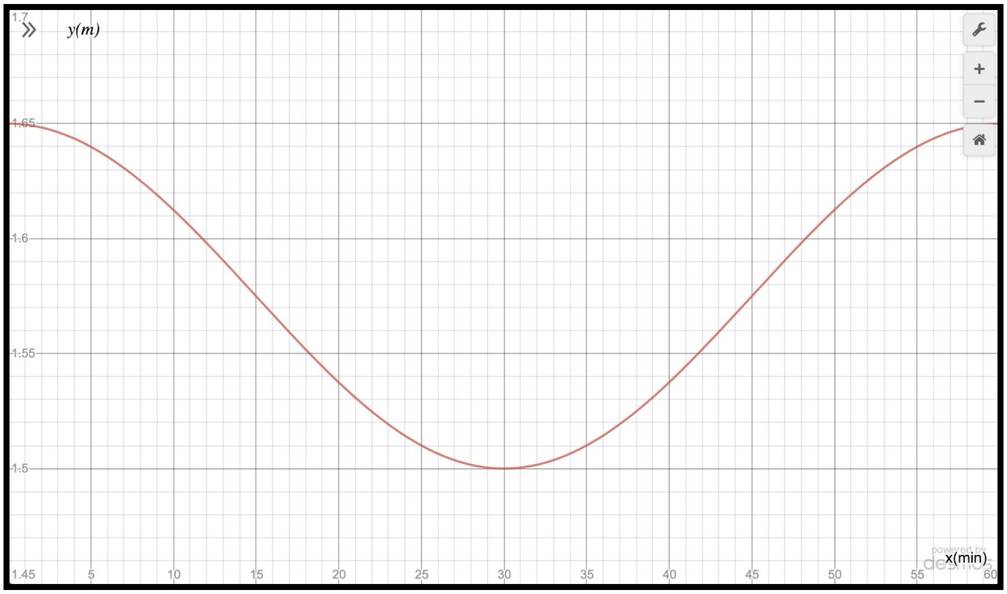
**<u>3.2 Graphing Using technology</u>**

- Graph of the analog clock under the condition that $\{x \mid 0 \le x \le 60,\ x \in R\}$

$f(x) = 0.075cos(\frac{\pi}{30}x) + 1.575$




$f(x) = 0.075sin(\frac{\pi}{30}(15 - x)) + 1.575$




4. **<u>Reflection</u>**

### 4.1 Reflection

The assignment provides me to think about how the real-life has dependent on the periodic functions. I also loved my time to invest to solve the assignment at how the statistical approach can be applied to clocks and daily life. I have raised the example of analog clocks, which had not been  thought that the mathematical concept is inherently applied into the concept of looking at the time. I was deeply inspired by how the analog clock has utilized the mathematical concepts while every part of daily life has now been influenced by the algebra concept. Clocks were groundbreaking inventions back then that provided and still provide critical and useful time to humans. Likewise, mathematics can be applied everywhere and has a huge effect on our lives. That is why I learned. As I mentioned in the previous part, since the minute-hand returned to its original or initial location after an hour, it was clearly a periodic feature. For me, converting the formulated cosine equation to a sine equation was difficult, before realizing that I could transform the equation using the trigonometric identity. This shows that I need more extra practice to memorize the rudimentary function including $cos(x) = sin(\frac{\pi}{2} - x)$ (to the level that I do not need to google to recall them).

**References**

Works Cited

Wikipedia Contributors. "History of Timekeeping Devices." *Wikipedia*, Wikimedia Foundation, 7 Feb. 2019, en.wikipedia.org/wiki/History_of_timekeeping_devices.