Unit Assignment: Operations on Functions

MHF4U

Virtual High School

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Jin Hyung Park

Question 1
Determine the composite function $f \circ g$ and $g \circ f$.
For each composite function state the domain and range.

a) $f(x) = 3x^2 - 5x + 6 \; and \; g(x) = x^2 + 3x$

This part should have been squared. (-0.5)

$f \circ g$	$g \circ f$
$f \circ g(x) = 3(x^2 + 3x) - 5(x^2 + 3x) + 6$ $= 3x^2 + 9x - 5x^2 - 15x + 6$ $= -2x^2 - 6x + 6$ Domain: $(-\infty, \infty)$, $\{x \mid x \in R\}$ Range: $(-\infty, \frac{21}{2}]$, $\{y \mid y \leq \frac{21}{2}\}$	$g \circ f(x) = (3x^2 - 5x + 6)^2 + 3(3x^2 - 5x + 6)$ $= 9x^4 - 30x^3 + 61x^2 - 60x + 36 + 9x^2 - 15x + 18$ $= 9x^4 - 30x^3 + 70x^2 - 75x + 54$ Domain: $(-\infty, \infty)$, $\{x \mid x \in R\}$ Range: $[27.09, \infty)$, $\{y \mid y \geq 27.09\}$

b) $f(x) = 2^x \; and \; g(x) = 3 - x$

$f \circ g$	$g \circ f$
$f \circ g(x) = 2^{(3-x)}$ Domain: $(-\infty, \infty)$, $\{x \mid x \in R\}$ Range: $(0, \infty)$, $\{y \mid y > 0\}$	$g \circ f(x) = 3 - 2^x$ Domain: $(-\infty, \infty)$, $\{x \mid x \in R\}$ Range: $(-\infty, 3)$, $\{y \mid y < 3\}$

c) $f(x) = sinx \; and \; g(x) = x$

$f \circ g$	$g \circ f$
$f \circ g(x) = sinx$ Domain: $(-\infty, \infty)$, $\{x \mid x \in R\}$ Range: $\{y \mid -1 \leq y \leq 1\}$	$g \circ f(x) = sinx$ Domain: $(-\infty, \infty)$, $\{x \mid x \in R\}$ Range: $\{y \mid -1 \leq y \leq 1\}$

Question 2.
Given the functions $f(x) = 10^x$ and $g(x) = 3x^2 + 10$ and $h(x) = x - 1$ what composition of functions would result in the following:

a) $y = 3(10^{2x}) + 10$

- $g \circ f(x) = 3(10^x)^2 + 10$
- $g \circ f(x) = 3(10^{2x}) + 10$
- Answer: the composition of $f(x)$ and $g(x)$ would result in the given function.

b) $y = 10^x - 1$

- $h \circ f(x) = (10^x) - 1$
- $h \circ f(x) = 10^x - 1$
- Answer: the composition of $h(x)$ and $f(x)$ would result in the given function.

c) $y = 3x^2 + 9$

- $h \circ g(x) = (3x^2 + 10) - 1$
- $h \circ g(x) = (3x^2 + 9)$
- Answer: the composition of $h(x)$ and $g(x)$ would result in the given function.

Question 3.

Given $h(x) = 2 - 3x$ and $g(x) = \frac{1}{3}x^2$, find $h^{-1}(x)$, $g^{-1}(x)$, $(g \circ h)^{-1}(x)$ and $(h^{-1} \circ g^{-1})(x)$. What can you conclude?

a) $h^{-1}(x)$

- $h(x) = 2 - 3x$
- $y = 2 - 3x$
- $x = 2 - 3y$
- $y = -\frac{x}{3} + \frac{2}{3}$
- $h^{-1}(x) = -\frac{x}{3} + \frac{2}{3}$

b) $g^{-1}(x)$

- $g(x) = \frac{1}{3}x^2$
- $x = \frac{1}{3}y^2$
- $y^2 = 3x$
- $y = \pm\sqrt{3x}$
- $g^{-1}(x) = \pm\sqrt{3x}$

c) $(g \circ h)^{-1}(x)$

- $y = \frac{1}{3}(2 - 3x)^2$
- $x = \frac{1}{3}(2 - 3y)^2$
- $3x = (2 - 3y)^2$
- $\pm\sqrt{3x} = 2 - 3y$
- $3y = \pm\sqrt{3x} - 2$
- $y = \frac{\pm\sqrt{3x} - 2}{3}$

d) $(h^{-1} \circ g^{-1})(x)$

- $y = (-\frac{x}{3} + \frac{2}{3}) \circ (\pm\sqrt{3x})$
- $y = (-\frac{\pm\sqrt{3x}}{3} + \frac{2}{3})$
- $y = \frac{\mp\sqrt{3x}-2}{3}$
- In addition, $(g \circ h)^{-1}(x) = (h^{-1} \circ g^{-1})(x)$, thus the value of d) is the same as c)

Question 4.
Suppose you need to choose between two car rental companies. You plan on driving for 8 hours at a constant speed of 60km/h.

Company 1 charges \$0.20/km and a daily fee of \$45.
Company 2 charges \$0.35/km and a daily fee of \$20.
Which company will give the lowest prices? Use composite functions to determine your answer.

- $d(t) = 60t$

Company 1	Company 2
$C(d) = 45 + 0.2d$ $C \circ d(t) = 45 + 0.2(60t) = 45 + 12t$ $t = 8$ $C(8) = 45 + 12 * 8 = 141$	$C(d) = 20 + 0.35d$ $C \circ d(t) = 20 + 0.35(60t) = 20 + 21t$ $t = 8$ $C(8) = 20 + 21 * 8 = 188$

- Answer: Company 1 will offer the lower prices.

Question 5.
From the following chart, choose two of the four function pairs.

a) Explain the meaning of $y = h(g(x))$ Volume is a function of time. (-1)

- When drawing a line that will be a radius, more time is spent to make the line longer. (which represents $g(x)$) The length of the line stretches the radius. (which represents $h(x)$) The longer the radius is, the wider the circle is. (which represents $h(g(x))$)
- When an airplane takes off, the higher the time it takes to go up, the higher the height of the plane is. (which represents $g(x)$) Height and air pressure are inversely proportional to each other. (which represents $h(x)$) Therefore, the higher the plane rises, the lower the pressure in the air. (which represents $h(g(x))$) Air pressure if a function of time.

b) Give a real life example of $y = h(g(x))$

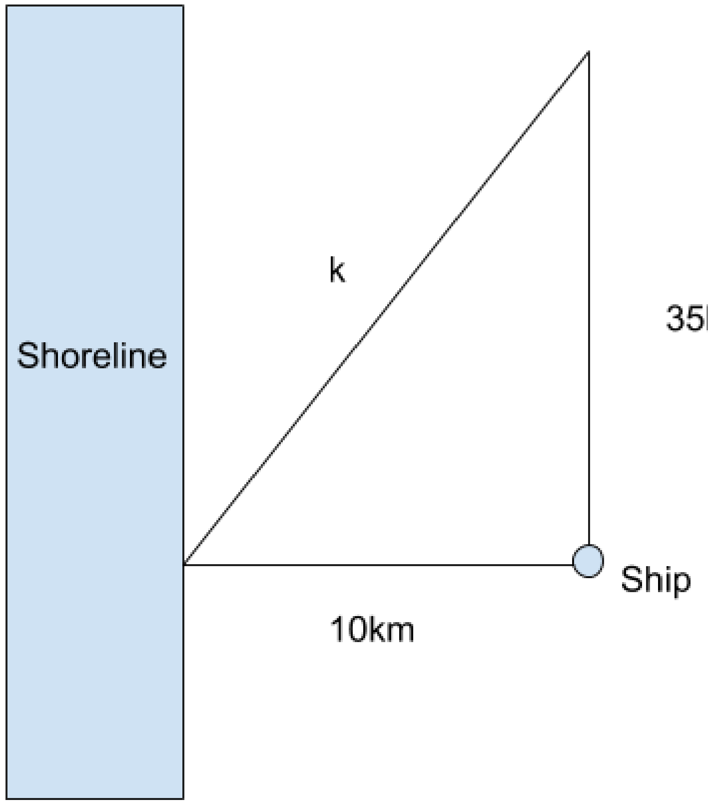
- There is a boy who draws a line. The line gets lengthened by 1cm per second.
- $P(t) = t$
  - Where P represents the length of the line in cm.
  - Where t represents the time in seconds.

- $I(p) = 2\pi p$
  - Where I represents the width of a circle in cm.
- $I(p(t)) = 2\pi t$
  - The circle gets 2π times wider per second.

- There is an airplane raising the height at 2km per 1 minute.
- $m(t) = 2t$
  - Where m represents the height in kilometers.
  - Where t represents the time in minutes.
- $F(m) =- 1.2m$
  - Where F represents air pressure which is inversely proportional to the height.
  - The unit of air pressure is Pa (Pascal).
- $F(m(t)) =- 1.2(2t) =- 2.4t$
  - This airplane will be released from its burden as $2.4Pa$ per minute.

Question 6.
Suppose a ship is sailing at a rate of 35km/h parallel to a straight shoreline. The ship is 10km from shore when it passes a lighthouse at 11 am.

a) Let k be the distance between the lighthouse and the ship. Let d be the distance from the ship that has travelled since 11 am. Express k as a function of d. Please include a diagram.



- t represents the time in house since 11 am
- $d(t) = 35t$
- $k(d) = \sqrt{100 + d^2}$

b) Express d as a function of t, the time elapsed since 11am.

- $d(t) = 35t$

c) Find $k \circ d$. What does this function represent?

- $h(t) = \sqrt{100 + (35t)^2}$
- $h(t) = \sqrt{100 + 1225t^2}$
- This function represents the distance between the lighthouse and the ship at t time since 11am.

Question 7.
Does the composition of functions display the commutative property? Give an example of each case to illustrate your answer.

When displays	When not displays

$f(x) = x + 1$, $g(x) = x - 1$ $f \circ g = (x-1) + 1 = x$ $g \circ f = (x+1) - 1 = x$ Thus, $f \circ g = g \circ f$	$f(x) = x + 5$, $g(x) = 4x$ $f \circ g = (4x) + 5 = 4x + 5$ $g \circ f = 4(x+5) = 4x + 20$ Thus, $f \circ g \neq g \circ f$