Vector Applications Unit Assignment
MCV4U Jin Hyung Park

1. Given $vector\ a\ = [2, 5, -7]$ and $vector\ b = [3, -6, -2]$, find

a) $vector\ a\ dot\ vector\ b$

$= [2, 5, -7] \bullet [3, -6, -2] = (2 * 3) + (5 * (-6)) + (-7) * (-2) = 6 - 30 + 14 = -24 + 14 = -10$

Thus, the answer is -10

b) A unit vector in the direction $vector\ b$

= $\widehat{h} = \frac{vector\ b}{|b|}$

= $\frac{[3, -6, -2]}{\sqrt{3^2+6^2+2^2}} = \frac{[3, -6, -2]}{\sqrt{9+36+4}} = \frac{[3, -6, -2]}{7}$

c) the angle between vector a and vector b

$cos\ \theta = \frac{vector\ a \bullet vector\ b}{|vector\ a||vector\ b|} = \frac{-10}{\sqrt{2^2+5^2+7^2} * \sqrt{3^2+6^2+2^2}}$

$cos\ \theta = -0.4617,\ \theta = 100^o$

d) A vector perpendicular to vector a

Let us say that $[vector\ a,\ b,\ c]\ is \perp to\ vector\ a$

$[vector\ a,\ b,\ c]\ *\ [2,\ 5,\ -7]\ = 0$

$2a + 5b - 7c = 0$

Substitute $a = 1, b = 1, c = 1$ to get the formula satisfied

$[1, 1, 1]\ and\ [2, 5, -7]\ are\ perpendicular\ each\ other$

2. A force $vector\ F\ = [-2, 1, 5]$ **in Newtons, pulls a sled through a displacement** $vector\ s = [-3, 5, 4]$ **in meters. The link between the dot product and geometric vectors and the calculation of work is** $Work = |vector\ F||vector\ s|cos\theta$

a) How much work is done on the sled by the force?

$Work = |vector\ F||vectorS|cos\theta$

As we learned in this lesson, If θ is the angle between the vectors a and b, then $a \bullet b = |a||b|cos\theta$.

Thus, we can replace the work formula with $Work = |vector\ F| \bullet |vector\ S|$.

As we can write the vectors as following, $vector\ F\ = -2i + j + 5k,\ vector\ s\ = -3i + 5j + 4k$

,

the following dot product calculation would be valid.

$(-2 * 3) + (1 * 5) + (5 * 4) = 6 + 5 + 20 = 31$

b) What is the minimum magnitude of force that could have been applied to the sled to obtain the same displacement? Explain your answer.

As stated earlier, $vector\ F\ \bullet vector\ S = |vector\ F||vectorS|\ cos\ \theta = 31$.

Thus, we can transpose an equation as the following. $vector\ F = \frac{31}{|vector\ S|cos\theta}$.

As we know that $|vector\ F|$ is smallest when the value of the denominator is largest.

To make the denominator the largest, we can render the case where $\theta = 0,\ cos\theta = 1$.

$$|vector\ F\ min| = \frac{31}{|vector\ s|} = \frac{31}{|(-3i+5j+4k)|} = \frac{31}{\sqrt{9+25+16}} = \frac{31}{\sqrt{50}} = \frac{31}{5\sqrt{2}}$$

3. Given that $vector\ a = [1, -3, 6]\ and\ vector\ b = [4, -5, -2]$, find

a) $vector\ a \times vector\ b$ and verify that it is perpendicular to both $vector\ a\ and\ vector\ b.$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 6 \\ 4 & -5 & -2 \end{vmatrix} = (6+30)\hat{i} - (-2-24)\hat{j} + (-5+12\hat{k})$$

$$= 36\hat{i} + 26\hat{j} + 7\hat{k}$$

As stated above, we can get $vector\ a \times vector\ b = 36\hat{i} + 26\hat{j} + 7\hat{k}$.

b) A $vector\ c$ such that $vector\ a \bullet (vector\ b \times vector\ c) = 0$. What is the relationship between the vectors $vector\ a$, $vector\ b$, and $vector\ c$ in this case, and why? Verify this.

If a vector c satisfy the relation $vector\ a \bullet (vector\ b \times vector\ c) = 0$, it means that we can say that $vector\ a,\ vector\ b,\ vector\ c$ are both coplanar. Since if $vector\ a \bullet (vector\ b \times vector\ c) = 0$, then we can say that $vector\ a$ is perpendicular to $(vector\ b \times vector\ c)$. Also, both $vector\ b\ and\ vector\ c$ are perpendicular to $(vector\ b \times vector\ c)$. Thus, each $vector\ a,\ vector\ b\ and\ vector\ c$ is perpendicular to the vector $(vector\ b \times vector\ c)$.

Thus, we can say that $vector\ a,\ vector\ b\ and\ vector\ c$ are coplanar.

4. Given $vector\ v = [3, 5, -4]\ and\ vector\ w = [4, -3, -2]$, find

a) $vector\ v \downarrow vector\ w$

To solve the projection v on w, $proj_w(V) = (\frac{v \bullet w}{|w|^2}) = \frac{12-15+8}{(4)^2+(-3)^2+(-2)^2}(4, -3, -2)$

$= \frac{5}{16+9+4}(4, -3, -2)$

$= (\frac{20}{29}, -\frac{15}{29}, -\frac{10}{29})$

b) $vector\ w \downarrow vector\ v$

To solve the projection w on v, $proj_v(W) = (\frac{v \bullet w}{|w|^2})v = \frac{12-15+8}{(3)^2+(5)^2+(-4)^2}(3, 5, -4)$

$= \frac{5}{9+25+16}(3, 5, -4)$

$= \frac{1}{10}(3, 5, -4)$

$= (\frac{3}{10}, \frac{5}{10}, -\frac{4}{10})$

$= (\frac{3}{10}, \frac{1}{2}, -\frac{2}{5})$

c) What does the magnitude of $vector\ w \downarrow vector\ v$ depend on?

$vector\ w \downarrow vector\ v\ = (\frac{3}{10}, \frac{1}{2}, -\frac{2}{5})$

Magnitude is equal to $\sqrt{\frac{9}{100} + \frac{1}{4} + \frac{4}{25}} = \sqrt{\frac{900+2500+1600}{10000}} = \sqrt{\frac{1}{2}}$

It depends on both $vector\ w\ and\ vector\ v$, as the formula is $\frac{v \bullet w}{|w|^2}$.

d) What does the direction of $vector\ w \downarrow vector\ v$ depend on?

$vector\ w \downarrow vector\ v\ = (\frac{3}{10}, \frac{1}{2}, -\frac{2}{5})$

$cos\ \theta = \frac{vector\ w \bullet vector\ v}{|vector\ w||vector\ v|} = \frac{(4i-3j-2k)(3i+5j-4k)}{(\sqrt{4^2+(-3)^2+(-2)^2})(\sqrt{3^2+5^2+(-4)^2})}$

$cos\ \theta = \frac{12-15+8}{\sqrt{29}\times\sqrt{50}} = \frac{5}{\sqrt{1H50}} = \frac{5}{37.0788}$

$\theta = cos^{-1}(\frac{5}{37.0788})$

5. Draw diagrams to explain the answers to the following questions.

a) Is it possible to have $vector\ a\ \downarrow vector\ b = 0$?

b) Is it possible to have $vector\ a\ \downarrow vector\ b\ undefined$?

c) Is it possible to have $vector\ a\ \downarrow vector\ b = vector\ b\ \downarrow vector\ a$

d) Explain why $vector\ a\ \downarrow vector\ c = vector\ a \downarrow (vector\ b\ \downarrow vector\ c)$.

6. Answer the following with either an explanation, a diagram or a proof.

a) If , what is the relationship between and ? If , what is the relationship between and ?

Date : / /

(a) We have $\vec{a}\cdot\vec{b} = \vec{a}\cdot\vec{c}$

$\Rightarrow \vec{a}\cdot\vec{b} - \vec{a}\cdot\vec{c} = 0$

$\Rightarrow \vec{a}\cdot(\vec{b}-\vec{c}) = 0$

$\Rightarrow \vec{a} = 0$ or, $\vec{b}-\vec{c} = 0$ or $\vec{a} \perp (\vec{b}-\vec{c})$

$\Rightarrow \vec{a} = 0$ or, $\vec{b} = \vec{c}$ or, $\vec{a} \perp (\vec{b}-\vec{c})$.

$\therefore \vec{b} = \vec{c}$ (relationship between $\vec{b}$ & $\vec{c}$) Ans

(b) We have $\vec{a}\times\vec{b} = \vec{a}\times\vec{c}$

$\Rightarrow \vec{a}\times\vec{b} - \vec{a}\times\vec{c} = \vec{0}$

$\Rightarrow \vec{a}\times(\vec{b}-\vec{c}) = \vec{0}$

$\Rightarrow \vec{a} = \vec{0}$ or, $\vec{b}-\vec{c} = \vec{0}$ or, $\vec{a} \parallel (\vec{b}-\vec{c})$

$\Rightarrow \vec{a} = \vec{0}$ or $\vec{b} = \vec{c}$ or, $\vec{a} \parallel (\vec{b}-\vec{c})$

$\therefore$ relationship between $\vec{b}$ & $\vec{c}$ is $\vec{b} = \vec{c}$.