**MDM4U - Probability Unit Assignment**
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1. **During a recent survey of ethnic backgrounds of 1000 people in a large city, 513 were Canadian, 148 were French, 72 were African and 56 were Asian and the remainder were from other groups. Calculate the probability that a person, selected at random from the population has:**

a. A canadian background
Probability (canadian): $\frac{Number\ of\ Favourable\ Outcomes}{Total\ Number\ of\ Possible\ Outcomes} = \frac{513}{1000} = 0.513$
Therefore, the probability that canadian is selected in random fashion is 0.513.

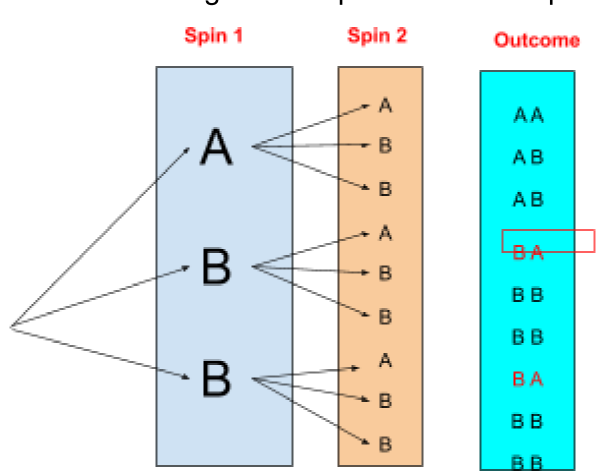
b. An african background
Probability (african): $\frac{Number\ of\ Favourable\ Outcomes}{Total\ Number\ of\ Possible\ Outcomes} = \frac{72}{1000} = 0.072$
Therefore, the probability that african is selected in random fashion in 0.072.

c. An “other” background
Remainder (Other groups): $1000 - (513 + 148 + 72 + 56) = 1000 - 789 = 211$
There are 211 people that are classified as “other” groups.
Probability (other groups): $\frac{Number\ of\ Favourable\ Outcomes}{Total\ Number\ of\ Possible\ Outcomes} = \frac{211}{1000} = 0.211$

**2. A spinner is divided into three equally sized regions as shown. The spinner is spun twice. For each probability you determine, express your answer as a fraction, decimal and percent.**

a. What is the probability of spinning A on the first spin?
Probability = $\frac{Number\ of\ Favourable\ Outcomes}{Total\ Number\ of\ Possible\ Outcomes} = \frac{1}{3}$
Since one region of the spinner should be recognized on the first spin, we can think of the problem as choosing one among three regions.

b. Draw a tree diagram to represent the sample space for both spins.



c. What is the probability of spinning A followed by B?

According to the tree diagram depicted above, there are two possible outcomes which are colored red (AB, AB). Thus, $Probability\ of\ A\ then\ B = \frac{Number\ of\ Outcomes\ that\ comes\ B\ after\ A}{Total\ Number\ of\ Possible\ Outcomes} = \frac{2}{9}$

d. What is the probability of getting the same letter on both spins?

The favourable outcomes that has same letter on both spins, which are colored as orange, are $\{A, A\}, \{B, B\}, \{B, B\}, \{B, B\}, \{B, B\}$ . Thus, the probability of getting same letter on spins is $\frac{5}{9}$.

**3. Explain the below.**

a. Explain the conditions where we would use it.
- A binomial distribution: the medium to be used when the number of successes and the number of trials is given in the first place. The model allows to calculate the probability of getting the number of success while repeating the given number of trials, with only the given result going to be success or failure.
- A geometric distribution: the medium to be used when the probability of success in a trial is present while the exact "nth" number of success is given. The model is well-known for utilizing to get the number of failures before the first success.
- A hypergeometric distribution: The model is used to pick a certain number of outcomes which is dependent on previous extraction from a given, but finite pool. It allows us to get the probability of the outcome that has the exact number of the type that we have an interest in.

b. Justify the formula used.
- A binomial distribution: $P(the\ number\ of\ successes; x) = {}_nC_x * p^x * (1-p)^{(n-x)}$
  - $n = the\ number\ of\ trials$
  - $x = the\ number\ of\ successes$
  - $p = the\ probability\ of\ getting\ success\ in\ a\ trial$
  - This formula is justified because the methodology of the binomial distribution is used to determine the probability of a certain number of "success" when the process is repeated over a certain number of trials.
- A geometric distribution: $P(the\ k^{th}\ trial) = (1-p)^{(k-1)} * p$
  - $k = the\ number\ of\ trials$
  - $p = the\ probability\ of\ success\ in\ a\ trial$
  - This formula is justified because a geometric distribution is utilized to decide the probability of each success after a specific, given number of failed trials.
- A hypergeometric distribution: $P(x) = \frac{kC_{x*}\ {}_{(n-k)}C_{(r-x)}}{nC_r}$
  - $n = the\ total\ number\ of\ items\ to\ be\ chosen\ from$
  - $k = the\ number\ of\ the\ type\ that\ we\ have\ an\ interest\ among\ outcomes$
  - $x = the\ number\ of\ the\ type\ that\ we\ have\ an\ interest\ that\ ends\ up\ in\ our\ sample$
  - This formula is justified since the methodology of the distribution is used to get the probability of the exact favourable outcomes when randomizing

performed trials. The formula is drawn by
$\{(favourable\ outcomes)\ X\ (unfavourable\ outcomes)\}\ /\ all\ combinations.$

c. Give an example of a situation in which it could be used.

- Example of binomial distribution: Jin is going to test the software that he made in order to submit the assignment. The probability of his program to pass the tests given by the professor is 50% because it is the first trial for him to code in the newly introduced language, Kotlin. He tests 15 cases. What is the probability that 5 are failing cases?
    - $P(x\ successes) = \ _nC_x * p^x * (1-p)^{(n-x)}$
    - $P(5\ failures) = {}_{15}C_5 * 0.5^5 * (1-0.5)^{15-5} = 0.0916$
    - Therefore, the probability that 5 cases out of the total 15 cases fail that Jin tests is about 0.0916 which is 9.16%.
- Example of geometric distribution: Airman Dion works for the fighter wing in the Air Force of South Korea as an enlisted officer. He can intercept a missile from North Korea in midair, with 90% of the probability for success. He has worked for three weeks in a row to serve the mission of counter striking against numerous ballistic missiles from Kim Jong Un's side. What is the probability that he fails his mission on the 21th day, after his phenomenal record for 20 days in a row?
    - $P(k^{th}\ trial) = (1-p)^{(k-1)} * p$
    - $P(21^{th}\ trial) = (1-0.10)^{(21-1)} * (0.10) = 0.0121$
    - Therefore, the probability that Dion will miss the missile on the 21th day is 0.0121, about 1.121%.
- Example of hypergeometric distribution: A random box contains numerous smart-devices including 10 iPhones, 12 iPads, and 21 Microsoft Surfaces. One Apple fanboy randomly picks 8 devices from the set without replacement. Find the probability that six out of 8 comes from Apple.
    - $P(x) = \frac{kCx *_{(n-k)} C_{(r-x)}}{nCr}$
    - $P(7/8\ chosen\ devices\ are\ made\ by\ Apple) = \frac{22C6*(43-22)C(8-6)}{43C8} = 0.1080503$
    - Therefore, the probability that the Apple fanboy picks six devices made by Apple out of 8 devices is 0.1080, about 10.8%.

**4. Two standard six-sided dice are rolled. One die is blue and the other is red.**

a. Create a table to represent the sample space
The following table represents the sample space.

| | Blue dice | | | | | | |
|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 |
| | 1 | 1 1 | 1 2 | 1 3 | 1 4 | 1 5 | 1 6 |
| Red dice | 2 | 2 1 | 2 2 | 2 3 | 2 4 | 2 5 | 2 6 |
| | 3 | 3 1 | 3 2 | 3 3 | 3 4 | 3 5 | 3 6 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | 4 | 4 1 | 4 2 | 4 3 | 4 4 | 4 5 | 4 6 |
| | 5 | 5 1 | 5 2 | 5 3 | 5 4 | 5 5 | 5 6 |
| | 6 | 6 1 | 6 2 | 6 3 | 6 4 | 6 5 | 6 6 |

**For each probability below, express the answer as a fraction, as a decimal, and as a percentage.**

c. What is the probability of rolling a sum greater than ten?

- In order for finding the probability of rolling a sum greater than ten, finding cells in the table with a sum greater than ten after adding the two numbers in the table.
- There are three cells coloured yellow below.

| | Blue dice | | | | | | |
|---|---|---|---|---|---|---|---|
| Red dice | | 1 | 2 | 3 | 4 | 5 | 6 |
| | 1 | 1 1 | 1 2 | 1 3 | 1 4 | 1 5 | 1 6 |
| | 2 | 2 1 | 2 2 | 2 3 | 2 4 | 2 5 | 2 6 |
| | 3 | 3 1 | 3 2 | 3 3 | 3 4 | 3 5 | 3 6 |
| | 4 | 4 1 | 4 2 | 4 3 | 4 4 | 4 5 | 4 6 |
| | 5 | 5 1 | 5 2 | 5 3 | 5 4 | 5 5 | 5 6 |
| | 6 | 6 1 | 6 2 | 6 3 | 6 4 | 6 5 | 6 6 |

$P(sum > 10) = \frac{3}{36} = 0.083 = 8.3\%$

d. What is the probability that the number on the red die is one larger than the number on the blue die?

| | Blue dice | | | | | | |
|---|---|---|---|---|---|---|---|
| Red dice | | 1 | 2 | 3 | 4 | 5 | 6 |
| | 1 | 1 1 | 1 2 | 1 3 | 1 4 | 1 5 | 1 6 |
| | 2 | 2 1 | 2 2 | 2 3 | 2 4 | 2 5 | 2 6 |
| | 3 | 3 1 | 3 2 | 3 3 | 3 4 | 3 5 | 3 6 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | 4 | 4 1 | 4 2 | 4 3 | 4 4 | 4 5 | 4 6 |
| | 5 | 5 1 | 5 2 | 5 3 | 5 4 | 5 5 | 5 6 |
| | 6 | 6 1 | 6 2 | 6 3 | 6 4 | 6 5 | 6 6 |

- In order for finding the probability that the number of red dice is larger than that of blue dice can be found by counting the cells above - 5 cells.

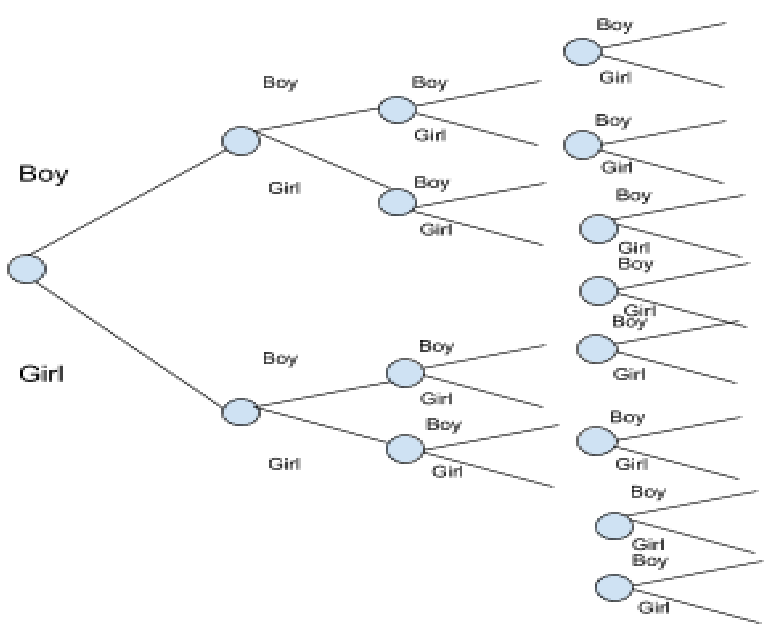
$P(\textit{number on red dice is one larger than number of blue dice}) = \frac{5}{36} = 0.138$

e. What is the probability that the sum of the two numbers is less than 11?

- It is much quicker to count the number of unfavourable outcomes and extract it from the total.
- Thus, the favourable outcomes would be $36(\textit{total}) - 3(\textit{greater than } 10) = 33$

$P(\textit{the sum of two numbers less than } 11) = \frac{33}{36}$

**5. Use a tree diagram to explain why the probability that a family with four children all have the same gender is 1/8. Assume that the probability of having a girl is equal to the probability of having a boy.**




The probability of having boys is $\frac{1}{16}$ and having girls is $\frac{1}{16}$. Thus, the probability of having boys and girls is $\frac{1}{16} + \frac{1}{16} = \frac{1}{8}$.

**6. In a library box, there are 8 novels, 8 biographies and 8 war history books. If Jack selects two books at random, what is the probability that the two books are of different types?**

Since there are three types of books, we can assume that the counting case of choosing only two novel books is $8C2.$ Since the number of novels, biographies and war history books is the same, the case is all the same. Thus, the answer would be the following.
$P(two\ books\ are\ of\ different\ types\ selecting\ two\ books\ at\ random) = \frac{(8C2)+(8C2)+(8C2)}{(24C2)} = \frac{84}{276}$

**7. The probability that Prasha will score above a 90 on a mathematics test is $\frac{4}{5}$. What is the probability that she will score above a 90 on exactly 3 of the 4 tests this quarter?**

Since the probability of getting the particular three tests scoring above a 90 is $(\frac{4}{5})^3 * (\frac{1}{5})$, multiplying it with the ways that she can get three out of four, $4C3.$ Thus, the answer is $4C3 * (\frac{4}{5})^3 * (\frac{1}{5}) = 0.4096$.

**8. A company manufacturing laptops believes that 5% of their computers are faulty. They take a sample of 30 computers. Showing your calculations, find the probability that:**

a. Two of the laptops are faulty.

Since the probability of getting two faulty laptops over the total of 30 is $(\frac{1}{20})^2 * (\frac{19}{20})^{28}$, multiplying it with the ways that one can get two out of thirty, $30C2$. Thus, the answer is $30C2 * (\frac{1}{20})^2 * (\frac{19}{20})^{28}$.

b. More than two of the laptops are faulty.

It is much quicker to count the number of unfavourable outcomes and extract it from the total. Since the probability of getting more than two of the faulty laptops is mutually exclusive than that of answer (a), we can get the probability by extracting answer (a) from the total probability (1). Thus, the answer is $1 - (30C2 * (\frac{1}{20})^2 * (\frac{19}{20})^{28})$.

**9. There are six cats and seven dogs in the local animal shelter. Four animals are chosen at random to visit a local school to educate the children on the great need for homes for these animals.**

a. What is the probability that exactly two of the animals are cats?

Since we need to choose exactly two to visit over six cats, the other two animals would naturally be dogs. Thus, multiplying each probability of two cats choosing from the total of six or two dogs selecting from the total of seven and dividing it with the probability of choosing two randomly from the total of thirteen animals, regardless of its species.
The answer would be the following: $\frac{(6C2)*(7C2)}{13C4} = \frac{315}{715} = \frac{63}{145}$

b. What is the probability that at least one cat is chosen?

To get the answer, we need to subtract the case where the cat is not selected at all while only the dog is chosen for visiting a local school from the probability of total(1). Thus, the answer would be the following: $(1 - \frac{(6C0)*(7C4)}{13C4}) = \frac{21}{715}$

**10. A certain hockey player scores on 18% of his shots.**

a. What is the probability he will score for the first time on his fifth shot?
Let us think a is 0.18. The probability of scoring for the first time on the fifth trial is getting the probability of failing four times in a row consecutively. To get the chance of failing on each of his shots, subtract a from the total probability which is 1. Thus, the answer would be the following.
$(1-0.18)^4 = 0.452 = 4.52\%$

b. What is the probability that it will take him fewer than 3 shots to score?
To get the probability, considering two cases where first shots were scored or first shots were failed while second shots did it. Thus, the answer would be the following.
$P(successful\ first\ shot) + P(failure\ of\ first\ shot\ while\ successful\ second\ shot) =$
$a + (1-a)a\ = 0.18 + 0.82 * 0.18 = 0.3276 = 32.76\%$

c. How many shots should he expect to take before scoring?

To begin with, getting the equation of the probability that player fails $n$ time before his first successful shots: $a(1-a)^n$. This equation introduces random variable $X$ that explains the amount of consecutive failures before his first scoring. $P(X = n) = a(1-a)^n$

Since each of possibilities is independent which means that the preceding calculation does not influence the subsequent calculation. Thus, we can think of this as a binomial distribution by writing down the following. $X \sim B(n, 0.18)$. To get a mathematical expectation from binomial distribution, think of the equation of the following. $E = np\ (n\ tries\ and\ p\ means\ possibilities)$ The expected number of tries for successful shots is n, and the expected number of the first successful goals in n tries is the following. $(E = 1) = n(0.18)$ , $1 = n(0.18)$. Thus, n is around 5.555 shots.

**11. In a multiple choice quiz, there are 6 questions. Each question has 5 possible answers. A student guesses at each question.**
a. Find the probability that the student gets 4 answers correct.
To find the probability, we need to acknowledge that the case is a binomial distribution since getting each question correct is an independent case. The probability of success, p, is $\frac{1}{5}$ since there is only one answer from five possible choices. Thus, we can get the probability by multiplying with the ways that one can get four out of six. $(6C4) * (\frac{1}{5})^4(\frac{4}{5})^2 = 1.536\%$

b. Find the probability that the student passes (i.e. gets at least 50%)
Let X is a random variable that stands for the number of questions that the student marks correct. In order for him to pass the exam, he needs to pass at least three questions (50%) out

of the total six questions. Thereby, we would need to solve the following by subtracting the unfavourable cases from the total possibilities(which is 1). $P(X \geq 3) = 1 - (1 \leq p \leq 2)$

- $P(X = 1) = 6C1 * (\frac{1}{5})^1 * (\frac{4}{5})^5$
- $P(X = 2) = 6C2 * (\frac{1}{5})^2 * (\frac{4}{5})^4$
- $P(X \geq 3) = 1 - (0.393216 + 0.24576) = 1 - 0.638976 = 0.361024$

<u>c. What is the student's expected number of correct answers?</u>

To get the expected number of correct answers, consider using the formula for solving mathematical expectation which is $E(X) = np$. We can write the equation as $X \sim B(6, \frac{1}{5})$, thus the answer would be the following. $E(X) = 6 * (\frac{1}{5}) = \frac{6}{5} = 1.12$