Trigonometric Differentiation and Applications Unit Assignment

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1. Find the derivatives of each of these functions.

a) $y = (x^4 + csc(x))^3$

$= 3(x^4 + csc(x))^2 \frac{d}{dx}[x^4 + csc(x)]$

$= 3(x^4 + csc(x))^2 (4x^3 + \frac{d}{dx}[csc(x)])$

Since $\frac{d}{dx}[csc(x)] =- csc(x)cot(x)$, We can write the following:

$= 3(x^4 + csc(x))^2 (4x^3 - csc(x)cot(x))$

b) $y = \frac{sec(4x)}{sin(x)}$

$= \frac{d}{dx}[\frac{\frac{1}{cos(4x)}}{sin(x)}] = \frac{d}{dx}[\frac{1}{cos(4x)} \bullet \frac{1}{sin(x)}]$

$= \frac{d}{dx}[sec(4x) \bullet csc(x)]$

$= sec(4x)\frac{d}{dx}[csc(x)] + csc(x)\frac{d}{dx}[sec(4x)]$

Since $\frac{d}{dx}[csc(x)] =- csc(x)cot(x)$, We can write as the following.

$= sec(4x)(- csc(x)cot(x)) + csc(x)\frac{d}{dx}[sec(4x)]$

Since $\frac{d}{dx}[sec(x)] = sec(x)tan(x)$, We can write as the following.

$=- sec(4x)csc(x)cot(x) + 4csc(x)sec(4x)tan(4x)$

c) $y = csc(\sqrt{3x^2 + 1})$

$= \frac{d}{dx}[csc(3x^2 + 1)^{1/2}]$

Since $\frac{d}{dx}[csc(x)] =- csc(x)cot(x)$, We can write as the following.

$=- csc(3x^2 + 1)^{\frac{1}{2}} cot(3x^2 + 1)^{\frac{1}{2}} (\frac{1}{2}(3x^2 + 1)^{-\frac{1}{2}}) \bullet 6x$

$=- \frac{cot(3x^2+1)^{\frac{1}{2}} csc(3x^2+1)^{\frac{1}{2}}}{2(3x^2+1)^{\frac{1}{2}}}(6x + 0)$

$=- \frac{3cot((3x^2+1)^{\frac{1}{2}}) csc((3x^2+1)^{\frac{1}{2}})x}{(3x^2+1)^{\frac{1}{2}}}$

d) $y = \frac{cos^2(x)}{ln(3x+4)}$

$= \frac{d}{dx}[\frac{cos^2(x)}{ln(3x+4)}]$

$= \frac{\frac{d}{dx}[cos^2(x)] \bullet ln(3x+4) - cos^2(x)\frac{d}{dx}[ln(3x+4)]}{ln^2(3x+4)}$

$= \frac{(-sin(2x)) \bullet ln(3x+4) - cos^2(x) \bullet \frac{1}{3x+4} \bullet \frac{d}{dx}[3x+4]}{ln^2(3x+4)}$

$= \frac{(-sin(2x))ln(3x+4) - \frac{cos^2(x)(3 \bullet \frac{d}{dx}[x] + \frac{d}{dx}[4])}{3x+4}}{ln^2(3x+4)}$

$= \dfrac{-sin(2x)ln(3x+4) - \frac{3cos^2(x)}{3x+4}}{ln^2(3x+4)}$

$= \dfrac{-sin(2x)ln(3x+4)(3x+4) - 3cos^2(x)}{(3x+4)ln^2(3x+4)}$

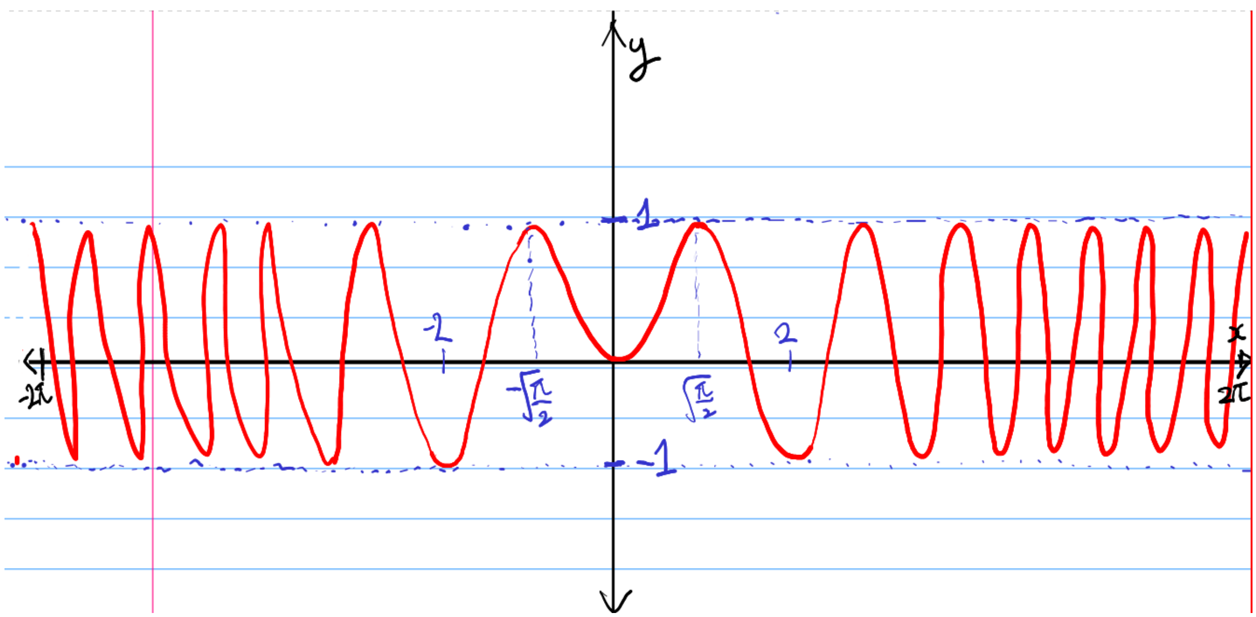
2. Use the process of implicit differentiation to find $\frac{dy}{dx}$ given that $sin(4x) + sin(2y) = 1$.

$\frac{d}{dx}(sin(4x) + sin(2y)) = \frac{d}{dx}(1)$.

$=> 4cos(4x) + 2cos(2y)\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-2cos(4x)}{cos(2y)}$

3. Using curve sketching methods, sketch the graph of the function $y = sin(x^2)$on the interval $-2\pi \leq x \leq 2\pi$. Make sure that you include all steps, charts, and derivations details.



The function is not periodic; but symmetric about the y-axis since $y(x) = y(-x)$.
The range of the function is $[-1, 1]$.

To differentiate the function, $y' = (cos(x^2))2x$
Maximum and minimum occurs when the first derivative is zero.

$2xcos(x^2) = 0,\ x = 0\ or\ cos(x^2) = 0$

$x^2 = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}...$

$y' = 0\ when\ x = 0, \pm\sqrt{\frac{\pi}{2}}, \pm\sqrt{\frac{3\pi}{2}}, \pm\sqrt{\frac{5\pi}{2}}, \pm\sqrt{\frac{7\pi}{2}}, \pm\sqrt{\frac{9\pi}{2}}...$

Using all the information above, we can draw the graph of $y = sin(x^2)$.

4. Find the equation of the tangent to the curve $y = \frac{sin(3x)}{cos(x)}$ at $\frac{\pi}{4}$.

$y(\frac{\pi}{4}) = \frac{sin(\frac{3\pi}{4})}{cos(\frac{\pi}{4})} = 1$

Thus, we find the point $(\frac{\pi}{4}, 1)$at first.

$\frac{dy}{dx}(\frac{sin(3x)}{cos(x)}) = \frac{3cos(3x)\bullet cos(x)+sin(3x)sin(x)}{cos^2(x)}$

We can substitute x with $\frac{\pi}{4}$ to get the highest point.

$\frac{dy}{dx}(\frac{sin(3x)}{cos(x)})|_{x=\frac{\pi}{4}} = \frac{3cos(\frac{3\pi}{4})cos(\frac{\pi}{4})+sin(\frac{3\pi}{4})sin(\frac{\pi}{4})}{cos^2(\frac{\pi}{4})} = \frac{-3\bullet 1+1\bullet 1}{1} =- 2$

Thus, $y'(\frac{\pi}{4}) =- 2$.

We can write the tangent curve as the following with given information.

$y - 1 =- 2(x - \frac{\pi}{4})$

$y =- 2x + \frac{\pi}{2} + 1$

5. The movement of the crest of a wave is modelled with the function $f(x) = 0.2cos(4t) + 0.3sin(5t)$. Find the maximum height of the wave and the time at which it occurs. (6 marks)

$f(x) = 0.2cos(4t) + 0.3sin(5t)$

To find the first derivative and get when the derivative is equal to zero.

$f'(x) = (- 0.2)4sin(4t) + 0.3(5)cos(5t) = 0$

$- 0.8sin(4t) =- 1.5cos(5t)$

$8sin(4t) = 15cos(5t)$

$\frac{sin(4t)}{cos(5t)} = \frac{15}{8}$

This is possible only when $t = \frac{\pi}{2} + 2n\pi$ since both $sin(4t)$ and $cos(5t)$ should be 1.

$When\, n = 0,\ t = \frac{\pi}{2} \ => \ 8sin(\frac{4\pi}{2}) = 8sin(2\pi) = 0,\ 15cos(\frac{5\pi}{2}) = 15(0) = 0$

Thus, we can conclude that max value of the first derivative can be found at $t = \frac{\pi}{2}$.

$f(x)|_{t=\frac{\pi}{2}} = 0.2cos(\frac{4\pi}{2}) + 0.3sin(\frac{5\pi}{2}) = 0.2cos(2\pi) + 0.3sin(2\pi + \frac{\pi}{2})$

$= 0.2 + 0.3 = 0.5$

Thus, the max of the function is $\frac{1}{2}$ which occurs at $t = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \frac{13\pi}{2}$...In general, $t = 2n\pi + \frac{\pi}{2}$

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6. Determine the 19th derivative of the function $y = sin(x)$. Fully explain the process you used to determine this. (4 marks)

To differentiate the given function, we can write the following.

$\frac{dy}{dx} = cos(x)$

Differentiate again and again and again.

$\frac{d^2y}{dx^2} = \frac{d}{dx}(cos(x)) =- sin(x)$

$\frac{d^3y}{dx^3} = \frac{d}{dx}(- sin(x)) =- cos(x)$

$\frac{d^4y}{dx^4} = \frac{d}{dx}(- cos(x)) = sin(x)$

Thus, $\frac{d^4y}{dx^4} = y$

Now, we know the fact that differentiating four times renders the same and given equation.

$\frac{d^4}{dx^4}(\frac{d^4y}{dx^4}) = \frac{d^4}{dx^4}(y) = \frac{d^4y}{dx^4}$

$\frac{d^8y}{dx^8} = y$

$\frac{d^4}{dx^4}(\frac{d^8y}{dx^8}) = \frac{d^4}{dx^4}(y) = \frac{d^4y}{dx^4}$

$\frac{d^{12}y}{dx^{12}} = y$

$\frac{d^4}{dx^4}(\frac{d^{12}y}{dx^{12}}) = \frac{d^4}{dx^4}(y) = \frac{d^4y}{dx^4}$

$\frac{d^{16}y}{dx^{16}} = y$

Thus, $\frac{d^{16}y}{dx^{16}} = sin(x)$

Differentiating x with one time by step-by-step method.

$\frac{d^{17}y}{dx^{17}} = \frac{dy}{dx} = cos(x)$

$\frac{d^{18}y}{dx^{18}} = \frac{d^2y}{dx^2} =- sin(x)$

$\frac{d^{19}y}{dx^{19}} = \frac{d^3y}{dx^3} =- cos(x)$

Thus, the 19th differentiating values of the given equation is -cos(x).