Unit Assignment: Trigonometric Functions and Graphs

MHF4U

Virtual High School

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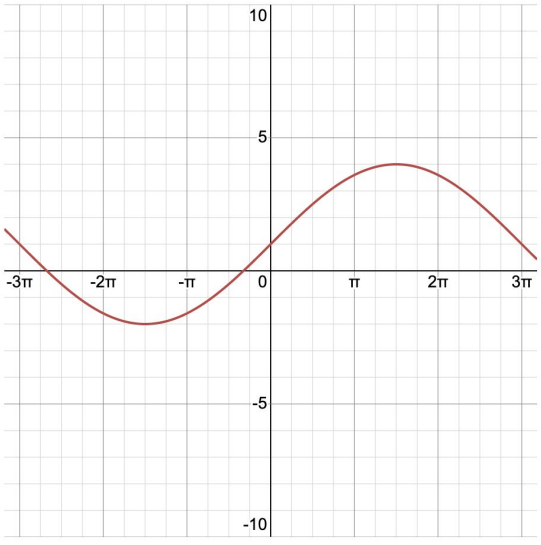
Question 1.

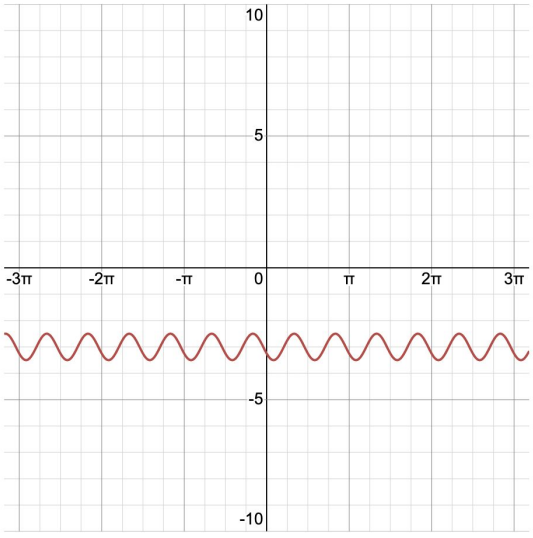
a) $f(x) = 3sin(\frac{1}{3}x) + 1$

- Use the form $asin(b(x-c)) + d$ to find the variables.
  - $a = 3$, $b = \frac{1}{3}$, $c = 0$, $d = 1$
- Domain: $(-\infty, \infty)$, $\{x|x \in R\}$
- Range: $[-2, 4]$, $\{y|-2 \leq y \leq 4\}$
  - Find the magnitude of the trig term of the function by taking the absolute value of the coefficient.
  - coefficient = 3
  - Find the lower bound of the range.
  - The lower bound of the range for sine is found by substituting the negative magnitude of the coefficient into the equation.
  - $y = -3 + 1$
  - $y = -2$
  - Find the upper bound of the range.
  - The upper bound of the range for sine is found by substituting the positive magnitude of the coefficient into the equation.
  - $y = 3 + 1$
  - $y = 4$
- Minimum and Maximum values:
  - local minimum: $(\frac{9\pi}{2} + 6\pi n, -2)$, when n *is any integer*
  - local maximum: $(\frac{3\pi}{2} + 6\pi n, 4)$, when n *is any integer*
- Period: $\frac{2\pi}{\frac{1}{3}} = 6\pi$
- Phase Shift: 0 to the right
- Amplitude: 3



b) $f(x) = -\frac{1}{2}cos(4x - \frac{\pi}{3}) - 3$

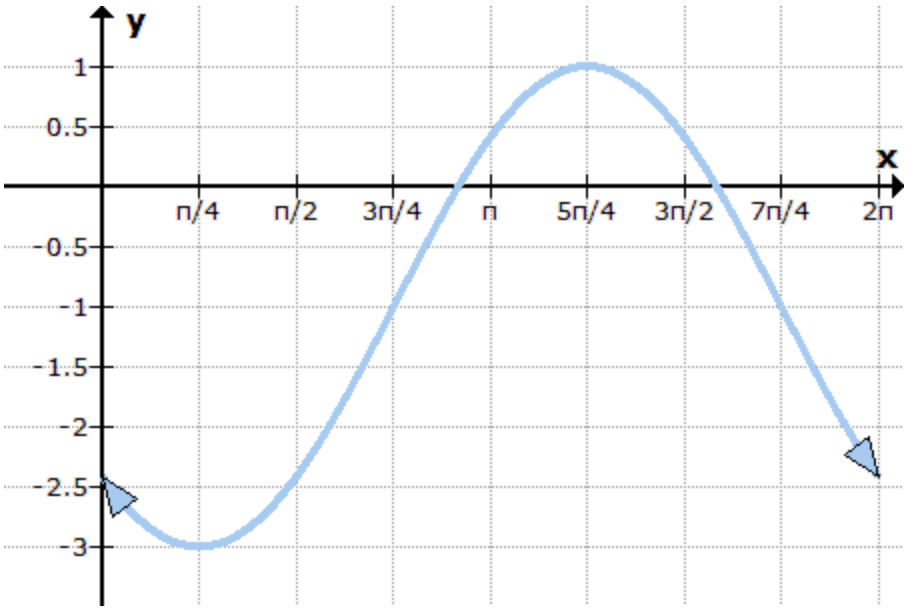
- Use the form $acos(b(x - c)) + d$ to find the variables.
  - $f(x) = -\frac{1}{2}cos(4(x - \frac{\pi}{12})) - 3$
  - $a = -\frac{1}{2}$, $b = 4$, $c = \frac{\pi}{12}$, $d = -3$
- Domain: $(-\infty, \infty)$, $\{x | x \in R\}$
- Range: $[-\frac{7}{2}, -\frac{5}{2}]$, $\{y \mid \frac{-7}{2} \leq y \leq \frac{-5}{2}\}$
  - Find the magnitude of the trig term of the function by taking the absolute value of the coefficient
  - magnitude = 1
  - Find the lower bound of the range.
  - The lower bound of the range for cosine is found by substituting the negative magnitude of the coefficient into the equation.
  - $y = -1 - \frac{1}{2} - 3$
  - $y = -\frac{9}{2}$
  - Find the upper bound of the range.
  - The upper bound of the range for cosine is found by substituting the positive magnitude of the coefficient into the equation.
  - $y = 1 - \frac{1}{2} - 3$
  - $y = -\frac{5}{2}$
- Minimum and Maximum values:
  - local minimum: $(\frac{\pi}{12} + \frac{\pi}{2}n, -\frac{7}{2})$, when $n \text{ is any integer}$
  - local maximum: $(\frac{\pi}{3} + \frac{\pi}{2}n, \frac{-5}{2})$, when $n \text{ is any integer}$
- Period: $\frac{2\pi}{4} = \frac{\pi}{2}$
- Phase Shift: $\frac{\pi}{12}$ to the right
- Amplitude: $\frac{1}{2}$



Question 3.

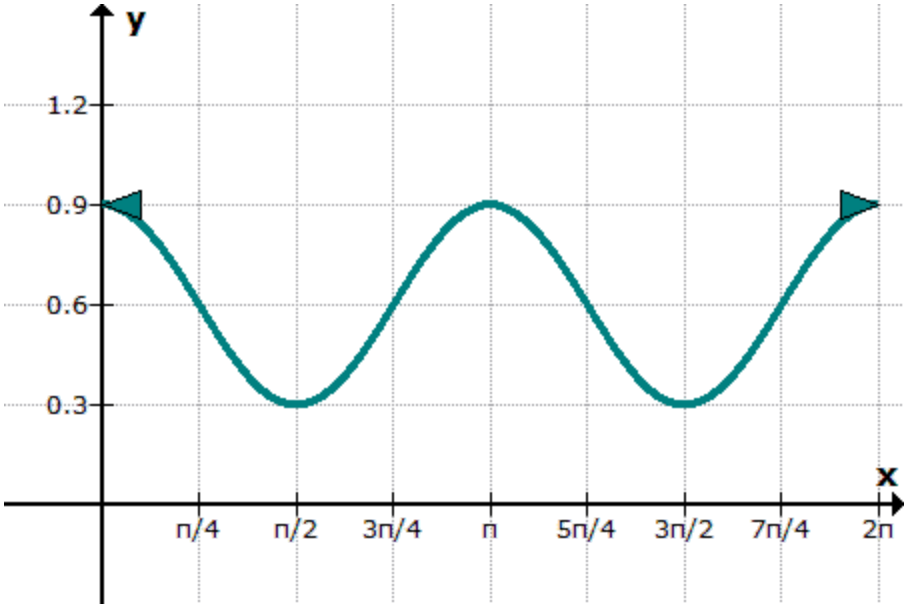
$f(x) = a\cos(k(x-d)) + c$ or $f(x) = a\sin(k(x-d)) + c$

1)



- Amplitude:
  - $\frac{maximum-minimum}{2} = \frac{1-(-3)}{2} = 2$
  - $a = 2$
- Axis of curve:
  - $y = -1$ which means that the graph is vertically shifted down by 1
  - $c = -1$
- Period:
  - Half of period: $\frac{5\pi}{4} - \frac{4\pi}{4} = \pi$
  - Period: 2π
  - $k = 1$
- Phase shift:
  - The maximum which happens at first located at: $\frac{5\pi}{4}$
  - We could horizontally shift the graph right by $\frac{5\pi}{4}$
  - $d = \frac{5\pi}{4}$
- The graph is $f(x) = 2\cos(x - \frac{5\pi}{4}) - 1$

2)



- Amplitude:
  - $\frac{minimum-maximum}{2} = \frac{0.9-0.3}{2} = 0.3$
  - $a = 0.3$
- Axis of curve:
  - $y = 0.6$ which means that the graph is vertically shifted up by 0.6
  - $c = 0.6$
- Period:
  - $\pi - 0 = \pi$
  - Period = π ,
  - $k = 2$
- Phase shift:
  - The maximum which happens first located at: 0
  - We could not horizontally shift the graph since the value is 0
  - $d = 0$
- The graph is $f(x) = 0.3cos(2x) + 0.6$

Question 4.

At its highest point, the second hand on a clock is 1.9 m above the ground. At its lowest point, the second hand is 1.6 m above the ground. The second hand starts at its highest point when the clock is started.

a) Let such function to be $h(t) = acos(k(t-d)) + c$

- Amplitude = $0.3/2 = 0.15\ m$
- $a = 0.15$

- Mid point of the function = $(1.9 + 1.6)/2 = 1.75$
- $c = 1.75$
- $period = 60s$
- $k = \frac{2\pi}{60} = \frac{\pi}{30}$
- The second hand starts at the maximum point
- $d = 0$
- $h(t) = 0.15[cos(\frac{\pi}{30}t)] + 1.75$

b)

- $h(10) = 1.75 + 0.15 * cos(\frac{\pi}{3}) = 1.825\ m$
- $h(35) = 1.75 + 0.15 * cos(\frac{35\pi}{30}) = 1.620\ m$