Mid-Unit Assignment: Engineering a Rollercoaster
MCV4U

Jin Hyung Park
2021.03.09

Question 1.

- Research a favourite, or a famous, rollercoaster. Write a short summary (2-3 paragraphs) of this coaster's general information; include its name, a picture, where it is located, when it was built, why you picked it and its physical characteristics. Make sure you note the height of the initial drop as the initial drop of the rollercoaster you will be designing in this assignment will model the initial drop of this rollercoaster you have researched. Also remember to cite your sources (1-2 sources).

(Samsung C&T)

I would choose T-express located in Yongin, South Korea. The giga roller coaster is made with wood, which is the first in Korea. T-express is operated by Everland, the subsidiary of Samsung group, sharing the same branch with Samsung electronics producing Galaxy smartphones. The Samsung-operated rides ranked the fourth in world's steepest wooden roller-coasters at the time of construction. It was publicly opened in 2008 and reached 3.33 million passengers just in two years("Everland, 330 million passengers to ride on wooden coaster so far", "Everland, the oldest wooden coaster in S. Korea."). According to the magazine called 'Amusement Today', T-express is positioned at 37th on the 'Top Wood Rollercoasters' of Golden Ticket Awards.(("The Golden Ticket Awards | Presented by Amusement Today")) T-express has the tallest height of 56m (184 feet) with 1.64km course, reaching speeds up to 116km/h. It also features 56m height with 45.99m initial drop.("Wooden Wonders! 5 Fantastic Wooden Roller Coasters.")

I chose T-express for my engineering in this assignment because the amusement ride had once been most admiring to me when I was young. The ride is the fastest, steepest and tallest rollercoaster in my country, which makes the roller coaster to be on my bucket list to ride. My

friends during elementary school were also eager to have a trip to Everland for a wooden coaster despite their fear. I am sure that you are going to be convinced, if you see this video clip on YouTube, that T-express is reserved for the bravest thrill-seekers to enjoy the maximum adrenalin.

Question 2.

- It may be useful now to do a little bit of research on the physics of roller coasters. You will not need to do any physics-style calculations in this assignment, but you will be expected to abide by the common rules. For example; a rollercoaster cannot be expected to defy gravity, it must be given momentum at the beginning of the track, and cannot reach as high a height at the end of its track as it can at the beginning. Write a short paragraph describing what you found and cite your sources (1-2 sources).

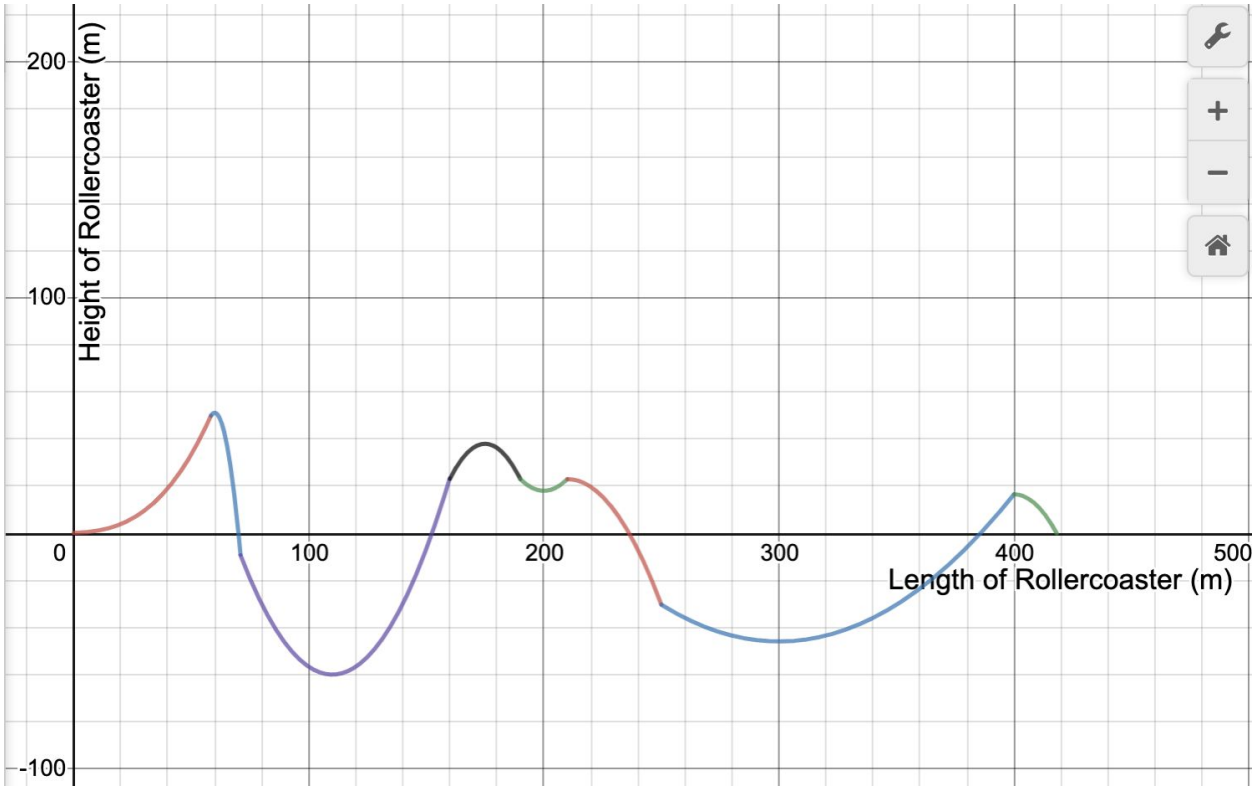
Rollercoasters run at high speeds and give a thrilling feeling. However, all the machines do is to just supply electricity to put the train on top of the rail. After that, even if the rides don't do anything, it naturally falls down due to gravity. This is due to the relationship between potential (height) and kinetic (speed) energy. The potential energy for gravity at a constant height changes to kinetic energy, increasingly speeding up. The addition of the potential energy and kinetic energy of an object is called mechanical energy, and the law of conservation of mechanical energy is that the sum of these dynamics energy is always constant.

When the potential energy is large, that is, when the object is high, the kinetic energy becomes smaller, and when the potential energy is small, the kinetic energy increases, which speeds up the object. Once the train is placed high to maximize the potential energy, the train moves down the set course gradually, decreasing the potential energy, and increasing the kinetic energy as it speeds up. The moment when you feel the maximum speed during the actual roller-coaster ride is when the train gets closer to the ground. While the roller coaster is in operation, assuming no air resistance and no friction between the rail and the train, the total amount of positional and kinetic energy remains constant.(“How Does a Roller Coaster Work?”)

In addition, the reason why the first drop of a roller-coaster is the biggest is that the kinetic energy can be exchanged back for gravitational potential energy the next time you climb the hill. The total energy could never be ramped up, but only decreased due to frictional losses, so the maximum hill the rides can climb is getting smaller and smaller.(“Q & A: First Hill on a Roller Coaster Is Biggest | Department of Physics | University of Illinois at Urbana-Champaign”)

Question 3.

- Now it is time to create your own roller-coaster with the following constraints involved.
    - a) Uses at least four different polynomial functions, pieced together.
    - b) Models the initial drop of the rollercoaster you researched.
    - c) Has at least 4 other drops.
    - d) Goes through an underground tunnel at least once (assuming y = 0 is the ground)
    - e) Any other features you feel would add to the ride and can be modelled with a polynomial function.



Link to the graph: https://www.desmos.com/calculator/gd5lhm8gz5

There are three steepest slopes on the designed roller-coaster graph. The first drop, which is the most steepest, is consist of $f_2(x), f_3(x)$ while the second drop is rendered by $f_4(x), f_5(x)$. The third drop is made up with $f_6(x)$. They are driven by a chain lift that mechanically pulls the train up onto the track.

The following is the screenshot of all piecewise functions.

1

$$f_1(x) = \frac{1}{20}\left(\frac{1}{300}(x+8.5)^3\right)\ \{0 < x \le 58.39\}$$

2

$$f_2(x) = -\left(\frac{1}{2}(x-60)^2\right) + 51.17705\ \{58.39 < x < 71\}$$

3

$$f_3(x) = \frac{1}{30}(x-110)^2 - 60.02295\{71 < x \le 160\}$$

4

$$f_4(x) = -\frac{1}{15}(x-175)^2 + 38\{160 < x \le 190\}$$

5

$$f_5(x) = \frac{1}{20}(x-200)^2 + 18\{190 < x \le 210\}$$

6

$$f_6(x) = -\frac{1}{30}(x-210)^2 + 23\{210 < x \le 250\}$$

7

$$f_7(x) = \frac{1}{160}(x-300)^2 - 45.958\{250 < x \le 400\}$$

8

$$f_8(x) = -\frac{1}{20}(x-400)^2 + 16.542\{400 < x \leq 418.189\}$$

Question 4.

For each of the pieces in your design:

- a) State the equation for the polynomial function used.
- b) Determine the degree of the function.
- c) State the domain and range of the restricted function.
- d) Determine if each unrestricted function piece has even or odd symmetry, or neither. If neither, determine which transformations, if any, could be applied to make it even or odd over its unrestricted domain?

$f_1(x) = \frac{1}{20}(\frac{1}{300}(x+8.5)^3)\ \{0 < x \leq 58.39\}$

- a) $y = \frac{1}{20}(\frac{1}{300}(x+8.5)^3)$
- b) Its degree is 3
- c) $\{x \mid 0 < x \leq 58.39\}$ and $\{y \mid 0 < y \leq 49.881\}$
- d) Neither - even or odd function, and also not available for rendering it to make them even or odd over its unrestricted domain. However, if the function is translated horizontally right by 8.5, the function becomes an odd function.

$f_2(x) = -(\frac{1}{2}(x-60)^2) + 51.17705\ \{58.39 < x \leq 71\}$

- a) $y = -(\frac{1}{2}(x-60)^2) + 51.17705$
- b) Its degree is 2
- c) $\{x \mid 58.39 < x \leq 71\}$ and $\{y \mid 49.881 < y \leq -9.323\}$
- d) Neither - Horizontal transition of 60 units to the left will make it even function.
    - Since the translation will make the function to be on the y-axis, the line symmetry would be around the y-axis.

$f_3(x) = \frac{1}{30}(x-110)^2 - 60.02295\ \{71 < x \leq 160\}$

- a) $y = \frac{1}{30}(x-110)^2 - 60.02295$
- b) Its degree is 2
- c) $\{x \mid 71 < x \leq 160\}$ and $\{y \mid -9.323 < y \leq 23.31\}$
- d) Neither - Horizontal translation of 110 units to the left will make it even function.
    - Since the translation will make the function to be on the y-axis, the line symmetry would be around the y-axis.

$f_4(x) = -\frac{1}{15}(x-175)^2 + 38\ \{160 < x \leq 190\}$

- a) $y = -\frac{1}{15}(x-175)^2 + 38$

- b) Its degree is 2
- c) $\{x \mid 160 < x \leq 190\}$ and $\{y \mid 23 \leq y < 23.31\}$
- d) Neither - Horizontal translation of 175 units to the left will make it even function.
    - Since the translation will make the function to be on the y-axis, the line symmetry would be around the y-axis.

$f_5(x) = \frac{1}{20}(x-200)^2 + 18 \ \{190 < x \leq 210\}$

- a) $y = \frac{1}{20}(x-200)^2 + 18$
- b) Its degree is 2
- c) $\{x \mid 190 < x \leq 210\}$ and $\{y \mid 23 \leq y < 23.31\}$
- d) Neither - Horizontal translation of 200 units to the left will make it even function.
    - Since the translation will make the function to be on the y-axis, the line symmetry would be around the y-axis.

$f_6(x) = -\frac{1}{30}(x-210)^2 + 23 \ \{210 < x \leq 236.238\}$

- a) $y = -\frac{1}{30}(x-210)^2 + 23$
- b) Its degree is 2
- c) $\{x \mid 210 < x \leq 236.238\}$ and $\{y \mid -30.333 \leq x < 23\}$
- d) Neither - Horizontal translation of 210 units to the left will make it even function.
    - Since the translation will make the function to be on the y-axis, the line symmetry would be around the y-axis.

$f_7(x) = \frac{1}{160}(x-300)^2 - 45.958 \ \{250 < x \leq 400\}$

- a) $y = \frac{1}{160}(x-300)^2 - 45.958$
- b) Its degree is 2
- c) $\{x \mid 250 < x \leq 400\}$ and $\{y \mid -30.333 < x \leq 16.542\}$
- d) Neither - Horizontal translation of 300 units to the left will make it even function.
    - Since the translation will make the function to be on the y-axis, the line symmetry would be around the y-axis.

$f_8(x) = -\frac{1}{20}(x-400)^2 + 16.542 \ \{400 < x \leq 418.189\}$

- a) $y = -\frac{1}{20}(x-400)^2 + 16.542$
- b) Its degree is 2
- c) $\{x \mid 400 < x \leq 418.189\}$ and $\{y \mid 0 \leq x < 16.542\}$
- d) Neither - Horizontal translation of 400 units to the left will make it even function.
    - Since the translation will make the function to be on the y-axis, the line symmetry would be around the y-axis.

Question 5.

Suppose you were asked to take the first three pieces of your rollercoaster and model them with a single function. Answer the following questions:

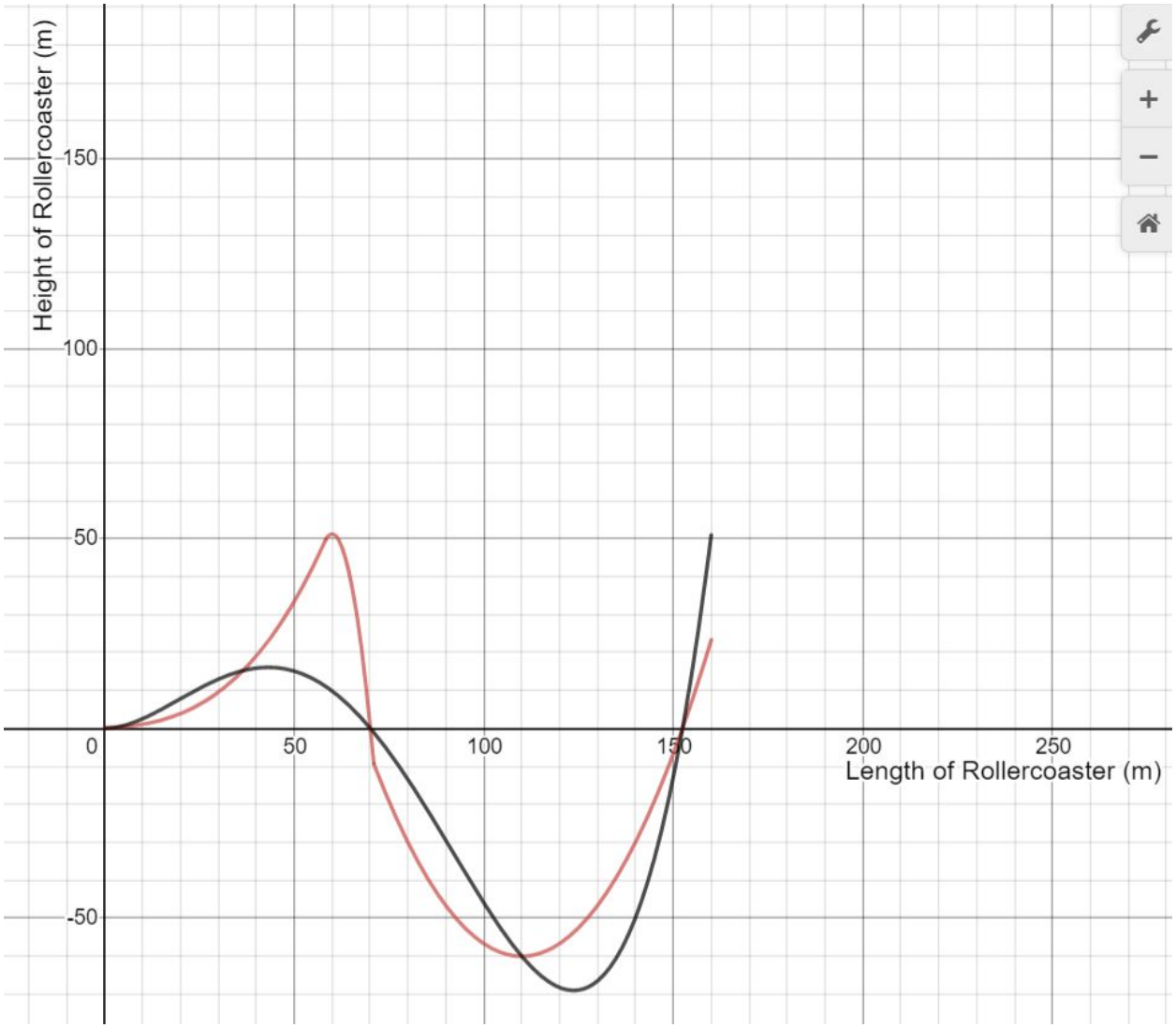
a) If you represent these three pieces with a single function, what degree do you think this function will be? Justify your reasoning.

- It would be a quartic function, the degree of four, since it would have three turns and 2 single roots (multiplicity of 1) and 1 double root (multiplicity of 2).

b) Determine an equation for this function. You can refer to the module Creating Polynomial Functions for examples of how to complete this question or use a more trial-and-error approach.

- The three functions, $f_1(x), f_2(x), f_3(x)$ passes x-axis when the following cases.
  - (0, 0) - multiplicity of 2
  - (70.117, 0) - multiplicity of 1
  - (152.435, 0) - multiplicity of 1
- Thus, we can write the function as the following: $f(x) = ax^2(x - 70.117)(x - 152.435)$
- We can appreciate the fact that the $f_3(x)$ passes (110,-60.023)
  - $-60.023 = a(110)^2(110 - 70.117)(110 - 152.435)$
  - $a = 0.00000293$
- $f(x) = 0.00000293x^2(x - 70.117)(x - 152.435)$

c) Graph the equation you created together with the first three functions. How similar is it? If there are any differences, why do you believe that is?



It is similar to the original function but slightly different because the three original functions are cubic or quadratic functions. It is difficult for a quartic function to resemble the original functions.

References

- Works Cited

- "Wooden Wonders! 5 Fantastic Wooden Roller Coasters." *Samsung C&T Newsroom*, 23 Nov. 2017, news.samsungcnt.com/wooden-wonders-5-fantastic-wooden-roller-coasters/.
- "Everland, the oldest wooden coaster in S. Korea.'" *Win News*, 18 Mar. 2008, www.newswin.kr/news/articleView.html?idxno=1278. Accessed 9 Mar. 2021.

- "How Does a Roller Coaster Work?" *BBC Bitesize*, www.bbc.co.uk/bitesize/topics/z72vrj6/articles/zm4cqp3.
- "Everland, 330 million passengers to ride on wooden coaster so far" *Kyunghyang Ilbo*, 14 Mar. 2010, news.khan.co.kr/kh_news/khan_art_view.html?art_id=201003141754561. Accessed 9 Mar. 2021.
- "Q & A: First Hill on a Roller Coaster Is Biggest | Department of Physics | University of Illinois at Urbana-Champaign." *Van.physics.illinois.edu*, van.physics.illinois.edu/qa/listing.php?id=31&t=first-hill-on-a-roller-coaster-is-biggest. Accessed 9 Mar. 2021.
- "The Golden Ticket Awards | Presented by Amusement Today." *The Golden Ticket Awards | Presented by Amusement Today*, 7 Sept. 2019, goldenticketawards.com/2019-top-wood/. Accessed 9 Mar. 2021.