Vectors Unit Assignment
MCV4U

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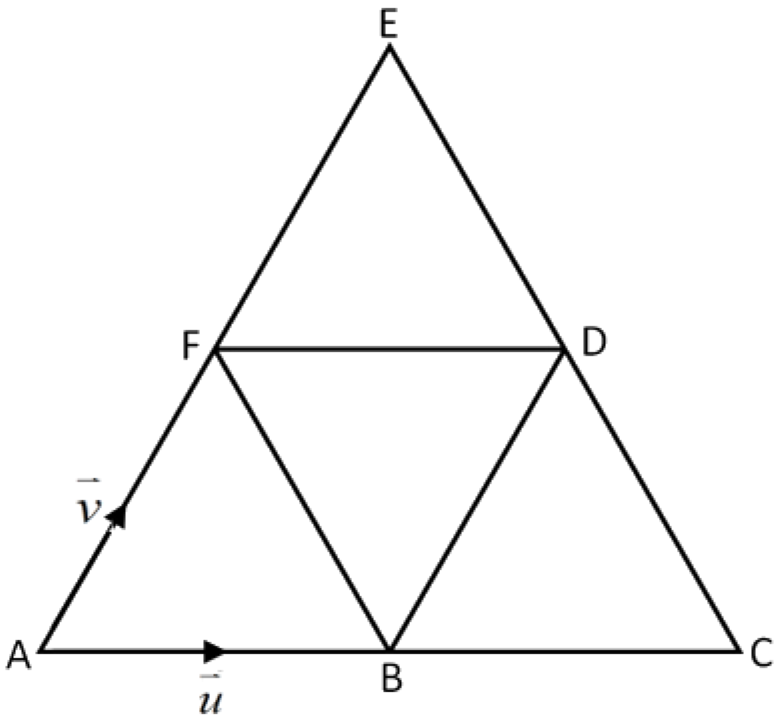
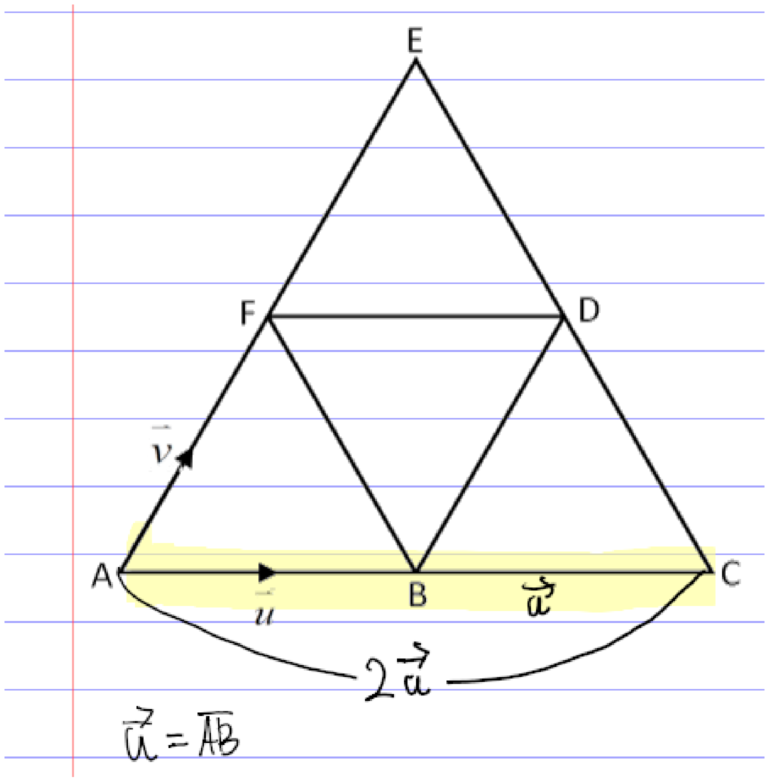
1. **State whether each quantity is a vector or scalar.**
    1. Speed
    - Because a scalar has only magnitude with no direction, speed is a scalar. $speed = \frac{distance}{time}$
    2. Velocity
    - Considering that velocity can be specified with magnitude in a designated direction, we can think that $velocity = \frac{vec(d)}{t}$has both magnitude and direction.
    3. Weight
    - Force is a product of mass, which is a scalar, and acceleration, which is a vector. When considering the formula that weight can be calculated by multiplying mass and acceleration, we can think that weight is a vector product taking both direction and magnitude.
    4. Mass
    - Mass itself is only a scalar quantity with only presenting magnitude. Mass does not change no matter where you are living and moving.
    5. Area
    - The area element is correctly defined only if its magnitude and direction are given. When it comes to the area of a parallelogram, we can appreciate the orientation of the area element while given as $vec(s) = vec(a) * vec(b)$(where $vec(a)\ and\ vec(b)$ are its two sides). Thus, the area element is a vector quantity.

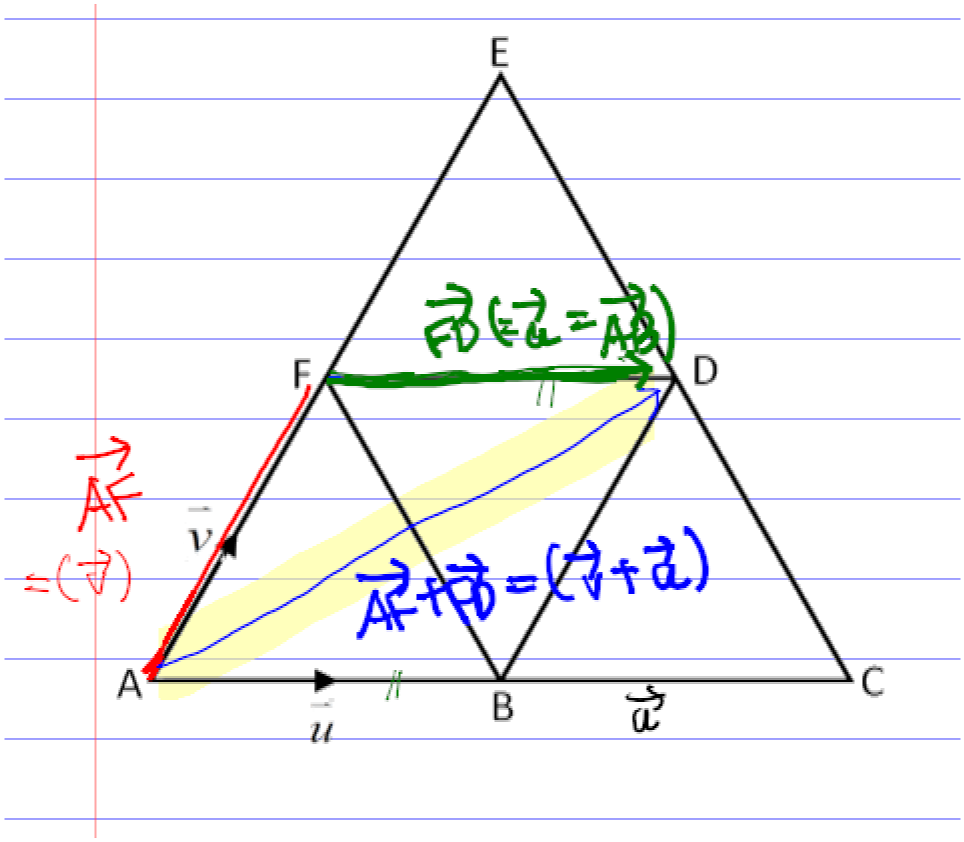
2. In the diagram, ACE is an equilateral triangle. B, D, and F are the midpoints of AC, CE, and EA. $\vec{AB} = \vec{u},\ \vec{AF} = \vec{v}$**. Write the following vectors in terms of** $\vec{u}$ **and** $\vec{v}$**.**



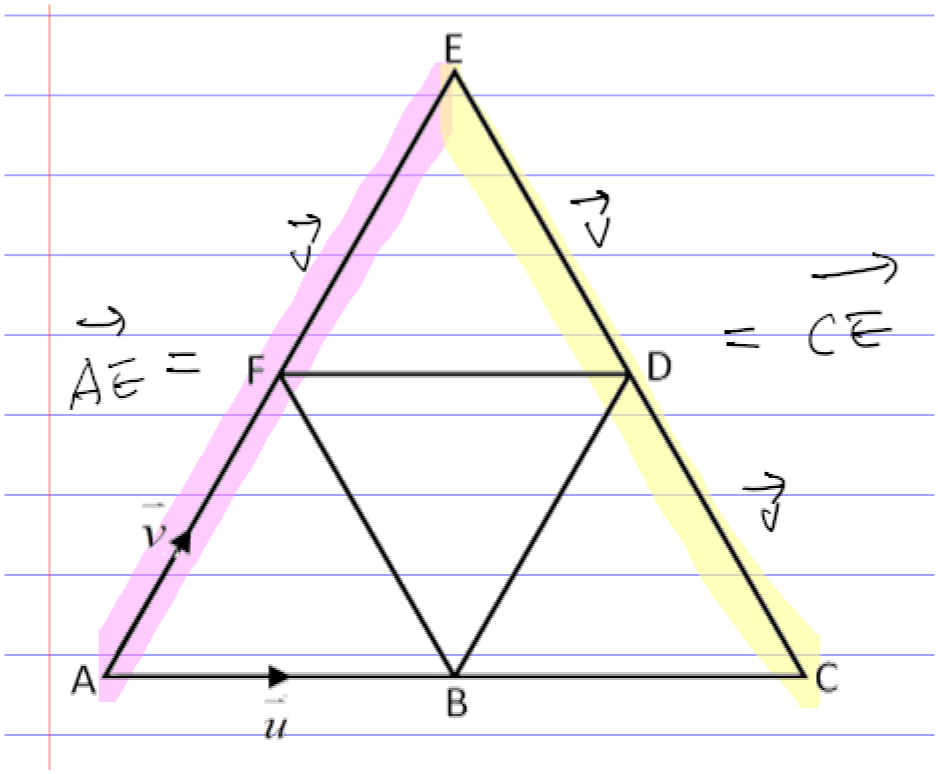
a. $\vec{AC} = 2\vec{AB} = 2\vec{u}$



b. $\vec{(}AD)=\vec{(}AF) + \vec{(}FD) = \vec{(}v) + \vec{(}u)$



c. $\rightarrow_{(CE)} = 2(\rightarrow_{v}) - 2(\rightarrow_{u})$



d. $\vec{(}EB) = ((\vec{(}EF) + \vec{(}FA)) + \vec{(}AB) = -\vec{(}v) - \vec{(}v) + \vec{(}u)$

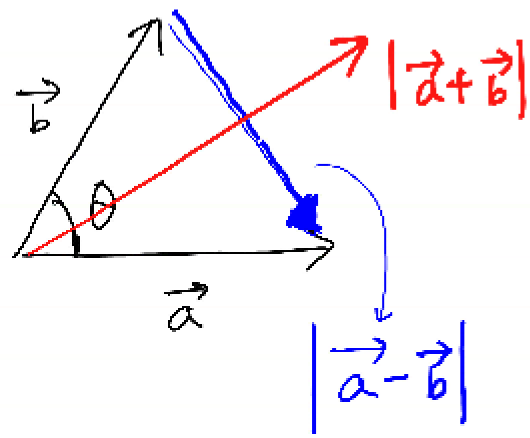
Give an example of a vector that is equal to:

e. $2\vec{(}v) = 2\overline{AF} = \overline{AE}$

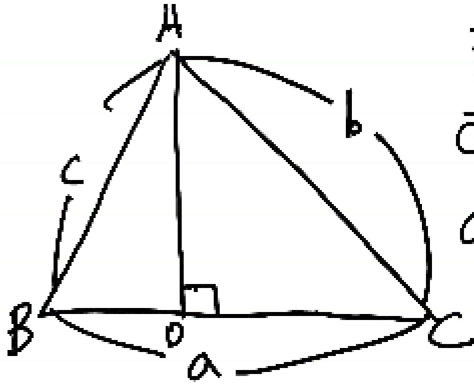
f. $\vec{(}u) - \vec{(}v) = \vec{(}\overline{FD}) + \vec{(}DC) = \vec{(}FC)$

3. Draw diagrams to show two vectors, vec(a) and vec(b), and the two vectors (vec(a)+vec(b)) and (vec(a)-vec(b)).

To begin with, let me define some elements that are required to solve the problems.



i) $|\vec{a}-\vec{b}|$ — 2nd law of cosine.



$\overline{BD} = c \cdot \cos B$

$\overline{CD} = b \cdot \cos C$

$a = c\cos B + b\cos C$

$a = c\cos B + b\cos C \xrightarrow{\times a} a^2 = ac\cos B + ab\cos C$

$b = a\cos C + c\cos A \xrightarrow{\times b} b^2 = ab\cos C + bc\cos A$

$c = b\cos A + a\cos B \xrightarrow{\times c} c^2 = bc\cos A + ac\cos B$

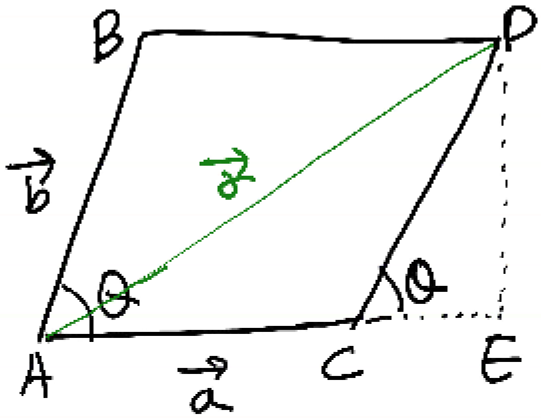
$(a^2 - b^2) = (ac\cos B + \cancel{ab\cos C}) - (\cancel{ab\cos C} + bc\cos A)$

$= ac\cos B + bc\cos A$

$(a^2 - b^2 + c^2) = (ac\cos B + \cancel{bc\cos A}) + (\cancel{bc\cos A} + ac\cos B)$

thus, $b^2 = a^2 + c^2 - 2ac\cos B$ ■

b. $|\vec{a}+\vec{b}|$



$\overline{CE} = \vec{b}\cdot\cos\theta$, $\overline{AE} = \vec{a}+(\vec{b}\cos\theta)$, $\overline{ED}=\vec{b}\sin\theta$

$\overrightarrow{AD} = \vec{\gamma} = (\vec{a}+\vec{b}\cos\theta)^2+(\vec{b}\sin\theta)^2$

According to pythagorean's law,

$$= (\vec{a}^2+2\vec{a}\vec{b}\cos\theta+\vec{b}\cos\theta^2)+(\vec{b}\sin\theta)^2$$

$$= (\vec{a}^2+2\vec{a}\vec{b}\cos\theta)+\vec{b}^2(\cos\theta^2+\sin\theta^2)$$

$(\because \cos\theta^2+\sin\theta^2=1)$

$$= \vec{a}^2+2\vec{a}\vec{b}\cos\theta+\vec{b}$$

a)

Let θbe the angle between $\vec{(a)}$ and $\vec{(b)}$ as shown in the attached figure. Thus, the range of θis to be $0 \le \theta \le \frac{\pi}{2}$.

$|\vec{a}+\vec{b}| = \sqrt{|\vec{a}|^2+|\vec{b}|^2+2|\vec{a}||\vec{b}|\cos\theta}$

$|\vec{a}-\vec{b}| = \sqrt{|\vec{a}|^2+|\vec{b}|^2-2|\vec{a}||\vec{b}|\cos\theta}$

Considering the given condition that is $|\overrightarrow{a}+\overrightarrow{b}|>|\overrightarrow{a}-\overrightarrow{b}|$,

$$\sqrt{|\overrightarrow{a}|^2+|\overrightarrow{b}|^2+2|\overrightarrow{a}||\overrightarrow{b}|\cos\theta}>\sqrt{|\overrightarrow{a}|^2+|\overrightarrow{b}|^2-2|\overrightarrow{a}||\overrightarrow{b}|\cos\theta}$$

$$\implies \cos\theta>-\cos\theta$$

$$\implies 2\cos\theta>1 \implies \cos\theta>\frac{1}{2} \implies \theta\in\left[0,\frac{\pi}{3}\right)$$

Thus, the answer is $0 \le \theta < \frac{\pi}{3}$.

b)

Considering the given condition that is $|\overrightarrow{a}+\overrightarrow{b}|<|\overrightarrow{a}-\overrightarrow{b}|$,

$$\sqrt{|\overrightarrow{a}|^2+|\overrightarrow{b}|^2+2|\overrightarrow{a}||\overrightarrow{b}|\cos\theta}<\sqrt{|\overrightarrow{a}|^2+|\overrightarrow{b}|^2-2|\overrightarrow{a}||\overrightarrow{b}|\cos\theta}$$

$$\implies \cos\theta<-\cos\theta$$

$$\implies 2\cos\theta<1 \implies \cos\theta<\frac{1}{2} \implies \theta\in\left(\frac{\pi}{3},\frac{\pi}{2}\right]$$

Thus, the answer is $\frac{\pi}{3} < \theta < \pi$.

c)

Considering the given condition that is $|\overrightarrow{a}+\overrightarrow{b}|=|\overrightarrow{a}-\overrightarrow{b}|$,

$$\sqrt{|\overrightarrow{a}|^2+|\overrightarrow{b}|^2+2|\overrightarrow{a}||\overrightarrow{b}|\cos\theta}=\sqrt{|\overrightarrow{a}|^2+|\overrightarrow{b}|^2+2|\overrightarrow{a}||\overrightarrow{b}|\cos\theta}$$

$$\implies \cos\theta=-\cos\theta$$

$$\implies \cos\theta=0 \implies \theta=\frac{\pi}{2}$$

Thus, the answer is $\theta = \frac{\pi}{2}$.

d)

Considering the given condition that is $|\overrightarrow{a}+\overrightarrow{b}|=|\overrightarrow{a}|+|\overrightarrow{b}|$,

$$\sqrt{|\overrightarrow{a}|^2+|\overrightarrow{b}|^2+2|\overrightarrow{a}||\overrightarrow{b}|\cos\theta}=|\overrightarrow{a}|+|\overrightarrow{b}|$$

$$\Longrightarrow \; |\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos\theta = |\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|$$

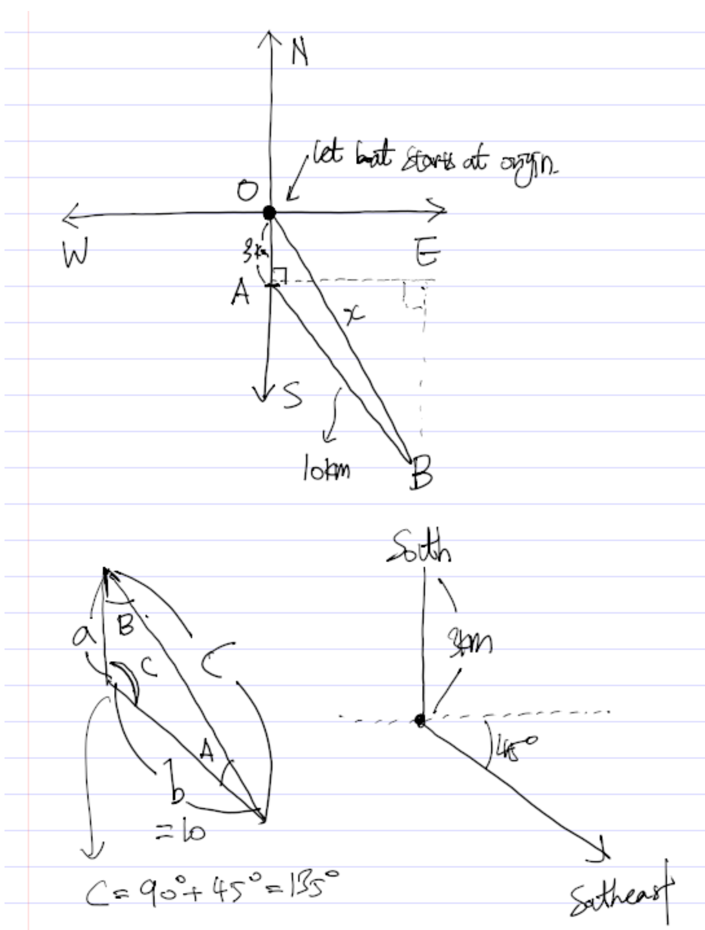
$$\Longrightarrow \; \cos\theta = 1 \; \Longrightarrow \; \theta = 0$$

Thus, the answer is $\theta = 0$.

e)

$|\vec{a} + \vec{b}| = \sqrt{|\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos\theta}$ would be minimum when $\theta = 180\ degree$.
Since $cos(180) =- 1$,
$|\vec{a} + \vec{b}| = \sqrt{|\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|} = ||a| - |b||$

4. A boat sails 3 km South, then 10 km Southeast. Use trigonometry to find the boat's distance and bearing from its starting point.



Using cosine theorem,

$x^2 = (a^2 + b^2) - 2abcos(c) = (10)^2 + (3)^2 - 2 * 10 * 3 * cos(90 + 45)$
$= 100 + 9 - 60 * cos(135) = 109 - 60(-cos45) = 109 + \frac{60}{\sqrt{2}}$

$x = 12.30km = boat's\ distance$

Using Lemi's theorem,

$\frac{c}{sinc} = \frac{b}{sinb} = \frac{a}{sina}$

$\frac{12.30}{sin(90+45)} = \frac{10}{sinb}, sinb = \frac{10}{12.30\sqrt{2}}$

$b = sin^{-1}[\frac{10}{12.30\sqrt{2}}]$

$b = sin^{-1}(0.57) = 34.75$

$180 - 34.75 = 145.25 = bearing\ from\ starting\ point$

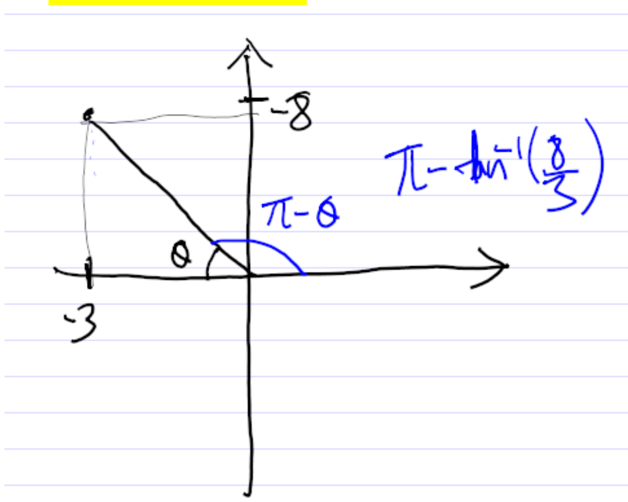
5. Convert the following vectors;

a) 75m/s on a bearing of 295 degrees to Cartesian form

- Since $\gamma = 75m/s$ while $\theta = 155\ degrees$, we can get [x, y] points by using trigonometry formulas.
- $x = \gamma cos\theta = 75cos(155) = 75 * (-0.487161) =- 36.5370$
- $y = \gamma sin\theta = 75sin(155) = 75 * (-0.873311) =- 65.4983$
- Thus, the answer is $(-36.5370, -65.4983)$.

b) [-3, 8] to direction/magnitude form

- $magnitude = \sqrt{(-3)^2 + (8)^2 = 9 + 64} = \sqrt{73} = 8.544$
- $tan\theta = \frac{8}{3}$
- $\theta = tan^{-1}(\frac{8}{3}) \approx 69^o$
- $direction = 69^o$



6. Express as a single vector.

a. $\vec{(PS)} + \vec{(SR)} = \vec{(PR)}$

b. $\vec{(EF)} - \vec{(DF)}$

$\vec{(EF)} + \vec{(FD)} = \vec{(ED)}$

$\boxed{\vec{(EF)} + \vec{(-FD)}) = \vec{(ED)}}$

c. $\vec{(MP)} - \vec{(QR)} + \vec{(NM)} + \vec{(RP)}$

$= \vec{(MP)} + \vec{(RQ)} + \vec{(NM)} + \vec{(PR)}$

$= \vec{(NM)} + \vec{(MP)} + \vec{(PR)} + \vec{(RQ)})$

$= \vec{(NP)} + \vec{(PR)} + \vec{(RQ)}$

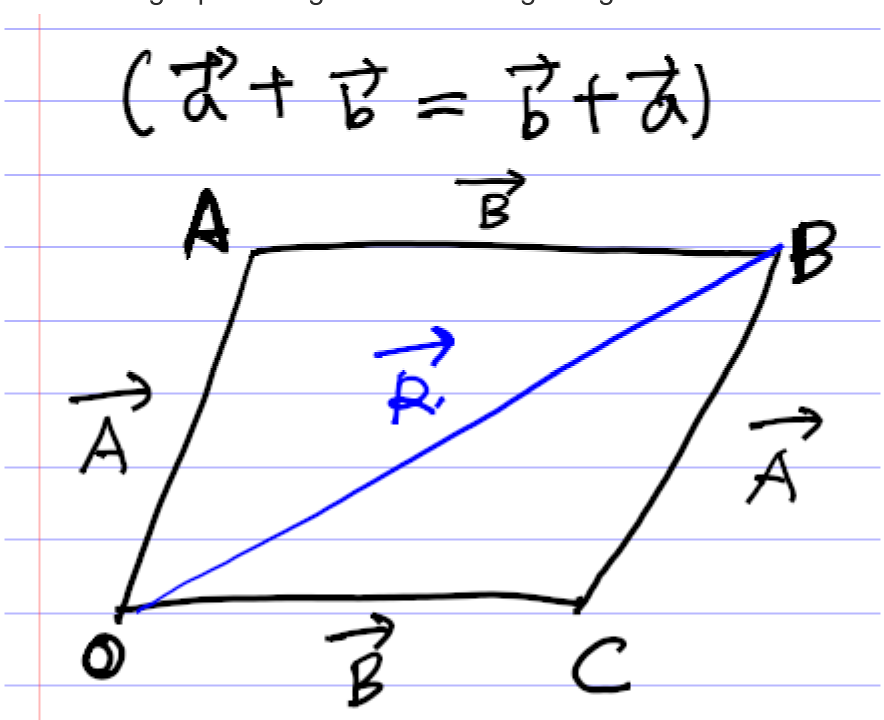
$= \vec{(NR)} + \vec{(RQ)}$

$\boxed{= \vec{(NQ)}}$

7. Demonstrate using diagrams

a) that vector addition is commutative: $\vec{(a)} + \vec{(b)} = \vec{(b)} + \vec{(a)}$

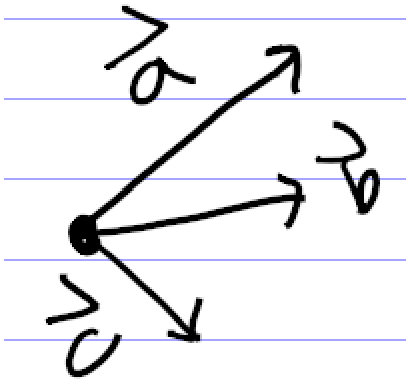
Constructing a parallelogram as following using two vectors as the adjacent sides.



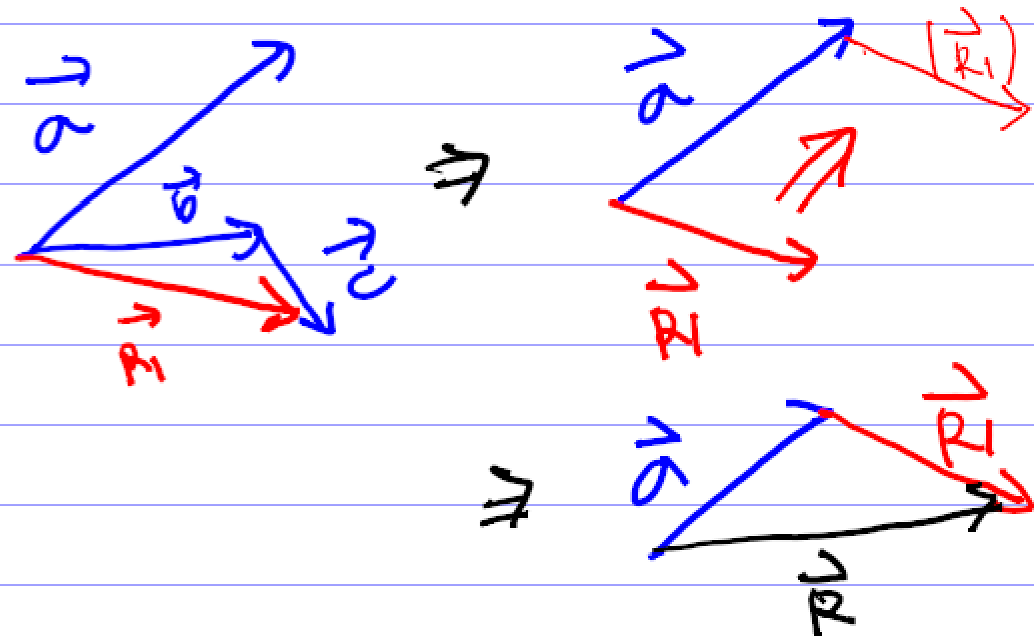
For the sake of convenience in typing, I will omit vector indication to each side below.
Using triangle law of vector addition in ΔOAB, $A + B = R$
and also using triangle law of vector addition in ΔOBC, $B + A = R$
Thus, $A + B = B + A$

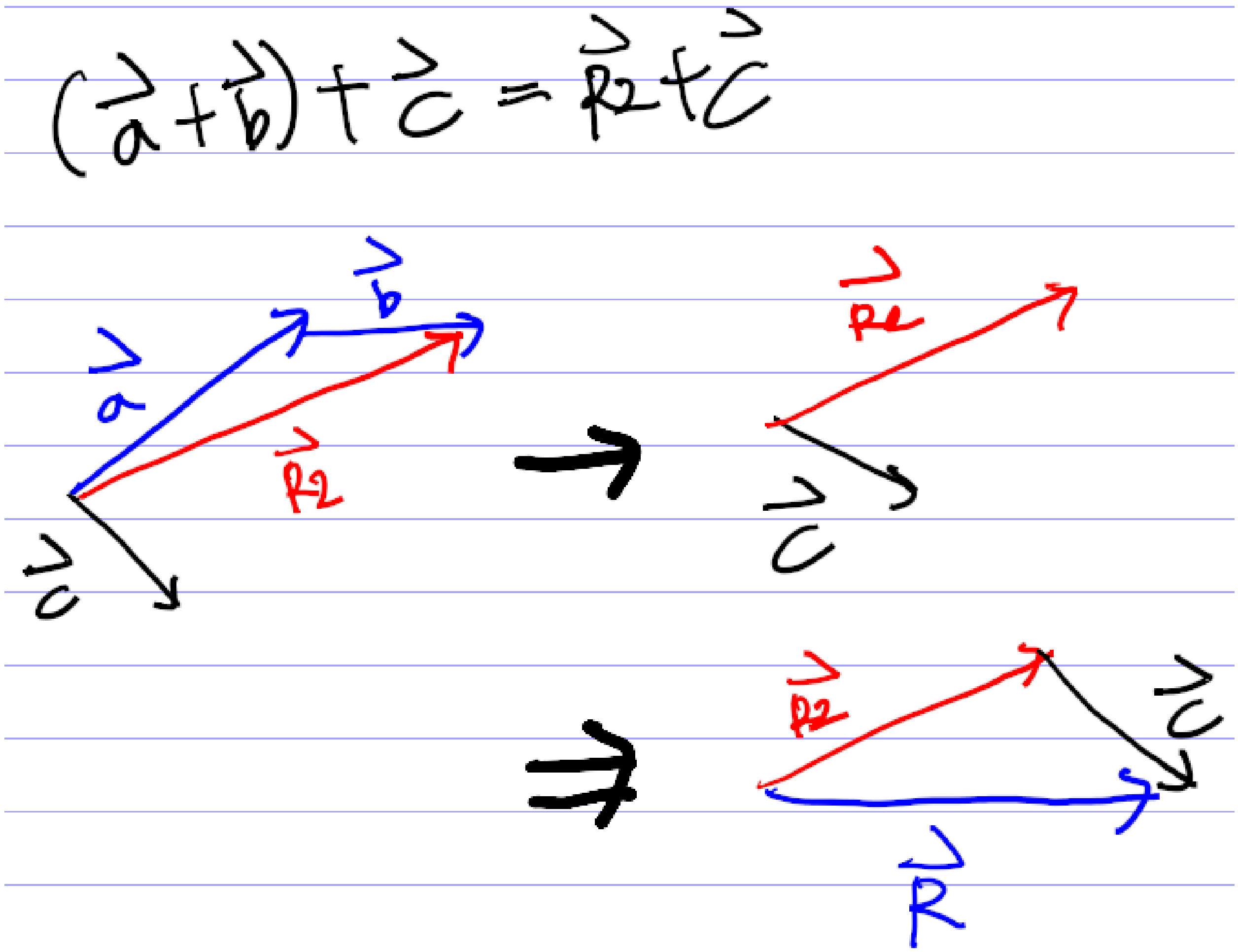
b) that vector addition is associative: $(\vec{}a) + ((\vec{}b) + (\vec{}c)) = ((\vec{}a) + (\vec{}b)) + (\vec{}c))$

Let $\rightarrow_{a}$, $\rightarrow_{b}$, $\rightarrow_{c}$ as the following.



$$\vec{a} + (\vec{b} + \vec{c}) = \vec{a} + \vec{R_1}$$





Thus, the resultant $\to_{R}$ is the same for $\to_{a} + (\to_{b} + \to_{c})$ and $(\to_{a} + \to_{b}) + \to_{c}$, which means that the vector addition is associative.

8. Research an example of the use of vectors, and explain how the mathematics is used, for example in engineering, computer animation, gaming, 3-D printing or GPS technology

A clean image that can be easily resized is needed for computer animation. Raster imaging was phased out in favor of vector graphics. The image's raster includes detail such as the number of pixels, colour, and placements. (“Vector vs Raster Graphics - GeeksforGeeks”) $.jpg,. gif,\ and\ .png$ are examples of raster imaging formats. Since raster imaging is readily warped and blurred when resized, it is unsuitable for computer graphics. Vector graphics, however, can be resized without losing accuracy. It defines the position of lines and forms using mathematical values (vectors). (Techquickie) (“Understanding Vector Graphics — Using SVG with CSS3 and HTML5 — Supplementary Material”)

$.ai,. eps,\ and. pdf$ are examples of vector image formats. Vector graphics do not degrade the image and allow the artist to work quickly on the file. In reality, most animators choose vector-based graphics because they are more detailed and take up less space than other types (“A Short Primer on Vector Animation”)

References

“A Short Primer on Vector Animation.” *Lifewire*, 2019, www.lifewire.com/introduction-to-vector-animation-140890.

Techquickie. “How Do Vector Graphics Work?” *YouTube*, 18 Oct. 2016, www.youtube.com/watch?v=W2xknX3k6FY. Accessed 15 Nov. 2019.

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“Vector vs Raster Graphics - GeeksforGeeks.” *GeeksforGeeks*, 8 Feb. 2018, www.geeksforgeeks.org/vector-vs-raster-graphics/.