Jin Hyung Park MCV4U Underlying Concepts of Calculus Unit Assignment

1. **Determine the following limits, if they exist!**

a) $\lim_{x\to 3}(3x^4 + 2x^2 - 5)$

- Since x goes close to 3, we can deem that x is almost 3. Thus substitute x with 3.
  $= (3 * 3^4 + 2 * 3^2 - 5) = 3^5 + 18 - 5 = 243 + 13 = 256$
- Thus, the limit exists while the answer is 256.

b) $\lim_{x\to 5}(\frac{4x}{x-5})$

- Since x goes close to 5, we can deem that x is almost 5. Thus substitute x with 5.
  $= (\frac{20}{0})$
- However, we can find that the denominator goes zero, which means that it is undefined.
- Thus, the limit does not exist.

c) $\lim_{x\to 4}(\frac{x^2 + x - 20}{8 - 2x})$

- To begin with, we need to subtract common factors between denominator and numerator.
- Thus, we need to factorize both of them.
  $= \lim_{x\to 4}(\frac{(x-4)(x+5)}{2(4-x)})$
  $= \lim_{x\to 4}(\frac{-(x+5)}{2})$
- Since x goes close to 4, we can deem that x is almost 4. Thus substitute x with 4.
  $= (\frac{-1}{2})$
- Thus, the limit exists while the answer is $-\frac{1}{2}$.

2. Evaluate the following limits (if they exist).

a) $\lim_{x\to 1}\frac{x^2 - 1}{x - 1}$

- To begin with, we need to subtract common factors between denominator and numerator.
- Thus, we need to factorize both of them.
  $\lim_{x\to 1}\frac{(x+1)(x-1)}{x-1}$
  $\lim_{x\to 1}(x+1)$
- Since x goes close to 1, we can deem that x is almost 1. Thus substitute x with 1.
  $= (2)$

b) $\lim_{x \to 0} \frac{\sqrt{x+16}-4}{x}$

- To begin with, we can appreciate that the value would become $\frac{0}{0}$, which is indeterminate. Thus, we need to avoid this by finding common factors to cancel between denominator and numerator.
- In order to do this, we need to multiply $\sqrt{x+16}+4$ to each denominator and numerator while keeping the same value as given above.
- Thus, the formula would be the following.

$$= \lim_{x \to 0} \frac{(x+16)-16}{x(\sqrt{x+16}+4)}$$

$$= \lim_{x \to 0} \frac{x}{x(\sqrt{x+16}+4)}$$

$$= \lim_{x \to 0} \frac{1}{(\sqrt{x+16}+4)}$$

$$= \frac{1}{8}$$

- Thus, the limit exists while the value is $\frac{1}{8}$.

c) $\lim_{x \to 64} \frac{\sqrt[3]{x}-4}{x-64}$

- We can use the following formula to solve the equation.

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$lim_{x->64} \frac{(x)^{\frac{1}{3}}-4}{(x^{\frac{1}{3}})^3-4^3}$$

$$= lim_{x->64} \frac{(x^{\frac{1}{3}}-4)}{(x^{\frac{1}{3}}-4)(x^{\frac{2}{3}}+16+4x^{\frac{1}{3}})}$$

$$= lim_{x->64} \frac{1}{x^{\frac{2}{3}}+16+4x^{\frac{1}{3}}}$$

$$= \frac{1}{(64)^{\frac{2}{3}}+16+4(64)^{\frac{1}{3}}}$$

$$= 48$$

- Thus, the limit exists while the value is 48.

3. From first principles (i.e. using the tangent slope method), find the slope of the following curves at the given value of x.

a) $f(x) = 2x^2 - 6x \; at \; x = 3$

- We can write differentiation, $f'(x)$, as the following.

  $f'(x) = \frac{d}{dx}(2x^2 - 6x)$

  $f'(x) = 4x - 6$

- According to the tangent slope method, $\frac{dy}{dx}$ is slope of the given, original curve.
  - Substitute x with 3

    $f'(3) = 4(3) - 6 = 12 - 6 = 6$

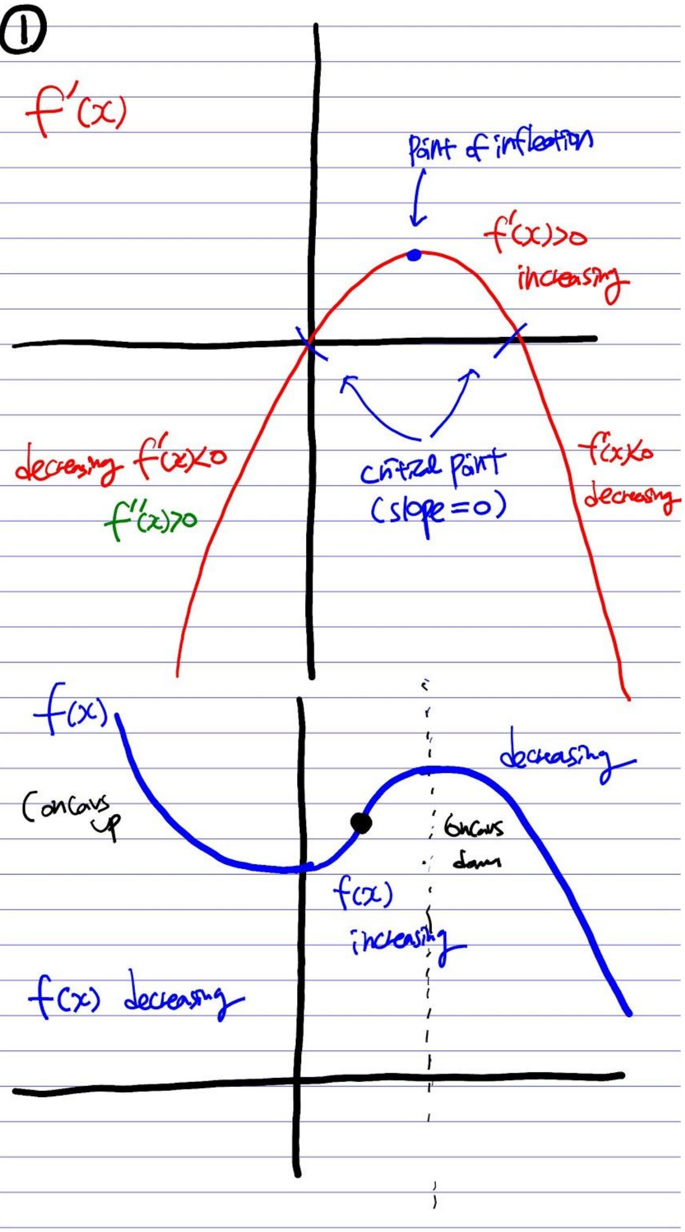
- Thus, the slope of $f(x) = 2x^2 - 6x \ at \ x = 3$ is 6.

b) $y = 3x^3 + 1 \ at \ x =- 4$

- We can differentiate by writing the $f'(x)$ as follows.
  $f'(x) = \frac{d}{dx}(3x^3 + 1)$
  $f'(x) = 9x^2$
- According to the tangent slope method, $\frac{dy}{dx}$ is slope of the given, original curve.
  - Substitute x with -4
  - $f'(4) = 9(-4^{\ 2}) = 9 \times 16 = 144$
- Thus, the slope of $f(x) = y = 3x^3 + 1 \ at \ x =- 4$ is 144.

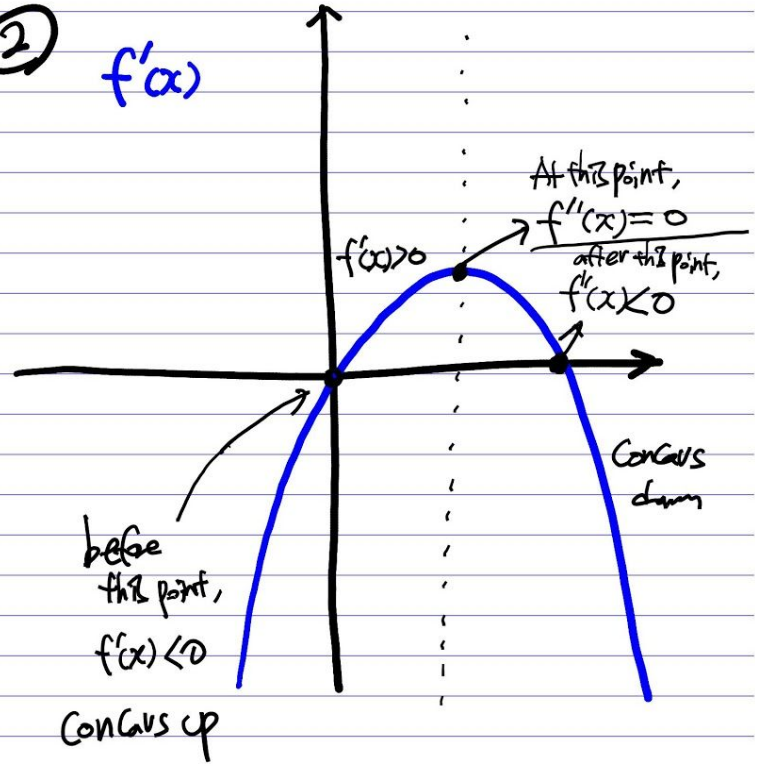
4. Look at the graph below of the derivative f(x). From this, make a sketch of the original function f(x) and of the second derivative f''(x), and explain your reasoning.

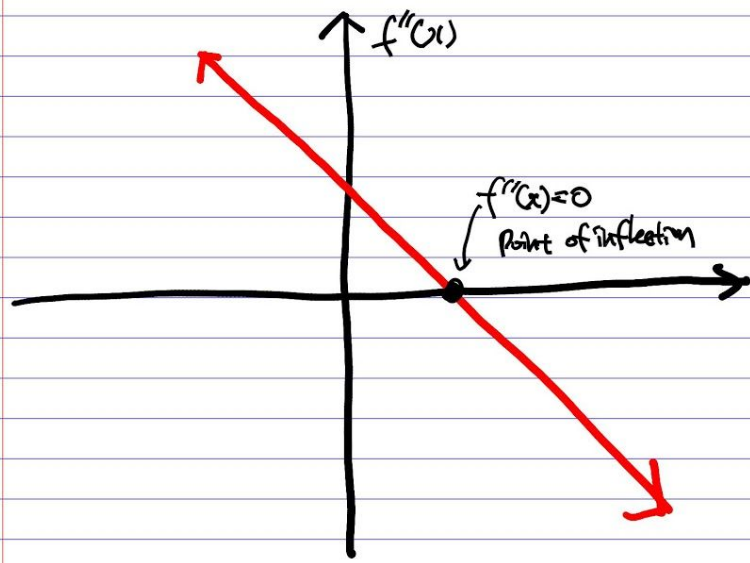
①



f'(x)
point of inflection
f'(x)>0
increasing
decreasing f'(x)<0
f''(x)>0
critical point
(slope=0)
f'(x)<0
decreasing
f(x)
decreasing
Concave up
Concave down
f(x)
increasing
f(x) decreasing

② $f'(x)$


f'(x)>0
At this point,
f''(x)=0
after this point,
f''(x)<0
Concavs
down
before
this point,
f'(x)<0
Concavs up


f''(x)
f''(x)=0
Point of inflection