Unit Assignment: Rational Functions and Inequalities

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Question 1.

1-1. What is a rational function?

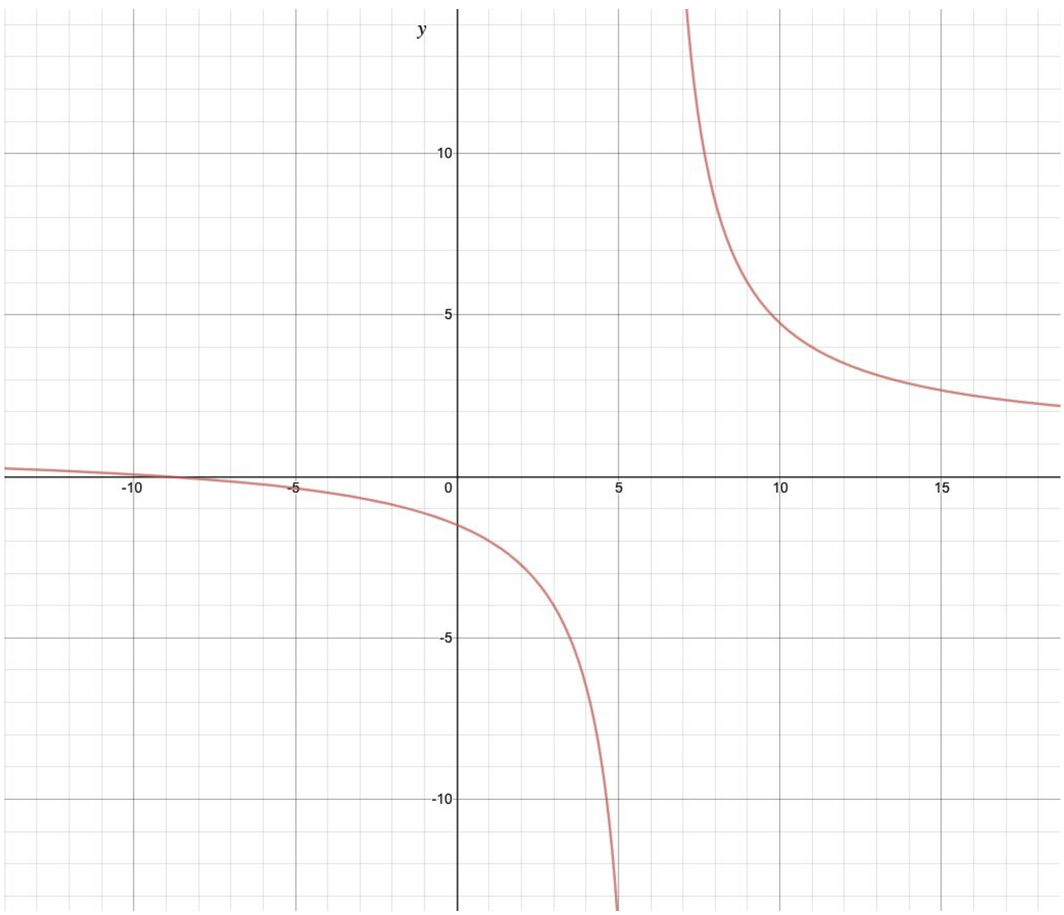
- A rational function has a ratio of two polynomial functions where its divider is not zero. The function has x-intercepts, y-intercepts, turning points, local maximum, and local minimum, and asymptotes.
- The domain of $f(x) = \frac{P(x)}{Q(x)}$ is all point of x where its denominator $Q(x)$ is not zero and $f(x),\ g(x)$ are polynomials.

1-2. How is it different from a polynomial?

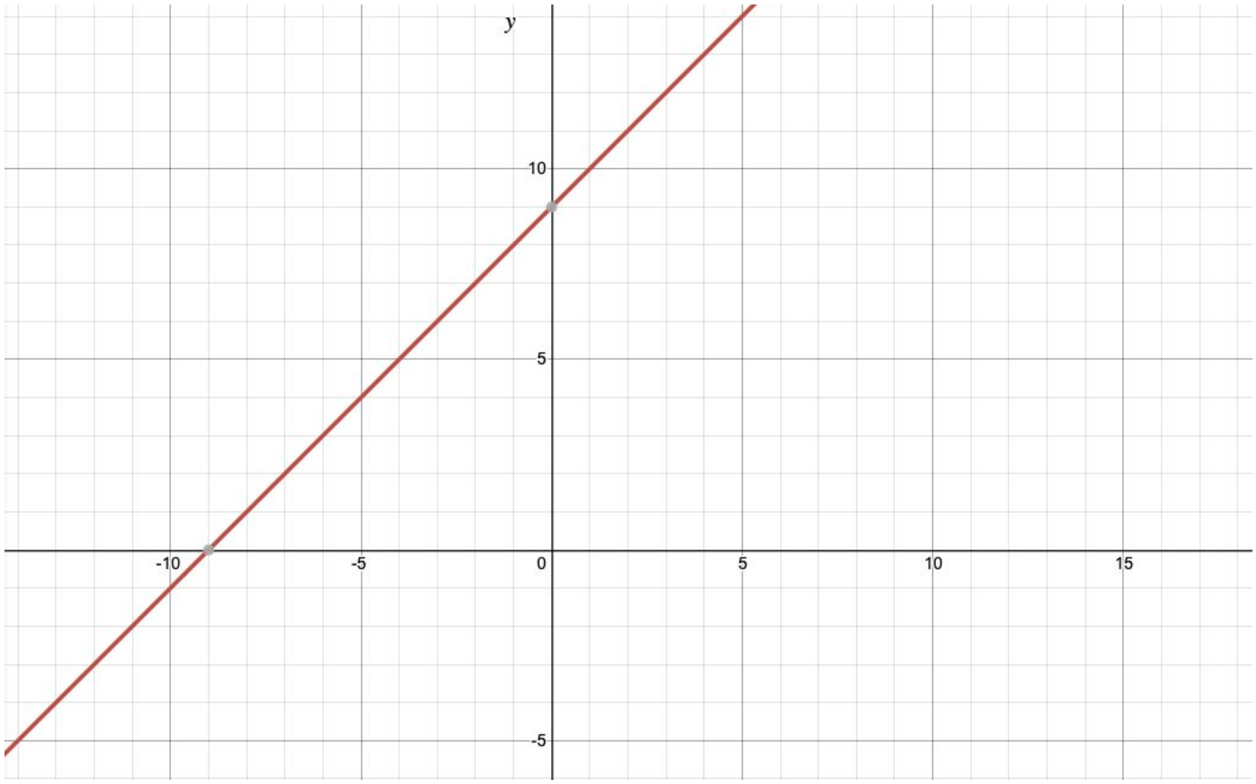
- A polynomial function is composed of one or multiple monomials while a rational function has a denominator, which is a polynomial function, to divide another polynomial function.

1-3. Provide a graph of each to demonstrate the difference.

- The graph of $f(x) = \frac{x+9}{x-6}$



- The graph of $f(x) = x + 9$



Question 2.

2-1. $f(x) = \frac{2x-1}{x+5}$

- x-intercept
  $0 = \frac{2x-1}{x+5}$
  $0 = 2x - 1$
  $1 = 2x$
  $x = \frac{1}{2}$
  Therefore, the x-intercept is $\frac{1}{2}$

- y-intercept
  $f(0) = \frac{2x-1}{x+5}$
  $f(0) = \frac{2(0)-1}{0+5}$
  $f(0) = \frac{-1}{5}$
  Therefore, the y-intercept is $\frac{-1}{5}$

- holes
  Factor: $\frac{2x-1}{x+5}$
  There is no hole since no common factor viable.

- Vertical asymptotes

Factor: $\frac{2x-1}{x+5}$
Thus, x = -5

- Horizontal and Oblique asymptotes
  The numerator and denominator have the same degree, thereby horizontal asymptotes are $y = \frac{a}{c}$ where a, c are both leading coefficient of the numerator and denominator.
  $y = \frac{2}{1}, y = 2$

- End behaviors
  i) when $x \to -5^-$
  $f(-5.001) = \frac{2*(-5.001)-1}{(-5.001)+5} = \frac{-11.002}{-0.001}$
  $y \to \infty$

  ii) when $x \to -5^+$
  $f(-4.999) = \frac{2*(-4.999)-1}{(-4.999)+5} = \frac{-10.998}{0.0001}$
  $y \to -\infty$

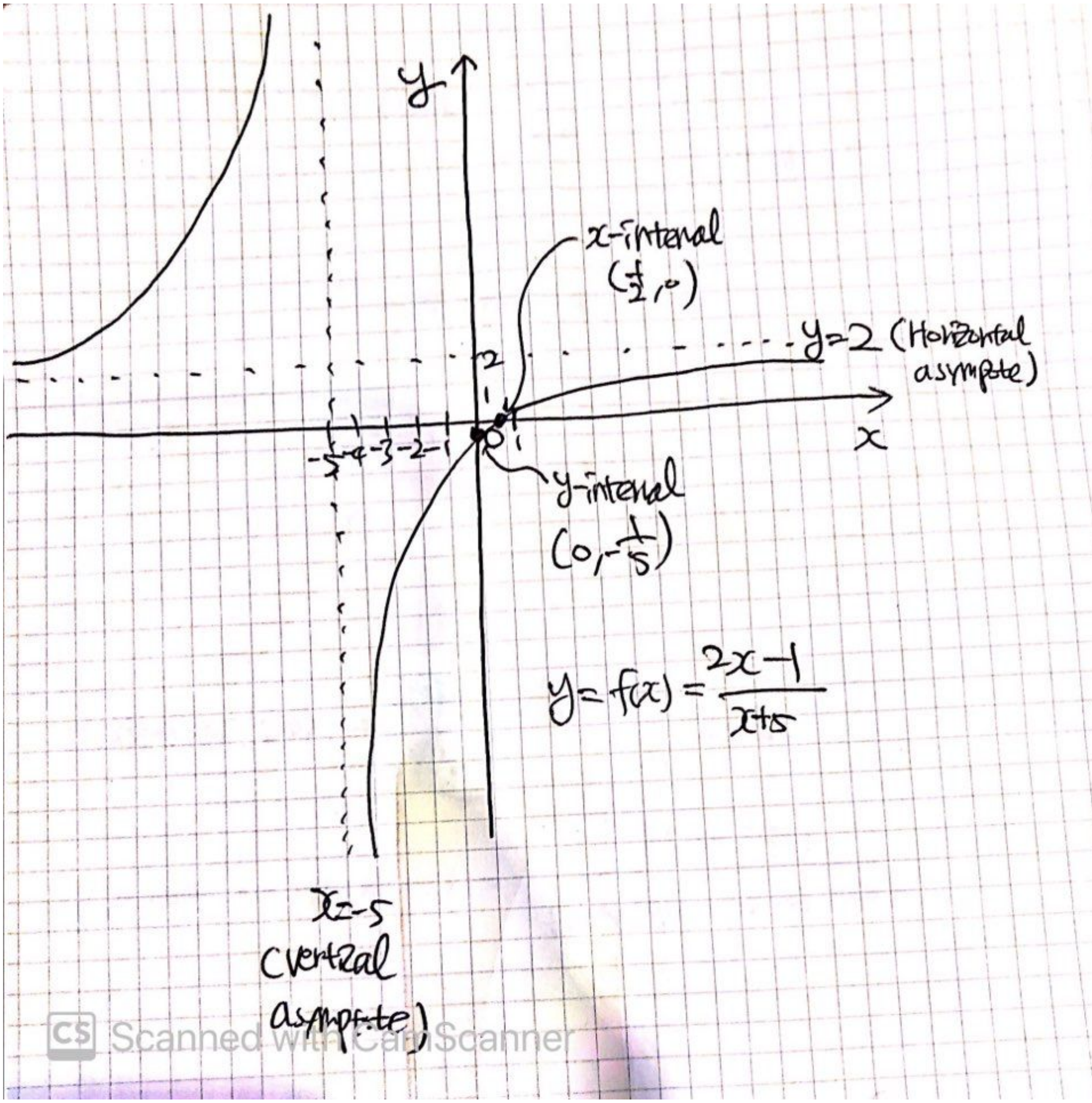
  iii) when $x \to \infty$
  $f(9999) = \frac{2*(9999)-1}{9999+5}$
  $y \to 2$

  iv) when $x \to -\infty$
  $f(-9999) = \frac{2*(-9999)-1}{(-9999)+5}$
  $y \to 2$

- Defining Intervals
  $f(x) = \frac{2x-1}{x+5}$

	$x < -5$	$-5 < x < \frac{1}{2}$	$x > \frac{1}{2}$
$2x - 1$	-	-	+
$x + 5$	+	+	+
f(x)	-	-	+

- Sketch by hand



2-2. $f(x) = \frac{x^2-9}{x}$

- x-intercept

  $0 = \frac{x^2-9}{x}$

  $0 = x^2 - 9$

  $9 = x^2$

  $x = 3 \ or \ x = -3$

- y-intercept

$f(x) = \frac{0-9}{0}$

It is not allowed to let the denominator equals zero. Thus, there is no y-intercept.

- Holes

  Since the nominator and denominator do not have a common factor, there is no hole.

- Vertical Asymptotes

  $f(x) = \frac{x^2-9}{x}$

  $x \neq 0$

  Thus, $x = 0$ is a vertical asymptote.

- Horizontal/Oblique Asymptotes

  The numerator has a larger degree than the denominator which means there are no horizontal asymptotes.

  To denote the following equation, $y = ax + b + \frac{e}{cx+d}\ (a \neq 0)$ has an oblique asymptote which is $y = ax + b$ while could get the same result by using long division.

  Thus, there is an oblique asymptote, which is $y = x$.

- End behaviours

  i) $x \to 0^-$

  $y = \frac{(-0.001)^2-9}{(-0.001)} = \frac{(0.000001)-9}{-0.001} > 0$

  $y \to \infty$

  ii) $x \to 0^+$

  $y = \frac{(0.001)^2-9}{(0.001)} = \frac{(0.000001)-9}{0.001} < 0$

  $y \to -\infty$

  iii) $x \to \infty$

  $f(999) = \frac{(999)^2-9}{999} > 0$

  $y \to \infty$

  iv) $x \to -\infty$

  $f(-999) = \frac{(-999)^2-9}{(-999)} < 0$

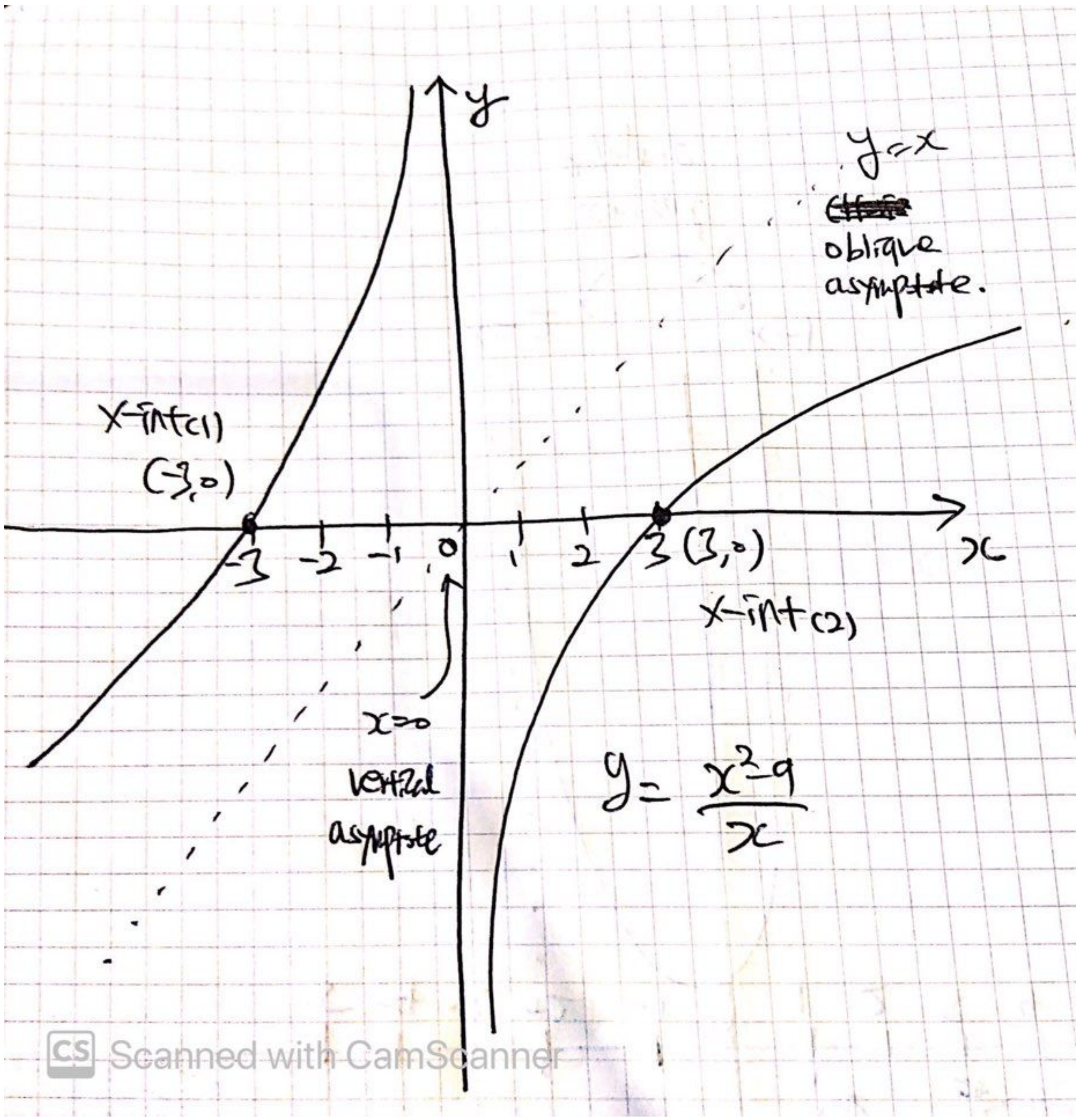
  $y \to -\infty$

- Defining Intervals

	$x < -3$	$-3 < x < 0$	$0 < x < 3$	$x > 3$

x	-	-	+	+
$x^2 - 9$	+	-	-	+
f(x)	-	+	-	+

- Sketch by hand



2-3. $f(x) = \frac{x-3}{x^3-2x^2-5x+6}$

$f(x) = \frac{(x-3)}{(x-1)(x+2)(x-3)}$

- x-intercept

  $0 = \frac{1}{(x-1)(x+2)}$

  $0 * (x-1)(x+2) = 1$ which is undefined.

  There is no x-intercept.

- y-intercept

  $f(x) = \frac{1}{(0-1)(0+2)} = -\frac{1}{2}$

- Holes

  Factor $f(x) = \frac{(x-3)}{(x-1)(x+2)(x-3)}$

  $(x-3)$ is the common factor

  $x = 3$

  Substitute $x = 3$ *to* $\frac{1}{(x-1)(x+2)}$

  $\frac{1}{(3-1)(3+2)} = \frac{1}{10}$

  Therefore hole is $(3,\ \frac{1}{10})$

- Vertical Asymptotes

  $(x-1)(x+2)(x-3) \neq 0$

  $x \neq -2,\ 1,\ 3$

  However, $x = 3$ is a hole value

  Therefore, the vertical asymptote is $-2,\ 1$

- Horizontal/Oblique Asymptotes

  The denominator has a bigger degree than the numerator

  The horizontal asymptote happens in $y = 0$

  There is no slant asymptote.

- End behaviors

  i) $x \to\ -2^-$

  $f(x) = \frac{(-2.001-3)}{(-2.001-1)(-2.001+2)(-2.001-3)} = \frac{1}{(-3.001)(-0.001)} > 0$

  $f(x) \to \infty$

  ii) $x \to\ -2^+$

  $f(x) = \frac{(-1.999-3)}{(-1.999-1)(-1.999+2)(-1.999-3)} = \frac{1}{(-2.999)(0.0001)} < 0$

  $f(x) \to\ -\infty$

iii) $x \to 1^-$

$f(x) = \frac{(0.999-3)}{(0.999-1)(0.999+2)(0.999-3)} = \frac{1}{(-0.001)(2.999)} < 0$

$f(x) \to -\infty$

iv) $x \to 1^+$

$f(x) = \frac{(1.001-3)}{(1.001-1)(1.001+2)(1.001-3)} = \frac{1}{(0.001)(3.001)} > 0$

$f(x) \to \infty$

v) $x \to \infty$

$f(999) = \frac{(999-3)}{(999-1)(999+2)(999-3)} > 0$

$f(x) \to 0$

vi) $x \to -\infty$

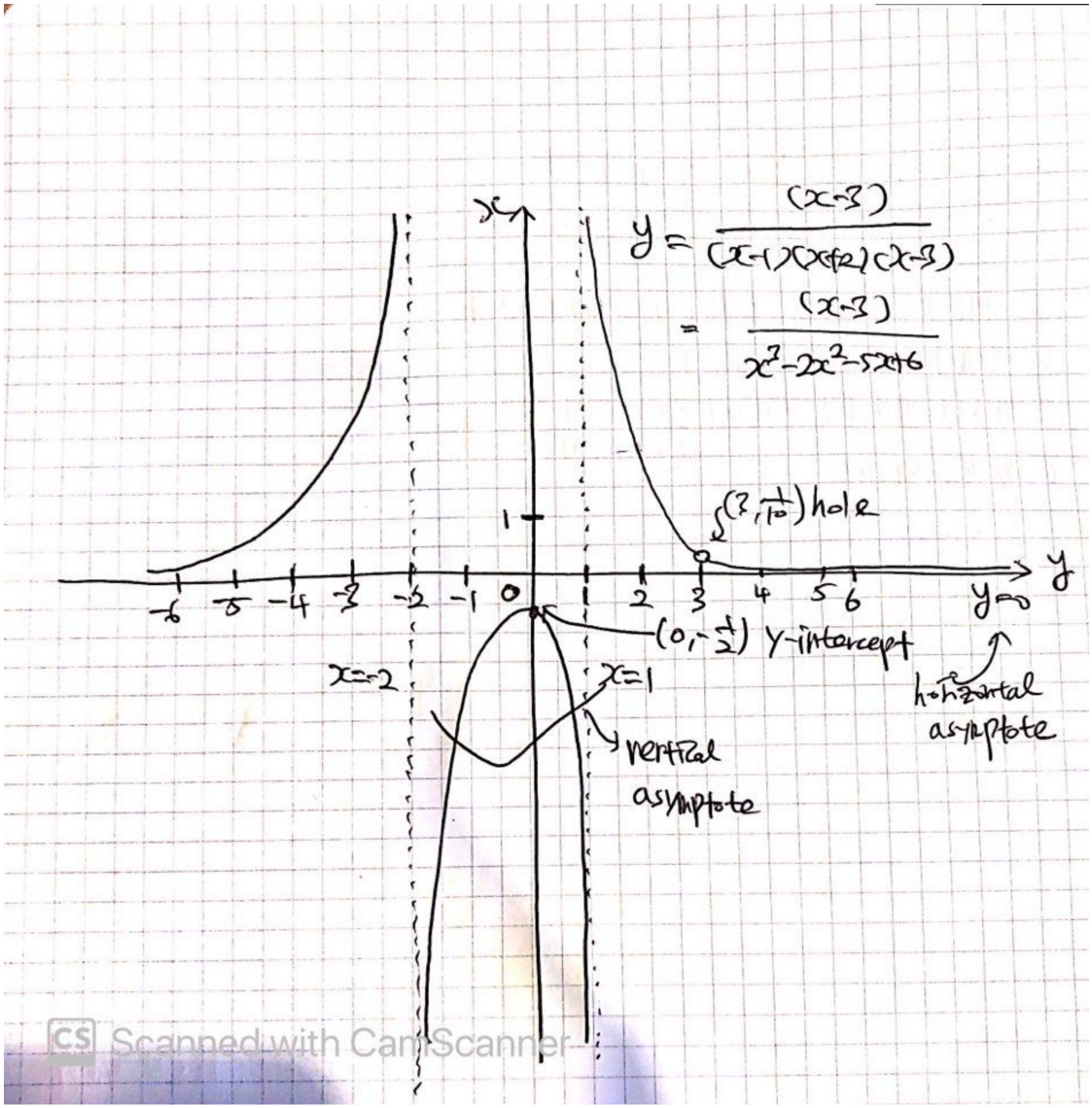
$f(-999) = \frac{(-999-3)}{(-999-1)(-999+2)(-999-3)} > 0$

$f(x) \to 0$

- Defining Intervals

	$x < -2$	$-2 < x < 1$	$x > 1$
$x+2$	-	+	+
$x-1$	-	-	+
f(x)	+	-	+

- Sketch by hand



Question 3.

a. $\frac{16x^2-9}{x^2+4x-12} = 0$

$\frac{(4x+3)(4x-3)}{(x-2)(x+6)} = 0$

To extract the value that makes the function undefined.

$(x-2) \neq 0,\ (x+6) \neq 0$

$x \neq 2,\ x \neq -6$

To get the value that makes the function zero.

$(4x+3) = 0$

$or\ (4x-3)=0$

$x=\frac{3}{4}\ or\ x=\ -\frac{3}{4}$

b. $x(2x-13)<\ -20$

$2x^2-13x\ <\ -20$

$2x^2-13x+20\ <\ 0$

$(2x-5)(x-4)\ <\ 0$

	$x<\frac{5}{2}$	$\frac{5}{2}<x<4$	$x>4$
• $(2x-5)$	-	+	+
$(x-4)$	-	-	+
	+	-	+

Therefore, $\frac{5}{2}<x<4$

c. $\frac{1}{r+3}>\frac{r+4}{r-2}+\frac{6}{r-2}$

$\frac{1}{r+3}>\frac{r+10}{r-2}$

$\frac{1}{r+3}-\frac{r+10}{r-2}>0$

$\frac{(r-2)-(r+10)(r+3)}{(r+3)(r-2)}>0$

$\frac{(r-2)-(r^2+13r+30)}{(r+3)(r-2)}>0$

$\frac{-r^2-13r-30+r-2}{(r+3)(r-2)}>0$

$\frac{-r^2-12r-32}{(r+3)(r-2)}>0$

$\frac{-(r^2+12r+32)}{(r+3)(r-2)}>0$

$\frac{-(r+4)(r+8)}{(r+3)(r-2)}>0$

$\frac{(r+4)(r+8)}{(r+3)(r-2)}<0$

	$r<-8$	$-8<r<-4$	$-4<r<-3$	$-3<r<2$	$r>2$
$(r-2)$	-	-	-	-	+
$(r+3)$	-	-	-	+	+
$(r+4)$	-	-	+	+	+
$(r+8)$	-	+	+	+	+

	+	-	+	-	+

The answer is $-8 < r < -4$ and $-3 < r < 2$

Question 4.

The following formula determines the minimum number of hours of studying required to attain a test score, x.

$p(x) = \frac{0.31x}{100.5-x}$

a. **How many hours of study are needed to score an 80?**

   $p(80) = \frac{0.31(80)}{100.5-80}$

   $p(80) = about\ 1.21$

   Therefore, 1.21 hours of study are needed to score an 80.

b. **What score can you achieve if you study for 6 hours?**

   $6 = \frac{0.31x}{100.5-x}$

   $6(100.5 - x) = 0.31x$

   $603 - 6x = 0.31x$

   $6.31x = 603$

   $x = 95.56$

   Therefore, I would score 95.56 after 6 hours of study.

c. **How many hours of study are needed to score the mark you would like to obtain in this course?**

   I would like to obtain 100, thus would put 100 into the equation.

   $P(100) = \frac{0.31(100)}{100.5-100}$

   $P = 62$

   Therefore, I should study for 62 hours to obtain 100.