

Trigonometric Differentiation and Applications Unit Assignment
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1. Find the derivatives of each of these functions.

a) $y = (x^4 + \csc(x))^3$

$$= 3(x^4 + \csc(x))^2 \frac{d}{dx} [x^4 + \csc(x)]$$

$$= 3(x^4 + \csc(x))^2 (4x^3 + \frac{d}{dx} [\csc(x)])$$

Since $\frac{d}{dx} [\csc(x)] = -\csc(x)\cot(x)$, We can write the following:

$$= 3(x^4 + \csc(x))^2 (4x^3 - \csc(x)\cot(x))$$

b) $y = \frac{\sec(4x)}{\sin(x)}$

$$= \frac{d}{dx} \left[\frac{1}{\sin(x)} \right] = \frac{d}{dx} \left[\frac{1}{\cos(4x)} \cdot \frac{1}{\sin(x)} \right]$$

$$= \frac{d}{dx} [\sec(4x) \cdot \csc(x)]$$

$$= \sec(4x) \frac{d}{dx} [\csc(x)] + \csc(x) \frac{d}{dx} [\sec(4x)]$$

Since $\frac{d}{dx} [\csc(x)] = -\csc(x)\cot(x)$, We can write as the following.

$$= \sec(4x)(-\csc(x)\cot(x)) + \csc(x) \frac{d}{dx} [\sec(4x)]$$

Since $\frac{d}{dx} [\sec(x)] = \sec(x)\tan(x)$, We can write as the following.

$$= -\sec(4x)\csc(x)\cot(x) + 4\csc(x)\sec(4x)\tan(4x)$$

c) $y = \csc(\sqrt{3x^2 + 1})$

$$= \frac{d}{dx} [\csc(3x^2 + 1)^{1/2}]$$

Since $\frac{d}{dx} [\csc(x)] = -\csc(x)\cot(x)$, We can write as the following.

$$= -\csc(3x^2 + 1)^{\frac{1}{2}} \cot(3x^2 + 1)^{\frac{1}{2}} \left(\frac{1}{2} (3x^2 + 1)^{-\frac{1}{2}} \right) \cdot 6x$$

$$= -\frac{\cot(3x^2 + 1)^{\frac{1}{2}} \csc(3x^2 + 1)^{\frac{1}{2}}}{2(3x^2 + 1)^{\frac{1}{2}}} (6x + 0)$$

$$= -\frac{3\cot((3x^2 + 1)^{\frac{1}{2}}) \csc((3x^2 + 1)^{\frac{1}{2}}) x}{(3x^2 + 1)^{\frac{1}{2}}}$$

d) $y = \frac{\cos^2(x)}{\ln(3x+4)}$

$$= \frac{d}{dx} \left[\frac{\cos^2(x)}{\ln(3x+4)} \right]$$

$$= \frac{\frac{d}{dx} [\cos^2(x)] \cdot \ln(3x+4) - \cos^2(x) \frac{d}{dx} [\ln(3x+4)]}{\ln^2(3x+4)}$$

$$= \frac{(-\sin(2x)) \cdot \ln(3x+4) - \cos^2(x) \cdot \frac{1}{3x+4} \cdot \frac{d}{dx} [3x+4]}{\ln^2(3x+4)}$$

$$\begin{aligned}
&= \frac{(-\sin(2x))\ln(3x+4) - \frac{\cos^2(x)(3 \cdot \frac{d}{dx}[x] + \frac{d}{dx}[4])}{3x+4}}{\ln^2(3x+4)} \\
&= \frac{-\sin(2x)\ln(3x+4) - \frac{3\cos^2(x)}{3x+4}}{\ln^2(3x+4)} \\
&= \frac{-\sin(2x)\ln(3x+4)(3x+4) - 3\cos^2(x)}{(3x+4)\ln^2(3x+4)}
\end{aligned}$$

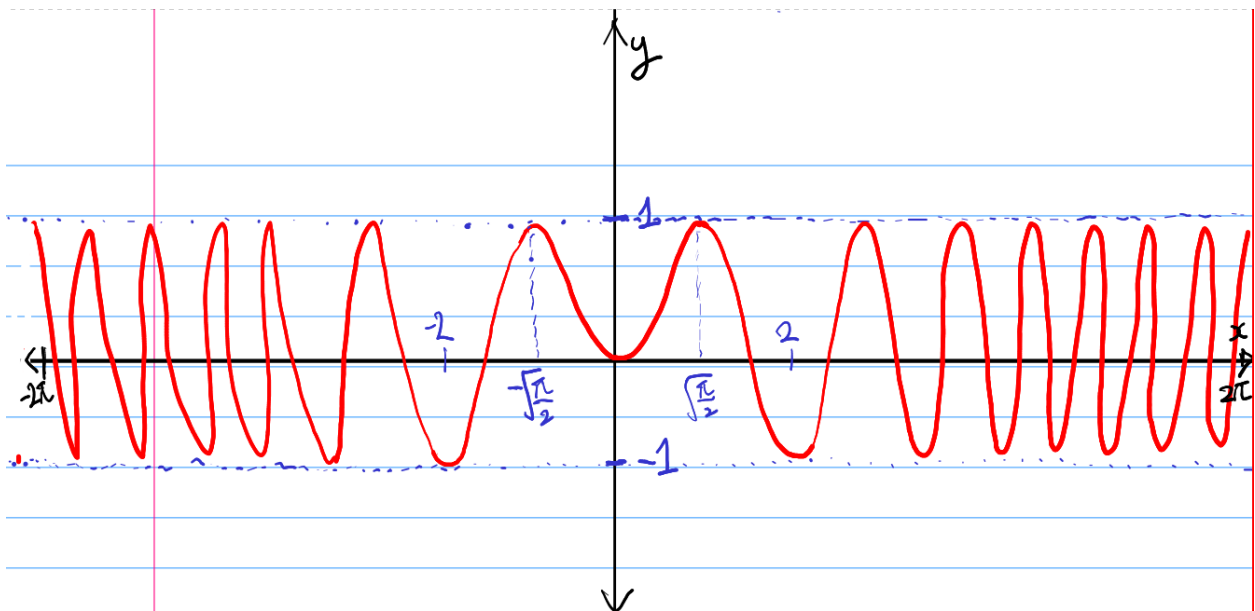
2. Use the process of implicit differentiation to find $\frac{dy}{dx}$ given that $\sin(4x) + \sin(2y) = 1$.

$$\frac{d}{dx}(\sin(4x) + \sin(2y)) = \frac{d}{dx}(1).$$

$$\Rightarrow 4\cos(4x) + 2\cos(2y)\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-2\cos(4x)}{\cos(2y)}$$

3. Using curve sketching methods, sketch the graph of the function $y = \sin(x^2)$ on the interval $-2\pi \leq x \leq 2\pi$. Make sure that you include all steps, charts, and derivations details.



The function is not periodic; but symmetric about the y-axis since $y(x) = y(-x)$.

The range of the function is $[-1, 1]$.

To differentiate the function, $y' = (\cos(x^2))2x$

Maximum and minimum occurs when the first derivative is zero.

$$2x\cos(x^2) = 0, x = 0 \text{ or } \cos(x^2) = 0$$

$$x^2 = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \dots$$

$$y' = 0 \text{ when } x = 0, \pm\sqrt{\frac{\pi}{2}}, \pm\sqrt{\frac{3\pi}{2}}, \pm\sqrt{\frac{5\pi}{2}}, \pm\sqrt{\frac{7\pi}{2}}, \pm\sqrt{\frac{9\pi}{2}} \dots$$

Using all the information above, we can draw the graph of $y = \sin(x^2)$.

4. Find the equation of the tangent to the curve $y = \frac{\sin(3x)}{\cos(x)}$ at $\frac{\pi}{4}$.

$$y\left(\frac{\pi}{4}\right) = \frac{\sin\left(\frac{3\pi}{4}\right)}{\cos\left(\frac{\pi}{4}\right)} = 1$$

Thus, we find the point $\left(\frac{\pi}{4}, 1\right)$ at first.

$$\frac{dy}{dx} \left(\frac{\sin(3x)}{\cos(x)} \right) = \frac{3\cos(3x) \cdot \cos(x) + \sin(3x)\sin(x)}{\cos^2(x)}$$

We can substitute x with $\frac{\pi}{4}$ to get the highest point.

$$\frac{dy}{dx} \left(\frac{\sin(3x)}{\cos(x)} \right) \Big|_{x=\frac{\pi}{4}} = \frac{3\cos\left(\frac{3\pi}{4}\right)\cos\left(\frac{\pi}{4}\right) + \sin\left(\frac{3\pi}{4}\right)\sin\left(\frac{\pi}{4}\right)}{\cos^2\left(\frac{\pi}{4}\right)} = \frac{-3 \cdot 1 + 1 \cdot 1}{1} = -2$$

Thus, $y'\left(\frac{\pi}{4}\right) = -2$.

We can write the tangent curve as the following with given information.

$$y - 1 = -2\left(x - \frac{\pi}{4}\right)$$

$$y = -2x + \frac{\pi}{2} + 1$$

5. The movement of the crest of a wave is modelled with the function

$f(x) = 0.2\cos(4t) + 0.3\sin(5t)$. **Find the maximum height of the wave and the time at which it occurs. (6 marks)**

$$f(x) = 0.2\cos(4t) + 0.3\sin(5t)$$

To find the first derivative and get when the derivative is equal to zero.

$$f'(x) = (-0.2)4\sin(4t) + 0.3(5)\cos(5t) = 0$$

$$-0.8\sin(4t) = 1.5\cos(5t)$$

$$8\sin(4t) = 15\cos(5t)$$

$$\frac{\sin(4t)}{\cos(5t)} = \frac{15}{8}$$

This is possible only when $t = \frac{\pi}{2} + 2n\pi$ since both $\sin(4t)$ and $\cos(5t)$ should be 1.

$$\text{When } n = 0, t = \frac{\pi}{2} \Rightarrow 8\sin\left(\frac{4\pi}{2}\right) = 8\sin(2\pi) = 0, 15\cos\left(\frac{5\pi}{2}\right) = 15(0) = 0$$

Thus, we can conclude that max value of the first derivative can be found at $t = \frac{\pi}{2}$.

$$f(x) \Big|_{t=\frac{\pi}{2}} = 0.2\cos\left(\frac{4\pi}{2}\right) + 0.3\sin\left(\frac{5\pi}{2}\right) = 0.2\cos(2\pi) + 0.3\sin\left(2\pi + \frac{\pi}{2}\right)$$

$$= 0.2 + 0.3 = 0.5$$

Thus, the max of the function is $\frac{1}{2}$ which occurs at $t = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \frac{13\pi}{2} \dots$. In general, $t = 2n\pi + \frac{\pi}{2}$

.

6. Determine the 19th derivative of the function $y = \sin(x)$. Fully explain the process you used to determine this. (4 marks)

To differentiate the given function, we can write the following.

$$\frac{dy}{dx} = \cos(x)$$

Differentiate again and again and again.

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(\cos(x)) = -\sin(x)$$

$$\frac{d^3y}{dx^3} = \frac{d}{dx}(-\sin(x)) = -\cos(x)$$

$$\frac{d^4y}{dx^4} = \frac{d}{dx}(-\cos(x)) = \sin(x)$$

$$\text{Thus, } \frac{d^4y}{dx^4} = y$$

Now, we know the fact that differentiating four times renders the same and given equation.

$$\frac{d^4}{dx^4} \left(\frac{d^4y}{dx^4} \right) = \frac{d^4}{dx^4} (y) = \frac{d^4y}{dx^4}$$

$$\frac{d^8y}{dx^8} = y$$

$$\frac{d^4}{dx^4} \left(\frac{d^8y}{dx^8} \right) = \frac{d^4}{dx^4} (y) = \frac{d^4y}{dx^4}$$

$$\frac{d^{12}y}{dx^{12}} = y$$

$$\frac{d^4}{dx^4} \left(\frac{d^{12}y}{dx^{12}} \right) = \frac{d^4}{dx^4} (y) = \frac{d^4y}{dx^4}$$

$$\frac{d^{16}y}{dx^{16}} = y$$

$$\text{Thus, } \frac{d^{16}y}{dx^{16}} = \sin(x)$$

Differentiating x with one time by step-by-step method.

$$\frac{d^{17}y}{dx^{17}} = \frac{dy}{dx} = \cos(x)$$

$$\frac{d^{18}y}{dx^{18}} = \frac{d^2y}{dx^2} = -\sin(x)$$

$$\frac{d^{19}y}{dx^{19}} = \frac{d^3y}{dx^3} = -\cos(x)$$

Thus, the 19th differentiating values of the given equation is $-\cos(x)$.