Virtual High School

Polynomial Unit 1 Assignment

MHF4U Unit Assignment

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MHF4U
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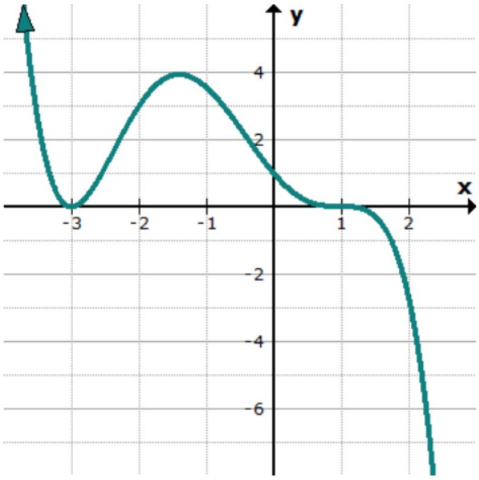
1. State end behavior, min & max number of turns, min & max number of x-intercepts and restrictions.
    a. Question 1 a) $f(x) = x^4 + 8x^3 + k$:Quartic
        i. End behavior
            1. As $x \to \infty, y \to \infty$
            2. As $x \to \infty, y \to \infty$
            3. Begin in quadrant 2 and end in quadrant 1
        ii. Minimum & Maximum number of $x - intercepts$
            1. Maximum : 4
            2. Minimum : 0
        iii. Minimum & Maximum number of turns
            1. Maximum : 3
            2. Minimum : 1
        iv. Domain & Range
            1. Domain $\{x | x \varepsilon | R\}$
            2. Range $\{y | y > minimum\ value, y \varepsilon | R\}$
    b. Question 1 b) $f(x) = x^6 + kx^4 - 9x^2 - 27$: 6th power
        i. End behavior
            1. As $x \to \infty, y \to \infty$
            2. As $x \to -\infty, y \to \infty$
            3. Begin in quadrant 2 and end in quadrant 1
        ii. Minimum & Maximum number of $x - intercepts$
            1. Maximum : 6
            2. Minimum : 0
        iii. Minimum & Maximum number of turns
            1. Maximum : 5
            2. Minimum : 1
        iv. Domain & Range
            1. Domain $\{x | x \varepsilon | R\}$
            2. Range $\{y | y > minimum\ value\ y \varepsilon | R\}$
    c. Question 1 c) $f(x) = -\frac{1}{2}x^7 - 441x^3 + k$: 7th power
        i. End behavior
            1. As $x \to \infty, y \to -\infty$
            2. As $x \to -\infty, y \to \infty$
            3. Begin in quadrant 2 and end in quadrant 4
        ii. Minimum & Maximum number of $x - intercepts$
            1. Maximum : 7
            2. Minimum : 1
        iii. Minimum & Maximum number of turns
            1. Maximum : 6
            2. Minimum : 0

iv. Domain & Range

1. Domain $\{x|\, x\varepsilon|R\}$
2. Range $\{y|\, y\varepsilon|R\}$

2. Using the graphs provided, determine the equation for the polynomial



a. Given Clues

i. $x-intercepts\ :\ -3,\ 1$

ii. $y-intercepts\ :\ 1$

iii. $Number\ of\ turns\ :\ 3 \rightarrow$ Degree has to be larger than 4

iv. $f(x)\ =\ a(x+3)^2(x-1)^3$

v. $f(0)\ =\ a(0+3)^2(x-1)^3$

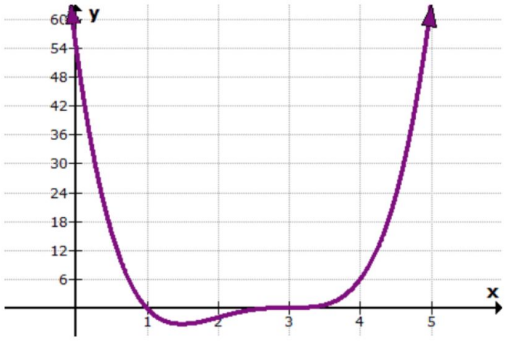
vi. $1\ =\ a(0+3)^2(x-1)^3$

vii. $1\ =\ a(9)(-1)$

viii. $\frac{1}{9}\ =\ -a$

ix. $-\frac{1}{9}\ =\ a$

x. $f(x)\ =-\frac{1}{9}(x+3)^2(x-1)^3$



b. Given Clues

i. $x-intercepts\ :\ 1,\ 3$

ii. $y-intercepts\ :\ 54$

iii. $Number\ of\ turns\ :\ 2 \rightarrow$ Degree has to be larger than 3

iv. Begin in quadrant 2, end in quadrant 1

v. $f(x)\ =\ a(x-1)(x-3)^3 \rightarrow$ Third power is added because the line on 3 is relatively not curved well compared to the line between 1 and 2

vi. $f(0)\ =\ a(0-1)(0-3)^3$

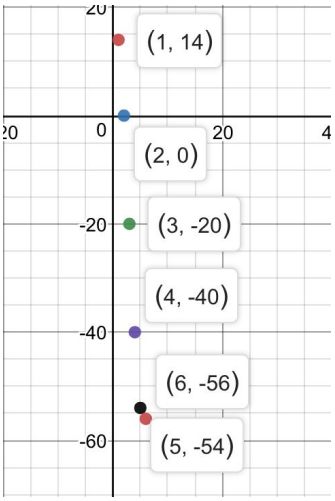
vii. $54\ =\ a(-1)(-27)$

viii. $54\ =\ a(27)$

ix. $\frac{54}{27}\ =\ a\ =\ 2$

x. $f(x)\ =\ 2(x-1)(x-3)^3$

3. Determine the equation of the function that would model this data



x	*y*	1st	2nd	3rd
1	14			
		-14		
2	0		-6	
		-20		6
3	-20		0	
		-20		6
4	-40		6	
		-14		6
5	-54		12	
		-2		
6	-56			

Cubic function

- $x-intercept\ :\ 2$
- $f(x)\ = ax^3 + bx^2 + cx + d$

When (2,0)

$0\ =\ 8a + 4b + 2c + d$ ==B

When (1,14)

$14 = a + b + c + d$ ==A

When (3,-20)

$-20 = 27a + 9b + 3c + d$ ==C

When (4,-40)

$-40 = 64a + 16b + 4c + d$ ==D

A-B

$-14\ =\ 7a + 3b + c$ ==E

A-C

$20 =- 19a - 5b - c$ ==F

$C - D$

$20 =- 37a - 7b - c$ ==G

E+F

$6 =- 12a - 2b$ ==H

E+G

$6 =- 30a - 4b$ ==I

$2H - I$

$6\ =\ 6a$

$a\ =\ 1$

Subs $a = 1$ into I

$36 =- 4b$

$b =- 9$

Subs $a = 1,\ b = -9$ into E

$-14 = 7 - 27 + c$

$c = 6$

Subs $a = 1, b = -9, c = 6$ into B

$14 = 1 - 9 + 6 + d$

$d = 16$

$f(x) = x^3 - 9x^2 + 6x + 16$

4. Even symmetry? Odd symmetry? Neither?

   Even symmetry : $f(x) = f(-x)$

   Odd symmetry : $f(-x) = -f(x)$

   Neither : Not fitting to either one

   $f(x) = 3x^3 - 144$

   i. $f(x) = f(1) = 3(1)^3 - 144 = -141$

   ii. Even : $f(-1) = 3(-1)^3 - 144 = -147 \neq f(1)$

   iii. Odd : $-f(1) = -(3(1)^3 - 144)) = 141 \neq f(-1)$

   iv. ∴Neither

   b. $f(x) = 8x^4 + x^2$

   i. $f(1) = 8(1)^4 + 1^2 = 9$

   ii. Even : $f(-1) = 8(-1)^4 + (-1)^2 = 9 = f(1)$

   iii. ∴ Even symmetry

   c. $f(x) = x^2 - x^6$

   i. $f(1) = 1^2 - 1^6 = 0$

   ii. Even : $f(-1) = (-1)^2 - (-1)^6 = 0 = f(1)$

   iii. ∴Even Symmetry

   d. $f(x) = x^5 + x^3 - x$

   i. $f(1) = 1^5 + 1^3 - 1 = 1$

   ii. Even : $f(-1) = (-1)^5 + (-1)^3 - (-1) = -3 \neq f(1)$

   iii. Odd : $-f(1) = -(1^5 + 1^3 - 1) = -1 \neq f(-1)$

   iv. ∴Neither

5. Factor completely
   a. $f(x) = x^3 - 13x + 12$

      x^2 -12

      x -1 Cross multiplication method

      i. $f(x) = (x^2 - 1)(x - 12)$

      ii. $f(x) = (x + 1)(x - 1)(x - 12)$

   b. $f(x) = -5x^3 + 17x^2 - 16x + 4$

      i. $f(x) = -x^2(5x - 17) - 4(4x - 1)$ Factoring by grouping

      ii. $f(x) = (-x^2 - 4)(5x - 17)(4x - 1)$

   c. $f(x) = 5x^4 - 22x^3 + 33x^2 - 20x + 4$

      i. $f(x) = (x - 2)(x - 1)^2(5x - 2)$

      ii. $f(x) = (x - 2)(x - 1)(x + 1)(5x - 2)$

6. The profit of a company can be modelled by the polynomial function $P(t) = -t^3 + 12t^2 - 21t + 10$, where *P* is the profit, in thousands of dollars, and *t* is the time, in years. When will the company make their maximum profit of $108,000?
   a. $P(t) = 108$ (Because the unit of this profit is represented thousand dollars

   b. $108 = -t^3 + 12t^2 - 21t + 10$

   c. Find factor *t cannot be negative because t represents time

      i. $If\ t = 1\ :\ -1 + 12 - 21 + 10 \neq 108$

      ii. $If\ t = 2\ :\ -8 + 48 - 42 + 10 \neq 108$

      iii. $If\ t = 3\ :\ -27 + 108 - 63 + 10 \neq 108$

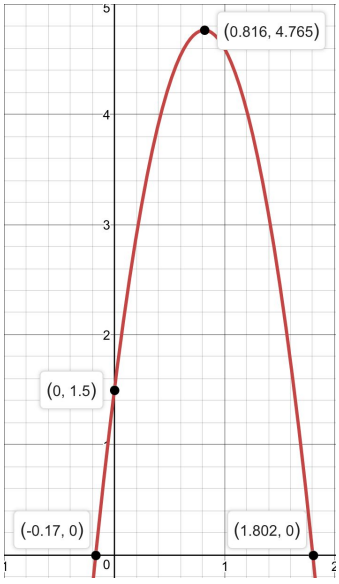
      iv. $If\ t = 4\ :\ -64 + 192 - 84 + 10 \neq 108$

      v. $If\ t = 5\ :\ -125 + 300 - 105 + 10 \neq 108$

      vi. $If\ t = 6\ :\ -216 + 432 - 126 + 10 \neq 108$

      vii. $If\ t = 7\ :\ -343 + 588 - 147 + 10 = 108$

   d. $\therefore t = 7$, The company would make max profit of $108,000 in 7 years.



7. $H(t) = -4.9t^2 + 8t + 1.5$

H is the height in meters and t is the time in seconds

a. Graph
   i. Domain & Range
      1. Domain $\{x| x\varepsilon|R\}$
      2. Range $\{y| y \leq 4.765\varepsilon|R\}$
   ii. At what point do you think the shotput was traveling fastest?
      1. When the height is closest to 0(Falling).
   iii. What factors did you use to make your inference?
      1. The slope look steepest.
   iv. Determine the average rate of change on a short interval near the point you chose
      1. Coordinates : $(1.700, 0.939),\ (1.65, 1.36)$
      2. $\frac{0.939-1.36}{1.700-1.65} = -8.42$

b. Determine the instantaneous rate of change at this point
   i. $Instantaneous\ rate\ of\ change = \frac{H2-H1}{t2-t1}$
   ii. Coordinates : $(1.802,\ 0),\ (1.801,\ 0.014)$
   iii. $\frac{0-0.014}{1.802-1.801} = -14$

c. Are the rates you calculated in a) and b) the same or different? Why do you think this is?
   i. The slope of part a) and part b) was different because as the object fall, acceleration will be added to the original speed. Then the object will have the highest speed when it's close to the ground.