MCV4U Vectors Unit Assignment
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1. **State whether each quantity is a vector or scalar.**
    1. Speed
    - Because a scalar has only magnitude with no direction, speed is a scalar. $speed = \frac{distance}{time}$
    2. Velocity
    - Considering that velocity can be specified with magnitude in a designated direction, we can think that $velocity = \frac{vec(d)}{t}$ has both magnitude and direction.
    3. Weight
    - Force is a product of mass, which is a scalar, and acceleration, which is a vector. When considering the formula that weight can be calculated by multiplying mass and acceleration, we can think that weight is a vector product taking both direction and magnitude.
    4. Mass
    - Mass itself is only a scalar quantity with only presenting magnitude. Mass does not change no matter where you are living and moving.
    5. Area
    - The area element is correctly defined only if its magnitude and direction are given. When it comes to the area of a parallelogram, we can appreciate the orientation of the area element while given as $vec(s) = vec(a) * vec(b)$ (where $vec(a)$ *and* $vec(b)$ are its two sides). Thus, the area element is a vector quantity.

2. In the diagram, ACE is an equilateral triangle. B, D, and F are the midpoints of AC, CE, and EA. $vec(AB) = vec(u),\ vec(AF) = vec(v)$ **. Write the following vectors in terms of** $vec(u)$ **and** $vec(v)$ **.**

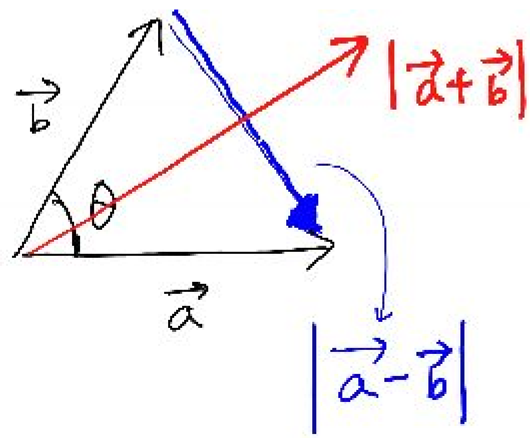
a. $vec(AC) = 2vec(AB) = 2vec(u)$
b. $\vec(AD) = \vec(AF) + \vec(FD) = \vec(v) + \vec(u)$
c. $\vec(CE) = \vec(AE) = 2\vec(v)$
d. $\vec(EB) = ((\vec(EF) + \vec(FA)) + \vec(AB) = -\vec(v) - \vec(v) + \vec(u)$

Give an example of a vector that is equal to:

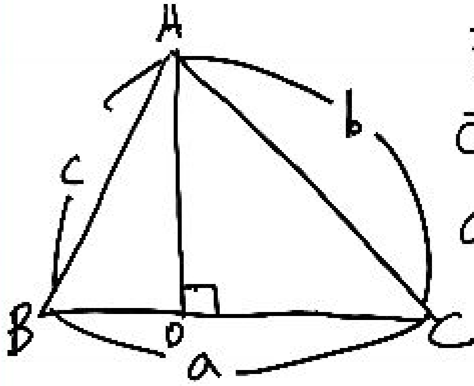
e. $2\vec(v) = 2\overline{AF} = \overline{AE}$
f. $\vec(u) - \vec(v) = \vec(\overline{FD}) + \vec(DC) = \vec(FC)$

3. Draw diagrams to show two vectors, vec(a) and vec(b), and the two vectors (vec(a)+vec(b)) and (vec(a)-vec(b)).

To begin with, let me define some elements that are required to solve the problems.



i) $|\vec{a}-\vec{b}|$ — 2nd law of cosine.



$\overline{BD} = c\cdot\cos B$

$\overline{CD} = b\cdot\cos C$

$a = c\cos B + b\cos C$

$$a = c\cos B + b\cos C \xrightarrow{\times a} a^2 = ac\cos B + ab\cos C$$

$$b = a\cos C + c\cos A \xrightarrow{\times b} b^2 = ab\cos C + bc\cos A$$

$$c = b\cos A + a\cos B \xrightarrow{\times c} c^2 = bc\cos A + ac\cos B$$

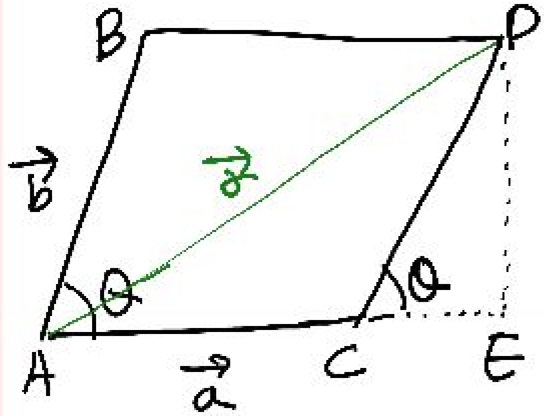
$$(a^2-b^2) = (ac\cos B + \cancel{ab\cos C}) - (\cancel{ab\cos C} + bc\cos A)$$

$$= ac\cos B + bc\cos A$$

$$(a^2-b^2+c^2) = (ac\cos B + \cancel{bc\cos A}) + (\cancel{bc\cos A} + ac\cos B)$$

thus, $b^2 = a^2+c^2-2ac\cos B$ ■

b. $|\vec{a}+\vec{b}|$



$\overline{CE} = \vec{b}\cdot\cos\theta$, $\overline{AE} = \vec{a}+(\vec{b}\cos\theta)$, $\overline{ED}=\vec{b}\sin\theta$

$\overrightarrow{AD} = \vec{x} = (\vec{a}+\vec{b}\cos\theta)^2+(\vec{b}\sin\theta)^2$

According to pythagoras's law,

$= (\vec{a}^2+2\vec{a}\vec{b}\cos\theta+\vec{b}\cos\theta^2)+(\vec{b}\sin\theta)^2$

$= (\vec{a}^2+2\vec{a}\vec{b}\cos\theta)+\vec{b}^2(\cos\theta^2+\sin\theta^2)$

$(\because \cos\theta^2+\sin\theta^2=1)$

$= \vec{a}^2+2\vec{a}\vec{b}\cos\theta+\vec{b}$

a)

Let θ be the angle between $\vec{(a)}$ and $\vec{(b)}$ as shown in the attached figure. Thus, the range of θ is to be $0 \le \theta \le \frac{\pi}{2}$.

$|\overrightarrow{a} + \overrightarrow{b}| = \sqrt{|\overrightarrow{a}|^2 + |\overrightarrow{b}|^2 + 2|\overrightarrow{a}||\overrightarrow{b}|\cos\theta}$

$|\overrightarrow{a} - \overrightarrow{b}| = \sqrt{|\overrightarrow{a}|^2 + |\overrightarrow{b}|^2 - 2|\overrightarrow{a}||\overrightarrow{b}|\cos\theta}$

Considering the given condition that is $|\overrightarrow{a} + \overrightarrow{b}| > |\overrightarrow{a} - \overrightarrow{b}|$,

$$\sqrt{|\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos\theta} > \sqrt{|\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|\cos\theta}$$
$$\implies \cos\theta > -\cos\theta$$
$$\implies 2\cos\theta > 1 \implies \cos\theta > \frac{1}{2} \implies \theta \in \left[0, \frac{\pi}{3}\right)$$

Thus, the answer is $0 \le \theta < \frac{\pi}{3}$.

b)

Considering the given condition that is $|\vec{a} + \vec{b}| < |\vec{a} - \vec{b}|$,

$$\sqrt{|\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos\theta} < \sqrt{|\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|\cos\theta}$$

$$\implies \cos\theta < -\cos\theta$$

$$\implies 2\cos\theta < 1 \implies \cos\theta < \frac{1}{2} \implies \theta \in \left(\frac{\pi}{3}, \frac{\pi}{2}\right]$$

Thus, the answer is $\frac{\pi}{3} < \theta < \frac{\pi}{2}$.

c)

Considering the given condition that is $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$,

$$\sqrt{|\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos\theta} = \sqrt{|\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos\theta}$$

$$\implies \cos\theta = -\cos\theta$$

$$\implies \cos\theta = 0 \implies \theta = \frac{\pi}{2}$$

Thus, the answer is $\theta = \frac{\pi}{2}$.

d)

Considering the given condition that is $|\vec{a} + \vec{b}| = |\vec{a}| + |\vec{b}|$,

$$\sqrt{|\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos\theta} = |\vec{a}| + |\vec{b}|$$

$$\implies |\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos\theta = |\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|$$

$$\implies \cos\theta = 1 \implies \theta = 0$$

Thus, the answer is $\theta = 0$.

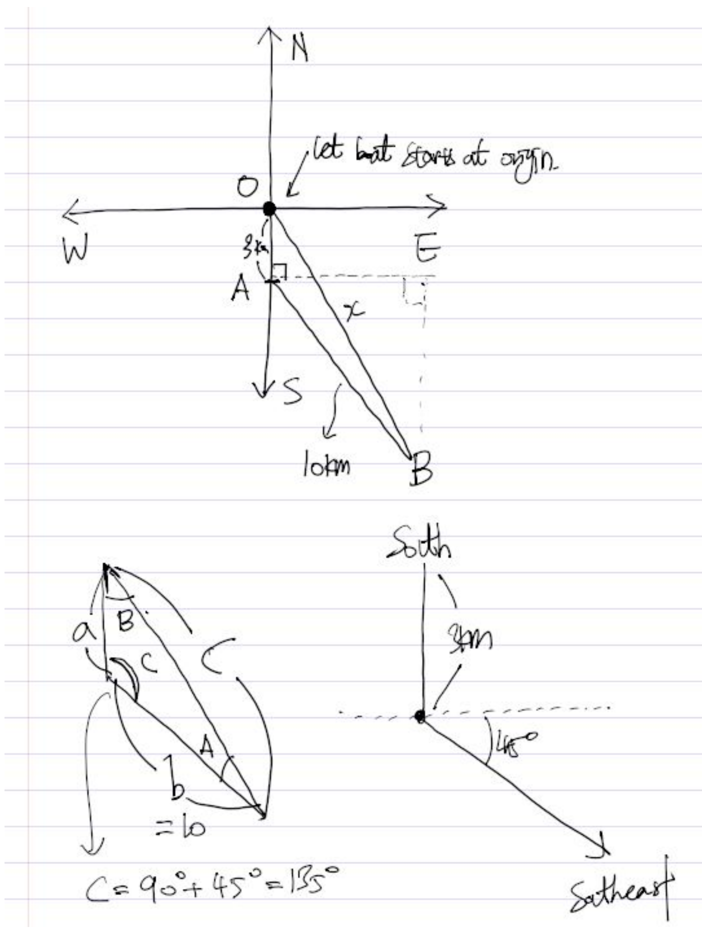
e)

$|\vec{a} + \vec{b}| = \sqrt{|\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos\theta}$ would be minimum when $\theta = 180\ degree.$

Since $cos(180) = -1,$

$|\vec{a} + \vec{b}| = \sqrt{|\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|} = ||a| - |b||$

4. A boat sails 3 km South, then 10 km Southeast. Use trigonometry to find the boat's distance and bearing from its starting point.



Using cosine theorem,

$x^2 = (a^2 + b^2) - 2abcos(c) = (10)^2 + (3)^2 - 2 * 10 * 3 * cos(90 + 45)$

$= 100 + 9 - 60 * cos(135) = 109 - 60(-cos45) = 109 + \frac{60}{\sqrt{2}}$

$x = 12.30km = boat's\ distance$

Using Lemi's theorem,

$\frac{c}{sinc} = \frac{b}{sinb} = \frac{a}{sina}$

$\frac{12.30}{sin(90+45)} = \frac{10}{sinb}$, $sinb = \frac{10}{12.30\sqrt{2}}$

$b = sin^{-1}[\frac{10}{12.30\sqrt{2}}]$

$b = sin^{-1}(0.57) = 34.75$

$180 - 34.75 = 145.25 = bearing\ from\ starting\ point$

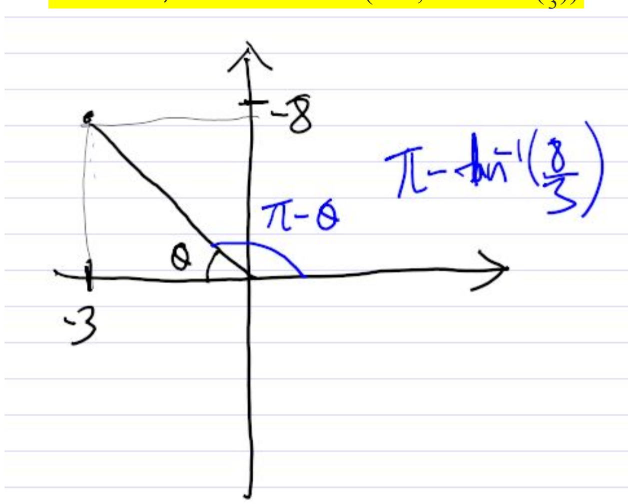
5. Convert the following vectors;

a) 75m/s on a bearing of 295 degrees to Cartesian form

- Since $\gamma = 75m/s$ while $\theta = 259\ degrees$, we can get [x, y] points by using trigonometry formulas.
- $x = \gamma cos\theta = 75cos(295) = 75 * 0.422618 = 31.6964$
- $y = \gamma sin\theta = 75sin(295) = 75 * (-0.906308) =- 67.9731$
- Thus, the answer is $(31.6964,\ -67.9731)$.

b) [-3, 8] to direction/magnitude form

- $magnitude = \sqrt{(-3)^2 + (8)^2} = 9 + 64 = \sqrt{73}$
- $tan\theta = \frac{8}{3}$
- $\theta = tan^{-1}(\frac{8}{3})$
- Thus, the answer is $(\sqrt{73}, \pi - tan^{-1}(\frac{8}{3}))$



6. Express as a single vector.

a. $(\vec{PS}) + (\vec{SR}) = (\vec{PR})$

b. $(\vec{EF}) - (\vec{DF})$

$(\vec{EF}) + (\vec{FD}) = (\vec{ED})$

$$\boxed{\overrightarrow{(EF)} + (\overrightarrow{(-FD)}) = \overrightarrow{(ED)}}$$

c. $\overrightarrow{(MP)} - \overrightarrow{(QR)} + \overrightarrow{(NM)} + \overrightarrow{(RP)}$

$= \overrightarrow{(MP)} + \overrightarrow{(RQ)} + \overrightarrow{(NM)} + \overrightarrow{(PR)}$

$= \overrightarrow{(NM)} + \overrightarrow{(MP)} + \overrightarrow{(PR)} + \overrightarrow{(RQ)})$

$= \overrightarrow{(NP)} + \overrightarrow{(PR)} + \overrightarrow{(RQ)}$

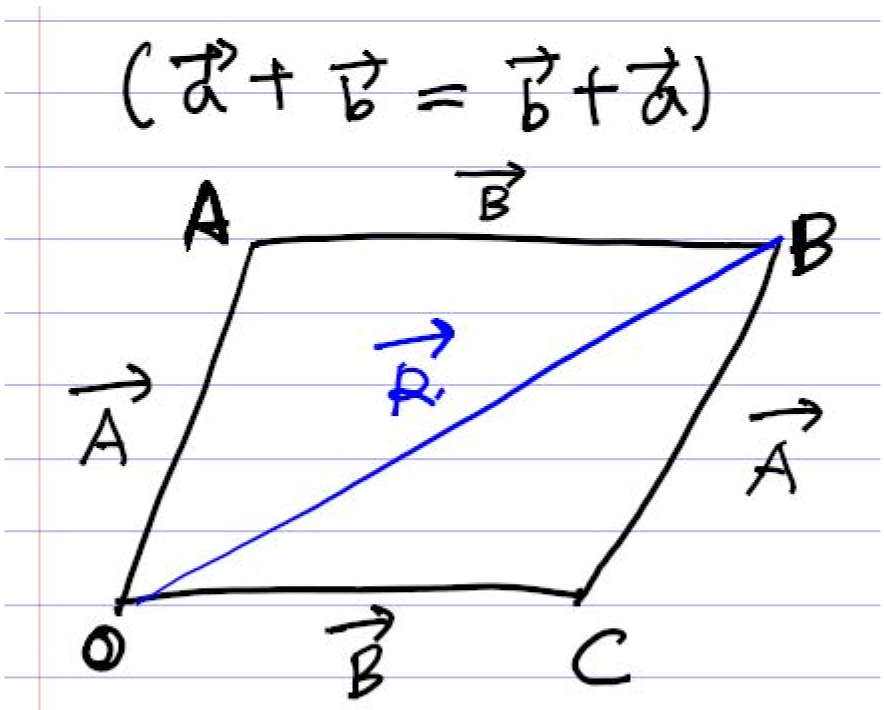
$= \overrightarrow{(NR)} + \overrightarrow{(RQ)}$

$\boxed{= \overrightarrow{(NQ)}}$

7. Demonstrate using diagrams

a) that vector addition is commutative: $\vec{(a)} + \vec{(b)} = \vec{(b)} + \vec{(a)}$

Constructing a parallelogram as following using two vectors as the adjacent sides.



For the sake of convenience in typing, I will omit vector indication to each side below.
Using triangle law of vector addition in ΔOAB, $A + B = R$
and also using triangle law of vector addition in ΔOBC, $B + A = R$
Thus, $A + B = B + A$

b) that vector addition is associative: $\vec{(a)} + (\vec{(b)} + \vec{(c)}) = (\vec{(a)} + \vec{(b)}) + \vec{(c)})$

For the sake of convenience in typing, I will omit vector indication to each side below.
Using triangle law of vector addition in ΔOAB, $\overline{OB} = A + B$ as similarly in ΔABC, $\overline{AC} = B + C$.

Now using the triangle law of vector addition in $\Delta OBC = R = (A + B) + C$.
Also, in ΔOAC we get, $\overline{R} = A + (B + C)$.
Thus, the following would be the same. $(A + B) + C = A + (B + C)$

8. Research an example of the use of vectors, and explain how the mathematics is used, for example in engineering, computer animation, gaming, 3-D printing or GPS technology

As you walk along the road, you can easily find electric poles that provide electricity throughout the city. You can imagine a curve of electric lines between the poles. If you hold the uniform lines at both ends, the lines will form symmetrical curves. This is Catenary. The most representative building using this catenary is the suspension bridge. Suspension bridges are often representative of each city as they are the first type of bridge that we can think of when we think of 'bridge'. For example, there are Yeongjong Bridge in Incheon, Korea, Gwangan Bridge, which creates beautiful scenery of Gwangalli Beach in Busan, Brooklyn Bridge, a symbol of New York, and Golden Gate Bridge in San Francisco. It is widely used in bridge construction because the shape of the suspension wire effectively disperses gravity, which is the force applied to the bridge so that the bridge can stand stably.