Combinatorics Unit Assignment

Jin Hyung Park

MDM4U
Virtual High School
2021.01.10

1. **A lot of national flags are made up of three horizontal stripes.**
    a. How many different three colour flags can be designed using green, blue, red, yellow, and black stripes if all three colours must be different?

Answer: When it comes to the top stripe, it has five candidates to be coloured. After the top stripe is filled with the color from green/blue/red/yellow and black, the middle stripe has four colour options to be filled with. At last, the bottom stripe has three possibilities. Thus, the answer is $5 \times 4 \times 3 = 60\ options.$

    b. How many of them contain a red stripe?

Answer: Imagine the scenario that three stripes do not fill red colour, which means the stripes only work with four different colours. When it comes to this, the number of possibilities is $4 \times 3 \times 2 = 24$. The case when containing a red stripe at least once can be solved by eradicating the aforementioned scenario from the total number of possibilities. Thus, the answer is $60 - 24 = 36\ flags.$

    c. How many extra possibilities are there if the top colour can be the same as the bottom colour?

Answer: When the top stripes and bottom stripes can be filled with the same colour, bottom stripe can have additional top stripe color plus the remaining three colours after middle stripe. In other words, top stripe can have 5 possibilities while middle stripe and bottom

stripe can have both 4 possibilities. Total is $5 \times 4 \times 4 = 80\ options$, while we can get the answer after subtracting the case when all three colours must be different.
Thus, the answer is 20 extra possibilities (80 options - 60 options).

2. **On his university application, Prashad must list his course choices in order of preference. He must choose four of the six courses available in his major discipline and three of the four courses offered in related subjects. In how many ways can Prashad list his course choices?**

Answer: To begin with, Prashad can choose any of six courses first. After that, he might choose the remaining courses - second, third … and last. When it comes to his non-major courses, he can choose three courses in a row among four related subjects.
Thus, the answer is $(6 \times 5 \times 4 \times 3) \times (4 \times 3 \times 2) \times 7! = 302400\ options.$

3. **Idi is creating a password for a website that has some strict requirements. The password must be 8 characters. Numbers and letters may be used, but may not be repeated.**
   a. How many different passwords are possible?

Answer: To begin with, we need to assume that the alphabet has 26 letters while there are 10 digits. Thus, the total 36 letters and numbers can be formatted in the password. The first character goes for 36 options, the second character goes for 35 options … The eighth character goes for 29 options.
Thus, We can get the answer as $36P8 = \frac{36!}{28!} = 1,220,096,908,800\ options.$

   b. How many are possible if the password must feature both numbers and letters?

We need to exclude the cases which are passwords that consist entirely of letters or digits from the total possibilities. The first group has $\frac{26!}{18!} = 62990928000\ members$ while the second contains $\frac{10!}{2!} = 1814400\ members$. Thus, the answer can be drawn by subtracting the aforementioned two numbers from the figure (a) which is to be $1157104166400\ possibilities.$

   c. How many are possible if the password must start with a letter?

We need to exclude all thoses where the first character is the number, which is $10 \times \frac{35!}{28!}$ while total cases figured in (a) can be written as $36 \times \frac{35!}{28!}$. One can get the answer by subtracting two numbers as the following $(36 - 10) \times \frac{35!}{28!} = 881181100800\ options.$

   d. How many are possible if the password must start with a letter and end with a number?

There is an additional restriction above figure (c) which is to be fixed with a number at the last character of password. We can reuse and extend the procedure of figure (c) which is to be $26 \times 10 \times \frac{34!}{28!} = 251766028800\ possibilities.$

4. **Marissa is doing a Tarot reading in which she must pick 6 cards from a deck of 72. The order of their selection is not important.**

a. How many different readings are possible?

We need to choose any of 6 cards from a deck of 72 involving restriction that does not allow duplication of cards. Thus, the answer is $72C6 = \frac{72!}{66!6!} = 156238908\ options$.

b. Marissa does not want to see the Fool card. There is one Fool card in the deck. How many of the possible readings do not feature the Fool card?

One can only consider 71 cards when choosing 6 cards from the deck, after subtracting one fool card that Marissa would not like to pick in the first place.
Thus, the answer is $71C6 = 143218999\ options$.

5. A committee of 5 people is to be chosen from a group of 8 women and 10 men.

a. How many different committees are possible?

Since there are a total of 18 people (8 women plus 10 men), we can choose 5 people among 8 candidates. Thus, the answer is $18C5 = 8568\ possibilities$.

b. How many are possible if the committee must feature both men and women?

Get the possibilities where the committee only features men or women. The number of those who are only men in the committee is $10C5 = 252\ possibilities$ while those who are only women in the committee is $8C5 = 56\ possibilities$. Then, subtract two numbers from the total number of possibilities featured in figure (a) which is $8586 - 252 - 56 = 8260\ possibilities$.

c. How many are possible if the committee must feature 3 women and 2 men?

We need to choose three from 8 women candidates while two from 10 men candidates.
Thus, we can write the formula as $8C3 \times 10C2 = 56 \times 45 = 2520\ possibilities$.

d. How many are possible if the committee must have more women than men?

Subtract the following options that the committee has more men than women from the number of possibilities solved in figure (a).

first case: 5 men with 0 women - $10C5 = 252\ possibilities$

second case: 4 men with 1 women - $10C4 \times 8C1 = 210 \times 8 = 1680$

third case: 3 men with 2 women - $10C3 \times 8C2 = 120 \times 28 = 3360$

Thus, the answer is the following as $8568 - 252 - 1680 - 3360 = 3276\ possibilities$.

6. A baseball team has 14 players.

a. How many 9-person batting orders are possible?

When it comes to the first order, we can choose any of players - 14 players. In second order, we are able to choose 13 players, after subtracting one player chosen in the first order.
Since we would like to pick 9 person orders, we can only consider till 9th order.
We can use the permutation by multiplying the following numbers together as $14 \times 13 \times 12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6$. Thus, the answer is 726485760 different possible orders.

b. How many batting orders are possible if Schierholtz is always in the starting line-up and always bats fourth?

If Schierholtz is always positioned in fourth order, we need to fix the player when considering the calculation for fourth order. For example, 13 possibilities go for first order, 12 possibilities go for the second order, 11 possibilities go for the third order, but the only one possibility - the room for Schierholtz - is designated for the fourth order, and 10 possibilities go for the fifth order, and subtracting the number by one until the total nine seats is set. Thus, We can draw the answer as $13 \times 12 \times 11 \times 1 \times 10 \ldots \times 6 = 51891540\, possibilities.$

7. Consider the word MATHEMATICS.

a. How many arrangements are there of the word MATHEMATICS?

Mathematics has eleven letters while the word can be arranged in $11!$ ways. Since the word could have repeating letters, we need to divide by the repetitions. As such, the answer becomes $\frac{11!}{2!2!2!} = 4989600\, possibilities.$

b. How many of these start with the letter M?

To begin with, we need to set the letter M in the first place. Since we do not consider duplicated letter M is mutually exclusive or different, fix the first order of letter by multiplying the $1.$ After that, multiplying the number of the remaining ten letters with removing the duplication by dividing them which is $\frac{10!}{2!2!}$ Thus, $907200\ of\ 4989600$ permutations start with the letter M.

c. How many of the arrangements in part (a) have the T's together?

Since we need to put the letters of T together, let us consider that there is arrangement that only one letter T in the word of "MATHEMAICS" exists which has 10 letters. Thus, multiplying the numbers by using the permutation to divide each of duplications from the total number of possibilities of $10!$. Thus, the answer can be drawn as $\frac{10!}{2!2!} = 907200\, possibilities.$

8. We have looked at situations in which we need to determine the number of possible routes between two places. We can look at the situation below as 9 steps, six of which must be East and three of which must be South.

This gives us 9!/(3!6!) possible routes. The calculation 9!/(3!6!) is equivalent to 9C3 (or 9C6).

Explain clearly why you could solve this question using combinations, and why this is equivalent to considering permutations with repeated items.

The order of this question is not important, so we will use a combination in this question. The equation (9C3) shows that the calculation should work in the original formula (n!)/(r!)(n-r). Knowing this, this would mean that 9C3 is the same as 9!/3! (9-3)!=84, 9!/3!6! This is also the same as 84. This proves that 9C3 is equivalent to considering the sequence of repeating items.

9. There are 8 parents and 43 students going on a school trip. Two groups are made, a large one with thirty students and five parents, and a small group with 13 students and three parents.

a. How many different ways can be the parents be chosen for the small group?

Since the small group only has three parents, we need to choose three from 8 parent candidates. Thus, the answer is $8C3 = 56\ possibilities.$

b. How many ways can the students be chosen for the large group if Stefan and Dylan must be in the small group?

To begin with, fix Stefan and Dylan in the small group. We need to consider that 2 students are already designated while 11 students need to be set afterwhile. This means that we need to choose eleven from 43 student candidates. After drawing the possibilities for a small group, we can think of the remaining students are automatically allotted to a large group. Thus, the answer is $41C11 = 41C30 = 3159461968\ possibilities.$

c. How many ways can the groups be arranged if Reena and both her parents must be in the small group?

We need to allocate Reena to the small group before beginning of calculation while doing the same for the parents. In other words, we only need to choose twelve students with three parents from each of the remaining candidates. Thus, the answer is $42C12 \times 6C1\ = 66348701328\ possibilities.$

10. Simplify each expression and write it without using factorial notation.

a. $\frac{(n+4)!}{(n+2)!}$

$\frac{(n+4)!}{(n+2)!} = \frac{1\times2\times3\times4\times5\times...\times(n-5)\times(n-4)\times(n-3)\times(n-2)\times(n-1)\times n\times(n+1)\times(n+2)\times(n+3)\times(n+4)}{1\times2\times3\times4\times5\times...\times(n-5)\times(n-4)\times(n-3)\times(n-2)\times(n-1)\times n\times(n+1)\times(n+2)}$

Since there are duplicates between the numerator and the denominator of the formula, we can cancel them by stating the following.

$(n+3)\times(n+4) = n^2 + 7n + 12$

b. $\frac{(n-r+1)!}{(n-r-2)!}$

$\frac{(n-r+1)!}{(n-r-2)!} = \frac{1\times2\times3\times4\times...\times(n-r-4)\times(n-r-3)\times(n-r-2)\times(n-r-1)\times(n-r)\times(n-r+1)}{1\times2\times3\times4\times...\times(n-r-4)\times(n-r-3)\times(n-r-2)}$

Since there are duplicates between the numerator and the denominator of the formula, we can cancel them by stating the following.

$(n-r-1)\times(n-r)\times(n-r+1)$

11. Investigate a lottery competition somewhere in the world. Explain how the lottery works, and what needs to happen for someone to win the jackpot, and at least one of the minor prizes.

a. Calculate the probability of winning each of the prizes you described, giving a full explanation of your work.

I have researched a Polish mini lotto in Lottoland.com, which costs 0.5 euros. To win the lottery, the player must guess all five of the 42 numbers in order to win 24,000 euros.

Second place is to guess four numbers to get 125 euros. The ways of choosing 5 numbers out of 42 numbers is $42C5 = 850668$. From the ways, we can render that the first place probability would be the following. $\frac{1}{850668}$. In addition, the second place probability would be the following also. $\frac{5C4 \times 37C1}{850668} = \frac{185}{850668}$

b. Consider the cost of playing. Do you think the prizes offered are fair? If not, why not, and why do you think people continue to play?

Using the above information, the estimated income for the first and second places is about 0.06 euros. Considering that the ticket price is 0.5 euros, the prizes offered are not fair. Despite the unfairness, I think people are playing lotto because they want to have a small hope of winning first place. The hope of winning first place is a positive stimulus every week for Proletarier.