Unit Assignment: Polynomial Functions

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Question 1. Stating the "End Behaviours, Maximum number of x-intercepts, Minimum number of x-intercepts, Maximum number of turns, Minimum number of turns, and Restrictions" of the following function.

a) $f(x) = -x^3 + kx^2 + 27x + 27$: cubic function

- end behaviour
  - $As\ x \to \infty,\ As\ y \to -\infty$
  - $As\ x \to -\infty,\ As\ y \to \infty$
  - This is a cubic function, which has opposite end behaviours.
  - The graph starts in the second quadrant and ends in the fourth quadrant.
- Minimum and Maximum number of x-intercepts
  - Maximum: 3 (when k = -100)
    - It could have a maximum of two relative max/min which means that the graph might touch x-axis(x-intercept) for three times.
  - Minimum: 1 (when k = 100)
- Minimum and Maximum number of turns
  - Maximum: 2
    - Since it is a cubic function, there can be a maximum of n-1 relative max and min when n represents the power of function. Thus, the maximum turn is (3-1) = 2.
  - Minimum: 0
    - In cubic function, there is no absolute relative maximum and minimum, which means that the graph might not have any turns.
- Domain and Range
  - Domain: $\{x|x \in |R\}$
  - Range: $\{y|y \in |R\}$

b) $f(x) = 4x^5 + 7x^4 + kx^3$: 5th function

$f(x) = x^3(4x^2 + 7x + k)$

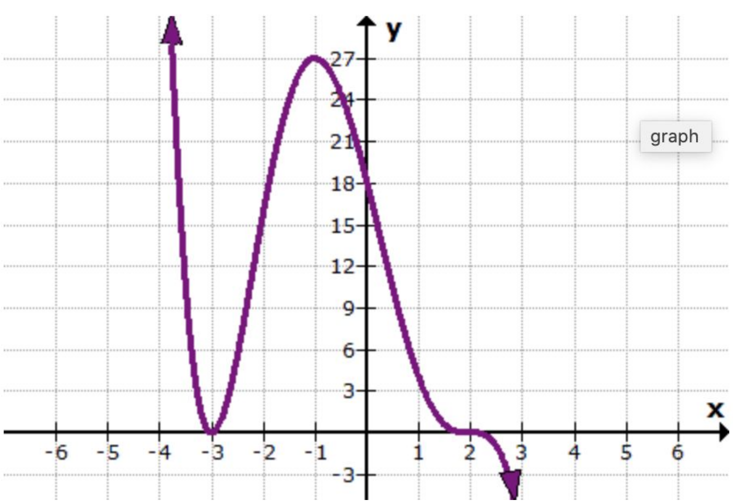
- end behavior:
  - $As\ x \to \infty,\ As\ y \to \infty$
  - This is a quintic function, which has opposite end behaviours.
  - The graph begins in the first quadrant and ends in the third quadrant.
- Minimum and Maximum number of x-intercepts
  - Maximum: 3 (when k = -100)
    - The function can be factored to $f(x) = x^3(4x^2 + 7x + k)$ which means that there is only one solution for $(x-0)^3$, and the left quadratic equation can have maximum two solutions. Thus, 1+2 = 3 is the answer for maximum x-intercepts.
  - Minimum: 1 (when k = +100)
    - When the function goes for both positive and negative infinity, the graph will touch only one $(0, 0)$ for x-axis.

- Minimum and Maximum number of turns
    - Maximum: 3
        - Although it is cubic function, the function can be factored as x^3 , it goes through the (0, 0), which means that only one turn is found in (0,0). Thus, the maximum turn is (4-2) = 2
    - Minimum: 1 (when x-intercept is found only in (0,0))

c) $f(x) = x^8 + kx^7 + 8x - 27$
$f(x) \ = \ x^7(x \ + k) + 8(x - \frac{27}{8}))$

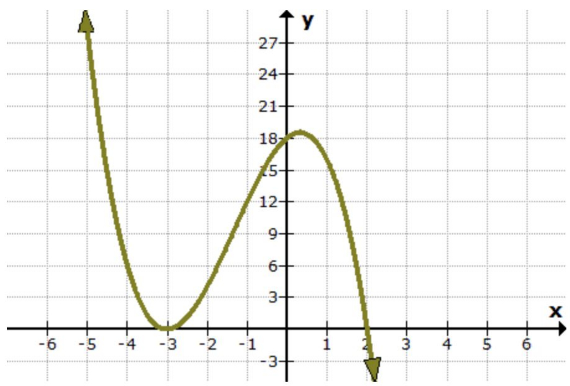
- end behaviour:
    - $As\ x \ \to \infty,\ As\ y \to \infty$
    - This is an even-degree function, which has the positive end behaviours.
    - The graph starts in the first quadrant and ends in the second quadrant.
- Maximum and minimum number of x-intercepts
    - The max number of x-intercepts is 2. When factoring the equation, it can have only two possible roots ($k\ and\ \frac{27}{8}$)
    - The min number of x-intercepts is 0, since an even-degree polynomial does not touch the x-axis.
- Maximum and minimum number of turns
    - The maximum number of turns is one since we have a maximum of two x-intercepts which means that the graph changes the behaviour once.
    - The minimum number of turns is the same as the maximum number, one, since the positive and negative behaviours have to be at least one turn to goes for positive indefinite.

2. Using the graphs provided, determine the equation for the polynomial function being represented. Show all of your work.



a. Given Clues

- x-intercepts: $-3,\ 2$
- y-intercepts: 18
- Number of turns: 3 which means that degree has to be larger than 4
- f(x) = $-a(x+3)^3(x-2)^3$
- f(0) = $-a(0+3)^2(0-2)^3$
- $27\ =-\ a(-1+3)^3(-1-2)^2$
- $27\ =-\ 4a * 9$
- $3\ =-4a,\ a\ =-\ \frac{3}{4}$
- Answer: $y\ =-\ \frac{3}{4}(x+3)^2(x-2)^3$

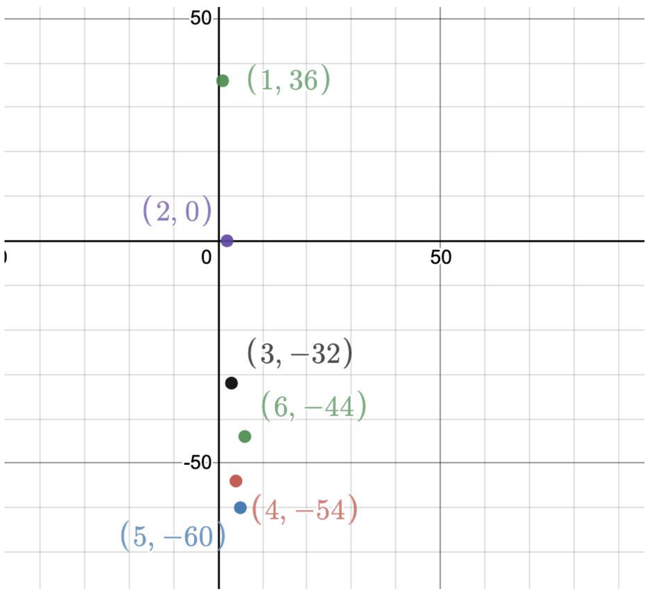


b. given clues
- x-intercepts: -3, 2
- y-intercept: 18
- Number of turns 2 which means that degree has to be larger than 3
- f(x) = $a(x+3)^2(x-2)$
- f(0) = $a(0+3)^2(0-2)$
- f(0) = $9a\ *\ (-2)\ =\ -18a\ =\ 18$
- Answer: f(x) = $-1(x+3)^2(x-2)$

Question 3. Using the table of values given, determine the equation of the function that would model this data.

<table>
<tr><th>x</th><th>f(x)</th><th>1st</th><th>2nd</th><th>3rd</th></tr>
<tr><td>1</td><td>36</td><td rowspan="4">-36
-32
-22</td><td rowspan="4">+4
+10</td><td rowspan="4">+6
+6
+6</td></tr>
<tr><td>2</td><td>0</td></tr>
<tr><td>3</td><td>-32</td></tr>
<tr><td>4</td><td>-54</td></tr>
</table>

5	-60	-6	+16	
6	-44	+16	+22	



- Cubic function
  - x-intercept: 2
  - f(x) = $ax^3 + bx^2 + cx + d$
- When (2, 0)
  - $0 = 8a + 4b + 2c + d$ == B
- When (1, 36)
  - $36 = 1a + 1b + 1c + d$ == A
- When (3, -32)
  - $-32 = 27a + 9b + 3c + d$ == C
- When (4, -54)
  - $-54 = 64a + 16b + 4c + d$ == D
- When (5, -60)
  - $-60 = 125a + 25b + 5c + d$ == E
- When (6, -44)
  - $-44 = 216a + 36b + 6c + d$ == F

- It is almost tedious to do calculations by substituting each of the aforementioned equations. Thus, I would like to factor $(2, 0)$ which means that when x = 2 it touches y = 0. We can start by writing the following: $y = (x-2)(ax^2 + bx + c)$

- Substitute (1, 36)
  - $36 \ = \ (1-2)(a+b+c)$
  - $36 =- a - b - c$
  - $c =- a - b - 36$ - G
- Substitute (3, -32)
  - $-32 = 9a + 3b + c$ -- H
- Substitute G to H
  - $-32 = 9a + 3b - a - b - 36$
  - $4 = 8a + 2b$
  - $2 = 4a + b$ -- I
- Substitute I to G
  - $b = 2 - 4a$
  - $c =- a - 2 + 4a - 36$
  - $c = 3a - 38$
- Thus, we can arrange $b,\ c$ as a
  - $b = 2 - 4a$
  - $c = 3a - 38$
  - $y = (x-2)(ax^2 + (2-4a)x + 3a - 38)$ -- J
- Substitute (4, -54) to J
  - $-54 = 2(16a + (2-4a)4 + 3a - 38)$
  - $-54 = 2(16a + 8 - 16a + 3a - 38)$
  - $-54 = 2(3a - 30)$
  - $-27 = 3a - 30$
  - $3 = 3a$
  - $1 = a$
- Thus, the answer is $f(x) = (x-2)(x^2 - 2a - 35)$

Question 4. Determine if the following polynomial functions have even or odd symmetry, or neither. Justify your reasoning.

Options:

- Even symmetry : f(x) = f(− x)
- Odd symmetry : f(− x) = − f(x)
- Neither

Variables:

(a) f(x) = $-x^3 + 3x$

- Even symmetry
  - $f(-x) =- (-x^3) + 3(-x) = x^3 - 3x$
  - $f(-x) \neq f(x)$

- Odd symmetry
    - $f(-x) =- f(x) =- (x^3 - 3x)$
    - $f(-x) = f(x)$
- Answer: Odd
- Additionally, the function has only odd powers of function, it should be an odd.

(b) f(x) = $x^4 + x^2 + x$

- Even symmetry
    - $f(-x) = (-x)^4 + (-x)^2 + (-x) = x^4 + x^2 - x$
    - $f(-x) \neq f(x)$
- Odd symmetry
    - $-f(x) =- x^4 - x^2 - x$
    - $f(-x) \neq- f(x)$
- Answer: Neither
- Additionally, the function has two types of(odd, even) powers of function, it should be neither.

(c) $f(x) = x^6 - x^4 - x^2$

- Even symmetry
    - $f(-x) = (-x)^6 - (-x)^4 - (-x)^2 = x^6 - x^4 - x^2$
    - $f(x) = f(-x)$
- Answer: Even
- Additionally, the function has only an even power of function, it should be even.

(d) $f(x) = \frac{1}{3}x^4 - \frac{2}{3}x^2$

- Even symmetry
    - $f(-x) = (-\frac{1}{3}x)^4 - (-\frac{2}{3}x)^2 = \frac{1}{3}x^4 - \frac{2}{3}x^2$
    - $f(x) = f(-x)$
- Thus, Even
- Additionally, the function has only an even power of function,, it should be even.

Question 5. Factor each of these polynomial functions completely. Show all of your work.

(a) $f(x) = 5x^3 - 17x^2 + 16x - 4$

- To perform the Synthetic division method, we can get the following.
    - enumerate a multiple of each polynomial: 5, -17, +16, -4
    - With multiplying 1, get the coefficients of quotients consecutively, 5, 12, 4.
    - The final remainder is 0, thereby we can get $f(x) = (x-1)(5x^2 - 12x + 4)$
    - Let us factor $5x^2 - 12x + 4$
        - perform the Synthetic division method, we can multiply 2.

        - With multiplying 2 respectively, we can get the coefficients of quotients consecutively, 5, -2.
        - The final remainder is 0, thereby we can get $f(x) = (x-2)(5x-2)$
- Thus, the answer is $f(x) = (x-1)(x-2)(5x-2)$

(b) $f(x) = x^3 - x^2 - 14x + 24$

- To perform the Synthetic division method, we can get the following.
    - enumerate a multiple of each polynomial: 1, -1, -14, 24
    - With multiplying 2 respectively, we can get the coefficients of the quotient consecutively, 1, 1, and -12.
    - The final remainder is 0, thereby we can get $(x-2)(x^2+x-12)$
    - Let us factor x^2+x-12
        - perform the Synthetic division method, we can multiply 3.
        - With multiplying 3 respectively, we can get the coefficients of the quotient consecutively, 1 and 4.
        - The final remainder is 0, thereby we can get $(x-3)(x-4)$
- Thus, the answer is $(x-2)(x-3)(x-4)$

(c) $f(x) = 2x^4 - 3x^3 - 24x^2 + 13x + 12$

- To perform the Synthetic division method, we can get the following.
    - enumerate a multiple of each polynomial: 2, -3, -24, 13, 12
    - With multiplying 1, we can get the coefficients of the quotient consecutively, 2, -1, -25, and -12.
    - The final remainder 0, thereby we can get $(x-1)(2x^3-x^2-25x-12)$
    - Let us factor $2x^3-x^2-25x-12$
        - perform the synthetic division method, we can multiply -3 to each coefficient respectively. We can get the quotients 2, -7, and -4.
        - The final remainder of this is 0, thereby we can get $(x+3)(2x^2-7x-4)$.
    - Let us factor $2x^2-7x-4$
        - perform the synthetic division method, we can multiply 4 to each coefficient respectively. We can get the quotients 2 and 1.
        - The final remainder of this is 0, thereby we can get $(x-4)(2x+1)$
    - Thus, the answer is $(x-1)(x+3)(x-4)(2x+1)$

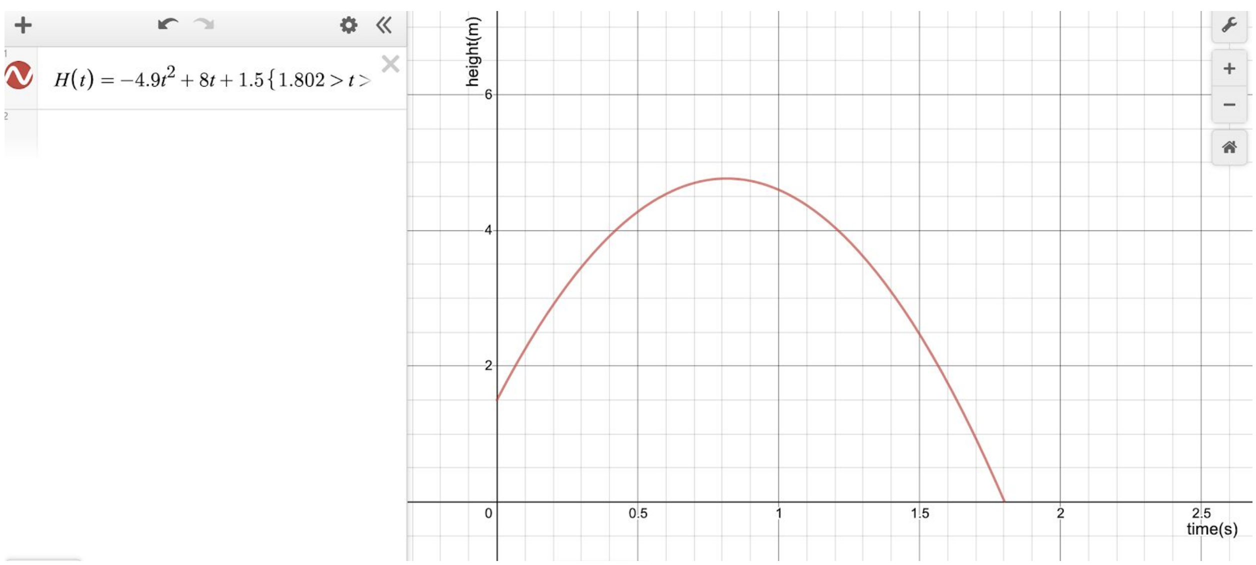
Question 6. Solve the problem following.

A packaging company has been asked to make cylindrical cans that can hold a volume of 49 πcm3. The company owns a machine that only makes cans that have a height which is the square of 6 less than the radius. To achieve the desired volume, what should the dimensions of the finished be?

- The following is the variable that is set to solve the problem.
    - radius = r
    - height = $h = (r-6)^2$
    - volume = $\pi r^2 h = \pi r^2 (r-6)^2 = 49\pi cm^3$
- Let us calculate to solve the function $r^2(r-6)^2 = 49$ which π is eliminated since left and right terms have the same value.
    - route each term both left and right. $r(r-6) = +7$
        - it could be -7 for the right side, but if then there would be no answer in Integer type.
        - $r(r-6) - 7 = 0$
        - $r^2 - 6r - 7 = 0$
        - $(r+1)(r-7) = 0$
- The answer would be the radius of 7cm, and the height of 1cm since -1 is not a viable answer for this question.

Question 7.
The height of a shotput can be modeled by the function: $H(t) = -4.9t + 8t + 1.5$
where H is the height in meters and t is the time in seconds.

(a) Graph this function using graphing technology. Ensure your axes are properly set up and state the domain and range that are appropriate for the situation. At what point do you think the shotput was traveling the fastest? What factors did you use to make your inference? Determine the average rate of change in a short interval near the point you chose.



- Domain & Range
    - Domain: $\{x | x \in |R,\ 0 < x < 1.802\},\ since\ time\ cannot\ be\ zero$

- Range: $\{y|0 \leq y \ \leq 4.765 \in |R\},\ since\ height\ cannot\ be\ zero$
- The point the shotput was traveling fastest
  - The moment when the height is getting close to the falling point, the moment before it touches the ground.
  - The reason behind the answer is the graph is that the speed gets accelerated at the most thanks to gravitational power.
- Determine the average rate of change in a short interval near the point.
  - (1.8, 0.024) and (1.801, 0.014)
  - $\frac{0.024-0.014}{1.8-1.801} =- 10$

(b) Determine the instantaneous rate of change at this point

- Instantaneous rate of change = $\frac{H2-H1}{t2-t1}$
- (1.801, 0.0143551) and (1.802, 0.0047004)
- $\frac{0.0143551-0.0047004}{1.801-1.802} =- 9.6547$

(c) Are the rates you calculated in a) and b) the same or different? Why do you think this is?

- The rates of (a) and (b) are different. The reason behind this is that acceleration to the original speed happens as the object falls down. The object will get the fastest speed when the moment before it touches the ground.