Image Denoising with Nonparametric Hidden Markov Trees

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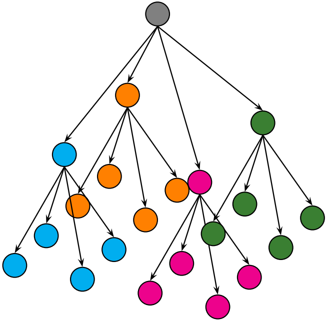
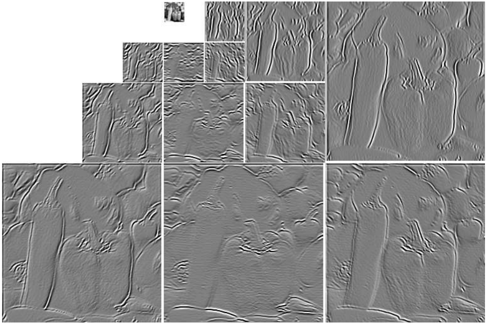
IEEE International Conference on Image Processing, 2007

Statistical Models for Wavelet Decompositions



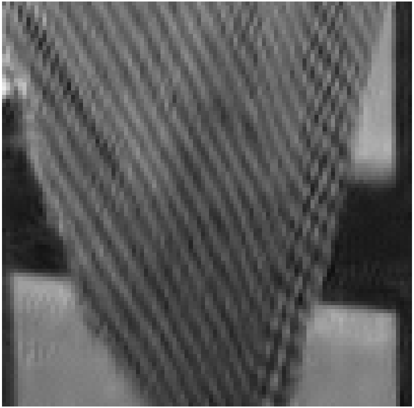


Goal: *Learn* good statistical models of natural images

Approach:
- Capture global statistics of wavelets via a *tree-structured* latent variable model
- Automatically adapt to data complexity via nonparametric, *Dirichlet process* priors

Denoising Images in Wavelet-Domain

Local Gaussian Scale Mixture
(Portilla et al. 2003)



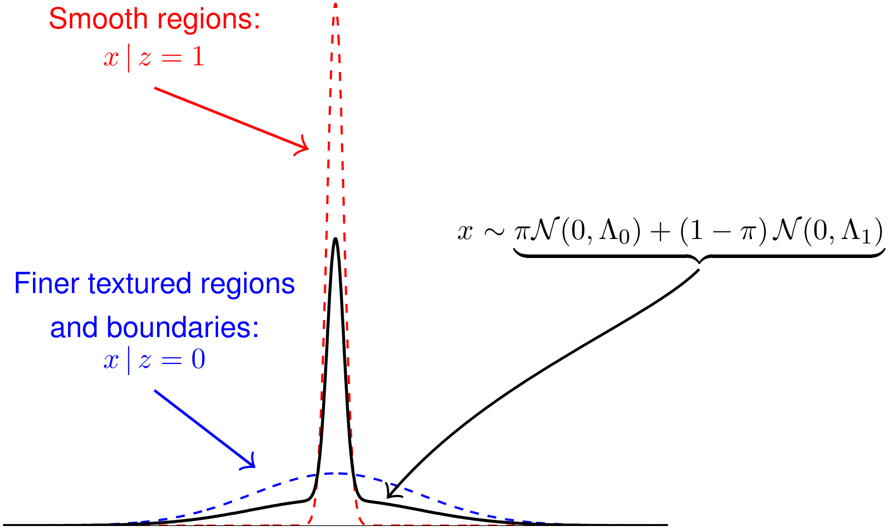
Hierarchical Dirichlet Process
Hidden Markov Tree

Apply learned statistics to *denoise* images

- model parameters are estimated *directly* from noisy image
- *global* statistics increase *robustness*

Modeling Wavelets Locally

A Two-State Mixture

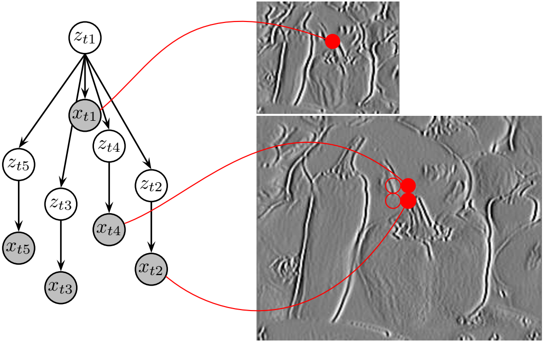


Modeling Wavelets Globally

Hidden Markov Tree Model (Crouse et al. 1998)

- Markov dependencies capture spatial relationships

- Models each orientation independently

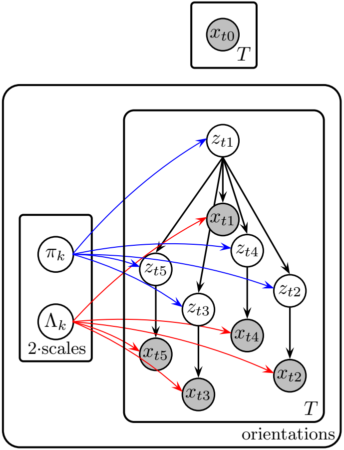


z_{ti}: hidden cluster assignment variable

x_{ti}: observed wavelet coefficient

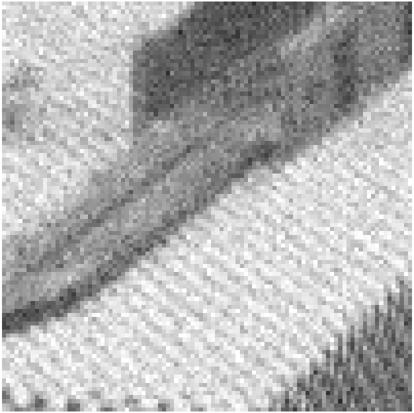
Modeling Wavelets Globally

Hidden Markov Tree Model (Crouse et al. 1998)

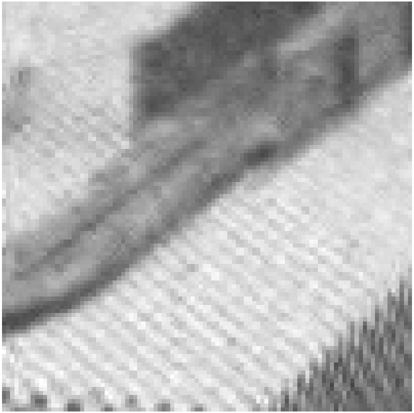


- Wavelet coefficients marginally distributed as mixtures of two zero-mean Gaussians
- Markov dependencies between hidden variables capture relationships among nearby coefficients
- Parameters chosen independently for each scale and orientation
- Successfully used for image denoising
- But...

Hidden Markov Tree (Crouse et al. 1998)



noisy



denoised

- Is two states per scale enough? How many is enough?
- Should I share states in the same pattern for all images?
- How should my model change for other wavelet decompositions?

Dirichlet Process Mixtures

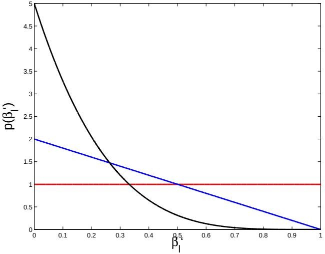
$$p(x \mid \beta, \Lambda_1, \Lambda_2, \ldots) = \sum_{k=1}^{\infty} \beta_k \mathcal{N}(x; 0, \Lambda_k)$$

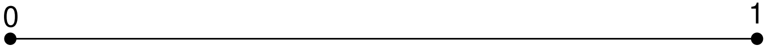
Control complexity using a *stick-breaking prior* for mixture weights:

$$\beta_k = \beta'_k \prod_{\ell=1}^{k-1} (1 - \beta'_\ell)$$

$$\beta'_\ell \sim \mathrm{Beta}(1, \gamma)$$

γ : concentration parameter





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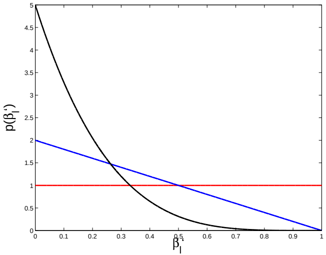

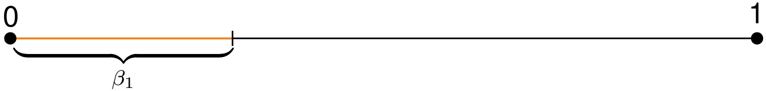

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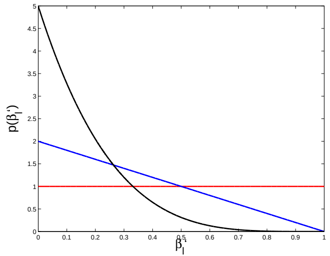

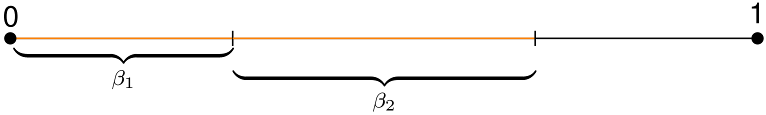

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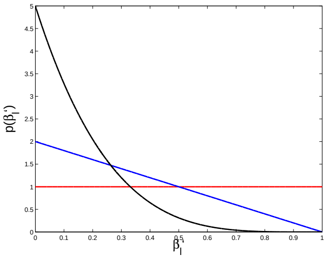

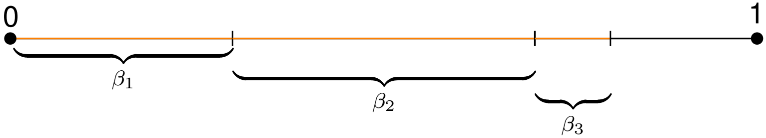

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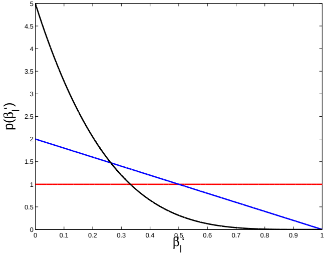

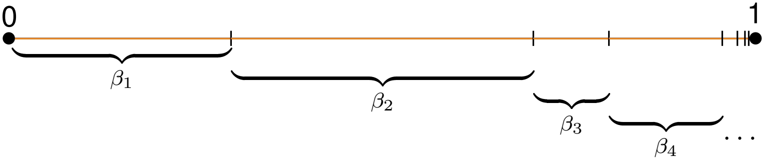

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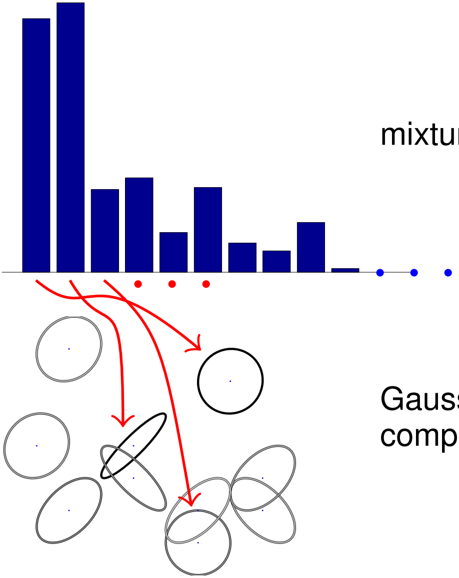
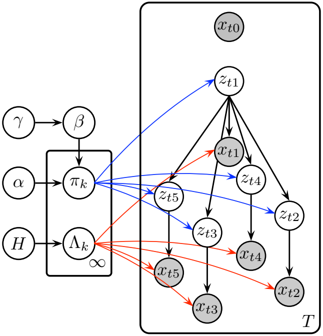
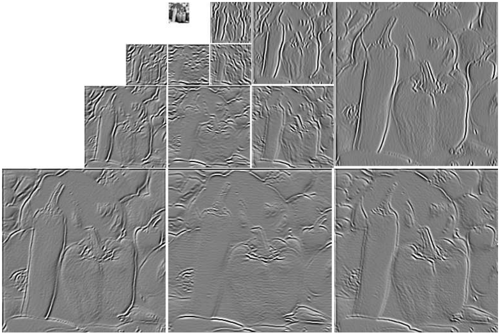


mixture weights (β_k)

Gaussian mixture components

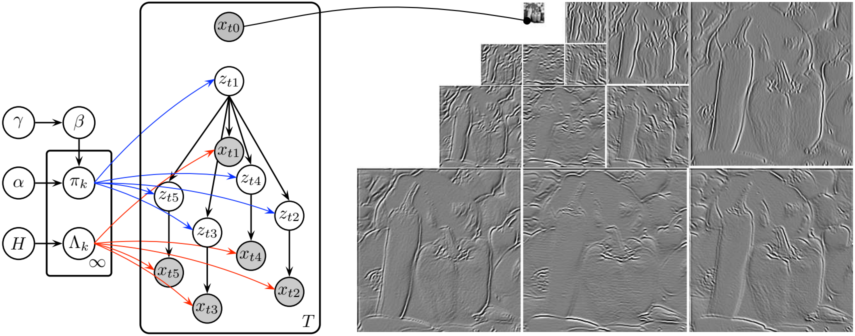
Hierarchical Dirichlet Process-Hidden Markov Tree





- Hidden states z_{ti} generate *vectors* of wavelet coefficients x_{ti} at *multiple* orientations.
- Wavelet coefficients marginally distributed as potentially *infinite* Dirichlet Process mixtures
- Hierarchical Dirichlet Process prior allows us to learn a potentially infinite set of *appearance patterns* from natural images

Hierarchical Dirichlet Process-Hidden Markov Tree



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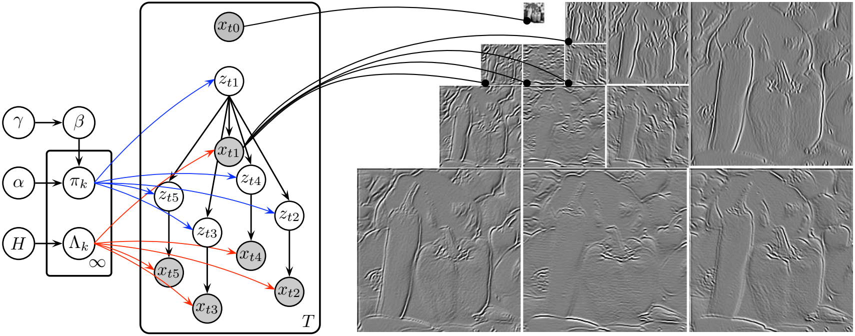

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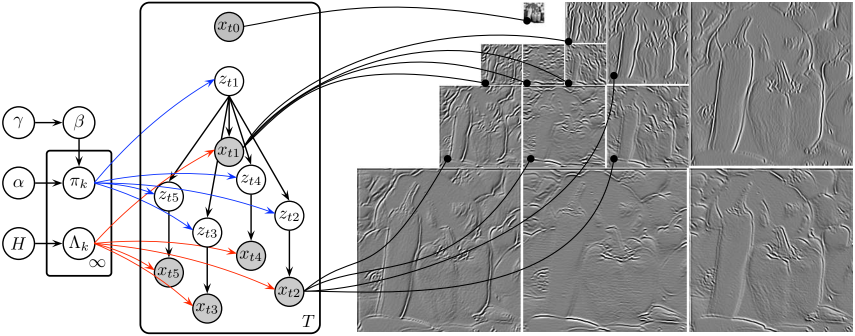

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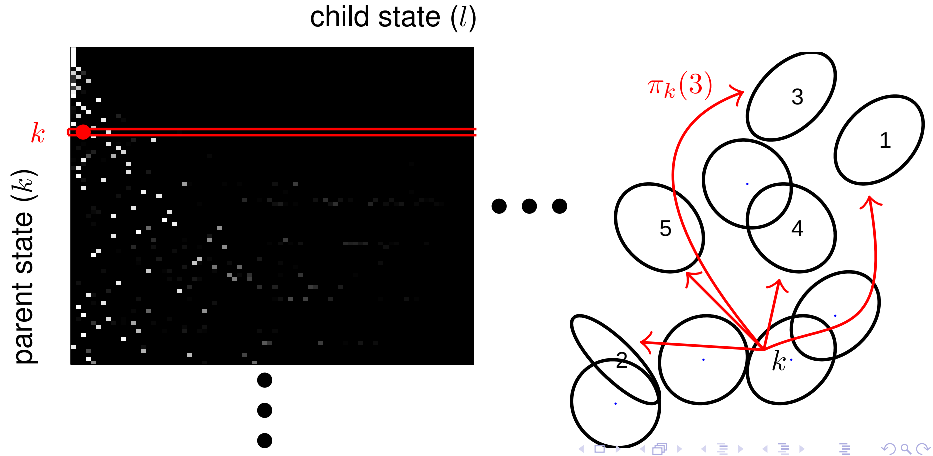
Need for *Hierarchical* Dirichlet Processes

- Each row of the transition matrix is a set of mixture proportions

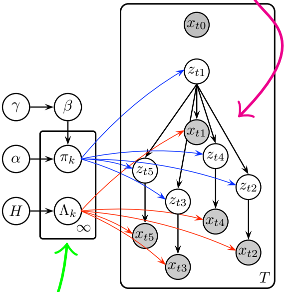
$$\pi_k(l) = Pr(z_{ti} = l \mid z_{\mathrm{Pa}(ti)} = k) \qquad k, l \in [1, \ldots, \infty)$$

- Hierarchical coupling ensures that child states reused by multiple parents



Learning HDP-HMT with a Collapsed Gibbs Sampler

Sample hidden assigments z



Consider

- states assigned to existing clusters
- a potential new state

(state cardinality determination)

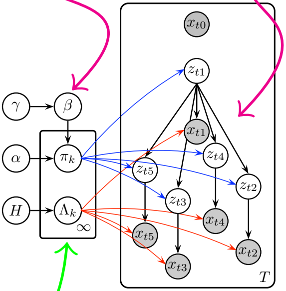
Marginalize state-specific parameters $\{\pi, \theta\}$

Rao–Blackwellization improves the efficiency and accuracy of MCMC methods.

Learning HDP-HMT with a Collapsed Gibbs Sampler

Sample global mixture proportions β

Sample hidden assigments z



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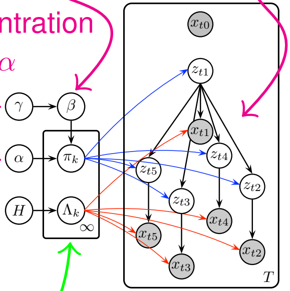
Learning HDP-HMT with a Collapsed Gibbs Sampler

Sample global mixture proportions β

Sample hidden assigments z

Sample concentration parameters γ, α



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The Overall Learning Algorithm to Denoise Images

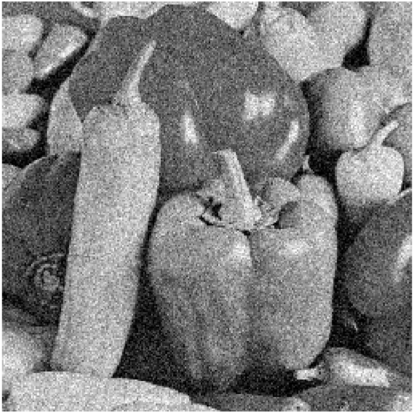
- Given a noisy image, learn HDP-HMT parameters by running the collapsed Gibbs sampler
- After burn-in, collect S samples $\theta^{(s)} = \{\pi_k^{(s)}, \Lambda_k^{(s)}\}_{k=1}^{K_s}$.
- Given each collected sample, a denoised image can be computed in closed form:

$$\mathbb{E}\big[w_{ti} \mid \mathbf{x}, \theta^{(s)}\big] = \sum_{k=1}^{K_s} \overbrace{p(z_{ti} = k \mid \mathbf{x}, \theta^{(s)})}^{\text{marginals from Belief Propagation}} \underbrace{\mathbb{E}\big[w_{ti} \mid x_{ti}, \Lambda_k^{(s)}\big]}_{\text{linear regression}}$$

- Average over samples of varying complexity to get posterior mean of detail coefficients

Results: Peppers, $\sigma = 25$

Binary HMT (Daubechies-4 Orthogonal Wavelets)

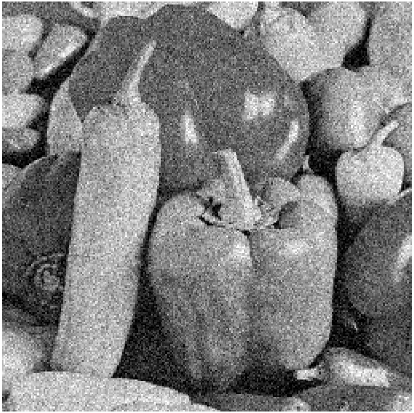


INPUT (20.18 dB)

OUTPUT (27.36 dB)

Results: Peppers, $\sigma = 25$

HDP-HMT (Daubechies-4 Orthogonal Wavelets)



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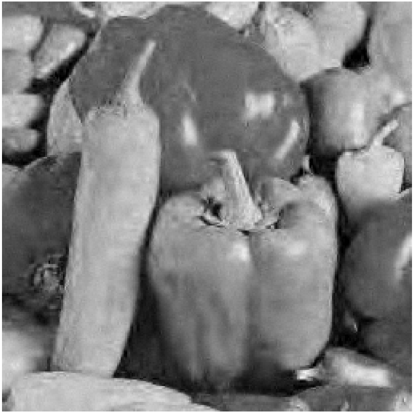
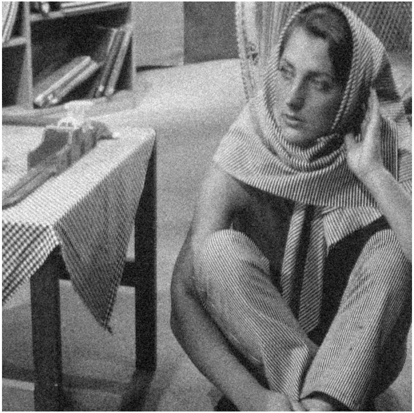


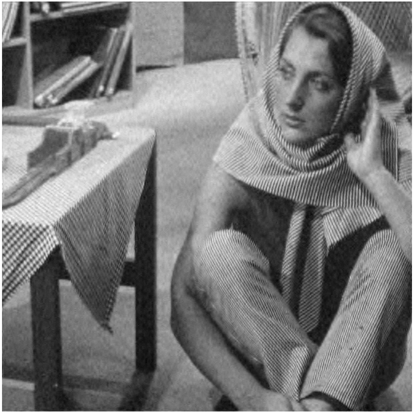
OUTPUT (28.09 dB)

Results: Barbara, $\sigma = 15$

Binary HMT (Daubechies-4 Orthogonal Wavelets)



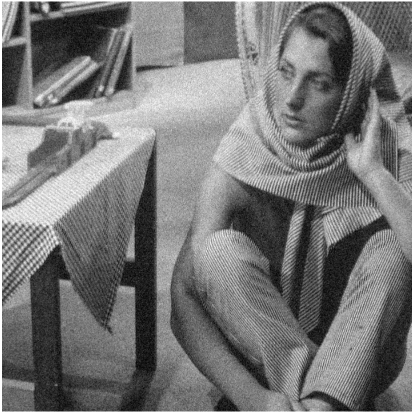
INPUT (24.61 dB)



OUTPUT (29.35 dB)

Results: Barbara, $\sigma = 15$

HDP-HMT (Daubechies-4 Orthogonal Wavelets)

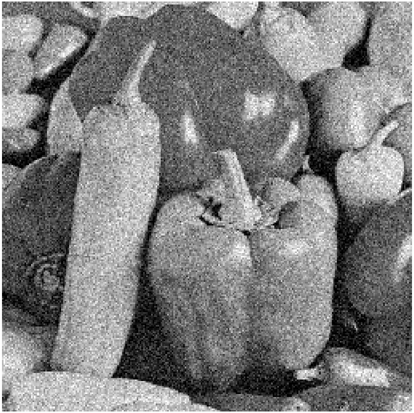


INPUT (24.61 dB)

OUTPUT (30.26 dB)

Results: Peppers, $\sigma = 25$

Local GSM by Portilla et. al. (2003) (7^{th}-Order Steerable Pyramids)

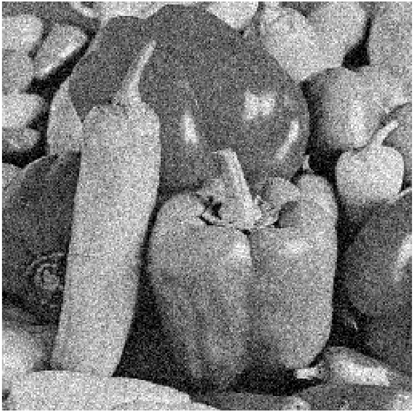


INPUT (20.18 dB)

OUTPUT (29.56 dB)

Results: Peppers, $\sigma = 25$

HDP-HMT (7^{th}-Order Steerable Pyramids)$\langle$Daubechies-4 Orthogonal Wavelets$\rangle$

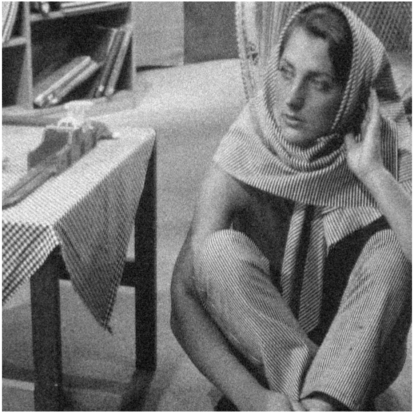


INPUT (20.18 dB)

OUTPUT (29.44 dB)$\langle$28.09 dB$\rangle$

Results: Barbara, $\sigma = 15$

Local GSM (7^{th}-Order Steerable Pyramids)



INPUT (24.61 dB)

OUTPUT (31.84dB)

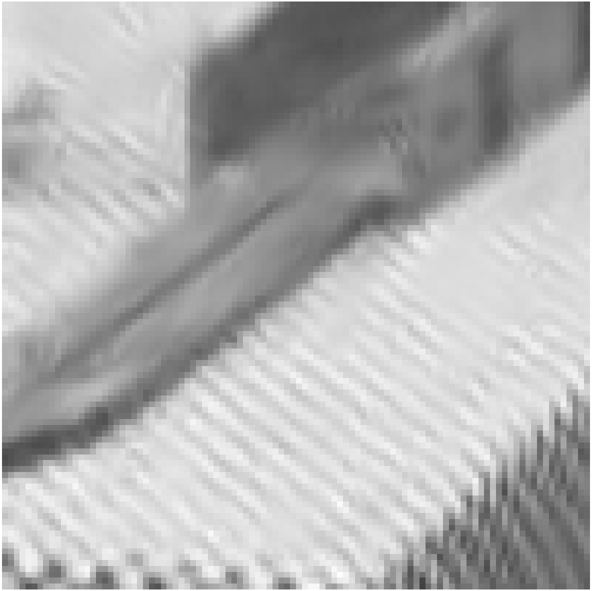
Results: Barbara, $\sigma = 15$

HDP-HMT (7^{th}-Order Steerable Pyramids)⟨Daubechies-4 Orthogonal Wavelets⟩

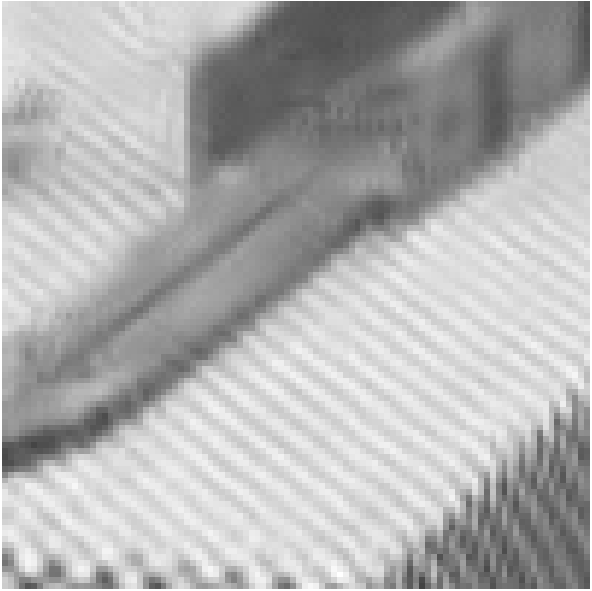
INPUT (24.61 dB)

OUTPUT (31.83 dB)⟨30.26 dB⟩

Local GSM

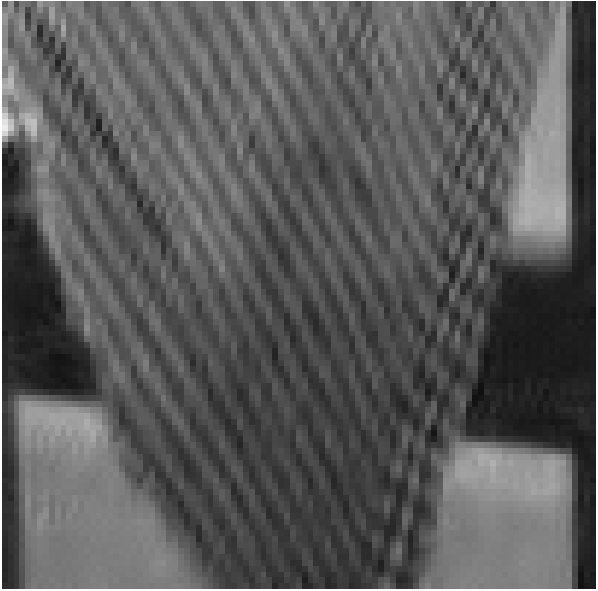


HDP-HMT



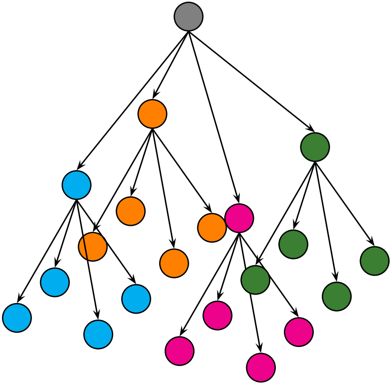
Local GSM

HDP-HMT

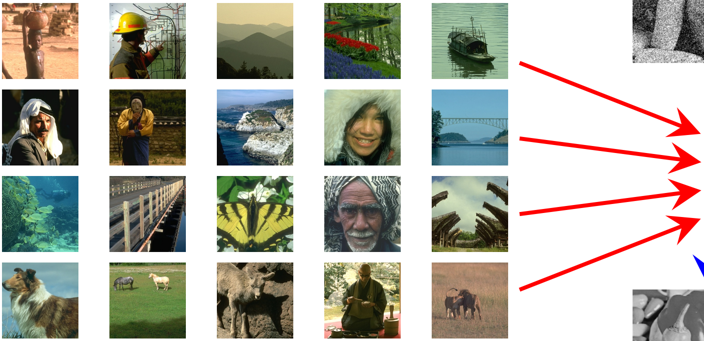


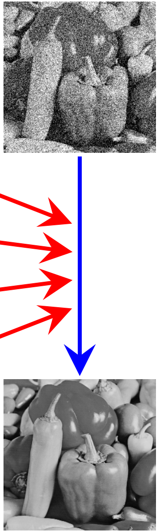
Summary

- Introduced a *nonparametric* Bayesian model for multiscale datasets with complex, non-local dependencies
- Developed an MCMC method which learns the potentially *infinite* set of HDP-HMT parameters
- Demonstrated effective *image denoising* via an empirical Bayesian approach



Ongoing Work





- Learn models from hundreds of natural images (to appear at ICCV 2007)
- Apply learned statistics of *clean* images to *denoise* images

http://www.cs.berkeley.edu/~kivinen

Joint Statistics of HDP-HMT Simulations

Peppers (Steerable Pyramid (Simoncelli et al. 1992) Detail Coefficients)