The Determinants of Bond-Stock Correlation: the Role of Trend Inflation and Monetary Policy

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Abstract

I show that Treasuries' role as hedge assets is determined by the level of trend inflation and the conduct of monetary policy, using a Generalized New Keynesian habit model. A novel prediction from the model is that when trend inflation is high, nominal bonds exhibit a positive correlation with stock returns, making them risky assets. As trend inflation rises, inflation exhibits more countercyclical pattern because any transitory inflation generates temporary output loss due to endogenous cost-push effects, which emerge in Generalized New Keynesian Phillips curve. When countercyclical inflation prevails, bond returns drop when stocks underperform, leading to a positive bond-stock correlation. The model explains the shift in US bond-stock correlation from positive to negative in 1997 as a consequence of stabilized trend inflation.

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When people begin anticipating inflation, it doesn't do you any good anymore, because any benefit of inflation comes from the fact that you do better than you thought you were going to do.

— Paul Volcker, *The First Measured Century* (2000)

1 Introduction

Recent financial markets have considered US Treasuries as a hedge, performing well when riskier assets like stocks decline. This perception has been supported by the negative correlation between Treasuries and stock returns since 1997. However, prior to 1997, the prevailing belief was that "long-term asset prices move (or should move) together, and indeed it is true that stock and bond returns are always positively correlated." (Campbell and Ammer 1993, p.17). This aligns with the positive correlation during that period (see figure 1). Therefore, the changing nature of Treasuries challenges the idea that the hedging property is a fundamental feature of US Treasuries.

In this paper, I build a Generalized New Keynesian (GNK) habit model with trend inflation following Ascari and Sbordone (2014) and emphasize the role of trend inflation and monetary policy in the correlation between nominal bonds and stocks. A novel prediction of the model is that, when trend inflation is high, long-term nominal bonds are less effective as hedges against stock market risks. This mechanism is absent in a textbook New Keynesian model (for instance, Galí 2015), or those with full indexation. But in my model, positive trend inflation and partial indexation amplify distortions arising from price dispersion. Here, shocks leading to transitory rise in inflation result in output losses through the demand misallocation due to these distortions. Consequently, inflation becomes more countercyclical, and nominal bonds yield lower returns when stocks do poorly.

At the heart of this mechanism lies distortions that stem from the inefficient distribution of relative prices. Under full indexation, non-optimizing firms adjust their prices according to past or steady state inflation, resulting in no price dispersion in steady state. However, if indexation is only partial, non-optimizing firms gradually deviate from optimal prices due to positive trend inflation. This leads to significant relative price dispersion, causing demand misallocation in the market.

Following a transitory inflationary shock, optimizing firms raise prices, while non-optimizing firms keep their old prices, leading to a wider dispersion in prices. Under monopolistic competition, lower-priced firms attract more demand, resulting in demand misallocation among intermediate goods and a subsequent decline in aggregate output[1]. This contributes to a more pronounced countercyclical inflation pattern.

This countercyclical inflation, in turn, has implications for the bond-stock relationship. When countercyclical inflation prevails, nominal bond prices decrease when stock prices are low, as high inflation coincides with lower output. As a result, bond-stock correlation becomes positive, making nominal bonds less effective as a hedge against stock price fluctuations. To sum it up, in cases where trend inflation is high, inflation is more likely to exhibit a countercyclical pattern, leading to riskier nominal bonds.

I begin by documenting that the shift in the bond-stock correlation coincided with the stabilization of trend inflation and long-term inflation expectations. Both trend inflation and long-term inflation expectations stayed above 4 percent until the mid-1990s, then settled at around 2.5 percent in 1997 in various measures. This shift from countercyclical to procyclical inflation occurred during the same period. Splitting the sample at 1997, inflation exhibited a negative correlation with the output gap before 1997 but a positive correlation thereafter.

Next, I establish a novel theoretical prediction for the bond-stock correlation and the cyclical behavior of inflation with respect to the level of trend inflation. In a stylized GNK model featuring Calvo pricing, as trend inflation increases, inflation tends to become more countercyclical due to the output loss accompanying transitory inflation. With high trend inflation, optimizing firms adjust prices, while non-optimizing firms remain locked into their previous price levels, leading to demand misallocation arising from price dispersion and aggregate output loss.

In this model, the underlying mechanism is summarized in the endogenous cost-push terms that emerge in a Generalized New Keynesian Phillips curve (GNKPC), including price dispersion and expected marginal costs from the pricing behaviors of non-optimizing and optimizing firms, respectively. These terms give rise to cost-push effects, whereby any inflationary shock leads to an output loss. Using this model, I show that demand shocks, usually associated with procyclical inflation, can induce a countercyclical pattern in inflation under sufficiently high trend inflation. Conventional exogenous cost-push shocks result in countercyclical inflation, with the impact being more pronounced due to the in-

[1] In Calvo model, pricing rule for non-optimizing firms is

$$P_t(i) = P_{t-1}\Big((1+\pi_{t-1})^{\mu}(1+\bar{\pi})^{(1-\mu)}\Big)^{\chi},$$

where $P_t(i)$ is the price of non-optimizing firm i, P_{t-1} is the aggregate price index in $t-1$, π_{t-1} is inflation in $t-1$, and $\bar{\pi}$ is steady state inflation. The partial indexation assumption in my paper is to assume that $\chi < 1$, whereas standard NK models assume $\chi = 1$ (Christiano, Eichenbaum, and Evans, 2005; Smets and Wouters, 2007).

fluence of endogenous cost-push effects.[2]

Furthermore, the extent of the endogenous cost-push effects varies with trend inflation. In cases with high trend inflation and partial indexation, non-optimizing prices diverge more rapidly from the optimal price, resulting in a more pronounced dispersion of prices. This, in turn, reinforces the impact of the endogenous cost-push effects. The theoretical finding from the stylized model suggests an important role of trend inflation on the cyclical dynamics of inflation and nominal bonds. Existing GNK models have focused on how the GNK framework endogenously increases the persistence and volatility of inflation, such as Cogley and Sbordone (2008) and Ascari and Ropele (2009). Building on this earlier contribution, this study establishes that trend inflation can markedly alter the effect of demand shocks on the cyclicality of inflation, which in turn determines bonds' hedging property.

Next, I quantitatively evaluate the significance of the endogenous cost-push effects in a GNK model with consumption habit preferences a la Campbell, Pflueger, and Viceira (2020). I estimate trend inflation and monetary policy rules separately for two sample periods (1965-1997 as period I and 1998-2019 as period II), while keeping all other parameters fixed across the two periods. The estimated parameters reveal a substantial decline in trend inflation[3] and a shift in monetary policy rules, characterized by increased persistence and reactions to inflation. The quantitative model elucidates the shift in the behavior of inflation and Treasuries as a consequence of stabilized trend inflation and the changes in monetary policy. Notably, the endogenous cost-push effects play a critical and quantitatively significant role in these dynamics.

In period I, with trend inflation estimated at 3.8%, the model shows amplified markup shocks and subdued demand shocks. As a result, inflation becomes countercyclical, and Treasuries and stocks move positively together. Accommodative monetary policy further intensifies the impact of endogenous cost-push effects by increasing inflation volatility. In contrast, period II features stable trend inflation at 2.0%, weakening the influence of endogenous cost-push effects and making nominal bonds a hedge due to the procyclical nature of inflation. Here, monetary policy has a limited role in shaping cyclical property of inflation and bonds as the endogenous cost-push effects are relatively small.

[2]Although analytical results for supply shocks are not explicitly characterized, it's worth noting that the impact on output loss is present for both demand and supply shocks. This is because the mechanism hinges on the endogenous cost-push factors within the GNKPC, which do not rely on particular types of shocks.

[3]The determination of or shift in trend inflation is the topic beyond the scope of this paper. Relatedly, Carvalho et al. (2023) show that in a NK model with endogenous long-term inflation expectations, trend inflation is determined by monetary policy and firms' long-term inflation expectations. In this regard, the change in trend inflation can happen by factors outside of the model in this paper.

Compared to existing estimates, such as those in Cogley and Sbordone (2008), my point estimates are in line with the average trend inflation in each period in those papers. Ascari and Sbordone (2014) further investigate how the level of trend inflation interacts with the transmission of TFP and monetary policy shocks. Building upon this literature, I extend the GNK model by incorporating demand shocks and habit preferences, and shed light on the signifcant influence of trend inflation on the shift in bond-stock correlation, as well as the cyclical behavior of inflation. The theoretical prediction from the stylized model survives: the decline in trend inflation has considerable effects on the transmission of demand shocks, which generate countercyclical inflation in period I, while countercyclical inflation in period II.

Model-implied counterfactual correlations further stress the importance of trend inflation for the sign-changing results. It suggests that trend inflation above 3% results in risky nominal bonds with countercyclical inflation, making trend inflation a crucial factor for these outcomes. The key takeaway from the quantitative model is that the stabilization of trend inflation primarily explains observed changes in these patterns. Furthermore, examining the decomposition of asset returns in accordance with Campbell and Shiller (1988), I find that the fluctuations in time-varying risk premiums contribute significantly to the shift in correlation.

This paper makes contributions to several branches of literature. First, my paper contributes to the growing body of the literature on the bond-stock correlation by highlighting the importance of trend inflation and monetary policy as crucial factors. Campbell, Pflueger, and Viceira (2020) construct and estimate a consumption-based asset pricing model with a New Keynesian Euler equation, highlighting the impact of changes in the output-inflation relationship on bond-stock correlation. Chernov, Lochstoer, and Song (2021) explore the real channel and identified regime changes between permanent and transitory consumption shocks as drivers of time-varying bond-stock correlation. Pflueger (2022), which is closely related to my paper, focus on the relative strength of supply shocks and monetary policy inertia as drivers of nominal bond beta. In contrast, my paper puts emphasis on the roles of trend inflation and monetary policy, shedding light on how long-run trend in inflation significantly influence the behavior of nominal assets.[4]

Second, my paper is also related to the expanding literature on New Keynesian asset pricing models. Existing studies in this domain have examined the relationship between monetary policy and risk premia (Rudebusch and Swanson, 2012; Gourio and Ngo,

[4]Far from exhaustive list of papers in this literature includes the papers cited above, Song (2017), Li et al. (2022) , David and Veronesi (2013), Baele, Bekaert, and Inghelbrecht (2010), Campbell, Sunderam, and Viceira (2009)

2020), the transmission mechanisms of monetary policy shocks through financial markets (Caramp and Silva, 2021; Kekre and Lenel, 2022), and the asset pricing implications of the stance of monetary policy (Bianchi, Lettau, and Ludvigson, 2022). While building upon this literature, my paper places greater emphasis on the role of trend inflation in shaping bond risk premia. It establishes a direct connection between inflation dynamics and risk premia, thereby uncovering a new advantage of low inflation—namely, the provision of safe and hedge assets at relatively low risk premia.

Lastly, this paper contributes to the extensive body of research on trend inflation, monetary policy, and inflation dynamics. [5] Building upon GNK framework, such as Ascari and Sbordone (2014), I introduce novel insights into macroeconomic dynamics and the impact of trend inflation. While the existing literature has predominantly focused on the effects of trend inflation on the volatility and persistence of inflation and on structural changes in exogenous shock processes as a potential source of inflation cyclicality (e.g., Smets and Wouters 2007; Pflueger 2022), my contribution lies in highlighting the role of trend inflation as a determinant of inflation cyclicality in the context of macroeconomic dynamics.[6]

The paper is organized as follows. Section 2 documents bond-stock correlation over time, the fall in trend inflation, and the change in the cyclicality of inflation. In section 3, I establish the analytical results using a stylized GNK model. In section 4, I describe quantitative GNK. model with consumption habit. Section 5 shows the quantitative model results. In section 6, I conclude.

2 Bond-stock correlation, trend inflation, and inflation cyclicality

Over the past couple of decades, US Treasuries have been perceived and served as hedge assets in financial markets. However, it's noteworthy to recognize that this perception of Treasuries as hedge assets wasn't widespread before the 1990s. In fact, leading up to the

[5]One of the first papers that deal with trend inflation is Ascari (2004). Subsequently, Ascari and Ropele (2007), Cogley and Sbordone (2008), Coibion and Gorodnichenko (2011) and Cogley, Primiceri, and Sargent (2010) study macroeconomic implications of trend inflation. The literature on the interaction of monetary policy and inflation dynamics includes Clarida, Gali, and Gertler (2000), Christiano, Eichenbaum, and Evans (2005) , Justiniano, Primiceri, and Tambalotti (2013), Bianchi (2013), Gust, Herbst, and López-Salido (2022), Gagliardone and Gertler (2023).

[6]In a medium-scale GNK framework, Ascari, Phaneuf, and Sims (2018) investigate welfare implications of trend inflation in business cycle frequency. They find that trend inflation may amplify or dampen the marginal efficiency of investment (MEI) shock, which is often suggested as a key driver of business cycle fluctuations (Justiniano, Primiceri, and Tambalotti, 2011).

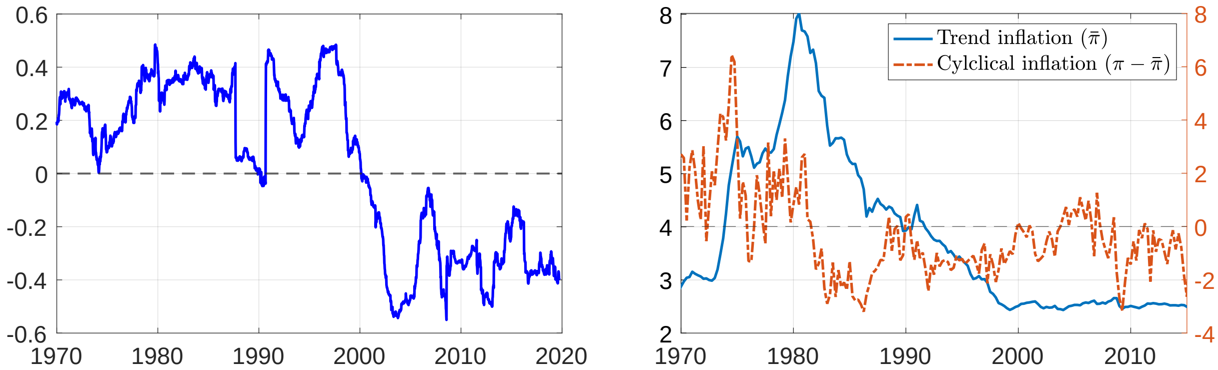


Figure 1: (a) Bond-stock correlation (left) and (b) trend inflation (right)

Notes: 3-year rolling correlations of bond and stock excess returns. Bond return (xr^b) is measured by one-quarter holding return of 5-year Treasury bonds excess of 3-month T-bills. Stock return (xr^s) is measured by one-quarter return from the value-weighted S&P 500 including dividends excess of 3-month T-bills. Inflation is the quarterly growth rate of GDP deflator. Trend inflation is the estimate of trend inflation by Carvalho et al. (2023).

late 1990s, US Treasuries exhibited significantly different performance characteristics, to the extent that it was considered a 'conventional wisdom' (Campbell and Ammer, 1993), that long-term Treasuries and stock prices tended to move in tandem.

The panel (a) of figure 1 shows 3-year rolling correlation of bond and stock returns.[7] The bond-stock relationship demonstrates substantial temporal fluctuations. During the initial three decades of the sample period, Treasuries exhibited a positive correlation with stocks. In other words, up until the late 1990s, Treasuries were perceived as risky assets due to their positive stock beta, aligning with the viewpoint expressed by Campbell and Ammer (1993)that nominal long-term bonds were indeed risky assets that moved in conjunction with other assets in the risk category.

However, a significant transformation occurred, marked by a sharp decline in correlation starting in 1997, which persisted negatively since 2000, with a few sporadic episodes of correlation spikes. Following this change in direction, Treasuries began to function as hedge assets not solely during times of financial crisis but also during normal market conditions. This observation indicates that the notion that the hedging characteristic of nominal long-term Treasuries is not an inherent, fixed attribute; rather, it exhibits time-varying behavior.

The observed shift in direction implies the presence of a low-frequency change in the relationship between nominal bonds and stock returns, thereby impacting the risk profile

[7]Correlations for other maturities are shown in Appendix B.

associated with long-term nominal bonds. This transformation is closely linked to a significant structural shift in trend inflation, characterized by the stabilization of trend inflation and the anchoring of long-term inflation expectations in the late 1990s.

In the right panel (b) of the figure 1, I have depicted trend inflation estimates from Carvalho et al. (2023) alongside cyclical inflation, representing the deviation of inflation from its trend. Here, I adopt the definition of trend inflation as established in the literature (Ascari and Sbordone 2014 and Carvalho et al. 2023), which is "the level to which inflation is expected to settle after short-run fluctuations die out, or $\bar{\pi}_t = \lim_{j\to\infty} E_t \pi_{t+j}$ (Ascari and Sbordone 2014, p.686)", Therefore, I do not distinguish between long-term inflation expectations and trend inflation.[8] As is widely recognized, following the Volcker disinflation in the late 1970s and early 1980s, inflation subsided during the 1980s. The era of the Great Moderation commenced in 1984 and persisted for more than two decades, concluding with the Global Financial Crisis.

However, the historical trajectory of trend inflation underwent a notably gradual transition. In a parallel fashion to actual inflation, trend inflation reached its peak in the early 1980s but experienced a sluggish descent compared to the pace of actual inflation decline. According to Carvalho et al. (2023) and Mertens (2016), the structural shift towards anchoring long-term inflation expectations materialized in 1997. This delay in the anchoring of long-term inflation expectations relative to actual inflation transpired due to several factors[9].

At the outset of the disinflationary policy, the credibility of the monetary regime shift was not firmly established, thus rendering the transition to anchored inflation expectations a gradual process, as discussed by Bianchi and Ilut (2017) and Goodfriend and King (2005). It was only after the preemptive monetary policy measures against inflation between 1994 and 1996 and the deliberation of a 2% inflation target during FOMC meetings that long-term inflation expectations became effectively anchored, as indicated by Bernanke et al. (2007) and Mishkin (2007).

The level of trend inflation, and consequently long-term inflation expectations, plays a critical role as a *forward-looking* macroeconomic variable impacting asset prices. When agents price nominal assets, they take into account expected inflation, which is affected by their long-term inflation expectations. This emphasis on future inflation matters for asset

[8]In empirical macro literature, these two often refer to the same object and are not distinguished (Beveridge and Nelson, 1981; Carvalho et al., 2023; Mertens, 2016). In terms of a model, trend inflation is equal to steady state inflation, which is also long-run inflation target.

[9]Those include the credibility of the Fed's low inflation target (Bordo and Schwartz, 1999; Benati and Goodhart, 2010) and lack of publicly available inflation target (Carvalho et al., 2023).

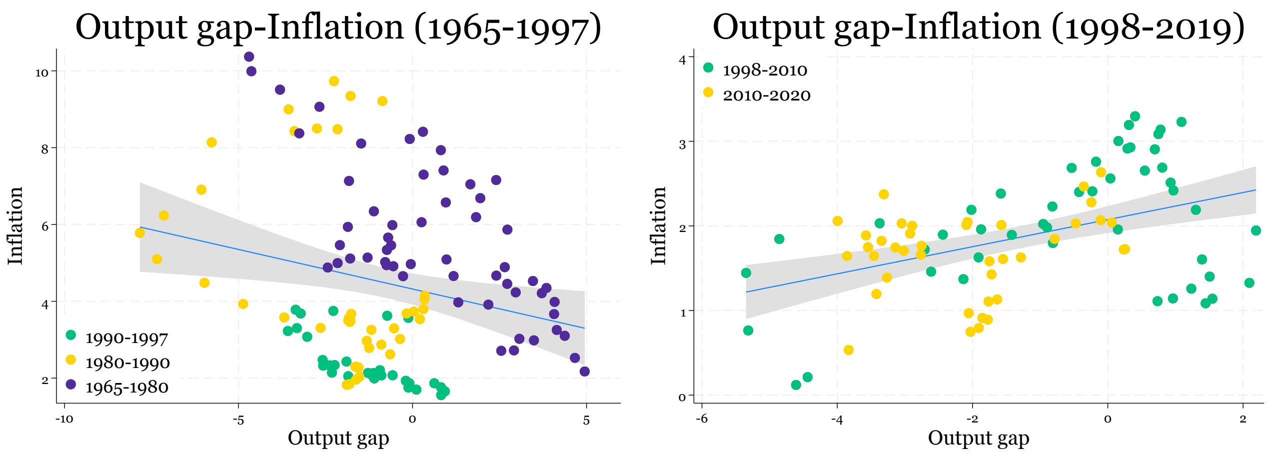


Figure 2: Output gap-inflation correlation

Notes: Blue line is the fitted line and the shaded area is 95% confidence interval. Inflation is measured by the annualized log difference in the GDP deflator. Output gap is the log difference between real GDP and potential GDP, estimated by CBO.

pricing, overshadowing the significance of past or present inflation levels. Given the empirical regularity that the persistence and volatility of inflation increase as inflation levels rise, the anchoring of long-term inflation expectations holds substantial implications for the behavior of nominal bond prices.

Figure 2 illustrates the historical relationship between inflation and the output gap, a macroeconomic fundamental with significant implications for the hedging characteristics of nominal bonds[10]. The cyclical nature of inflation plays a pivotal role: if inflation is procyclical, meaning it positively correlates with the output gap, higher output levels tend to coincide with elevated inflation on average. This leads to higher stock returns and lower bond returns, resulting in a negative correlation. Consequently, holding nominal bonds allows agents to hedge against stock market risks because lower stock returns align with lower inflation, thereby boosting bond returns. Conversely, countercyclical inflation implies a positive correlation between bonds and stocks since high inflation accompanies lower output levels.

The left panel of the figure depicts the relationship between inflation and the output gap for the period spanning 1965 to 1997. During the Great Inflation (1965-1980, represented by purple dots), inflation exhibited a countercyclical pattern, which is well-documented as a period of stagflation. Remarkably, this pattern persisted until 1997[11]. In the subsequent

[10] In appendix D.3, I also show the inflation-output growth, and inflation-unemployment relationship. All these relationships show the same pattern: high inflation in economic downturn before 1997 but the opposite since 1998.

[11] Campbell, Pflueger, and Viceira (2020) formally test the structural break in output gap-inflation cor-

two decades, inflation continued to display a countercyclical trend until the late 1990s. This period saw 'stagflationary' stock returns, where stock returns exhibited negativity following inflation surprises (Knox and Timmer, 2023).

In contrast, the right panel reveals the post-1998 relationship between the output gap and inflation. During this period, the US economy was characterized by low output gaps and modest inflation. Inflation remained stable and fluctuated around a 2 percent target level. Significantly, procyclical inflation has prevailed, positively correlating with the output gap.

The existing body of literature often emphasizes the dominant role of demand shocks in the latter period. However, a lingering question pertains to the source of countercyclical inflation during the Great Moderation. Empirical estimates suggest a limited impact of supply shocks on inflation during the 1980s and 1990s (Kilian, 2009; Shapiro et al., 2022). Yet, the decline in trend inflation can elucidate the cyclicality of inflation within a model where trend inflation has real consequences. This explanation doesn't necessarily hinge on the dominant role of supply shocks in driving countercyclical inflation during the Great Moderation. Within a single analytical framework, the reduction in trend inflation also elucidates the shift in bond-stock correlation, resulting from alterations in the cyclicality of inflation.

3 The role of trend inflation in Generalized New Keynesian model

In this section, I analyze how trend inflation affects inflation and asset pricing dynamics using a stylized Generalized New Keynesian (GNK) model. The primary aim of this section is to elucidate the fundamental mechanism by which trend inflation influences the cyclical nature of inflation and asset returns. My specific focus centers on the transmission of demand shocks, or perturbations to the IS curve, while the same mechanism applies to other shocks, such as markup shocks.[12]

The model is based on the GNK model of Ascari and Sbordone (2014) with Calvo pricing, positive trend inflation ($\bar{\pi} \geq 0$), and partial indexation. Unlike the standard New Keynesian model, which has a single NKPC equation, the supply side of the model is char-

relation and estimate 2001:Q2 as a structural break date.

[12]As for markup shocks, the same mechanism in this section applies and amplifies the effects of exogenous cost-push shocks. The model IRFs can be found in the appendix. Results for markup shocks are reported in appendix.

acterized by three equations: the GNKPC and two equations that capture the evolution of price dispersion and the expected marginal costs. These latter two components function in a manner akin to cost-push shocks on the GNKPC and are thus denoted as the endogenous cost-push terms.

3.1 A Generalized New Keynesian model

IS curve and Monetary Policy A continuum of identical and infinitely-lived households with log preferences makes consumption and labor supply choices. Consequently, the demand side is summarized by the following IS curve, utilizing the Euler equation of households

$$\tilde{y}_t = E_t\tilde{y}_{t+1} - (\hat{i}_t - E_t\hat{\pi}_{t+1}) + \zeta_t \tag{1}$$

$$\log(\zeta_{t+1}) = \rho_\zeta \log(\zeta_t) + \epsilon^\zeta_{t+1}, \quad \epsilon^\zeta_{t+1} \sim^{iid} N(0, \sigma^2_\zeta) \tag{2}$$

where β is household's discount factor, $\tilde{y}_t$ denotes the log-deviation of output gap from its steady state, and $\hat{\pi}_t$ is the deviation of inflation rate from its steady state ($\bar{\pi}$), and $\hat{i}_t$ denotes the deviation of short-term nominal interest rate. ζ_t represents a demand shock through household's Euler equation.[13] A central bank sets a short-term nominal interest rate (i_t) following a simple Taylor rule:

$$\hat{i}_t = \phi_\pi\hat{\pi}_t + \phi_y\tilde{y}_t \tag{3}$$

where $\hat{\pi}$ is the deviation of π_t from its long-run target, which is equal to trend inflation. ϕ_π and ϕ_y denote a central bank's response to inflation and output gap.

Production The production side of the economy consists of final output and intermediate goods firms. In each period t, the final consumption good Y_t is produced by perfectly competitive firms using a continuum of each intermediate goods $Y_t(i),\ i \in [0,1]$ as inputs. Final good producer has access to a CES production technology that aggregates the continuum of intermediate goods into the single final good:

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di\right]^{\frac{\epsilon-1}{\epsilon}}. \tag{4}$$

[13] Examples of this type of shocks in the literature is the convenience yield shocks in Krishnamurthy and Vissing-Jorgensen (2012) and financial shocks in international finance, for instance Itskhoki and Mukhin (2021). The demand shocks are also used in many recent asset pricing models to introduce wedges between bonds and other assets (Gourio and Ngo, 2020; Pflueger, 2023).

where $\epsilon > 1$ is the elasticity of substitution. Final good producer's profit maximization and the zero profit condition for the competitive market yield the following condition for the final good price P_t:

$$P_t = \Big[\int_0^1 P_t(i)^{1-\epsilon} di\Big]^{\frac{1}{1-\epsilon}} \tag{5}$$

and the downward sloping demand function for each intermediate good i:

$$Y_t(i) = \Big(\frac{P_t(i)}{P_t}\Big)^{-\epsilon} Y_t \tag{6}$$

A continuum of monopolistically competitive firms produce a different variety i. Each intermediate goods firm i has access to a CRS technology

$$Y_t(i) = A_t L_t(i) \tag{7}$$

where $L_t(i)$ denote the quantity of labor hired by firm i. A_t is a neutral exogenous technology that is common across firms.

Every intermediate goods firm is subject to the Calvo pricing friction. In each period t, a fraction ξ_p of intermediate goods firms are not allowed to change its price optimally. The remaining fraction of firms, $1-\xi_p$, can reset their price optimally to maximize the present value of its future expected cash flow. In each period, profits are distributed to households. Specifically, optimizing firms set their price $P_t(i)$ to solve the following expected profit maximizing problem:

$$\max_{P_t(i)} E_t \sum_{j=0}^{\infty} \xi_p^j \Lambda_{t,t+j} \Big[\frac{P_t(i)}{P_{t+j}} Y_{t+j}(i) - w_{t+s} L_{t+j}(i)\Big]. \tag{8}$$

$$\text{subject to} \quad Y_t(i) = \Big(\frac{P_t(i)}{P_t}\Big)^{-\epsilon} Y_t \tag{9}$$

$$Y_t(i) = A_t L_t(i) \tag{10}$$

Firm's first order condition can be expressed as follows:

$$p_t^*(i) = \frac{\epsilon}{\epsilon - 1} \frac{x_{1,t}}{x_{2,t}} \tag{11}$$

$$x_{1,t} = mc_t Y_t^{1-\sigma} + \xi_p \beta E_t \Big[x_{1,t+1} \Pi_{t+1}^{\epsilon}\Big] \tag{12}$$

$$x_{2,t} = Y_t^{1-\sigma} + \xi_p \beta E_t \Big[x_{2,t+1} \Pi_{t+1}^{\epsilon-1}\Big] \tag{13}$$

where $p_t^*(i) = P_t^*(i)/P_t$ is the optimal relative reset price that is common across optimizing firms, and $mc_t = w_t/A_t$ denotes an economy-wide real marginal cost. Non-optimizing firms keep their previous price without any indexation[14], hence

$$P_t(i) = P_{t-1}(i). \tag{14}$$

To investigate how positive trend inflation influences firms' pricing decisions, it is insightful to examine the determination of the optimal reset price (p_t^*) as outlined in equation (11). It is important to note that $x_{1,t}$ is present discounted costs and $x_{2,t}$ is present discounted revenue. Both of these economic factors are influenced by inflation, but the extent of their exposure differs. Specifically, cost conditions are more susceptible to inflation, meaning that costs increase at a faster rate than revenue. This suggests that as inflation is anticipated to persist in the distant future, it has a more pronounced negative impact on firms' expectations regarding their future cost conditions compared to revenue. Consequently, this leads to an increase in the reset price. Thus, as trend inflation rises, firms' pricing decisions become more forward-looking, as they assign greater importance to future cost conditions over future revenue.

Conversely, non-optimizing firms maintain their old prices. In a scenario with zero steady-state inflation or full price indexation, this has only a limited impact on real outcomes because even non-optimizing prices, on average, closely align with the optimal price. However, when steady-state inflation is positive, firms with unchanged prices gradually deviate from the optimal price over time. These unchanged prices are lower than the new prices, and this drifting effect accumulates as time passes. Consequently, non-optimizing firms' prices becomes increasingly backward-looking.

When these two effects are combined, it becomes apparent that, in comparison to the case of zero steady-state inflation or full indexation, firms' pricing behavior exhibits both more forward and backward-looking aspects. This represents a crucial characteristic of the Generalized New Keynesian Phillips curve (GNKPC) and will be explicitly evident in the equilibrium relationship outlined below.

[14] In standard New Keynesian models, pricing rule for non-optimizing firms is

$$P_t(i) = P_{t-1}\Big((1+\pi_{t-1})^{\mu}(1+\bar{\pi})^{(1-\mu)}\Big)^{\chi},$$

where $\chi = 1$ (Christiano, Eichenbaum, and Evans, 2005; Smets and Wouters, 2007). In this case, there is no price dispersion in steady state, and $\bar{\pi}$ has very limited role and mostly pins down average growth rate of nominal variables. This is a convenient assumption to derive single equation New Keynesian Phillips curve. But Cogley and Sbordone (2008) find that once trend inflation is taken into account, there is no need for indexation. The partial indexation assumption in my paper is to assume that $\chi < 1$, which implies non-zero price dispersion and first-order effect from the fluctuations in price disperison.

Aggregation Aggregate output can be obtained by aggregating each intermediate firm's demand for labor from (7) and demand function for each intermediate good (6):

$$L_t = \int_0^1 L_t(i)di = \int_0^1 \frac{Y_t(i)}{A_t}di = \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} di \frac{Y_t}{A_t} = \exp(\Delta_t)\frac{Y_t}{A_t}, \tag{15}$$

where the second equality is from the labor demand (7), the third equality from the demand for firm i, (6), and the last equality is from the definition of the price dispersion ($\exp(\Delta_t) \equiv \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} di$). Rewrite the aggregate production as

$$Y_t = \exp(-\Delta_t)A_t L_t. \tag{16}$$

Equation (16) shows the inverse relationship between aggregate output and the price dispersion. When price dispersion increases, demand for intermediate goods reallocates and it causes inefficiency and raises real wage, hence the marginal cost.

This is summarized by Δ_t, which is akin to a negative productivity shock (Ascari, Castelnuovo, and Rossi, 2011). Because of the nominal rigidity, price dispersion is increasing in inflation and as a result, there exists output loss in the long-run under positive trend inflation and no price indexation (Ascari and Ropele, 2009). In addition, in the short-run, fluctuation in inflation drives fluctuation in price dispersion, which generates output loss. The short-run output loss is due to the endogenous cost-push effects in the Generalized New Keynesian model (Alves, 2014).

Log-linearizing the equilibrium conditions around a positive steady state inflation $\bar{\pi}$ yields the following equations.

$$\hat{\pi}_t = \kappa \hat{mc}_t + \beta E_t \hat{\pi}_{t+1} + \beta\bar{\pi}(1-\tilde{\xi}_p)E_t\hat{\psi}_{t+1}, \tag{17}$$

$$\hat{\Delta} = \frac{\epsilon\tilde{\xi}_p}{1-\tilde{\xi}_p}\bar{\pi}\hat{\pi}_t + \tilde{\xi}_p\pi\hat{\Delta}_{t-1}, \tag{18}$$

$$\hat{\psi}_t = (1-\tilde{\xi}_p\beta\bar{\Pi})\hat{mc}_t + \tilde{\xi}_p\beta\bar{\Pi}\epsilon\hat{\pi}_t + +\tilde{\xi}_p\beta\bar{\Pi}E_t\hat{\psi}_{t+1}, \tag{19}$$

$$\hat{mc}_t = \Big(\varphi + 1\Big)\tilde{y}_t + \varphi\hat{\Delta}_t \tag{20}$$

where $\kappa \equiv \frac{(1-\tilde{\xi}_p)(1-\tilde{\xi}_p\beta\bar{\Pi})}{\tilde{\xi}_p}$, $\tilde{\xi}_p \equiv \xi_p(1+\bar{\pi})^{\epsilon-1}$, and $\bar{\Pi} \equiv 1+\bar{\pi}$.

Equation (17) shows the Phillips curve relationship in the model. In standard NK model with zero inflation, a New Keynesian Phillips curve (NKPC) is a single equation that characterizes a relationship between output gap and inflation. In fact, this is a special case of more general setting. When steady state inflation is zero, forward- and backward-looking forces cancel out each other up to the first order, hence price dispersion is constant up to the first order. Consequently, it ceases to be relevant for equilibrium considerations and is omitted from the equilibrium conditions in three-equation NK models.

However, under the GNK framework, which encompasses positive inflation and no indexation, a Generalized New Keynesian Phillips curve (GNKPC) includes additional terms that reflects the distortions in supply side, through which price dispersion has first-order effect on equilibrium. These terms include backward looking price dispersion (18) and forward looking cost conditions (20). Recall the objective function of optimizing firms in (8). When these firms maximize expected profits, they take into consideration future cost conditions, which are influenced by the future trajectory of inflation and real wages. Hence, as trend inflation increases, the reset price p_t^* becomes increasingly forward looking, as in (19), and this dynamics is manifested in the GNKPC outlined in equation (17).

In contrast, non-optimizing firms adhere to their previous prices (refer to equation 14). But unlike in the zero trend inflation case, these prices drift away from the optimal reset price over time, which is increasing over time due to positive $\bar{\pi}$. Given that the probability of re-optimization is stochastic, the distribution of old prices at time t is determined by the price index at time $t-1$. This implies that the evolution of the price dispersion in (18) is a weighted average of the past inflation.

Equations (17) through (20) summarize the supply side dynamics of the model and illustrate its distinctive feature. Notably, the terms in equation (17) give rise to endogenous cost-push effects. By substituting out marginal cost, equation (17) can be expressed as follows:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa(\varphi + 1)\tilde{y}_t + \underbrace{\kappa\varphi\hat{\Delta}_t + \bar{\pi}\gamma E_t \sum_{s=1}^{\infty} (\beta\tilde{\xi}_p)^s \widehat{mc}_{t+s}^p}_{\textbf{Endogenous cost-push term}}, \tag{21}$$

where $\gamma = \beta\left(1-\tilde{\xi}_p(1+\bar{\pi})^{-1}\right)$, and $\widehat{mc}_t^p = (1-\beta\tilde{\xi}_p)(\varphi\Delta_t+(\varphi+1)\tilde{y}_t)+\beta\tilde{\xi}_p\epsilon\hat{\pi}_t$ is marginal cost with respect to p_t^*. That is, marginal change in cost when a firm increases the relative price by one unit. The third and fourth terms on the right hand side are the terms that emerge under positive $\bar{\pi}$ and in the absence of indexation, or the *the endogenous cost-push* terms. It is worth noting that both price dispersion and real marginal cost ($\varphi\hat{\Delta}_t + (\varphi + 1)\tilde{y}_t$) are increasing in temporary inflationary pressure. Consequently, when an inflationary shock impacts the economy, it engenders supply-side effects due to price distortion arising from forward- and backward-looking forces.[15] Greater price dispersion prompts a reallocation

[15]Phillips curve featuring backward-looking terms is also studied in Gali and Gertler (1999). This reflects

of demand across intermediate goods firms due to the downward-sloping demand curve. But the aggregate output is a weighted average of each intermediate inputs, hence the price dispersion leads to a fall in aggregate output.

The endogenous cost-push effect stands as the pivotal feature of the GNK model. It signifies the output loss attributed to trend inflation, both in steady-state conditions and in the short run. In steady state, the presence of positive trend inflation without indexation leads to greater price dispersion than 1. From the aggregate production function where price dispersion serves as a negative shock in (16), this culminates in steady state output loss. In the short-run, any transitory inflationary pressure triggers a deviations in price dispersion and marginal cost from its steady state values. Consequently, through the GNKPC in equation (17), this inflationary pressure shifts the supply curves, akin to cost-push shocks. This, in turn, results in a temporary output loss in the short run. This generates temporary output loss in the short-run.

3.2 Macroeconomic dynamics

In this subsection, I analyze the influence of trend inflation on inflation and output dynamics. Ultimately, I show how the conditional correlation between inflation and output gap can switch its sign depending on the level of trend inflation.

For analytical tractability and simplicity, assume $\varphi = 0$, i.e. labor choice is indivisible. Note that under this assumption, price dispersion is irrelevant for Phillips curve (see equation (17) and (18)) because non-adjusting firms can use labor margin to fully absorb temporary inflation, hence the equilibrium consists of four equations (1), (3), (17), and (19). In this equilibrium, all variables are forward-looking and thus free variables. By substituting out the nominal interest rate $\hat{i}_t$ (3) into IS equation (1), the system of equations (1), (17), and (19) can be rewritten in the following matrix form

$$x_t = BE_t x_{t+1} + \varepsilon_t, \tag{22}$$

where $x_t = [\tilde{y}_t, \hat{\pi}_t, \hat{\psi}_t]'$. In this case, determinacy of rational expectation equilibrium can be achieved when all eigenvalues of B lie inside the unit circle (Blanchard and Kahn, 1980).

They derive the hybrid New Keynesian Phillips curve which depends on previous inflation ($\hat{\pi}_{t-1}$) and future expected inflation ($E_t\hat{\pi}_{t+1}$). The underlying pricing behavior in their model is that among $1-\xi$ fraction of adjusting firms, a fraction $1-\omega$ of firms are 'forward looking' and reoptimize prices considering future inflation while the remaining ω fraction of firms adjust their prices but non-optimally index their prices to the past inflation. Hence, in each period, $(1-\xi)\omega$ fraction of firms adopt the past inflation, which introduces persistence in the Phillips curve. The expression in (17) is even more generalized, and it includes all the past inflation and expected inflation.

The condition for determinacy under zero inflation collapses to the textbook determinacy condition $\kappa(\phi_\pi - 1) + (1-\beta)\phi_y > 0$ when $\bar{\Pi} = 1$, hence ψ_t becomes irrelevant for determinacy (Bullard and Mitra, 2002; Woodford, 2003). But with a positive trend inflation ($\bar{\pi} \geq 0$), the condition requires more aggressive reaction to inflation because of the expectation term, ψ_t. Ascari and Ropele (2009) provides characterization of the sufficient and necessary condition for determinacy, which is also depends on the level of trend inflation ($\bar{\pi}$). They show that the determinacy region in GNK model shrinks with $\bar{\pi}$, which requires higher ϕ_π or lower ϕ_y.[16] The following lemma characterizes the equilibrium dynamics in the model as a linear function of ζ_t when a demand shock is the only source of uncertainty.

Lemma 1 (Equilibrium dynamics)**.** *Suppose the sufficient and necessary condition for a unique rational expectations equilibrium holds. Further assume that the economy is initially in steady state. At the beginning of period t, an unexpected inflation target shock ε_t hits the economy. Then, the equilibrium dynamics can be characterized by the following forms*

$$[\hat{\pi}_t, \tilde{y}_t, \hat{i}_t, \hat{\psi}_t]' = [\Gamma_\pi, \Gamma_y, \Gamma_i, \Gamma_\psi]'\zeta_t \tag{23}$$

for some $\Gamma_\pi(\bar{\pi}),\ \Gamma_y(\bar{\pi}),\ \Gamma_i(\bar{\pi}),\ \Gamma_\psi(\bar{\pi})$.

Proof. See Appendix A. □

Lemma 1 confirms that all endogenous variables $(\hat{\pi}_t, \tilde{y}_t, \hat{i}_t, \hat{\psi}_t)$ are free variables and can be expressed a linear function of the state variable, ζ_t. The following propositions show how trend inflation and monetary policy affect macroeconomic dynamics conditional on demand shocks. The results show that trend inflation dampens the effects of demand shocks on output, and amplifies the effects on inflation, which determines the sign of output gap-inflation covariance.

Proposition 1 (Inflation Response)**.** *Suppose the sufficient and necessary condition for a unique rational expectations equilibrium holds. Further assume that the economy is initially in steady state. Then, Inflation response is increasing in $\bar{\pi}$, or $\Gamma_\pi(\bar{\pi})$ is increasing in $\bar{\pi}$, i.e. $\frac{d\Gamma_\pi(\bar{\pi})}{d\bar{\pi}} > 0$. Also, active monetary policy reduces inflation response, i.e. $\Gamma_\pi(\bar{\pi})$ is decreasing in ϕ_π, i.e. $\frac{d\Gamma_\pi(\bar{\pi})}{d\phi_\pi} < 0$*

Proof. See Appendix A. □

[16] In appendix D.2, I display a determinacy region for a GNK model with standard calibration.

Proposition 1 uncovers the effect of trend inflation on inflation responses following demand shocks. First, the impact effect on inflation becomes more pronounced as $\bar{\pi}$ increases. This heightened response occurs as a consequence of the forward and backward looking forces, which gain strength with higher $\bar{\pi}$. In the presence of positive trend inflation, old prices drift away from optimal price over time. Consequently, whenever possible, firms need to 'catch up' the price gap between the old prices and optimal price. Additionally, those optimizing firms take into account future inflation, which is on average positive. This forward-looking perspective prompts firms to proactively set higher prices that reflect anticipated future cost conditions. Accordingly, the immediate effect on inflation, $\Gamma_{\pi}(\bar{\pi})$, rises with $\bar{\pi}$.

On the other hand, this impact response diminishes as the inflation reaction coefficient ϕ_{π} decreases. When a central bank responds more aggressively to inflation, firms profit increases much less by raising prices. This takes place because the increase in nominal interest rate reallocate current consumption toward future consumption. Consequently, the optimal price is reduced with higher ϕ_{π}, which dampens the initial inflation response.

Proposition 2 (Output Response)**.** *Suppose the conditions in proposition 1 hold. Then output response is decreasing in $\bar{\pi}$, or $\Gamma_y(\bar{\pi})$ is increasing in $\bar{\pi}$, i.e.* $\frac{d\Gamma_y(\bar{\pi})}{d\bar{\pi}} < 0$

Proof. See Appendix A. □

Proposition 2 highlights the relationship between trend inflation and its impact on output. Specifically, it demonstrates that the output response decreases as trend inflation increases, indicating the output loss caused by trend inflation. This loss arises from the inefficiencies resulting from both nominal and real rigidity. As shown proposition 1, increase in inflation follows positive demand shocks. With Calvo nominal friction, this leads to a rise in price dispersion because only a portion of firms can adjust their prices. This price distortion interacts with real rigidity in the model in the form of the monopolistic competition. Intermediate goods firms face downward sloping demand curve due to their market power. Consequently, firms with lower relative prices experience greater demand, while those with higher relative prices face reduced demand. Since the final consumption good is a weighted average of intermediate goods, aggregate output decreases as price dispersion increases.[17]

[17] Ascari and Sbordone (2014) show that there exists a negative long-run relationship between output and trend inflation. That is, steady state output and steady state inflation has negative relationship. Compare to their long-run relationship, my results show that there exists short-run negative relationship between output and inflation when trend inflation is high.

In the Calvo model, the output loss can be summarized by the price dispersion in aggregate production function.[18] By aggregating individual intermediate good production function, aggregate output can be expressed as[19] $Y_t = s_t^{-1} A_t L_t$, where Y_t is output, A_t is aggregate productivity, and L_t is labor. This expression reveals that price dispersion is similar to a negative productivity shocks to aggregate output. Therefore, shocks that increase price dispersion contribute to a decline in aggregate output through the above channel.

Proposition 3 (Output Gap-Inflation Covariance)**.** *Suppose the conditions in proposition 1 holds. Further suppose demand shocks are persistent, or* $\rho_\zeta > \rho_\zeta^*$ *for some* $\rho_\zeta^* \in (0,1)$ *and* $\Gamma_\pi > \Gamma_\pi^* \equiv \frac{1}{\phi_\pi - \rho_\pi}$*. Then, there exists* $\bar{\pi}_0$*, above which output responses to demand shocks becomes negative, i.e.* $\Gamma_y(\bar{\pi}) < 0$ *for* $\bar{\pi} > \bar{\pi}_0$*. Consequently, output gap-inflation covariance conditional on demand shocks can be characterized by*

$$Cov(\hat{y}_t, \hat{\pi}_t) = \Gamma_y(\bar{\pi})\Gamma_\pi(\bar{\pi})\frac{\sigma_\zeta^2}{1-\rho_\zeta^2} \begin{cases} > 0 & \textit{if} \quad \bar{\pi} \leq \bar{\pi}_0 \\ < 0 & \textit{if} \quad \bar{\pi} > \bar{\pi}_0 \end{cases} \tag{24}$$

where $\bar{\pi}_0 > 0$.

Proof. See Appendix A. □

Proposition 3 provides insights into the relationship between the output gap and inflation covariance and sheds light on how trend inflation influences inflation's cyclical behavior. As previously established, higher trend inflation leads to a diminishing output response. In addition to this, there exists a threshold $\bar{\pi}_0$ above which the output response turns negative. Higher trend inflation amplifies the endogenous cost-push effects, resulting in output loss. If this effect is sufficiently strong, it can outweigh the initial expansionary effect of positive demand shocks. As a result, the covariance between the output gap and inflation shifts from positive to negative, making inflation countercyclical.

It is important to note that this sign-changing outcome hinges on a substantial inflation response ($\Gamma_\pi > \Gamma_\pi^*$). This reflects the endogenous cost-push mechanism, where inflation-induced price dispersion leads to an aggregate output decline. This is the key result that rationalizes the sign switching pattern observed in the correlation between output and

[18] From the definition of price dispersion, it is a cross-sectional distribution of prices in a give period. But one can show that it is also a weighted average of past inflation. This is because in each period, ξ fraction of non-adjusting firms are stuck at the previous prices, and $\xi(1-\xi)$ fraction of non-adjusting firms are stuck at the prices in two-periods before, ... Therefore, the price dispersion in the current period not only summarizes the current cross-sectional distribution, but also all the previous inflation.

[19] This aggregate output can be derived from integrating production functions of individual intermediate goods producers

inflation, which subsequently translates into the bond-stock correlation change discussed later.

In appendix D.4, a graphical representation of this proposition is provided. The figure illustrates that as $\bar{\pi}$ increases, inflation responses become stronger and more persistent, while output gap responses decline. Around 3.5% of $\bar{\pi}$, the output response turns negative under standard calibration. In appendix D.5, similar impulse response functions are presented for varying values of ϕ_π. In this scenario, inflation response decreases with ϕ_π, but the behavior of the output gap response is more nuanced. When $\bar{\pi}$ is low, the output gap response decreases with ϕ_π but when $\bar{\pi}$ is high, it increases. This divergence is attributed to the dominance of endogenous cost-push effects at higher $\bar{\pi}$ levels, resulting in a decline in the output gap following positive demand shocks. However, by implementing a more aggressive inflation response, a central bank can mitigate these endogenous cost-push effects, leading to smaller output losses.

3.3 Asset price dynamics with trend inflation

Consider a (unlevered) consumption claim and two period nominal bonds to keep the analysis simple.[20] A consumption claim is defined as a claim on aggregate consumption, hence output in the model, which does not expire. In each period, a claim pays out dividend Y_t. Each claim can be traded in financial markets at an ex-dividend price P_t^s. In equilibrium, the price of the consumption claim is given by $P_t^s = E_t m_{t+1}(Y_t + P_{t+1}^s)$, where m_{t+1} is the households' stochastic discount factor (SDF). When households have log utility (i.e. $\sigma = 1$), log-deviation of stock returns from steady state ($\hat{r}_{t+1}^s$) is same as output growth

$$\hat{r}_{t+1}^s = \Delta \hat{y}_{t+1} \tag{25}$$

Next, two period nominal bonds are default-free zero coupon bonds that pay one 'dollar' at maturity. Denote the price of this bond $P_t^{(2)}$ then the equilibrium price of long-term bonds satisfy $P_t^{(2)} = E_t m_{t+1} P_{t+1}^{(1)} / \pi_{t+1}$, where $P_t^{(1)}$ is the price of one period nominal bond. By linearizing the equation around the steady state, I have the following equation for the return on two period nominal bonds (r_{t+1}^b)

$$\hat{r}_{t+1}^b = \hat{p}_{t+1}^{(1)} - \hat{p}_t^{(2)} - \hat{\pi}_{t+1} \tag{26}$$

where $p_t^{(1)}$ and $p_t^{(2)}$ are log prices of one and two period bonds, respectively. The following

[20] The analysis can be easily extended to n period nominal bonds.

proposition characterizes bond-stock covariance in this example economy.

Proposition 4 (Bond-stock covariance)**.** *Suppose the sufficient and necessary condition for a unique rational expectations equilibrium holds, and the economy is initially in steady state. Further assume that* $\rho_\zeta > \max\{\bar{\rho}_\zeta, \rho_\zeta^*\}$*. Then bond-stock return covariance conditional on demand shocks can be characterized by*

$$Cov(xr^s_{t+1}, xr^b_{t+1}) = \Gamma_y(\bar{\pi})(\Gamma_y(\bar{\pi})(1-\rho_\zeta) - \Gamma_\pi(\bar{\pi})(1+\rho_\zeta))\frac{\sigma_\zeta^2}{1-\rho_\zeta^2}. \tag{27}$$

Also, $\Gamma_y(\bar{\pi})(1-\rho_\pi) - \Gamma_\pi(\bar{\pi})(1+\rho_\pi) < 0$*, which implies bond-stock covariance is negatively associated with output gap-inflation covariance. Formally,*

$$Cov(xr^s_{t+1}, xr^b_{t+1}) < 0 \quad \textit{if and only if} \quad \Gamma_y(\bar{\pi}) > 0. \tag{28}$$

Therefore, there exists $\bar{\pi}_0 > 1$ *such that*

$$\begin{aligned} & Cov(xr^s_{t+1}, xr^b_{t+1}) < 0 \qquad \textit{if} \quad \bar{\pi} \leq \bar{\pi}_0 \\ \textit{and} \quad & Cov(xr^s_{t+1}, xr^b_{t+1}) > 0 \qquad \textit{if} \quad \bar{\pi} > \bar{\pi}_0 \end{aligned}$$

Proof. See Appendix A. □

Proposition 4 sheds light on the role of trend inflation on the covariance between the two asset prices, which is inherently tied to the output gap-inflation relationship and trend inflation (Proposition 3). Importantly, for sufficiently persistent demand shocks, the covariance of nominal bonds and stocks undergoes a sign change when the correlation between output and inflation shifts, with a caveat that the threshold $\bar{\pi}_0$ may not be the same for bond-stock and output gap-inflation covariances in general.

In low trend inflation with the limited endogenous cost-push effects, nominal long-term bonds act as hedges because demand shocks induce rise in inflation to be aligned with higher output, resulting in procyclical inflation that boosts stock returns despite low bond returns.

In contrast, high trend inflation leads to risky nominal bonds positively correlated with stocks due to amplified endogenous cost-push effects that link positive demand shocks to a large output loss. In such cases, future inflation coincides with lower output, preventing nominal bonds from hedging consumption risks because inflation exhibits countercyclical pattern and nominal assets lose value concurrently with falling stock returns. Thus, the level of trend inflation plays a critical role in determining both the cyclical behavior of inflation and the hedging property of nominal bonds.

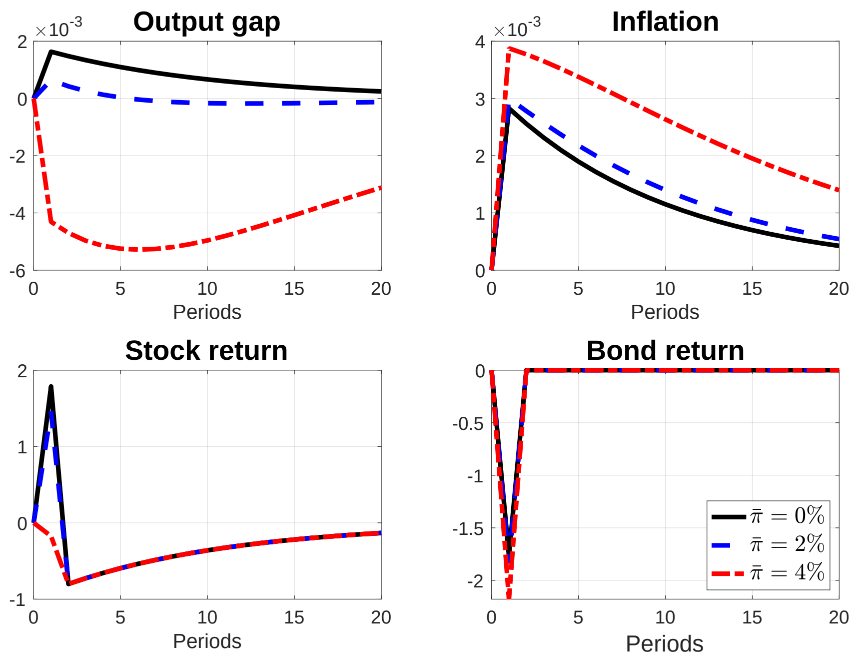


Figure 3: IRFs of output gap, inflation, real stock and bond return to a positive demand shock (ζ_t)

Notes: This figure is based on the following standard calibration – $\beta = 0.995$, $\sigma = 1$, $\varphi = 1$, $\xi = 0.75$, $\epsilon = 11$, ϕ_π=1.5, $\phi_y = 0.1$, $\rho_\zeta = 0.9$.

In figure 3, I illustrate a theoretical prediction using a standard calibration from the literature. The figure shows IRFs of output gap, inflation and excess returns on the two assets following a positive demand shock for different trend inflation levels: $\bar{\pi} = 0$% (black solid line), 2% (blue dashed line), and 4% (red dotted line). In a zero-inflation case ($\bar{\pi} = 0$%), a positive demand shock leads to increased output gap and inflation, indicating an expansionary effect. As a result, stock returns increase with higher output, while inflation erodes real value of nominal bonds.

Under moderate inflation ($\bar{\pi} = 2$%) the expansionary effect weakens but remains, resulting in a smaller increase in stock returns. Inflation still exhibits a procyclical pattern, driving a negative bond-stock covariance, albeit to a lesser extent. Under a higher inflation ($\bar{\pi} = 4$%), the effect of an inflationary shock is reversed due to the significant endogenous cost-push effects. A positive demand shock pushes inflation higher but it ultimately leads to a fall in output gap due to demand misallocation. As a result of the recessionary effect, consumption claim performs poorly, experiencing negative returns. Therefore, with high trend inflation, a temporary yet persistent increase in demands has a stagflationary effect similar to cost-push shocks and generates countercyclical inflation. Risk-averse households prefer holding short-term bonds since nominal long-term bonds do not hedge consumption risks. Consequently, this leads to a positive bond-stock covariance due to shifts in supply curve of the economy.[21]

4 Quantitative GNK model with habit preferences

In this section, I describe a GNK model with consumption habit to quantitatively evaluate the importance of the endogenous cost-push effects. The quantitative model is building on the GNK model in section 3 and adopts habit preferences following Campbell, Pflueger, and Viceira (2020). The supply side and the description of monetary policy are similar to the previous section, and I describe it in detail in appendix B. The analytical results in the previous section still applies: trend inflation generates more countercyclical inflation by dampening demand shocks and amplifying supply shocks. The habit preferences introduce time-varying risk premia for asset prices in the model hence improve the asset pricing dynamics of the model, which turns out to be important for risk premia for bonds and stocks, hence for the correlation.

4.1 Habit preferences

Preferences Time is discrete and denoted by $t = 0,\ 1, \cdots$. The economy is populated by a continuum of households, which are identical and infinitely lived. A representative household consumes, supplies labor and earns labor income. The household also decides how much to save in each class of assets: short-term bonds, long-term bonds and consumption claim.

The representative household maximizes expected utility (29), subject to (30)

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{(C_t - X_t)^{1-\sigma} - 1}{1-\sigma} - \chi \frac{H_t^{1+\nu}}{1+\nu} \right) \tag{29}$$

$$P_t C_t + \frac{D_t}{\zeta_t} + \frac{P_t^B}{\zeta_t}(B_t - \kappa B_{t-1}) = W_t H_t + (1+i_t) D_{t-1} + B_{t-1} \tag{30}$$

where C_t is consumption, H_t is labor supply, W_t is nominal wage, D_t/P_t is the real holdings

[21]In the appendix figure 14, I also report the same set of IRFs to cost-push shocks. It shows that with higher trend inflation, cost-push shocks are amplified by the endogenous cost-push terms. After cost-push shocks hit, temporary inflation shifts GNKPC even further, contributing to larger output and inflation response.

of short-term bonds, $P_t^B B_t/P_t$ is the real holdings of long-term bonds,[22]. Here, X_t is the level of external habit, σ is the curvature of the utility function for consumption, $\beta \in (0,1)$ is a time discount factor, ν are the inverse of the Frisch elasticity of labor supply, and $\chi > 0$.

In the consumption habit literature, it is common to define the surplus consumption ratio $S_t = (C_t - X_t)/C_t$, which is the fraction of consumption relevant to utility. It should be noted that habit is external so that households do not internalize the habit when making saving and labor choice. Relative risk aversion can be expressed as an inverse function of the surplus consumption ratio: $-U_{CC}C/U_C = \sigma/S_t$. Because the surplus consumption ratio is increasing in the level of consumption, relative risk aversion in the model not only is time-varying, but also increases in the time of low level of consumption relative to habit.

Surplus consumption dynamics Following Campbell, Pflueger, and Viceira (2020), I assume that the log of surplus consumption ratio is characterized by a heteroskedastic AR(1) process:

$$s_{t+1} = (1-\theta_0)\bar{s} + \theta_0 s_t + \theta_1 \hat{c}_t + \theta_2 \hat{c}_{t-1} + \lambda(s_t)\epsilon_{c,t+1} \tag{31}$$

$$\epsilon_{c,t+1} = \hat{c}_{t+1} - E_t \hat{c}_{t+1}, \tag{32}$$

where $\bar{s}$ is the steady state value of the log surplus consumption and $\epsilon_{c,t+1}$ is a shock to consumption. The sensitivity function $\lambda(s_t)$ has the following form (Campbell and Cochrane, 1999)

$$\lambda(s_t) = \begin{cases} \frac{1}{\bar{S}}\sqrt{1-2(s_t-\bar{s})} - 1 & s_t \leq s_{max} \\ 0 & s_t \geq s_{max} \end{cases} \tag{33}$$

$$\bar{S} = \sigma_c \sqrt{\frac{\sigma}{1-\theta_0}} \tag{34}$$

$$\bar{s} = \log(\bar{S}) \tag{35}$$

$$s_{max} = \bar{s} + 0.5(1-\bar{S}^2) \tag{36}$$

Equation (33) shows that the surplus consumption is more sensitive to the fluctuation in consumption when the surplus consumption is low, or consumption is close to habit. Hence, the negative relationship between s_t and $\lambda(s_t)$ is crucial to generate countercyclical price of risk.

[22] In the model, new issuance of long-term bonds is $B_t - \kappa B_{t-1}$, and each unit of long-term bonds pays off the cash flows of $1,\ \kappa,\ \kappa^2, \cdots$. The structure of the long-term bonds will be discussed further below.

Stochastic discount factor and Euler equation Household's Euler equation for short-term bonds is

$$1 = \zeta_t E_t[m_{t+1}(1+i_t)(1+\pi_{t+1})^{-1}], \tag{37}$$

where ζ_t represents bond demand shock following a stationary AR(1) process:

$$\log(\zeta_{t+1}) = \rho_\zeta \log(\zeta_t) + \epsilon^\zeta_{t+1}, \quad \epsilon^\zeta_{t+1} \sim^{iid} N(0, \sigma^2_\zeta). \tag{38}$$

As discussed in the previous section, ζ_t is bond demand shock (henceforth "demand shocks") which has an interpretation of shocks to convenience yield (Krishnamurthy and Vissing-Jorgensen, 2012), or liquidity shocks (Itskhoki and Mukhin, 2021; Gourio and Ngo, 2020; Pflueger, 2023). [23]

The stochastic discount factor (SDF), m_{t+1}, is

$$\begin{aligned} m_{t,t+1} &= \beta \frac{u_1(C_{t+1}, H_{t+1})}{u_1(C_t, H_t)} \\ &= \beta exp(-\sigma(\Delta s_{t+1} + \Delta c_{t+1})). \end{aligned}$$

Combining (39) with household Euler equation (37) and the surplus consumption process (31) and (33) yields the following Euler equation

$$\hat{c}_t = \frac{1}{1-\theta_1} E_t \hat{c}_{t+1} + \frac{\theta_2}{1-\theta_1} \hat{c}_{t-1} - \frac{1}{\sigma(1-\theta_1)}(i_t - E_t \pi_{t+1} - \zeta_t) \tag{39}$$

4.2 Asset prices

I model nominal long-term bonds as n-period zero coupon bonds that pays $1 at maturity. The price of n-period nominal bonds, $P^{(n)}_t$ is recursively defined as follows:

$$P^{(n)}_t = \zeta_t E_t m_{t+1}(1+\pi_{t+1})^{-1} P^{(n-1)}_{t+1}, \; P^{(0)}_t \equiv 1 \tag{40}$$

[23] Fisher (2015) shows that the demand for safe and liquid assets as in Krishnamurthy and Vissing-Jorgensen (2012) can rationalize the risk premium shock in Smets and Wouters (2007). The convenience benefit can be rationalized by the notion that Treasury securities are highly liquid and safe, which can reduce transaction costs that could be incurred. ζ_t is a shock to the convenience function, which makes the convenience benefits time-varying. In the current specification, risk premium shocks affect short-term and long-term bonds in the same manner. It is a simplifying assumption but also reflects the fact that Treasuries are very liquid even for longer maturity securities.

The yield to maturity on this n-period bond with continuous compounding is defined

$$y_t^{(n)} = -\frac{1}{n}\log P_t^{(n)} \tag{41}$$

Then, the one period return on n-period bond is

$$1 + R_{t+1}^{(n)} = -(n-1)y_{t+1}^{(n-1)} + ny_t^{(n)} \tag{42}$$

I model stocks as a levered claim on aggregate consumption, which is conventional in asset pricing literature (Campbell, Pflueger, and Viceira, 2020). Let $P_{n,t}^C$ denote the price of a claim that pays consumption C_{t+n} at $t+n$, and zero otherwise. Then the price of this claim can be defined following the recursion

$$\frac{P_{n,t}^C}{C_t} = E_t m_{t+1} \frac{C_{t+1}}{C_t} \frac{P_{n-1,t}^C}{C_{t+1}} \tag{43}$$

Then the price of a claim to all future consumption, P_t^C, satisfies

$$\frac{P_t^C}{C_t} = \sum_{n=1}^{\infty} \frac{P_{n,t}^C}{C_t} \tag{44}$$

As in (Campbell, Pflueger, and Viceira, 2020), I assume that in each period t, an aggregate firm buys P_t^C and sells δP_t^C worth of equity to investors. Thus the leverage ratio of this aggregate firm is $1/\delta$, which I assume constant over time.

The gross dividend to equity holders, D_{t+1}^{δ}, is the cash flow to the firm net of payments to bondholders, $(1-\delta)P_t^C(1+r_t)$, and new equity financing, δP_{t+1}^C

$$D_{t+1}^{\delta} = C_{t+1} + P_{t+1}^C - (1-\delta)P_t^C(1+i_t) - \delta P_{t+1}^C, \tag{45}$$

where r_t is real interest rate. Then, the price of the claim to the levered dividend claim (45) is $P_t^{\delta} = \delta P_t^C$. Finally, the gross return on the levered claim is

$$1 + R_{t+1}^{\delta} = \frac{D_{t+1}^{\delta} + P_{t+1}^{\delta}}{P_t^{\delta}} = \frac{D_{t+1}^{\delta} + \delta P_{t+1}}{\delta P_t} \tag{46}$$

$$= \frac{1}{\delta}\frac{C_{t+1} + P_{t+1}^C}{P_t^C} - \frac{1-\delta}{\delta}(1+i_t) \tag{47}$$

4.3 Calibration and parameter estimation

In this subsection, I discuss empirical approach to estimate model parameters. The data used to estimate model parameters are quarterly US time series of quarterly growth on log real GDP per capita and log CPI, and federal funds rate. The sample period runs from 1965Q1 to 2019Q4. I collected all data series from the St. Louis Fed website.

Table 1: Calibrated parameters

Parameters	Value	Description
A. Macro parameters		
β	0.995	Discount rate
φ	2	Frisch elasticity
ξ_p	0.65	Price rigidity
λ_p	0.10	Steady-state price markup
χ_p	0.25	Price indexation
α	0.33	Capital share
B. Habit parameters		
θ_0	0.87	Persistence of surplus consumption
θ_1	-0.05	Dependence on consumption
θ_2	0.02	Dependence on lagged consumption
δ	2/3	Leverage
C. Shock processes		
ρ_A	0.97	Persistence of TFP shock
ρ_ζ	0.90	Persistence of bond demand shock
ρ_λ	0.95	Persistence of cost push shock
σ_a	0.007	Std of TFP shock
σ_ζ	0.0023	Std of bond demand shock
σ_λ	0.004	Std of cost push shock
σ_{MP}	0.0015	Std of monetary policy shock

As in Christiano, Eichenbaum, and Evans (2005), I partition model parameters into four groups. The first group of parameters consist of β, φ, λ_p, ξ_p, χ_p, γ, κ. Parameters values are listed in Panel A of table 1. These parameters are calibrated using external information. First, discount rate β is set to 0.995, implying steady state real rate of around 2 percent. φ is 2 so that Frisch elasticity of labor supply is 0.5 (Smets and Wouters, 2007).

Table 2: Estimated parameters

Parameters	1965Q1-1997Q4:Q4	1998Q1-2019Q4	Description
$\bar{\pi}$	3.8	2.0	Trend inflation
ϕ_π	2.5	3.1	MP inflation
ϕ_y	0.13	0.09	MP output gap
ρ_R	0.79	0.85	MP persistence

Maturity of nominal long-term bonds is set to 20 quarters (5 years) to match the average duration of the outstanding US Treasuries. Steady state markup is calibrated to 10 percent which is in line with the literature.

The degree of price stickiness, ξ_p, is set to 0.65, aligning with Nakamura et al. (2018), indicating firms change prices every 8 months on average during the Great Inflation. The degree of price indexation χ_p is calibrated to 0.25, the upper bound of empirical estimates (Cogley and Sbordone, 2008; Ascari, Castelnuovo, and Rossi, 2011)[24]. While there exists limited empirical support for price indexation, it is widely used in workhorse New Keynesian models for simplifying equilibrium conditions. Here, I am tying my hands by choosing low price stickiness and high indexation, which dampens the endogenous cost-push effects. The quantitative results make clear that even with this choice, the endogenous cost-push effects accounts for the majority of the shift in correlations.

Panel B of the table displays the second group of parameters which determine the external habit preferences. These parameters are directly taken from Campbell, Pflueger, and Viceira (2020). θ_0 governs the persistence of surplus consumption so any deviation of SDF from steady state would have longer effects on asset prices with higher θ_0. Both θ_1 and θ_2 determine the backward looking components of Euler equation, giving more consumption smoothing incentives to households. In the following sections, it will be clear that by the strength of habit, the model generates substantial amount of risk premia, due to time-varying risk aversion.

In Panel C of table 1, I show the third group of parameters, which describe exogenous shock processes. These parameters are taken from the literature. Persistence and volatility of bond demand shocks, cost push shocks, TFP shocks and conventional monetary policy shocks follow the estimates of Smets and Wouters (2007). One exception is the persistence

[24]According to Cogley and Sbordone (2008), once steady-state inflation is taken into account, estimates of the NKPC reject price indexation, and its median estimates is 0, with 90 percent confidence interval (0, 0.15).

of bond demand shocks, which follows the calibration of Coibion, Gorodnichenko, and Wieland (2012). Persistent bond demand shocks can match the high persistence of yield spread in financial markets.

It is worth emphasizing that I fix the shock processes across both subperiods intentionally, as it allows me to highlight how different levels of trend inflation affect the correlations. In contrast, some other papers assumes structural changes in exogenous shock processes (for example Campbell, Pflueger, and Viceira 2020; Pflueger 2022). Compared to the existing literature, my approach emphasizes the role of trend inflation and monetary policy on the cyclical behavior inflation and bond returns.

Parameters related to monetary policy rule ($\phi_\pi,\ \phi_y,\ \rho_R$) and trend inflation ($\bar{\pi}$) are estimated to minimize a measure of the distance between data and model-implied moments. I estimate a vector of four parameters, $\omega = (\phi_\pi,\ \phi_y,\ \rho_R,\ \bar{\pi})$, for each subperiod using SMM. Let $\Psi(\omega)$ denote the mapping from a vector of parameters ω to the model-implied moments, and $\hat{\Psi}$ denote the data counterpart. Then, the objective function of the SMM is

$$J = \min_{\omega}[\Psi(\omega) - \hat{\Psi}]' V^{-1} [\Psi(\omega) - \hat{\Psi}] \tag{48}$$

V is a symmetric positive definite matrix. Similar to Basu and Bundick (2017), I choose V to be a diagonal matrix which normalizes the distance between data and model-implied distance. The data moments $\hat{\Psi}$ used in estimation include standard deviation of GDP growth, inflation and federal funds rate, and AR(1) of each variable.

Table 2 presents the estimated parameters, and there are several points to discuss. Trend inflation is estimated at 3.8% annually, slightly below the sample average, while for the second period, it is estimated at 2.0%, aligning with the Federal Reserve's current inflation target. These estimates are consistent with existing range of estimates. For instance, Clarida, Gali, and Gertler (2000) estimates $\bar{\pi}$ at 4.2% for Pre-Volcker and 3.58 for Volcker-Greenspan era, Coibion and Gorodnichenko (2011) find a range of trend inflation estimates in the US, gradually stabilizing over time from its peak at 8% in 1980, to 2% level since the late 1990s, and Cogley and Sbordone (2008) find find smoother trend inflation estimates, hitting 4.5% in 1978 and stabilizing in the late 90s.

Taylor rule parameters are also in line with the previous estimates in trend inflation literature (Ascari, Castelnuovo, and Rossi, 2011). The inflation coefficient (ϕ_π) is estimated at 2.5 and 3.1 for each subperiod, aligning with existing estimates and exhibiting a similar increasing trend across the two periods. Moreover, the persistence of the monetary policy rule increases over time, from 0.79 to 0.85. It is worth noting that the inflation coefficient in a New Keynesian model with trend inflation is often higher than in models with zero inflation due to the shrinking determinacy region.

5 Quantitative analysis

5.1 Model-implied moments

Table 3 summarizes data and model-implied macro moments. The estimated model successfully matches the targeted macro moments for both periods. The model captures significant fall in macroeconomic volatility (output, inflation and interest rates) and inflation persistence from period I to period II, reflecting the impact of lower trend inflation and less accommodative monetary policy rule.

Panel A of table 4 reports key asset pricing moments. The estimated model does a good job of matching these moments, even though any asset pricing moment is neither targeted nor used in estimation. Stock risk premia are higher in period I due to greater macroeconomic volatility but decrease in period II. Sharpe ratio implied by the model is also close to its data counterpart. Price-dividend ratios from data and the model are close, although model price-dividend ratio is more volatile. 1-year stock return predictability is the regression coefficient of 1-year excess stock return onto lagged price-dividend ratio, following Campbell and Cochrane (1999). While pre-1997 data shows little predictability, the model shows some degree of predictability. In post-1998 period, both data and model predictability imply predictability of price-dividend ratio.

Nominal bonds also exhibit substantial risk premia, especially in period I, driven by positive trend inflation and stagflation risks. This also translates into steeper yield curve slope. 1-year bond return predictability is the regression coefficient of 1-year excess bond return onto lagged yield spread, following Campbell and Shiller (1991). The model-implied bond returns feature significant predictability by the slope of the yield curve in both period.

Panel B reports output gap-inflation and bond-stock correlations, which is untargeted in the estimation. The model predicts sign-switching in both correlations due to the substantial impact of high trend inflation on the endogenous cost-push factors and cyclicality of inflation. This change in cyclicality of inflation and macroeconomic volatility can be attributed to the stabilized trend inflation and shifts in monetary policy.

The shift in trend inflation and monetary policy regime significantly affects bond-stock correlation. In period I, the model-implied nominal bond-stock correlation in period I is positive, close to data (0.35 in data and 0.52 in model). High trend inflation in this period leads to countercyclical inflation, resulting in low nominal bond returns when stocks perform poorly. But as trend inflation stabilizes and monetary policy becomes more ag-

Table 3: Data and model-implied macro moments

	Period I: 1965-1997		Period II: 1998-2019	
	Data	Model	Data	Model
$\sigma(y)$	1.64	1.49	1.11	1.19
$\sigma(\pi)$	2.49	2.28	1.24	1.19
$\sigma(R)$	3.38	3.48	2.10	2.04
$\rho(y)$	0.87	0.84	0.89	0.68
$\rho(\pi)$	0.89	0.84	0.53	0.65
$\rho(R)$	0.94	0.98	0.97	0.94

Notes: Output data is detrended with HP filter. Inflation is measured by the log-difference in CPI. Interest rate is the Federal Funds rate. Model-implied moments are the simulated moments using 50,000 simulated data.

gressive, the correlation turns strongly negative (-0.57 in data vs. -0.47 in model). This shift indicates that the endogenous cost-push effects play a crucial role in explaining the majority of the sign change.

These results align with the empirical literature on the structural break in trend inflation and long-term inflation expectations. The anchoring of long-term inflation expectations and the stabilization of trend inflation, starting around 1997 (Ball and Mazumder, 2019; Carvalho et al., 2023)[25], have led to procyclical inflation and the ability of nominal bonds to hedge stocks. The quantitative GNK habit model supports this narrative, emphasizing the critical role of stable trend inflation and anchored long-term inflation expectations in the observed sign-switching.

5.2 The role of trend inflation and monetary policy

Figure 4 illustrates the model-implied bond-stock and output gap-inflation correlations for each pair of ϕ_π and $\bar{\pi}$, with the estimates of ϕ_y and ρ_R from period I. In the figure, the red region represents positive correlations, while the blue region indicates negative correlations in both panels. The point in the upper left reflects the period I estimates of ϕ_π

[25]Without publicly announced inflation target, long-term inflation expectation was not anchored until 1997. After the preemptive tightening of the FOMC in the early 1990s, long-term expectations are anchored at the current 2% target rate, which contributes to stabilized trend inflation.

Table 4: Data and model-implied asset pricing moments

	Period I: 1965-1997		Period II: 1998-2019	
	Data	Model	Data	Model
A. Asset pricing moments				
a. Stocks				
$E(xr^s)$	6.56	8.41	8.87	7.21
$\sigma(xr^s)$	16.45	14.91	18.62	14.71
$E(\log(pd))$	3.41	3.79	3.72	3.80
$\sigma(\log(pd))$	0.34	0.46	0.19	0.48
1-year predictability stock	-0.01	-0.15	-0.35	-0.16
b. Bonds				
$E(xr^b)$	1.83	2.41	2.12	1.15
$\sigma(xr^b)$	7.83	12.37	5.01	6.48
$E(y^{(20)} - y^{(4)})$	0.58	1.26	0.90	1.01
$\sigma(y^{(20)} - y^{(4)})$	0.84	0.78	0.71	0.55
1-year predictability bond	2.36	0.74	1.53	1.71
B. Correlations				
$\rho(\tilde{y}, \pi)$	-0.25	-0.71	0.40	0.65
$\rho(r^b, r^s)$	0.35	0.52	-0.57	-0.47

Notes: Bond-stock correlation in data is calculated using quarterly returns on 5-year zero coupon Treasuries and S&P 500 returns including dividends, excess of 3-month T-bill rates. Price-dividend ratio is the real price of S&P 500 divided by real dividends. Both data are collected from Robert Shiller's web site. 1-year predictability of stock is the regression coefficient of log price-dividend ratio on one year excess return on stocks. $y^{(20)} - y^{(4)}$ is the yield difference between 5 year nominal bonds and 1 year nominal bonds. 1-year predictability of bond is the regression coefficient of the yield spread on one year excess return on bonds. Model-implied moments are the simulated moments using 50,000 simulated data.

and $\bar{\pi}$ and the one in the lower right presents the period II estimates. From both panels, it is evident that both of bond-stock and output gap-inflation correlation exhibit substantial

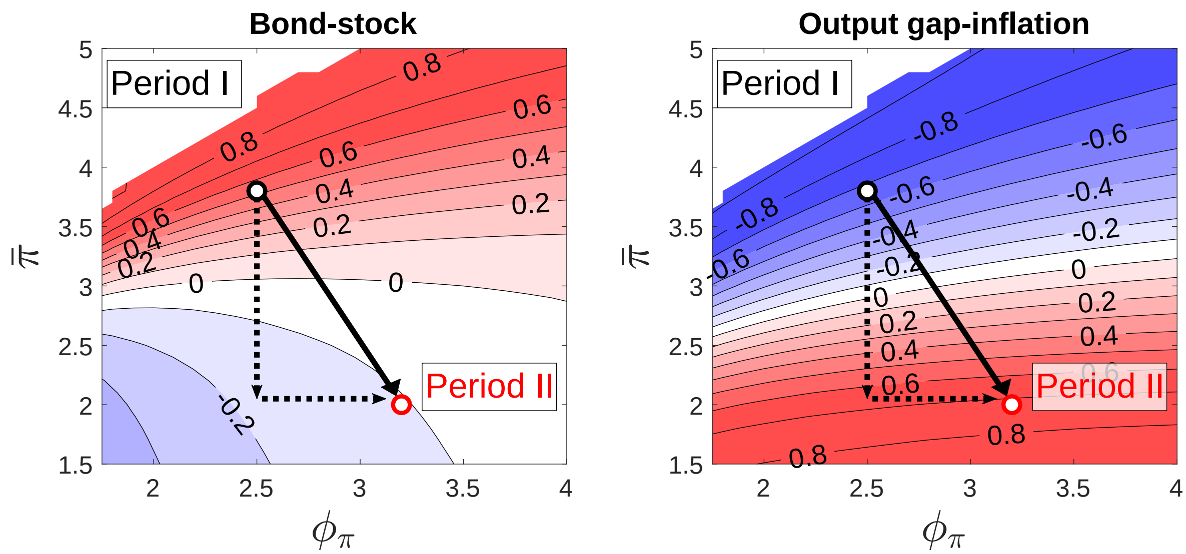


Figure 4: Model-implied bond-stock (left) and output gap-inflation (right) correlations

Notes: This model-implied correlation is with varying $\bar{\pi}$ and ϕ_π, while other parameters are fixed at period I estimates, i.e. $\phi_y = 0.13$ and $\rho_R = 0.79$. Period is from 1965:Q1 to 1997:Q4, and period II is from 1998:Q1 to 2019:Q4.

variation as $\bar{\pi}$ and ϕ_π change.

Several important points arise from the figure. First, as trend inflation ($\bar{\pi}$) increases, the gap between the two lines narrows, making correlations more sensitive to both $\bar{\pi}$ and ϕ_π. This suggests that smaller changes in $\bar{\pi}$ or ϕ_π can have a larger impact on these correlations, indicating increasing significance of the endogenous cost-push effects as trend inflation rises. Notably, aggressive policy reaction to inflation attenuates the endogenous cost-push effects, which has bigger impact under high inflation.

But the majority of the sign change in both correlations can be attributed to the change in trend inflation. From the period I estimates to the period II estimates, the decline in trend inflation by 1.8 percent is associated with the fall in bond-stock correlation by around 0.6, which is sufficient enough to switch the sign. In the model, the level of trend inflation has large impact on the significance of the endogenous cost-push terms because it determines the loadings of these terms, as can be seen in equations (17)-(20). In particular, the effect of trend inflation is nonlinear, and further amplified by the degree of real rigidity from monopolistic competition, which is represented by the elasticity of substitution, ϵ[26]. As proposition 2 and 3 shows, this alters the cyclical behavior of inflation and this shift

[26] Ascari (2004) discuss that steady state price dispersion and the consequent output loss are nonlinear in the level of trend inflation, and the output loss is decreasing fast as trend inflation increases.

in macroeconomic fundamental results in the change in bond-stock relationship. Consequently, the sign-change in correlations between the two periods is primarily driven by the decline in $\bar{\pi}$ from 3.8 percent to 2.0 percent.

Second, the impact of inflation reaction coefficient ϕ_π varies with trend inflation levels. n regions with high trend inflation (red in the left figure), a more aggressive monetary policy reduces bond-stock correlation and makes nominal bonds less risky. In the blue region (right figure), higher ϕ_π makes inflation less countercyclical. This is because strict monetary policy can mitigate the sizable endogenous cost-push effects in this region by reducing inflation volatility. Conversely, in low trend inflation environment where nominal bonds serve as hedges, a higher ϕ_π has a small positive effect on bond-stock correlation. This is because aggressive policy measures primarily dampen demand shocks more than other supply shocks, like markup shocks, leading to procyclical inflation. In summary, the influence of the inflation coefficient ϕ_π on bond-stock correlation is more pronounced in high trend inflation settings and less significant in low trend inflation environments.

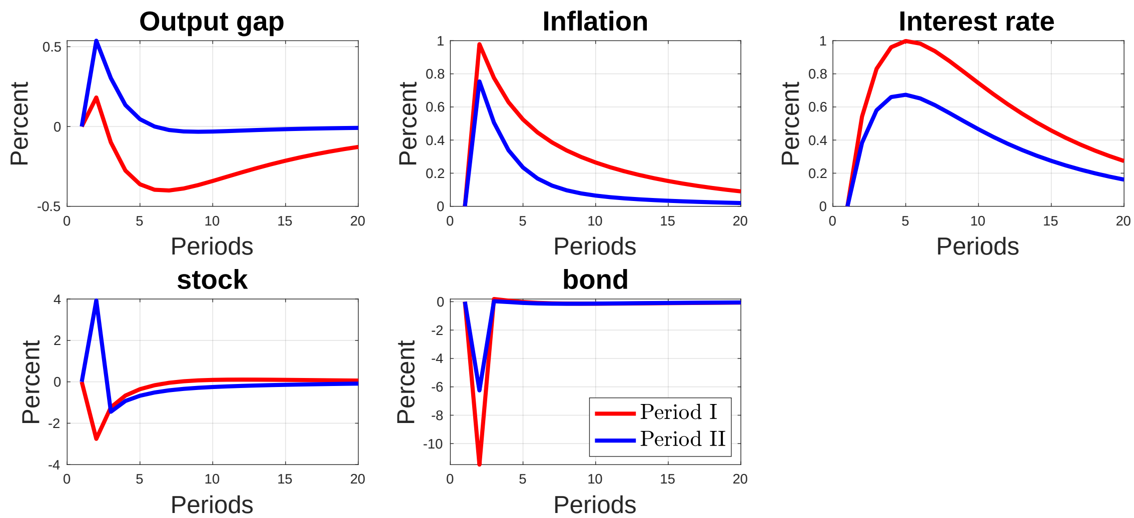
The model-implied correlations in figure 4 sheds lights on the implications of trend inflation for asset pricing and macro dynamics. Traditional New Keynesian models primarily focus on the inflation coefficient in the Taylor rule or the relative strength of supply and demand shocks to explain correlation shifts (Campbell, Pflueger, and Viceira, 2020; Pflueger, 2023). These models attribute the shift in hedging property of Treasuries to the change in exogenous shock processes. In contrast, my model provides new insights about the role of trend inflation as a determinant of bond-stock correlation and cyclicality of inflation, which has not received much attention in the literature. The GNK habit model coherently explains the shifts in the cyclical behavior of inflation and nominal bond returns as a consequence of stabilized trend inflation. As trend inflation and long-term inflation expectations became anchored in the late 1990s, inflation exhibited procyclical pattern, while nominal bonds became hedge assets in the absence of stagflation risks.

5.3 Impulse responses and the role of trend inflation

Here, I investigate how the endogenous cost-push effects alter the responses of endogenous variables to structural shocks in each period by the IRFs of key variables to demand and markup shocks. Figure 5 shows the impulse responses of macroeconomic variables and asset prices following a one standard deviation demand and markup shocks.

The first row of panel (a) displays the IRFs of output gap, inflation and short-term interest rate in each period, following a positive demand shock. In period I, after a positive demand shock hits (red lines), output gap, inflation and interest rate rise simultaneously.

(a) Impulse responses to demand shocks



(b) Impulse responses to markup shocks

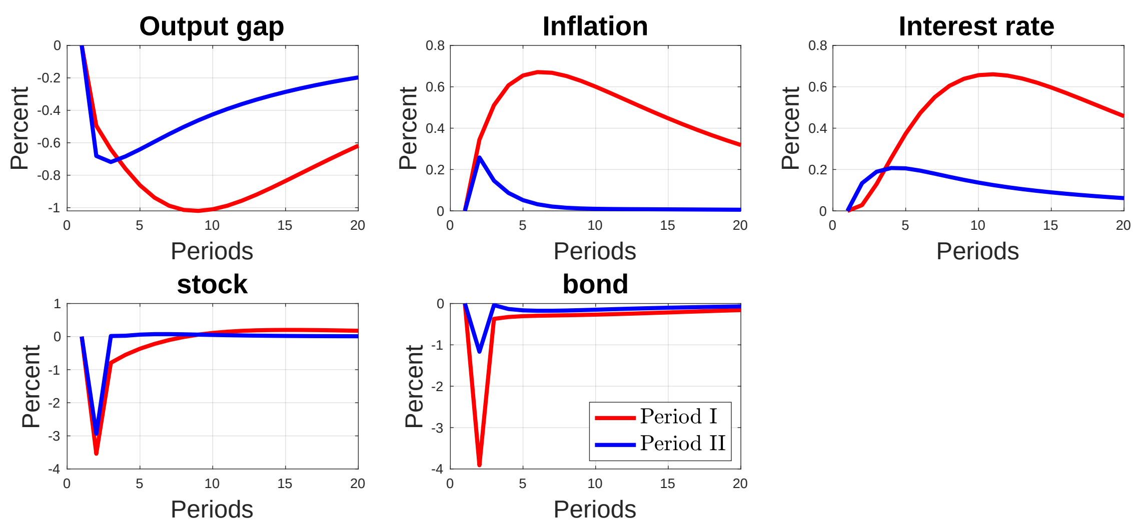


Figure 5: Impulse responses to one standard deviation demand and price markup shock.

Notes: Period I is 1965-1997 and Period II is 1998-2019.

But its expansionary effect is very short-lived. Starting from the second period after the shock realization, output gap remains below steady state level, while inflation slowly goes back to steady state from above. This is consistent with the analytical results in the previous section, which shows that under high trend inflation, demand shocks can generate countercyclical inflation.

In contrast, period II shows strongly procyclical inflation following demand shocks. Low trend inflation is associated with significantly weaker endogenous cost-push effects, and the IRFs are much similar to the zero inflation (or full indexation) case. Initial and subsequent path of output gap to demand shocks are positive and approaches steady state from above. Inflation response is still strong and persistent but less than that under high trend inflation.

The second row of panel (a) shows asset pricing dynamics. Nominal bond returns fall in both periods after the shock because of inflationary pressure following positive demand shocks. After a positive demand shock, excess bond return falls around 10 percent in period I, while it decreases by 6 percent in period II. Due to the substantial endogenous cost-push effects from trend inflation, both inflation and bond return show more pronounced responses in the earlier period.

High trend inflation also alters the responses of stock returns. Because demand shocks generate recession in period I, stock returns fall from the impact. As a result, under high trend inflation with less aggressive monetary policy rule, stock and bond returns are positively correlated conditional on demand shocks. In period II, however, stock returns increase strongly so that bond-stock returns exhibit a negative correlation after demand shocks. Hence, the different impact effects provide hedging opportunity from holding nominal bonds.

The first row of panel (b) reports the same set of IRFs to markup shocks Price markup shocks (red dashed) drive large and prolonged contraction in output gap with persistent rise in inflation. Trough occurs around 8 quarters after the shock and it shows slow recovery. Inflation increases from the impact and peak occurs around at 6 quarters.

When trend inflation falls and monetary policy becomes more reactive to inflation, the effect of price markup shocks weaken, although it still generates stagflation. The responses of output and inflation is less pronounced and decay much faster. This is because the significance of the endogenous cost-push terms are attenuated. However, inflation is countercyclical in both periods.

Turning to asset price responses, stock returns drop significantly on impact due to lower expected dividend streams. Nominal bond returns also decrease due to inflation, but much more so in period I. In both cases, price markup shocks result in a positive bond-stock correlation because the macroeconomic dynamics following markup shocks exhibit countercyclical inflation. Hence, markup shocks always predict risky nominal bonds.

To better visualize the consequence of the endogenous cost-push channel, I illustrate the same set of IRFs under period I parameters but with full price indexation ($\chi_p = 1$) in appendix D.6[27]. This means even if steady state inflation is at its period I estimates of 4.1%, firms are allowed to automatically adjust their prices according to previous inflation rate. Under full indexation, steady state price dispersion is zero and it has very limited impact on macroeconomic dynamics, hence the strength of the endogenous cost-push terms is much weaker.

As expected, responses to demand shocks are strongly expansionary and cause procyclical inflation. Output rises on impact and goes back to steady state from above. Inflation reacts less strongly and less persistent than that with partial price indexation.[28] Risky asset prices increase and nominal long-term bonds exhibit negative returns. The exercise in this subsection clearly points out that it is not just how a central bank sets its policy rate or the mix of demand and supply shocks, but average level of inflation that matters for the cyclicality of inflation and nominal bond risks.

5.4 Volatility of markup shocks and price indexation

Can volatile markup shocks alone explain the sign changing pattern? In literature, volatility and countercyclical feature of inflation in 1970s are often considered as the result of large supply shocks, such as oil shocks. But there is less clear evidence on the prevailing supply shocks in 80s or 90s. In this subsection, I answer the question whether the New Keynesian model asset pricing model in this paper predicts the observed strong positive relationship between bond and stocks mainly based on supply shocks.

For this exercise, I recalculate the model-implied bond-stock correlation as the volatility of markup shocks vary. Panel (a) of table 5 shows the results of this counterfactual exercise. The table shows the counterfactual bond-stock correlations when the standard deviation of price markup shocks is at the baseline, and two and three times bigger than the baseline. I also compare the counterfactual correlations under period I monetary pol-

[27]Full indexation is often used in the literature to capture internal persistence of inflation and analytical tractability (Christiano, Eichenbaum, and Evans, 2005).

[28]This provides one solution to deal with the lack of internal persistence of inflation in NK models. In the literature, price indexation is often used to fit the empirical persistence of inflation (Gali and Gertler, 1999; Christiano, Eichenbaum, and Evans, 2005). But this non-optimizing pricing behavior is ad-hoc and not well supported by empirical evidence on price setting behavior of firms (Bils and Klenow, 2004; Nakamura and Steinsson, 2008)

Table 5: Bond-stock correlation counterfactuals

(a) Bond-stock correlations with different volatility of cost-push shocks

	Period I rule, $\bar{\pi}$ =2.1			Period II rule, $\bar{\pi}$ =2.1		
std of markup shocks (σ_λ)	σ_λ	$2\sigma_\lambda$	$3\sigma_\lambda$	σ_λ	$2\sigma_\lambda$	$3\sigma_\lambda$
$\rho(r^b, r^s)$	-0.40	-0.30	-0.19	-0.41	-0.25	-0.21

Notes: Baseline calibration is $\sigma_\lambda = 0.004$.

(b) Bond-stock correlations with different degree of price indexation

	Period I rule, $\bar{\pi}$ =4.1			Period II rule, $\bar{\pi}$ =2.1		
price indexation (χ_p)	0.25	0.5	0.75	0.25	0.5	0.75
$Corr(xr^b, xr^s)$	0.52	-0.23	-0.41	-0.47	-0.48	-0.49

Notes: Baseline calibration is $\lambda_p = 0.25$.

icy rule with $\bar{\pi} = 2.1$ and period II rule with $\bar{\pi} = 2.1$. This comparison shows how much bond-stock correlation change can possibly be explained by only considering time-varying volatility of markup shocks.

As markup shocks become more volatile, bond-stock correlation increases as expected. Under monetary policy rule in period I, unrealistically high markup shocks are required to obtain the sign change. As a comparison, Smets and Wouters (2007) estimate that the volatility of markup shocks before Volcker is twice the volatility of the shocks after Volcker.[29] Even if markup shocks become even less volatile after 2000, three time bigger markup shocks in 1980s that are needed to generate the sign change are unrealistically high, considering the existing estimates. The similar conclusion emerges even considering Taylor rule after 2000.

This exercise clearly shows that without positive trend inflation, the model requires unrealistically high volatility of cost-push shocks, which is beyond the existing estimates from literature. It is also consistent with the empirical literature that studies the role of

[29]The estimates of markup shock volatility range from 0.1 to 0.3 in Smets and Wouters (2007).

supply shocks for the relationship between inflation and macroeconomic output. For instance, Dräger, Lamla, and Pfajfar (2016) show using supply and demand shocks from Kilian (2009) that there is limited evidence that supply shocks were dominant in the late 80s and 90s. Also, Shapiro et al. (2022) decompose inflation into supply and demand driven component, which shows no particularly dominant role of supply shocks. Therefore, without the mechanism through the endogenous cost-push factors in GNKPC, supply shocks alone may not be enough to explain the sign-changing correlation.[30]

5.5 The role of time-varying risk premia

The external habit preferences augmented in the model introduce substantial time-varying risk premia, which have considerable effects on asset pricing dynamics. To investigate the role of time-varying risk premia on the nominal bond and stock risks, I decompose the covariance between bond and stock returns into cash flow, real rate, and risk premium news using the classic Campbell-Shiller decomposition (Campbell and Shiller, 1988).

Table 6 shows the decomposition of bond-stock covariance before and after 1997. In both periods, risk neutral returns (the sum of cash flow and real rate news) are positively correlated to each other. This is quantitatively large, so that bond-stock covariance is positive. But in period II, covariance between risk premia components becomes larger and dominates risk neutral parts, switching the sign.

This decomposition highlights the role of time-varying risk premia on the correlation change. The table shows that substantial amount of covariance change is due to risk premium news. It amplifies the effects from the change in inflation dynamics through the SDF. This is because when cyclicality of inflation changes, the information about the future paths of output and inflation also change, hence asset prices move differently. Therefore, substantial time-varying risks are crucial to explain the observed sign shift.

6 Conclusion

Historical observation reveals that the hedging property is not an inherent feature of Treasuries. In this paper, I show that trend inflation is a key to understand Treasuries' hedging feature. Using a stylized GNK model, I show that there exists a rich interaction between trend inflation, monetary policy and the cyclicality of inflation, which determines nomi-

[30]This can be also observed in Pflueger (2022). The author estimates that inflation is almost driven by supply shocks in 1980s and demand shocks are negligible. But 2000s show the opposite pattern where demand shocks dominate supply and monetary shocks.

Table 6: Campbell-Shiller decomposition

	Period I: 1965-1997		
	Cash Flow	Real Rate	Risk Premium
Cash Flow	-10.47	166.89	-85.78
Real Rate	74.72	1.24	2.51
Risk Premium	-46.68	-140.55	86.16

	Period II: 1998-2019		
	Cash Flow	Real Rate	Risk Premium
Cash Flow	14.01	133.23	-90.65
Real Rate	4.88	-11.32	21.65
Risk Premium	-31.01	-164.10	109.26

nal bond risks. With high trend inflation, demand shocks generate stagflation through its interaction with inefficient price dispersion. In this case, bond risk premia are high and comove with equity premia.

I fit the quantitative model to US data to match macroeconomic moments before and after 1997 separately. The estimates of trend inflation has decreased from around 3.8% to 2%, which accounts for a large portion of the sign-change in bond-stock correlation and inflation-output relationship. My paper explains why the bond-stock correlation changed in 1997. After long fighting against inflation, long-term inflation expectations are anchored at the Fed's target with stable inflation. It eliminated future inflation risks and changed inflation dynamics, from countercyclical to procyclical.

Important policy implication of this paper is that nominal long-term bonds are not necessarily hedge assets. What makes it hedge is th future paths of inflation and real economic conditions. The role of central bank is crucial because a central bank has a control over the macroeconomic dynamics through its inflation target. By convincingly setting low inflation target, a central bank not only stabilizes macroeconomy but also provides safe and hedge assets in financial markets.

Another important implication of my results is that the common finding in the literature that demand shocks have driven macroeconomic dynamics since 1997 could be the result of the stabilization of trend inflation. The model shows that given exogenous shock processes, the level of trend inflation matters for inflation cyclicality and nominal bond risks. A central bank with credible inflation target can shape inflation more procyclical and use its conventional monetary policy tool to stabilize real outcome and inflation.

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A Proof

Proof of Lemma 1. First, guess each endogenous variable as a linear function of ζ_t

$$[\hat{\pi}_t, \tilde{y}_t, \hat{i}_t, \hat{\psi}_t]' = [\Gamma_\pi, \Gamma_y, \Gamma_i, \Gamma_\psi]'\zeta_t \tag{49}$$

Then, from the IS equation (1),

$$y_t = y_{t+1} - \phi_\pi \pi_t + E_t \pi_{t+1} + \zeta_t \tag{50}$$

$$\Gamma_y(1-\rho_\zeta) = \Gamma_\pi(\rho_\zeta - \phi_\pi) + 1 \tag{51}$$

$$\therefore \quad \Gamma_y = \frac{1-\Gamma_\pi(\phi_\pi - \rho_\zeta)}{1-\rho_\zeta} \tag{52}$$

Next, from (19),

$$\psi_t = (1-\xi\beta\bar{\Pi}^\epsilon)y_t + \epsilon\beta\xi\bar{\Pi}^\epsilon E_t\pi_{t+1} + \xi\beta\bar{\Pi}^\epsilon E_t\psi_{t+1} \tag{53}$$

$$= X_t + \xi\beta\bar{\Pi}^\epsilon E_t X_{t+1}, \qquad \text{where } X_t = (1-\xi\beta\bar{\Pi}^\epsilon)y_t + \epsilon\beta\xi\bar{\Pi}^\epsilon E_t\pi_{t+1} \tag{54}$$

$$= \frac{1}{1-\rho_\zeta\xi\beta\bar{\Pi}^\epsilon} X_t \tag{55}$$

$$= \frac{1}{1-\rho_\zeta\xi\beta\bar{\Pi}^\epsilon}\Big[(1-\xi\beta\bar{\Pi}^\epsilon)\Gamma_y + \epsilon\beta\xi\bar{\Pi}^\epsilon\rho_\zeta\Gamma_\pi\Big]\zeta_t \tag{56}$$

Now, from (17),

$$\pi_t = \bar{\beta}\rho_\zeta\Gamma_\pi\zeta_t + \bar{\kappa}\Gamma_y\zeta_t + (\bar{\pi}-1)\bar{\gamma}\frac{\rho_\zeta}{1-\rho_\zeta\xi\beta\bar{\Pi}^\epsilon}\Big[(1-\xi\beta\bar{\Pi}^\epsilon)\Gamma_y + \epsilon\beta\xi\bar{\Pi}^\epsilon\rho_\zeta\Gamma_\pi\Big]\zeta_t \tag{57}$$

$$\Gamma_\pi = \bar{\beta}\rho_\zeta\Gamma_\pi + \bar{\kappa}\Gamma_y + \frac{\rho_\zeta(\bar{\pi}-1)\bar{\gamma}}{1-\rho_\zeta\xi\beta\bar{\Pi}^\epsilon}\Big[(1-\xi\beta\bar{\Pi}^\epsilon)\Gamma_y + \epsilon\beta\xi\bar{\Pi}^\epsilon\rho_\zeta\Gamma_\pi\Big] \tag{58}$$

Substituting out Γ_y using (56),

$$\Gamma_\pi(\bar{\Pi}) = \frac{\frac{\bar{\kappa}}{1-\rho_\zeta} + \frac{\bar{\gamma}\rho_\zeta\phi_\pi(\bar{\Pi}-1)(1-\xi\beta\bar{\Pi}^\epsilon)}{(1-\rho_\zeta\xi\beta\bar{\Pi}^\epsilon)(1-\rho_\zeta)}}{1-\bar{\beta}\rho_\zeta + \frac{\bar{\kappa}(\phi_\pi-\rho_\zeta)}{1-\rho_\zeta} + \frac{\bar{\gamma}\rho_\zeta(\bar{\Pi}-1)}{1-\rho_\zeta\xi\beta\bar{\Pi}^\epsilon}\left(\frac{(1-\xi\beta\bar{\Pi}^\epsilon)(\phi_\pi-\rho_\zeta)}{1-\rho_\zeta} - \epsilon\xi\beta\bar{\Pi}^\epsilon\rho_\zeta\right)}. \tag{59}$$

Then, using (56), (58) and (3)

$$\Gamma_y(\bar{\Pi}) = \frac{1}{1-\rho_\zeta} - \frac{\phi_\pi - \rho_\zeta}{1-\rho_\zeta}\Gamma_\pi \tag{60}$$

$$\Gamma_i(\bar{\Pi}) = \phi_\pi(\Gamma_\pi - 1) + \phi_y\Gamma_y \tag{61}$$

$$\Gamma_\psi(\bar{\Pi}) = \frac{(1-\xi\beta\bar{\Pi}^\epsilon)\Gamma_y + \epsilon\xi\beta\bar{\Pi}^\epsilon\rho_\zeta\Gamma_\pi}{1-\rho_\zeta\xi\beta\bar{\Pi}^\epsilon} \tag{62}$$

$\square$

Proof of Proposition. From (59), $\frac{d\Gamma_\pi}{d\bar{\pi}} > 0$ if and only if $\frac{df}{d\bar{\pi}} > 0$ where

$$f(\bar{\pi}) = \frac{(\bar{\pi}-1)(1-\xi\beta\bar{\pi}^\epsilon)}{1-\rho_\zeta\xi\beta\bar{\pi}^\epsilon} \tag{63}$$

Then,

$$\frac{df}{d\bar{\pi}} = \frac{1}{(1-\rho_\zeta\xi\beta\bar{\Pi}^\epsilon)} \tag{64}$$

$$\frac{df}{d\bar{\pi}} > 0 \tag{65}$$

$$\Leftrightarrow (1-\xi\beta\bar{\Pi}^\epsilon)(1-\rho_\zeta\xi\beta\bar{\Pi}^\epsilon) > (1-\rho_\zeta)(\bar{\Pi}-1)\epsilon\xi\beta\bar{\Pi}^{\epsilon-1} \tag{66}$$

$$\Leftrightarrow -(1-\xi\beta\bar{\Pi}^\epsilon)\xi\beta\bar{\Pi}^\epsilon\rho_\zeta + (1-\xi\beta\bar{\Pi}^\epsilon) > (1-\rho_\zeta)(\bar{\Pi}-1)\epsilon\xi\beta\bar{\Pi}^{\epsilon-1} \tag{67}$$

$$\Leftrightarrow \rho_\zeta > \frac{\xi\beta\bar{\Pi}^{\epsilon-1}((\bar{\Pi}-1)\epsilon+\bar{\Pi})-1}{\xi\beta\bar{\Pi}^{\epsilon-1}((\bar{\Pi}-1)\epsilon-(1-\xi\beta\bar{\Pi}^\epsilon)\bar{\Pi})} \equiv \rho_\zeta^* \tag{68}$$

Therefore, if $\rho_\zeta > \rho_\zeta^*$, then $\frac{d\Gamma_\pi}{d\bar{\pi}} > 0$.

For the result 5,

$$\frac{d\Gamma_\pi}{d\phi_\pi} < 0 \tag{69}$$

$$\Leftrightarrow \frac{1}{\phi_\pi}\Big(\frac{\bar{\kappa}\phi_\pi}{1-\rho_\zeta} + \frac{\bar{\gamma}\rho_\zeta\phi_\pi(\bar{\Pi}-1)(1-\xi\beta\bar{\Pi}^\epsilon)}{(1-\rho_\zeta\xi\beta\bar{\Pi}^\epsilon)(1-\rho_\zeta)}\Big)\Big(1-\bar{\beta}\rho_\zeta + \frac{\bar{\kappa}(\phi_\pi-\rho_\zeta)}{1-\rho_\zeta} + \frac{\bar{\gamma}\rho_\zeta(\bar{\Pi}-1)}{1-\rho_\zeta\xi\beta\bar{\Pi}^\epsilon}\Big(\frac{(1-\xi\beta\bar{\Pi}^\epsilon)(\phi_\pi-\rho_\zeta)}{1-\rho_\zeta} \tag{70}$$

$$-\Big(\frac{\bar{\kappa}\phi_\pi}{1-\rho_\zeta} + \frac{\bar{\gamma}\rho_\zeta\phi_\pi(\bar{\Pi}-1)(1-\xi\beta\bar{\Pi}^\epsilon)}{(1-\rho_\zeta\xi\beta\bar{\Pi}^\epsilon)(1-\rho_\zeta)}\Big)\Big(\frac{\bar{\kappa}}{1-\rho_\zeta} + \frac{\bar{\gamma}\rho_\zeta(\bar{\Pi}-1)}{1-\rho_\zeta\xi\beta\bar{\Pi}^\epsilon}\frac{(1-\xi\beta\bar{\Pi}^\epsilon)}{1-\rho_\zeta}\Big) < 0 \tag{71}$$

Therefore, $\frac{d\Gamma_\pi}{d\phi_\pi} < 0$.

Next,

$$\Gamma_\pi|_{\bar{\Pi}=1} = \frac{\frac{\bar{\kappa}\phi_\pi}{1-\rho_\zeta}}{1-\bar{\beta}\rho_\zeta + \frac{\bar{\kappa}(\phi_\pi-\rho_\zeta)}{1-\rho_\zeta)}} > 1 \tag{72}$$

$$\Leftrightarrow \frac{\bar{\kappa}\phi_\pi}{1-\rho_\zeta} > 1-\bar{\beta}\rho_\zeta + \frac{\bar{\kappa}(\phi_\pi-\rho_\zeta)}{(1-\rho_\zeta)} \tag{73}$$

$$\Leftrightarrow \bar{\kappa}(\phi_\pi-\rho_\zeta) > (1-\bar{\beta}\rho_\zeta)(1-\rho_\zeta) \tag{74}$$

$$0 > \rho_\zeta^2 - (1+\bar{\beta}+\bar{\kappa})\rho_\zeta + 1 \tag{75}$$

$$\rho_\zeta > \frac{1+\bar{\beta}+\bar{\kappa} - \sqrt{(1+\bar{\beta}+\bar{\kappa})^2 - 4\bar{\beta}}}{2\bar{\beta}} \equiv \bar{\rho}_\zeta \tag{76}$$

Therefore, if $\rho_\zeta > \max\{\bar{\rho}_\pi, \rho_\zeta^*\}$, then $\Gamma_\pi > 1$. From (61), $\Gamma_i > 0$ under the assumption that $\phi_y \geq 0$.

From the above result, when $\rho_\zeta > \rho_\zeta^*$,

$$\frac{d\Gamma_y}{d\bar{\Pi}} = -\frac{\phi_\pi - \rho_\zeta}{1-\rho_\zeta}\frac{d\Gamma_\pi}{d\bar{\Pi}} < 0 \tag{77}$$

Furthermore, if $\Gamma_\pi > \frac{1}{\phi_\pi - \rho_\zeta}$, then $\Gamma_y < 0$.

Lastly,

$$Cov(\hat{y}_t, \hat{\pi}_t) = \Gamma_y(\bar{\Pi})\Gamma_\pi(\bar{\Pi})\frac{\sigma_\pi^2}{1-\rho_\zeta^2} \tag{78}$$

is immediate from the lemma. □

Proof of Proposition 3. Excess return on consumption claim is

$$\hat{xr}_{t+1}^s = \hat{r}_{t+1}^s - \hat{r}_t = \Delta\hat{y}_{t+1} + E_t\hat{m}_{t+1} = y_{t+1} - E_t y_{t+1} \tag{79}$$

and excess return on two period nominal bonds is

$$\hat{xr}_{t+1}^b = \hat{r}_{t+1}^b - \hat{i}_t = E_{\Delta(t+1)}\hat{m}_{t+2} - E_{\Delta(t+1)}\hat{\pi}_{t+2} - \hat{\pi}_{t+1}, \tag{80}$$

The SDF is give by

$$\hat{m}_{t+1} = -\Delta y_{t+1} \tag{81}$$

Then, using the lemma,

$$Cov(xr_{t+1}^s, xr_{t+1}^b) = \Gamma_y(\bar{\Pi})(\Gamma_y(\bar{\Pi})(1-\rho_\zeta) - \Gamma_\pi(\bar{\Pi})(1+\rho_\zeta))\frac{\sigma_\pi^2}{1-\rho_\zeta^2}. \tag{82}$$

□

B Description of the quantitative GNK model

B.1 Final good and intermediate goods producers

In each period t, the final consumption good Y_t is produced by perfectly competitive firms using a continuum of each intermediate goods $Y_t(i),\ i \in [0,1]$ as inputs. Final good producers have access to a CES production technology that aggregates the continuum of intermediate goods into the single final good:

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{1}{1+\lambda_{p,t}}} di \right]^{1+\lambda_{p,t}}. \tag{83}$$

where $\lambda_{p,t}$ represents the desired markup over marginal costs for intermediate goods firms. I label this innovation as price markup shocks as in Smets and Wouters (2007). Final good producers' profit maximization and the zero profit condition for the competitive market yield the following condition for the final good price P_t:

$$P_t = \left[\int_0^1 P_t(i)^{-\frac{1}{\lambda_{p,t}}} di \right]^{-\lambda_{p,t}} \tag{84}$$

and the downward sloping demand function for each intermediate good i:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\frac{1+\lambda_{p,t}}{\lambda_{p,t}}} Y_t \tag{85}$$

The demand function for each intermediate good will be used to derive price dispersion, which is central to understand the inflation risk channel.

A continuum of monopolistically competitive firms produce a different variety i. Each intermediate goods firm i has access to a technology

$$Y_t(i) = A_t H_t(i)^{1-\alpha} \tag{86}$$

where $H_t(i)$ denotes the quantity of labor hired by firm i. A_t is a neutral exogenous technology that is common across firms. I assume that $\lambda_{p,t}$ and $\log A_t$ follow stationary AR(1) processes

$$\log(1+\lambda_{p,t}) = (1-\rho_p)\log(1+\lambda_p) + \rho_p \log(1+\lambda_{p,t-1}) + \epsilon_t^p, \quad \epsilon_t^p \sim^{iid} N(0,\sigma_p^2), \tag{87}$$

$$\log A_t = \rho_A \log A_{t-1} + \epsilon_t^A, \quad \epsilon_t^A \sim^{iid} N(0,\sigma_A^2). \tag{88}$$

Every intermediate goods firm is subject to the Calvo-type friction. In each period t, a fraction ξ_p of intermediate goods firms are not allowed to change its price optimally and instead, they reset the price according to the indexation rule

$$P_t(i) = P_{t-1}(i)(1+\bar{\pi})^{\chi_p} \tag{89}$$

where χ_p is the degree of price indexation. In general, $\chi_p < 1$ and therefore, there exists price dispersion in steady state, which is crucial for the existence of the endogenous cost-push factors. The remaining fraction of firms, $1-\xi_p$, can reset their price optimally to maximize the present value of its future expected cash flow. Because households own firms, realized profits will be distributed to households. Specifically, optimizing firms set their price $P_t(i)$ solve the following expected profit maximizing problem:

$$\max E_t \sum_{s=0}^{\infty} (\xi_p \beta)^s \Big[P_t(i) \Big(\prod_{k=1}^{s} \pi_{t+k-1}^{\chi_p} \Big) Y_{t+s}(i) - W_{t+s} H_{t+s}(i) \Big] \tag{90}$$

where $m_{t,t+s}$ is the SDF. Solving this optimization problem, the aggregate price level is determined by

$$P_t = \Big[(1-\xi_p)(P_t^r)^{-\frac{1}{\lambda_{p,t}}} + \xi_p(\pi_{t-1}P_{t-1})^{-\frac{1}{\lambda_{p,t}}} \Big]^{-\lambda_{p,t}} \tag{91}$$

where P_t^r is the optimal reset price for adjusting firms, and P_{t-1} is the average price level in the previous period.

B.2 Monetary policy

A monetary authority sets the short-term interest rate R_t according to the following Taylor-type policy rule:

$$1 + i_t = \rho_R(1 + i_{t-1}) + (1-\rho_R)\Big(1 + \bar{i} + \phi_\pi(\pi_t - \bar{\pi}) + \phi_y \tilde{y}_t\Big) + \epsilon_t^{MP} \tag{92}$$

where $\bar{i}$ is the steady state short-term interest rate, π_t^* is the inflation target set by the monetary authority, and ϵ_t^{MP} is an iid monetary policy shock, which follows $\epsilon_t^{MP} \sim N(0, \sigma_{MP}^2)$. Parameters ϕ_π, and ϕ_y determine its response to inflation, output growth, and output gap, respectively.

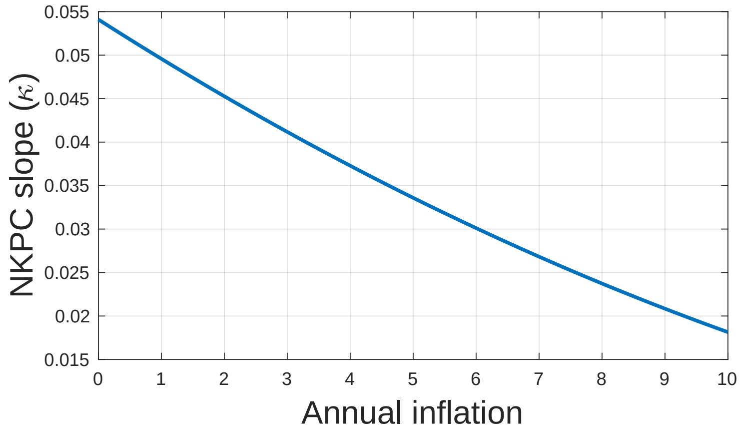

Figure 6: Model implied NKPC slope

C Macroeconomic volatility and trend inflation

The stabilization of inflation and other macroeconomic volatility has been a central question in macroeconomics. Unlike conventional explanation based on the change in the slope of Phillips curve as in Stock and Watson (2020), the results in this paper emphasizes the importance of the anchoring of trend inflation at low level. When trend inflation is high, it introduces substantial inflation risks and it contributes to inflation and output volatility. As analyzed in the previous section, this is the effects of trend inflation on NKPC as a disturbance term. With high inflation, any deviation of inflation from steady state is observationally equivalent to disturbances to NKPC, amplifying the initial inflationary pressure.

Figure 6 depicts the model implied NKPC slope. It shows that the slope of NKPC turns out to be decreasing in trend inflation, but still brings about macroeconomic instability. This is because the instability implied by the model is caused by trend inflation expectation and the changes in the 'intercept' of Phillips curve. In this sense, what matters to the inflation-real outcome relationship is not the slope but trend inflation. This is consistent with the recent empirical findings of Hazell et al. (2022). They use state-level price data to estimate the slope of Phillips curve and find that the sharp drop in inflation is due to the anchoring of trend inflation. The theoretical and quantitative results in this paper provides economic mechanism that supports their empirical findings.

In another strand of literature, there is a long debate whether the responsiveness of monetary policy to inflation has changed over time.[31] The estimates in this paper on trend inflation adds a new angle on this debate. When trend inflation is high, given the set of monetary policy reaction function, macroeconomic volatilities, especially inflation volatility, are increasing functions of $\bar{\pi}$. It is also noteworthy that impulse responses to inflation target shocks resemble the reaction of monetary rule under indeterminacy (i.e. violation of Taylor principle). Together with high trend inflation, the consequence of inflationary shocks is deterioration of price stability and prolonged recession (see figure 5). In this regard, the observed price instability during 1970s can be also understood as dire outcome of high trend inflation with persistent inflationary pressure, such as increase in target inflation (Cogley and Sargent, 2005), which doesn't necessarily need to be in indeterminate region.

[31] Clarida, Gali, and Gertler (2000) estimate the responsiveness parameter ϕ_π is smaller then 1 for the pre-Volcker (before 1983) but much higher afterwards.This, "policy mistake" view stresses that US monetary policy was less responsive to inflation in the 1960s and 1970s. For detailed discussion, see Clarida, Gali, and Gertler (2000), Lubik and Schorfheide (2004), Coibion and Gorodnichenko (2011) and Boivin and Giannoni (2006). Relatedly, Ascari and Ropele (2007) study the implication of non-zero inflation target on macroeconomic stability. They find that once inflation target is above 4%, determinate equilibrium is very unlikely, based on standard calibration of New Keynesian model. Benati and Goodhart (2010) also point out that high average inflation during 1960s and 1970s could the economy to remain in indeterminate region. In contrast, Sims and Zha (2006) find that the parameter has been stable over time, and instead, volatilities of shocks has decreased, which contributes to low inflation. This "bad luck" view explanation argues that the instability during the Great Inflation can be attributed to the change in the volatility of the exogenous shocks. For further detail, see Sims and Zha (2006), Smets and Wouters (2007), Justiniano and Primiceri (2008).

D Additional results

D.1 SVAR evidence with identification by sign restriction

In this section, I estimate a structural vector autoregression (SVAR) model with sign restriction, and provide a 'model-free' evidence on the state-dependent effects of demand shocks. The IRFs to positive demand shocks show procyclical inflation (positive comovement between inflation and output) for 2000-2019 period, but countercyclical inflation for 1965-1999. In the next section, I will provide theoretical mechanism that predicts the state-dependent effects of demand shocks, which is determined by long-run inflation.

Here, I use the term 'demand shocks' to refer to a certain type of shocks that push up IS curve. This type of shocks includes discount rate shocks, bond demand shocks (Krishnamurthy and Vissing-Jorgensen, 2012) and inflation target shocks (Cogley, Primiceri, and Sargent, 2010; Justiniano, Primiceri, and Tambalotti, 2013). [32] These shocks generate observationally equivalent impulse response functions because following a shock, both inflation and interest rate move in the same direction. But its effect on output is state-dependent and it will be shown that long-run inflation is a key to determine its expansionary effect.

To identify demand shocks separately from supply and monetary shocks, I exploit the sign restriction framework following **?**. Identification is achieved by the sign restriction on the impact effect of each shock on three macro aggregates: output, inflation and short-term interest rates. Identification of demand shocks can be achieved by imposing the assumption that the signs of the impact effect on output, inflation and short-term interest rate are the same.[33] To deal with supply shocks (eg. oil shocks in 1970s) that could cause the stagflationary outcome and monetary shocks, I explicitly identify supply shocks and monetary shocks by imposing sign restrictions that are predicted by theory.

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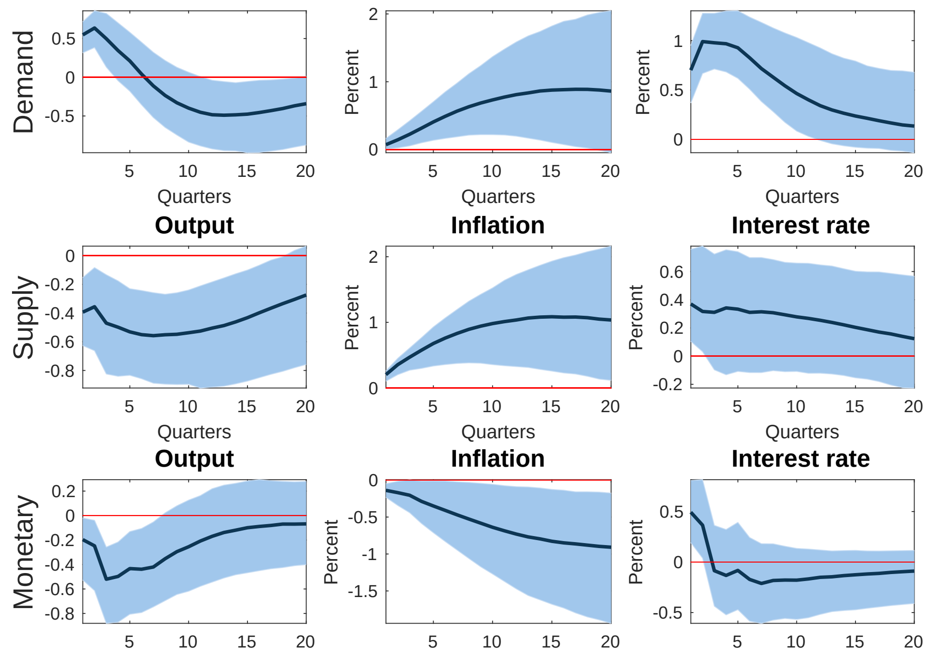
[32] Inflation target shocks are a stochastic shock to a central bank's interest rate rule (for instance, see equation (**??**)). This is a modeling device to introduce low frequency movement in inflation (Cogley, Primiceri, and Sargent, 2010) and is not a demand shock in a conventional way, but I categorize this shock into 'demand shocks' in the paper. This is because unlike conventional monetary policy shocks that generate negative comovement between inflation and interest rate, inflation target shocks induce positive comovement between inflation and interest rate, the so-called 'Neo-Fisherian' effect (**?**). If the persistence of inflation target shock is sufficiently high, then it has the Neo-Fisherian effect. When a positive inflation target shock hits, both inflation and short-term nominal rates increase, while real interest rate falls, hence a temporary violation of Taylor principle. This boosts up aggregate demand, which makes it expansionary. Note that the observed responses of output, inflation and interest rate are indistinguishable from those to more conventional demand shocks. For detailed analysis, see section 3.

[33] Theoretical prediction of a workhorse New Keynesian model when long-run inflation target (i.e. steady state inflation) is not zero will be discussed in the next section. If the persistence of inflation target shock is sufficiently high, then it increases both inflation and nominal short-term interest rates.

(a) Impulse responses to identified shocks in period I (1965-1999)



(b) Impulse responses to identified shocks in period I (2000-2019)

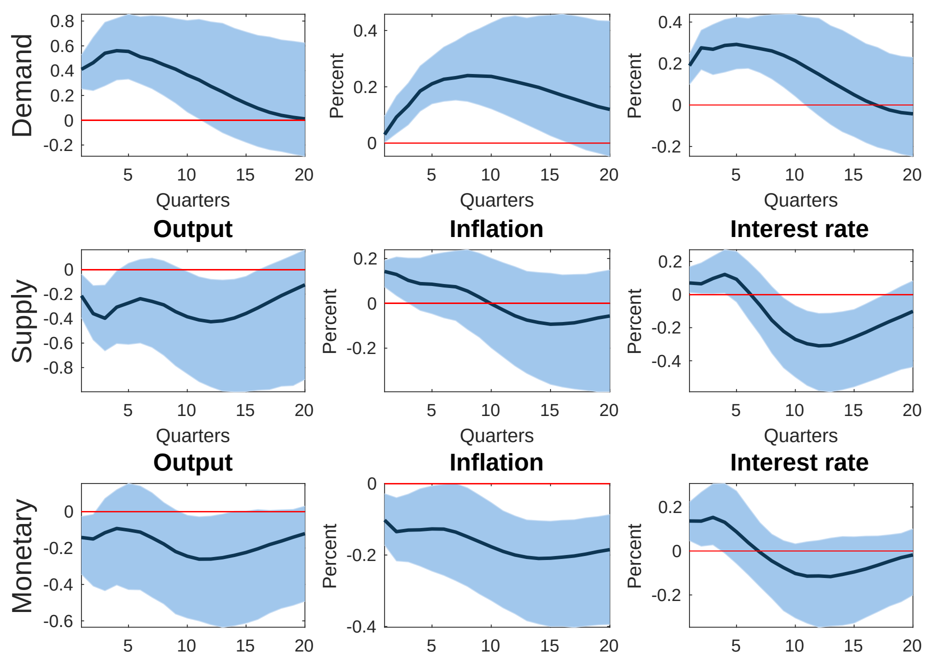


Figure 7: Impulse responses to structural shocks identified by sign restrictions

Notes: Impulse responses to standard deviation structural shocks identified by sign restrictions. Identification scheme is described in the main test. The SVAR model is estimated by OLS. The shaded area represents 90% confidence interval. GDP is log real GDP per capita. Inflation is measured using the annualized log difference in GDP deflator. Short-term interest rate is the effective federal funds rate.

Table 7: Sign restriction on structural shocks

	Demand	Supply	Monetary
Output	$+$	$-$	$-$
Inflation	$+$	$+$	$-$
Interest rate	$+$	$+$	$+$

Notes: Sign restrictions on each shock. The restrictions are only imposed on impact.

I estimate the SVAR model using quarterly US date spanning two subperiods based on a break date, from 1965Q1 to 1999Q4 and 2000Q1 to 2019Q4 with one year lag. I collected macro data (GDP, GDP deflator and the federal funds rate) from the St. Louis Fed website. I define the vector of three variables $Y_t \equiv (\log GDP_t, \log P_t, R_t)'$ where $\log GDP_t$ is log real GDP per capita, $\log P_t$ is log GDP deflator, R_t is the effective federal funds rate. For drawing random rotation matrix, I use uninformative uniform distributions following **?**.

Figure 7 displays the estimated median impulse responses of output, inflation, and nominal short-term interest rates following a one standard deviation innovation to each shock for each sample period. Panel (a) shows the IRFs in period I spanning 1965-1999. All impulse responses have the predicted sign on impact. First consider supply shocks in the second row. Supply shocks decrease output by 0.5 percent and increase inflation and interest rate by 1 percent and 0.4 percent, respectively, so that inflation dynamics exhibit countercyclical movement. Monetary policy shocks also show the predicted responses. Contractionary monetary shock drives fall in output and inflation. Responses are statistically significant at 90% confidence level. However, inconsistent with the prediction from basic economic theory, demand shocks drive countercyclical inflation. Following a demand shock, all of output, inflation and interest rate increase and those responses are statistically significant *at first*. From the second quarter after the shock hits, output starts to fall and drops below zero, and after three years, output response is -0.5% and it is statistically significant. Therefore, in a period of high long-run inflation, not only supply shocks but also demand shocks induce negative comovement between output and inflation, hence countercyclical inflation.

Panel (b) shows the same set of IRFs in period II from 2000 to 2019. Output responses for supply and monetary shocks are similar, but weaker and less significant. Inflation responses are also weaker. In particular, inflation driven by supply shocks is small and much less persistent. In period I, supply shocks drive large and persistent inflation with 1 percent increase at peak, lasting 20 quarters. But in period II, peak occurs on impact and it is less than 0.2 percent, which goes back to zero after 10 quarters. Demand shocks generate even different impulse responses. In the second period, positive demand shocks are strongly expansionary, which has 0.6 increase in output at peak and lasts 20 quarters. In contrast, similar to IRFs following supply shocks, inflation responses are weaker and less persistent. Peak effect on inflation is merely 0.2 percent, and shows faster decaying rate. Taken together, demand shocks drive procyclical inflation in period II when long-run inflation is well anchored, while other shocks have similar but weaker responses, compared to their counterparts in period I.

The key empirical finding from the SVAR results is that it is the demand shocks that have state-dependent effects across two periods. In the earlier sample, positive demand shocks are followed by fall in output, which implies countercyclical inflation. As a result, in this period, inflation is hardly procyclical. Whereas, in the recent sample period, demand shocks have expansionary effects for the entire horizon. Remember that the cyclicality of inflation has an important implication for nominal bond risks. If inflation is countercyclical, then nominal long-term bonds lose its value when marginal utility of households is high, so that it provides less incentive for households to hold nominal bonds during recession.

The finding in this section points out that the macroeconomic dynamics driven by demand shocks have changed and it can account for the countercyclical inflation before 2000. Based on this empirical finding, it is imperative to understand the economic mechanism behind the state-dependence of demand shocks. In the remaining part of the paper, I analytically show that a workhorse New Keynesian model with long-run inflation can explain this empirical finding, and more importantly, policy choice of long-run inflation explains the majority of the sign change in bond-stock correlation and inflation cyclicality.

D.2 Determinacy region of the GNK model

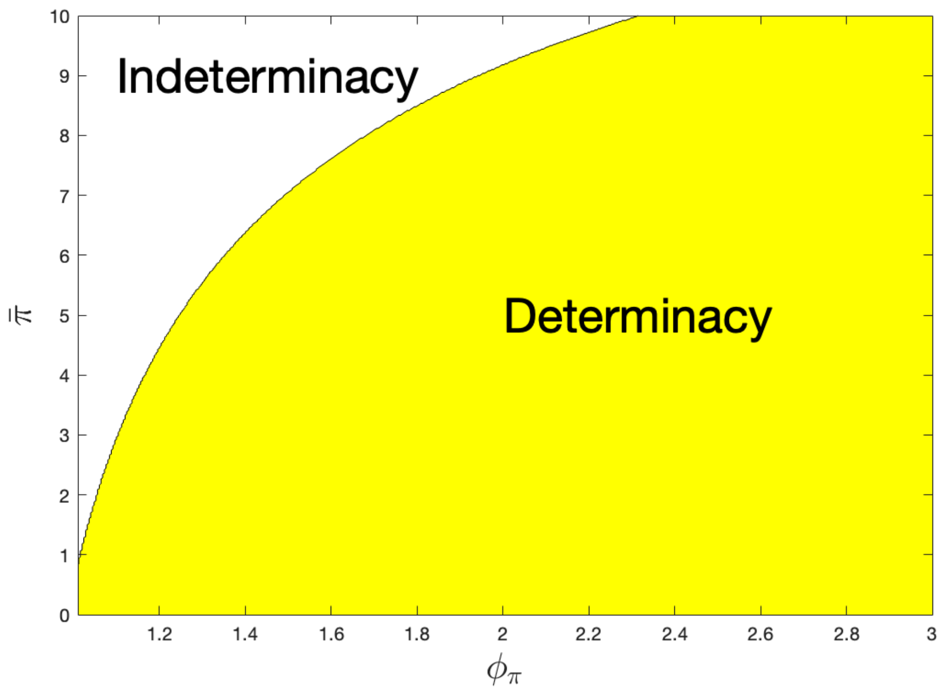


Figure 8: Determinacy region of the GNK model

Notes: This figure is based on the following standard calibration – $\beta = 0.99$, $\sigma = 1$, $\varphi = 0$, $\xi_p = 0.75$, $\epsilon = 7$, $\phi_y = 0.15$.

To get a sense of the determinacy region with $\bar{\pi} \geq 0$, figure 8 depicts the determinacy region over ϕ_π and $\bar{\pi}$ plane with a textbook calibration of the model. Similar to the case in zero trend inflation, it is more likely to achieve determinacy when ϕ_π is higher. Higher $\bar{\pi}$, however, reduces determinacy region and it is increasingly shrinking determinacy area. Consistent with Ascari and Ropele (2009) and Coibion and Gorodnichenko (2011), when $\bar{\pi}$ is above zero, the basic Taylor principle breaks down and the minimum inflation response by a central bank becomes larger.

D.3 Additional scatter plot

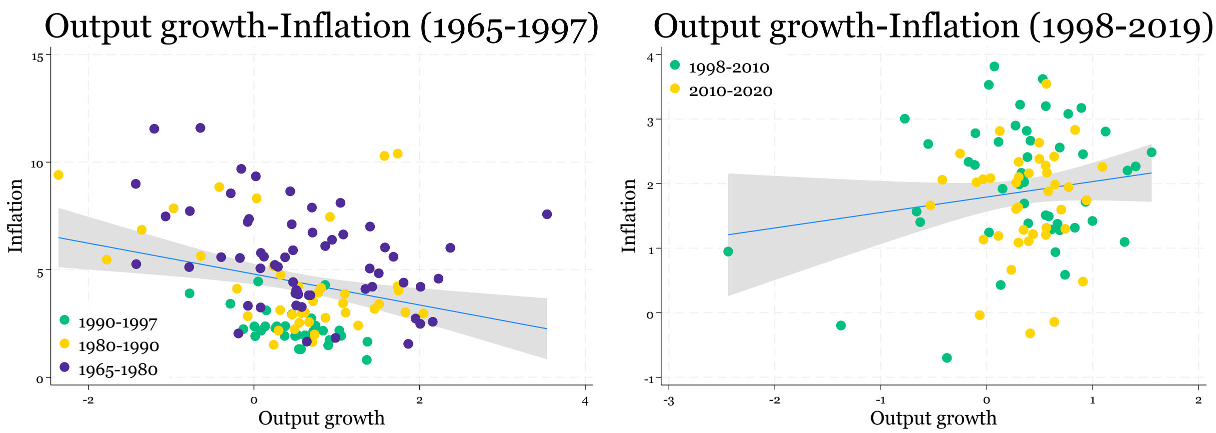


Figure 9: Output growth-inflation

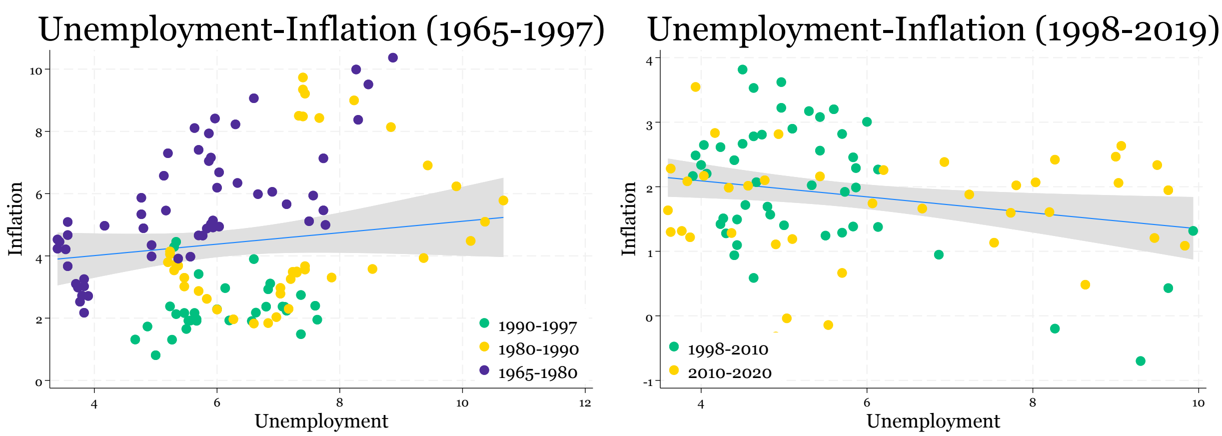


Figure 10: Unemployment-inflation

D.4 IRFs for different $\bar{\pi}$

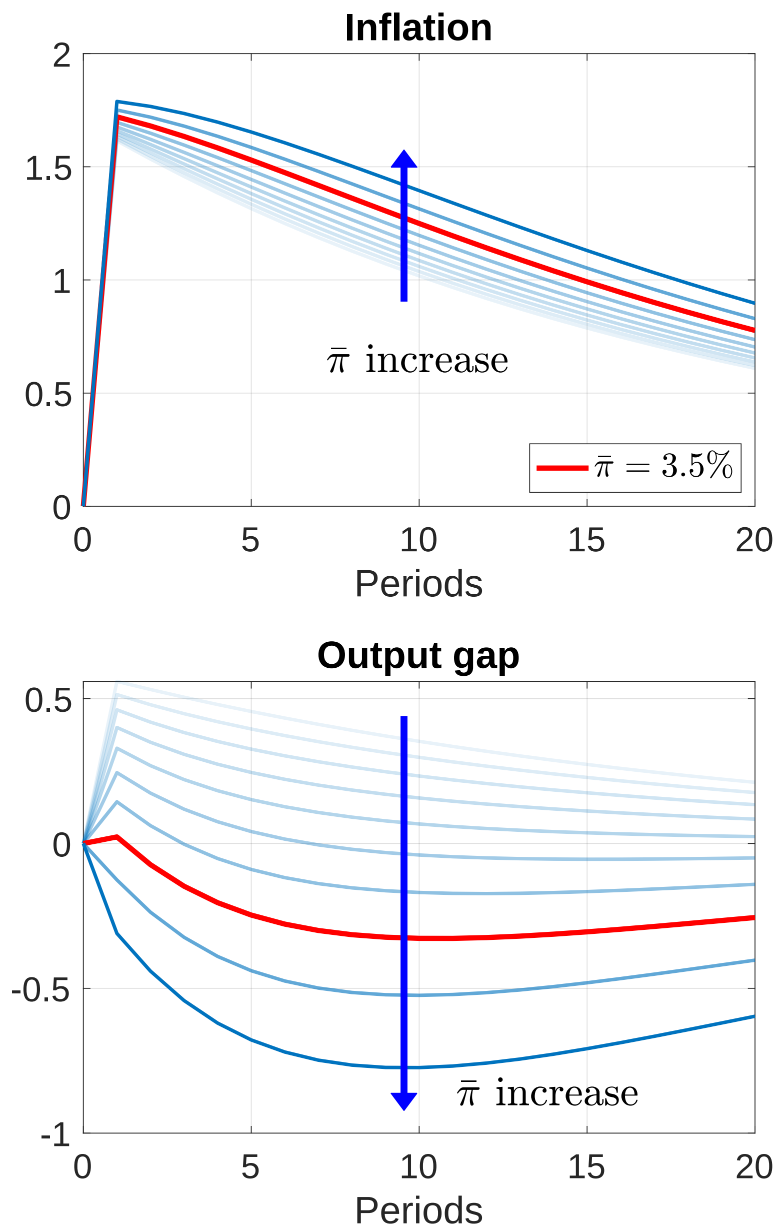


Figure 11: IRFs for different $\bar{\pi}$

D.5 IRFs for different π_ϕ

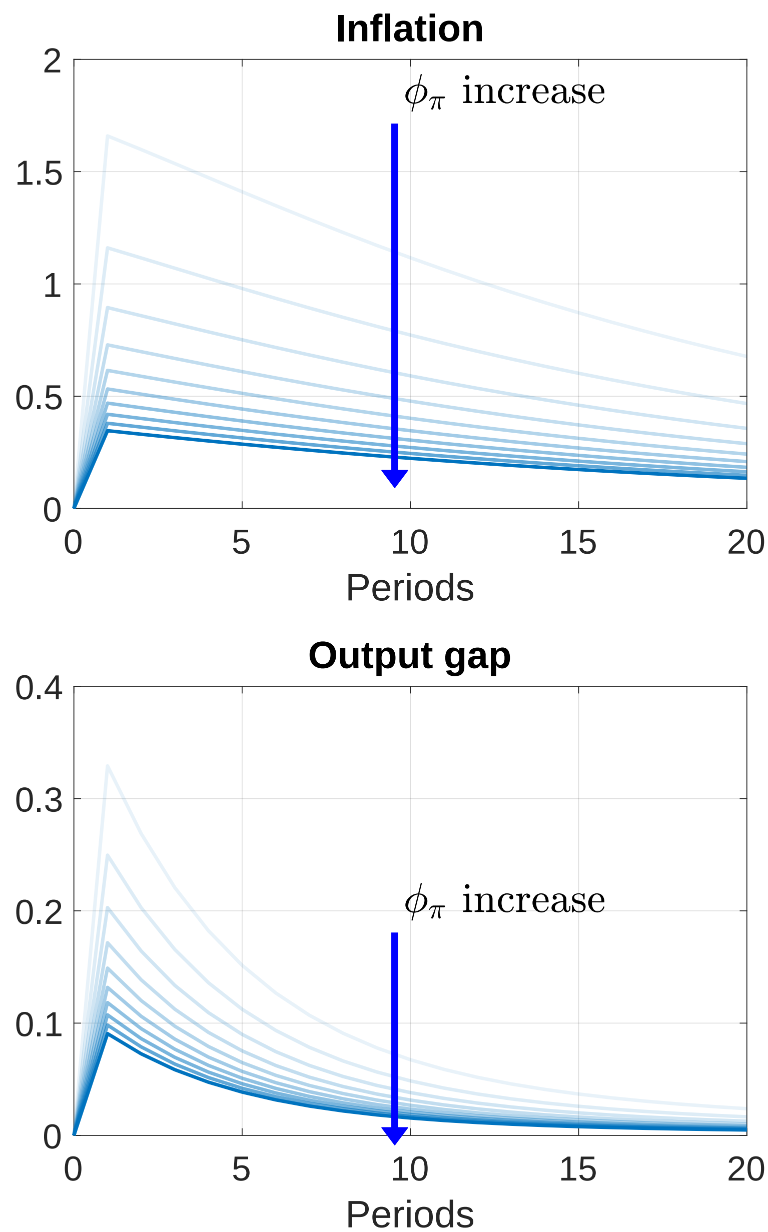


Figure 12: IRFs for different $\bar{\pi}$ when $\bar{\pi} = 2\%$

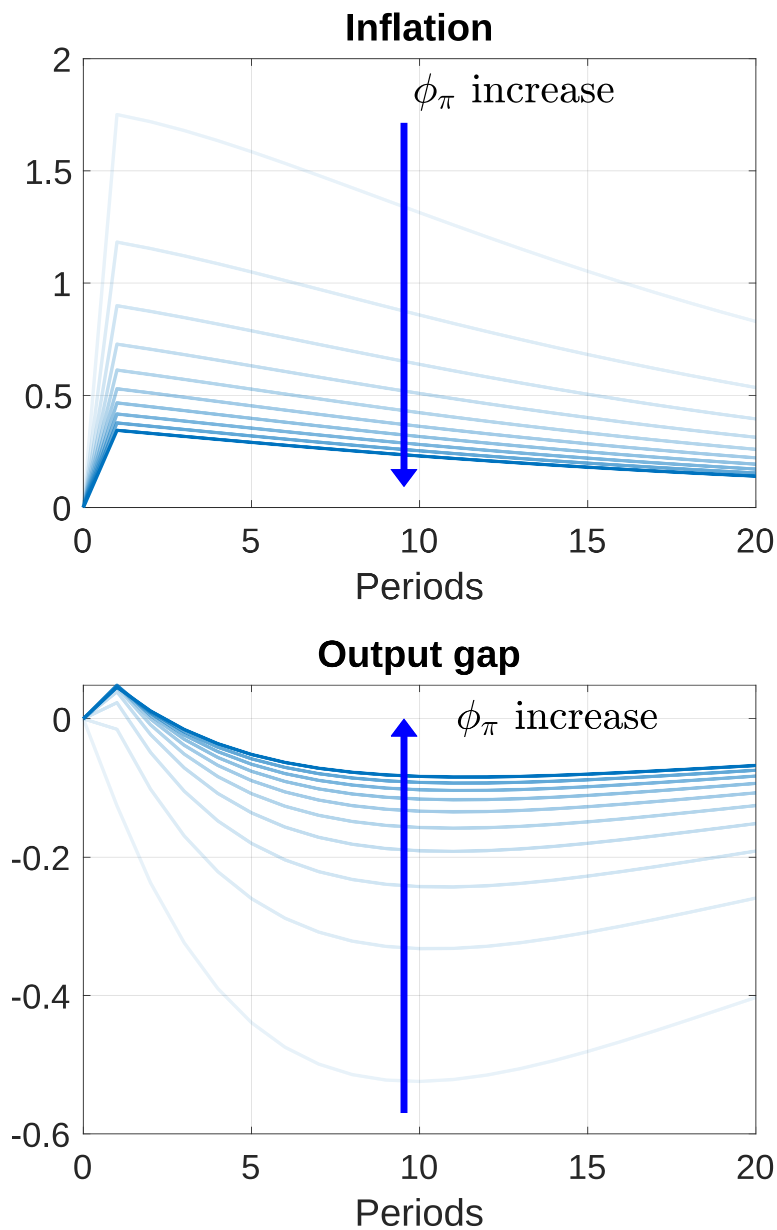


Figure 13: IRFs for different $\bar{\pi}$ when $\bar{\pi} = 4\%$

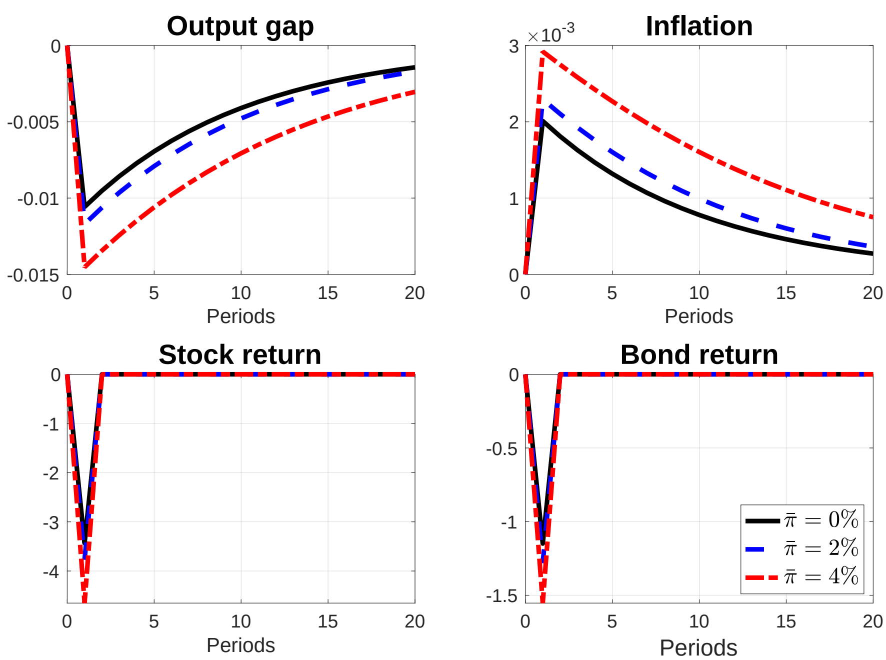


Figure 14: IRFs of output gap, inflation, real stock and bond return to a positive cost-push shock

Notes: Notes: This figure is based on the following standard calibration – $\beta = 0.995$, $\sigma = 1$, $\varphi = 1$, $\xi = 0.75$, $\epsilon = 11$, ϕ_π=1.5, $\phi_y = 0.1$, $\rho_\zeta = 0.9$.

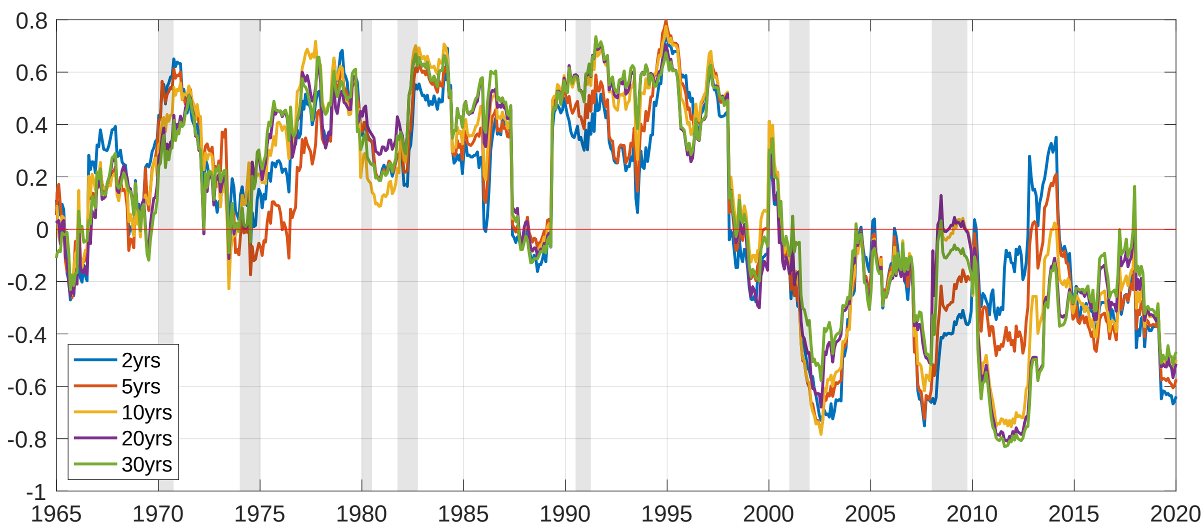


Figure 15: Bond-stock correlation for different maturity

Notes: 3-year rolling correlations and betas of bond and stock excess returns. Bond return (xr^b) is measured by one-quarter holding return of 2/5/10/20/30-year Treasury bonds excess of 3-month T-bills. Stock return (xr^s) is measured by one-quarter return from the value-weighted S&P 500 including dividends excess of 3-month T-bills. The shaded area indicates the recession date identified by NBER.

D.6 IRFs with full price indexation

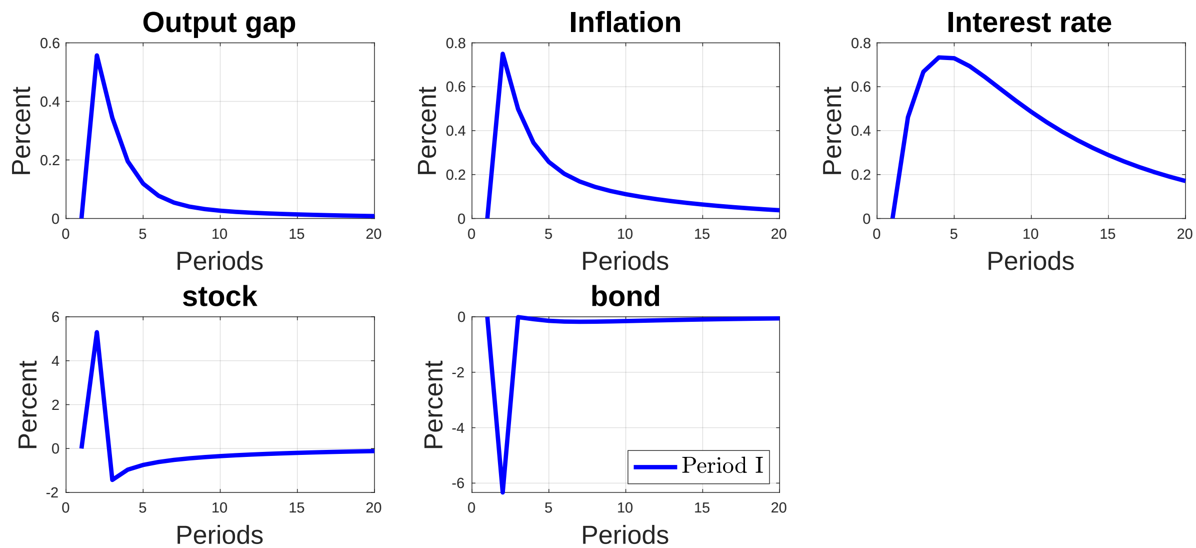


Figure 16: IRF to demand shocks with full indexation

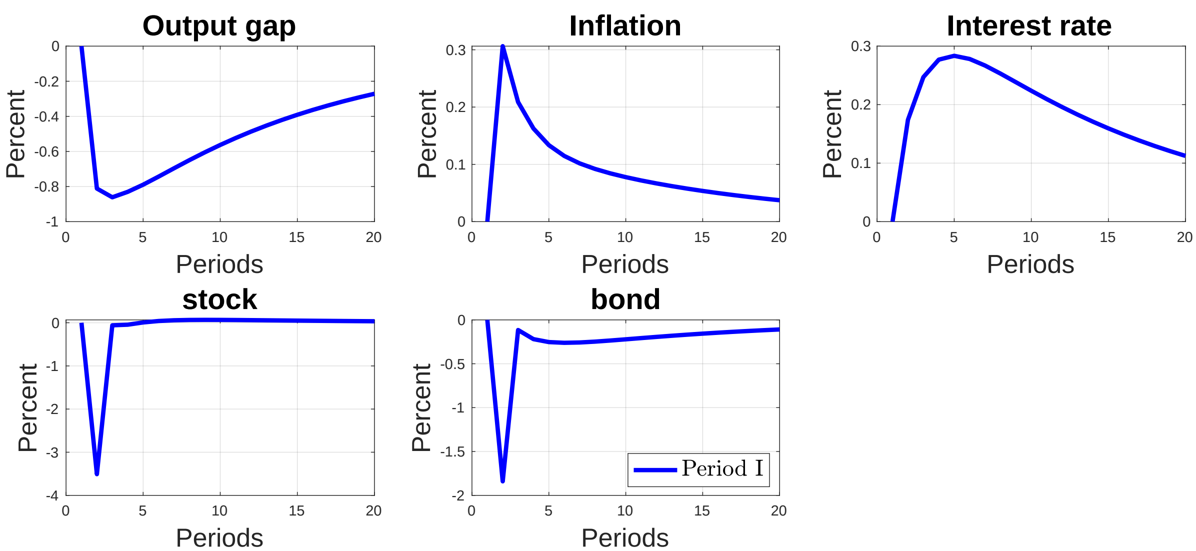


Figure 17: IRF to markup shocks with full indexation