Wed

Name	M#

S --- salt in g
C --- conc
V --- volume

$V_1(0) = 100L$
$C_1(0) = 0\ \%$
Capacity $10^3 L$

$V_2(0) = 200L$
$C_2(0) = 0\ \%$
Capacity $10^3 L$

$V_3(0) = 200L$
$C_3(0) = 0\ \%$
Capacity $10^3 L$

$10 g/L$
$2 L/m$

$4 L/m$
$6 L/m$
$1 L/m$
$3 L/m$
$2 L/m$
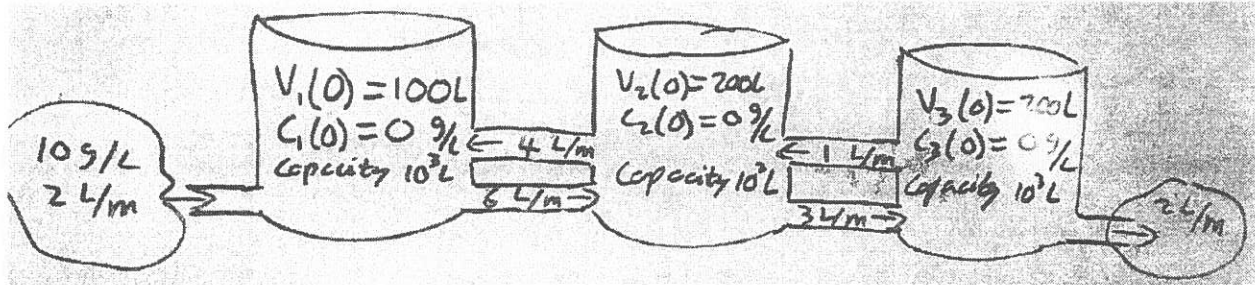
Write down the ODE for the tank system.

check Vol In = Vol out

$V_1(t) = 100$
$V_2(t) = 200$
$V_3(t) = 200$

$C_1 = S_1/100$
$C_2 = S_2/200$
$C_3 = S_3/200$

$S_1' = 10(2) - 6C_1 + 4C_2$
$S_2' = \quad 0 \quad + 6C_1 - (4+3)C_2 + 1C_3$
$S_3' = \quad 0 \quad + 0C_1 + 3C_2 - 2C_3$

$$S' = \begin{bmatrix} -6/100 & 4/200 & 0 \\ 6/100 & -7/200 & 1/200 \\ 0 & 3/200 & -2/200 \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \\ S_3 \end{bmatrix} + \begin{bmatrix} 20 \\ 0 \\ 0 \end{bmatrix}$$

3 g/L
2 L/m

$V_1(0) = 100$ L
$C_1(0) = 1$ g/L
Capacity 10^3 L

← 5 L/m

$V_2(0) = 200$ L
$C_2(0) = 2$ g/L
Capacity 10^2 L

← 2 L/m

$V_3(0) = 700$ L
$C_3(0) = 3$ g/L
Capacity 10^3 L

2 L/m

7 L/m →

4 L/m →
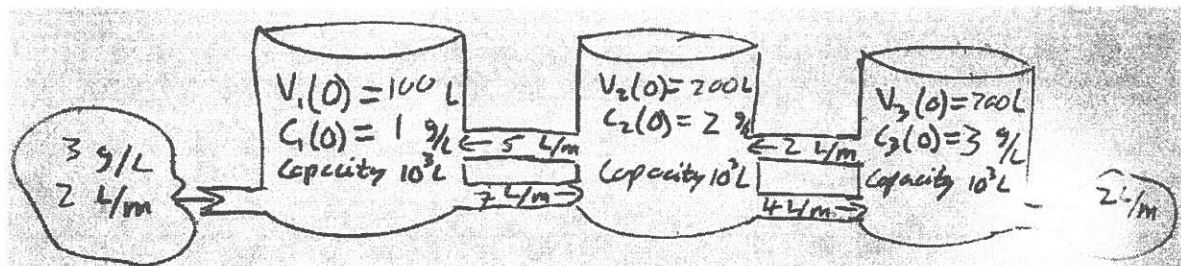
1) Write down the ODE for the tanks system in matrix for $y' = A.y + f$ and $y(0) = y_0$

$$A = \begin{pmatrix} -7/100 & 5/200 & 0 \\ 7/100 & -9/200 & 2/700 \\ 0 & 4/200 & -4/700 \end{pmatrix}, \quad f = \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix}, \text{ and } y_0 = \begin{pmatrix} 100 \\ 200 \\ 2100 \end{pmatrix}$$

$C_1 = S_1/V_1$ etc.

Volume   inflows/outflows   match

$V_1(t) = 100$ L
$V_2(t) = 200$ L
$V_3(t) = 700$ L

$S_1 \cdots$ Salt in tank$_1$
$S_2 \cdots$ " " " 2
$S_3 \cdots$ salt in tank 3

C_1 cmc in tanks
C_2
C_3

$S_1' = (3)(2) + 5C_2 - 7C_1$
$S_2' = 0 + 7C_1 - 9C_2 + 2C_3$
$S_3' = 0 + 4C_2 - 4C_3$

• rewrite in $S_{1,2,3}$
• rearrange in matrix form

$$S' = \begin{bmatrix} -7 S_1/100 & + 5 S_2/200 & + 0 S_3 & + 6 \\ 7/100 S_1 & - 9/200 S_2 & + 2/700 S_3 & + 0 \\ 0 S_1 & + 4/200 S_2 & - 4/700 S_3 & + 0 \end{bmatrix}$$

$$S' = \begin{bmatrix} -7/100 & 5/200 & 0 \\ 7/100 & -9/200 & 2/700 \\ 0 & 4/200 & -4/700 \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \\ S_3 \end{bmatrix} + \begin{bmatrix} 6 \\ 0 \\ 0 \end{bmatrix}$$

$$y_0 = \begin{bmatrix} 1 (100) \\ 2 (200) \\ 3 (700) \end{bmatrix}$$

Tank 3 has 3(700) grams of salt at t=0.