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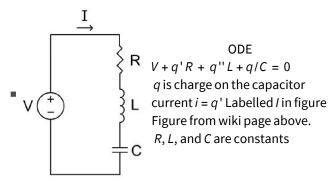
M♯

Models

- Mixing in Tanks https://xkcd.com/2974/
 - y' = Ay + f with $y(0) = y_0$ given.
 - Conservation of volume v and salt s for each tank
 - v'(t) = rate in rate out
 - s'(t) = ratein rateout
- Probes in Earth-Moon system. CR3BP https://orbital-mechanics.space/the-n-body-problem/circular-restricted-three-body-problem.html

■ With
$$M = 1/82.45$$
 and $E = 1 - M$ and $r_1(x, y) = \sqrt{(x + M)^2 + y^2}$ and $r_2 = \sqrt{(x - E)^2 + y^2}$
 $X'' = 2y' + x - \frac{E(x + M)}{r_1^3} - \frac{M(x - E)}{r_2^3}$ with $y(0), x(0), x'(0), y'(0)$ given $y'' = -2x' + y - \frac{Ey}{r_1^3} - \frac{My}{r_2^3}$

- Follows from Newton's law of gravitation in a rotating coordinate system.
- RLC circuits https://en.wikipedia.org/wiki/RLC_circuit
 - Circuit Rules: https://en.wikipedia.org/wiki/Kirchhoff%27s_circuit_laws
 - Current *i* is the rate of change of charge *q* (usually on a capacitor) through a wire.
 - Voltage drops due to resistors $(V_R = i R)$, capacitors $(V_C = \frac{q}{c})$, inductors $(V_I = L \operatorname{di}/\operatorname{dt})$, and voltage sources sum to zero around a circuit loop.
 - Currents sum to zero at a junction.



- Population models https://en.wikipedia.org/wiki/Population_model
 - Growth of population p
 - p'(t) = birth rate death rate

- Predator (wolf w) prey (moose m) https://en.wikipedia.org/wiki/Lotka%E2%80%93Volterra_equations
 - $\mathbf{w}'(t) = \text{birth rate} \text{death rate}$ (Assume wolf birth rate proportional to wolf and moose population)
 - = m'(t) = birth rate death rate (Assume moose death rate proportional to wolf and moose population)

First order ODE: Analytical Solutions

■ A <u>separable</u> scalar ODE has the form y'(t) = q(t)h(y).

You solved these in Calculus II by separating variables and integrating

$$dy/dt = g(t) h(y) \Longrightarrow \int \frac{dy}{h(y)} = \int g(t) dt + C$$

An initial condition $y(0) = y_0$ would let you solve for the one constant of integration.

- A <u>linear</u> 1st order ODE has the standard form y'(t) + p(t)y(t) = f(t). Solve by computing the IF $v(t) = e^{\int_0^t p(\tau) d\tau}$ and solve the separable ODE (v, v)' = vf.
- A 1st order ODE of the form $\frac{\partial f}{\partial t}$ dt + $\frac{\partial f}{\partial y}$ dy = 0 is <u>exact</u> with general solution f(t, y) = C.
 - M(t, y) dt + N(t, y) dy = 0 is exact if and only if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial t}$
 - To solve compute $f(t, y) = \int M(t, y) dt + h(y) \Longrightarrow h'(y) = N(t, y) \frac{\partial}{\partial y} \left[\int M(t, y) dt \right]$ then integrate h'(y) and write down the general solution f(t, y) = C
- 2.7 Substitution. Some ODEs look nicer in other variables. For example
 - 2.7.4 Bernoulli: $y' + p(t) y = q(t) y^n$. The substitution $u = y^{-n}$ gives a linear equation in u.

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