Name

M♯

- An Ordinary Differential Equation ODE is an equation containing ordinary (not partial) derivatives.
  - We need to find and plot a function $y(t)$ that solves (or approximately solves) the ODE.
  - An ODE describing a physical situation is an ODE model.
  - A constant solution of an ODE model is an equilibrium solution of the ODE.
  - Almost all science and engineering is described by ODE models. They are flexible, cheap and accurate.
  - The ODE model describes how the system changes. They need Initial Conditions IC that say how the system starts.
  - An Initial Value Problem IVP is an ODE with a suitable IC to give a single solution.
  - Almost all interesting ODEs are systems because they involve more than one unknown function.
  - An ODE is autonomous if time t does not appear explicitly.
- The order of an ODE is the highest derivative appearing.
  - ODEs describing water tank mixing are 1st order.
    - Suitable ICs are the state of the tanks at $t = 0$.
  - Most ODEs describing motion are second order. The a in $f = m\,a$ is a second derivative!
    - Suitable ICs are all positions and velocities at $t = 0$.
- An ODE is non-linear in y if there are any nonlinear terms in y and its derivatives e.g. y^3, sin(y), $y\,y'$, y_1^2, etc.
- An ODE is linear in y if there are no nonlinear terms in y and its derivatives. Note: $t^2\,y''$ is linear in y
  - Nonlinear ODE models can be approximated by linear ODE models.
  - Any first order linear ode can be written as $a_1(t)\,y' + a_0(t)\,y = b(t)$
  - Standard form is $y' + p(t)\,y = f$
  - You know how to solve the homogeneous linear ODE $y' + p(t)\,y = 0$.
- Solutions of IVPs can be plotted. Our first two examples of ODE models are
  - Mixing in Tanks https://xkcd.com/2974/
    - $y' = A\,y + f$ with $y(0) = y_0$ given.
  - Probes in Earth-Moon system. CR3BP https://orbital-mechanics.space/the-n-body-problem/circular-restricted-three-body-problem.html
    - With $M = 1/82.45$ and $E = 1 - M$ and $r_1(x, y) = \sqrt{(x+M)^2 + y^2}$ and $r_2 = \sqrt{(x-E)^2 + y^2}$

$$\begin{aligned} x'' &= 2\,y' + x - \tfrac{E(x+M)}{r_1^3} - \tfrac{M(x-E)}{r_2^3} \\ y'' &= -2\,x' + y - \tfrac{E\,y}{r_1^3} - \tfrac{M\,y}{r_2^3} \end{aligned} \quad \text{with } y(0),\ x(0),\ x'(0),\ y'(0) \text{ given}$$

- Linear algebra is the language of ODE systems.
    - You need understand what solving a linear system $A x = b$ means.
        - You need to be able to verify solutions.
        - You need to understand how to compute all solutions.
    - You need understand what $A v = \lambda v$ means.
        - You need to be able to verify partial solutions.
        - You need to be able to compute eigenvalues and eigenvectors.