Name

$M\#$

Models

- Mixing in Tanks https://xkcd.com/2974/
  - $y' = A\,y + f$ with $y(0) = y_0$ given.
  - Conservation of volume V and salt S for each tank
    - $v'(t)$ = rate in – rate out
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- Probes in Earth-Moon system. CR3BP https://orbital-mechanics.space/the-n-body-problem/circular-restricted-three-body-problem.html
  - With $M = 1/82.45$ and $E = 1 - M$ and $r_1(x, y) = \sqrt{(x+M)^2 + y^2}$ and $r_2 = \sqrt{(x-E)^2 + y^2}$
    $$\begin{aligned} x'' &= 2\,y' + x - \frac{E(x+M)}{r_1^3} - \frac{M(x-E)}{r_2^3} \\ y'' &= -2\,x' + y - \frac{E\,y}{r_1^3} - \frac{M\,y}{r_2^3} \end{aligned}$$ with $y(0)$, $x(0)$, $x'(0)$, $y'(0)$ given
- RLC circuits https://en.wikipedia.org/wiki/RLC_circuit
  - https://en.wikipedia.org/wiki/File:RLC_series_circuit_v1.svg
  - Lots of possible configurations are. We are going to learn how to think about such circuits!
  - Circuit Rules: https://en.wikipedia.org/wiki/Kirchhoff%27s_circuit_laws
    - Current i is the rate of change of charge q through a wire.
    - Voltage drops due to resistors ($V_R = i\,R$), capacitors ($V_c = \frac{q}{C}$), inductors ($V_l = L\,\mathrm{di}/\mathrm{dt}$), and voltage sources sum to zero around a circuit loop.
    - Currents sum to zero at a junction
- Population models https://en.wikipedia.org/wiki/Population_model
  - Growth of population p
    - $p'(t)$ = birth rate – death rate
  - Predator (wolf w) prey (moose m) https://en.wikipedia.org/wiki/Lotka%E2%80%93Volterra_equations
    - $w'(t)$ = birth rate – death rate (Assume wolf birth rate proportional to wolf and moose population)
    - $m'(t)$ = birth rate – death rate (Assume moose death rate proportional to wolf and moose population)

First order ODE: Analytical Solutions

- A <u>separable</u> scalar ODE has the form $y'(t) = g(t)\,h(y)$.

You solved these in Calculus II by separating variables and integrating

$$dy/dt = g(t)\,h(y) \Longrightarrow \int \frac{dy}{h(y)} = \int g(t)\,dt + C$$

An initial condition $y(0) = y_0$ would let you solve for the one constant of integration.

- A <u>linear</u> 1st order ODE has the standard form $y'(t) + p(t)\,y(t) = f(t)$. Solve by computing the IF $v(t) = e^{\int_0^t p(\tau)\,d\tau}$ and solve the separable ODE $(v\,y)' = v\,f$.
- A 1st order ODE of the form $\frac{\partial f}{\partial t}\,dt + \frac{\partial f}{\partial y}\,dy = 0$ is <u>exact</u> with general solution $f(t, y) = C$.
  - $M(t, y)\,dt + N(t, y)\,dy = 0$ is exact if and only if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial t}$
  - To solve compute $f(t, y) = \int M(t, y)\,dt + h(y) \Longrightarrow h'(y) = N(t, y) - \frac{\partial}{\partial y}\left[\int M(t, y)\,dt\right]$ then integrate $h'(y)$ and write down the general solution $f(t, y) = C$
- 2.7 Substitution. Some ODEs look nicer in other variables. For example
  - 2.7.4 Bernoulli: $y' + p(t)\,y = q(t)\,y^n$. The substitution $u = y^{-n}$ gives a linear equation in u.