

Stat 154: Elementary Statistics

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Ch 2: **Descriptive Statistics**

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Overview

1 Measures of Position

- Quantiles
- Quartiles
- Deciles
- Percentiles
- Box-Plot

Quartiles

- Three partitioning measurements when the overall distribution is divided into four equal proportions.
- The three quartiles are denoted as Q_1 , Q_2 and Q_3 .
- Q_1 (the first quartile) is the value for which 25 % of the measurements are lower or equal.
- Q_2 (the second quartile or the median) is the value for which 50 % of the measurements are lower or equal.
- Q_3 (the third quartile) is the value for which 75 % of the measurements are lower or equal.

Quartiles

EXAMPLE 2.23 Find Q_1 , Q_2 and Q_3 of the data: 15, 13, 6, 5, 12, 50, 22, 18

STEP 1 Ordered data: 5, 6, 12, 13, 15, 18, 22, 50

STEP 2 $Q_2 = \text{Median of the whole data} = \frac{(n+1)}{2} \text{th value} = 4.5 \text{th value} = \frac{13 + 15}{2} = 14$

STEP 3 $Q_1 = \text{Median of the lower half data} = 9$

STEP 4 $Q_3 = \text{Median of the upper half data} = 20$

Figure: Quartiles

Percentiles

- The partitioning measurements when the overall distribution is divided into 100 equal proportions.
- For example, P_{17} (the seventeenth percentile) is the value for which 17 % of the measurements are lower or equal.
- It can be computed as below
 - 1 Order the data from the smallest to the largest
 - 2 Compute the locator, $L = (n + 1)p$, where n is the sample size and p is the proportion corresponding to the percentage of interest.
 - 3 Locate the L th measurement in the ordered data and interpolate whenever L is not an integer.

Percentiles

EXAMPLE 2.24

A random sample of 7 National Basketball Association (NBA) players' heights (in feet) consists of 6.52, 6.39, 6.78, 7.12, 6.23, 6.68, 6.94. Find the 25th percentile.

STEP 1 Ordered data: 6.23, 6.39, 6.52, 6.68, 6.78, 6.94, 7.12

STEP 2 $L = (n + 1)p = (7 + 1)0.25 = 2\text{nd value}$

STEP 3 $P_{25} = Q_1 = 6.39$

EXAMPLE 2.25

Find the second decile (D_2) from the NBA data given in example 2.24.

STEP 1 Ordered data: 6.23, 6.39, 6.52, 6.68, 6.78, 6.94, 7.12

STEP 2 $L = (n + 1)p = (7 + 1)0.2 = 1.6$

STEP 3 $D_2 = 6.23 + (6.39 - 6.23)0.6 = 6.326$

EXAMPLE 2.26

Find the seventeenth percentile (P_{17}) from the NBA data given in example 2.24.

STEP 1 Ordered data: 6.23, 6.39, 6.52, 6.68, 6.78, 6.94, 7.12

STEP 2 $L = (n + 1)p = (7 + 1)0.17 = 1.36$

STEP 3 $P_{17} = 6.23 + (6.39 - 6.23)0.36 = 6.2876$

Figure: Percentiles

Box-Plot

- The box-plot uses measures of positions to determine the shape of the distribution and to identify the **outlier(s)**.
- Box-plots are based on the five number summary: minimum, Q_1 , Median, Q_3 , Maximum.
- An outlier is an observation that lies outside the overall pattern of a distribution.
- All the values outside the interval $(Q_1 - 1.5 \times IQR, Q_3 + 1.5 \times IQR)$ are considered outliers.
- $IQR = \text{InterquartileRange} = Q_3 - Q_1$.

Box-Plot

EXAMPLE 2.27

To construct the box-plot for the NBA players' heights data, $Median = 6.68$, $Q_1 = 6.39$, in computing Q_3 , $L = (n + 1)p = (7 + 1)0.75 = 6$, and $Q_3 = 6.94$. $IQR =$ Interquartile Range $= Q_3 - Q_1 = 6.94 - 6.39 = 0.55$. $LF = Q_1 - 1.5 \times IQR = 6.39 - 1.5(0.55) = 5.565$ and $UF = Q_3 + 1.5 \times IQR = 6.94 + 1.5(0.55) = 7.765$. There is no data value below 5.565 or above 7.765. So, there is no outlier in the data.

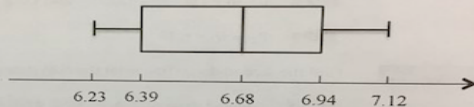


FIGURE 2.10 • Box-plot for example 2.27.

EXAMPLE 2.28

Check for outliers in the data set: 5, 13, 6, 12, 18, 15, 50, 22

STEP 1 Ordered data: 5, 6, 12, 13, 15, 18, 22, 50

STEP 2 First we find Q_1 and Q_3 . In this case, by interpolation, $Q_1 = 7.5$ and $Q_3 =$

STEP 3 Find $IQR = Q_3 - Q_1$. In this case, $IQR = 13.5$

$$(Q_1 - 1.5 \cdot IQR) = -12.75$$

$$(Q_3 + 1.5 \cdot IQR) = 41.25$$

Hence, there is only one value outside; $(-7.5, 36.5)$ is 50.

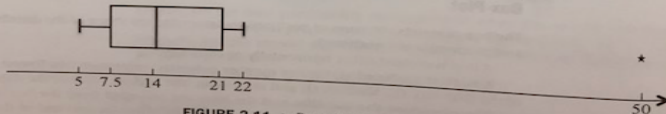


FIGURE 2.11 • Box-plot for example 2.28.

References



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Kendall Hunt