Problems 1.3

1-3. *List the binary, octal, and hexadecimal numbers from 16 to 31.

Solution 1.3 :

Dec	16	17	18	19	20	21	22	23
Bin	10000	10001	10010	10011	10100	10101	10110	10111
Oct	20	21	22	23	24	25	26	27
Hex	10	11	12	13	14	15	16	17

24	25	26	27	28	29	30	31
11000	11001	11010	11011	11100	11101	11110	11111
30	31	32	33	34	35	36	37
18	19	1A	1B	1C	1D	1E	1F

Problems 1.9

1-9. *Convert the following numbers from the given base to the other three bases listed in the table:

Solution 1.9 :

Decimal	Binary	Octal	Hexadecimal
369.3125	101110001.0101	561.24	171.5
189.625	10111101.101	275.5	BD.A
214.625	11010110.101	326.5	D6.A
62407.625	1111001111000111.101	171707.5	F3C7.A

Problems 1.12

1-12. Perform the following binary multiplications :

(a) 1010 * 1100

(b) 0110 * 1001

(c) 1111001 * 011101

Solution 1.12 :

a)

$$\begin{array}{r} 1010 \\ \times\ 1100 \\ \hline 0000 \\ 0000 \\ 1010 \\ +\ 1010 \\ \hline 1111000 \end{array}$$

b)

$$\begin{array}{r} 0110 \\ \times\ 1001 \\ \hline 0110 \\ 0000 \\ 0000 \\ +\ 0110 \\ \hline 110110 \end{array}$$

c)

$$\begin{array}{r} 1111001 \\ \times\ 011101 \\ \hline 1111001 \\ 0000000 \\ 1111001 \\ 1111001 \\ +\ 1111001 \\ \hline 110110110101 \end{array}$$

1-13. +Division is composed of multiplications and subtractions. Perform the binary division 1010110 , 101 to obtain a quotient and remainder.

Solution 1.13 :

$$
\begin{array}{cccc|ccccccc}
 & & & & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
\cline{5-11}
1 & 0 & 1 & & 1 & 0 & 1 & 0 & 1 & 1 & 0 \\
 & & & & 1 & 0 & 1 & & & & \\
\cline{5-7}
 & & & & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\
 & & & & & & & & 1 & 0 & 1 \\
\cline{8-11}
 & & & & & & & & 0 & 0 & 1
\end{array}
$$

Problems 1.16

1-16. *In each of the following cases, determine the radix r:

(a) $(BEE)_{\boldsymbol{r}} = (2699)_{10}$

(b) $(365)_{\boldsymbol{r}} = (194)_{10}$

Solution 1.16：

(a) for $(BEE)r = (2699)_{10}$

get: $11 \times r^2 + 14 \times r^1 + 14 \times r^0 = 2699$

and $11 \times r^2 + 14 \times r - 2685 = 0$

square root： $r = 15$ or $r \approx -16.27$

Deserve： $r = 15$

(b) for $(365)r = (194)_{10}$

get $3 \times \boldsymbol{r}^2 + 6 \times r^1 + 5 \times r^0 = 194$

and $3 \times \boldsymbol{r}^2 + 6 \times r - 189 = 0$

square root： $r = -9$ or $r = 7$

choose: $r = 7$

Problems 1.18

1-18. *Find the binary representations for each of the following BCD numbers:

(a) 0100 1000 0110 0111

(b) 0011 0111 1000.0111 0101

Solution 1.18 :

a) $(0100\ 1000\ 0110\ 0111)_{BCD} = (4867)_{10} = (1001100000011)_2$

b) $(0011\ 0111\ 1000.0111\ 0101)_{BCD} = (378.75)_{10} = (101111010.11)_2$

Problems 1.19

1-19. *Represent the decimal numbers 715 and 354 in BCD.

Solution 1.19 :

$$(715)_{10} = (0111\ 0001\ 0101)_{BCD}$$

$$(354)_{10} = (0011\ 0101\ 0100)_{BCD}$$

Problems 1.28

1-28. Using the procedure given in Section 1-7, find the hexadecimal Gray code.

Solution 1.28 :

Gray Code for Hexadecimal Digits

Hex	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
Gray	0000	0001	0011	0010	0110	0111	0101	0100	1100	1101	1111	1110	1010	1011	1001	1000

$$\begin{array}{r} 0\ x_3\,x_2\,x_1\,x_0 \\ \oplus\ x_3\,x_2\,x_1\,x_0 \\ \hline G_3G_2G_1G_0 \end{array}$$

2-1. *Demonstrate by means of truth tables the validity of the following identities:

(a) DeMorgan's theorem for three variables: $\overline{XYZ} = \overline{X} + \overline{Y} + \overline{Z}$

……

Solution 2.1 :

(a)

X	Y	Z	XYZ	$\overline{XYZ}$	$\overline{X}+\overline{Y}+\overline{Z}$
0	0	0	0	1	1
0	0	1	0	1	1
0	1	0	0	1	1
0	1	1	0	1	1
1	0	0	0	1	1
1	0	1	0	1	1
1	1	0	0	1	1
1	1	1	1	0	0

Problems 2.2

2-2. *Prove the identity of each of the following Boolean equations, using algebraic manipulation:

(a) $\overline{X}\,\overline{Y} + \overline{X}Y + XY = \overline{X} + Y$

(b) $\overline{A}B + \overline{B}\,\overline{C} + AB + \overline{B}C = 1$

(c) $Y + \overline{X}Z + X\overline{Y} = X + Y + Z$

(d) $\overline{X}\,\overline{Y} + \overline{Y}Z + XZ + XY + Y\overline{Z} = \overline{X}\,\overline{Y} + XZ + Y\overline{Z}$

Solution 2.2 :

(a)

$$\overline{X}\,\overline{Y} + \overline{X}Y + XY = (\overline{X}\,\overline{Y} + \overline{X}Y) + (\overline{X}Y + XY)$$
$$= \overline{X}(\overline{Y} + Y) + Y(\overline{X} + X)$$
$$= \overline{X} + Y$$

(b)

$$Y + \overline{X}Z + X\overline{Y} = Y + X\overline{Y} + \overline{X}Z$$
$$= (Y + X)(Y + \overline{Y}) + \overline{X}Z$$
$$= Y + X + \overline{X}Z$$
$$= Y + (X + \overline{X})(X + Z)$$
$$= X + Y + Z$$

Problems 2.3

2-3. +Prove the identity of each of the following Boolean equations, using algebraic manipulation:

(a) $AB\overline{C} + B\overline{C}\,\overline{D} + BC + \overline{C}D = B + \overline{C}D$

(b) $WY + \overline{W}Y\overline{Z} + WXZ + \overline{W}X\overline{Y} = WY + \overline{W}X\overline{Z} + \overline{X}Y\overline{Z} + X\overline{Y}Z$

(c) $A\overline{D} + \overline{A}B + \overline{C}D + \overline{B}C = (\overline{A} + \overline{B} + \overline{C} + \overline{D})(A + B + C + D)$

Solution 2.3 :

(a)

$$
\begin{aligned}
& AB\overline{C} + B\overline{CD} + BC + \overline{C}D \\
&= AB\overline{C} + B(\overline{C} + \overline{D}) + BC + \overline{C}D \\
&= AB\overline{C} + B\overline{C} + B\overline{D} + BC + \overline{C}D \\
&= AB\overline{C} + B(\overline{C} + C) + B\overline{D} + \overline{C}D \\
&= AB\overline{C} + B + B\overline{D} + \overline{C}D \\
&= B(1 + A\overline{C} + \overline{D}) + \overline{C}D \\
&= B + \overline{C}D
\end{aligned}
$$

(c)

$$
\begin{aligned}
& A\overline{D} + \overline{A}B + \overline{C}D + \overline{B}C \\
&= \overline{\overline{A\overline{D} + \overline{A}B + \overline{C}D + \overline{B}C}} \\
&= \overline{(\overline{A} + D)(C + \overline{D})(A + \overline{B})\overline{B}C} \\
&= \overline{(\overline{A}C + \overline{A}\,\overline{D} + CD)(B + \overline{C})(A + \overline{B})} \\
&= \overline{(\overline{A}BC + \overline{A}B\overline{D} + BCD + \overline{A}\,\overline{C}\,\overline{D})(A + \overline{B})} \\
&= \overline{ABCD + \overline{A}B\overline{C}\,\overline{D}} \\
&= (\overline{A} + \overline{B} + \overline{C} + \overline{D})(A + B + C + D)
\end{aligned}
$$

Problems 2.6

2-6. Simplify the following Boolean expressions to expressions containing a minimum number of literals:

(a) $\bar{A}\bar{C} + \bar{A}BC + \bar{B}C$

(b) $(\overline{A + B + C}) \cdot \overline{ABC}$

(c) $AB\bar{C} + AC$

(d) $\bar{A}\bar{B}D + \bar{A}\bar{C}D + BD$

(e) $(A + B)(A + C)(A\bar{B}C)$

Solution 2.6 :

$$b) \qquad (\overline{A+B+C}) \bullet \overline{ABC}$$

$$= \bar{A}\,\bar{B}\,\bar{C} \bullet \overline{ABC}$$

$$= \bar{A}\,\bar{B}\,\bar{C} \bullet (\bar{A} + \bar{B} + \bar{C})$$

$$= \bar{A}\,\bar{B}\,\bar{C}$$

$$d) \qquad \bar{A}\bar{B}D + \bar{A}\bar{C}D + BD \quad = D(\bar{A}\bar{B} + B) + \bar{A}\bar{C}D$$

$$= \bar{A}D + DB + \bar{A}\bar{C}D \quad = \bar{A}D(1 + \bar{C}) + DB$$

$$= \bar{A}D + DB \quad = D(\bar{A} + B)$$

Problems 2.10

2-10. *Obtain the truth table of the following functions, and express each function in sum-of-minterms and product-of-maxterms form:

(a) $(XY + Z)(Y + XZ)$

(b) $(\overline{A} + B)(\overline{B} + C)$

(c) $WX\overline{Y} + WX\overline{Z} + WXZ + Y\overline{Z}$

Solution 2.10 :

(a)

XYZ	F
000	0
001	0
010	0
011	1
100	0
101	1
110	1
111	1

$$F = (XY + Z)(Y + XZ)$$

$$= (X + Z)(Y + Z)(Y + X)(Y + Z)$$

$$= (X + Z) + Y\overline{Y})(Y + Z + X\overline{X})(Y + X + Z\overline{Z})$$

$$= (X + Y + Z)(X + Z + \overline{Y})(Y + Z + X)$$

$$(Y + Z + \overline{X})(Y + X + Z)(Y + X + \overline{Z})$$

POM $$= (X + Y + Z)(X + \overline{Y} + Z)(\overline{X} + Y + Z)(X + Y + \overline{Z})$$

SOM $$= \overline{X}YZ + X\overline{Y}Z + XY\overline{Z} + XYZ$$

Solution 2.10 :

(c)

XYZ	F
0000	0
0001	0
0010	1
0011	0
0100	0
0101	0
0110	1
0111	0
1000	0
1001	0
1010	1
1011	0
1100	1
1101	1
1110	1
1111	1

SOM

$$\boxed{\overline{W}\,\overline{X}Y\overline{Z} + \overline{W}XY\overline{Z} + W\overline{X}Y\overline{Z} + WX\overline{Y}\,\overline{Z} + WX\overline{Y}Z + WXY\overline{Z} + WXYZ}$$

POM

$$\boxed{\begin{array}{l}(W + X + Y + Z)(W + X + Y + \overline{Z})(W + X + \overline{Y} + \overline{Z}) \\ (W + \overline{X} + Y + Z)(W + \overline{X} + Y + \overline{Z})(W + \overline{X} + \overline{Y} + \overline{Z}) \\ (\overline{W} + X + Y + Z)(\overline{W} + X + Y + \overline{Z})(\overline{W} + X + \overline{Y} + \overline{Z})\end{array}}$$

Problems 2.11

2-11. For the Boolean functions E and F, as given in the following truth table:

(a) List the minterms and maxterms of each function.

(c) List the minterms of E + F and E • F.

(d) Express E and F in sum-of-minterms algebraic form.

XYZ	E	F
0 0 0	0	1
0 0 1	1	0
0 1 0	1	1
0 1 1	0	0
1 0 0	1	1
1 0 1	0	0
1 1 0	1	0
1 1 1	0	1

Solution 2.11 :

$$a) \quad E = \sum m(1,2,4,6) = \prod M(0,3,5,7) \qquad F = \sum m(0,2,4,7) = \prod M(1,3,5,6)$$

$$c) \quad E + F = \sum m(0,1,2,4,6,7) \qquad E \bullet F = \sum m(2,4)$$

$$d) \quad E = \overline{X}\,\overline{Y}Z + \overline{X}Y\overline{Z} + X\overline{Y}\,\overline{Z} + XY\overline{Z}$$

$$= \overline{X}\,\overline{Y}Z + X\overline{Z} + Y\overline{Z}$$

$$F = \overline{X}\,\overline{Y}\,\overline{Z} + \overline{X}Y\overline{Z} + X\overline{Y}\,\overline{Z} + XYZ$$

$$= \overline{Y}\,\overline{Z} + \overline{X}\,\overline{Z} + XYZ$$

Problems 2.12

2-12. *Convert the following expressions into sum-of-products and product-of-sums forms:

(b) $\overline{X} + X(X + \overline{Y})(Y + \overline{Z})$

Solution 2.11 :

b) $\overline{X} + X(X + \overline{Y})(Y + \overline{Z}) = (\overline{X} + X)(\overline{X} + (X + \overline{Y})(Y + \overline{Z}))$

$= \boxed{(\overline{X} + X + \overline{Y})(\overline{X} + Y + \overline{Z}) \quad \text{p.o.s.}}$

$= \boxed{(1 + \overline{Y})(\overline{X} + Y + \overline{Z}) = \overline{X} + Y + \overline{Z} \quad \text{s.o.p.}}$

Problems 2.15

2-15. *Optimize the following Boolean expressions using a map:

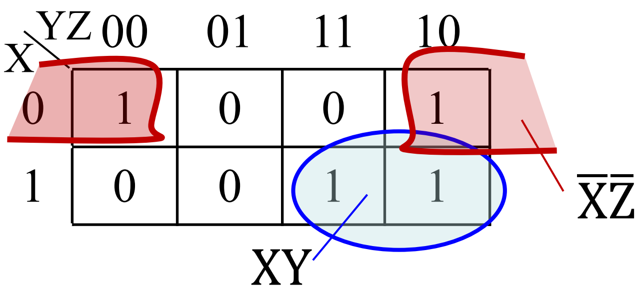
(a) $\overline{X}\,\overline{Z} + Y\overline{Z} + XYZ$

(b) $\overline{A}B + \overline{B}C + \overline{A}\,\overline{B}\,\overline{C}$

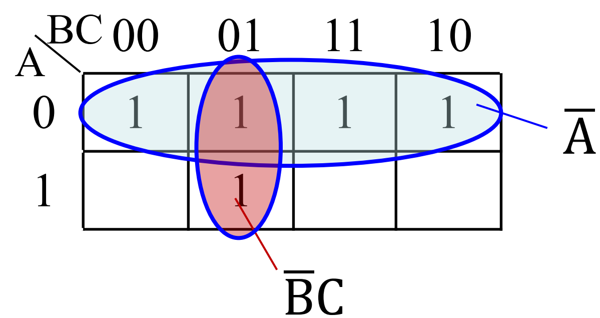
(c) $\overline{A}\,\overline{B} + A\overline{C} + \overline{B}C + \overline{A}B\overline{C}$

Solution 2.15 :

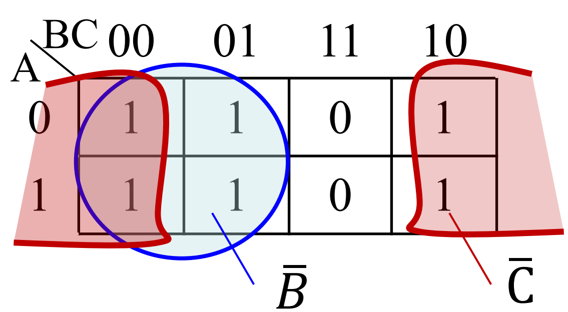
(a)



(b)



(c)



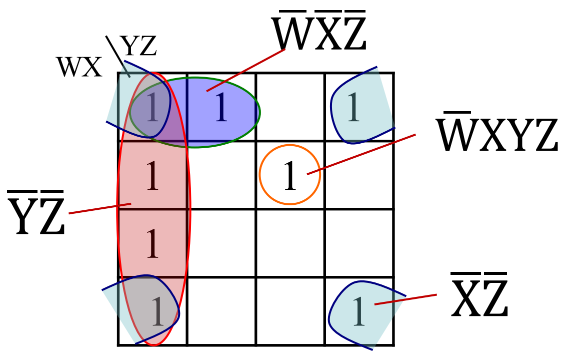
2-17. Optimize the following Boolean functions, using a map:

(a) F(W, X, Y, Z) = Σm(0, 1, 2, 4, 7, 8, 10, 12)

(b) F(A, B, C, D) = Σm(1, 4, 5 , 6, 10, 11, 12, 13, 15)

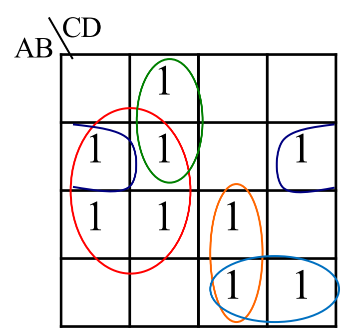
Solution 2.17 :

a)



$$F = \overline{Y}\,\overline{Z} + \overline{X}\,\overline{Z} + \overline{W}\,\overline{X}\,\overline{Z} + \overline{W}XYZ$$

b) $F = B\overline{C} + \overline{A}\,\overline{C}D + \overline{A}B\overline{D} + ACD + A\overline{B}C$



Problems 2.19

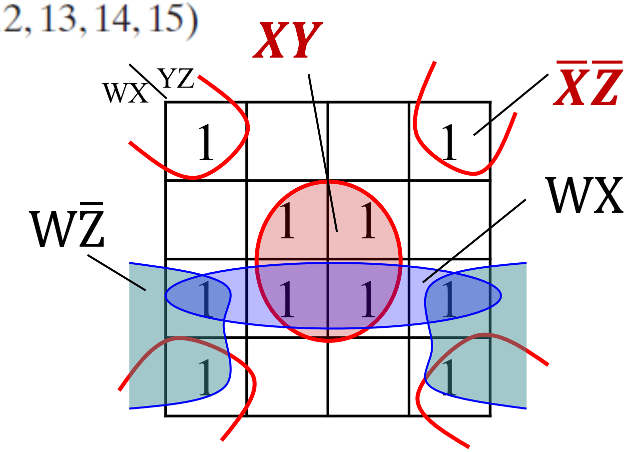
2-19. *Find all the prime implicants for the following Boolean functions, and determine which are essential:

(a) $F(W, X, Y, Z) = \Sigma m\,(0, 2, 5, 7, 8, 10, 12, 13, 14, 15)$

(b) $F(A, B, C, D) = \Sigma m\,(0, 2, 3, 5, 7, 8, 10, 11, 14, 15)$

(c) $F(A, B, C, D) = \Sigma m\,(1, 3, 4, 5, 9, 10, 11, 12, 13, 14, 15)$

Solution 2.19：



a) $Prime = XZ,\ WX,\ \bar{X}\bar{Z},\ W\bar{Z}$

$Essential = XZ,\ \bar{X}\bar{Z}$

b) $Prime = CD,\ AC,\ \bar{B}\bar{D},\ \bar{A}BD,\ \bar{B}C$

$Essential = AC,\ \bar{B}\bar{D},\ \bar{A}BD$

c) $Prime = AB,\ AC,\ AD,\ B\bar{C},\ \bar{B}D,\ \bar{C}D$

$Essential = AC,\ B\bar{C},\ \bar{B}D$

2-22. *Optimize the following expressions in (1) sum-of-products and (2) product-of-sums forms:

(a) $A\overline{C} + \overline{B}D + \overline{A}CD + ABCD$

Solution 2.22 :

(1) SOP： $A\bar{C} + CD + \bar{B}D$

$=A(\bar{C} + BCD) + \bar{B}D + \bar{A}CD$

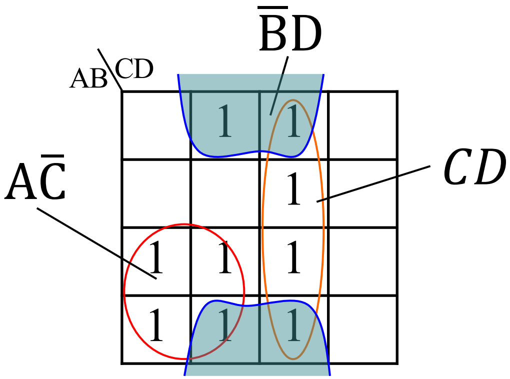
$=A\bar{C} + ABD + \bar{B}D + \bar{A}CD$

$=A\bar{C} + AD + \bar{B}D + \bar{A}CD$

$=A\bar{C} + AD + \bar{B}D + CD$

$=A\bar{C} + AD + CD + \bar{B}D$

$=A\bar{C} + CD + \bar{B}D$



(2) POS： $(\bar{C} + D)(A + D)(A + \bar{B} + C)$

Problems 2.25

2-25. *Optimize the following Boolean functions F together with the don't-care conditions d. Find all prime implicants and essential prime implicants, and apply the selection rule.

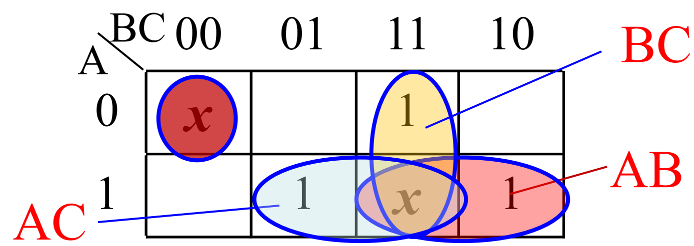
(a) F(A, B, C) = Σm (3, 5, 6), d(A, B, C) = Σm (0, 7)
(b) F(W, X, Y, Z) = Σm (0, 2, 4, 5, 8, 14, 15), d(W, X, Y, Z)= Σm (7, 10, 13)
(c) F(A, B, C, D) = Σ*m* (4, 6, 7, 8, 12, 15), d(A, B, C, D) = Σm(2, 3, 5, 10, 11, 14)

Solution 2.25 :

(a) *Primes*: $AB, AC, BC, \bar{A}\bar{B}\bar{C}$;

Essential : AB, AC, BC;

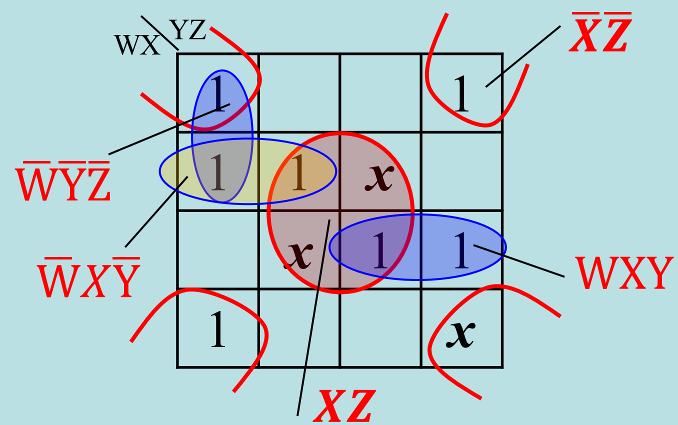
$$F = AB + AC + BC$$



(b) *Primes*: $\bar{X}\bar{Z}, XZ, \bar{W}X\bar{Y}, WXY, \bar{W}\bar{Y}\bar{Z}$

Essential : $\bar{X}\bar{Z}$

$$F = \bar{X}\bar{Z} + \bar{W}X\bar{Y} + WXY$$

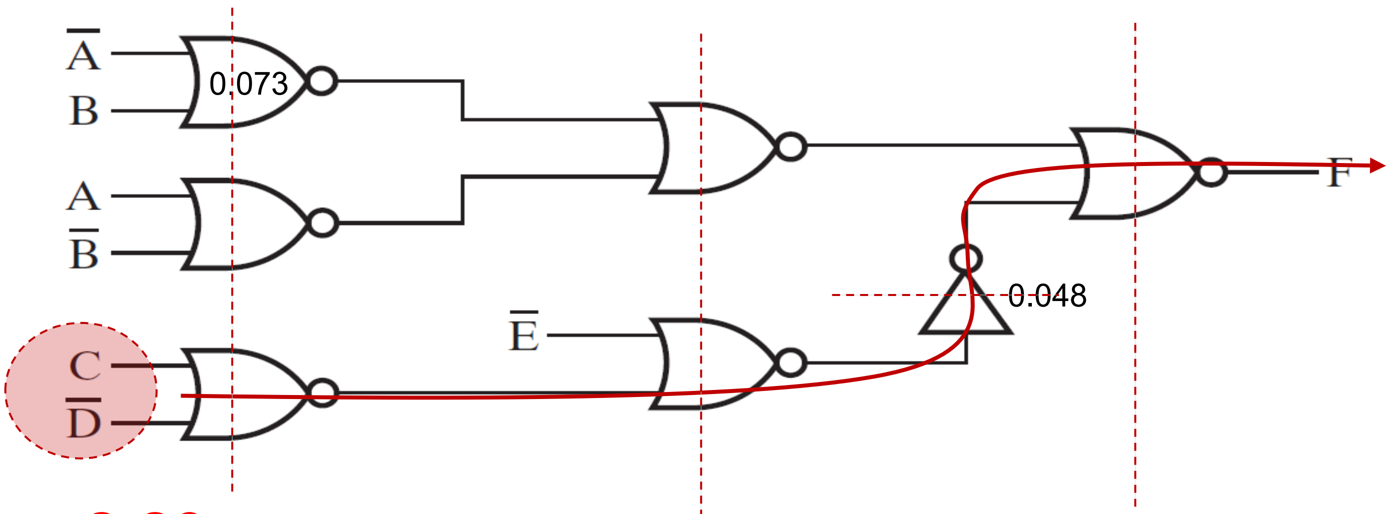


(c) *Primes*: $\bar{A}B, C, A\bar{D}, B\bar{D}$

Essential: $C, A\bar{D}$

$$F = C + A\bar{D}\ (B\bar{D} \text{ or } \bar{A}B)$$

2-29. *The NOR gates in Figure 2-39 have propagation delay t_{pd} = 0.073ns and the inverter has a propagation delay t_{pd} = 0.048ns. What is the propagation delay of the longest path through the circuit ?



Solution 2.29 :

The longest path is from input C or $\overline{D}$.

longest delay = 0.073 ns + 0.073 ns + 0.048 ns + 0.073 ns = 0.267 ns

Problems 2.30

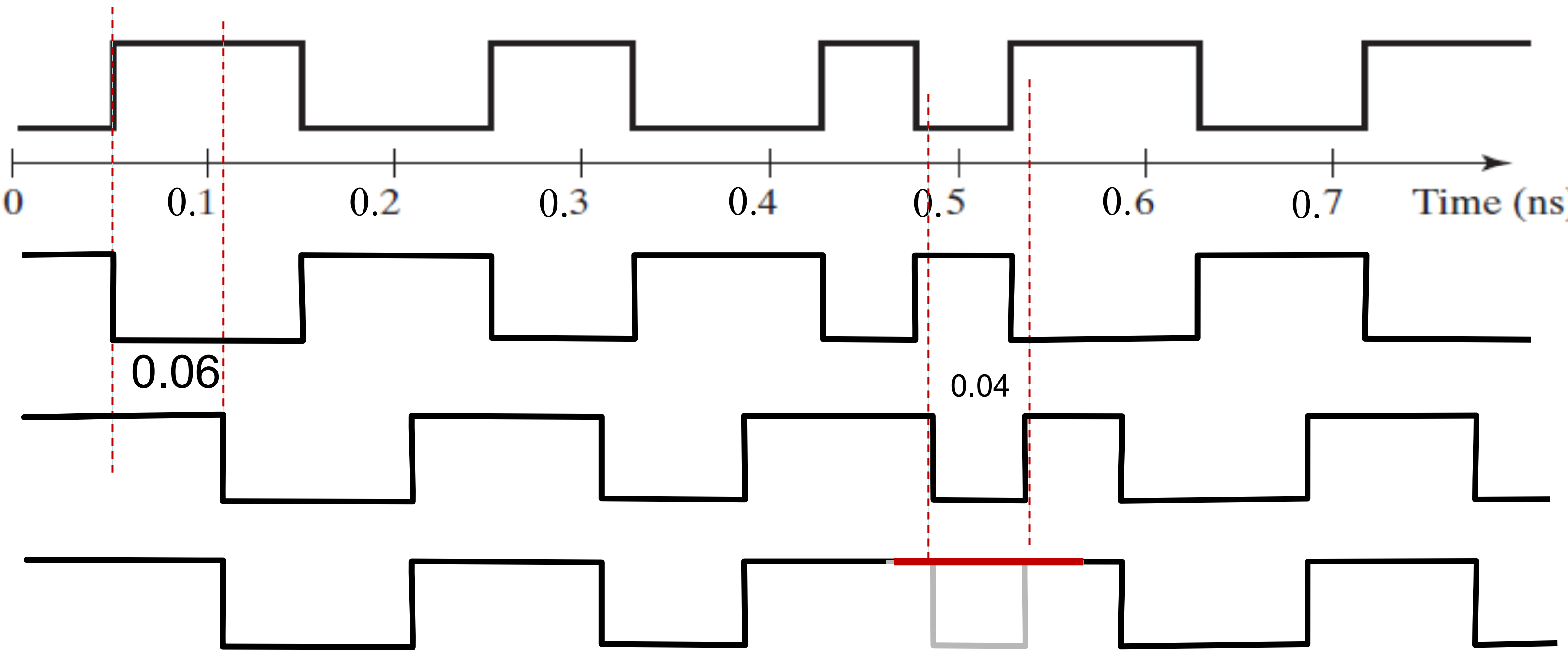
2-30. The waveform in Figure 2-40 is applied to an inverter. Find the output of the inverter, assuming that

(a) It has no delay.

(b) It has a transport delay of 0.06 ns.

(c) It has an inertial delay of 0.06 ns with a rejection time of 0.04 ns.

Solution 2.30 :



Problems 2.31

2-31. Assume that t_{pd} is the *average* of t_{PHL} and t_{PLH}. Find the delay from each input to the output in Figure 2-41 by

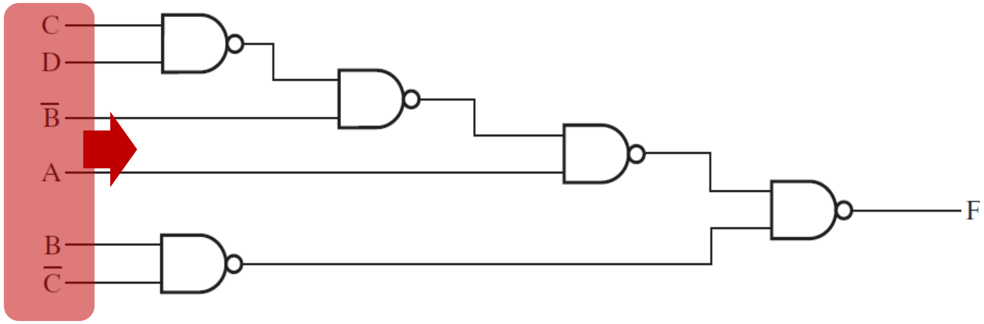
(a) Finding t_{PHL} and t_{PLH} for each path, assuming $t_{PHL} = 0.20$ ns and $t_{PLH} = 0.36$ ns for each gate. From these values, find t_{pd} for each path.

(b) Using $t_{pd} = 0.28$ ns for each gate.

(c) Compare your answers from parts (a) and (b) and discuss any differences.

Solution 2.31 :

path	(a) Delay t_{pd}	(b) Delay t_{pd}
C	1.12ns	1.12ns
D	1.12ns	1.12ns
$\overline{B}$	0.84ns	0.84ns
A	0.56ns	0.56ns
B	0.56ns	0.56ns
$\overline{C}$	0.56ns	0.56ns



(c) They are the same.