Math 741 Assignment 22 (Hand-In)

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10.4.5(H) solution: There are 8 outcomes and one unkown parameter, thus $t = 8, s = 1$.
The maximum likelihood estimate is

$$\hat{\lambda} = \frac{130 + 41 + 25 + 8 + 2 + 3 + 1 + 1}{0.5 \cdot 130 + 1.5 \cdot 41 + 2.5 \cdot 25 + 3.5 \cdot 8 + 4.5 \cdot 2 + 5.5 \cdot 3 + 6.5 \cdot 1 + 7.5 \cdot 1}$$

$$= \frac{211}{256.5} = 0.8226$$

A test can be formulated,

$$H_0: \text{ The data follows an exponential model, } f_Y(y) = 0.8226e^{-0.8226y}, y > 0$$

$$H_1: \text{ The data does not follow an exponential model, } f_Y(y) \neq 0.8226e^{-0.8226y}, y > 0$$

with $\alpha = 0.05$. A table can be formulated,

i	class	k_i	$\hat{p}_i$	$n\hat{p}_i$
1	$0-1$	130	0.5607	118.3077
2	$1-2$	41	0.2463	51.9693
3	$2-3$	25	0.1082	22.8302
4	$3-4$	8	0.0475	10.0225
5	$4-5$	2	0.0209	4.4099
6	$5-6$	3	0.0092	1.9412
7	$6-7$	1	0.00403	0.85033
8	$7-8$	1	0.00177	0.37347

where k_i is observed frequency, $\hat{p}_i$ is estimated probability of each outcomes occurs, it is calculated by $\int_{y_1}^{y_2} 0.8226e^{-0.8226y} dy$, and $n\hat{p}_i$ is estimated expected frequency.

i	class	k_i	$\hat{p}_i$	$n\hat{p}_i$
1	$0-1$	130	0.5607	118.307
2	$1-2$	41	0.2463	51.9693
3	$2-3$	25	0.1082	22.8302
4	$3-4$	8	0.0475	10.0225
5	$4-8$	7	0.0373	7.8703

with $t = 5$.

$$\chi_0^2 = \sum_{i=1}^{5} \frac{[k_1 - n\hat{p}_i]^2}{n\hat{p}_i} = 4.1816$$

$$p - value = 1 - P(0 \leq \chi^2_{t-1-s} \leq 4.1816) = 0.2425$$

since $p - value = 0.2425 > \alpha = 0.05 \implies$ Fail to reject H_0. Hence, there is enough evidence to say the data follows an exponential model.

10.4.8(H) solution: The random variables represents the pdf of $f_Y(y) = 1, 0 \leq y \leq 1$. $y_e = y_{\max} = 0.985$ and $t = 100, s = 0$. A test can be formulated,

$$H_0 : f_Y(y) = 0.985, 0 \leq y \leq 1$$

$$H_1 : f_Y(y) \neq 0.985, 0 \leq y \leq 1$$

with $\alpha = 0.05$. $E = n\hat{p} = 100 \cdot 0.985 = 98.5$.

$$\chi_0^2 = \frac{(100 - 98.5)^2}{98.5} = 0.022843$$

$$p - value = 1 - P(0 \leq \chi^2_{t-1-s} \leq 0.022843) \approx 1$$

since $p - value = 1 > \alpha = 0.05 \implies$ Fail to reject H_0. Hence, there is enough evidence to say the data represents the uniforming distribution. Since $n\hat{p}_i \geq 5$ need to be ensured, a new table need to be formulated,