# Math 741 Assignment 21 (Hand-In)

Arnold Jiadong Yu

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10.3.8(H) solution: From the information given, a test can be formulated

$$H_0: \text{ These data are compatible witht he model that}$$
$$\text{each World Series game is an independent Bernoulli trial with}$$
$$p = P(\text{AL wins}) = P(\text{NL wins}) = \frac{1}{2}$$
$$H_1: \text{ These data are not compatible witht he model that}$$
$$\text{each World Series game is an independent Bernoulli trial with}$$
$$p = P(\text{AL wins}) = P(\text{NL wins}) = \frac{1}{2}$$

with $\alpha = 0.10, t = 4, n = 50$. A table also can be formulated,

| $i$ | Number of Games | Number of Years($k_i$) | $p_i$ | $np_i$ |
|---|---|---|---|---|
| 1 | 4 | 9 | 1/8 | 152.7 |
| 2 | 5 | 11 | 1/4 | 152.7 |
| 3 | 6 | 8 | 5/16 | 152.7 |
| 4 | 7 | 22 | 5/16 | 152.7 |

where

$$p_1 = P(X = 4) = (1/2)^4 + (1/2)^4 = 1/8$$
$$p_2 = P(X = 5) = \binom{4}{1}(1/2)^5 + \binom{4}{1}(1/2)^5 = 1/4$$
$$p_3 = P(X = 6) = \binom{5}{2}(1/2)^6 + \binom{4}{1}(1/2)^6 = 5/16$$

$$p_4 = P(X=7) = \binom{6}{3}(1/2)^7 + \binom{4}{1}(1/2)^7 = 5/16$$

Therefore,

$$\chi_0^2 = \sum_{i=1}^{4} \frac{[k_i - np_i]^2}{np_i} = 7.712$$

$$p - value = 1 - P(0 \leq \chi_{t-1}^2 \leq 7.712) = 0.05235$$

since $p-value = 0.05235 < \alpha = 0.10 \implies$ Reject $H_0$. Hence, there is enough evidence to say that these data are not compatible witht he model that each World Series game is an independent Bernoulli trial with $p = P(\text{AL wins}) = P(\text{NL wins}) = \frac{1}{2}$.

10.3.11(H) solution: Given,

$$f_Y(y) = \begin{cases} \frac{1}{9}y^2 & 0 < y \leq 3 \\ 0 & o.w. \end{cases}$$

Then

$$F_Y(y) = \begin{cases} \frac{1}{27}y^3 & 0 < y \leq 3 \\ 0 & o.w. \end{cases}$$

A table can be formulated,

| $i$ | class | $k_i$ | $p_i$ | $np_i$ |
|---|---|---|---|---|
| 1 | $0 < y \leq 1$ | 8 | 1/27 | 50/27 |
| 2 | $1 < y \leq 2$ | 16 | 7/27 | 350/27 |
| 3 | $2 < y \leq 3$ | 26 | 19/27 | 950/27 |

where $k_i$ is the observed frequency, $p_i$ is the probability with $P(y_1 < Y \leq y_2) = F(y_2) - F(y_1)$, and $np_i$ is expected frequency. And $t = 3$
Since $np_1 \leq 5$, we need to pool the table. A new table need to be formulated,

| $i$ | class | $k_i$ | $p_i$ | $np_i$ |
|---|---|---|---|---|
| 1 | $0 < y \leq 2$ | 24 | 8/27 | 400/27 |
| 2 | $2 < y \leq 3$ | 26 | 19/27 | 950/27 |

A test can be formulated,

$$H_0: \text{ The data is consistent with } f_Y(y)$$

$$H_1: \text{ The data is not consistent with } f_Y(y)$$

with $\alpha = 0.05$ and $t = 2$. Then

$$\chi_0^2 = \sum_{i=1}^{2} \frac{[k_i - np_i]^2}{np_i} = 8.09263$$

$$p - value = 1 - P(0 \leq \chi_{t-1}^2 \leq 8.09263) = 0.0044446$$

since $p - value = 0.0044446 < \alpha = 0.05 \implies$ Reject $H_0$. Hence, there is enough evidence to say that the data is not consistent with $f_Y(y)$.