Math 741 Assignment 18 (Quiz)

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9.5.2. solution: Given $s_p = 11.2$, then we can assume that $\sigma_1^2 = \sigma_2^2$. We can use two-sample t test on the dataset. Enter the dataset into calculator and use 2-sample t test with 95% interval,

$$n_1 = 5, n_2 = 7$$

$$\bar{x}_1 = 83.96, \bar{x}_2 = 84.843$$

$$s_1 = 15.74462.s_2 = 6.55715, s_p = 11.1783$$

95% interval is $(-15.4667, 13.701)$.

9.5.3. solution: Assume the variances are the same, then it is same as question 9.5.2

$$n_1 = 5, n_2 = 7$$

$$\bar{x}_1 = 83.96, \bar{x}_2 = 84.843$$

$$s_1 = 15.74462.s_2 = 6.55715, s_p = 11.1783$$

99% interval is $(-21.6269, 19.8612)$.
Assume the variances are different,

$$n_1 = 5, n_2 = 7$$

$$\bar{x}_1 = 83.96, \bar{x}_2 = 84.843$$

$$s_1 = 15.74462, s_2 = 6.55715,$$

99% interval is $(-30.9772, 29.2115)$.

9.5.5. solution: Since $E(\bar{X}-\bar{Y})=\mu_X-\mu_Y$ and $\mathrm{Var}(\bar{X}-\bar{Y})=\frac{\sigma_X^2}{n}+\frac{\sigma_Y^2}{m}$, then

$$P(-z_{\alpha/2}\le \frac{(\bar{X}-\bar{Y})-(\mu_X-\mu_Y)}{\sqrt{\frac{\sigma_X^2}{n}+\frac{\sigma_Y^2}{m}}}\le z_{\alpha/2})=1-\alpha$$

$$P(-z_{\alpha/2}\cdot\sqrt{\frac{\sigma_X^2}{n}+\frac{\sigma_Y^2}{m}}\le(\bar{X}-\bar{Y})-(\mu_X-\mu_Y)\le z_{\alpha/2}\cdot\sqrt{\frac{\sigma_X^2}{n}+\frac{\sigma_Y^2}{m}})=1-\alpha$$

$$P((\bar{X}-\bar{Y})-z_{\alpha/2}\cdot\sqrt{\frac{\sigma_X^2}{n}+\frac{\sigma_Y^2}{m}}\le(\mu_X-\mu_Y)\le(\bar{X}-\bar{Y})+z_{\alpha/2}\cdot\sqrt{\frac{\sigma_X^2}{n}+\frac{\sigma_Y^2}{m}})=1-\alpha$$

Therefore, the $(1-\alpha)\%$ confidence interval is

$$((\bar{x}-\bar{y})-z_{\alpha/2}\cdot\sqrt{\frac{\sigma_X^2}{n}+\frac{\sigma_Y^2}{m}},(\bar{x}-\bar{y})+z_{\alpha/2}\cdot\sqrt{\frac{\sigma_X^2}{n}+\frac{\sigma_Y^2}{m}})$$

9.5.8. solution: Since $\frac{S_Y^2/\sigma_Y^2}{S_X^2/\sigma_X^2}\sim F(m-1,n-1)$, then

$$P(F_{\alpha/2,m-1,n-1}\le\frac{S_Y^2/\sigma_Y^2}{S_X^2/\sigma_X^2}\le F_{1-\alpha/2,m-1,n-1})=1-\alpha$$

$$P(F_{\alpha/2,m-1,n-1}\le\frac{S_Y^2}{S_X^2}\cdot\frac{\sigma_X^2}{\sigma_Y^2}\le F_{1-\alpha/2,m-1,n-1})=1-\alpha$$

$$P(F_{\alpha/2,m-1,n-1}\cdot\frac{S_X^2}{S_Y^2}\le\frac{\sigma_X^2}{\sigma_Y^2}\le F_{1-\alpha/2,m-1,n-1})\cdot\frac{S_X^2}{S_Y^2}=1-\alpha$$

Therefore, the $(1-\alpha)\%$ confidence interval is

$$(\frac{S_X^2}{S_Y^2}F_{\alpha/2,m-1,n-1},\frac{S_X^2}{S_Y^2}F_{1-\alpha/2,m-1,n-1})$$

9.5.11. solution: Since the ratio has approximately a standard normal distribution, then

$$P(-z_{\alpha/2}\le\frac{(\frac{X}{n}-\frac{Y}{m})-(p_X-p_Y)}{\sqrt{\frac{(X/n)(1-X/n)}{n}+\frac{(Y/m)(1-Y/m)}{m}}}\le z_{\alpha/2})=1-\alpha$$

$$P(-z_{\alpha/2}\sqrt{\frac{(X/n)(1-X/n)}{n}+\frac{(Y/m)(1-Y/m)}{m}} \le$$

$$(\frac{X}{n}-\frac{Y}{m})-(p_X-p_Y) \le z_{\alpha/2}\sqrt{\frac{(X/n)(1-X/n)}{n}+\frac{(Y/m)(1-Y/m)}{m}}) = 1-\alpha$$

$$P((\frac{X}{n}-\frac{Y}{m})-z_{\alpha/2}\sqrt{\frac{(X/n)(1-X/n)}{n}+\frac{(Y/m)(1-Y/m)}{m}} \le$$

$$(p_X-p_Y) \le (\frac{X}{n}-\frac{Y}{m})+z_{\alpha/2}\sqrt{\frac{(X/n)(1-X/n)}{n}+\frac{(Y/m)(1-Y/m)}{m}}) = 1-\alpha$$

Therefore, the $(1-\alpha)\%$ confidence interval is

$$\Big((\frac{X}{n}-\frac{Y}{m})-z_{\alpha/2}\sqrt{\frac{(X/n)(1-X/n)}{n}+\frac{(Y/m)(1-Y/m)}{m}},$$

$$(\frac{X}{n}-\frac{Y}{m})+z_{\alpha/2}\sqrt{\frac{(X/n)(1-X/n)}{n}+\frac{(Y/m)(1-Y/m)}{m}}\Big)$$