Math 430 Assignment 2

Arnold Jiadong Yu

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Exercise 1.3

Solution: The problem is to minimize a cost function of the form $\mathbf{c}'\mathbf{x}+f(\mathbf{d}'\mathbf{x})$, subject to the linear constraints $\mathbf{Ax} \geq \mathbf{b}$. f is a piecewise convex function defined in graph and

$$f_1(x) = -x + 1$$

$$f_2(x) = 0$$

$$f_3(x) = 2x - 4$$

Note, $\mathbf{x}, x$ are just dummy variables. Let $y = \max_{i=1,2,3} f_i(x)$, the linear programming formulation is

$$\text{minimize } \mathbf{c}'\mathbf{x} + y$$

$$\text{subject to } \mathbf{Ax} \geq \mathbf{b}$$

$$y \geq -\mathbf{d}'\mathbf{x} + 1$$

$$y \geq 0$$

$$y \geq 2\mathbf{d}'\mathbf{x} - 4$$

Therefore, the standard form is

$$\text{minimize } \mathbf{c}'\mathbf{x} + y$$

$$\text{subject to } \mathbf{Ax} \geq \mathbf{b}$$

$$y + \mathbf{d}'\mathbf{x} \geq 1$$

$$y \geq 0$$

$$y - 2\mathbf{d}'\mathbf{x} \geq -4$$

Exercise 1.10

Solution: Let x_i be the units of product which company produced in ith month, d_i be the units of product which must be delivered at the end of ith month. c_1 is inventory cost per month for each unit, and the cost of switching to a new production level is $c_2|x_{i+1} - x_i|$. Want to minimize the total production and inventory cost. Let a_i be the units of product stored in inventory in ith month.

The units in inventory adds up with the units of product produced must not be less than the units of product for delivery. Moreover, units in inventory, units of product produced, units of product for delivery should be non-negative and zero units in inventory in zero month. Therefore,

$$a_0 = 0$$

$$x_i + a_{i-1} \geq d_i \text{ for } i = 1, 2, 3, ..., 12$$

$$a_i = x_i + a_{i-1} - d_i \text{ for } i = 1, 2, 3, ..., 12$$

$$a_i \geq 0, x_i \geq 0, d_i \geq 0 \text{ for } i = 1, 2, 3, ..., 12$$

The linear programming formulation is

$$\text{minimize} \sum_{i=1}^{11} c_1 a_i + \sum_{i=1}^{11} c_2 |x_{i+1} - x_i|$$

$$\text{subject to } a_0 = 0$$

$$x_i + a_{i-1} \geq d_i \text{ for } i = 1, 2, 3, ..., 12$$

$$a_i = x_i + a_{i-1} - d_i \text{ for } i = 1, 2, 3, ..., 12$$

$$a_i \geq 0, x_i \geq 0, d_i \geq 0 \text{ for } i = 1, 2, 3, ..., 12$$

Let $y_i = |x_{i+1} - x_i|$. The standard linear programming formulation is

$$\text{minimize} \sum_{i=1}^{11} c_1 a_i + \sum_{i=1}^{11} c_2 y_i$$

$$\text{subject to } a_0 = 0$$

$$x_i + a_{i-1} \geq d_i \text{ for } i = 1, 2, 3, ..., 12$$

$$a_i = x_i + a_{i-1} - d_i \text{ for } i = 1, 2, 3, ..., 12$$

$$a_i \geq 0, x_i \geq 0, d_i \geq 0 \text{ for } i = 1, 2, 3, ..., 12$$

$$y_i \geq x_{i+1} - x_i \text{ for } i = 1, 2, 3, ..., 12$$

$$y_i \geq -x_{i+1} + x_i \text{ for } i = 1, 2, 3, ..., 12$$

Exercise 1.15

Solution: (a) Let x, y be units of first and second product produced. The profit is $9x + 8y - 1.2x - 0.9y = 7.8x + 7.1y$. The linear programming formulation is

$$\text{maximize } 7.8x + 7.1y$$

$$\text{subject to } \frac{1}{4}x + \frac{1}{3}y \leq 90$$

$$\frac{1}{8}x + \frac{1}{3}y \leq 80$$

$$x \geq 0, y \geq 0$$

(b)(i) This is easy to incorporate. Let z be the overtime assembly labor, then the profit is $7.8x + 7.1y - 7z$. The linear programming formulation will change to

$$\text{maximize } 7.8x + 7.1y - 7z$$

$$\text{subject to } \frac{1}{4}x + \frac{1}{3}y \leq 90 + z$$

$$\frac{1}{8}x + \frac{1}{3}y \leq 80$$

$$z \leq 50$$

$$x \geq 0, y \geq 0, z \geq 0$$

Now we are solving x, y, z with 6 constraints.

(ii)This is not easy to incorporate since it is not guaranteed if the profit can be maximized with discount or without discount. There also might be a trade off between. First we want to find out the solution for part(a) and check when the profit is maximized, will the cost of raw material exceed \$300? The solution for part(a) is $x = 360, y = 0$. The profit is $7.8x + 7.1y = 2808$ and the cost for raw material is $1.2x + 0.9y = 432 > 300$ Therefore, the profit can be maximized with discount. Let us formulate the profit with discount. The profit is $9x+8y-0.9\cdot(1.2x+0.9y) = 7.92x+7.19y$. The linear programming formulation is

$$\begin{aligned} \text{maximize } & 7.92x + 7.19y \\ \text{subject to } & \frac{1}{4}x + \frac{1}{3}y \leq 90 \\ & \frac{1}{8}x + \frac{1}{3}y \leq 80 \\ & 1.2x + 0.9y > 300 \\ & x \geq 0, y \geq 0 \end{aligned}$$

The maximum is when $x = 360, y = 0$. The profit is $7.92x + 7.19y = 2851.2$. Compare this profit with previous profit. We obtain that maximum profit can be produced when discount of raw material is applied.