Math 470 Assignment 28

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10.4.2 Let A, B be compact subsets of X. Prove that $A \cup B$ and $A \cap B$ are compact.

proof: Let A, B be compact subsets of X, $\mathcal{V} = \{V_\alpha\}_{\alpha \in \mathbf{N}}$ a collection subsets of X and every V_α is open in X. W.L.O.G, let $A \subset \{V_\alpha\}_{\alpha \in [1,N]}$ and $B \subset \{V_\alpha\}_{\alpha \in [N+1,M]}$ where $0 < N < M \in \mathbf{N}$. Therefore, $A \cup B = \{V_\alpha\}_{\alpha \in [1,N]} \cup \{V_\alpha\}_{\alpha \in [N+1,M]} \subset \{V_\alpha\}_{\alpha \in [1,M]}$. Hence $A \cup B$ is compact. $A \cap B \subset A \subset \{V_\alpha\}_{\alpha \in [1,N]}$. Hence $A \cap B$ is also compact.

10.4.3 Suppose that $E \subseteq \mathbf{R}$ is compact and nonempty. Prove that $\sup E, \inf E \in E$.

proof: Suppose that $E \subseteq \mathbf{R}$ is compact and nonempty. Then E is closed and bounded. E has finite supremum and finite infimum since E is bounded. Let $\sup E = x_{sup}$, then choose $x_1, x_2, ... \in E$, such that $x_n \to x_{sup}$ as $n \to \infty$. Since E is closed, then $x_{sup} \in E$. This implies $\sup E \in E$. The proof of $\inf E \in E$ is trivial.