Math 741 Assignment 16 (Hand-In)

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9.3.6.(H) solution: Let σ_1^2 and σ_2^2 are the variances of American League teams fans changes and National League teams fans changes, then the test can be formulated as following

$$H_0 : \sigma_1^2 = \sigma_2^2$$
$$H_1 : \sigma_1^2 \neq \sigma_2^2$$

let $\alpha = 0.05$. Enter the data into calculator and use 2-sample F Test, then

$$n_1 = 12, n_2 = 14$$
$$\bar{x}_1 = -12.833, \bar{x}_2 = -15.143$$
$$s_1 = 16.5685, s_2 = 19.9687$$
$$F_0 = \frac{s_1^2}{s_2^2} = 0.688444, P - value = 0.541978$$

since $P - value = 0.541978 > \alpha = 0.05 \implies$ fail to reject H_0. Therefore, they have the same variance. As a result, we can used the pooled two-sample t test.

9.3.10. (H) solution: Let $X_1, ..., X_n$ and $Y_1, ..., Y_n$ be independent random samples from normal distributions with means μ_1 and μ_2 and standard deviations σ_1 and σ_2 and unbiased standard deviations estimator S_1 and S_2. We will derive the λ of $\sigma_X^2 = \sigma_Y^2$ against $\sigma_X^2 \neq \sigma_Y^2$, since for one-sided it is just to change the subscript notation from $\alpha/2$ to α.

If $\sigma_X^2 = \sigma_Y^2 = \sigma^2$, then

$$\hat{\sigma}^2 = \frac{1}{n+m}\Big(\sum_{i=1}^{n}(x_i - \bar{x})^2 + \sum_{i=1}^{m}(y_i - \bar{y})^2\Big) = \frac{n}{\text{ffl}}$$

$$L(\Omega_0) = \prod_{i=1}^{n+m} \frac{1}{\sqrt{2\pi}\hat{\sigma}} e^{-\frac{1}{2\hat{\sigma}^2}\left(\sum_{i=1}^{n}(x_i-\bar{x})^2+\sum_{i=1}^{m}(y_i-\bar{y})^2\right)}$$

$$L(\Omega_0) = \left(\frac{1}{2\pi\hat{\sigma}^2}\right)^{(n+m)/2} e^{-\frac{n+m}{2}}$$

If $\sigma_X^2 \neq \sigma_Y^2$, then

$$\hat{\sigma_X}^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2, \hat{\sigma_Y}^2 = \frac{1}{m}\sum_{i=1}^{m}(y_i-\bar{y})^2$$

$$L(\Omega) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}\sigma_X} e^{-\frac{1}{2\sigma_X^2}\sum_{i=1}^{n}(x_i-\bar{x})^2} \cdot \prod_{i=1}^{m} \frac{1}{\sqrt{2\pi}\sigma_Y} e^{-\frac{1}{2\sigma_Y^2}\sum_{i=1}^{m}(y_i-\bar{y})^2}$$

$$L(\Omega) = \left(\frac{1}{2\pi\sigma_X}\right)^{n/2} e^{-n/2} \cdot \left(\frac{1}{2\pi\sigma_Y}\right)^{m/2} e^{-m/2}$$

Moreover,

$$\lambda = \frac{L(\Omega_0)}{L(\Omega)} = \frac{\left(\frac{1}{2\pi\hat{\sigma}^2}\right)^{(n+m)/2} e^{-\frac{n+m}{2}}}{\left(\frac{1}{2\pi\sigma_X}\right)^{n/2} e^{-n/2} \cdot \left(\frac{1}{2\pi\sigma_Y}\right)^{m/2} e^{-m/2}} = \frac{\sigma_X^{n/2}\sigma_Y^{m/2}}{\hat{\sigma}^{(n+m)/2}}$$

Therefore,

$$\lambda = \frac{(\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2)^{n/2}(\frac{1}{m}\sum_{i=1}^{m}(y_i-\bar{y})^2)^{m/2}}{(\frac{1}{n+m})^{(n+m)/2}\left(\sum_{i=1}^{n}(x_i-\bar{x})^2+\sum_{i=1}^{m}(y_i-\bar{y})^2\right)^{(n+m)/2}}$$

$$\lambda = \frac{(n+m)^{(n+m)/2}}{n^{n/2}m^{m/2}} \frac{(\sum_{i=1}^{n}(x_i-\bar{x})^2)^{n/2}(\sum_{i=1}^{m}(y_i-\bar{y})^2)^{m/2}}{\left(\sum_{i=1}^{n}(x_i-\bar{x})^2+\sum_{i=1}^{m}(y_i-\bar{y})^2\right)^{(n+m)/2}}$$

It is known that under H_0, the statistic follows:

$$F_0 = \frac{S_Y^2}{S_X^2}$$

with $(m-1, n-1)$ df.
Then the GLRT for $H_0 : \sigma_X^2 = \sigma_Y^2$ versus $H_1 : \sigma_X^2 > \sigma_Y^2$ if $\frac{S_Y^2}{S_X^2} \leq F_{\alpha,m-1,n-1}$.