Math 741 Assignment 6 (Hand-In)

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5.6.2.(H)
solution: In order to show $\hat{p}^*$ is not sufficient for p, we need to find an counter example. Assume $\hat{p}^* = X_1 + 2X_2 + 3X_3 = 3$, then

$$P(X_1 = 1, X_2 = 1, X_3 = 0 | X_1+2X_2+3X_3 = 3) = \frac{P(X_1 = 1, X_2 = 1, X_3 = 0)}{P(X_1 + 2X_2 + 3X_3 = 3)}$$

$$= \frac{P(X_1 = 1, X_2 = 1, X_3 = 0)}{P(X_1 = 1, X_2 = 1, X_3 = 0) + P(X_1 = 0, X_2 = 0, X_3 = 1)} = \frac{p^2(1-p)}{p^2(1-p) + p(1-p)^2}$$

$$= \frac{p}{p + (1-p)} = p$$

Since the conditional probability depends on p, it is not sufficient for p.
5.6.6.(H)
solution: Let $Y_1, ..., Y_n \sim f_Y(y;\theta)$ where

$$f_Y(y;\theta) = \begin{cases} \theta y^{\theta-1} & 0 \le y \le 1 \\ 0 & o.w. \end{cases}$$

Then,

$$L(\theta) = \prod_{i=1}^n \theta y_i^{\theta-1} = \theta^n \Big(\prod_{i=1}^n y_i\Big)^{\theta-1}$$

Since $W = \prod_{i=1}^n Y_i$, then

$$\theta^n \Big(\prod_{i=1}^n y_i\Big)^{\theta-1} = \theta^n (w)^{\theta-1} = [\theta^n (w)^{\theta-1}] \cdot 1$$

where $g[h(y_1, ..., y_n); \theta] = \theta^n (w)^{\theta-1}$ and $b(y_1, ..., y_n) = 1$. Hence, $W = \prod_{i=1}^n Y_i$ is sufficient statistic for θ.

$$\ln L(\theta) = n \ln \theta + (\theta - 1)(\ln y_1 + ... + \ln y_n)$$

$$\frac{\partial}{\partial \theta} \ln L(\theta) = \frac{n}{\theta} + \sum_{i=1}^n \ln y_i$$

Let $\frac{\partial}{\partial \theta} \ln L(\theta) = 0$, then

$$\frac{n}{\hat{\theta}} + \sum_{i=1}^n \ln y_i = 0 \implies \hat{\theta} = -\frac{n}{\ln \prod_{i=1}^n y_i} = -\frac{n}{\ln w}$$

Therefore, the maximum likelihood estimator of θ is a function of W.