Math 470 Assignment 25

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10.3.1. Find the interior, closure, and boundary of each of the following subsets of $\mathbf{R}$.

a) $E = \{1/n : n \in \mathbf{N}\}$

proof: E is a sequences of points apporching 0 from 1 by $\frac{1}{n}$. Every points are isolated, then $E^o = \emptyset$, $\overline{E} = E \cup \{0\}$ and $\partial E = E \cup \{0\}$

b)$E = \bigcup_{n=1}^{\infty} \left(\frac{1}{n+1}, \frac{1}{n}\right)$

proof: $E = (\frac{1}{2}, 1) \cup (\frac{1}{3}, \frac{1}{2}) \cup ....$ For every open interval $\left(\frac{1}{n+1}, \frac{1}{n}\right)$ is open as $n \to \infty$. Then the $E^o = E$. $\overline{E} = [0, 1]$ and $\partial E = \{1/n : n \in \mathbf{N}\} \cup \{0\}$.

c)$E = \bigcup(-n, n)$

proof: When $n \to \infty$, $E = (-\infty, \infty)$. Hence $E^o = \overline{E} = \mathbf{R}$ and $\partial E = \emptyset$.

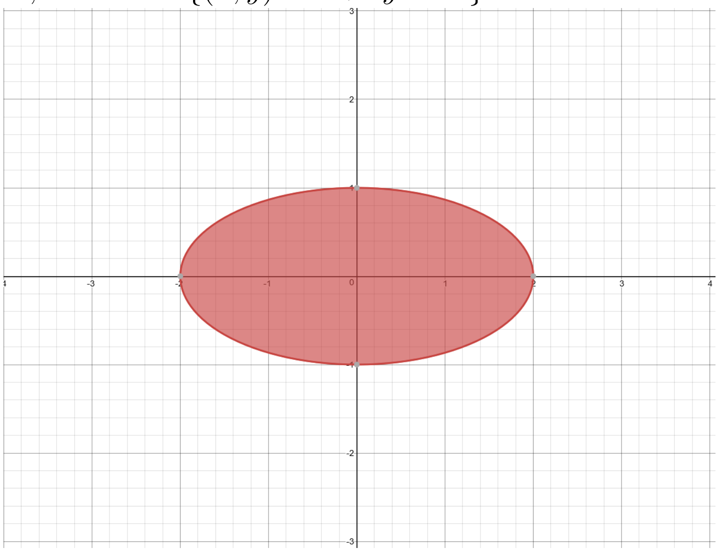
d)$E = \mathbf{Q}$

proof: By definition of rationals. Every points are isolated. Then $E^o = \emptyset$, $\overline{E} = \mathbf{R}$ and $\partial E = \mathbf{R}$.

10.3.2. Identify which of the following sets are open, which are closed, and which are neither. Find E^o, $\overline{E}$, and ∂E and sketch E in each case.

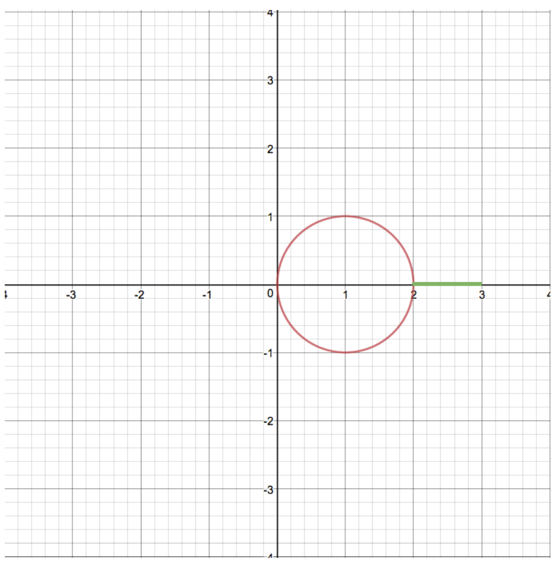
a)$E = \{(x, y) : x^2 + 4y^2 \leq 1\}$

proof: $E^c = \{(x, y) : x^2 + 4y^2 > 1\}$ which is open. Then E is closed. $\overline{E} = E$, and $E^o = \{(x, y) : x^2 + 4y^2 < 1\}$. Therefore $\partial E = \{(x, y) : x^2 + 4y^2 = 1\}$.



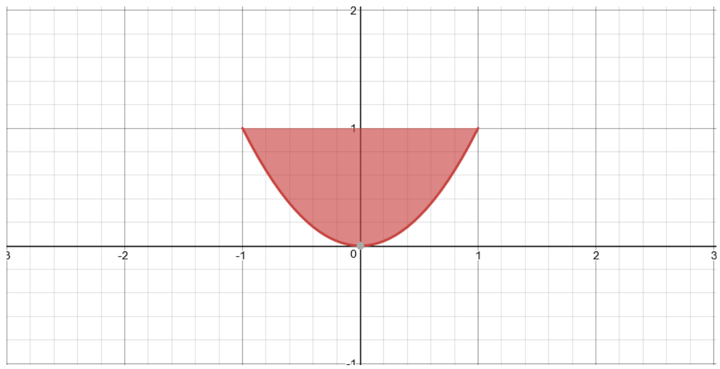
b)$E = \{(x, y) : x^2 - 2x + y^2 = 0\} \cup \{(x, 0) : x \in [2, 3]\}$

proof: $x^2 - 2x + y^2 = 0 \Rightarrow (x - 1)^2 + y^2 = 1$. A circle center at $(1, 0)$ with radius 1. E is closed, since $E^c = \{(x, y) : x^2 - 2x + y^2 < 0\} \cup \{(x, y) : x^2 - 2x + y^2 > 0\} \backslash \{(x, 0) : x \in [2, 3]\}$ is open. $E^o = \emptyset$, $\overline{E} = E$ and $\partial E = E$.



c)$E = \{(x, y) : y \geq x^2, 0 \leq y < 1\}$

proof: E includes all points on $y = x^2$ and below $y = 1$. Therefore, E is neither open nor closed. $E^o = \{(x, y) : y > x^2, 0 < y < 1\}$, $\overline{E} = \{(x, y) : y \geq x^2, 0 \leq y \leq 1\}$ and $\partial E = \{(x, y) : y = x^2, 0 \leq y \leq 1\} \cup \{(x, 1) : -1 \leq x \leq 1\}$.



d)$E = \{(x, y) : x^2 - y^2 < 1, -1 < y < 1\}$

proof: By definition, E is open. Therefore, $E^o = E$, $\overline{E} = \{(x, y) : x^2 - y^2 \leq 1, -1 \leq y \leq 1\}$ and $\partial E = \{(x, y) : x^2 - y^2 = 1, -1 \leq y \leq 1\} \cup \{(x, y) : -\sqrt{2} \leq x \leq \sqrt{2}, y = -1 \text{ or } 1\}$.