Math 741 Assignment 9 (Quiz)

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6.3.2.
solution: From the problem, we can formulate the following,

$$H_0 : p = 0.67$$

$$H_1 : p \neq 0.67$$

where $n = 35, x = 18, \alpha = 0.05$. Then

$$z = \frac{x - np_0}{\sqrt{np_0(1-p_0)}} = \frac{18 - 35 \cdot 0.67}{\sqrt{35 \cdot 0.67 \cdot 0.33}} = -1.95915$$

And $z_{\alpha/2} = 1.95996, -z_{\alpha/2} = -1.95996$. Therefore,

$$-1.95996 < -1.95915 < 1.95996 \implies \text{ Fail to reject } H_0$$

There is not sufficient evidence to support the claim that the proportion of right-pawed mice found in the A/HeJ sample was significantly different from what was know about the A/J strain.

6.3.3.
solution: From the problem, we can formulate the following,

$$H_0 : p = 0.65$$

$$H_1 : p < 0.65$$

where $n = 120, x = 72, \alpha = 0.05$. Then

$$z = \frac{x - np_0}{\sqrt{np_0(1-p_0)}} = \frac{72 - 120 \cdot 0.65}{\sqrt{120 \cdot 0.65 \cdot 0.35}} = -1.14834$$

And $z_\alpha = z_{0.05} = 1.64485 \implies -z_{0.05} = -1.64485$. Since $-1.14834 \not< -1.64485$, this implies fail to reject H_0. There is not sufficient evidence to reject the claim that the proportion of his male proportions has remained the same.

6.3.4.(H)

solution: Given,

$$H_0 : p = 0.45$$

$$H_1 : p > 0.45$$

where $n = 200, \alpha = 0.14$. WTF x value such that H_0 will be rejected.

Since this is a right-tail test,

$$z = \frac{x - np_0}{\sqrt{np_0(1-p_0)}} = \frac{x - 200 \cdot 0.45}{\sqrt{200 \cdot 0.45 \cdot 0.55}}$$

And $z_\alpha = z_{0.14} = 1.0803$. In order to reject H_0, $z > z_\alpha$ must hold. Therefore,

$$\frac{x - 200 \cdot 0.45}{\sqrt{200 \cdot 0.45 \cdot 0.55}} > 1.0803 \implies x > 97.6006$$

Hence, the smallest number of successes that will cause H_0 to be rejected is 98.

6.3.6.(H)

solution: Let p denote the proportion of death in the month preceding their birth month, if people do not postpone their deaths, it should be $\frac{1}{12}$. Therefore,

$$H_0 : p = 1/12$$

$$H_1 : p < 1/12$$

where $n = 348, x = 16, \alpha = 0.05$. Then

$$z = \frac{x - np_0}{\sqrt{np_0(1-p_0)}} = \frac{16 - 348 \cdot \frac{1}{12}}{\sqrt{348 \cdot \frac{1}{12} \cdot \frac{11}{12}}} = -2.52138$$

And $-z_\alpha = -z_{0.05} = -1.64485$. Since $-2.52138 < -1.64485$, reject H_0. In conclusion, there is enough evidence to conclude that celebrities postpone their deaths with significance level 0.05.

6.3.7.
solution: Given,

$$H_0 : p = 0.5$$

$$H_1 : p > 0.5$$

with $n = 7$. Since we are testing against $p > 0.5$. i.e.$k = 4, 5, 6, 7$ need to be considered.

$$\alpha = P(K \geq 4) = P(K = 4) + P(K = 5) + P(K = 6) + P(K = 7)$$

$$= 0.2734375 + 0.1640625 + 0.0546875 + 0.0078125 = 0.5$$

Therefore, reject H_0 if $k \geq 4$ with $\alpha = 0.5$.

$$\alpha = P(K \geq 5) = P(K = 5) + P(K = 6) + P(K = 7)$$

$$= 0.1640625 + 0.0546875 + 0.0078125 = 0.2265625$$

Therefore, reject H_0 if $k \geq 5$ with $\alpha = 0.2265625$.

$$\alpha = P(K \geq 6) = P(K = 6) + P(K = 7)$$

$$= 0.0546875 + 0.0078125 = 0.0625$$

Therefore, reject H_0 if $k \geq 6$ with $\alpha = 0.0625$.

$$\alpha = P(K \geq 7) = P(K = 7)$$

$$= 0.0078125$$

Therefore, reject H_0 if $k \geq 7$ with $\alpha = 0.0078125$.
6.3.9.
solution: Given,

$$H_0 : p = 0.75$$

$$H_1 : p < 0.75$$

with $n = 7$ and decision rule "Reject H_0 if $k \leq 3$. Therefore,

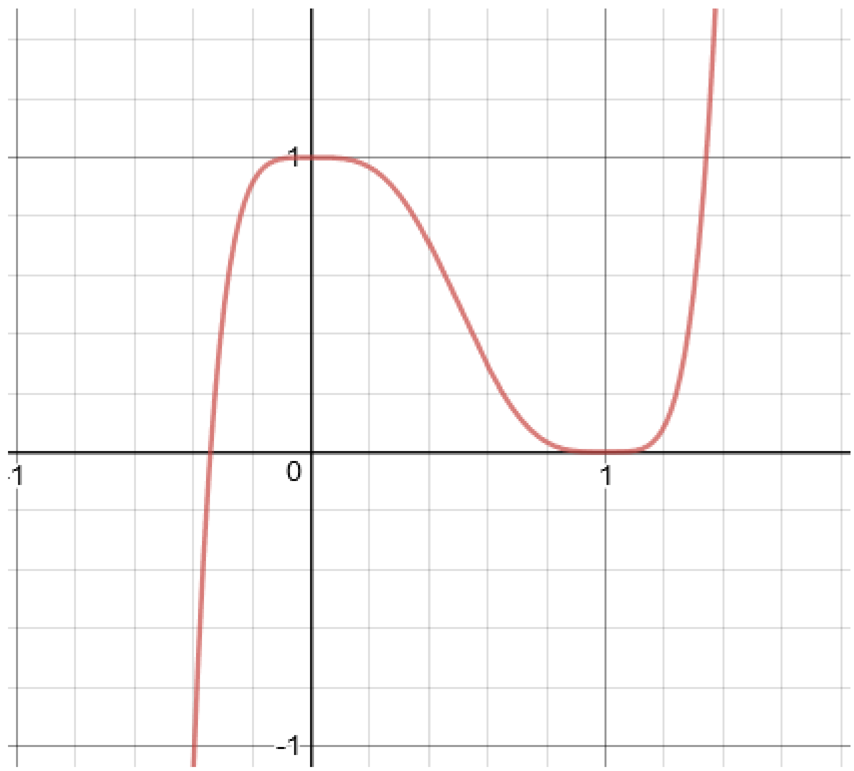
$$\alpha = P(K \leq 3) = P(K = 0) + P(K = 1) + P(K = 2) + P(K = 3)$$

$$= 0.000061 + 0.0012817 + 0.0115356 + 0.057678 = 0.070493$$

a) The test level of significance is $\alpha = 0.070493$.
b)

$$\alpha = P(K \le 3) = \binom{7}{0}(1-p)^7 + \binom{7}{1}p(1-p)^6 + \binom{7}{2}p^2(1-p)^5 + \binom{7}{3}p^3(1-p)^4$$

$$= (1-p)^7 + 7p(1-p)^6 + 21p^2(1-p)^5 + 35p^3(1-p)^4$$



Only consider the interval [0,1] on both axis.