Math 741 Assignment 14 (Hand-In)

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7.5.6.(H) solution: To generate the question, it is to find smallest n such that

$$P(\frac{(n-1)S^2}{\sigma^2} < 2(n-1)) \geq 0.95$$

$$n = 2, P(\frac{(n-1)S^2}{\sigma^2} < 2(n-1)) = 0.8427 \leq 0.95$$

$$n = 3, P(\frac{(n-1)S^2}{\sigma^2} < 2(n-1)) = 0.8647 \leq 0.95$$

$$n = 4, P(\frac{(n-1)S^2}{\sigma^2} < 2(n-1)) = 0.8884 \leq 0.95$$

$$n = 5, P(\frac{(n-1)S^2}{\sigma^2} < 2(n-1)) = 0.9084 \leq 0.95$$

$$n = 6, P(\frac{(n-1)S^2}{\sigma^2} < 2(n-1)) = 0.9248 \leq 0.95$$

$$n = 7, P(\frac{(n-1)S^2}{\sigma^2} < 2(n-1)) = 0.9380 \leq 0.95$$

$$n = 8, P(\frac{(n-1)S^2}{\sigma^2} < 2(n-1)) = 0.9488 \leq 0.95$$

$$n = 9, P(\frac{(n-1)S^2}{\sigma^2} < 2(n-1)) = 0.9576 \geq 0.95$$

Hence, the smallest n is 9.

7.5.14.(H) solution: a)

$$M_{Y_i}(t) = E(e^{ty_i}) = \int_0^\infty e^{ty_i}(1/\theta)e^{-y_i/\theta}dy_i$$

$$= \frac{1}{\theta}\int_0^\infty e^{y_i(t-1/\theta)}dy_i = \frac{1}{\theta}\frac{e^{y_i(t-1/\theta)}}{t-1/\theta}|_0^\infty = \frac{1}{1-\theta t}$$

Let

$$X = \frac{2n\bar{Y}}{\theta} = \frac{2n\frac{\sum_{i=1}^n Y_i}{n}}{\theta} = \frac{2Y_1}{\theta} + ... + \frac{2Y_n}{\theta}$$

$$M_X(t) = \prod_{i=1}^n M_{2Y_i/\theta}(t) = \prod_{i=1}^n M_{Y_i}(\frac{2t}{\theta})$$

$$= \prod_{i=1}^n (\frac{1}{1-2t}) = (\frac{1}{1-2t})^n$$

Since the moment generating function for chi square distribution is $(\frac{1}{1-2t})^{n/2}$ with n df, then $\frac{2n\bar{Y}}{\theta}$ is a chi square distribution with $2n$ df.

b)

$$P(\chi^2_{\alpha/2,2n} \leq \frac{2n\bar{Y}}{\theta} \leq \chi^2_{1-\alpha/2,2n}) = 1-\alpha$$

$$P(\frac{\chi^2_{\alpha/2,2n}}{2n\bar{Y}} \leq \frac{1}{\theta} \leq \frac{\chi^2_{1-\alpha/2,2n}}{2n\bar{Y}}) = 1-\alpha$$

$$P(\frac{2n\bar{Y}}{\chi^2_{\alpha/2,2n}} \geq \theta \geq \frac{2n\bar{Y}}{\chi^2_{1-\alpha/2,2n}}) = 1-\alpha$$

Therefore, the $100(1-\alpha)\%$ CI is $(\frac{2n\bar{Y}}{\chi^2_{1-\alpha/2,2n}}, \frac{2n\bar{Y}}{\chi^2_{\alpha/2,2n}})$