Math 741 Assignment 17 (Quiz)

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April 26, 2019

9.4.3. solution: Let p_1 and p_2 be the proportion of successful in "witched" wells and "non-witched" wells. Then we can formulate the test as following,

$$H_0 : p_1 = p_2$$

$$H_1 : p_1 \neq p_2$$

$x_1 = 24$ and $x_2 = 27$ be the successes and $n_1 = 29$ and $n_2 = 32$ are trials of each group. Let $\alpha = 0.05$,

$$\hat{p}_1 = 24/29, \hat{p}_2 = 27/32, \hat{p}_p = \frac{24+27}{29+32} = \frac{51}{61}$$

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\hat{p}_p(1-\hat{p}_p)}{n_1} + \frac{\hat{p}_p(1-\hat{p}_p)}{n_2}}} = -0.1703$$

$$P - value = 1 - P(-0.1703 < Z < 0.1703) = 0.864774$$

Since $P - value = 0.864774 > \alpha = 0.05 \implies$ Fail to reject H_0. Therefore, there is enough evidence to say the successful rate of both groups are the same.

9.4.4. solution: Let p_1 and p_2 denote the true probabilities of "Saucer on ground" in Spain and not in Spain. The test is

$$H_0 : p_1 = p_2$$

$$H_1 : p_1 \neq p_2$$

with $x_1 = 38, n_1 = 53 + 38 = 91$ and $x_2 = 412, n_2 = 705 + 412 = 1117$. Let $\alpha = 0.01$,

$$\hat{p}_1 = 38/91, \hat{p}_2 = 412/1117, \hat{p}_p = \frac{38+412}{91+1117} = \frac{225}{604}$$

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\hat{p}_p(1-\hat{p}_p)}{n_1} + \frac{\hat{p}_p(1-\hat{p}_p)}{n_2}}} = 0.9247$$

$$P - value = 1 - P(-0.9247 < Z < 0.9247) = 0.355122$$

Since $P - value = 0.355122 > \alpha = 0.01 \implies$ Fail to reject H_0. Therefore, there is enough evidence to say they have the same probabilities.

9.4.5. solution: Let p_1 and p_2 denote the probability of mitigation rate before and after the new policies, then we can formulate the test as following

$$H_0 : p_1 = p_2$$

$$H_1 : p_1 > p_2$$

with $x_1 = 1033, n_1 = 1675$ and $x_2 = 344, n_2 = 660$. Let $\alpha = 0.01$,

$$\hat{p}_1 = 1033/1675, \hat{p}_2 = 344/660, \hat{p}_p = \frac{1033+344}{1675+660} = \frac{1377}{2335}$$

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\hat{p}_p(1-\hat{p}_p)}{n_1} + \frac{\hat{p}_p(1-\hat{p}_p)}{n_2}}} = 4.1946$$

$$P - value = P(4.1946 < Z) = 0.000013675$$

Since $P - value = 0.000013675 < \alpha = 0.01 \implies$ Reject H_0. Therefore, there is enough evidence to say that mitigation rate dropped after the new policies.

9.4.6. solution:

$$H_0 : p_X = p_Y$$

$$H_1 : p_X \neq p_Y$$

with $x = 60, n = 100$ and $y = 48, m = 100$.

$$\hat{p}_X = 0.6, \hat{p}_Y = 0.48, \hat{p}_p = 0.54$$

$$z = 1.702513$$

$$P - value = 1 - P(-1.702513 < Z < 1.702513) = 0.08866$$

The P-value would be 0.08866.

9.4.8.(H) solution: Let p_1 and p_2 denote divorce rate of first and second breeds. Then we can formulate the test as following,

$$H_0 : p_1 = p_2$$

$$H_1 : p_1 \neq p_2$$

with $x_1 = 175, n_1 = 609$ and $x_2 = 100, n_2 = 160$. Let $\alpha = 0.05$,

$$\hat{p}_1 = 175/609, \hat{p}_2 = 100/160, \hat{p}_p = \frac{175 + 100}{609 + 160} = \frac{275}{769}$$

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\hat{p}_p(1-\hat{p}_p)}{n_1} + \frac{\hat{p}_p(1-\hat{p}_p)}{n_2}}} = -7.8298$$

$$P - value = 1 - P(-7.8298 < Z < 7.8298) = 0$$

Since $P - value = 0 < \alpha = 0.01 \implies$ Reject H_0. Therefore, there is enough evidence to say that the difference in the two divorce rate is statistically significant.

9.4.9. solution: Let p_1 and p_2 denote batted rate for the first and second season. Then

$$H_0 : p_1 = p_2$$

$$H_1 : p_1 \neq p_2$$

with $x_1 = 78, n_1 = 300$ and $x_2 = 50, n_2 = 200$. Let $\alpha = 0.05$,

$$\hat{p}_1 = 0.260, \hat{p}_2 = 0.250, \hat{p}_p = \frac{78 + 50}{300 + 200} = 0.256$$

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\hat{p}_p(1-\hat{p}_p)}{n_1} + \frac{\hat{p}_p(1-\hat{p}_p)}{n_2}}} = 0.20605$$

$$P - value = 1 - P(-0.20605 < Z < 0.20605) = 0.836752$$

Since $P - value = 00.836752 > \alpha = 0.05 \implies$ Fail to reject H_0. Therefore, there is enough evidence to say that the player is right.