Math 470 Assignment 22

Arnold Jiadong Yu

April 2, 2018

1. For $a_1, a_2, ..., a_n \in \mathbb{R}$, prove that

$$(\frac{\sum_{i=1}^n a_i}{n})^2 \leq \frac{\sum_{i=1}^n a_i^2}{n}$$

proof: Need to show $(\frac{\sum_{i=1}^n a_i}{n})^2 \leq \frac{\sum_{i=1}^n a_i^2}{n}$, but this is the same to show

$$(a_1 + a_2 + a_3 + ... + a_n)^2 \leq n(a_1^2 + a_2^2 + a_3^2 + ... + a_n^2)$$

Let $x, y \in \mathbb{R}^n$, then $x = (x_1, ..., x_n)$ and $y = (y_1, ..., y_n)$. By Cauchy-Schwarz Inequity,

$$|x \cdot y| \leq ||x|||y|| \Rightarrow |x \cdot y|^2 \leq ||x||^2|y||^2$$

Force $x_1 = a_1, x_2 = a_2, ..., x_n = a_n$ and $y_1 = y_2 = y_3 = ... = y_n = 1$. Then

$$|x \cdot y|^2 \leq ||x||^2|y||^2 \Rightarrow (a_1+a_2+a_3+...+a_n)^2 \leq \underbrace{(1+1+...+1)}_{n}(a_1^2+a_2^2+a_3^2+...+a_n^2)$$

$$= n(a_1^2 + a_2^2 + a_3^2 + ... + a_n^2)$$

That is

$$(\frac{\sum_{i=1}^n a_i}{n})^2 \leq \frac{\sum_{i=1}^n a_i^2}{n}$$

.