Math 741 Assignment 3 (Quiz)

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5.3.3

solution: Let M represent male distribution and F for female. Then $M \sim N(\mu, \sigma^2), F \sim N(\mu, \sigma^2)$. Let $\bar{m}$ be the average for M, and $\bar{f}$ for F. Given $\bar{m} = 80$. NTF μ for $\bar{f} \sim N(\mu, \sigma^2/n)$ for $\sigma = 8$.

$$\bar{f} = \frac{52 + 69 + 73 + 88 + 87 + 56}{6} = \frac{425}{6} \approx 70.83, n = 6$$

The confidence interval is 0.95, i.e $1 - \alpha = 0.95$. Therefore, $\alpha/2 = 0.025$. A 100(1-α)% confidence interval for μ, is the range of number

$$(\bar{f} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}, \bar{f} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}})$$

And $z_{0.025} = 1.96$ for standard normal. Then

$$(\bar{f} - z_{0.025} \cdot \frac{\sigma}{\sqrt{n}}, \bar{f} + z_{0.025} \cdot \frac{\sigma}{\sqrt{n}}) = (\frac{425}{6} - 1.96 \cdot \frac{8}{\sqrt{6}}, \frac{425}{6} + 1.96 \cdot \frac{8}{\sqrt{6}}) \approx (64.432, 77.245)$$

Since $\bar{m} = 80 \notin (64.432, 77.245)$, we believe that males and females metabolize methylmercury are not at the same rate.

5.3.5

solution: Given confidence interval is 0.95, $\sigma = 14.3$. NTF n s.t. $2z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} < 3.06$. Then

$$z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} < 1.53 \Rightarrow z_{0.025} \cdot \frac{\sigma}{\sqrt{n}} < 1.53$$

$$\Rightarrow 1.96 \cdot 14.3/1.53 < \sqrt{n} \Rightarrow n > (1.96 \cdot 14.3/1.53)^2 \approx 335.58$$

Therefore, n need to be at least 336 to guarantee that the length of the 95% confidence interval for μ will be less than 3.06.

5.3.8

solution: It is unique because it is the only 95% confidence interval that is centered at sample mean $\bar{y}$.

5.3.10

solution: By Theorem 5.3.1, a 100(1-α)% confidence interval for p is a set of number

$$\left[\frac{k}{n} - z_{\alpha/2} \cdot \sqrt{\frac{(k/n)(1-k/n)}{n}}, \frac{k}{n} + z_{\alpha/2} \cdot \sqrt{\frac{(k/n)(1-k/n)}{n}}\right]$$

where $\frac{k}{n}$ is given by $\frac{192}{540} = \hat{p} = 0.356$ and $1 - \alpha = 0.95$ implies $z_{\alpha/2} = 1.96$. Then we can simply the interval above.

$$(0.356 - 1.96 \cdot \sqrt{\frac{0.356 \cdot (1 - 0.356)}{540}}, 0.356 + 1.96 \cdot \sqrt{\frac{0.356 \cdot (1 - 0.356)}{540}})$$

$$= (0.3156, 0.3964)$$

5.3.11

solution:By Theorem 5.3.1, a 100(1-α)% confidence interval for p is a set of number

$$\left[\frac{k}{n} - z_{\alpha/2} \cdot \sqrt{\frac{(k/n)(1-k/n)}{n}}, \frac{k}{n} + z_{\alpha/2} \cdot \sqrt{\frac{(k/n)(1-k/n)}{n}}\right]$$

where $\frac{k}{n}$ is given by $\frac{281}{1015} = \hat{p} = 0.2768$ and $1 - \alpha = 0.90$ implies $z_{\alpha/2} = 1.645$. Then we can simply the interval above.

$$(0.2768 - 1.645 \cdot \sqrt{\frac{0.2768 \cdot (1 - 0.2768)}{1015}}, 0.2768 + 1.645 \cdot \sqrt{\frac{0.2768 \cdot (1 - 0.2768)}{1015}})$$

$$= (0.2537, 0.2999)$$

Parameter p: Proportion of viewers that see less than a quarter of the advertisements during the game.

5.3.14

solution: Given $(0.57, 0.63)$ is a 50% confidence interval. By Theorem 5.3.1, we can obtain margin of error

$$z_{\alpha/2} \cdot \sqrt{\frac{(k/n)(1-k/n)}{n}} = \frac{1}{2}(0.63 - 0.57) = 0.03$$

Moreover, p lies in the center of 50% confidence interval which is $0.57+0.03 = 0.60 = \frac{k}{n}$. And $z_{\alpha/2} = z_{0.25} = 0.6745$. Then

$$0.6745 \cdot \sqrt{\frac{0.60 \cdot (1-0.60)}{n}} = 0.03 \Rightarrow n = (0.6 \cdot 0.4)/(\frac{0.03}{0.6745})^2 \approx 121$$

5.3.18

solution: Let $g(p) = p(1-p) = p - p^2$, then

$$g'(p) = 1 - 2p, g''(p) = -2 < 0$$

Let $g'(p) = 0$ solve for p. We can get $p = 0.5$. Since the second derivative is alwasy less than zero, $p = 0.5$ is maximum. As a result, $p(1-p) \leq 0.5 \cdot (1-0.5) = 0.25$ for $0 < p < 1$.

5.3.22

solution: Given $k = 126, n = 350$.

(a) A 90% confidence interval is

$$\left[\frac{k}{n} - z_{\alpha/2} \cdot \sqrt{\frac{(k/n)(1-k/n)}{n}}, \frac{k}{n} + z_{\alpha/2} \cdot \sqrt{\frac{(k/n)(1-k/n)}{n}}\right]$$

where $z_{\alpha/2} = z_{0.05} = 1.645$. Therefore, the interval is

$$(126/350 - 1.645 \cdot \sqrt{\frac{(126/350) \cdot (1-126/350)}{350}}, 126/350 + 1.645 \cdot \sqrt{\frac{(126/350) \cdot (1-126/350)}{350}})$$

$$= (0.3178, 0.4022)$$

(b) The finite correction factor is $\frac{N-n}{n-1}$. Therefore, the new interval is

$$\left[\frac{k}{n} - z_{\alpha/2} \cdot \sqrt{\frac{(k/n)(1-k/n)}{n}}\sqrt{\frac{N-n}{N-1}}, \frac{k}{n} + z_{\alpha/2} \cdot \sqrt{\frac{(k/n)(1-k/n)}{n}}\sqrt{\frac{N-n}{N-1}}\right]$$

which is

$$(\frac{126}{350}-1.645\cdot\sqrt{\frac{(\frac{126}{350})\cdot(1-\frac{126}{350})}{350}}\sqrt{\frac{3000-350}{3000-1}},\frac{126}{350}+1.645\cdot\sqrt{\frac{(\frac{126}{350})\cdot(1-\frac{126}{350})}{350}}\sqrt{\frac{3000-350}{3000-1}})$$

$$=(0.3203,0.3997)$$

5.3.24

solution: Given $\hat{p}_A=0.52,\hat{p}_B=0.48$, then the interval esitmation for A and B are

$$(\hat{p}_A-0.05,\hat{p}_A+0.05)=(0.48,0.57)_A,(\hat{p}_B-0.05,\hat{p}_B+0.05)=(0.43,0.53)_B$$

If two candidates are tied, both of them will need to have 50% of sample favors. Since 0.50 lies in both $(0.48,0.57)_A,(0.43,0.53)_B$. It makes sense to claim that the two candidates are tied.

5.3.26

solution: Given $\hat{p}=\frac{X}{n}\leq 0.4$. A 99% of confidence interval gives $z_{\alpha/2}=z_{0.005}=2.576$. By Theorem 5.3.1, we can obtain margin of error is

$$z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}=0.05\Rightarrow\frac{\hat{p}(1-\hat{p})}{n}=(\frac{0.05}{2.576})^2$$

$$\Rightarrow n=\hat{p}(1-\hat{p})/(\frac{0.05}{2.576})^2\Rightarrow n\geq 0.4\cdot 0.6/(\frac{0.05}{2.576})^2\approx 637.03$$

i.e. the smallest of n is 638.