Math 741 Assignment 22 (Quiz)

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10.4.1 solution: There are 4 outcomes and one unknown parameter, thus $t = 4, s = 1$.
The maximum likelihood estimate is

$$\hat{p} = \frac{0 \cdot 30 + 1 \cdot 56 + 2 \cdot 73 + 3 \cdot 41}{3 \cdot (30 + 56 + 73 + 41)} = 0.5417$$

A table can be formulated,

i	k_i	$\hat{p}_i$	$n\hat{p}_i$
0	30	0.09734	19.5
1	56	0.34279	68.6
2	73	0.40241	80.5
3	41	0.15746	31.5

where k_i is observed frequency, $\hat{p}_i$ is estimated probability of each outcomes occurs, it is calculated by $\binom{3}{i}\hat{p}^i(1-\hat{p})^{3-i}$, and $n\hat{p}_i$ is estimated expected frequency.
A test can be formulated,

$$H_0 : p_0 = 0.09734, p_1 = 0.34279, p_2 = 0.40241, p_3 = 0.15746$$

$$H_1 : \text{ At least one } p_i \text{ is different}, i = 0, 1, 2, 3$$

with $\alpha = 0.05$.

$$d_1 = \sum_{i=1}^{4} \frac{[k_1 - n\hat{p}_i]^2}{n\hat{p}_i} = 11.60$$

$$p - value = 1 - P(0 \le \chi^2_{t-1-s} \le 11.60) = 0.00303$$

since $p-value = 0.00303 < \alpha = 0.05 \implies$ Reject H_0. Hence, there is enough evidence to say the number saying "yes" is not binomially distributed.

10.4.2 solution: There are 5 outcomes and one unknown parameter, thus $t = 5, s = 1$.
The maximum likelihood estimate is

$$\hat{\lambda} = \frac{0 \cdot 59 + 11 \cdot 27 + 2 \cdot 9 + 3 \cdot 14 \cdot 0}{59 + 27 + 9 + 1} = 0.5$$

A table can be formulated,

i	k_i	$\hat{p}_i$	$n\hat{p}_i$
0	59	0.60653	58.2
1	27	0.30327	29.1
2	9	0.07582	7.3
3	1	0.01264	1.2
4	0	0.00158	0.2

where k_i is observed frequency, $\hat{p}_i$ is estimated probability of each outcomes occurs, it is calculated by $\frac{\hat{\lambda}^i e^{-\hat{\lambda}}}{i!}, i = 0, 1, 2, 3, 4$, and $n\hat{p}_i$ is estimated expected frequency.
Since $n\hat{p}_i \geq 5$ need to meet for each category, then the last two category need to be pooled. A new table can be formulated,

i	k_i	$\hat{p}_i$	$n\hat{p}_i$
0	59	0.60653	58.2
1	27	0.30327	29.1
2	10	0.0.07582	8.7

A test can be formulated,

$$H_0 : p_0 = 0.60653, p_1 = 0.30327, p_2 = 0.0.07582$$

$$H_1 : \text{ At least one } p_i \text{ is different}, i = 0, 1, 2$$

with $\alpha = 0.01, t = 3, s = 1$.

$$d_1 = \sum_{i=1}^{3} \frac{[k_1 - n\hat{p}_i]^2}{n\hat{p}_i} = 0.36$$

$$p-value = 1-P(0 \le \chi^2_{t-1-s} \le 0.36) = 0.5485$$

since $p-value = 0.5485 > \alpha = 0.05 \implies$ Fail to reject H_0. Hence, there is enough evidence to say the data can be described by Poisson pdf.

10.4.3 solution: Same as 10.4.2 trivial.

10.4.4 solution: Same as 10.4.2 trivial.

10.4.5(H) solution: There are 8 outcomes and one unkown parameter, thus $t=8, s=1$.

The maximum likelihood estimate is

$$\hat{\lambda} = \frac{130+41+25+8+2+3+1+1}{0.5\cdot 130+1.5\cdot 41+2.5\cdot 25+3.5\cdot 8+4.5\cdot 2+5.5\cdot 3+6.5\cdot 1+7.5\cdot 1}$$

$$= \frac{211}{256.5} = 0.8226$$

A test can be formulated,

H_0 : The data follows an exponential model, $f_Y(y) = 0.8226e^{-0.8226y}, y>0$

H_1 : The data does not follow an exponential model, $f_Y(y) \neq 0.8226e^{-0.8226y}, y>0$

with $\alpha = 0.05$. A table can be formulated,

i	class	k_i	$\hat{p}_i$	$n\hat{p}_i$
1	$0-1$	130	0.5607	118.3077
2	$1-2$	41	0.2463	51.9693
3	$2-3$	25	0.1082	22.8302
4	$3-4$	8	0.0475	10.0225
5	$4-5$	2	0.0209	4.4099
6	$5-6$	3	0.0092	1.9412
7	$6-7$	1	0.00403	0.85033
8	$7-8$	1	0.00177	0.37347

where k_i is observed frequency, $\hat{p}_i$ is estimated probability of each outcomes occurs, it is calculated by $\int_{y_1}^{y_2} 0.8226e^{-0.8226y}dy$, and $n\hat{p}_i$ is estimated expected frequency.

i	class	k_i	$\hat{p}_i$	$n\hat{p}_i$
1	$0-1$	130	0.5607	118.307
2	$1-2$	41	0.2463	51.9693
3	$2-3$	25	0.1082	22.8302
4	$3-4$	8	0.0475	10.0225
5	$4-8$	7	0.0373	7.8703

with $t = 5$.

$$\chi_0^2 = \sum_{i=1}^{5} \frac{[k_1 - n\hat{p}_i]^2}{n\hat{p}_i} = 4.1816$$

$$p - value = 1 - P(0 \leq \chi^2_{t-1-s} \leq 4.1816) = 0.2425$$

since $p - value = 0.2425 > \alpha = 0.05 \implies$ Fail to reject H_0. Hence, there is enough evidence to say the data follows an exponential model.
10.4.7 solution: Same as 10.4.1 trivial.

10.4.8(H) solution: The random variables represents the pdf of $f_Y(y) = 1, 0 \leq y \leq 1$. $y_e = y_{\max} = 0.985$ and $t = 100, s = 0$. A test can be formulated,

$$H_0 : f_Y(y) = 0.985, 0 \leq y \leq 1$$

$$H_1 : f_Y(y) \neq 0.985, 0 \leq y \leq 1$$

with $\alpha = 0.05$. $E = n\hat{p} = 100 \cdot 0.985 = 98.5$.

$$\chi_0^2 = \frac{(100 - 98.5)^2}{98.5} = 0.022843$$

$$p - value = 1 - P(0 \leq \chi^2_{t-1-s} \leq 0.022843) \approx 1$$

since $p - value = 1 > \alpha = 0.05 \implies$ Fail to reject H_0. Hence, there is enough evidence to say the data represents the uniforming distribution. Since $n\hat{p}_i \geq 5$ need to be ensured, a new table need to be formulated,

10.4.11 solution: Same as 10.4.1 trivial.