Math 470 Assignment 27

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10.3.8 Let Y be a subspace of X.
a)Show that a set V is open in Y if and only if there is an open set U in X such that $V = U \cap Y$.
b)Show that a set E is closed in Y if and only if there is a closed set A in X such that $E = A \cap Y$.

proof: a) ($\Rightarrow$) Let Y be a subspace of X and V be an open set in Y, then $V \subseteq Y \subseteq X$. Let $\delta > 0$ and $x \in V$, then $B_\delta(x) \subseteq V \subseteq Y \subseteq X$. Moreover, $x \in X$. Let $U = \bigcup_{x\in V} B_\delta(x)$, then U is open in X. Hence $V = \bigcup_{x\in V} B_\delta(x) \cap Y = U \cap Y$.
($\Leftarrow$) Let Y be a subspace of X, U be an open set in X and $V = U \cap Y$. Then $V \subseteq U$ and U is open in X implies $\forall\delta > 0$ and $x \in V$, $B_\delta(x) \subseteq V \subseteq U$. Moreover $V \subseteq Y$, then $B_\delta(x) \cap Y \subseteq V$. That is V is open set in Y.
b)Let Y be a subspace of X and a set E is closed in Y. Then $Y\backslash E$ is open. By part a), $Y\backslash E = U \cap Y$ for some open U in X. Let $A = Y\backslash U$, then $E = A \cap Y$ and A is closed. The converse is trivial.

10.3.10 Let V be the subset of X.
a)Prove that V is open in X if and only if there is a collection of open balls $\{B_\alpha : \alpha \in A\}$ such that

$$V = \bigcup_{\alpha\in A} B_\alpha$$

b) What happens to this result if *open* is replaced by *closed*?

proof: a) ($\Rightarrow$) Let V be a subset of X, and V is open in X. Let $\epsilon > 0$ and $x \in V$, then $B_\epsilon(x) \subseteq V$. Moreover, $\cup_{x\in V} B_\epsilon(x) \supseteq V$. On the other hand, $\cup_{x\in V} B_\epsilon(x) \subseteq V$ since every $B_\epsilon(x) \subseteq V$. Hence $V = \cup_{x\in V} B_\epsilon(x)$

($\Leftarrow$) By Theorem 10.31 i) A collection of open sets in X is open. Then let each ball from $\{B_\alpha : \alpha \in A\}$to be a open set, then $V = \cup_{\alpha\in A} B_\alpha$ is open in X.

b) If open is replaced by closed, then the close subset V will be decomposited into smaller closed balls, which also can be understood as closed sets.

10.3.11 Let $E \subseteq X$ be closed.
a) Prove that $\partial E \subseteq E$.
b) Prove that $\partial E = E$ if and only if $E^0 = \emptyset$.

c) Show that b) is false if E is not closed.

proof: a) By Theorem 10.39, $\partial E = \overline{E} \backslash E^0$. Here $E = \overline{E}$. By definition, $\partial E \subseteq E$.
b) Suppose $\partial E = E$. By Theorem 10.39, $\partial E = \overline{E} \backslash E^0 = E \backslash E^0 = E$, then $E^0 = \emptyset$. The converse is trivial.
c) Let $E = \mathbf{Q}$, then $E^0 = \emptyset$. But $E = \mathbf{Q} \neq \partial E = \mathbf{R}$.