Math 430 Assignment 12

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10.1

solution:

To model this, we introduce a binary variable z_i such that $z_i = 1$ for active constraints and $z_i = 0$ otherwise. There are at least k active constraints for non-feasible $\mathbf{x}$ i.e. the sum of all z_i is greater and equal to k. Therefore, the constrainst are as below

$$\mathbf{a}_i^{`}\mathbf{x} \geq z_i b_i + (1 - z_i)f, \forall i$$

$$\sum_{i=1}^{m} z_i \geq k$$

$$0 \leq z_i \leq 1, z_i \in \mathbb{Z}$$

10.2

solution:

Let each p_i be a binary variable, if $p_i = 1$ then the player is selected, zero otherwise. Select 12 players means $\sum_{i=1}^{20} p_i = 12$. At least 3 play makers means $\sum_{i=1}^{5} p_i \geq 3$. At least 4 shoot guards means $\sum_{i=4}^{11} p_i \geq 4$. At least 4 forwards means $\sum_{i=9}^{16} p_i \geq 4$. At least 3 centers means $\sum_{i=16}^{20} p_i \geq 3$. At least 2 from the NCAA means $p_4 + p_8 + p_{15} + p_{20} \geq 2$. While rebounding, assists, scoring average, height, and defense ability gives constraints such as $\sum_{i=1}^{20} r_i p_i \geq r, \sum_{i=1}^{20} a_i p_i \geq a, \sum_{i=1}^{20} s_i p_i \geq s, \sum_{i=1}^{20} h_i p_i \geq h, \sum_{i=1}^{20} d_i p_i \geq d$. Moreover, $p_5 \neq p_9, p_2 = p_{19}, p_1 + p_7 + p_{12} + p_{16} \leq 3$. As a result, the model of integer programming should be

$$\text{maximize } \sum_{i=1}^{20} s_i p_i$$

$$\text{subject to } \sum_{i=1}^{20} p_i = 12, \sum_{i=1}^{5} p_i \geq 3, \sum_{i=4}^{11} p_i \geq 4, \sum_{i=9}^{16} p_i \geq 4, \sum_{i=16}^{20} p_i \geq 3$$

$$\sum_{i=1}^{20} r_i p_i \geq r, \sum_{i=1}^{20} a_i p_i \geq a, \sum_{i=1}^{20} s_i p_i \geq s, \sum_{i=1}^{20} h_i p_i \geq h, \sum_{i=1}^{20} d_i p_i \geq d$$

$$p_4 + p_8 + p_{15} + p_{20} \geq 2, p_5 \neq p_9, p_2 = p_{19}, p_1 + p_7 + p_{12} + p_{16} \leq 3$$

$$0 \leq p_1, ..., p_{20} \leq 1, p_1, ..., p_{20} \in \mathbb{Z}$$

10.3

solution:

Let's introduce variable z_i which represents time played for each player, then $0 \leq z_i \leq u_i, \forall i$ and $\sum_{i=1}^{20} z_i p_i = 40 \cdot 5 = 200$. Moreover, $\sum_{1=1}^{20} p_i = 5$ since only 5 players will play at the same time. Then the integer programming changed to

$$\text{maximize } \sum_{i=1}^{20} s_i p_i$$

$$\text{subject to } \sum_{i=1}^{20} p_i = 5, \sum_{i=1}^{5} p_i = 1, 1 \leq \sum_{i=4}^{11} p_i \leq 2, 1 \leq \sum_{i=9}^{16} p_i \leq 2,$$

$$\sum_{i=4}^{11} p_i \neq \sum_{i=9}^{16} p_i, \sum_{i=16}^{20} p_i = 1$$

$$\sum_{i=1}^{5} z_i p_i = 40, \sum_{i=4}^{11} z_i p_i = 60, \sum_{i=9}^{16} z_i p_i = 60, \sum_{i=16}^{20} z_i p_i = 40$$

$$0 \leq z_i \leq u_i, \forall i, \sum_{i=1}^{20} z_i p_i = 200$$

$$\sum_{i=1}^{20} r_i p_i \geq r, \sum_{i=1}^{20} a_i p_i \geq a, \sum_{i=1}^{20} s_i p_i \geq s, \sum_{i=1}^{20} h_i p_i \geq h, \sum_{i=1}^{20} d_i p_i \geq d$$

$$p_4 + p_8 + p_{15} + p_{20} \geq 2, p_5 \neq p_9, p_2 = p_{19}, p_1 + p_7 + p_{12} + p_{16} \leq 3$$

$$0 \leq p_1, ..., p_{20} \leq 1, p_1, ..., p_{20} \in \mathbb{Z}$$

10.5

solution:

Assume the truck travels just once, therefore there is no other scalar mulitples of the variables. We also can put mulitple items in the same box as long as they don't exceed the capacity of the box. We also need to assume every variable is nonnegative, otherwise it will not make sence. We introduce two binary variable x_{ij}, y_i where $x_{ij} = 1$ if the item j is in the box i, zero otherwise. $y_i = 1$ if the box i is in the truck, zero otherwise. Moreover, $\sum_{i=1}^{m} x_{ij} = 1, \forall j$, $\sum_{j=1}^{n} a_j x_{ij} \leq b_i y_i, \forall i$. We also can not exceed the capacity of the truck which mean $\sum_{i=1}^{m} b_i y_i \leq Q$. As a result, we can formulate a linear programming problem, i.e.

$$\text{minimize } C \text{ (just a constant)}$$

$$\text{subject to } \sum_{i=1}^{m} x_{ij} = 1, \forall j$$

$$\sum_{j=1}^{n} a_j x_{ij} \leq b_i y_i, \forall i$$

$$\sum_{i=1}^{m} b_i y_i \leq Q$$

$$0 \leq x_{ij} \leq 1, 0 \leq y_i \leq 1, x_{ij}, y_i \in \mathbb{Z}$$

the move is possible if and only if the above integer programming problem is feasible.