# Math 741 Assignment 14 (Quiz)

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7.5.1. solution:
a)23.6848
b)4.6052
c)2.70039

7.5.3. solution:
a) $y = 2.0879$.
b) $y = 7.2609$.
d) $y = 14.0414$.
d) $y = 17.5388$.

7.5.5. solution: Given $n = 200$, $P(\chi^2_n \leq \chi^2_{p,n}) = p$ and

$$P(\chi^2_{200} \leq y) = 0.95 \implies P(Z \leq z_p^*) = 0.95 \implies z_p^* = 1.645$$

Then

$$\chi^2_{p,n} = \chi^2_{0.95,200} = 200 \cdot \left(1 - \frac{2}{9 \cdot 200} + 1.645\sqrt{\frac{2}{9 \cdot 200}}\right)^3 = 233.996$$

$$\implies y = 233.996$$

7.5.6.(H) solution: To generate the question, it is to find smallest $n$ such that

$$P(\frac{(n-1)S^2}{\sigma^2} < 2(n-1)) \geq 0.95$$

$$n = 2, P(\frac{(n-1)S^2}{\sigma^2} < 2(n-1)) = 0.8427 \leq 0.95$$

$$n=3, P(\frac{(n-1)S^2}{\sigma^2}<2(n-1))=0.8647\leq 0.95$$
$$n=4, P(\frac{(n-1)S^2}{\sigma^2}<2(n-1))=0.8884\leq 0.95$$
$$n=5, P(\frac{(n-1)S^2}{\sigma^2}<2(n-1))=0.9084\leq 0.95$$
$$n=6, P(\frac{(n-1)S^2}{\sigma^2}<2(n-1))=0.9248\leq 0.95$$
$$n=7, P(\frac{(n-1)S^2}{\sigma^2}<2(n-1))=0.9380\leq 0.95$$
$$n=8, P(\frac{(n-1)S^2}{\sigma^2}<2(n-1))=0.9488\leq 0.95$$
$$n=9, P(\frac{(n-1)S^2}{\sigma^2}<2(n-1))=0.9576\geq 0.95$$

Hence, the smallest $n$ is 9.

7.5.9. solution:

a) By Theorem 7.5.1, $100(1-\alpha)\%$ CI for $\sigma$ is the set of values

$$[\sqrt{\frac{(n-1)s^2}{\chi^2_{1-\alpha/2,n-1}}},\sqrt{\frac{(n-1)s^2}{\chi^2_{\alpha/2,n-1}}}]$$

Enter the data into calculator and obtain,

$$s=27.2467, n=16$$

$$\chi^2_{0.975,15}=27.48839, \chi^2_{0.025,15}=6.26214$$

Then the 95% CI is $(20.1273, 42.1695)$.

b) Left side,

$$(0,\sqrt{\frac{(n-1)s^2}{\chi^2_{\alpha,n-1}}})=(0,\sqrt{\frac{(15)(27.48839)^2}{\chi^2_{0.05,15}}})=(0,39.4517)$$

Right side,

$$(\sqrt{\frac{(n-1)s^2}{\chi^2_{1-\alpha,n-1}}},\infty)=(\sqrt{\frac{(15)(27.48839)^2}{\chi^2_{0.95,15}}},\infty)=(21.2632,\infty)$$

7.5.13. solution:By Theorem 7.5.1, $100(1-\alpha)\%$ CI for $\sigma$ is the set of values

$$[\sqrt{\frac{(n-1)s^2}{\chi^2_{1-\alpha/2,n-1}}}, \sqrt{\frac{(n-1)s^2}{\chi^2_{\alpha/2,n-1}}}]$$

Then

$$(51.47, 261.90) = (\frac{(n-1)s^2}{\chi^2_{0.95,n-1}}, \frac{(n-1)s^2}{\chi^2_{0.05,n-1}})$$

Let $n = 10$, then

$$\frac{9 \cdot s^2}{\chi^2_{0.95,9}} = 51.47 \implies s = 9.8366$$

7.5.14.(H) solution: a)

$$M_{Y_i}(t) = E(e^{ty_i}) = \int_0^\infty e^{ty_i}(1/\theta)e^{-y_i/\theta}dy_i$$

$$= \frac{1}{\theta}\int_0^\infty e^{y_i(t-1/\theta)}dy_i = \frac{1}{\theta}\frac{e^{y_i(t-1/\theta)}}{t-1/\theta}|_0^\infty = \frac{1}{1-\theta t}$$

Let

$$X = \frac{2n\bar{Y}}{\theta} = \frac{2n\frac{\sum_{i=1}^n Y_i}{n}}{\theta} = \frac{2Y_1}{\theta} + ... + \frac{2Y_n}{\theta}$$

$$M_X(t) = \prod_{i=1}^n M_{2Y_i/\theta}(t) = \prod_{i=1}^n M_{Y_i}(\frac{2t}{\theta})$$

$$= \prod_{i=1}^n(\frac{1}{1-2t}) = (\frac{1}{1-2t})^n$$

Since the moment generating function for chi square distribution is $(\frac{1}{1-2t})^{n/2}$, then $\frac{2n\bar{Y}}{\theta}$ is a chi square distribution with $2n$ df.
b)

$$P(\chi^2_{\alpha/2,2n} \le \frac{2n\bar{Y}}{\theta} \le \chi^2_{1-\alpha/2,2n}) = 1-\alpha$$

$$P(\frac{\chi^2_{\alpha/2,2n}}{2n\bar{Y}} \le \frac{1}{\theta} \le \frac{\chi^2_{1-\alpha/2,2n}}{2n\bar{Y}}) = 1-\alpha$$

$$P(\frac{2n\bar{Y}}{\chi^2_{\alpha/2,2n}} \ge \theta \ge \frac{2n\bar{Y}}{\chi^2_{1-\alpha/2,2n}}) = 1-\alpha$$

Therefore, the $100(1-\alpha)\%$ CI is $(\frac{2n\bar{Y}}{\chi^2_{1-\alpha/2,2n}}, \frac{2n\bar{Y}}{\chi^2_{\alpha/2,2n}})$

7.5.15. solution: Let $S$ denoted as standard deviation from potassium-argon method and $\sigma_0$ is from the older procedure using lead. Therefore, $s = 27.1, \sigma_0 = 30.4$. The test can be formulated as following,

$$H_0 : \sigma = 30.4$$

$$H_1 : \sigma < 30.4$$

with $\alpha = 0.05$.

$$\text{P-value} = P(0 < \chi^2_{18} < \frac{18s^2}{\sigma_0}) = 0.28964 > \alpha = 0.05$$
$$\implies \text{Fail to reject } H_0$$

Hence, the potassium-argon method doesn't have a smaller standard deviation than the older procedure.