Math 741 Assignment 8 (Handin)

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6.2.6.(H)
solution: Given,

$$H_0 : \mu = 30$$

$$H_1 : \mu \neq 30$$

with critical region $(29.9, 30.1)$ and $n = 16, \sigma = 6.0$. Then

$$\alpha = P(29.9 < \bar{Y} < 30.1) = P(\frac{29.9 - 30}{6.0/\sqrt{16}} < Z < \frac{30.1 - 30}{6.0/\sqrt{16}})$$

$$= P(-\frac{1}{15} < Z < \frac{1}{15}) = 0.0532$$

The interval $(29.9, 30.1)$ is a bad choice because it contains the μ_0 value in the interval. Even if we get $\bar{y} = 30$, H_0 will be rejected anyway. This should not happen.
We calculate the standard normal interval $(-z_{\alpha/2}, z_{\alpha/2}) = (-1.933, 1.933)$ which H_0 will be failed to reject. Then we transform it into regular interval

$$(-z_{\alpha/2} * \frac{\sigma}{\sqrt{n}} + \mu_0, z_{\alpha/2}\frac{\sigma}{\sqrt{n}} + \mu_0) = (27.1005, 32.8995)$$

Hence, $C = \{\bar{y} : \bar{y} < 27.1005 \text{ or } \bar{y} > 32.8995\}$.
6.2.8.(H)
solution: a) Given

$$H_0 : \mu = 120$$

$$H_1 : \mu < 120$$

with $\bar{y} = 114.2, n = 25, \sigma = 18, \alpha = 0.08$.

$$\text{P-value} = P(Z < \frac{\bar{y} - \mu_0}{\sigma/\sqrt{n}}) = P(Z < -1.61111) = 0.05358$$

Since P-value $< \alpha$, there is sufficient evidence to recommend that H_0 to be rejected.

b) Given

$$H_0 : \mu = 42.9$$

$$H_1 : \mu \neq 42.9$$

with $\bar{y} = 45.1, n = 16, \sigma = 3.2, \alpha = 0.01$.

$$\text{P-value} = P(Z < \frac{\bar{y} - \mu_0}{\sigma/\sqrt{n}} \text{ or } Z > -\frac{\bar{y} - \mu_0}{\sigma/\sqrt{n}})$$

$$= P(Z < -2.75) + P(Z > 2.75) = 1 - P(< -2.75 < Z < 2.75) = 0.005960$$

Since P-value $< \alpha$, there is sufficient evidence to recommend that H_0 to be rejected.

c) Given

$$H_0 : \mu = 14.2$$

$$H_1 : \mu > 14.2$$

with $\bar{y} = 15.8, n = 9, \sigma = 4.1, \alpha = 0.13$.

$$\text{P-value} = P(Z > \frac{\bar{y} - \mu_0}{\sigma/\sqrt{n}}) = 1 - P(Z < \frac{\bar{y} - \mu_0}{\sigma/\sqrt{n}}) = 1 - P(Z < 1.171) = 0.1209$$

Since P-value $< \alpha$, there is sufficient evidence to recommend that H_0 to be rejected.

6.2.10.(H)

solution: With the given condition, we can formulate the question as following. H_0 represents the stress of final exams doesn't elevate the blood pressures of freshmen women, where H_1 represents the the stress of final exams elevates the blood pressures of freshmen women.

$$H_0 : \mu = 120$$

$$H_1 : \mu > 120$$

with $\bar{x} = 125.2, n = 50, \sigma = 12$. Assume $\alpha = 0.05$, we can calculate the z-value

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{125.2 - 120}{12/\sqrt{50}} = 3.064$$

$$\text{P-value} = P(Z > 3.064) = 1 - P(Z < 3.064) = 0.00109$$

Since P-value $< \alpha$, then reject H_0. There is sufficient evidence to support the claim that the stress of final exams elevates the blood pressures of freshmen women.