Math 741 Assignment 10 (Hand-In)

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6.4.4. (H)
solution: Given,

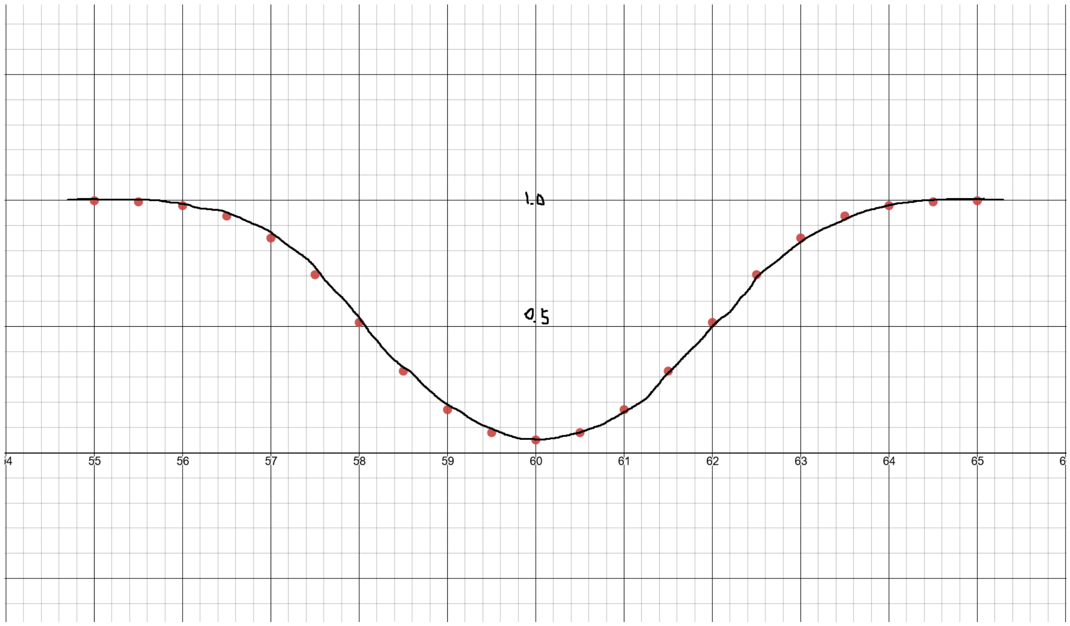
$$H_0 : \mu = 60$$
$$H_1 : \mu \neq 60$$

with $n = 16, \sigma = 4, \alpha = 0.05$. Then the compliment of the critical region (acceptable region) is

$$(-z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}}+\mu_0, z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}}+\mu_0) = (-1.96\cdot\frac{4}{4}+60, 1.96\cdot\frac{4}{4}+60) = (58.04, 61.96)$$

$$\Pi(\beta) = P(\text{ Reject } H_0|H_1 \text{ is true}) = P(\bar{Y} < 58.04) + P(\bar{Y} > 61.96)$$

μ	$\Pi(\beta)$	μ	$\Pi(\beta)$
55	0.998817	65	0.998817
55.5	0.994457	64.5	0.994457
56	0.979325	64	0.979325
56.5	0.938220	63.5	0.938220
57	0.850830	63	0.850830
57.5	0.705406	62.5	0.705406
58	0.515991	62	0.515991
58.5	0.323028	61.5	0.323028
59	0.170066	61	0.1700660
59.5	0.079092	60.5	0.079092



6.4.10. (H)

solution: Given,

$$f_Y(y) = \begin{cases} \frac{1}{\lambda} e^{-y/\lambda} & y > 0 \\ 0 & o.w. \end{cases}$$

Then, $F_Y(y) = \int_y^\infty \frac{1}{\lambda} e^{-t/\lambda} dt = e^{-y/\lambda}$ Moreover,

$$H_0 : \lambda = 1$$

$$H_1 : \lambda > 1$$

with critical region $[3.20, \infty)$.

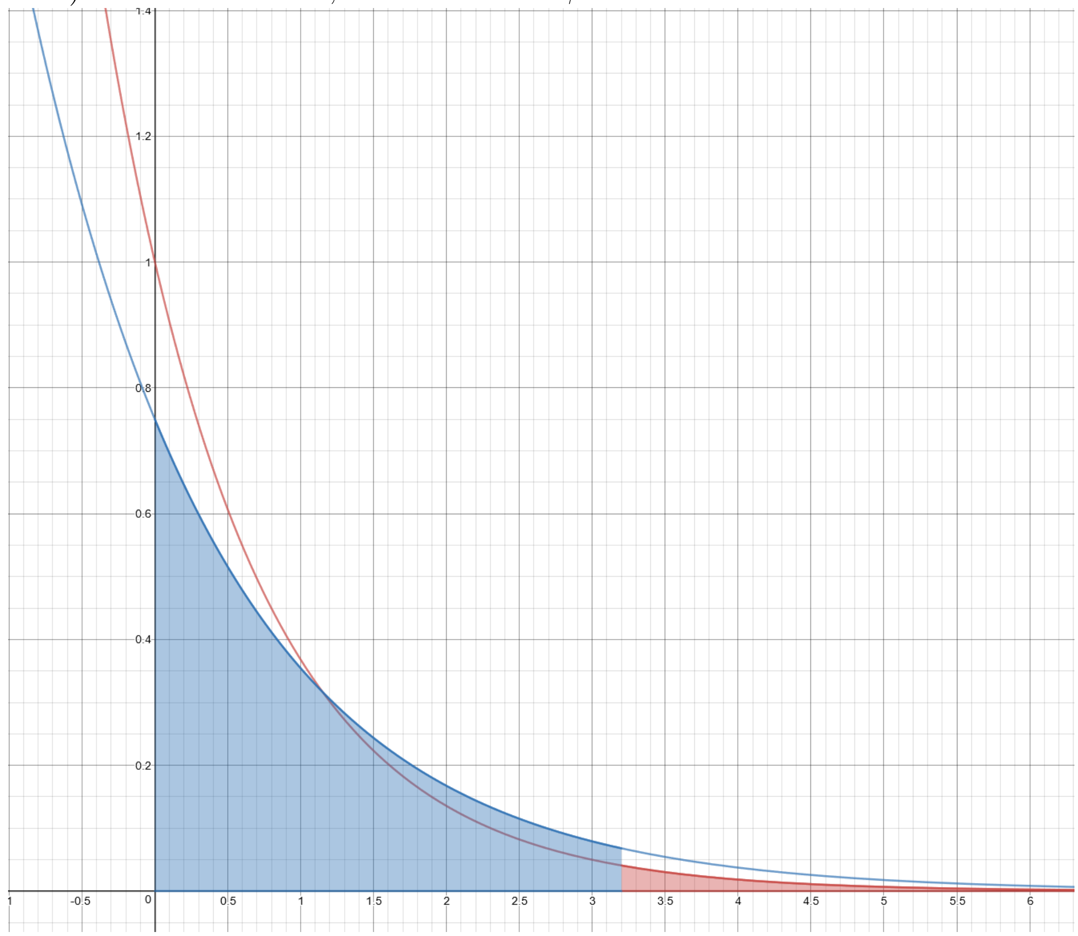
a)

$$\text{Type I error } = P(Y \geq 3.20 | H_0 \text{ is true }) = \int_{3.20}^{\infty} e^{-y} dy = -e^{-y}|_{3.2}^{\infty} = 0.04076$$

b)

$$\text{Type II error } = P(Y < 3.20 | H_1 \text{ is true }) = \int_0^{3.20} \frac{3}{4} e^{-3y/4} dy = -e^{-3y/4}|_0^{3.2} = 0.9093$$

c) The red area is α, the blue area is β.



6.4.16. (H)

solution: Given X_1 a binomial random variable with $n = 2$, X_2 a binomial random variable with $n = 4$,

$$H_0 : p_{X_1} = p_{X_2} = \frac{1}{2}$$

$$H_1 : p_{X_1} = p_{X_2} > \frac{1}{2}$$

with $X = X_1 + X_2$ and reject H_0 if $k \geq 5$. Then

$$\alpha = P(X \geq 5|H_0 \text{ is true }) = P(X = 5|H_0 \text{ is true }) + P(X = 6|H_0 \text{ is true })$$

$$P(X_1 = 2, X_2 = 3|H_0 \text{ is true }) + P(X_1 = 1, X_2 = 4|H_0 \text{ is true }) + P(X_1 = 2, X_2 = 4|H_0 \text{ is true })$$

$$= P(X_1 = 2|H_0 \text{ is true })P(X_2 = 3|H_0 \text{ is true })$$
$$+P(X_1 = 1|H_0 \text{ is true })P(X_2 = 4|H_0 \text{ is true })$$
$$+P(X_1 = 2|H_0 \text{ is true })P(X_2 = 4|H_0 \text{ is true })$$
$$= \binom{2}{2}(\frac{1}{2})^2(\frac{1}{2})^{2-2} \cdot \binom{4}{3}(\frac{1}{2})^3(\frac{1}{2})^{4-3} + \binom{2}{1}(\frac{1}{2})^1(\frac{1}{2})^{2-1} \cdot \binom{4}{4}(\frac{1}{2})^4(\frac{1}{2})^{4-4}$$
$$+\binom{2}{2}(\frac{1}{2})^2(\frac{1}{2})^{2-2} \cdot \binom{4}{4}(\frac{1}{2})^4(\frac{1}{2})^{4-4} = (\frac{1}{2})^2 \cdot 4 \cdot (\frac{1}{2})^4 + 2 \cdot (\frac{1}{2})^2 \cdot (\frac{1}{2})^4 + (\frac{1}{2})^2(\frac{1}{2})^4$$
$$= 7 \cdot (\frac{1}{2})^6 = 0.109375$$

Hence, $\alpha = 0.190375$.

6.4.20. (H)

solution: Given,

$$f_Y(y) = \begin{cases} \lambda e^{-\lambda y} & y > 0 \\ 0 & o.w. \end{cases}$$

and

$$H_0 : \lambda = 1$$
$$H_1 : \lambda < 1$$

with reject H_0 if $y \geq \ln 10$. Then

$$\beta = P(\text{ Fail to reject } H_0|H_1 \text{ is true}) = P(Y < \ln 10|H_1 \text{ is true})$$
$$= \int_0^{\ln 10} \lambda e^{-\lambda y} dy = 1 - e^{-\ln 10 \lambda} = 1 - (\frac{1}{10})^{\lambda}, \lambda < 1$$