Math 741 Assignment 16 (Quiz)

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9.3.2. solution: Let σ_1^2 denoted uncertainty of 30-year fixed and σ^2 denoted uncertainty for ARM, we can formulate the test as following

$$H_0 : \sigma_1^2 = \sigma_2^2$$

$$H_1 : \sigma_1^2 < \sigma_2^2$$

with $\alpha = 0.10$. Enter the data into calculator and use 2-sample F Test, then

$$n_1 = 8, n_2 = 5$$

$$\bar{x}_1 = 5.375, \bar{x}_2 = 4.625$$

$$s_1 = 0.31339, s_2 = 0.50775$$

$$F_0 = 0.38095, P - value = 0.12548$$

since $P - value = 0.12548 > \alpha = 0.1 \implies$ fail to reject H_0. In conclusion, there is enough evidence to say that both 30-year fixed and ARM have the same uncertainty.

9.3.3. solution: a) σ_1^2 and σ_2^2 are the variances of the scores of mothers of normal children and scores of mothers of schizophrenic children, then the test can be formulated as following

$$H_0 : \sigma_1^2 = \sigma_2^2$$

$$H_1 : \sigma_1^2 \neq \sigma_2^2$$

with $\alpha = 0.05$. Enter the data into calculator and use 2-sample F Test, then

$$n_1 = 20, n_2 = 20$$

$$\bar{x}_1 = 3.55, \bar{x}_2 = 2.1$$

$$s_1 = 1.87715, s_2 = 1.55259$$

$$F_0 = 1.46179, P - value = 0.415506$$

since $P - value = 0..415506 > \alpha = 0.05 \implies$ fail to reject H_0. There is enough evidence to say that both variances are the same.
b)

$$H_0 : \mu_0 = \mu_1$$

$$H_1 : \mu_0 \neq \mu_1$$

with $\alpha = 0.05$. Enter the data into calculator and use 2-sample t Test, then

$$n_1 = 20, n_2 = 20$$

$$\bar{x}_1 = 3.55, \bar{x}_2 = 2.1$$

$$s_1 = 1.87715, s_2 = 1.55259, s_p = 1.72253$$

$$t_0 = 2.66196, P - value = 0.011324(pooled)$$

since $P - value = 0.011324 < \alpha = 0.05 \implies$ reject H_0. Therefore, there is enough evidence to same they have difference means.

9.3.6.(H) solution: Let σ_1^2 and σ_2^2 are the variances of American League teams fans changes and National League teams fans changes, then the test can be formulated as following

$$H_0 : \sigma_1^2 = \sigma_2^2$$

$$H_1 : \sigma_1^2 \neq \sigma_2^2$$

let $\alpha = 0.05$. Enter the data into calculator and use 2-sample F Test, then

$$n_1 = 12, n_2 = 14$$

$$\bar{x}_1 = -12.833, \bar{x}_2 = -15.143$$

$$s_1 = 16.5685, s_2 = 19.9687$$

$$F_0 = 0.688444, P - value = 0.541978$$

since $P - value = 0.541978 > \alpha = 0.05 \implies$ fail to reject H_0. Therefore, they have the same variance. As a result, we can used the pooled two-sample t test.

9.3.9. solution: If $\sigma_1^2 = \sigma_2^2 = \sigma^2$, then

$$\hat{\sigma}^2 = \frac{1}{n+m}\Big(\sum_{i=1}^{n}(x_i-\bar{x})^2 + \sum_{i=1}^{m}(y_i-\bar{y})^2\Big)$$

$$L(\Omega_0) = \prod_{i=1}^{n+m}\frac{1}{\sqrt{2\pi}\hat{\sigma}}e^{-\frac{1}{2\hat{\sigma}^2}\Big(\sum_{i=1}^{n}(x_i-\bar{x})^2+\sum_{i=1}^{m}(y_i-\bar{y})^2\Big)}$$

$$L(\Omega_0) = \Big(\frac{1}{2\pi\hat{\sigma}^2}\Big)^{(n+m)/2}e^{-\frac{n+m}{2}}$$

If $\sigma_1^2 \neq \sigma_2^2$, then

$$\hat{\sigma_1}^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2, \hat{\sigma_2}^2 = \frac{1}{m}\sum_{i=1}^{m}(y_i-\bar{y})^2$$

$$L(\Omega) = \prod_{i=1}^{n}\frac{1}{\sqrt{2\pi}\sigma_1}e^{-\frac{1}{2\sigma_1^2}\sum_{i=1}^{n}(x_i-\bar{x})^2}\cdot\prod_{i=1}^{m}\frac{1}{\sqrt{2\pi}\sigma_2}e^{-\frac{1}{2\sigma_2^2}\sum_{i=1}^{m}(y_i-\bar{y})^2}$$

$$L(\Omega) = \Big(\frac{1}{2\pi\sigma_1}\Big)^{n/2}e^{-n/2}\cdot\Big(\frac{1}{2\pi\sigma_2}\Big)^{m/2}e^{-m/2}$$

Moreover,

$$\lambda = \frac{L(\Omega_0)}{L(\Omega)} = \frac{\Big(\frac{1}{2\pi\hat{\sigma}^2}\Big)^{(n+m)/2}e^{-\frac{n+m}{2}}}{\Big(\frac{1}{2\pi\sigma_1}\Big)^{n/2}e^{-n/2}\cdot\Big(\frac{1}{2\pi\sigma_2}\Big)^{m/2}e^{-m/2}} = \frac{\sigma_1^{n/2}\sigma_2^{m/2}}{\hat{\sigma}^{(n+m)/2}}$$

Therefore,

$$\lambda = \frac{(\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2)^{n/2}(\frac{1}{m}\sum_{i=1}^{m}(y_i-\bar{y})^2)^{m/2}}{(\frac{1}{n+m})^{(n+m)/2}\Big(\sum_{i=1}^{n}(x_i-\bar{x})^2+\sum_{i=1}^{m}(y_i-\bar{y})^2\Big)^{(n+m)/2}}$$

$$\lambda = \frac{(n+m)^{(n+m)/2}}{n^{n/2}m^{m/2}}\frac{(\sum_{i=1}^{n}(x_i-\bar{x})^2)^{n/2}(\sum_{i=1}^{m}(y_i-\bar{y})^2)^{m/2}}{\Big(\sum_{i=1}^{n}(x_i-\bar{x})^2+\sum_{i=1}^{m}(y_i-\bar{y})^2\Big)^{(n+m)/2}}$$

9.3.10. (H) solution: Let $X_1, ..., X_n$ and $Y_1, ..., Y_n$ be independent random samples from normal distributions with means μ_1 and μ_2 and standard deviations σ_1 and σ_2 and unbiased standard deviations estimator S_1 and S_2. We will derive the λ of $\sigma_X^2 = \sigma_Y^2$ against $\sigma_X^2 \neq \sigma_Y^2$, since for one-sided it is just to change the subscript notation from $\alpha/2$ to α.