Math 741 Assignment 8 (Quiz)

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6.2.2.

solution: With the given condition, we can formulate the question as following,

$$H_0 : \mu = 95$$

$$H_1 : \mu \neq 95$$

with $n = 22, \sigma = 15, \alpha = 0.06$. Want to find $\bar{y}$ such that H_0 is rejected.
The confidence interval that H_0 will not be rejected is $z \in (-z_{\alpha/2}, z_{\alpha/2}) = (-1.8808, 1.8808)$ for standard normal. Therefore, we reject the null hypothesis if $z < -1.8808$ or $z > 1.8808$.
$\bar{y} < -1.8808 * \frac{\sigma}{\sqrt{n}} + \mu = -1.8808 * \frac{15}{\sqrt{22}} + 95 = 88.985$ or $\bar{y} > 1.8808 * \frac{\sigma}{\sqrt{n}} + \mu = 1.8808 * \frac{15}{\sqrt{22}} + 95 = 101.015$.
Thus, H_0 is rejected if $\bar{y} < 88.985$ or $\bar{y} > 101.015$.

6.2.5.

solution: No. For the right-tail H_1, we need to find z_α. If H_1 is two sided, we need to find $z_{\alpha/2}$. Since it is the same α, $z_{\alpha/2} > z_\alpha$. For any $\mu_0 \in (z_\alpha, z_{\alpha/2})$, H_0 will be rejected if H_1 is right-tail and H_0 will fail to be rejected if H_1 is two-sided.

6.2.6.(H)

solution: Given,

$$H_0 : \mu = 30$$

$$H_1 : \mu \neq 30$$

with interval $(29.9, 30.1)$ and $n = 16, \sigma = 6.0$. Then

$$\alpha = P(29.9 < \bar{x} < 30.1) = P(\frac{29.9 - 30}{6.0/\sqrt{16}} < z < \frac{30.1 - 30}{6.0/\sqrt{16}})$$

$$= P(-0.2/3 < z < 0.2/3) = 0.0532$$

The interval $(29.9, 30.1)$ is a bad choice because it contains the μ value in the interval. Even if we get $\mu_0 = 30$, H_0 will be rejected which should not happen.
We calculate the standard normal interval $(-z_{\alpha/2}, z_{\alpha/2}) = (-1.933, 1.933)$. Then we transform it into regular interval

$$(-z_{\alpha/2} * \frac{\sigma}{\sqrt{n}} + \mu, z_{\alpha/2}\frac{\sigma}{\sqrt{n}} + \mu) = (27.1005, 32.8995)$$

Hence, $C = \{\bar{y} : \bar{y} < 27.1005 \text{ or } \bar{y} > 32.8995\}$.

6.2.7.

solution: Given

$$H_0 : \mu = 12.6$$

$$H_1 : \mu \neq 12.6$$

with $\alpha = 0.05$ and $n = 30, \sigma = 0.4$.

a)

$$\bar{x} = \frac{12.3 + 12.7 + ... + 12.9}{30} = \frac{382.6}{30} = 12.757$$

$z_{\alpha/2} = 1.96$, then

$$(-z_{\alpha/2} * \frac{\sigma}{\sqrt{n}} + \mu, z_{\alpha/2}\frac{\sigma}{\sqrt{n}} + \mu) = (12.4569, 12.7431)$$

Since $\bar{x} \notin (12.4569, 12.7431)$, then reject H_0. Hence, there is sufficient evidence to recommend that the machine be adjusted.
b) Assumed the distribution is normal. n might be too small for the distribution to be normal.

6.2.8.(H)

solution: a) Given

$$H_0 : \mu = 120$$

$$H_1 : \mu < 120$$

with $\bar{y} = 114.2, n = 25, \sigma = 18, \alpha = 0.08$.

$$\text{P-value} = P(z < \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}) = P(z < -1.61111) = 0.05358$$

Since P-value $< \alpha$, there is sufficient evidence to recommend that H_0 to be rejected.

b) Given

$$H_0 : \mu = 42.9$$

$$H_1 : \mu \neq 42.9$$

with $\bar{y} = 45.1, n = 16, \sigma = 3.2, \alpha = 0.01.$

$$\text{P-value} = P(z < \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \text{ or } z > -\frac{\bar{x} - \mu}{\sigma/\sqrt{n}})$$

$$= P(z < -2.75 \text{ or } z > 2.75) = 1 - P(< -2.75 < z < 2.75) = 0.005960$$

Since P-value $< \alpha$, there is sufficient evidence to recommend that H_0 to be rejected.

c) Given

$$H_0 : \mu = 14.2$$

$$H_1 : \mu > 14.2$$

with $\bar{y} = 15.8, n = 9, \sigma = 4.1, \alpha = 0.13.$

$$\text{P-value} = P(z > \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}) = 1 - P(z < \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}) = 1 - P(z < 1.171) = 0.1209$$

Since P-value $< \alpha$, there is sufficient evidence to recommend that H_0 to be rejected.

6.2.10.(H)

solution: With the given condition, we can formulate the question as following. H_0 represents the stress of final exams doesn't elevate the blood pressures of freshmen women, where H_1 represents the the stress of final exams elevates the blood pressures of freshmen women.

$$H_0 : \mu = 120$$

$$H_1 : \mu > 120$$

with $\bar{x} = 125.2, n = 50, \sigma = 12$. Assume $\alpha = 0.05$,

$$\text{P-value} = P(\bar{x} > 120) = P(z > \frac{120 - 125.2}{12/\sqrt{50}}) = 1 - P(z < 3.064) = 0.00109$$

Since P-value $< \alpha$, then reject H_0. There is sufficient evidence to support the claim that the stress of final exams elevates the blood pressures of freshmen women.

6.2.11.

solution: With the given condition, we can formulate the question as following. H_0 represents the average cost of a hypothetical "food basket" in east Tennessee in July would be \$145.75, where H_1 the average cost of a hypothetical "food basket" in east Tennessee in July would not be \$145.75.

$$H_0 : \mu = 145.75$$

$$H_1 : \mu \neq 145.75$$

with $\bar{x} = 149.75, n = 25, \sigma = 9.5, \alpha = 0.05$.

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{25}} = 2.105$$

$$\text{P-value} = P(z < -2.105) + P(z > 2.105) = 2P(z < -2.105) = 0.0353$$

Since P-value $< \alpha$, then reject H_0. There is sufficient evidence to support the claim that the difference between the economists' prediction and the sample mean statistically significant.