Math 741 Assignment 15 (Quiz)

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9.2.2. solution: Given the information, we can formulate the test as following, μ_1 and μ_2 denoted average weight loss for Atkin diet and Zone diet. Since variances are unknown of both group, first test is to test if the variances are the same

$$H_0 : \sigma_1^2 = \sigma_2^2$$
$$H_1 : \sigma_1^2 \neq \sigma_2^2$$

with $\alpha = 0.05$. Use 2-sample F Test,

$$P - value = 0.00000267$$

Since $P - value = 0.00000267 < \alpha = 0.05 \implies$ Reject H_0. As a result, there is enough evidence to say that the variance of two groups are not the same. Next test is as following,

$$H_0 : \mu_1 = \mu_2$$
$$H_1 : \mu_1 < \mu_2$$

with $\bar{x}_1 = -4.7, \bar{x}_2 = -1.6$ and $s_1 = 7.05, s_2 = 5.36$ $n_1 = 77, n_2 = 79$ and $\alpha = 0.05$.

$$t_0 = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(-4.7)_1 - (-1.6)}{\sqrt{\frac{7.05^2}{77} + \frac{6.36^2}{79}}} = -3.0860$$

$$\nu = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\sigma_1^4}{n_1^2(n_1-1)} + \frac{\sigma_2^4}{n_2^2(n_2-1)}} = 141.880 \approx 142$$

$$P - value = P(t_{142} \leq -3.0860) = 0.001220$$

Since $P-value = 0.001220 < \alpha = 0.05 \implies$ Reject H_0. As a result, there is enough evidence to say that Atkins diet is statistically significant.

9.2.3. solution: Let μ_1 and μ_2 be the mean of men and women population. Since variances are unknown of both group, first test is to test if the variances are the same

$$H_0 : \sigma_1^2 = \sigma_2^2$$
$$H_1 : \sigma_1^2 \neq \sigma_2^2$$

with $\alpha = 0.05$. Use 2-sample F Test,

$$P - value = 0.2191753$$

Since $P-value = 0.2191753 > \alpha = 0.05 \implies$ Reject H_0. As a result, there is enough evidence to say that the variance of two groups are the same. Next test is as following,

We can formulate the test as following

$$H_0 : \mu_1 = \mu_2$$

$$H_1 : \mu_1 > \mu_2$$

with

$\bar{x}_1 = 189.0, \bar{x}_2 = 177.2$ and $s_1 = 34.2, s_2 = 33.3$ $n_1 = 476, n_2 = 592$ and $\alpha = 0.05$.

$$t_0 = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{189.0 - 177.2}{\sqrt{\frac{34.2^2}{476} + \frac{33.3^2}{592}}} = 5.67049$$
$$\nu = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\sigma_1^4}{n_1^2(n_1-1)} + \frac{\sigma_2^4}{n_2^2(n_2-1)}} = 1005.5625 \approx 1006$$
$$P - value = P(t_{1006} \geq 5.67049) = 0.000000009299$$

Since $P-value = 0.000000009299 < \alpha = 0.05 \implies$ Reject H_0. As a result, there is enough evidence to say the lower average for the women statistically significant.

9.2.5. solution: Let μ_1 and μ_2 be the mean of students receiving no college credit and students receiving college credit. We can formulate the test as following

$$H_0 : \mu_1 = \mu_2$$

$$H_1 : \mu_1 \neq \mu_2$$

with

$\bar{x}_1 = 4.17, \bar{x}_2 = 4.61$ and $s_1 = 3.70, s_2 = 4.28$ $n_1 = 93, n_2 = 28$ and $\alpha = 0.01$.

$$t_0 = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{4.17 - 4.61}{\sqrt{\frac{3.70^2}{93} + \frac{4.61^2}{28}}} = -0.4622094$$

$$\nu = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\sigma_1^4}{n_1^2(n_1-1)} + \frac{\sigma_2^4}{n_2^2(n_2-1)}} = 38.068 \approx 38$$

$$P - value = 1 - P(-0.4622094 \leq t_{38} \leq 0.4622094) = 0.64654$$

Since $P - value = 0.64654 > \alpha = 0.05 \implies$ Fail to reject H_0. As a result, there is enough evidence to say there is no significant difference between the two group.

9.2.6. solution: Let μ_1 and μ_2 be the average life spans of Authors Noted for Alcohol Abuse and Authors Not Noted for Alcohol Abuse. We can formulate the test as following

$$H_0 : \mu_1 = \mu_2$$

$$H_1 : \mu_1 < \mu_2$$

with $\alpha = 0.05$. Enter the data and use two-sample t test (not pooled).

$$n_1 = 9, n_2 = 12$$

$$\bar{x}_1 = 65.2222, \bar{x}_2 = 75.5$$

$$s_1 = 8.5408, s_2 = 16.7196$$

$$t = -1.83414, P - value = 0.04203$$

since $P - value = 0.04203 < \alpha = 0.05 \implies$ Reject H_0. As a result, there is enough evidence to say that Authors Not Noted for Alcohol Abuse have a higher average life span.

9.2.9. solution: Let μ_1 and μ_2 be the average of Single and Married People who uses coupons regularly. Then we can formulate the test as following,

$$H_0 : \mu_1 = \mu_2$$

$$H_1 : \mu_1 \neq \mu_2$$

with

$\bar{x}_1 = 3.10, \bar{x}_2 = 2.43$ and $s_1 = 1.469, s_2 = 1.350$ $n_1 = 31, n_2 = 57$ and $\alpha = 0.05$.

$$t_0 = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{3.10 - 2.43}{\sqrt{\frac{1.469^2}{31} + \frac{1.350^2}{57}}} = 2.10213$$

$$\nu = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\sigma_1^4}{n_1^2(n_1-1)} + \frac{\sigma_2^4}{n_2^2(n_2-1)}} = 57.4 \approx 57$$

$$P - value = 1 - P(-2.10213 \leq t_{57} \leq 2.10213) = 0.03997$$

Since $P - value = 0.03997 < \alpha = 0.05 \implies$ Reject H_0. As a result, there is enough evidence to say there is observed difference significant between two groups.

9.2.10. solution: Let μ_1 and μ_2 be the average of regular and a more expensive brand dries hours. Then we can formulate the test as following

$$H_0 : \mu_1 = \mu_2 + 1$$

$$H_1 : \mu_1 < \mu_2 + 1$$

with

$\bar{x}_1 = 2.1, \bar{x}_2 = 1.6$ and $s_1 = 12/60, s_2 = 16/60$ $n_1 = 10, n_2 = 10$ and $\alpha = 0.05$.

$$t_0 = \frac{\bar{x}_1 - (\bar{x}_2 + 1)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{2.1 - (1.6 + 1)}{\sqrt{\frac{(12/60)^2}{10} + \frac{(16/60)^2}{10}}} = -4.7434$$

$$\nu = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\sigma_1^4}{n_1^2(n_1-1)} + \frac{\sigma_2^4}{n_2^2(n_2-1)}} = 16.69 \approx 17$$

$$P - value = P(t_{17} \leq -4.7434) = 0.000094$$

Since $P - value = 0.000094 < \alpha = 0.05 \implies$ Reject H_0. As a result, there is enough evidence to say that the more expensive brand doesn't dry one hour quicker than the regular one.

9.2.12.(H) solution: Suppose that

$$H_0 : \mu_1 = \mu_2$$

$$H_1 : \mu_1 \neq \mu_2$$

with $\sigma_1^2 = 17.6, \sigma_2^2 = 22.9$ and $n_1 = 10, n_2 = 20$ and $\bar{x}_1 = 81.6, \bar{x}_2 = 79.9$, then

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = 0.997415$$

$$P - value = 1 - P(-0.997415 \leq z \leq 0.997145) = 0.31856$$

Therefore, the p-value is 0.31856 with the observed Z ratio.

9.2.13. solution: Let $\bar{x}_1 = 33$ and $\sigma_1 = 6$ for the first route by interstate, $\bar{x}_2 = 35$ and $\sigma_2 = 5$ for the second route by driving. WTF $P(\bar{x}_1 - \bar{x}_2 > 0)$ at $n = 1$ Since both have approximately normally distributed, the difference is also approximately normally distributed.

$$E(X_1 - X_2) = E(X_1) - E(X_2) = 33 - 35 = -2$$

$$\text{Var}(X_1 - X_2) = \text{Var}(X_1) + \text{Var}(X_2) = 5^2 + 6^2 = 61$$

$$P(X_1 - X_2 > 0) = P(z > \frac{0 - (-2)}{\sqrt{61}}) = 0.39895$$

For $n = 10$,

$$E(X_1 - X_2) = E(X_1) - E(X_2) = 33 - 35 = -2$$

$$\text{Var}(X_1 - X_2) = \text{Var}(X_1) + \text{Var}(X_2) = 5^2/10 + 6^2/10 = 6.1$$

$$P(X_1 - X_2 > 0) = P(z > \frac{0 - (-2)}{\sqrt{6.1}}) = 0.20903$$

a) On a given day, the probability of driving is faster is 0.39895.
b) Driving of an entire week would yield a lower probability 0.20903.

9.2.14.(H) solution: Let n X_i's and m Y_i's be normally distributed, then

$$E(X_i) = \mu_X, E(\bar{X}) = \frac{1}{n}\sum_{i=1}^{n} E(X_i) = \mu_X$$

$$E(Y_i) = \mu_Y, E(\bar{Y}) = \frac{1}{m}\sum_{i=1}^{m} E(Y_i) = \mu_Y$$

$$\mathrm{Var}(X_i) = \sigma_X^2, \mathrm{Var}(\bar{X}) = \frac{1}{n^2}\sum_{i=1}^{n}\mathrm{Var}(X_i) = \frac{\sigma_X^2}{n}$$

$$\mathrm{Var}(Y_i) = \sigma_Y^2, \mathrm{Var}(\bar{Y}) = \frac{1}{m^2}\sum_{i=1}^{m}\mathrm{Var}(Y_i) = \frac{\sigma_Y^2}{m}$$

Since they are independent,

$$E(\bar{X} - \bar{Y}) = E(\bar{X}) - E(\bar{Y}) = \mu_X - \mu_Y$$

$$\mathrm{Var}(\bar{X} - \bar{Y}) = \mathrm{Var}(\bar{X}) + \mathrm{Var}(\bar{Y}) = \frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}$$

Therefore,

$$Z = \frac{\big(\bar{X} - \bar{Y}) - E(\bar{X} - \bar{Y}\big)}{\sqrt{\mathrm{Var}(\bar{X} - \bar{Y})}} = \frac{\big(\bar{X} - \bar{Y}) - (\mu_X - \mu_Y)}{\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}}} \sim Z(0,1)$$

9.2.15. solution: WTS $E(s_p^2) = \sigma^2$. Since S^2 is unbiased estimator for σ^2, then $E(S_X^2) = E(S_Y^2) = \sigma^2$.

$$E(S_p^2) = E(\frac{(n-1)S_X^2 + (m-1)S_Y^2}{n+m-2} = \frac{(n-1)E(S_X^2) + (m-1)E(S_Y^2)}{n+m-2}$$

$$= \frac{(n-1)\sigma^2 + (m-1)\sigma^2}{n+m-2} = \sigma^2$$

Hence proved.