Math 741 Assignment 15 (Hand-In)

Arnold Jiadong Yu

May 5, 2019

9.2.12.(H) solution: Suppose that

$$H_0 : \mu_1 = \mu_2$$

$$H_1 : \mu_1 \neq \mu_2$$

with $\sigma_1^2 = 17.6, \sigma_2^2 = 22.9$ and $n_1 = 10, n_2 = 20$ and $\bar{x}_1 = 81.6, \bar{x}_2 = 79.9$, then

$$z_0 = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = 0.84474$$

$$P - value = 1 - P(-0.84474 \leq z \leq 0.84474) = 0.39826$$

Therefore, the p-value is 0.39826 with the observed Z ratio.

9.2.14.(H) solution: Let n X_i's and m Y_i's be normal iid samples and X_i's and Y_i's are also independent, then

$$E(X_i) = \mu_X, E(\bar{X}) = \frac{1}{n}\sum_{i=1}^{n} E(X_i) = \mu_X$$

$$E(Y_i) = \mu_Y, E(\bar{Y}) = \frac{1}{m}\sum_{i=1}^{m} E(Y_i) = \mu_Y$$

$$\text{Var}(X_i) = \sigma_X^2, \text{Var}(\bar{X}) = \frac{1}{n^2}\sum_{i=1}^{n} \text{Var}(X_i) = \frac{\sigma_X^2}{n}$$

$$\text{Var}(Y_i) = \sigma_Y^2, \text{Var}(\bar{Y}) = \frac{1}{m^2}\sum_{i=1}^{m} \text{Var}(Y_i) = \frac{\sigma_Y^2}{m}$$

Since they are independent,

$$E(\bar{X} - \bar{Y}) = E(\bar{X}) - E(\bar{Y}) = \mu_X - \mu_Y$$

$$\mathrm{Var}(\bar{X} - \bar{Y}) = \mathrm{Var}(\bar{X}) + \mathrm{Var}(\bar{Y}) = \frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}$$

Therefore,

$$Z = \frac{\big(\bar{X} - \bar{Y}) - E(\bar{X} - \bar{Y}\big)}{\sqrt{\mathrm{Var}(\bar{X} - \bar{Y})}} = \frac{\big(\bar{X} - \bar{Y}) - (\mu_X - \mu_Y\big)}{\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}}} \sim Z(0, 1)$$