Math 470 Assignment 10

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7.1.1. a) Prove that $\frac{x}{n} \to 0$ uniformly, as $n \to \infty$, on any closed interval $[a, b]$.

b) Prove that $\frac{1}{nx} \to 0$ pointwise but not uniformly on $(0, 1)$ as $n \to \infty$.

proof:

a) Let $f_n(x) = \frac{x}{n}$ and $f(x) = 0$, thus $f_n(x) \to f(x)$. Let $\epsilon > 0$, choose $N > \frac{max\{|a|,|b|\}}{\epsilon}$. Then $n \geq N \in \mathbb{N} \Rightarrow \frac{1}{n} \leq \frac{1}{N}$, also $|x| \leq max\{|a|, |b|\}$. Thus

$$|f_n(x) - f(x)| = |\frac{x}{n} - 0| = \frac{|x|}{n} \leq \frac{|x|}{N} \leq \frac{max\{|a|, |b|\}}{N} < \epsilon.$$

Hence $\frac{x}{n} \to 0$ converges uniformly for $\forall x \in [a, b]$.

b)Let $f_n(x) = \frac{1}{nx}$ and $f(x) = 0$, thus $f_n(x) \to f(x)$. Let $\epsilon > 0$, choose $N > \frac{1}{x\epsilon}$. Then $n \geq N \in \mathbb{N} \Rightarrow \frac{1}{n} \leq \frac{1}{N}$ and $x \in (0, 1)$ which x is positive. Thus

$$|f_n(x) - f(x)| = |\frac{1}{nx} - 0| = \frac{1}{nx} \leq \frac{1}{Nx} < \epsilon$$

Hence $\frac{1}{nx} \to 0$ converges pointwise.

Assume it converges uniformly, then for every $\epsilon > 0$, there is an $N \in \mathbb{N}$ such that $\frac{1}{Nx} < 1$ for every $x \in (0, 1)$. Since it works for all x, let's choose $x = \frac{1}{10N} \in (0, 1)$, then $\frac{1}{Nx} = 10 > 1$, there is a contradiction.Then it doesn't converge uniformly. The choice of N depends on ϵ and x.