Math 470 Assignment 34

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Q1. Suppose X is metric space. E is a (path) connected subset of X.
a) Prove or give a counterexample. $\overline{E}$ is (path) connected.
b) Prove or give a counterexample. ∂E is (path) connected.

proof: 1) With path.
a) No. Let $X = \mathbb{R}^2$, $E = \{(x, \sin\frac{1}{x}) : x \in (0,1]\}$. E is path connected. $\overline{E} = \{(x, \sin\frac{1}{x}) : x \in (0,1]\} \cup \{(0,y) : y \in [-1,1]\}$ and $\overline{E}$ is not path connected.
b) No. Let $X = \mathbb{R}$, and $E = (a,b)$ with $a < b$ be a path connected subset of $\mathbb{R}$. Then $\partial E = \{a\} \cup \{b\}$ which is not path connected.
2) Without path.
a) Yes. Prove by contradiction. Suppose X is metric space. E is a connected subset of X. Suppose $\overline{E}$ is disconnected, then $\exists$ nonempty open subsets A and B such that $A \cap B = \emptyset$, $A \cup B = \overline{E}$. i.e. for some $C, D \in \partial E$, $C \cap D = \emptyset$ and $C \cup D = \partial E$. W.L.O.G, let $C \in A, D \in B$. Therefore, $\{A \backslash C\} \cap \{B \backslash D\} = \emptyset$ and $\{A \backslash C\} \cup \{B \backslash D\} = E$. i.e. E is disconnected. It contradicts with E is connected. Hence $\overline{E}$ is connected.
b) No. Same example as 1b.