Electrodynamics
Problem Set 7

董建宇 2019511017

April 12

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1.1

Considering if there only exists the top large plate, using Ampere's Law, we could get that

$$B(s)2L = \mu_0 \sigma L v.$$

So the magnitude of the field produced by the top plate with positive charge density is constant $\frac{1}{2}\mu_0\sigma v$.

Similarly, the magnitude of the field produced by the below plate with negative charge density is the same as top plate but the direction is different. So the magnetic field between the plates is

$$B = \mu_0 \sigma v$$

the direction is inside the page. And the magnetic field above and below them is 0.

1.2

Using the Lorentz force law, we could determine the force per unit area is

$$F = \sigma \vec{v} \times \vec{B}.$$

B is the magnetic field produced by the below plate. So the magnitude of the magnetic force per unit area is

$$F = \frac{\mu_0 \sigma^2 v^2}{2}.$$

And its direction is up.

1.3

We could calculate the electric field produced by the below plate is

$$E = \frac{\sigma}{2\epsilon_0}.$$

So if the magnetic force balance the electrical force, we have that the force per unit area is the same. So

$$\frac{\mu_0 \sigma^2 v^2}{2} = \frac{\sigma^2}{2\epsilon_0}$$

So we could get that

$$v = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = c$$

So the magnetic force balance the electrical force when the speed is the speed of light c.

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By the definition using the cylindrical coordinate, we could get that the magnetic field is

$$\vec{B} = \nabla \times \vec{A} = \frac{1}{s}\frac{\partial(sA)}{\partial s}\hat{z} = \frac{A}{s}\hat{z}.$$

Using Ampere's Law, we could get that the current density is

$$\vec{J} = \frac{1}{\mu_0}\nabla \times \vec{B} = \frac{1}{\mu_0}\left(-\frac{\partial}{\partial s}\frac{A}{s}\right)\hat{\varphi} = \frac{A}{\mu_0 s^2}\hat{\varphi}.$$

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3.1

According to Example 5.11, we get the magnetic field inside the spherical shell is uniform

$$\vec{B}_{in} = \frac{2}{3}\mu_0 \sigma R \omega \hat{z}$$

Using the result in Example 5.11, we could get the magnetic field outside the spherical shell is

$$\vec{B}_{out} = \nabla \times \vec{A} = \frac{\mu_0 R^4 \omega \sigma}{3r^3}\left(2\cos\theta\hat{r} + \sin\theta\hat{\theta}\right)$$

Considering a surface S, let $\vec{B}_S$ be the magnetic field produced by itself and $\vec{B}'$ be the magnetic field produced by others. Then using the boundary conditions we could get that

$$\begin{cases} \vec{B}_S + \vec{B}' = \lim\limits_{r\to R^+} \vec{B} \\ -\vec{B}_S + \vec{B}' = \lim\limits_{r\to R^-} \vec{B} \end{cases}$$

So we could get that

$$\vec{B}' = \frac{2\mu_0 R\omega\sigma}{3}\cos\theta\hat{r} - \frac{1}{6}\mu_0 R\omega\sigma\sin\theta\hat{\theta}$$

So the total force is

$$\vec{F} = \iint \vec{v} \times \vec{B}\, dq = \int_0^{\frac{\pi}{2}} \mu_0 R^2\omega^2 \sin\theta\sigma^2 2\pi R\sin\theta R\, d\theta \left(\frac{2}{3}\cos\theta\hat{\theta} + \frac{1}{6}\sin\theta\hat{r}\right)$$

It is easy to get that the force is along z-axis from symmetry. So we could get that

$$\vec{F} = \int_0^{\frac{\pi}{2}} -\mu_0 R^4 \pi\sigma^2\omega^2 \sin^3\theta\cos\theta\, d\theta\hat{z} = -\frac{1}{4}\mu_0\pi\sigma^2\omega^2 R^4\hat{z}$$

3.2

We could calculate the electric field in total space is

$$\vec{E} = \begin{cases} 0, & for\ r \leqslant R, \\ \frac{\sigma R^2}{\epsilon_0 r^2}\hat{r}, & for\ r \geqslant R. \end{cases}, \quad \vec{E}_{ave} = \frac{1}{2}\frac{\sigma}{\epsilon_0}\hat{r}$$

So the force is

$$\vec{F}_z = \int \sigma \left|\vec{E}_{ave}\right| \cos\theta R^2 \sin\theta\, d\theta\, d\varphi\hat{z} = \frac{\pi\sigma^2 R^2}{\epsilon_0}\int_0^{\frac{\pi}{2}} \sin\theta\cos\theta\, d\theta\hat{z} = \frac{\pi\sigma^2 R^2}{2\epsilon_0}\hat{z}$$