3月9日 第二周 周一作业 作业

Section 1.3

2. (a) EA is defined and the size is (1×1).
   (b) AB^T is not defined.
   (c) $B^T(A+E^T)$ is defined and the size is (6×1).
   (d) $2A+C$ is not defined
   (e) $(C^T+D)B^T$ is defined and the size is (2×3)
   (f) $CD+B^TE^T$ is not defined
   (g) $(BD^T)E^T$ is defined and the size is (3×6)
   (h) $DC+EA$ is not defined

3. (h) $A-A=\begin{bmatrix}0&0\\0&0\\0&0\end{bmatrix}$

   (i) $tr(D)=5$

   (j) $D-3E=\begin{bmatrix}-17&2&-7\\2&-3&-5\\-9&-1&-5\end{bmatrix}$ $tr(D-3E)=-25$

   (k) $4\,tr(7B)=4\times42=168$

   (l) $tr(A)$ is not defined

Section 1.4

1. (a) $B+C=\begin{bmatrix}8&-5&-2\\1&8&6\\7&-2&15\end{bmatrix}$ 则 $A+(B+C)=\begin{bmatrix}10&-6&1\\1&12&11\\5&-1&19\end{bmatrix}$

   $A+B=\begin{bmatrix}10&-4&-2\\0&5&7\\2&-6&10\end{bmatrix}$ 则 $(A+B)+C=\begin{bmatrix}10&-6&1\\1&12&11\\5&-1&19\end{bmatrix}$

   即 $A+(B+C)=(A+B)+C$

(b) $AB=\begin{bmatrix}28&-28&6\\20&-31&38\\0&-21&36\end{bmatrix}$ 则 $(AB)C=\begin{bmatrix}-10&-222&26\\83&-67&278\\87&33&240\end{bmatrix}$

$BC=\begin{bmatrix}-18&-62&-33\\7&17&22\\11&-27&38\end{bmatrix}$ 则 $A(BC)=\begin{bmatrix}-10&-222&26\\83&-67&278\\87&33&240\end{bmatrix}$

即 $(AB)C=A(BC)$

(c) $(a+b)C=\begin{bmatrix}0&6&-9\\-3&-21&-12\\-9&-15&-27\end{bmatrix}$ $aC+bC=\begin{bmatrix}0&-8&12\\4&28&16\\12&20&36\end{bmatrix}+\begin{bmatrix}0&14&-21\\-7&-49&-28\\-21&-35&-63\end{bmatrix}=\begin{bmatrix}0&6&-9\\-3&-21&-12\\-9&-15&-27\end{bmatrix}$

即 $(a+b)C=aC+bC$

(d) $a(B-C)=\begin{bmatrix}32&-4&-32\\-4&-24&-8\\4&-48&-12\end{bmatrix}$ $aB-aC=\begin{bmatrix}32&-12&-20\\0&4&8\\16&-28&24\end{bmatrix}-\begin{bmatrix}0&-8&12\\4&28&16\\12&20&36\end{bmatrix}=\begin{bmatrix}32&-4&-32\\-4&-24&-8\\4&-48&-12\end{bmatrix}$

则 $a(B-C)=aB-aC$

2.(a) $BC=\begin{bmatrix}-18&-62&-33\\7&17&22\\11&-27&38\end{bmatrix}$ $a(BC)=\begin{bmatrix}-72&-248&-132\\28&68&88\\44&-108&152\end{bmatrix}$

$aB=\begin{bmatrix}32&-12&-20\\0&4&8\\16&-28&24\end{bmatrix}$ $(aB)C=\begin{bmatrix}-72&-248&-132\\28&68&88\\44&-108&152\end{bmatrix}$

$aC=\begin{bmatrix}0&-8&12\\4&28&16\\12&20&36\end{bmatrix}$ $B(aC)=\begin{bmatrix}-72&-248&-132\\28&68&88\\44&-108&152\end{bmatrix}$

即 $a(BC)=(aB)C=B(aC)$

(b) $A(B-C)=\begin{bmatrix}2&-1&3\\0&4&5\\-2&1&4\end{bmatrix}\begin{bmatrix}8&-1&-8\\-1&-6&-2\\1&-12&-3\end{bmatrix}=\begin{bmatrix}20&-32&-23\\1&-84&-23\\-13&-52&2\end{bmatrix}$

$AB-AC=\begin{bmatrix}28&-28&6\\20&-31&38\\0&-21&36\end{bmatrix}-\begin{bmatrix}8&4&29\\19&53&61\\13&31&34\end{bmatrix}=\begin{bmatrix}20&-32&-23\\1&-84&-23\\-13&-52&2\end{bmatrix}$

则 $A(B-C)=AB-AC$

(c) $(B+C)A=\begin{bmatrix}8&-5&-2\\1&8&6\\7&-2&15\end{bmatrix}\begin{bmatrix}2&-1&3\\0&4&5\\-2&1&4\end{bmatrix}=\begin{bmatrix}20&-30&-9\\-10&37&67\\-16&0&71\end{bmatrix}$

$BA+CA=\begin{bmatrix}26&-25&-11\\-4&6&13\\-4&-26&1\end{bmatrix}+\begin{bmatrix}-6&-5&2\\-6&31&54\\-12&26&70\end{bmatrix}=\begin{bmatrix}20&-30&-9\\-10&37&67\\-16&0&71\end{bmatrix}$

则 $(B+C)A=BA+CA$

(d) $a(bC)=4\begin{bmatrix}0&14&-21\\-7&-49&-28\\-21&-35&-63\end{bmatrix}=\begin{bmatrix}0&56&-84\\-28&-196&-112\\-84&-140&-252\end{bmatrix}$

$(ab)C=(-28)\begin{bmatrix}0&-2&3\\1&7&4\\3&5&9\end{bmatrix}=\begin{bmatrix}0&56&-84\\-28&-196&-112\\-84&-140&-252\end{bmatrix}$

则 $a(bC)=(ab)C$

3 (a) $(A^T)^T=\begin{bmatrix}2&0&-2\\-1&4&1\\3&5&4\end{bmatrix}^T=\begin{bmatrix}2&-1&3\\0&4&5\\-2&1&4\end{bmatrix}$

则 $(A^T)^T=A$

(b) $(A+B)^T = \begin{bmatrix} 10 & -4 & -2 \\ 0 & 5 & 7 \\ 2 & -6 & 10 \end{bmatrix}^T = \begin{bmatrix} 10 & 0 & 2 \\ -4 & 5 & -6 \\ -2 & 7 & 10 \end{bmatrix}$

$A^T + B^T = \begin{bmatrix} 2 & 0 & -2 \\ -1 & 4 & 1 \\ 3 & 5 & 4 \end{bmatrix} + \begin{bmatrix} 8 & 0 & 4 \\ -3 & 1 & -7 \\ -5 & 2 & 6 \end{bmatrix} = \begin{bmatrix} 10 & 0 & 2 \\ -4 & 5 & -6 \\ -2 & 7 & 10 \end{bmatrix}$

则 $(A+B)^T = A^T + B^T$

(c) $(aC)^T = \begin{bmatrix} 0 & -8 & 12 \\ 4 & 28 & 16 \\ 12 & 20 & 36 \end{bmatrix}^T = \begin{bmatrix} 0 & 4 & 12 \\ -8 & 28 & 20 \\ 12 & 16 & 36 \end{bmatrix}$

$a\,C^T = 4 \cdot \begin{bmatrix} 0 & 1 & 3 \\ -2 & 7 & 5 \\ 3 & 4 & 9 \end{bmatrix} = \begin{bmatrix} 0 & 4 & 12 \\ -8 & 28 & 20 \\ 12 & 16 & 36 \end{bmatrix}$

则 $(aC)^T = a\,C^T$

(d) $(AB)^T = \begin{bmatrix} 28 & -28 & 6 \\ 20 & -31 & 38 \\ 0 & -21 & 36 \end{bmatrix}^T = \begin{bmatrix} 28 & 20 & 0 \\ -28 & -31 & -21 \\ 6 & 38 & 36 \end{bmatrix}$

$B^T A^T = \begin{bmatrix} 8 & 0 & 4 \\ -3 & 1 & -7 \\ -5 & 2 & 6 \end{bmatrix} \begin{bmatrix} 2 & 0 & -2 \\ -1 & 4 & 1 \\ 3 & 5 & 4 \end{bmatrix} = \begin{bmatrix} 28 & 20 & 0 \\ -28 & -31 & -21 \\ 6 & 38 & 36 \end{bmatrix}$

则 $(AB)^T = B^T A^T$

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Prove theorem 1.4.5:

If $ad-bc\neq 0$, the we can easily get

$$\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}=\begin{bmatrix} a & b \\ c & d \end{bmatrix}\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

which says $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is invertible and $A^{-1}=\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Then we assume A is invertible and $A^{-1}=\begin{bmatrix} x & z \\ y & w \end{bmatrix}$

We have $AA^{-1}=\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x & z \\ y & w \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$\begin{cases} ax+by=1 \\ az+bw=0 \\ cx+dy=0 \\ cz+dw=1 \end{cases}$$ The augmented matrix is $\begin{bmatrix} a & b & 0 & 0 & 1 \\ 0 & 0 & a & b & 0 \\ c & d & 0 & 0 & 0 \\ 0 & 0 & c & d & 1 \end{bmatrix}$

We can get both a and b can't be zero at the same time, and both a and c can't be zero at the same time.

①. If $a=0$, then we have $b,c\neq 0$ such that $ad-bc\neq 0$.

We can easily get $x=-\frac{d}{bc}$ $y=\frac{1}{b}$ $z=\frac{1}{c}$ $w=0$

Therefore, $A^{-1}=\begin{bmatrix} x & z \\ y & w \end{bmatrix}=\begin{bmatrix} -\frac{d}{bc} & \frac{1}{c} \\ \frac{1}{b} & 0 \end{bmatrix}$

It satisfy $A^{-1}=\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ when $a=0$

②. If $a\neq 0$ add $-\frac{c}{a}$ times of row① to row③ and add $-\frac{c}{a}$ times of row② to row④, then we get

$$\begin{bmatrix} a & b & 0 & 0 & 1 \\ 0 & 0 & a & b & 0 \\ 0 & d-\frac{bc}{a} & 0 & 0 & -\frac{c}{a} \\ 0 & 0 & 0 & d-\frac{bc}{a} & 1 \end{bmatrix}$$

From row ④ we have $d - \frac{bc}{a} \neq 0$, which says $ad-bc \neq 0$

$$x = \frac{d}{ad-bc} \qquad y = \frac{-c}{ad-bc} \qquad z = \frac{-b}{ad-bc} \qquad w = \frac{a}{ad-bc}$$

$$A^{-1} = \begin{bmatrix} \frac{d}{ad-bc} & \frac{-b}{ad-bc} \\ \frac{-c}{ad-bc} & \frac{a}{ad-bc} \end{bmatrix} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Finally, it's easy to check $A^{-1}A = AA^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Section 1.4

4. $A = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix} \qquad A^{-1} = \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$

8. Assume $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then $A^{-1} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

10 $A = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix} \qquad A^T = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix} \qquad A^{-1} = \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$

Then $(A^T)^{-1} = \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix} \qquad (A^{-1})^T = \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix}$

We get $(A^T)^{-1} = (A^{-1})^T$

18 (a) $A^3 = AAA = \begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 12 & 1 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 28 & 1 \end{bmatrix}$

(e) $p(A) = 2A^2 - A + I_2 = 2\begin{bmatrix} 4 & 0 \\ 12 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 20 & 1 \end{bmatrix}$

Section 1.5

5 (a) Exchange row① and row②.

$$EA = \begin{bmatrix} 3 & -6 & -6 & -6 \\ -1 & -2 & 5 & -1 \end{bmatrix}$$

(b) Add -3 times of row② to row③.

$$EA = \begin{bmatrix} 2 & -1 & 0 & -4 & -4 \\ 1 & -3 & -1 & 5 & 3 \\ -1 & 9 & 4 & -12 & -10 \end{bmatrix}$$

(c) Add 4 times of row③ to row①

$$EA = \begin{bmatrix} 13 & 28 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

7. (a) $E = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

(b) $E = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

(c) $E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$

(d) $E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$