3月16日 第三周周一作业

Section 1.5

21. 解.

$$\left[\begin{array}{cccc:cccc} 2 & -4 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 2 & 12 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & -4 & -5 & 0 & 0 & 0 & 1 \end{array}\right]$$

row① $\times (-\frac{1}{2}) \xrightarrow{+}$ row②

row③ $\times \frac{1}{2}$

row① $\times \frac{1}{2}$

$$\left[\begin{array}{cccc:cccc} 1 & -2 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 4 & 12 & 0 & -\frac{1}{2} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & -1 & -4 & -5 & 0 & 0 & 0 & 1 \end{array}\right]$$

row② $\times \frac{1}{4}$

row④ $\times (-1)$

$$\left[\begin{array}{cccc:cccc} 1 & -2 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 1 & 3 & 0 & -\frac{1}{8} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 1 & 4 & 5 & 0 & 0 & 0 & -1 \end{array}\right]$$

row② $\times (-1) \xrightarrow{+}$ row④

$$\left[\begin{array}{cccc:cccc} 1 & -2 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 1 & 3 & 0 & -\frac{1}{8} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 & 5 & \frac{1}{8} & -\frac{1}{4} & 0 & -1 \end{array}\right]$$

row③ $\times (-1) \xrightarrow{+}$ row④

row④ $\times \frac{1}{5}$

row③ $\times (-3) \xrightarrow{+}$ row②

$$\left[\begin{array}{cccc:cccc} 1 & -2 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -\frac{1}{8} & \frac{1}{4} & -\frac{3}{2} & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 & \frac{1}{40} & -\frac{1}{20} & -\frac{1}{10} & -\frac{1}{5} \end{array}\right]$$

row② $\times 2 \xrightarrow{+}$ row①

$$\left[\begin{array}{cccc:cccc} 1 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & -3 & 0 \\ 0 & 1 & 0 & 0 & -\frac{1}{8} & \frac{1}{4} & -\frac{3}{2} & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 & \frac{1}{40} & -\frac{1}{20} & -\frac{1}{10} & -\frac{1}{5} \end{array}\right]$$

即原矩阵的逆为

$$\begin{bmatrix} \frac{1}{4} & \frac{1}{2} & -3 & 0 \\ -\frac{1}{8} & \frac{1}{4} & -\frac{3}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ \frac{1}{40} & -\frac{1}{20} & -\frac{1}{10} & -\frac{1}{5} \end{bmatrix}$$

Section 1.6

17.

$$\begin{aligned} x_1 - x_2 + 3x_3 + 2x_4 &= b_1 \\ -2x_1 + x_2 + 5x_3 + x_4 &= b_2 \\ -3x_1 + 2x_2 + 2x_3 - x_4 &= b_3 \\ 4x_1 - 3x_2 + x_3 + 3x_4 &= b_4 \end{aligned}$$

$$\left[\begin{array}{cccc:c} 1 & -1 & 3 & 2 & b_1 \\ -2 & 1 & 5 & 1 & b_2 \\ -3 & 2 & 2 & -1 & b_3 \\ 4 & -3 & 1 & 3 & b_4 \end{array}\right]$$

对系数矩阵 A: $\begin{bmatrix} 1 & -1 & 3 & 2 \\ -2 & 1 & 5 & 1 \\ -3 & 2 & 2 & -1 \\ 4 & -3 & 1 & 3 \end{bmatrix}$

注意到 row① = row③ + row④

row② = 2 row③ + row④

即 $b_1 = b_3 + b_4$

$b_2 = 2b_3 + b_4$

22. Proof. ①. If $(QA)\vec{x} = \vec{0}$ has just the trivial solution:

Since Q is an invertible $n \times n$ matrix, we have Q^{-1}

Left multiply Q^{-1} to $(QA)\vec{x} = \vec{0}$.

Then we have $Q^{-1}(QA)\vec{x} = \vec{0}$,

$Q^{-1}Q = I_n$ Then $A\vec{x} = \vec{0}$ has just the trivial solution.

② If $A\vec{x} = \vec{0}$ has just the trivial solution:

Since Q is an invertible $n \times n$ matrix, Q doesn't have zero line.

Then, left multiply Q to $A\vec{x} = \vec{0}$. We get $(QA)\vec{x} = \vec{0}$ also has just the trivial solution.

Section 1.7

10. $A = \begin{bmatrix} -6 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ $\begin{bmatrix} -6 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

$A^2 = \begin{bmatrix} 36 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 25 \end{bmatrix}$ $A^{-2} = \begin{bmatrix} \frac{1}{36} & 0 & 0 \\ 0 & \frac{1}{9} & 0 \\ 0 & 0 & \frac{1}{25} \end{bmatrix}$

$A^{-k} = \begin{bmatrix} \frac{1}{(-6)^k} & 0 & 0 \\ 0 & \frac{1}{3^k} & 0 \\ 0 & 0 & \frac{1}{5^k} \end{bmatrix}$

21. Not invertible. Because the first column of the given matrix is zero. Then it can't become unit matrix. So it is not invertible.

3月18日 第三周 周三作业

Section 1.7

32. (a). Since A is an nxn symmetric matrix. According to Theorm 1.7.3 : If A and b are nxn symmetric matrices, AB is also an nxn symmetric matrix.

$A^2 = AA$ such that A^2 is symmetric.

$(A^2)^T = (AA)^T = A^T A^T = AA = A^2$

(b). Since A is an nxn symmetric matrix. We have $A^T = A$

$$(2A^2 - 3A + I)^T = (2A^2)^T - (3A)^T + I^T = 2(A^2)^T - 3A^T + I^T$$
$$= 2A^2 - 3A + I$$

So $2A^2 - 3A + I$ is symmetric

34. Assume $A = \begin{bmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{bmatrix}$ $A^2 = \begin{bmatrix} a_1^2 & 0 & 0 \\ 0 & a_2^2 & 0 \\ 0 & 0 & a_3^2 \end{bmatrix}$

$$A^2 - 3A - 4I = \begin{bmatrix} a_1^2 - 3a_1 - 4 & 0 & 0 \\ 0 & a_2^2 - 3a - 4 & 0 \\ 0 & 0 & a_3^2 - 3a_3 - 4 \end{bmatrix} = \vec{0}$$

即 $a_1^2 - 3a_1 - 4 = 0$ $a_2^2 - 3a_2 - 4 = 0$ $a_3^2 - 3a_3 - 4 = 0$

令 $x^2 - 3x - 4 = 0$ 得 $x = -1$ 或 $x = 4$

则 A 的可能结果 $\begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ $\begin{bmatrix} 4 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ $\begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ $\begin{bmatrix} 4 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

Section 2.1

3.
$$A = \begin{bmatrix} 4 & -1 & 1 & 6 \\ 0 & 0 & -3 & 3 \\ 4 & 1 & 0 & 14 \\ 4 & 1 & 3 & 2 \end{bmatrix}$$

(a). $M_{13} = \begin{vmatrix} 0 & 0 & 3 \\ 4 & 1 & 14 \\ 4 & 1 & 2 \end{vmatrix} = 12 - 12 = 0$

$C_{13} = (-1)^{1+3} M_{13} = 0$

(b). $M_{23} = \begin{vmatrix} 4 & -1 & 6 \\ 4 & 1 & 14 \\ 4 & 1 & 2 \end{vmatrix} = 8 - 56 + 24 - 24 - (-8) - 56 = -96$

$C_{23} = (-1)^{2+3} M_{23} = 96$

(c). $M_{22} = \begin{vmatrix} 4 & 1 & 6 \\ 4 & 0 & 14 \\ 4 & 3 & 2 \end{vmatrix} = 0 + 56 + 72 - 0 - 168 - 8 = -48$

$C_{22} = (-1)^{2+2} M_{22} = -48$

(d) $M_{21} = \begin{vmatrix} -1 & 1 & 6 \\ 1 & 0 & 14 \\ 1 & 3 & 2 \end{vmatrix} = 0 + 14 + 18 - 0 - (-42) - 2 = 72$

$C_{21} = (-1)^{2+1} M_{21} = -72$

25. $A = \begin{bmatrix} 3 & 3 & 0 & 5 \\ 2 & 2 & 0 & -2 \\ 4 & 1 & -3 & 0 \\ 2 & 10 & 3 & 2 \end{bmatrix}$

$$\det(A) = (-3)(-1)^{3+3} \begin{vmatrix} 3 & 3 & 5 \\ 2 & 2 & -2 \\ 2 & 10 & 2 \end{vmatrix} + 3\cdot(-1)^{4+3} \begin{vmatrix} 3 & 3 & 5 \\ 2 & 2 & -2 \\ 4 & 1 & 0 \end{vmatrix}$$

$$= -3\cdot[12+(-12)+100-20-(-60)-12] - 3\cdot[0+(-24)+10-40-(-6)-0]$$

$$= -384 + 144$$

$$= -240$$

即 $\det(A) = -240$

33. $$\begin{vmatrix} \sin\theta & \cos\theta & 0 \\ -\cos\theta & \sin\theta & 0 \\ \sin\theta-\cos\theta & \sin\theta+\cos\theta & 1 \end{vmatrix} = 1\cdot(-1)^{3+3}\cdot \begin{vmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{vmatrix} = \sin^2\theta - (-\cos^2\theta) = \sin^2\theta + \cos^2\theta = 1$$

The determinat is independent of θ.