第十一周周一作业 5月11日.

Section 5.1

6 (c) $\begin{bmatrix} -2 & 0 & 1 \\ -6 & -2 & 0 \\ 19 & 5 & -4 \end{bmatrix}$ The characteristic equation is $\det(\lambda I - A) = 0$

$(\lambda+2)^2(\lambda+4) + 30 - 19(\lambda+2) = 0$

$\lambda^3 + 8\lambda^2 + \lambda + 8 = 0$

(d) $\begin{bmatrix} -1 & 0 & 1 \\ -1 & 3 & 0 \\ -4 & 13 & -1 \end{bmatrix}$ The characteristic equation is $\det(\lambda I - A) = 0$

$(\lambda+1)^2(\lambda-3) + 13 + 4(\lambda-3) = 0$

$\lambda^3 - \lambda^2 - \lambda - 2 = 0$

7. (c) $\lambda^3 + 8\lambda^2 + \lambda + 8 = 0$ We have $\lambda_1 = -8$

$(\lambda+8)(\lambda^2+1) = 0$

(d) $\lambda^3 - \lambda^2 - \lambda - 2 = 0$ We have $\lambda_1 = 2$

$(\lambda-2)(\lambda^2+\lambda+1) = 0$

8. (c) $\lambda = -8$ $(\lambda I - A)\vec{x} = \begin{bmatrix} -6 & 0 & -1 \\ 6 & -6 & 0 \\ -19 & -5 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0}$

$\therefore \vec{x} = t \begin{bmatrix} 1 \\ 1 \\ -6 \end{bmatrix}$ The basis for eigenspace corresponding to $\lambda = -8$: $\begin{bmatrix} 1 \\ 1 \\ -6 \end{bmatrix}$

(d). $\lambda = 2$ $(\lambda I - A)\vec{x} = \begin{bmatrix} 3 & 0 & -1 \\ 1 & -1 & 0 \\ 4 & -13 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0}$

$\therefore \vec{x} = t \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$ The basis for eigenspace corresponding to $\lambda = 2$: $\begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$

16. (a). $\det(\lambda I - A) = p(\lambda) = \lambda^3 - 2\lambda^2 + \lambda + 5$

Assume $\lambda = 0$ $\det(-A) = (-1)^3 \det(A) = -\det(A) = 5$

$\therefore \det(A) = -5$

(b). $\det(\lambda I - A) = p(\lambda) = \lambda^4 - \lambda^3 + 7$

Assume $\lambda = 0$ $\det(-A) = (-1)^4 \det(A) = \det(A) = 7$

$\therefore \det(A) = 7$

17. (a) Prove: We have A is an $n \times n$ matrix.

Assume $A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$

The characteristic polynomial is $\det(\lambda I - A) = \begin{vmatrix} \lambda - a_{11} & -a_{12} & \cdots & -a_{1n} \\ -a_{21} & \lambda - a_{22} & \cdots & -a_{2n} \\ \vdots & \vdots & & \vdots \\ -a_{n1} & -a_{n2} & \cdots & \lambda - a_{nn} \end{vmatrix}$

So we get $\det(\lambda I - A) = \lambda^n + c_1 \lambda^{n-1} + \cdots + c_n$

Then, the degree of the characteristic polynomial is n.

(b). According to (a) the characteristic polynomial is

$$\det(\lambda I - A) = \begin{vmatrix} \lambda - a_{11} & -a_{12} & \cdots & -a_{1n} \\ -a_{21} & \lambda - a_{22} & \cdots & -a_{2n} \\ \vdots & \vdots & & \vdots \\ -a_{n1} & -a_{n2} & \cdots & \lambda - a_{nn} \end{vmatrix} = (\lambda - a_{11})(\lambda - a_{12}) \cdots (\lambda - a_{nn}) + C_2(\lambda) = \lambda^n + C_1(\lambda) + C_2(\lambda)$$

The degree of $C_1(\lambda)$ and $C_2(\lambda)$ are less than n.

Then the coefficient of λ^n in characteristic polynomial is 1

Section 5.2

1. $\det(A) = -1$ $\det(B) = -2$

$\det(A) \neq \det(B)$ $\therefore$ A and B are not similar matrices.

2. $\det(A) = 18$ $\det(B) = 14$

$\det(A) \neq \det(B)$ $\therefore$ A and B are not similar matrices.

3. $\det(A) = 1$ $\det(B) = 0$

$\det(A) \neq \det(B)$ $\therefore$ A and B are not similar matrices.

4 $\operatorname{rank}(A) = 1$ $\operatorname{rank}(B) = 2$

$\operatorname{rank}(A) \neq \operatorname{rank}(B)$ $\therefore$ A and B are not similar matrices.

第十一周周三作业 5月13日.

Section 5.2

5. $\lambda^2(\lambda-1)(\lambda-2)^3=0$ We have $\lambda=0$, $\lambda=1$ or $\lambda=2$.

When $\lambda=0$ the possible dimensions for eigenspaces of A is 1 or 2

When $\lambda=1$ the possible dimensions for eigenspaces of A is 1

When $\lambda=2$ the possible dimensions for eigenspaces of A is 1 or 2 or 3

6 (a). $\det(\lambda I-A)=\begin{vmatrix}\lambda-4 & 0 & -1\\ -2 & \lambda-3 & -2\\ -1 & 0 & \lambda-4\end{vmatrix}=(\lambda-3)(\lambda-4)^2-(\lambda-3)=(\lambda-3)^2(\lambda-5)$

Assume $\det(\lambda I-A)=0$ we have $\lambda=3$ or $\lambda=5$

So the eigenvalues of A is 3 or 5.

(b). When $\lambda=3$, $(\lambda I-A)\vec{x}=\begin{pmatrix}-1 & 0 & -1\\ -2 & 0 & -2\\ -1 & 0 & -1\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\vec{0}$ $\vec{x}=t\begin{pmatrix}0\\1\\0\end{pmatrix}+s\begin{pmatrix}1\\0\\-1\end{pmatrix}$ rank $=2$

When $\lambda=5$, $(\lambda I-A)\vec{x}=\begin{pmatrix}1 & 0 & -1\\ -2 & 2 & -2\\ -1 & 0 & 1\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\vec{0}$ $\vec{x}=t\begin{pmatrix}1\\2\\1\end{pmatrix}$ rank $=1$

(c). A is diagonalizable. Because for any λ the geometric multiplicity equals to algebraic multiplicity.

16 $\det(\lambda I-A)=\begin{vmatrix}\lambda-19 & 9 & 6\\ -25 & \lambda+11 & 9\\ -17 & 9 & \lambda+4\end{vmatrix}=(\lambda-19)(\lambda+11)(\lambda+4)-81\times17-6\times225+17\times6(\lambda+11)-81(\lambda-19)+225(\lambda+4)$

$=(\lambda-1)^2(\lambda-2)=0$

$\therefore \lambda=1$ or $\lambda=2$

When $\lambda=1$ $(\lambda I-A)\vec{x}=\begin{pmatrix}-18 & 9 & 6\\ -25 & 12 & 9\\ -17 & 9 & 5\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\vec{0}$ $\vec{x}=t\begin{pmatrix}1\\ \frac{4}{3}\\ 1\end{pmatrix}$ rank $=1$

When $\lambda=2$ $(\lambda I-A)\vec{x}=\begin{pmatrix}-17 & 9 & 6\\ -25 & 13 & 9\\ -17 & 9 & 6\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\vec{0}$ $\vec{x}=t\begin{pmatrix}1\\1\\ \frac{4}{3}\end{pmatrix}$ rank $=1$

$\because$ When $\lambda=1$. the geometric multiplicity doesn't equal to algebraic multiplicity

$\therefore$ A is not diagonalizable.

22. $\det(\lambda I - A) = \begin{vmatrix} \lambda-1 & 0 \\ 1 & \lambda-2 \end{vmatrix} = (\lambda-1)(\lambda-2)$

So we have $\lambda=1$ or $\lambda=2$

When $\lambda=2$ $(\lambda I - A)\vec{x} = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \vec{0}$ $\quad \vec{x} = t\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $\quad \vec{p}_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

When $\lambda=1$ $(\lambda I - A)\vec{x} = \begin{pmatrix} 0 & 0 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \vec{0}$ $\quad \vec{x} = t\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\quad \vec{p}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$P = (\vec{p}_1 \ \vec{p}_2) = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$ $\quad P^{-1} = \frac{1}{-1}\begin{pmatrix} 1 & -1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 1 \\ 1 & 0 \end{pmatrix}$

$D = P^{-1}AP = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$ $\quad A = PDP^{-1}$

$A^{10} = PD^{10}P^{-1} = \begin{pmatrix} 1 & 0 \\ -1023 & 1024 \end{pmatrix}$