5月27日 第十三周 周三作业.

Section 6.3

5. (a): $\langle p_1, p_2 \rangle = \frac{2}{3}\times\frac{2}{3} - \frac{2}{3}\times\frac{1}{3} - \frac{1}{3}\times\frac{2}{3} = 0$ $\quad \langle p_1, p_3 \rangle = \frac{2}{3}\times\frac{1}{3} - \frac{2}{3}\times\frac{2}{3} + \frac{1}{3}\times\frac{2}{3} = 0$ $\quad \langle p_2, p_3 \rangle = \frac{2}{3}\times\frac{1}{3} + \frac{1}{3}\times\frac{2}{3} - \frac{2}{3}\times\frac{2}{3} = 0$

(b): $\langle p_1, p_2 \rangle = 1\times0 + 0\times\frac{1}{\sqrt{2}} + 0\times\frac{1}{\sqrt{2}} = 0$ $\quad \langle p_1, p_3 \rangle = 1\times0 + 0\times0 + 0\times1 = 0$ $\quad \langle p_2, p_3 \rangle = 0\times0 + \frac{1}{\sqrt{2}}\times0 + \frac{1}{\sqrt{2}}\times1 = \frac{\sqrt{2}}{2}$

So the set (a) of polynomials are orthonormal.

6. (a). $\left\langle \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & \frac{2}{3} \\ \frac{1}{3} & -\frac{2}{3} \end{bmatrix} \right\rangle = 0$ $\quad \left\langle \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & \frac{2}{3} \\ -\frac{2}{3} & \frac{1}{3} \end{bmatrix} \right\rangle = 0$ $\quad \left\langle \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & \frac{1}{3} \\ \frac{2}{3} & \frac{2}{3} \end{bmatrix} \right\rangle = 0$

$\left\langle \begin{bmatrix} 0 & \frac{2}{3} \\ \frac{1}{3} & -\frac{2}{3} \end{bmatrix}, \begin{bmatrix} 0 & \frac{2}{3} \\ -\frac{2}{3} & \frac{1}{3} \end{bmatrix} \right\rangle = \frac{4}{9} - \frac{2}{9} - \frac{2}{9} = 0$ $\quad \left\langle \begin{bmatrix} 0 & \frac{2}{3} \\ \frac{1}{3} & -\frac{2}{3} \end{bmatrix}, \begin{bmatrix} 0 & \frac{1}{3} \\ \frac{2}{3} & \frac{2}{3} \end{bmatrix} \right\rangle = \frac{2}{9} + \frac{2}{9} - \frac{4}{9} = 0$ $\quad \left\langle \begin{bmatrix} 0 & \frac{2}{3} \\ -\frac{2}{3} & \frac{1}{3} \end{bmatrix}, \begin{bmatrix} 0 & \frac{1}{3} \\ \frac{2}{3} & \frac{2}{3} \end{bmatrix} \right\rangle = \frac{2}{9} - \frac{4}{9} + \frac{2}{9} = 0$

(b). $\left\langle \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \right\rangle = 0$ $\quad \left\langle \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} \right\rangle = 0$ $\quad \left\langle \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & -1 \end{bmatrix} \right\rangle = 0$

$\left\langle \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} \right\rangle = 0$ $\quad \left\langle \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & -1 \end{bmatrix} \right\rangle = 0$ $\quad \left\langle \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & -1 \end{bmatrix} \right\rangle = 0$

So the sets (a) and (b) of matrices are both orthonormal.

9. Proof: $\langle \vec{v}_1, \vec{v}_2 \rangle = -\frac{3}{5}\times\frac{4}{5} + \frac{4}{5}\times\frac{3}{5} = 0$ $\quad \langle \vec{v}_1, \vec{v}_3 \rangle = 0$ $\quad \langle \vec{v}_2, \vec{v}_3 \rangle = 0$

$\|\vec{v}_1\| = \sqrt{\langle \vec{v}_1, \vec{v}_1 \rangle} = 1$ $\quad \|\vec{v}_2\| = \sqrt{\langle \vec{v}_2, \vec{v}_2 \rangle} = 1$ $\quad \|\vec{v}_3\| = \sqrt{\langle \vec{v}_3, \vec{v}_3 \rangle} = 1$

So $\vec{v}_1, \vec{v}_2, \vec{v}_3$ form an orthonormal basis for R^3 with the Euclidean inner product.

(b). $\vec{u} = \langle \vec{u}, \vec{v}_1 \rangle \vec{v}_1 + \langle \vec{u}, \vec{v}_2 \rangle \vec{v}_2 + \langle \vec{u}, \vec{v}_3 \rangle \vec{v}_3$

$= -\frac{37}{5}\vec{v}_1 - \frac{9}{5}\vec{v}_2 + 4\vec{v}_3$

16(b). $\|\vec{v}_1\| = \sqrt{\langle \vec{v}_1, \vec{v}_1 \rangle} = 1$ $\quad \|\vec{v}_2\| = \sqrt{\langle \vec{v}_2, \vec{v}_2 \rangle} = 1$

$\text{proj}_W \vec{x} = \langle \vec{x}, \vec{v}_1 \rangle \vec{v}_1 + \langle \vec{x}, \vec{v}_2 \rangle \vec{v}_2$

$= \vec{v}_1 + 2\vec{v}_2$

So $\text{proj}_W \vec{x} = \left(\frac{3}{2}, \frac{3}{2}, -\frac{1}{2}, -\frac{1}{2}\right)$

25. $\vec{v}_1 = \vec{u}_1 = (1, 1, 1)$

$\vec{v}_2 = \vec{u}_2 - \text{proj}_{W_1}\vec{u}_2 = (1, 1, -1)$

$\vec{v}_3 = \vec{u}_3 - \text{proj}_{W_2}\vec{u}_3 = (2, -1, 0)$

29. (e).

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \\ 0 & 3 & 1 \end{bmatrix}$$

Assume $\vec{v}_1 = (1, 1, 0)$ $\quad \vec{v}_2 = (2, 1, 3)$ $\quad \vec{v}_3 = (1, 1, 1)$

$\det(A) = 1+3+0-0-2-3 = -1 \neq 0$ So $\vec{v}_1, \vec{v}_2, \vec{v}_3$ are linearly independent.

$\vec{u}_1 = (1,1,0)$ $\quad \vec{u}_2 = \vec{v}_2 - \mathrm{proj}_{W_1}\vec{v}_2 = (\frac{1}{2}, -\frac{1}{2}, 3)$ $\quad \vec{u}_3 = \vec{v}_3 - \mathrm{proj}_{W_2}\vec{v}_3 = (\frac{3}{19}, -\frac{3}{19}, -\frac{1}{19})$

$\vec{u}_1' = \frac{\vec{u}_1}{\|\vec{u}_1\|} = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0)$ $\quad \vec{u}_2' = \frac{\vec{u}_2}{\|\vec{u}_2\|} = (\frac{1}{\sqrt{38}}, -\frac{1}{\sqrt{38}}, \frac{6}{\sqrt{38}})$ $\quad \vec{u}_3' = \frac{\vec{u}_3}{\|\vec{u}_3\|} = (\frac{3}{\sqrt{19}}, -\frac{3}{\sqrt{19}}, -\frac{1}{\sqrt{19}})$

Then $$Q = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{38}} & \frac{3}{\sqrt{19}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{38}} & -\frac{3}{\sqrt{19}} \\ 0 & \frac{6}{\sqrt{38}} & -\frac{1}{\sqrt{19}} \end{bmatrix}$$

$$R = \begin{bmatrix} \sqrt{2} & \frac{3}{2}\sqrt{2} & \sqrt{2} \\ 0 & \frac{\sqrt{38}}{2} & \frac{3\sqrt{38}}{19} \\ 0 & 0 & -\frac{1}{\sqrt{19}} \end{bmatrix}$$

$$A = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{38}} & \frac{3}{\sqrt{19}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{38}} & -\frac{3}{\sqrt{19}} \\ 0 & \frac{6}{\sqrt{38}} & -\frac{1}{\sqrt{19}} \end{bmatrix} \begin{bmatrix} \sqrt{2} & \frac{3\sqrt{2}}{2} & \sqrt{2} \\ 0 & \frac{\sqrt{38}}{2} & \frac{3\sqrt{38}}{19} \\ 0 & 0 & -\frac{\sqrt{19}}{19} \end{bmatrix}$$