4月8日 第六周周三作业

Section 3.5

3. $\vec{u}=(-6,4,2)$ $\vec{v}=(3,1,5)$

解: $\vec{n}=\vec{u}\times\vec{v}=(18,36,-18)$ then $\vec{n}\cdot\vec{u}=0$ $\vec{n}\cdot\vec{v}=0$ such that $\vec{n}\perp\vec{u}$ and $\vec{n}\perp\vec{v}$

12. $\overrightarrow{P_1P_2}=(2,2)=\overrightarrow{P_4P_3}=(2,2)$ $\overrightarrow{P_1P_4}=(4,0)=\overrightarrow{P_2P_3}=(4,0)$

$S=\|\overrightarrow{P_1P_2}\times\overrightarrow{P_1P_4}\|=8$

The area of the parallelogram with the given vertices is 8.

13. $\overrightarrow{AB}=(1,4)$ $\overrightarrow{AC}=(-3,2)$

$S=\frac{1}{2}\|\overrightarrow{AB}\times\overrightarrow{AC}\|=\frac{1}{2}\times 14=7$

The area of the triangle with the given vertices is 7.

20. Assume $\vec{n}=\vec{u}\times\vec{v}=(-1,-1,3)$ $\vec{n}\cdot\vec{w}=-1+1+0=0$ such that $\vec{n}\perp\vec{w}$

$\vec{n}\perp\vec{u}$ $\vec{n}\perp\vec{v}$ $\vec{n}\perp\vec{w}$ so $\vec{u},\vec{v},\vec{w}$ lie in the same plane

31. (a) Assume $\vec{n}_1=\vec{u}\times(\vec{v}\times\vec{w})$ $\vec{u},\vec{v},\vec{w}\neq\vec{0}$

Then we have $\vec{u}\times(\vec{v}\times\vec{w})=(\vec{u}\cdot\vec{w})\vec{v}-(\vec{u}\cdot\vec{v})\vec{w}$ according to Thm 3.5.1

So $\vec{n}_1=(\vec{u}\cdot\vec{w})\vec{v}-(\vec{u}\cdot\vec{v})\vec{w}$ which shows that $\vec{n}_1=\vec{u}\times(\vec{v}\times\vec{w})$ lies in the plane determined by $\vec{v}$ and $\vec{w}$

(b) Assume $\vec{n}_2=(\vec{u}\times\vec{v})\times\vec{w}$

Then we have $\vec{n}_2=(\vec{u}\cdot\vec{w})\vec{v}-(\vec{v}\cdot\vec{w})\vec{u}$ according to Thm 3.5.1

which shows that $\vec{n}_2=(\vec{u}\times\vec{v})\times\vec{w}$ lies in the plane determined by $\vec{u}$ and $\vec{v}$.

34. Prove: Because of $\vec{u}\cdot\vec{v}\neq 0$, we have $\|\vec{u}\|\,\|\vec{v}\|\neq 0$ and $\cos\theta\neq 0$. We know $\vec{u}\cdot\vec{v}=\|\vec{u}\|\,\|\vec{v}\|\cos\theta$ so $\cos\theta=\frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\,\|\vec{v}\|}\neq 0$

We also have $\|\vec{u}\times\vec{v}\|=\|\vec{u}\|\,\|\vec{v}\|\sin\theta$ so $\sin\theta=\frac{\|\vec{u}\times\vec{v}\|}{\|\vec{u}\|\,\|\vec{v}\|}$

Then $\tan\theta=\frac{\sin\theta}{\cos\theta}=\frac{\|\vec{u}\times\vec{v}\|}{(\vec{u}\cdot\vec{v})}$