第十五周 周一作业 6月8日

Section 7.3

6. $Q_A(\vec{x}) = 5x_1^2 + 2x_2^2 + 4x_3^2 + 4x_1x_2$

$\therefore A = \begin{bmatrix} 5 & 2 & 0 \\ 2 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ $\quad \det(\lambda I - A) = \begin{vmatrix} \lambda-5 & -2 & 0 \\ -2 & \lambda-2 & 0 \\ 0 & 0 & \lambda-4 \end{vmatrix} = (\lambda-1)(\lambda-4)(\lambda-6) = 0$ We have $\lambda=1$ or $\lambda=4$ or $\lambda=6$

When $\lambda=1$ $(\lambda I - A)\vec{x} = \begin{bmatrix} -4 & -2 & 0 \\ -2 & -1 & 0 \\ 0 & 0 & -3 \end{bmatrix}\vec{x} = \vec{0}$ $\quad \vec{x} = t\begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}$

When $\lambda=4$ $(\lambda I - A)\vec{x} = \begin{bmatrix} -1 & -2 & 0 \\ -2 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}\vec{x} = \vec{0}$ $\quad x = t\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

When $\lambda=6$ $(\lambda I - A)\vec{x} = \begin{bmatrix} 1 & -2 & 0 \\ -2 & 4 & 0 \\ 0 & 0 & 2 \end{bmatrix}\vec{x} = \vec{0}$ $\quad x = t\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$

$\vec{v}_1 = \begin{bmatrix} \frac{1}{\sqrt{5}} \\ -\frac{2}{\sqrt{5}} \\ 0 \end{bmatrix}$ $\quad \vec{v}_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ $\quad \vec{v}_3 = \begin{bmatrix} \frac{2}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \\ 0 \end{bmatrix}$

$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{5}} & 0 & \frac{2}{\sqrt{5}} \\ -\frac{2}{\sqrt{5}} & 0 & \frac{1}{\sqrt{5}} \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$ $\quad Q = y_1^2 + 4y_2^2 + 6y_3^2$

8. $Q_A(\vec{x}) = 2x_1^2 + 5x_2^2 + 5x_3^2 + 4x_1x_2 - 4x_1x_3 - 8x_2x_3$

$\therefore A = \begin{bmatrix} 2 & 2 & -2 \\ 2 & 5 & -4 \\ -2 & -4 & 5 \end{bmatrix}$ $\quad \det(\lambda I - A) = \begin{vmatrix} \lambda-2 & -2 & 2 \\ -2 & \lambda-5 & 4 \\ 2 & 4 & \lambda-5 \end{vmatrix} = (\lambda-1)^2(\lambda-10) = 0$ We have $\lambda=1$ or $\lambda=10$.

When $\lambda=1$: $(\lambda I - A)\vec{x} = \begin{bmatrix} -1 & -2 & 2 \\ -2 & -4 & 4 \\ 2 & 4 & -4 \end{bmatrix}\vec{x} = \vec{0}$ $\quad \vec{x} = t\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + s\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$ $\quad \vec{v}_1 = (-2, 1, 0)$

$\vec{v}_2 = (2, 0, 1)$

When $\lambda=10$: $(\lambda I - A)\vec{x} = \begin{bmatrix} 8 & -2 & 2 \\ -2 & 5 & 4 \\ 2 & 4 & 5 \end{bmatrix}\vec{x} = \vec{0}$ $\quad \vec{x} = t\begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$ $\quad \vec{v}_3 = (1, 2, -2)$

$\vec{u}_1 = (-2, 1, 0)$ $\quad \vec{u}_2 = \vec{v}_2 - \text{proj}_{W_1}\vec{v}_2 = (\frac{2}{5}, \frac{4}{5}, 1)$ $\quad \vec{u}_3 = \vec{v}_3 - \text{proj}_{W_2}\vec{v}_3 = (1, 2, -2)$

$\vec{u}_1' = (-\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}, 0)$ $\quad \vec{u}_2' = (\frac{2}{3\sqrt{5}}, \frac{4}{3\sqrt{5}}, \frac{5}{3\sqrt{5}})$ $\quad \vec{u}_3' = (\frac{1}{3}, \frac{2}{3}, -\frac{2}{3})$

$\therefore \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -\frac{2}{\sqrt{5}} & \frac{2}{3\sqrt{5}} & \frac{1}{3} \\ \frac{1}{\sqrt{5}} & \frac{4}{3\sqrt{5}} & \frac{2}{3} \\ 0 & \frac{5}{3\sqrt{5}} & -\frac{2}{3} \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$ $\quad Q = y_1^2 + y_2^2 + 10y_3^2$

11. (a) $2x^2+5y^2=20 \Rightarrow \frac{x^2}{10}+\frac{y^2}{4}=1$ 椭圓.

(b). $x^2-y^2-8=0 \Rightarrow \frac{x^2}{8}-\frac{y^2}{8}=1$ 双曲线.

(c). $7y^2-2x=0 \Rightarrow y^2=\frac{2}{7}x$ 抛物线.

(d). $x^2+y^2-25=0 \Rightarrow x^2+y^2=25$ 圓.

15. $11x^2+24xy+4y^2-15=0$

$A=\begin{bmatrix} 11 & 12 \\ 12 & 4 \end{bmatrix}$ $\det(\lambda I-A)=\begin{vmatrix} \lambda-11 & -12 \\ -12 & \lambda-4 \end{vmatrix}=(\lambda+5)(\lambda-20)=0$ We have $\lambda=-5$ or $\lambda=20$.

When $\lambda=-5$ $(\lambda I-A)\vec{x}=\begin{bmatrix} -16 & -12 \\ -12 & -9 \end{bmatrix}\vec{x}=\vec{0}$ $\vec{x}=t\begin{bmatrix} 3 \\ -4 \end{bmatrix}$

When $\lambda=20$ $(\lambda I-A)\vec{x}=\begin{bmatrix} 9 & -12 \\ -12 & 16 \end{bmatrix}\vec{x}=\vec{0}$ $\vec{x}=t\begin{bmatrix} 4 \\ 3 \end{bmatrix}$

$\vec{u}_1=\begin{pmatrix} \frac{3}{5} \\ -\frac{4}{5} \end{pmatrix}$ $\vec{u}_2=\begin{pmatrix} \frac{4}{5} \\ \frac{3}{5} \end{pmatrix}$ $P=\begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ -\frac{4}{5} & \frac{3}{5} \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ -\frac{4}{5} & \frac{3}{5} \end{bmatrix}\begin{bmatrix} x' \\ y' \end{bmatrix}$ $-5x'^2+20y'^2-15=0$

$(x')^2-4(y')^2+3=0$ $\theta=\arctan\frac{4}{3}$

25. (a) $A=\begin{bmatrix} 5 & -2 \\ -2 & 5 \end{bmatrix}$ $\det(\lambda I-A)=\begin{vmatrix} \lambda-5 & 2 \\ 2 & \lambda-5 \end{vmatrix}=(\lambda-3)(\lambda-7)=0$

We have $\lambda=3>0$ or $\lambda=7>0$

So A is positive definite.

(b) $A=\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

$\det(A_1)=2>0$ $\det(A_2)=\begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix}=3>0$ $\det(A_3)=\begin{vmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & 5 \end{vmatrix}=15>0$

So A is positive definite.

Section 7.4.

6. $f(x,y,z) = 2x^2+y^2+z^2+2xy+2xz \qquad x^2+y^2+z^2=1$

$A=\begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \qquad \det(\lambda I - A) = \begin{vmatrix} \lambda-2 & -1 & -1 \\ -1 & \lambda-1 & 0 \\ -1 & 0 & \lambda-1 \end{vmatrix} = \lambda\cdot(\lambda-1)(\lambda-3) = 0$ We have $\lambda_1=3 \quad \lambda_2=1 \quad \lambda_3=0.$

When $\lambda=\lambda_1=3 \qquad (\lambda I-A)\vec{x} = \begin{bmatrix} 1 & -1 & -1 \\ -1 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}\vec{x}=0 \qquad \vec{x} = t\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} \qquad \vec{u}_1 = \left(\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$

When $\lambda=\lambda_3=0 \qquad (\lambda I-A)\vec{x} = \begin{bmatrix} -2 & -1 & -1 \\ -1 & -1 & 0 \\ -1 & 0 & -1 \end{bmatrix}\vec{x}=0 \qquad \vec{x} = t\begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} \qquad \vec{u}_3 = \left(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right)$

$\therefore$ Maximum : 3 at $\left(\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$ and $\left(-\frac{2}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}\right)$

minimum : 0 at $\left(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right)$ and $\left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

第十五周周三作业 6月10日.

Section 7.4

7. $4x^2+8y^2=16$ Then. $(\frac{x}{2})^2+(\frac{y}{\sqrt{2}})^2=1$

Assume $x=2x_1$ $y=\sqrt{2}y_1$ $z=xy=2\sqrt{2}x_1y_1$ $x_1^2+y_1^2=1$

$A=\begin{bmatrix}0 & \sqrt{2}\\ \sqrt{2} & 0\end{bmatrix}$ $\det(\lambda I-A)=\begin{vmatrix}\lambda & -\sqrt{2}\\ -\sqrt{2} & \lambda\end{vmatrix}=\lambda^2-2=0$ We have $\lambda=\sqrt{2}$ or $\lambda=-\sqrt{2}$.

When $\lambda=\sqrt{2}$ $(\lambda I-A)\vec{x}=\begin{bmatrix}\sqrt{2} & -\sqrt{2}\\ -\sqrt{2} & \sqrt{2}\end{bmatrix}\vec{x}=0$ $\vec{x}=t\begin{bmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{bmatrix}$

When $\lambda=-\sqrt{2}$ $(\lambda I-A)\vec{x}=\begin{bmatrix}-\sqrt{2} & -\sqrt{2}\\ -\sqrt{2} & -\sqrt{2}\end{bmatrix}\vec{x}=0$ $\vec{x}=t\begin{bmatrix}\frac{1}{\sqrt{2}}\\ -\frac{1}{\sqrt{2}}\end{bmatrix}$

$\therefore$ Maximum $z=\sqrt{2}$ at $(\sqrt{2},1)$ and $(-\sqrt{2},-1)$

minimum $z=-\sqrt{2}$ at $(\sqrt{2},-1)$ and $(-\sqrt{2},1)$

11. (a). $f(x,y)=4xy-x^4-y^4$

$f'_x=4y-4x^3$ $f'_y=4x-4y^3$

When $(x,y)=(0,0)$, $(1,1)$ or $(-1,-1)$, $f'_x=f'_y=0$

So $f(x,y)$ has critical points at $(0,0)$, $(1,1)$ and $(-1,-1)$

(b). $f_{xx}=-12x^2$ $f_{xy}=4$ $f_{yx}=4$ $f_{yy}=-12y^2$

$H(0,0)=\begin{pmatrix}0 & 4\\ 4 & 0\end{pmatrix}$ $\det(H(0,0))=-16<0$ $\therefore (0,0)$ is a saddle point.

$H(1,1)=\begin{pmatrix}-12 & 4\\ 4 & -12\end{pmatrix}$ $\det(H(1,1))=128>0$ $-12<0$

$\therefore (1,1)$ is a relative maxima point.

$H(-1,-1)=\begin{pmatrix}-12 & 4\\ 4 & -12\end{pmatrix}$ $\det(H(-1,-1))=128>0$ $-12<0$

$\therefore (-1,-1)$ is a relative maxima point.

So f has relative maxima at $(1,1)$ and $(-1,-1)$

and a saddle point at $(0,0)$.

Prove Thm 7.5.4 (a)(d)(e)

(a) $\Rightarrow$ (d): $\because$ A is unitary $\therefore$ $A^{-1} = A^*$ Then $A^*A = I$

Assume $A = [\ \vec{u}_1 \ \ \vec{u}_2 \cdots \vec{u}_n\ ]$ $A^* = \begin{bmatrix} \bar{\vec{u}}_1 \\ \bar{\vec{u}}_2 \\ \vdots \\ \bar{\vec{u}}_n \end{bmatrix}$ $A^*A = \begin{bmatrix} \langle \bar{\vec{u}}_1, \vec{u}_1 \rangle & \langle \bar{\vec{u}}_1, \vec{u}_2 \rangle & \cdots & \langle \bar{\vec{u}}_1, \vec{u}_n \rangle \\ \langle \bar{\vec{u}}_2, \vec{u}_1 \rangle & \langle \bar{\vec{u}}_2, \vec{u}_2 \rangle & \cdots & \langle \bar{\vec{u}}_2, \vec{u}_n \rangle \\ \vdots & \vdots & & \vdots \\ \langle \bar{\vec{u}}_n, \vec{u}_1 \rangle & \langle \bar{\vec{u}}_n, \vec{u}_2 \rangle & \cdots & \langle \bar{\vec{u}}_n, \vec{u}_n \rangle \end{bmatrix} = I$

$\therefore \langle \bar{\vec{u}}_i, \vec{u}_i \rangle = 1 \quad \forall i \in [1, n] \quad i = N_+ \qquad \langle \bar{\vec{u}}_i, \vec{u}_j \rangle = 0 \quad \forall i \in [1, n] \ i \in N_+ \ j \in [1, n] \ j \in N_+$ and $j \neq i$

$\therefore$ $\vec{u}_1, \vec{u}_2, \cdots \vec{u}_n$ are linearly independent.

So $\vec{u}_1, \vec{u}_2 \cdots \vec{u}_n$ form an orthonormal set in C^n.

(d) $\Rightarrow$ (a): Assume $A = [\ \vec{u}_1 \ \ \vec{u}_2 \cdots \vec{u}_n\ ]$ $A^* = \begin{bmatrix} \bar{\vec{u}}_1 \\ \bar{\vec{u}}_2 \\ \vdots \\ \bar{\vec{u}}_n \end{bmatrix}$ $A^*A = \begin{bmatrix} \langle \bar{\vec{u}}_1, \vec{u}_1 \rangle & \langle \bar{\vec{u}}_1, \vec{u}_2 \rangle & \cdots & \langle \bar{\vec{u}}_1, \vec{u}_n \rangle \\ \langle \bar{\vec{u}}_2, \vec{u}_1 \rangle & \langle \bar{\vec{u}}_2, \vec{u}_2 \rangle & \cdots & \langle \bar{\vec{u}}_2, \vec{u}_n \rangle \\ \vdots & \vdots & & \vdots \\ \langle \bar{\vec{u}}_n, \vec{u}_1 \rangle & \langle \bar{\vec{u}}_n, \vec{u}_2 \rangle & \cdots & \langle \bar{\vec{u}}_n, \vec{u}_n \rangle \end{bmatrix}$

$\because$ The column vectors of A form an orthonormal set in C^n

$\therefore \langle \bar{\vec{u}}_i, \vec{u}_i \rangle = 1 \quad \forall i \in [1, n] \quad i = N_+ \qquad \langle \bar{\vec{u}}_i, \vec{u}_j \rangle = 0 \quad \forall i \in [1, n] \ i \in N_+ \ j \in [1, n] \ j \in N_+$ and $j \neq i$

$\therefore A^*A = I$ Then $A^*AA^{-1} = IA^{-1}$

$\therefore A^* = A^{-1}$ A is unitary

(a) $\Rightarrow$ (e): $\because$ A is unitary $\therefore A^* = A^{-1}$ $AA^* = I$

Assume $A = \begin{bmatrix} \vec{v}_1 \\ \vec{v}_2 \\ \vdots \\ \vec{v}_n \end{bmatrix}$ $A^* = [\ \bar{\vec{v}}_1 \ \ \bar{\vec{v}}_2 \ \cdots \ \bar{\vec{v}}_n\ ]$ $AA^* = \begin{bmatrix} \langle \vec{v}_1, \bar{\vec{v}}_1 \rangle & \langle \vec{v}_1, \bar{\vec{v}}_2 \rangle & \cdots & \langle \vec{v}_1, \bar{\vec{v}}_n \rangle \\ \langle \vec{v}_2, \bar{\vec{v}}_1 \rangle & \langle \vec{v}_2, \bar{\vec{v}}_2 \rangle & \cdots & \langle \vec{v}_2, \bar{\vec{v}}_n \rangle \\ \vdots & \vdots & & \vdots \\ \langle \vec{v}_n, \bar{\vec{v}}_1 \rangle & \langle \vec{v}_n, \bar{\vec{v}}_2 \rangle & \cdots & \langle \vec{v}_n, \bar{\vec{v}}_n \rangle \end{bmatrix} = I$

$\therefore \langle \vec{v}_i, \bar{\vec{v}}_i \rangle = 1 \quad \forall i \in [1, n] \quad i = N_+ \qquad \langle \vec{v}_i, \bar{\vec{v}}_j \rangle = 0 \quad \forall i \in [1, n] \ i \in N_+ \ j \in [1, n] \ j \in N_+$ and $j \neq i$

$\therefore$ $\vec{v}_1, \vec{v}_2, \cdots \vec{v}_n$ are linearly independent.

So $\vec{v}_1, \vec{v}_2, \cdots \vec{v}_n$ form an orthonormal set in C^n.

(e) $\Rightarrow$ (a): Assume $A = \begin{bmatrix} \vec{v}_1 \\ \vec{v}_2 \\ \vdots \\ \vec{v}_n \end{bmatrix}$ $A^* = [\ \bar{\vec{v}}_1 \ \ \bar{\vec{v}}_2 \ \cdots \ \bar{\vec{v}}_n\ ]$ $AA^* = \begin{bmatrix} \langle \vec{v}_1, \bar{\vec{v}}_1 \rangle & \langle \vec{v}_1, \bar{\vec{v}}_2 \rangle & \cdots & \langle \vec{v}_1, \bar{\vec{v}}_n \rangle \\ \langle \vec{v}_2, \bar{\vec{v}}_1 \rangle & \langle \vec{v}_2, \bar{\vec{v}}_2 \rangle & \cdots & \langle \vec{v}_2, \bar{\vec{v}}_n \rangle \\ \vdots & \vdots & & \vdots \\ \langle \vec{v}_n, \bar{\vec{v}}_1 \rangle & \langle \vec{v}_n, \bar{\vec{v}}_2 \rangle & \cdots & \langle \vec{v}_n, \bar{\vec{v}}_n \rangle \end{bmatrix}$

$\because$ The row vectors of A form an orthonormal set in C^n

$\therefore \langle \vec{v}_i, \bar{\vec{v}}_i \rangle = 1 \quad \forall i \in [1, n] \quad i = N_+ \qquad \langle \vec{v}_i, \bar{\vec{v}}_j \rangle = 0 \quad \forall i \in [1, n] \ i \in N_+ \ j \in [1, n] \ j \in N_+$ and $j \neq i$

$\therefore AA^* = I \quad A^{-1}AA^* = A^{-1}I$

$\therefore A^* = A^{-1}$ A is unitary.

Section 7.5

3. $A = \begin{bmatrix} 1 & i & 2-3i \\ x & -3 & 1 \\ x & x & 2 \end{bmatrix}$

$A^* = \begin{bmatrix} 1 & x & x \\ -i & -3 & x \\ 2+3i & 1 & 2 \end{bmatrix}$ $\qquad A^* = A$

$\therefore A = \begin{bmatrix} 1 & i & 2-3i \\ -i & -3 & 1 \\ 2+3i & 1 & 2 \end{bmatrix}$

7. $A = \begin{bmatrix} 3 & 2-3i \\ 2+3i & -1 \end{bmatrix}$ $\qquad A^* = A$

$\det(\lambda I - A) = \begin{vmatrix} \lambda-3 & -2+3i \\ -2-3i & \lambda+1 \end{vmatrix} = (\lambda-1)^2 - 17 = 0$ We have $\lambda_1 = 1+\sqrt{17}$ or $\lambda_2 = 1-\sqrt{17}$

λ_1, λ_2 are real numbers.

Assume $\vec{v}_1$ is the eigenvector whe the eigenvalue is λ_1 $\quad A\vec{v}_1 = \lambda_1\vec{v}_1$

$\vec{v}_2$ is the eigenvector whe the eigenvalue is λ_2. $\quad A\vec{v}_2 = \lambda_2\vec{v}_2$

$A\vec{v}_1 \cdot \vec{v}_2 = \vec{v}_1 \cdot (A^*\vec{v}_2) = \vec{v}_1 (A\vec{v}_2)$

Then $\lambda_1 \vec{v}_1 \cdot \vec{v}_2 = \vec{v}_1 \cdot (\lambda_2 \vec{v}_2) = \lambda_2 \vec{v}_1 \cdot \vec{v}_2$ $\quad \lambda_1 \neq 0 \quad \lambda_2 \neq 0 \quad \therefore \vec{v}_1 \cdot \vec{v}_2 = 0$

$\therefore$ The eigenvectors from different eigenspaces are orthogonal.

10. $A = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2}(1+i) & \frac{1}{2}(1+i) \end{bmatrix}$

$\forall \vec{x} = \begin{bmatrix} a+bi \\ c+di \end{bmatrix}$ $\qquad A\vec{x} = \begin{bmatrix} \frac{1}{\sqrt{2}}[(a+c)+(b+d)i] \\ \frac{1}{2}[(b+c-a-d)+(c+d-a-b)i] \end{bmatrix}$

$\|\vec{x}\| = \sqrt{\langle \vec{x}, \bar{\vec{x}} \rangle} = \sqrt{a^2+b^2+c^2+d^2}$

$\|A\vec{x}\| = \sqrt{\langle A\vec{x}, \overline{A\vec{x}} \rangle} = \sqrt{\frac{1}{2}[(a+c)^2+(b+d)^2] + \frac{1}{4}(b+c-a-d)^2 + \frac{1}{4}(c+d-a-b)^2}$

$= \sqrt{\frac{1}{2}(a^2+b^2+c^2+d^2) + ac+bd + \frac{1}{4}(a^2+b^2+c^2+d^2) + \frac{1}{2}(bc+ad-ab-ac-bd-cd) + \frac{1}{4}(a^2+b^2+c^2+d^2) + \frac{1}{2}(cd+ab-ac-ad-bc-bd)}$

$= \sqrt{a^2+b^2+c^2+d^2}$

$\therefore \|A\vec{x}\| = \|\vec{x}\| \quad \therefore$ A is unitary

$$A^{-1} = A^* = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{2}+\frac{1}{2}i \\ \frac{1}{\sqrt{2}} & \frac{1}{2}-\frac{1}{2}i \end{bmatrix}$$

17. $A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & -1 & -1+i \\ 0 & -1-i & 0 \end{bmatrix} \qquad A^* = \begin{bmatrix} 5 & 0 & 0 \\ 0 & -1 & -1+i \\ 0 & -1-i & 0 \end{bmatrix} = A$

$\det(\lambda I - A) = \begin{vmatrix} \lambda-5 & 0 & 0 \\ 0 & \lambda+1 & 1-i \\ 0 & 1+i & \lambda \end{vmatrix} = (\lambda-5)(\lambda-1)(\lambda+2) = 0$ We have $\lambda=5$, $\lambda=1$, $\lambda=-2$.

When $\lambda=5$ $(\lambda I - A)\vec{x} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 6 & 1-i \\ 0 & 1+i & 5 \end{bmatrix} \vec{x} = 0 \qquad \vec{x} = t\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

When $\lambda=1$ $(\lambda I - A)\vec{x} = \begin{bmatrix} -4 & 0 & 0 \\ 0 & 2 & 1-i \\ 0 & 1+i & 1 \end{bmatrix} \vec{x} = 0 \qquad \vec{x} = t \cdot \begin{bmatrix} 0 \\ \frac{-1+i}{2} \\ 1 \end{bmatrix}$

When $\lambda=-2$ $(\lambda I - A)\vec{x} = \begin{bmatrix} -7 & 0 & 0 \\ 0 & -1 & 1-i \\ 0 & 1+i & -2 \end{bmatrix} \vec{x} = 0 \qquad \vec{x} = t\begin{bmatrix} 0 \\ 1-i \\ 1 \end{bmatrix}$

$\vec{u}_1 = (1,0,0) \qquad \vec{u}_2 = (0, \frac{-1+i}{\sqrt{6}}, \frac{2}{\sqrt{6}}) \qquad \vec{u}_3 = (0, \frac{1-i}{\sqrt{3}}, \frac{1}{\sqrt{3}})$

$\therefore P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{-1+i}{\sqrt{6}} & \frac{1-i}{\sqrt{3}} \\ 0 & \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{bmatrix} \qquad D = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$