第九周 周日作业 4月26日

Section 4.7

3 (e) $A = [\vec{v}_1 \vdots \vec{v}_2 \vdots \vec{v}_3 \vdots \vec{v}_4]$

Assume $A\vec{x} = \vec{b}$ $\quad \vec{b} = x_1\vec{v}_1 + x_2\vec{v}_2 + x_3\vec{v}_3 + x_4\vec{v}_4$

Then
$$\begin{aligned} x_1 + 2x_2 \qquad + x_4 &= 4 \\ x_2 + 2x_3 + x_4 &= 3 \\ x_1 + 2x_2 + x_3 + 3x_4 &= 5 \\ x_2 + 2x_3 + 2x_4 &= 7 \end{aligned} \qquad \left[\begin{array}{cccc:c} 1 & 2 & 0 & 1 & 4 \\ 0 & 1 & 2 & 1 & 3 \\ 1 & 2 & 1 & 3 & 5 \\ 0 & 1 & 2 & 2 & 7 \end{array}\right]$$

According to Gaussion Elimination:
$$\begin{bmatrix} 1 & 0 & 0 & 0 & -26 \\ 0 & 1 & 0 & 0 & 13 \\ 0 & 0 & 1 & 0 & -7 \\ 0 & 0 & 0 & 1 & 4 \end{bmatrix}$$

$$\vec{b} = -26\vec{v}_1 + 13\vec{v}_2 - 7\vec{v}_3 + 4\vec{v}_4 \qquad \begin{bmatrix} 4 \\ 3 \\ 5 \\ 7 \end{bmatrix} = -26\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + 13\begin{bmatrix} 2 \\ 1 \\ 2 \\ 1 \end{bmatrix} - 7\begin{bmatrix} 0 \\ 2 \\ 1 \\ 2 \end{bmatrix} + 4\begin{bmatrix} 1 \\ 1 \\ 3 \\ 2 \end{bmatrix}$$

6 (c) $A = \begin{bmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{bmatrix}$

Assume $A\vec{x} = \vec{0}$ We have
$$\begin{aligned} x_1 + 4x_2 + 5x_3 + 2x_4 &= 0 \\ 2x_1 + x_2 + 3x_3 \qquad &= 0 \\ -x_1 + 3x_2 + 2x_3 + 2x_4 &= 0 \end{aligned} \qquad \left[\begin{array}{cccc:c} 1 & 4 & 5 & 2 & 0 \\ 2 & 1 & 3 & 0 & 0 \\ -1 & 3 & 2 & 2 & 0 \end{array}\right]$$

According to Gaussion Elimination:
$$\left[\begin{array}{cccc:c} 1 & 0 & 1 & -\frac{2}{7} & 0 \\ 0 & 1 & 1 & \frac{4}{7} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \qquad x_1 = -r + \tfrac{2}{7}s \quad x_2 = -r - \tfrac{4}{7}s \quad x_3 = r \quad x_4 = s$$

$(x_1, x_2, x_3, x_4) = (-r + \frac{2}{7}s, -r - \frac{4}{7}s, r, s) = r(-1, -1, 1, 0) + s(\frac{2}{7}, -\frac{4}{7}, 0, 1)$

So a basis for the mull space of A is $(-1, -1, 1, 0)$ $(\frac{2}{7}, -\frac{4}{7}, 0, 1)$

7 (c). $\begin{bmatrix} 1 & 2 & 4 & 5 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Bases for the row space of A $\vec{r}_1 = [1 \ 2 \ 4 \ 5]$ $\vec{r}_2 = [0 \ 1 \ -3 \ 0]$ $\vec{r}_3 = [0 \ 0 \ 1 \ -3]$ $\vec{r}_4 = [0 \ 0 \ 0 \ 1]$

Bases for the colume space of A $\vec{c}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ $\vec{c}_2 = \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ $\vec{c}_3 = \begin{bmatrix} 4 \\ -3 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ $\vec{c}_4 = \begin{bmatrix} 5 \\ 0 \\ -3 \\ 1 \\ 0 \end{bmatrix}$

12 (c). $\begin{bmatrix} 1 & -2 & 4 & 0 & -7 \\ -1 & 3 & -5 & 4 & 18 \\ 5 & 1 & 9 & 2 & 2 \\ 2 & 0 & 4 & -3 & -8 \end{bmatrix}$

According to Gaussion Elimination $\begin{bmatrix} 1 & -2 & 4 & 0 & -7 \\ 0 & 1 & -1 & 4 & 11 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

So the subset is $\vec{v}_1 = (1, -1, 5, 2)$ $\vec{v}_2 = (-2, 3, 1, 0)$ $\vec{v}_4 = (0, 4, 2, -3)$ $\vec{v}_5 = (-7, 18, 2, -8)$

$\vec{v}_3 = 2\vec{v}_1 - \vec{v}_2 = (4, -5, 9, 4)$

第九周周一作业 4月27日.

Section 4.8

2 (C) $A = \begin{bmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{bmatrix}$

According to Gaussion Elimination : $\begin{bmatrix} 1 & 4 & 5 & 2 \\ 0 & 1 & 1 & \frac{4}{7} \\ 0 & 0 & 0 & 0 \end{bmatrix}$

rank (A) = 2 nullity = 2

rank (A) + nullity (A) = 2+2 = 4

∴ The values satisfy Formula 4

6. The largest possible value for rank (A) is min {m. n}

The smallest possible value for nullity (A) is n - min {m, n}

10. ($\Rightarrow$). ∵ rank (A) = 2 ∴ Assume R is the matrix after Gaussion Elimination. Then R has 2 "first 1".

$R = \begin{bmatrix} a'_{11} & a'_{12} & a'_{13} \\ a'_{21} & a'_{22} & a'_{23} \end{bmatrix}$

There are three possibilities : ① $a'_{11} = a'_{22} = 1$ $a'_{21} = 0$ $\begin{vmatrix} a'_{11} & a'_{12} \\ a'_{21} & a'_{22} \end{vmatrix} = 1$

② $a'_{11} = a'_{23} = 1$ $a'_{21} = a'_{22} = 0$ $\begin{vmatrix} a'_{11} & a'_{13} \\ a'_{21} & a'_{23} \end{vmatrix} = 1$

③. $a'_{12} = a'_{23} = 1$ $a'_{11} = a'_{21} = a'_{22} = 0$ $\begin{vmatrix} a'_{12} & a'_{13} \\ a'_{22} & a'_{23} \end{vmatrix} = 1$

∴ One or more of the determinants $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$, $\begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix}$. $\begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$ is nozero.

($\Leftarrow$) : A is a 2×3 matrix so we know the number of "first 1" after Gaussion Elimination can be 0, 1, 2

Assume R is the matrix after Gaussion Elimination:

$R = \begin{bmatrix} a'_{11} & a'_{12} & a'_{13} \\ a'_{21} & a'_{22} & a'_{23} \end{bmatrix}$

If the number of "first 1" is 0 or 1, then $a'_{21} = a'_{22} = a'_{23} = 0$

$\therefore \begin{vmatrix} a'_{11} & a'_{12} \\ a'_{21} & a'_{22} \end{vmatrix} = 0 \quad \begin{vmatrix} a'_{11} & a'_{13} \\ a'_{21} & a'_{23} \end{vmatrix} = 0 \quad \begin{vmatrix} a'_{12} & a'_{13} \\ a'_{22} & a'_{23} \end{vmatrix} = 0$

$\therefore$ All of the three determinants $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$ $\begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix}$ $\begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$ equal to zero

$\therefore$ The number of "first 1" after Gaussion Elimination can only be 2.

$\therefore$ rank $(A) = 2$

18 (a). The rank of A is at most 3.

Because $\text{rank}(A_{3\times5}) \le \min\{3,5\} = 3$

(b). The nullity of A is at most 5.

Because $\text{rank}(A_{3\times5}) + \text{nullity}(A_{3\times5}) = 5 \quad \therefore \text{nullity}(A_{3\times5}) \le 5$

(c). The rank of A^T is at most 3

Becaust A^T is a 5×3 matrix $\quad \text{rank}(A^T) \le \min\{5,3\} = 3$

(d) The nullity of A^T is at most 3

Because A^T is a 5×3 matrix $\quad \text{rank}(A^T) + \text{nullity}(A^T) = 3 \quad \therefore \text{nullity}(A^T) \le 3$

第九周周三作业 4月29日.

Section 4.9

4 The domain of T is R^3, the codomain of T is R^2.

$T(\vec{x}) = (-2, 2)$

7 (a). $\forall \vec{u}, \vec{v} \in R^3 \quad \lambda \in R$

$T(\vec{0}) = (0, 0)$

$T(\lambda\vec{u}) = (0, 0) = \lambda T(\vec{u})$

$T(\vec{u}+\vec{v}) = (0, 0) = T(\vec{u}) + T(\vec{v})$

$T(\vec{u}-\vec{v}) = (0, 0) = T(\vec{u}) - T(\vec{v})$

$\therefore$ (a) $T(x, y, z) = (0, 0)$ is a matrix transformation.

(b). $\exists \lambda = 2 \in R \quad \forall \vec{u} \in R^3$

$T(\lambda\vec{u}) = (1, 1) \qquad \lambda T(\vec{u}) = (2, 2)$

$\therefore T(\lambda\vec{u}) \neq \lambda T(\vec{u})$

$\therefore$ (b) $T(x, y, z) = (1, 1)$ is not a matrix transformation

(c) $\forall \vec{u} = (x_1, y_1, z_1) \quad \vec{v} = (x_2, y_2, z_2) \in R^3 \quad \lambda \in R$

$T(\vec{0}) = (0, 0)$

$T(\vec{u}+\vec{v}) = (3(x_1+x_2) - 4(y_1+y_2), 2(x_1+x_2) - 5(z_1+z_2)) = T(\vec{u}) + T(\vec{v})$

$T(\lambda\vec{u}) = (3\lambda x_1 - 4\lambda y_1, 2\lambda x_1 - 5\lambda z_1) = \lambda T(\vec{u})$

$T(\vec{u}-\vec{v}) = (3(x_1-x_2) - 4(y_1-y_2), 2(x_1-x_2) - 5(z_1-z_2)) = T(\vec{u}) - T(\vec{v})$

$\therefore$ (c) $T(x, y, z) = (3x - 4y, 2x - 5z)$ is a matrix transformation.

(d) $\exists \vec{u} = (0, 1, 0) \quad \vec{v} = (0, 2, 0) \in R^3$

$T(\vec{u}+\vec{v}) = (9, 0) \qquad T(\vec{u}) + T(\vec{v}) = (5, 0)$

$\therefore T(\vec{u}+\vec{v}) \neq T(\vec{u}) + T(\vec{v})$

$\therefore$ (d) $T(x, y, z) = (y^2, z)$ is not a matrix transformation.

(e) $T(\vec{0}) = (-1, 0) \neq \vec{0}$

$\therefore$ (e) $T(x, y, z) = (y-1, x)$ is not a matrix transformation.

So (a) (c) are matrix transformations; (b) (d) (e) are not matrix transformations

11 (b) $T(\vec{e}_1) = (7, 0, -1)$ $T(\vec{e}_2) = (2, -1, 0)$ $T(\vec{e}_3) = (-1, 1, 0)$ $T(\vec{e}_4) = (1, 0, 0)$

$$\therefore T = \begin{pmatrix} 7 & 2 & -1 & 1 \\ 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$

15 (a) $T_{(a)}(2, -5, 3) = (2, -5, -3)$

(b) $T_{(b)}(2, -5, 3) = (2, 5, 3)$

(c) $T_{(c)}(2, -5, 3) = (-2, -5, 3)$

17 (a) $T(x, y, z) = (x, y, 0)$

$T(-2, 1, 3) = (-2, 1, 0)$

(b) $T(x, y, z) = (x, 0, z)$

$T(-2, 1, 3) = (-2, 0, 3)$

(c) $T(x, y, z) = (0, y, z)$

$T(-2, 1, 3) = (0, 1, 3)$

19. (a) $T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$

$T(x, y, z) = (x, \frac{\sqrt{3}}{2}y - \frac{1}{2}z, \frac{1}{2}y + \frac{\sqrt{3}}{2}z)$

$T(-2, 1, 2) = (-2, \frac{\sqrt{3}-2}{2}, \frac{1+2\sqrt{3}}{2})$

(b) $T = \begin{pmatrix} \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \\ 0 & 1 & 0 \\ -\frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \end{pmatrix}$

$T(x, y, z) = (\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}z, y, -\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}z)$

$T(-2, 1, 2) = (0, 1, 2\sqrt{2})$

(c) $T = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$T(x, y, z) = (-y, x, z)$

$\therefore T(-2, 1, 2) = (-1, -2, 2)$