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Section 2.2

10. $\begin{bmatrix} 3 & 6 & -9 \\ 0 & 0 & -2 \\ -2 & 1 & 5 \end{bmatrix}$

解. $\begin{vmatrix} 3 & 6 & -9 \\ 0 & 0 & -2 \\ -2 & 1 & 5 \end{vmatrix} = -\begin{vmatrix} 3 & 6 & -9 \\ -2 & 1 & 5 \\ 0 & 0 & -2 \end{vmatrix} = -\begin{vmatrix} 3 & 6 & -9 \\ 0 & 5 & -1 \\ 0 & 0 & -2 \end{vmatrix} = -3\times5\times(-2) = 30$

14. $\begin{bmatrix} 1 & -2 & 3 & 1 \\ 5 & -9 & 6 & 3 \\ -1 & 2 & -6 & -2 \\ 2 & 8 & 6 & 1 \end{bmatrix}$

解: $\begin{vmatrix} 1 & -2 & 3 & 1 \\ 5 & -9 & 6 & 3 \\ -1 & 2 & -6 & -2 \\ 2 & 8 & 6 & 1 \end{vmatrix} = \begin{vmatrix} 1 & -2 & 3 & 1 \\ 0 & 1 & -9 & -2 \\ 0 & 0 & -3 & -1 \\ 0 & 12 & 0 & -1 \end{vmatrix} = \begin{vmatrix} 1 & -2 & 3 & 1 \\ 0 & 1 & -9 & -2 \\ 0 & 0 & -3 & -1 \\ 0 & 0 & 108 & 23 \end{vmatrix} = \begin{vmatrix} 1 & -2 & 3 & 1 \\ 0 & 1 & -9 & -2 \\ 0 & 0 & -3 & -1 \\ 0 & 0 & 0 & -13 \end{vmatrix} = 1\times1\times(-3)\times(-13)$

$= 39$

28. (a). $\det\begin{bmatrix} 0 & 0 & a_{13} \\ 0 & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = a_{13}(-1)^{1+3}\cdot\begin{vmatrix} 0 & a_{22} \\ a_{31} & a_{32} \end{vmatrix} = a_{13}\cdot(0-a_{22}\cdot a_{31}) = -a_{13}a_{22}a_{31}$

(b). $\det\begin{bmatrix} 0 & 0 & 0 & a_{14} \\ 0 & 0 & a_{23} & a_{24} \\ 0 & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} = a_{14}(-1)^{1+4}\begin{vmatrix} 0 & 0 & a_{23} \\ 0 & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \end{vmatrix} = -a_{14}\cdot a_{23}(-1)^{1+3}\cdot\begin{vmatrix} 0 & a_{32} \\ a_{41} & a_{42} \end{vmatrix} = -a_{14}a_{23}\cdot(-a_{32}a_{41})$

$= a_{14}a_{23}a_{32}a_{41}$

32.
$$\begin{vmatrix} a_1 & b_1+ta_1 & c_1+rb_1+sa_1 \\ a_2 & b_2+ta_2 & c_2+rb_2+sa_2 \\ a_3 & b_3+ta_3 & c_3+rb_3+sa_3 \end{vmatrix} \underset{=}{\overset{(-t)\cdot \text{line①} \xrightarrow{+} \text{line②}}{(-s)\ \text{line①} \xrightarrow{+} \text{line③}}} \begin{vmatrix} a_1 & b_1 & c_1+rb_1 \\ a_2 & b_2 & c_2+rb_2 \\ a_3 & b_3 & c_3+rb_3 \end{vmatrix}$$

$$\underset{=}{(-r)\ \text{line②} \xrightarrow{+} \text{line③}} \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

Then, $\begin{vmatrix} a_1 & b_1+ta_1 & c_1+rb_1+sa_1 \\ a_2 & b_2+ta_2 & c_2+rb_2+sa_2 \\ a_3 & b_3+ta_3 & c_3+rb_3+sa_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ is confirmed.

34.
令 $A = \begin{bmatrix} a & b & b & b \\ b & a & b & b \\ b & b & a & b \\ b & b & b & a \end{bmatrix}$

$$\det A = a\begin{vmatrix} a & b & b \\ b & a & b \\ b & b & a \end{vmatrix} - b\begin{vmatrix} b & b & b \\ b & a & b \\ b & b & a \end{vmatrix} + b\begin{vmatrix} b & b & b \\ a & b & b \\ b & b & a \end{vmatrix} - b\begin{vmatrix} b & b & b \\ a & b & b \\ b & a & b \end{vmatrix}$$

$$= a(a^3+b^3+b^3-3ab^2) - b(a^2b+b^3+b^3-b^3-2ab^2)$$
$$+ b(b^2a+b^2a+b^3-b^3-a^2b-b^3) - b(b^3+b^3+a^2b-ab^2-ab^2-b^3)$$

$$= a^4+2ab^3-3a^2b^2-a^2b^2-b^4+2ab^3-b^4+2ab^3-a^2b^2-b^4+2ab^3-a^2b^2$$

$$= a^4-6a^2b^2+8ab^3-3b^4$$

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Section 2.3

18.
$$A=\begin{bmatrix} 1 & 2 & 0 \\ k & 1 & k \\ 0 & 2 & 1 \end{bmatrix} \qquad \det(A) = 1-4k$$

If A is invertible, $\det(A) \neq 0$.
Then we have $k \neq \frac{1}{4}$.

20.
$$A=\begin{bmatrix} 2 & 0 & 3 \\ 0 & 3 & 2 \\ -2 & 0 & -4 \end{bmatrix} \qquad \det(A) = -6$$ Such that A is invertible.

$$\operatorname{adj}(A) = \begin{bmatrix} -12 & 0 & -9 \\ -4 & -2 & -4 \\ 6 & 0 & 6 \end{bmatrix} \qquad A^{-1} = \frac{1}{\det A}\operatorname{adj}(A) = \begin{bmatrix} 2 & 0 & \frac{3}{2} \\ \frac{2}{3} & \frac{1}{3} & \frac{2}{3} \\ -1 & 0 & -1 \end{bmatrix}$$

30.
$$A=\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \det(A) = \cos^2\theta + \sin^2\theta = 1 \neq 0$$ Such that A is invertible

$$\operatorname{adj} A = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

According to Theorem 2.3.6

$$A^{-1} = \frac{1}{\det A}\operatorname{adj}(A) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

35.
$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \qquad \det A = -7$$

(a). $\det(3A) = 3^3 \cdot \det A = -189$

(b). $\det(A^{-1}) = \frac{1}{\det A} = -\frac{1}{7}$

(c). $\det(2A^{-1}) = 2^3 \cdot \det(A^{-1}) = -\frac{8}{7}$

(d). $\det((2A)^{-1}) = \det(\frac{1}{2}A^{-1}) = (\frac{1}{2})^3 \det(A^{-1}) = -\frac{1}{56}$

(e).
$$\det \begin{bmatrix} a & g & d \\ b & h & e \\ c & i & f \end{bmatrix} = ahf + ceg + bdi - cdh - bfg - aie$$

$$\det A = aie + bfg + cdh - ahf - ceg - bdi$$

则 $\det \begin{bmatrix} a & g & d \\ b & h & e \\ c & i & f \end{bmatrix} = -\det A = 7$

38. Proof: If a square matrix A is invertible, we have $\det A \neq 0$.
Then $\det(A^T) = \det A \neq 0$.
Such that $\det(AA^T) = \det(A)\det(A^T) \neq 0$.
Which says AA^T is invertible.

On the other hand:
If AA^T is invertible, we have $\det(AA^T) \neq 0$ and $\det(AA^T) = \det(A) \cdot \det(A^T)$
Such that $\det(A) \neq 0$ and $\det(A^T) \neq 0$

Which says A is invertible.

So a square matrix A is invertible if and only if A^TA is invertible.

39. If A is a square matrix, then $\det(A^TA) = \det(A^T)\cdot\det(A)$

$\det(AA^T) = \det(A)\cdot\det(A^T)$

Then, we have $\det(A^TA) = \det(AA^T)$