第十周 周三 作业.

Section 4.10

4 (a) $T_1 = \begin{pmatrix} 4 & 0 & 0 \\ -2 & 1 & 0 \\ -1 & -3 & 0 \end{pmatrix}$ $T_2 = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 0 & -1 \\ 4 & 0 & -1 \end{pmatrix}$

(b) $T_2 \cdot T_1 = \begin{pmatrix} 0 & 2 & 0 \\ 1 & 3 & 0 \\ 17 & 3 & 0 \end{pmatrix}$ $T_1 \cdot T_2 = \begin{pmatrix} 4 & 8 & 0 \\ -2 & -4 & -1 \\ -1 & -2 & 3 \end{pmatrix}$

(c) $T_1(T_2(x_1, x_2, x_3)) = (4x_1 + 8x_2, \ -2x_1 - 4x_2 - x_3, \ -x_1 - 2x_2 + 3x_3)$

$T_2(T_1(x_1, x_2, x_3)) = (4x_1 + 2x_2, \ x_1 + 3x_2, \ 17x_1 + 3x_2)$

8. (a). $E_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$ $E_2 = \begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$T = k E_2 E_1 = \begin{pmatrix} \frac{\sqrt{3}}{8} & -\frac{\sqrt{3}}{16} & \frac{1}{16} \\ \frac{1}{8} & \frac{3}{16} & \frac{\sqrt{3}}{16} \\ 0 & \frac{1}{8} & \frac{\sqrt{3}}{8} \end{pmatrix}$

(b). $E_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$ $E_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $E_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$T = E_3 E_2 E_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

(c) $\bar{E}_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$ $E_2 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$ $E_3 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$T = E_3 E_2 E_1 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{pmatrix}$

14 (a) One-to-one.

$T^{-1} = \begin{pmatrix} 1 & -2 & 4 \\ -1 & 2 & -3 \\ -1 & 3 & -5 \end{pmatrix}$ $T^{-1}(w_1, w_2, w_3) = (w_1 - 2w_2 + 4w_3, \ -w_1 + 2w_2 - 3w_3, \ -w_1 + 3w_2 - 5w_3)$

(c) One-to-one

$T^{-1} = \begin{pmatrix} -\frac{3}{2} & -\frac{3}{2} & \frac{11}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{3}{2} \\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \end{pmatrix}$ $T^{-1}(w_1, w_2, w_3) = (-\frac{3}{2}w_1 - \frac{3}{2}w_2 + \frac{11}{2}w_3, \ \frac{1}{2}w_1 + \frac{1}{2}w_2 - \frac{3}{2}w_3, \ -\frac{1}{2}w_1 + \frac{1}{2}w_2 - \frac{1}{2}w_3)$

Section 4.11

12. (b) $A = \begin{bmatrix} 1 & 4 \\ 2 & 9 \end{bmatrix}$ $(x+4y, \ y)$

$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}$

先沿 x 轴 斜移 y 坐标 4 倍；然后 沿 y 轴 斜移 x 坐标 的 2 倍。

14. (a). $E_1 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ $E_2 = \begin{pmatrix} 5 & 0 \\ 0 & 1 \end{pmatrix}$ $E_3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$T = E_3 \, E_2 \, E_1 = \begin{pmatrix} 0 & 1 \\ -5 & 0 \end{pmatrix}$

(b) $E_1 = \begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$ $E_2 = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}$ $E_3 = \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$

$T = E_3 \, E_2 \, E_1 = \begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{3-6\sqrt{3}}{2} & \frac{3\sqrt{3}+6}{2} \end{pmatrix}$

16. $L: \ y = -4x + 3$ $\vec{r} = (0,3) + t\,(1,-4)$

$A \, \vec{r}_0 = \begin{bmatrix} 4 & -3 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} 0 \\ 3 \end{bmatrix} = \begin{bmatrix} -9 \\ -6 \end{bmatrix}$

$A \, \vec{r}_0 = \begin{bmatrix} 4 & -3 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ -4 \end{bmatrix} = \begin{bmatrix} 16 \\ 11 \end{bmatrix}$

$L': \quad \vec{r} = (-9, -6) + t\,(16, 11)$