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section 1.1

12(c) $\begin{bmatrix} 1 & 2 & 3 & 4 \\ -4 & -3 & -2 & -1 \\ 5 & -6 & 1 & 1 \\ -8 & 0 & 0 & 3 \end{bmatrix}$ $\begin{cases} x_1 + 2x_2 + 3x_3 = 4 \\ -4x_1 - 3x_2 - 2x_3 = -1 \\ 5x_1 - 6x_2 + x_3 = 1 \\ -8x_1 = 3 \end{cases}$

13(c)
$$\begin{aligned} 2x_2 - 3x_4 + x_5 &= 0 \\ -3x_1 - x_2 + x_3 &= -1 \\ 6x_1 + 2x_2 - x_3 + 2x_4 - 3x_5 &= 6 \end{aligned}$$
$$\begin{bmatrix} 0 & 2 & 0 & -3 & 1 & 0 \\ -3 & -1 & 1 & 0 & 0 & -1 \\ 6 & 2 & -1 & 2 & -3 & 6 \end{bmatrix}$$

16. Proof. Suppose the liner system is

$$LS(1) \begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m \end{cases}$$

and the solution is denoted by V

case(1): We multiply the equation $a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n = b_i$ through a nonzero element c and get a new liner system.

$$LS(2) \begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \vdots \\ ca_{i1}x_1 + ca_{i2}x_2 + \cdots + ca_{in}x_n = c \cdot b_i \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m \end{cases}$$

and the solution of $LS(2)$ is denoted by V_1

As $V=(s_1, s_2, \cdots s_n)$ is the solution of $LS(1)$, it's easy to check that $V=(s_1, s_2 \cdots s_n)$ is also the solution of $LS(2)$ which says $V \subseteq V_1$

On the other hand, if we multiply the equation

$$c a_{i1}x_1 + c a_{i2}x_2 + \cdots + c \cdot a_{in}x_n = b_i$$

through $\frac{1}{c}$ $(c \neq 0)$ we can easily get $V_1 \subseteq V$.

Then we get $V = V_1$

case (2) We interchange i-th equation and j-th equation $(i<j)$ of the $LS(1)$ and get a new liner system $LS(3)$

$$LS(3) \begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ \vdots \\ a_{j1}x_1 + a_{j2}x_2 + \cdots + a_{jn}x_n = b_j \\ \vdots \\ a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n = b_i \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m \end{cases}$$

The solution of $LS(3)$ is denoted by V_2

Then we interchange the j-th equation and the i-th equation and we get $LS(1)$.

So it's easy to check $V = V_2$

case(3) Add c times of the i-th equation to the j-th equation of $LS(1)$ and we get a new liner system called $LS(4)$, whose solution is V_3

$$LS(4)\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ \vdots \\ (ca_{i1}+a_{j1})x_1 + (ca_{i2}+a_{j2})x_2 + \cdots + (ca_{in}+a_{jn})x_n = b_j \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m \end{cases}$$

It's easy to check $V=(s_1, s_2, \cdots s_n)$ is also the solution of $LS(4)$
Then $V \subseteq V_3$

Add $(-c)$ times of i-th equation to the j-th equation in $LS(4)$
We can easily get $V_3 \subseteq V$

Then we have $V = V_3$

section 1.2

2. (a) Row echelon (b) Neither (c) Row echelon
(d). Row echelon (e) Neither (f) Neither
(g) Both

3. (a) $\begin{bmatrix} 1 & -3 & 4 & 7 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 5 \end{bmatrix}$ $\text{row}3 \times (-2) \xrightarrow{+} \text{row}2$ $\begin{bmatrix} 1 & -3 & 4 & 7 \\ 0 & 1 & 0 & -8 \\ 0 & 0 & 1 & 5 \end{bmatrix}$

$\text{row}3 \times (-4) \xrightarrow{+} \text{row}1$ $\begin{bmatrix} 1 & -3 & 0 & -13 \\ 0 & 1 & 0 & -8 \\ 0 & 0 & 1 & 5 \end{bmatrix}$ $\text{row}2 \times 3 \xrightarrow{+} \text{row}1$ $\begin{bmatrix} 1 & 0 & 0 & -37 \\ 0 & 1 & 0 & -8 \\ 0 & 0 & 1 & 5 \end{bmatrix}$

$x_1 = -37$ $x_2 = -8$ $x_3 = 5$

4. (c) $\begin{bmatrix} 1 & 6 & 0 & 0 & 3 & -2 \\ 0 & 0 & 1 & 0 & 4 & 7 \\ 0 & 0 & 0 & 1 & 5 & 8 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$$\begin{aligned} x_1 + 6x_2 \qquad\qquad + 3x_5 &= -2 \\ x_3 \qquad + 4x_5 &= 7 \\ x_4 + 5x_5 &= 8 \end{aligned}$$

$x_1 = -6s - 3t - 2$ $x_2 = s$ $x_3 = 7 - 4t$ $x_4 = 8 - 5t$ $x_5 = t$

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Section 1.2

26. $x + 2y + z = 2$
$2x - 2y + 3z = 1$
$x + 2y - (a^2-3)z = a$

$$\begin{bmatrix} 1 & 2 & 1 & 2 \\ 2 & -2 & 3 & 1 \\ 1 & 2 & 3-a^2 & a \end{bmatrix} \text{row}① \times (-2) \overset{+}{\rightsquigarrow} \text{row}②$$

$$\begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & -6 & 1 & -3 \\ 1 & 2 & 3-a^2 & a \end{bmatrix} \begin{matrix} \text{row}① \times (-1) \overset{+}{\rightsquigarrow} \text{row}③ \\ \text{row}② \times (-\frac{1}{6}) \end{matrix} \begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & 1 & -\frac{1}{6} & \frac{1}{2} \\ 0 & 0 & 2-a^2 & a-2 \end{bmatrix}$$

If $a = \pm\sqrt{2}$, there are no solutions;
If $a \neq \pm\sqrt{2}$ there is exactly one solution.

Section 1.3

1. (a) BA is not defined.
(b). AC + D is defined, and the size of the resulting matrix is (4x2)
(c) AE + B is not defined.
(d) AB + B is not defined.
(e) E(A+B) is defined, and the size of the resulting matrix is (5x5)
(f) E(AC) is defined, and the size of the resulting matrix is (5x2)

3 (a) $D+E = \begin{bmatrix} 7 & 6 & 5 \\ -2 & 1 & 3 \\ 7 & 3 & 7 \end{bmatrix}$ (b) $D-E = \begin{bmatrix} -5 & 4 & -1 \\ 0 & -1 & -1 \\ -1 & 1 & 1 \end{bmatrix}$

(c) $5A = \begin{bmatrix} 15 & 0 \\ -5 & 10 \\ 5 & 5 \end{bmatrix}$ (d) $-7C = \begin{bmatrix} -7 & -28 & -14 \\ -21 & -7 & -35 \end{bmatrix}$

(e). 2B - C is not defined (f) $4E - 2D = \begin{bmatrix} 22 & -6 & 8 \\ -2 & 4 & 6 \\ 10 & 0 & 4 \end{bmatrix}$

(g) $-3(D+2E) = \begin{bmatrix} -39 & -21 & -24 \\ 9 & -6 & -15 \\ -33 & -12 & -30 \end{bmatrix}$

$(AB)_{ij} = \sum_{k=1}^{r} a_{ik} \cdot b_{kj}$

5(a) $AB = \begin{bmatrix} 12 & -3 \\ -4 & 5 \\ 4 & 1 \end{bmatrix}$

(b) BA is not defined

(c) $(3E)D = \begin{bmatrix} 42 & 108 & 75 \\ 12 & -3 & 21 \\ 36 & 78 & 63 \end{bmatrix}$

(d) $(AB)C = \begin{bmatrix} 3 & 45 & 9 \\ 11 & -11 & 17 \\ 7 & 17 & 13 \end{bmatrix}$

(e) $A(BC) = \begin{bmatrix} 3 & 45 & 9 \\ 11 & -11 & 17 \\ 7 & 17 & 13 \end{bmatrix}$