第九周周二作业　4月28日

习题 11.1

1(2) $x=3t \quad y=3t^2 \quad z=2t^3$ 从 $(0,0,0)$ 到 $(3,3,2)$

解 $L=\int_0^1 \sqrt{9+36t^2+36t^4}\,dt=\int_0^1 (6t^2+3)\,dt = 2t^3+3t\Big|_0^1 = 5$

(5). $4ax=(y+z)^2$ 与 $4x^2+3y^2=3z^2$ 的交线 从原点到 $M(x,y,z)$ $(a>0,\ z\geq 0)$

解 $y(x)=\sqrt{ax}-\frac{1}{3\sqrt{a}}x^{\frac{3}{2}} \qquad z(x)=\sqrt{ax}+\frac{1}{3\sqrt{a}}x^{\frac{3}{2}}$

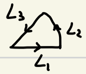
$y'(x)=\frac{1}{2}\frac{\sqrt{a}}{\sqrt{x}}-\frac{\sqrt{x}}{2\sqrt{a}} \qquad z'(x)=\frac{\sqrt{a}}{2\sqrt{x}}+\frac{\sqrt{x}}{2\sqrt{a}}$

$L=\int_0^{x_0}\sqrt{1+y'^2_{(x)}+z'^2_{(x)}}\,dx=\int_0^{x_0}\sqrt{\frac{a}{2x}+\frac{x}{2a}+1}\,dx=\int_0^{x_0}\left(\sqrt{\frac{a}{2x}}+\sqrt{\frac{x}{2a}}\right)dx=\sqrt{2a}\cdot\sqrt{x}+\frac{\sqrt{2}}{3\sqrt{a}}x^{\frac{3}{2}}\Big|_0^{x_0}$

$=\sqrt{2ax_0}+\frac{\sqrt{2}}{3\sqrt{a}}x_0^{\frac{3}{2}}$

$\therefore$ 从原点到 $M(x,y,z)$ 距离为 $\sqrt{2}z$

2. (6). $\int_L e^{\sqrt{x^2+y^2}}ds$ 　L: 由 $r=a$ 　$\varphi=0$, $\varphi=\frac{\pi}{4}$ 围成的区域边界



解: $\int_L e^{\sqrt{x^2+y^2}}ds=\int_{L_1}e^{\sqrt{x^2+y^2}}ds+\int_{L_2}e^{\sqrt{x^2+y^2}}ds+\int_{L_3}e^{\sqrt{x^2+y^2}}ds$

$\int_{L_1}e^{\sqrt{x^2+y^2}}ds=\int_0^a e^x dx = e^a-1$

$\int_{L_2}e^{\sqrt{x^2+y^2}}ds=\int_0^{\frac{\pi}{4}}e^a\cdot a\,dt=\frac{\pi}{4}ae^a$

$\int_{L_3}e^{\sqrt{x^2+y^2}}ds=\int_0^{\frac{\sqrt{2}a}{2}}e^{\sqrt{2}x}\sqrt{2}\,dx = e^a-1$

$\therefore \int_L e^{\sqrt{x^2+y^2}}ds=\frac{\pi}{4}ae^a+2e^a-2$

(11) $\int_L x^2 ds$　　L：圆周 $x^2+y^2+z^2=a^2$　　$x+y+z=0$

令 $x=-\frac{\sqrt{2}}{2}u-\frac{\sqrt{6}}{6}v+\frac{\sqrt{3}}{3}w$　　$y=\frac{\sqrt{2}}{2}u-\frac{\sqrt{6}}{6}v+\frac{\sqrt{3}}{3}w$　　$z=\frac{\sqrt{6}}{3}v+\frac{\sqrt{3}}{3}w$

$\left|\frac{\partial(x,y,z)}{\partial(u,v,w)}\right|=1$　　L: $u^2+v^2+w^2=a^2$　　$\sqrt{3}w=0$

$u=a\cos\theta$　　$v=a\sin\theta$　　$w=0$　　$\theta\in[0,2\pi]$

$$\int_L x^2 ds=\int_0^{2\pi}|a|^3\frac{1}{6}(\sqrt{3}\cos\theta+\sin\theta)^2 d\theta=\frac{2\pi}{3}|a|^3$$

(12). 由(11)可知 $\int_L (xy+yz+zx)ds=\int_0^{2\pi}|a|^3\cos\theta\sin\theta d\theta=0$

3. 解. 由题意 $\rho=\frac{k}{x^2+y^2+z^2}$　　当 $(x,y,z)=(1,0,1)$ 时 $\rho=1$　　$\therefore k=2$

即 $\rho=\frac{2}{x^2+y^2+z^2}$　　$x'(t)=e^t(\cos t-\sin t)$　　$y'(t)=e^t(\sin t+\cos t)$　　$z'(t)=e^t$

$$\int_L \rho\, ds=\int_0^{t_0}\frac{1}{e^{2t}}\sqrt{x'^2(t)+y'^2(t)+z'^2(t)}\,dt=\int_0^{t_0}\frac{\sqrt{3}}{e^t}dt=\sqrt{3}(1-e^{-t_0})$$

$\therefore$ 弧的质量为 $\sqrt{3}(1-e^{-t_0})$

习题 11.2

1.(4). 联立 $\begin{cases} x^2+y^2+z^2=3a^2 \\ x^2+y^2=2az \end{cases}$ $(z\geq 0)$ $\therefore$ $z=a$ 即 交线为 $x^2+y^2=2a^2$ $S=S_1+S_2$

对于 S_1:

令 $x=\sqrt{3}a\sin\theta\cos\varphi$ $y=\sqrt{3}a\sin\theta\sin\varphi$ $z=\sqrt{3}a\cos\theta$

$z\geq a$ 得 $\theta\in[0,\cos^{-1}\frac{\sqrt{3}}{3}]$

$\vec{r}(\theta,\varphi)=(\sqrt{3}a\sin\theta\cos\varphi,\ \sqrt{3}a\sin\theta\sin\varphi,\ \sqrt{3}a\cos\theta)$

$\vec{r}_\theta'=(\sqrt{3}a\cos\theta\cos\varphi,\ \sqrt{3}a\cos\theta\sin\varphi,\ -\sqrt{3}a\sin\theta)$ $\vec{r}_\varphi'=(-\sqrt{3}a\sin\theta\sin\varphi,\ \sqrt{3}a\sin\theta\cos\varphi,\ 0)$

$E=\vec{r}_\theta'^2=3a^2\cos^2\theta\cos^2\varphi+3a^2\cos^2\theta\sin^2\varphi+3a^2\sin^2\theta=3a^2$

$G=\vec{r}_\varphi'^2=3a^2\sin^2\theta\sin^2\varphi+3a^2\sin^2\theta\cos^2\varphi=3a^2\sin^2\theta$

$F=\vec{r}_\theta'\cdot\vec{r}_\varphi'=-3a^2\sin\theta\cos\theta\sin\varphi\cos\varphi+3a^2\sin\theta\cos\theta\sin\varphi\cos\varphi+0=0$

$S_1=\iint_{D_1}\sqrt{EG-F^2}\,d\theta\,d\varphi=\iint_{D_1}3a^2\sin\theta\,d\theta\,d\varphi=3a^2\int_0^{2\pi}d\varphi\int_0^{\cos^{-1}\frac{\sqrt{3}}{3}}\sin\theta\,d\theta=(6-2\sqrt{3})\pi a^2$

对于 S_2: $\vec{r}=(x,y,\frac{x^2+y^2}{2a})$

$E=1+z_x'^2=1+\frac{x^2}{a^2}$ $G=1+z_y'^2=1+\frac{y^2}{a^2}$ $F=z_x'\cdot z_y'=\frac{xy}{a^2}$

$S_2=\iint_{D_2}\sqrt{EG-F^2}\,dx\,dy=\iint_{D_2}\sqrt{1+\frac{x^2+y^2}{a^2}}\,dx\,dy$ D_2: $x^2+y^2=2az$ $0\leq z\leq a$

令 $x=r\cos\theta$ $y=r\sin\theta$ $r\in[0,\sqrt{2}a]$ $\theta\in[0,2\pi]$ $\left|\frac{\partial(x,y)}{\partial(r,\theta)}\right|=r$

$\therefore S_2=\iint_{D_2'}\sqrt{1+\frac{r^2}{a^2}}\,r\,dr\,d\theta=\int_0^{2\pi}d\theta\cdot\int_0^{\sqrt{2}a}r\cdot\sqrt{1+\frac{r^2}{a^2}}\,dr=2\pi\cdot\frac{3\sqrt{3}-1}{3}a^2=\frac{6\sqrt{3}-2}{3}\pi a^2$

$S=S_1+S_2=\frac{16}{3}\pi a^2$

(18). 曲面 $(x^2+y^2+z^2)^2=2a^2xy$ 的全部

解：令 $x=r\sin\theta\cos\varphi \quad y=r\sin\theta\sin\varphi \quad z=r\cos\theta$

则 $r^4=a^2r^2\sin^2\theta\sin 2\varphi$ 得 $r=a\sin\theta\sqrt{\sin 2\varphi}$ $\quad \theta\in[0,\pi] \quad \varphi[0,\frac{\pi}{2}],[\pi,\frac{3\pi}{2}]$

$\vec{r}=(a\sin^2\theta\sqrt{\sin2\varphi}\cos\varphi,\ a\sin^2\theta\sqrt{\sin2\varphi}\sin\varphi,\ a\sin\theta\cos\theta\sqrt{\sin2\varphi})$

$\vec{r}_\theta'=(2a\sin\theta\cos\theta\sqrt{\sin2\varphi}\cos\varphi,\ 2a\sin\theta\cos\theta\sqrt{\sin2\varphi}\sin\varphi,\ a\sqrt{\sin2\varphi}\cos2\theta)$

$\vec{r}_\varphi'=\left(a\sin^2\theta\left(\frac{\cos2\varphi}{\sqrt{\sin2\varphi}}\cos\varphi-\sqrt{\sin2\varphi}\sin\varphi\right),\ a\sin^2\theta\left(\frac{\cos2\varphi}{\sqrt{\sin2\varphi}}\sin\varphi+\sqrt{\sin2\varphi}\cos\varphi\right),\ a\sin\theta\cos\theta\frac{\cos2\varphi}{\sqrt{\sin2\varphi}}\right)$

$E=\vec{r}_\theta'^2=a^2\sin^22\theta\sin2\varphi\cos^2\varphi+a^2\sin^22\theta\sin2\varphi\sin^2\varphi+a^2\sin2\varphi\cos^22\theta=a^2\sin2\varphi$

$G=\vec{r}_\varphi'^2=a^2\sin^4\theta\left(\frac{\cos^22\varphi\cos^2\varphi}{\sin2\varphi}+\sin2\varphi\sin^2\varphi-\sin2\varphi\cos2\varphi+\frac{\cos^22\varphi\sin^2\varphi}{\sin2\varphi}+\sin2\varphi\cos^2\varphi+\cos2\varphi\sin2\varphi\right)+a^2\sin^2\theta\cos^2\theta\frac{\cos^22\varphi}{\sin2\varphi}$

$=\frac{a^2\sin^2\theta}{\sin2\varphi}(\sin^2\theta+\cos^2\theta\cos^22\varphi)$

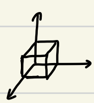
$F=\vec{r}_\theta'\cdot\vec{r}_\varphi'=a^2\sin^2\theta\sin2\theta(\cos2\varphi\cos^2\varphi-\sin2\varphi\sin\varphi\cos\varphi)+a^2\sin^2\theta\sin2\theta(\cos2\varphi\sin^2\varphi+\sin2\varphi\sin\varphi\cos\varphi)+a^2\sin\theta\cos\theta\cos2\theta\cos2\varphi$

$=a^2\sin^2\theta\sin2\theta\cos2\varphi+a^2\sin\theta\cos\theta\cos2\varphi\cos2\theta=a^2\cos2\varphi\sin\theta(\sin\theta\sin2\theta+\cos\theta\cos2\theta)$

$S=\iint_D\sqrt{EG-F^2}\,d\theta\,d\varphi=\iint_D a^2\sin^2\theta\,d\theta\,d\varphi=\int_0^\pi a^2\sin^2\theta\,d\theta\cdot\left(\int_0^{\frac{\pi}{2}}d\varphi+\int_\pi^{\frac{3\pi}{2}}d\varphi\right)=\frac{1}{2}\pi^2a^2$

则面积为 $\frac{1}{2}\pi^2a^2$

2(1). $\iint_S(x+y+z)\,dS \quad S$: 立方体 $0\le x\le1,\ 0\le y\le1,\ 0\le z\le1$ 全表面



解：$D_1: x=0 \quad y\in[0,1] \quad z\in[0,1] \qquad D_2: x=1 \quad y\in[0,1] \quad z\in[0,1]$

$D_3: y=0 \quad x\in[0,1] \quad z\in[0,1] \qquad D_4: y=1 \quad x\in[0,1] \quad z\in[0,1]$

$D_5: z=0 \quad x\in[0,1] \quad y\in[0,1] \qquad D_6: z=0 \quad x\in[0,1] \quad y\in[0,1]$

$\iint_S(x+y+z)\,dS=\iint_{D_1}(y+z)\,dy\,dz+\iint_{D_2}(1+y+z)\,dy\,dz+\iint_{D_3}(x+z)\,dx\,dz+\iint_{D_4}(1+x+z)\,dx\,dz+\iint_{D_5}(x+y)\,dx\,dy+\iint_{D_6}(1+x+y)\,dx\,dy$

$=1+2+1+2+1+2=9$

a,0)

(4). $\iint_S (xy+yz+zx)\,ds$　　S: 锥面 $z=\sqrt{x^2+y^2}$ 被柱面 $x^2+y^2=2ax$ 所截下部分.

解: 令 $x=r\cos\theta$　$y=r\sin\theta$　$z=r$　　$r\in[0,2a]$　$\theta\in[-\frac{\pi}{2},\frac{\pi}{2}]$

即 $\vec{r}=(r\cos\theta,\ r\sin\theta,\ r)$

$\vec{r}_r'=(\cos\theta,\ \sin\theta,\ 1)$　　$\vec{r}_\theta'=(-r\sin\theta,\ r\cos\theta,\ 0)$

$E=\vec{r}_r'^{\,2}=2$　　$G=\vec{r}_\theta'^{\,2}=r^2$　　$F=\vec{r}_r'\cdot\vec{r}_\theta'=0$

$\therefore ds=\sqrt{EG-F^2}\,dr\,d\theta=\sqrt{2}\,r\,dr\,d\theta$

$$\iint_S (xy+yz+zx)\,ds=\iint_D r^2(\cos\theta\sin\theta+\sin\theta+\cos\theta)\cdot\sqrt{2}\,r\,dr\,d\theta=\int_0^{2a}\sqrt{2}\,r^3dr\cdot\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(\frac{1}{2}\sin2\theta+\sin\theta+\cos\theta\right)d\theta$$

$$=4\sqrt{2}a^4\cdot2=8\sqrt{2}a^4$$

(7). $\iint_S |xyz|\,dS$　　S: 曲面 $z=x^2+y^2$ 介于 $z=0$　$z=1$ 间部分.

解: 令 $x=r\cos\theta$　$y=r\sin\theta$　$z=r^2$　　$r\in[0,1]$　$\theta\in[0,\frac{\pi}{2}]$

$\vec{r}=(r\cos\theta,\ r\sin\theta,\ r^2)$

$\vec{r}_r'=(\cos\theta,\ \sin\theta,\ 2r)$　　$\vec{r}_\theta'=(-r\sin\theta,\ r\cos\theta,\ 0)$

$E=\vec{r}_r'^{\,2}=1+4r^2$　　$G=\vec{r}_\theta'^{\,2}=r^2$　　$F=\vec{r}_r'\cdot\vec{r}_\theta'=0$

$\therefore dS=\sqrt{EG-F^2}\,dr\,d\theta=r\cdot\sqrt{1+4r^2}\,dr\,d\theta$

$$\iint_S |xyz|\,ds=4\iint_D r^4\sin\theta\cos\theta\cdot r\sqrt{1+4r^2}\,dr\,d\theta=2\int_0^{\frac{\pi}{2}}\sin2\theta\,d\theta\cdot\int_0^1 r^5\sqrt{1+4r^2}\,dr=2\cdot\left(\frac{25\sqrt{5}}{168}-\frac{1}{840}\right)$$

$$=\frac{25\sqrt{5}}{84}-\frac{1}{420}$$

4、证明: $G:\ \vec{r}=(x,\ y,\ -\frac{Ax+By+D}{C})$

$\vec{r}_x'=(1,0,-\frac{A}{C})\qquad \vec{r}_y'=(0,1,-\frac{B}{C})$

$E=\vec{r}_x'^{\,2}=1+\frac{A^2}{C^2}\qquad G=\vec{r}_y'^{\,2}=1+\frac{B^2}{C^2}\qquad F=\vec{r}_x'\cdot\vec{r}_y'=\frac{AB}{C^2}$

$dS_G=\sqrt{EG-F^2}\,dx\,dy=\sqrt{1+\frac{A^2}{C^2}+\frac{B^2}{C^2}}\,dx\,dy$

$G_1:\ \vec{r}=(x,y,0)$

$\vec{r}_x'=(1,0,0)\qquad \vec{r}_y'=(0,1,0)$

$E=\vec{r}_x'^{\,2}=1\qquad G=\vec{r}_y'^{\,2}=1\qquad F=\vec{r}_x'\cdot\vec{r}_y'=0$

$dS_{G_1}=\sqrt{EG-F^2}\,dx\,dy=dx\,dy$

$$\frac{G\text{的面积}}{G_1\text{的面积}}=\frac{\iint_D dS_G}{\iint_D dS_{G_1}}=\frac{\iint_D\sqrt{1+\frac{A^2}{C^2}+\frac{B^2}{C^2}}\,dx\,dy}{\iint_D dx\,dy}=\sqrt{\frac{A^2+B^2+C^2}{C^2}}$$

第九周周四作业 4月30日.

习题 11.3

1.(1). $\int_L (x^2+y^2)\,dx + (x^2-y^2)\,dy$. L: 曲线 $y=1-|1-x|$ 从 $(0,0)$ 到 $(2,0)$

$L_1: \quad y=x \quad x\in[0,1] \qquad L_2: \quad y=2-x \quad x\in[1,2] \qquad dy=-dx$

$$\int_L (x^2+y^2)dx + (x^2-y^2)dy = \int_{L_1} (x^2+y^2)dx + (x^2-y^2)dy + \int_{L_2} (x^2+y^2)dx + (x^2-y^2)dy$$

$$= \int_0^1 2x^2 dx + \int_1^2 [x^2+(2-x)^2 - x^2 + (2-x)^2]\,dx$$

$$= \frac{2}{3} + \frac{2}{3} = \frac{4}{3}$$

(2). $\int_L \frac{dx+dy}{|x|+|y|}$ L: 沿顶点为 $A(1,0)$ $B(0,1)$ $C(-1,0)$ $D(0,-1)$ 的正方形逆时针一周的路径。

$L_1: \ y=1-x \quad x\in[0,1] \qquad L_2: \ y=x+1 \quad x\in[-1,0] \qquad L_3: \ y=-x-1 \quad x\in[-1,0] \qquad L_4: \ y=x-1 \quad x\in[0,1]$

$$\int_L \frac{dx+dy}{|x|+|y|} = -\int_{L_1} \frac{dx+dy}{|x|+|y|} - \int_{L_2} \frac{dx+dy}{|x|+|y|} + \int_{L_3} \frac{dx+dy}{|x|+|y|} + \int_{L_4} \frac{dx+dy}{|x|+|y|}$$

$$= -\int_0^1 0\,dx - \int_{-1}^0 2\,dx + \int_{-1}^0 0\,dx + \int_0^1 2\,dx$$

$$= 0 - 2 + 0 + 2 = 0$$

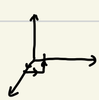
(3). $\int_L \frac{-x\,dx + y\,dy}{x^2+y^2}$ L: 圆周 $x^2+y^2=a^2$ 沿逆时针方向一周路径

解. 令 $x = a\cos t$ $\quad y = a\sin t$ $\quad dx = -a\sin t\,dt$ $\quad dy = a\cos t\,dt$

$$\int_L \frac{-x\,dx+y\,dy}{x^2+y^2} = \int_0^{2\pi} \frac{a^2\sin t\cos t\,dt + a^2\sin t\cos t\,dt}{a^2} = \int_0^{2\pi} \sin 2t\,dt = 0$$

(4) $\int_L y^2dx + xy\,dy + xz\,dz$ $\quad L$: 从 $O(0,0,0)$ 到 $A(1,0,0)$ 到 $B(1,1,0)$ 到 $C(1,1,1)$ 的折线段.

$L_1 = \overline{OA} \quad L_2 = \overline{AB} \quad L_3 = \overline{BC}$



$$\int_L y^2dx+xydy+xzdz=\int_{L_1}y^2dx+xydy+xzdz+\int_{L_2}y^2dx+xydy+xzdz+\int_{L_3}y^2dx+xydy+xzdz$$

$$=0+\int_0^1 y\,dy+\int_0^1 z\,dz=1$$

(5) $\int_L e^{x+y+z}dx+e^{x+y+z}dy+e^{x+y+z}dz$ L: $x=\cos\varphi$ $y=\sin\varphi$ $z=\frac{\varphi}{\pi}$ 从 $A(1,0,0)$ 到 $B(0,1,\frac{1}{2})$

$dx=-\sin\varphi\,d\varphi$ $dy=\cos\varphi\,d\varphi$ $dz=\frac{1}{\pi}$

原式 $=\int_0^{\frac{\pi}{2}} e^{\cos\varphi+\sin\varphi+\frac{\varphi}{\pi}}(-\sin\varphi+\cos\varphi+\frac{1}{\pi})\,d\varphi$

$$=e^{\cos\varphi+\sin\varphi+\frac{\varphi}{\pi}}\Big|_0^{\frac{\pi}{2}}=e^{\frac{3}{2}}-e$$

$(x-1)^2+$

(6) $\int_L y\,dx+z\,dy+x\,dz$ L: $x+y=2$ 与 $x^2+y^2+z^2=2(x+y)$ 交线 从原点看到顺时针方向.

解：令 $x=1+\sqrt{2}\sin\theta\cos\varphi$ $y=1+\sqrt{2}\sin\theta\sin\varphi$ $z=\sqrt{2}\cos\theta$

$x+y=2$ 得 $\sqrt{2}\sin\theta(\sin\varphi+\cos\varphi)=0$ $\because \sin\theta\neq0$ $\therefore \sin\varphi+\cos\varphi=0$ $\therefore \sin\varphi=\frac{\sqrt{2}}{2}$ $\cos\varphi=-\frac{\sqrt{2}}{2}$

$x=1+\sin\theta$ $y=1-\sin\theta$ $z=\sqrt{2}\cos\theta$ $\theta\in[0,2\pi]$

$dx=\cos\theta\,d\theta$ $dy=-\cos\theta\,d\theta$ $dz=-\sqrt{2}\sin\theta\,d\theta$

$$\int_L y\,dx+z\,dy+x\,dz=\int_0^{2\pi}(1-\sin\theta)\cos\theta\,d\theta-\sqrt{2}\cos^2\theta\,d\theta-\sqrt{2}\sin\theta(1+\sin\theta)\,d\theta$$

$$=\int_0^{2\pi}(-\sqrt{2}+\cos\theta-\sqrt{2}\sin\theta-\frac{1}{2}\sin2\theta)\,d\theta=-2\sqrt{2}\pi$$

2. 解：$\int_L \vec{v}\cdot d\vec{r}=\int_L(y+z)dx+(z+x)dy+(x+y)dz$

$dx=a\sin2t\,dt$ $dy=2a\cos2t\,dt$ $dz=-a\sin2t\,dt$

$\int_L(y+z)dx+(z+x)dy+(x+y)dz$

$$= \int_0^{\pi} \left\{ \left[a \sin 2t + a\left(\frac{1}{2}+\frac{1}{2}\cos 2t\right)\right] \cdot a \sin 2t + a \cdot 2a \cos 2t + \left[a\left(\frac{1}{2}-\frac{1}{2}\cos 2t\right) + a \sin 2t\right] \cdot (-a \sin 2t) \right\} dt$$

$$= \int_0^{\pi} a^2 \left(2\cos 2t + \frac{1}{2}\sin 4t\right) dt$$

$$= 0$$

3. 解 $|\vec{F}| = k\sqrt{x^2+y^2}$ $\quad \vec{F} = k \cdot (-x\,\vec{i} - y\,\vec{j}) = -kx\,\vec{i} - ky\,\vec{j}$

令 $x = a\cos\theta \quad y = b\sin\theta \qquad dx = -a\sin\theta\, d\theta \qquad dy = b\cos\theta\, d\theta$

$$W = \int_L \vec{F} \cdot d\vec{r} = \int_0^{\frac{\pi}{2}} (a^2 \sin\theta\cos\theta - b^2 \sin\theta\cos\theta)\, d\theta$$

$$= \int_0^{\frac{\pi}{2}} \frac{a^2-b^2}{2} \sin 2\theta\, d\theta$$

$$= \frac{a^2-b^2}{2}$$