介观物理第七次作业

董建宇 202328000807038

Problem II.5 Assuming $g(L_0) = g_0$, where L_0 is a certain microscopic length scale, find the solution $g(L)$ to the scaling equation (148) at $d = 2$ by using Eq. (154). The results can be found in Eq. (167).

$$\frac{d\ln g}{d\ln L} = \beta[g(L)]. \tag{148}$$

$$\beta(g) = d - 2 - \frac{a}{g}. \tag{154}$$

$$g(L) = g_0 - \frac{1}{\pi^2}\ln\left(\frac{L}{L_0}\right). \tag{167}$$

Proof. 方程 (148) 可以写作:

$$\frac{d\ln g}{d\ln L} = \frac{L}{d}\frac{dg}{dL} = \beta[g(L)].$$

其中，利用了 $d\ln g = \frac{1}{g}dg$。当 $d = 2$ 时，$\beta(g) = -\frac{a}{g}$。即有:

$$-\frac{dg}{a} = \frac{dL}{L}.$$

两侧不定积分，可得:

$$-\frac{g}{a} = \ln L + C.$$

其中 C 为常数。当 $L = L_0$ 时，有:

$$-\frac{g_0}{a} = \ln L_0 + C.$$

从而确定常数 C 为

$$C = -\frac{g_0}{a} - \ln L_0.$$

从而有:

$$g(L) = g_0 - a\ln\left(\frac{L}{L_0}\right) = g_0 - \frac{1}{\pi^2}\ln\left(\frac{L}{L_0}\right).$$

利用了 $a = \frac{1}{\pi^2}$。 □

Problem II.6 Find the solution in Eq. (170).

$$\sigma(L) = g_0 L_0 - a(L - L_0). \tag{170}$$

Proof. 当 $d = 1$ 时，$\beta(g) = -1 - \frac{a}{g}$，即方程 (148) 可化为:

$$-\frac{dg}{g+a} = \frac{dL}{L}.$$

两侧不定积分，可得:

$$-\ln(g+a) = \ln L + C_2.$$

其中，C_2 为积分常数，当 $L = L_0$ 时，有:

$$-\ln(g_0 + a) = \ln L_0 + C_2.$$

从而确定积分常数 C_2 为:

$$C_2 = -\ln(g_0 + a) - \ln L_0.$$

则 $d = 1$ 时，微分方程的解为:

$$\ln\left(\frac{g + a}{g_0 + a}\right) = \ln\left(\frac{L_0}{L}\right).$$

两侧取 e 指数，可得:

$$\frac{g + a}{g_0 + a} = \frac{L_0}{L}.$$

即:

$$g = \frac{L_0}{L}(g_0 + a) - a.$$

则电导率为:

$$\sigma(L) = gL = g_0 L_0 - a(L - L_0).$$

□