PHYS1106 物理原理I第四次作□

董建宇

TOTAL POINTS

77 / 82

QUESTION 1

1 第一□ 12 / 12

✓ **- 0 pts** *Correct*

- 2 pts Click here to replace this description.

- 4 pts Click here to replace this description.

- 1 pts Click here to replace this description.

QUESTION 2

2 第二□ 24 / 24

✓ **- 0 pts** *Correct*

- 2 pts Click here to replace this description.

- 4 pts Click here to replace this description.

- 6 pts Click here to replace this description.

- 1 pts Click here to replace this description.

QUESTION 3

3 第三□ 10 / 10

✓ **- 0 pts** *Correct*

- 2 pts 。

QUESTION 4

4 第四□ 11 / 12

- 0 pts Correct

✓ **- 1 pts** *Click here to replace this description.*

- 2 pts 少□

- 4 pts Click here to replace this description.

QUESTION 5

5 第五□ 12 / 12

✓ **- 0 pts** *Correct*

- 2 pts 少□ 或 缺少□明

- 1 pts 答案□□或缺少□明

QUESTION 6

6 第六□ 8 / 12

- 0 pts Correct

- 2 pts 瑕疵或 □

✓ **- 4 pts** □果 □

- 8 pts 少□

- 7 pts Click here to replace this description.

1 第一题 **12 / 12**

✓ **- 0 pts** *Correct*

- 2 pts Click here to replace this description.

- 4 pts Click here to replace this description.

- 1 pts Click here to replace this description.

2 第二 24 / 24

✓ **- 0 pts** *Correct*

- 2 pts Click here to replace this description.

- 4 pts Click here to replace this description.

- 6 pts Click here to replace this description.

- 1 pts Click here to replace this description.

3 第三 **10 / 10**

✓ **- 0 pts** *Correct*

- 2 pts 。

4 第四 **11 / 12**

- **- 0 pts** Correct
- ✓ **- 1 pts** *Click here to replace this description.*
- **- 2 pts** 少
- **- 4 pts** Click here to replace this description.

5 第五 **12 / 12**

✓ **- 0 pts** *Correct*

- 2 pts 少 或 缺少明

- 1 pts 答案或缺少明

6 第六 8 / 12

- **- 0 pts** Correct
- **- 2 pts** 瑕疵或
- ✓ **- 4 pts** 果
- **- 8 pts** 少
- **- 7 pts** Click here to replace this description.

gradescope

物理原理　第四次作业

第一题：机械波练习题.

1. 第一个振动：$y_1 = A_1\sin(\omega t+\varphi)$　　第二个振动　$y_2 = A_2\sin(\omega t+\varphi+\theta)$

合振动：$y = A\sin(\omega t+\varphi+\frac{\pi}{6}) = A\cdot[\frac{\sqrt{3}}{2}\sin(\omega t+\varphi)+\frac{1}{2}\cos(\omega t+\varphi)]$

$A = 20\,cm$　　$A_1 = 10\sqrt{3}\,cm = \frac{\sqrt{3}}{2}\cdot A$　　$\therefore A_2 = \frac{1}{2}A = 10\,cm$

$\sin(\omega t+\varphi+\theta) = \cos(\omega t+\varphi)$　得 $\theta = \frac{\pi}{2}$

答：第二个振动振幅 10cm　两简谐振动相位差 $\frac{\pi}{2}$

2. $y = A\cos(\frac{2\pi}{\lambda}x+\frac{\pi}{2})\cdot\cos\omega t$

(1). ① 振动~~势能和~~动能总是为零，即 $\cos(\frac{2\pi}{\lambda}x+\frac{\pi}{2}) = 0$　　即 $\frac{2\pi}{\lambda}x+\frac{\pi}{2} = (2n+1)\cdot\frac{\pi}{2}$　$n = (0,1,2\cdots)$

即 $x = \frac{n\lambda}{2}$　$(n=0,1,2\cdots)$

② 振动势能总是为零　即 $|\cos(\frac{2\pi}{\lambda}x+\frac{\pi}{2})| = 1$　　$\frac{2\pi}{\lambda}x+\frac{\pi}{2} = n\pi$　$(n=1,2,3\cdots)$

得 $x = (\frac{n}{2}-\frac{1}{4})\lambda$　$(n=1,2,3\cdots)$

(2). $E_k = \int_0^m \frac{1}{2}v^2 dm$　　$dm = \rho\,dx$　　$V = \frac{\partial y}{\partial t} = -A\omega\cos(\frac{2\pi}{\lambda}x+\frac{\pi}{2})\sin\omega t$

$E_k = \int_0^{\frac{\lambda}{2}} \frac{1}{2}v^2\rho\,dx = \frac{1}{2}A^2\omega^2\rho\sin^2\omega t\,\frac{\lambda}{2\pi}\int_0^{\frac{\lambda}{2}}\sin^2(\frac{2\pi}{\lambda}x)\,d(\frac{2\pi}{\lambda}x) = \frac{1}{8}A^2\omega^2\rho\lambda\sin^2\omega t$

$E_p = \int\frac{1}{2}dk\,y^2 = \int_0^m \frac{1}{2}\omega^2 dm\,y^2 = \frac{1}{2}\omega^2\rho\int_0^{\frac{\lambda}{2}} y^2 dx = \frac{1}{8}A^2\omega^2\lambda\rho\cos^2\omega t$

$E = E_k+E_p = \frac{1}{8}A^2\omega^2\lambda\rho$

3. ~~S_1: $y_{10} = A\cos\frac{2\pi}{T}t$　则 S_1 发出简谐波为　$y_1 = A\cos(\frac{2\pi}{T}t-\frac{2\pi}{\lambda}x)$~~

~~S_2: $y_{20} = A\cos\frac{2\pi}{T}(t+\frac{T}{4})$　则 S_2 发出简谐波为　$y_2 = A\cos(\frac{2\pi}{T}t+\frac{\pi}{2}$　$y_2 = A\cos[\frac{2\pi}{T}(t+\frac{T}{4})$~~

~~$y = y_{10}+y_{20} = A\cdot\cos\frac{2\pi}{T}t + A\cos(\frac{2\pi}{T}t+\frac{\pi}{2}) = A\sqrt{2}\cos(\frac{2\pi}{T}t+\frac{\pi}{4}) = \sqrt{2}A\cos(\frac{2\pi}{T}\cdot t+\frac{\pi}{4})$~~

~~$y_2 = A\cdot\cos[\frac{2\pi}{T}(t+\frac{T}{4})+\frac{2\pi}{\lambda}(x-\frac{5}{4}\lambda)] = A\cdot\sin(\frac{2\pi}{T}t-\frac{2\pi}{\lambda}x)$~~

3. 解. S_1: $y_{10} = A\cos\frac{2\pi}{T}\cdot t$ 左侧 $y_{1l} = A\cos(\frac{2\pi}{T}t + \frac{2\pi}{\lambda}\cdot x)$ 右侧 $y_{1r} = A\cos(\frac{2\pi}{T}\cdot t - \frac{2\pi}{\lambda}x)$

S_2: $y_{20} = A\cos\frac{2\pi}{T}\cdot(t+\frac{T}{4})$ 左侧 $y_{2l} = A\cos[\frac{2\pi}{T}(t+\frac{T}{4}) + \frac{2\pi}{\lambda}(x-\frac{5}{4}\lambda)]$ 右侧 $y_{2r} = A\cos[\frac{2\pi}{T}(t+\frac{T}{4}) - \frac{2\pi}{\lambda}(x-\frac{5}{4}\lambda)]$

(1). $y_{1r} + y_{2l} = A\{\cos(\frac{2\pi}{T}\cdot t - \frac{2\pi}{\lambda}x) + \cos[\frac{2\pi}{T}(t+\frac{T}{4}) + \frac{2\pi}{\lambda}(x-\frac{5}{4}\lambda)]\}$.

$= 2A\cos(\frac{2\pi}{T}t-\pi)\cos(-\frac{2\pi}{\lambda}x+\pi) = 2A\cos(\frac{2\pi}{T}\cdot t)\cdot\cos(\frac{2\pi}{\lambda}x)$

(2) 右侧 $y_{1r} + y_{2r} = A\{\cos(\frac{2\pi}{T}\cdot t - \frac{2\pi}{\lambda}x) + \cos[\frac{2\pi}{T}(t+\frac{T}{4}) - \frac{2\pi}{\lambda}(x-\frac{5}{4}\lambda)]\}$

$= 2A\cos(\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x + \frac{3}{2}\pi)\cdot\cos(-\frac{3}{2}\pi) = 0$

左侧 $y_{1l} + y_{2l} = A\{\cos(\frac{2\pi}{T}t + \frac{2\pi}{\lambda}x) + \cos[\frac{2\pi}{T}(t+\frac{T}{4}) + \frac{2\pi}{\lambda}(x-\frac{5}{4}\lambda)]\}$

$= 2A\cos(\frac{2\pi}{T}t + \frac{2\pi}{\lambda}x - \pi)\cdot\cos\pi = 2A\cos(\frac{2\pi}{T}t + \frac{2\pi}{\lambda}x)$

第二题. 相对论练习题

(1). 由题意, 在离子参考系中发出光子. 以加速器系为 K 系 离子参考系为 K′ 系.

由洛伦兹变换: $V_0 = \dfrac{v+c}{1+\frac{vc}{c^2}} = c$

即光子相对加速器速度为 c.

(2) 以其中一个飞船 A 为参考系 K. 则恒星为 K′ 系 K′ 相对 K 以 $V = 0.8c$ 运动.

在 K′ 系中, 只有另一个飞船 B 以 $v' = 0.8c$ 运动.

由洛伦兹变换在 K 系中 B 速度 $v = \dfrac{v'+V}{1+\frac{v'V}{c^2}} = 0.976c$

即两飞船相对速度为 0.976c

(3). 由洛伦兹变换. 其中 $\beta = \frac{u}{c} = 0.96$ $\gamma = \dfrac{1}{\sqrt{1-\beta^2}} = \dfrac{25}{7} \approx 3.57$

$ct_1' = \gamma(ct_1 - \beta x_1)$

$x_1' = \gamma(x_1 - \beta ct_1)$

$ct_2' = \gamma(ct_2 - \beta x_2)$

$x_2' = \gamma(x_2 - \beta ct_2)$

$t_1 = 0s$ $x_1 = 100$

$t_2 = 10s$ $x_2 = 100 + v(t_2 - t_1)$ m

得 $t_1' = -1.14 \times 10^{-6}$ s $x_1 = 357$ m $t_2' = 2.11$ s $x_2' = 2.14 \times 10^8$ m.

由洛伦兹变换 $v' = \dfrac{v-u}{1-\frac{vu}{c^2}} = 0.338c$

粒子相对 S′ 系速度 0.338c

(4). 设恒星系为 K 系 飞船为 K′ 系. 在 K′ 系中设小球初始位置为 0 初始时间为 0

则 $l_0 = v \cdot t'$

在 K 系中小球速度为 $V_K = \dfrac{u+v}{1+\frac{uv}{c^2}}$ 飞船长度 $l_K = l_0 \cdot \sqrt{1-\frac{u^2}{c^2}}$

则 $ct_K = \gamma(ct' + \beta x')$ $\gamma = \dfrac{1}{\sqrt{1-\frac{u^2}{c^2}}}$ $\beta = \frac{u}{c}$ $x' = l_0$

则 $t_K = \dfrac{\frac{l_0}{v} + \frac{u}{c^2} l_0}{\sqrt{1-\frac{u^2}{c^2}}}$

(5). $m_1 = \dfrac{m_0}{\sqrt{1-(\frac{0.1c}{c})^2}}$ $W = m_1c^2 - m_0c^2 = \cancel{8.23} \times 10^{-16}$ J 4.12×10^{-16} J m_0 为电子质量

$m_2 = \dfrac{m_0}{\sqrt{1-(\frac{0.9c}{c})^2}}$ $m_3 = \dfrac{m_0}{\sqrt{1-(\frac{0.99c}{c})^2}}$ $W = m_3c^2 - m_2c^2 = 3.93 \times 10^{-13}$ J (b).

$$x' = l_0 \cos\theta_0 \qquad y' = l_0 \sin\theta_0$$

$$x = x' \cdot \sqrt{1-\frac{v^2}{c^2}} \qquad y = y'$$

$$l = \sqrt{x^2+y^2} = l_0 \cdot \sqrt{\sin^2\theta_0 + \left(1-\frac{v^2}{c^2}\right)\cos^2\theta_0}$$

$$\theta = \arctan \frac{\sin\theta_0}{\cos\theta_0 \cdot \sqrt{1-\frac{v^2}{c^2}}}$$

第三题.

解. 设在S系中经过时间t 到达坐标x $\quad x = \frac{1}{2}a_0 t^2$

由洛伦兹变换 $\quad ct' = \gamma(ct-\beta x)$

$$x' = \gamma(x-\beta ct)$$

$$dt' = \frac{dt - \frac{u}{c^2}dx}{\sqrt{1-\frac{u^2}{c^2}}} = \frac{dt\left(1-\frac{u}{c^2}\frac{dx}{dt}\right)}{\sqrt{1-\frac{u^2}{c^2}}} = \frac{dt\left(1-\frac{u}{c^2}v_x\right)}{\sqrt{1-\frac{u^2}{c^2}}}$$

$$v_x' = \frac{v_x - u}{1-\frac{v_x u}{c^2}} \qquad v_x' = \frac{a_0 \cdot t - u}{1-\frac{a_0 u}{c^2}t} \qquad dv_x' = \frac{\cancel{a_0\, dt}\; a_0\left(1-\frac{u^2}{c^2}\right)}{\left(1-\frac{a_0 u}{c^2}t\right)^2} \cdot dt$$

$$\text{则}\ a_x' = \frac{dv_x'}{dt'} = \frac{\frac{a_0\left(1-\frac{u^2}{c^2}\right)}{\left(1-\frac{a_0 u}{c^2}t\right)^2}dt}{\frac{\left(1-\frac{u}{c^2}a_0 t\right)}{\sqrt{1-\frac{u^2}{c^2}}}dt} = \frac{\left(1-\frac{u^2}{c^2}\right)^{\frac{3}{2}}}{\left(1-\frac{u a_0}{c^2}t\right)^3} \cdot a_0$$

第四题.

解(1) 光从B到C ~~再到B~~ $ct_1=\sqrt{L_{BC}^2+u^2t_1^2}$ 得 $t_1=\frac{L_{BC}}{\sqrt{c^2-u^2}}$ 从B到C再到B用时 $2t_1=\frac{2L_{BC}}{\sqrt{c^2-u^2}}$

光从B到E $L_{BE}+ut_{21}=ct_{21}$ 得 $t_{21}=\frac{L_{BE}}{c-u}$ $L=ut_{22}+ct_{22}$ $t_{22}=\frac{L_{BE}}{c+u}$ $t_2=t_{21}+t_{22}=\frac{2cL_{BE}}{c^2-u^2}$

(2). 令 $2t_1=t_2$ 得 $L_{BE}=\sqrt{1-\frac{u^2}{c^2}}\cdot L_{BC}$ 即BE臂长缩短.

(3). ~~由洛伦兹变换~~ $\cancel{ct_{31}=\gamma(ct'_B-\beta x'_B)}$

由洛伦兹变换 $y=L$ $t_{BC}=\frac{L}{\sqrt{c^2-u^2}}$ 光从B到C到B用时 $2t_{BC}=\frac{2L}{\sqrt{c^2-u^2}}$

$x_B=\gamma(x'_B-\beta ct')$ $x_E=\gamma(x'_B+L-\beta ct')$ $x_{sBE}=x_E-x_B=\frac{L}{\sqrt{1-\frac{u^2}{c^2}}}$

从B到E再到B用时 $\frac{2x_{sBE}}{c}=\frac{2L}{\sqrt{c^2-u^2}}=2t_{BC}$

即在S系中看到光从B到C再到B与光从B到E再到B用时相等.

(4). 假设存在这样惯性系S'' 相对S以速度V沿x轴正方向运动. B在S''系的~~原~~原点

由洛伦兹变换 $ct_B=\gamma''(ct''_B-\cancel{\beta x})$ $ct_E=\gamma''[c(t''_B+\frac{L}{c})-\beta L]$

若 $t_E<t_B$ 则 $\beta>1$ 即 $V>c$ 与相对论假设矛盾

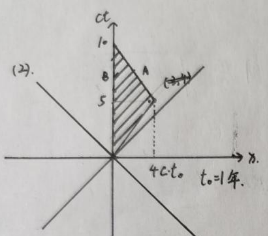
∴不存在惯性系S''使得光到C或E在从B发出前.

第五题 双生子佯谬及时空时.

(1). $\gamma=\frac{1}{\sqrt{1-(\frac{0.8c}{c})^2}}=\frac{5}{3}$ 由时间膨胀 $t=\gamma t'$ 其中 $t=10y$ 得 $t'=6y$

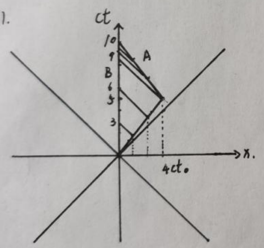
即地球经过10年而宇航员A只过了6年 A只送出6次红包 B送出了10次.

(2).



即A在抵达半人马座α星时收到一个红包 余下9个在返回途中收到.

(3).



由洛伦兹变换 $\cancel{t=\gamma t'}$

$\cancel{ct=\gamma(ct'-\beta x)}$

~~得 t=~~ $\cancel{0.8ct+ct=}$

$t_1=\frac{1}{3}\cdot 4ct_0\cdot\frac{1}{c}+\gamma t_0=3t_0=3$年.

$t_2=\frac{2}{3}4ct_0\cdot\frac{1}{c}+2\gamma t_0=6t_0=6$年

$t_3=4ct_0\cdot\frac{1}{c}+3\gamma t_0=9t_0=9$年.

即前9年每3年收到一个红包 最后一年收到3个红包.

第六题. 相对论性的动量和能量.

(1). 系统动量守恒 $\frac{m_0}{\sqrt{1-\frac{V_0^2}{c^2}}}\cdot V_0=\frac{m_1}{\sqrt{1-\frac{V_1^2}{c^2}}}\cdot V_1$

系质量守恒 $\frac{m_0}{\sqrt{1-\frac{V_0^2}{c^2}}}+m_0=\frac{m_1}{\sqrt{1-\frac{V_1^2}{c^2}}}$

得 $\frac{V_1}{V_0}=\frac{1}{1+\sqrt{1-\frac{V_0^2}{c^2}}}$ $\frac{m_1}{m_0}=\sqrt{2\frac{1-(\frac{V_0}{c})^2+\sqrt{1-(\frac{V_0}{c})^2}}{1-(\frac{V_0}{c})^2}}$ 令 $\beta=\frac{V_0}{c}$ 则 $\frac{m_1}{m_0}=\sqrt{2\frac{1-\beta^2+\sqrt{1-\beta^2}}{1-\beta^2}}$

(2). 设初粒子静质量为 m_α. 初速度为0 分裂为两静质量为 m_β 的粒子. 速度大小为 V_β.

质量守恒 $m_\alpha=\frac{2m_\beta}{\sqrt{1-\frac{V_\beta^2}{c^2}}}$

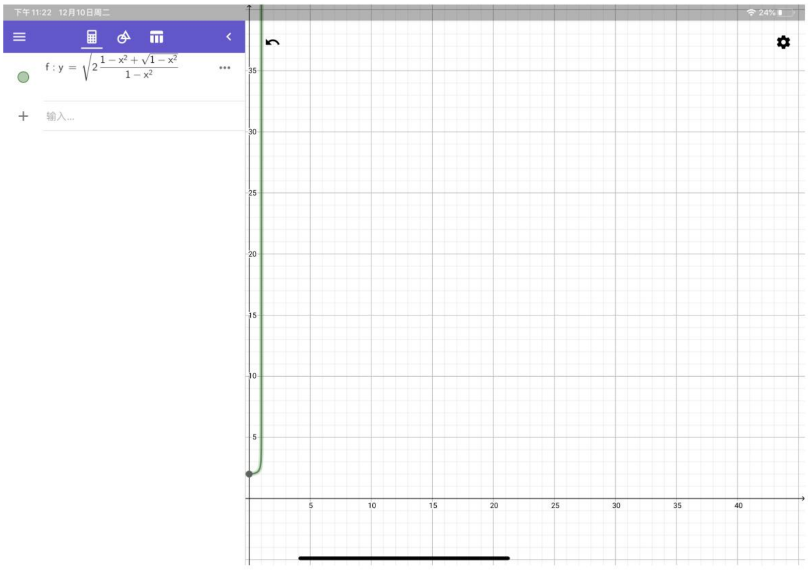
即 $m_\alpha>2m_\beta$ 粒子分裂时其静质量减小.

(3). 能量守恒 $\frac{E_0}{\sqrt{1-\frac{V^2}{c^2}}} = \cancel{2E_0}\ 2E_{h\nu}$ 得 $V=\frac{\sqrt{871}}{40}c$ $\frac{V}{c}=\sqrt{1-\left(\frac{E_0}{2E_{h\nu}}\right)^2}$

E_0为π介子静能 $E_{h\nu}$是每个光子能量.

$$\frac{\frac{E_0}{c^2}}{\sqrt{1-\frac{V^2}{c^2}}}\cdot V = \frac{2E_{h\nu}}{c}\cdot\cos\theta$$

得 $\theta=\arccos\left(\frac{\sqrt{871}}{40}\right)=42.45°$



（第一问图）

其中 y=m1/m0 x=V0/c

当 x 趋向于 0 时 y 趋向于 2，即碰后质量和等于初始两粒子质量和

当 x 趋向于 1 时 y 趋向于正无穷，即碰后质量和远大于两粒子静止质量和。