HW 4.

4-3 解. 当波前被遮住一半时 新的振幅 $A' = \frac{1}{2}A_0$.

则新的光强 $I' = \frac{1}{4}I_0$

即几何阴影边缘上的光强是自由传播时的 $\frac{1}{4}$. 比自由传播小 $\frac{3}{4}$ 倍.

4-5 解: 全自由传播时 振幅为 A_0. 光强为 I_0.

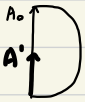
加有衍射屏时 振幅为 A' 光强为 I'

a. 

$\frac{A'}{A_0} = \sqrt{2}$ $\frac{I'}{I_0} = 2$

b. 

$\frac{A'}{A_0} = \sqrt{2}$ $\frac{I'}{I_0} = 2$

c. 

$\frac{A'}{A_0} = \frac{1}{2}$ $\frac{I'}{I_0} = \frac{1}{4}$

d 

$\frac{A'}{A_0} = 1$ $\frac{I'}{I_0} = 1$

e. 

$\frac{A'}{A_0} = \sqrt{5}$ $\frac{I'}{I_0} = 5$

e. $\frac{A'}{A_0} = \frac{1}{4}$ $\frac{I'}{I_0} = \frac{1}{16}$

4-9. 解. (1). $\rho_1 = \sqrt{f\lambda} = 0.566\ mm$

$\rho_k = \sqrt{k}\,\rho_1$ 以 ρ_k 为半径画出一系列同心环. 然后交替遮盖住奇数半波带.

(2) $\frac{A'}{A_0} = \sqrt{\frac{I'}{I_0}}$

$A' = k A_0$

得 $k = \sqrt{1000} \simeq 32$

$\rho_{32} = 3.18\ mm$ 即有效面积半径为 3.18 mm.

4-14. 解. 主极大半角宽 $\Delta\theta = \frac{\lambda}{a} = \frac{\Delta l}{2f}$

得 $a = \frac{2f\lambda}{\Delta l} = 63\,\mu m$

即细丝直径 63 μm.

4-17. 解. (1). $\delta y_m = \frac{0.61\lambda}{N.A}$

令 $\lambda = 550\ nm$ 得 $\delta y_m = 250\ nm$

(2). $\delta y_e = S_0 \delta\theta_e$　　S_0为明视距离　$\delta\theta_e$为最小分辨角.

得 $\delta y_e = 72.5\mu m$

$M = \frac{\delta y_e}{\delta y_m} = 290$

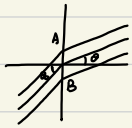
有效放大率为290.

(3). $\Delta = \frac{f_0 f_E}{S_0} \cdot M$

得$\Delta = 0.111\ m$

即光学筒长为0.111 m.

4-23. 解：考虑，衍射角为θ的一束光线，波前从B到A两侧光线光程差为



$\Delta L = a\cdot(\sin\theta - \sin\theta_0)$　　a为|AB|间距离

则相位差 $\Delta\varphi = \frac{2\pi}{\lambda}\Delta L = \frac{2\pi}{\lambda} a(\sin\theta - \sin\theta_0)$

场点合振幅为 $a_\beta = 2R\sin\alpha' = a_0 \frac{\sin\alpha'}{\alpha'}$　　$\alpha' = \frac{\Delta\varphi}{2} = \frac{\pi a}{\lambda}(\sin\theta\cdot\sin\theta_0)$

同理，多缝衍射 场点总振幅为 $A = a_\beta \frac{\sin N\beta'}{\sin\beta'}$　　$\beta' = \frac{\pi d}{\lambda}(\sin\theta - \sin\theta_0)$

$I = A^2 = I_0 \cdot (\frac{\sin\alpha'}{\alpha'})^2 \cdot (\frac{\sin N\beta'}{\sin\beta'})^2$

4-25解.　$A_{\theta 1} = a_\theta(1 + \cos\delta + \cos 3\delta)$

$A_{\theta 2} = a_\theta(\sin\delta + \sin 3\delta)$

$I_\theta = A_{\theta 1}^2 + A_{\theta 2}^2$

$= a_\theta^2 \cdot [3 + 2(\cos\delta + \cos 2\delta + \cos 3\delta)]$

$= I_0^2 (\frac{\sin\alpha}{\alpha})^2 \cdot [3 + 2(\cos 2\beta + \cos 4\beta + \cos 6\beta)]$

$\alpha = \frac{\pi a \sin\theta}{\lambda}$　　$\beta = \frac{\pi d\sin\theta}{\lambda}$

4-27. 解. (1). 遮住偶数缝. 则衍射屏变为一块缝宽为 a. 缝间距 $d=6a$ 的光栅.

$$\alpha=\frac{\pi a\sin\theta}{\lambda}\qquad \beta=\frac{\pi d\sin\theta}{\lambda}=\frac{6\pi a\sin\theta}{\lambda}$$

$$I=I_0\left(\frac{\sin\alpha}{\alpha}\right)^2\cdot\left(\frac{\sin N\beta}{\sin\beta}\right)^2$$

(2). 遮住奇数缝. 则衍射屏变为一块缝宽为 a. 缝间距 $d=6a$ 的光栅.

$$\alpha=\frac{\pi a\sin\theta}{\lambda}\qquad \beta=\frac{\pi d\sin\theta}{\lambda}=\frac{6\pi a\sin\theta}{\lambda}$$

$$I=I_0\left(\frac{\sin\alpha}{\alpha}\right)^2\cdot\left(\frac{\sin N\beta}{\sin\beta}\right)^2$$

(3). 全开放.

把两条间距为 $2a$ 的缝视作一个衍射单元. 则由间距 $d=6a$ 的 N 个衍射单元组成衍射屏.

$$\alpha=\frac{\pi a\sin\theta}{\lambda}\qquad \beta'=\frac{\pi d'\sin\theta}{\lambda}=2\frac{\pi a\sin\theta}{\lambda}\qquad u(\theta)=\frac{\sin\alpha}{\alpha}\frac{\sin2\beta'}{\sin\beta'}=2\frac{\sin\alpha}{\alpha}\cos2\alpha$$

$$\beta=\frac{\pi d\sin\theta}{\lambda}=6\frac{\pi a\sin\theta}{\lambda}\qquad N(\theta)=\frac{\sin N\beta}{\sin\beta}=\frac{\sin 6N\alpha}{\sin 6\alpha}$$

$$I(\theta)=I_0^2u^2(\theta)N^2(\theta)=4I_0^2\left(\frac{\sin\alpha}{\alpha}\cos2\alpha\right)^2\cdot\left(\frac{\sin 6N\alpha}{\sin 6\alpha}\right)^2$$

4-32. 解. (1) $\delta\lambda=\frac{\lambda}{R}$ λ 为闪耀波长.

得 $\delta\lambda=0.0051\ nm$

即能分辨的谱线间隔最小值为 $0.0051\ nm$.

(2). $D_\theta=\frac{D_1}{f}$

得 $\frac{1}{D_\theta}=0.244\ nm/分$. $D_\theta=4.10\times10^6\ 分/mm$ 即色散本领为 $4.10\times10^6\ 分/mm$

(3). $2d\sin\theta_b=\lambda_b$

得 $\theta_b=0.22$

即光栅的闪耀角为 $0.22\ rad$ 光栅方向与光栅平面的法线成 $0.22\ rad$.

4-34. 解. (1). 光栅: $R_1=\frac{D}{d}=3\times10^4$

棱镜 $R_2=b\frac{dn}{d\lambda}=3\times10^3$

法-珀干涉仪: $R_3=\frac{2\pi nh\cos\theta_k}{\lambda}\frac{\sqrt{R}}{1-R}$ 令 $\lambda=500nm$ $\cos\theta_k=1$ $n=1$ 得 $R_3=6\times10^5$.

得 $R_3>R_1>R_2$.

(2). 光栅: $D_1=\frac{1}{d\cos\theta_1}=2.2\ 分/nm$

棱镜: $D_2=\frac{2\sin\frac{\alpha}{2}}{\sqrt{1-n^2\sin^2\frac{\alpha}{2}}}\frac{dn}{d\lambda}=0.31\ 分/nm$.

法-珀腔: $D_3 = \frac{k}{2nh\sin\theta_k} = \frac{1}{\lambda\tan\theta_k} = 39\,分/nm.$

$D_3 > D_1 > D_2.$

(3). 光栅: $\lambda_M < d = 1700\,nm$ $\Delta\lambda = \lambda_M - \frac{1}{2}\lambda_M = 850nm$

故自由光谱范围 $850\,nm \sim 1700\,nm.$

棱镜: 无自由光谱范围限制.

法-珀腔: $k\lambda = (k-1)(\lambda+\Delta\lambda)$

得 $\Delta\lambda \simeq \frac{\lambda}{k} = \frac{\lambda^2}{2nh}$

当 $\lambda = 550\,nm$ 时 $\Delta\lambda = 0.003nm.$