第四周周五作业　3月27日.

1. 6.37 *Off-center hole* *

A long copper rod 8 cm in diameter has an off-center cylindrical hole, as shown in Fig. 6.43, down its full length. This conductor carries a current of 900 amps flowing in the direction "into the paper." What is the direction, and strength in gauss, of the magnetic field at the point P that lies on the axis of the outer cylinder?

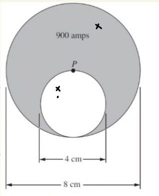


Figure 6.43.

解: 假设空心圆柱筒内同时有大小相等方向相反的电流密度 $J=\frac{I_0}{\pi[R^2-(\frac{R}{2})^2]}$

则P点磁感应强度可视为大圆筒垂直纸面向里的电流与小圆筒垂直纸面向外的电流在P点产生的磁感应强度的叠加。

大圆筒内电流在圆筒内部磁感应强度B:

由安培环路定理　$B\,2\pi r=\mu_0 J\cdot\pi r^2$　(当 $r\le R$ 时)　得 $B=\frac{\mu_0}{2}J\cdot r$

当 $r=0$ 时 $B_1=0$　即大圆筒内电流在P点生磁感应强度为0.

由安培环路定理得　$B_2\cdot 2\pi\cdot\frac{R}{2}=\mu_0 J\cdot\pi(\frac{R}{2})^2$　得 $B_2=\frac{\mu_0}{4}JR$

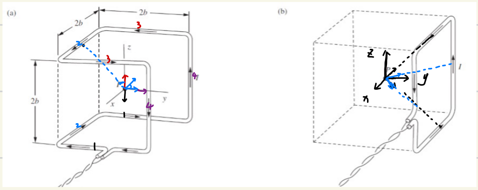
代入数据得　$B_2=1.5\times10^{-3}T$　方向水平向左、

$\vec{B}_p=\vec{B}_1+\vec{B}_2$　　得 P点磁感应强度大小 $1.5\times10^{-3}T$ 方向水平向左.

2 6.46 *Field from a wire frame* *

(a) Current I flows around the wire frame in Fig. 6.45(a). What is the direction of the magnetic field at P, the center of the cube?

(b) Show by using superposition that the field at P is the same as if the frame were replaced by the single square loop shown in Fig. 6.45(b).



解. (a). 如图(a) 根据对称性 标号为1的两部分导线 与标号为3的两部分 导线 在P点 处产生磁感应强度大小相等,方向相反. 故相互抵消.

根据对称性:

标号为2的两部分导线 在P产生的磁场沿y轴正方向.

标号为4的两部分导线 在P产生的磁场沿y轴正方向.

综上所述、P点磁感应强度 沿y轴正方向.

(b). 图a中: 由对称性可知道 标号为2、4 的两组导线在P点产生场强相同.

不妨 大小均为B_0。

同理. 图b中 每一部分导线 在P处产生磁感应强度 大小 为B_0 且每个$\vec{B_0}$除沿y轴方向分量全部互相抵消

则 $B_{pa} = 4B_0 \cdot \cos\theta = 4B_0 \cdot \frac{\sqrt{2}}{2} = 2\sqrt{2}B_0$ 方向沿y轴正方向.

$B_{pb} = 4B_0 \cos\alpha = 4B_0 \frac{\sqrt{2}}{2} = 2\sqrt{2}B_0$ 方向沿y轴正方向.

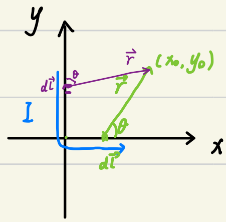
即 $\vec{B}_{pa} = \vec{B}_{pb}$

3.

6.52 *Right-angled wire* **

A wire carrying current I runs down the y axis to the origin, thence out to infinity along the positive x axis. Show that the magnetic field at any point in the xy plane (except right on one of the axes) is given by

$$B_z = \frac{\mu_0 I}{4\pi}\left(\frac{1}{x} + \frac{1}{y} + \frac{x}{y\sqrt{x^2+y^2}} + \frac{y}{x\sqrt{x^2+y^2}}\right). \qquad (6.98)$$



解: 由毕奥-萨法尔定律: $d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{l}\times\vec{r}}{r^3}$

考虑x轴上电流在(x_0,y_0)处产生的磁感应强度。则 $r=\sqrt{(x-x_0)^2+y_0^2}$

如图 $|d\vec{l}\times\vec{r}| = |d\vec{l}|\,|\vec{r}|\sin\theta = dx\cdot y_0$

则 $dB = \frac{\mu_0 I}{4\pi}\cdot\frac{dx\, y_0}{[(x-x_0)^2+y_0^2]^{\frac{3}{2}}}$

两侧积分 $B_{z_1} = \frac{\mu_0 I}{4\pi}\int_0^{+\infty}\frac{y_0\,dx}{[(x-x_0)^2+y_0^2]^{3/2}} = \frac{\mu_0 I}{4\pi}\, y_0\int_0^{+\infty}\frac{dx}{[(x-x_0)^2+y_0^2]^{3/2}}$

令 $t = x-x_0$ 则 $dt = dx$ $\quad \int_0^{+\infty}\frac{dx}{[(x-x_0)^2+y_0^2]^{3/2}} = \int_{-x_0}^{+\infty}\frac{dt}{(t^2+y_0^2)^{\frac{3}{2}}} = \frac{t}{y_0^2\sqrt{t^2+y_0^2}}\Big|_{-x_0}^{+\infty} = \frac{1}{y_0^2} - \frac{-x_0}{y_0^2\sqrt{x_0^2+y_0^2}}$

则 $B_{z_1} = \frac{\mu_0 I}{4\pi}\left(\frac{1}{y_0} + \frac{x_0}{y_0\sqrt{x_0^2+y_0^2}}\right)$

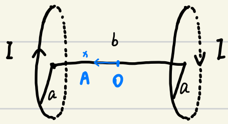
同理 y轴正半轴在(x_0,y_0)处产生磁感应强度 $B_{z_2} = \frac{\mu_0 I}{4\pi}\left(\frac{1}{x_0} + \frac{y_0}{x_0\sqrt{x_0^2+y_0^2}}\right)$

则在(x_0,y_0)处总磁感应强度为 $B_z = B_{z_1} + B_{z_2} = \frac{\mu_0 I}{4\pi}\left(\frac{1}{x_0} + \frac{1}{y_0} + \frac{x_0}{y_0\sqrt{x_0^2+y_0^2}} + \frac{y_0}{x_0\sqrt{x_0^2+y_0^2}}\right)$

则在任意一点(x,y) $(xy\neq 0)$处 $B_z = \frac{\mu_0 I}{4\pi}\left(\frac{1}{x} + \frac{1}{y} + \frac{x}{y\sqrt{x^2+y^2}} + \frac{y}{x\sqrt{x^2+y^2}}\right)$

4. 6.55 *Helmholtz coils* **

One way to produce a very uniform magnetic field is to use a very long solenoid and work only in the middle section of its interior. This is often inconvenient, wasteful of space and power. Can you suggest ways in which two short coils or current rings might be arranged to achieve good uniformity over a limited region? *Hint:* Consider two coaxial current rings of radius a, separated axially by a distance b. Investigate the uniformity of the field in the vicinity of the point on the axis midway between the two coils. Determine the magnitude of the coil separation b that for given coil radius a will make the field in this region as nearly uniform as possible.



解. 考察中轴线上距两环形电流轴线中点x处A点的磁感应强度.

$$B_{A_1}=\frac{\mu_0 I}{2}\frac{a^2}{[a^2+(\frac{b}{2}-x)^2]^{3/2}} \qquad B_{A_2}=\frac{\mu_0 I}{2}\frac{a^2}{[a^2+(\frac{b}{2}+x)^2]^{3/2}}$$

$$B_A=B_{A_1}+B_{A_2}=\frac{\mu_0 I a^2}{2}\left\{\frac{1}{[a^2+(\frac{b}{2}-x)^2]^{3/2}}+\frac{1}{[a^2+(\frac{b}{2}+x)^2]^{3/2}}\right\}$$

$$\frac{dB_A}{dx}=\frac{\mu_0 I a^2}{2}\left\{3\left(\frac{b}{2}-x\right)\cdot\left[a^2+\left(\frac{b}{2}-x\right)^2\right]^{-\frac{5}{2}}-3\left(\frac{b}{2}+x\right)\left[a^2+\left(\frac{b}{2}+x\right)^2\right]^{-\frac{5}{2}}\right\}$$

显然 $\left.\frac{dB_A}{dx}\right|_{x=0}=0$

$$\frac{d^2B_A}{dx^2}=\frac{\mu_0 I a^2}{2}\left\{-3\left[a^2+\left(\frac{b}{2}-x\right)^2\right]^{-\frac{5}{2}}+15\left(\frac{b}{2}-x\right)^2\cdot\left[a^2+\left(\frac{b}{2}-x\right)^2\right]^{-\frac{7}{2}}-3\left[a^2+\left(\frac{b}{2}+x\right)^2\right]^{-\frac{5}{2}}+15\left(\frac{b}{2}+x\right)^2\left[a^2+\left(\frac{b}{2}+x\right)^2\right]^{-\frac{7}{2}}\right\}$$

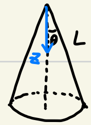
令 $\left.\frac{d^2B_A}{dx^2}\right|_{x=0}=0$ 得. $30\,\frac{b^2}{4}\cdot\left(a^2+\frac{b^2}{4}\right)^{-\frac{7}{2}}-6\left(a^2+\frac{b^2}{4}\right)^{-\frac{5}{2}}=0$

得 $b^2=a^2$ 即 $b=a$

对于给定的环形电流半径a. 两环形导线间距 $b=a$ 时 中轴线上附近的有限区域内磁场有较好均匀性.

5. 6.56 *Field at the tip of a cone* **

A hollow cone (like a party hat) has vertex angle 2θ, slant height L, and surface charge density σ. It spins around its symmetry axis with angular frequency ω. What is the magnetic field at the tip?



解：如图从顶点沿竖直向下建立坐标系

dzB

则在坐标z处. 圆半径 $R = z\tan\theta$

该圆弧通电后在顶点处产生磁感应强度大小为 $dB_z = \frac{\mu_0 I R^2}{2(R^2+z^2)^{\frac{3}{2}}}$

设在坐标z处有一微小增量 dz　则 $dq = (\omega R\cdot dt)\cdot\frac{dz}{\cos\theta}\cdot\sigma$

$$I = \frac{dq}{dt} = \frac{\omega\sigma\sin\theta}{\cos^2\theta}\cdot z\,dz$$

$$则\ dB_z = \frac{\mu_0}{2}\frac{\omega\sigma\sin\theta}{\cos^2\theta} z\,dz\,\frac{z^2\tan^2\theta}{z^3}\cdot\cos^3\theta = \frac{\mu_0\omega\sigma\sin^3\theta}{2\cos\theta}\,dz$$

$$则\ B_z = \int_0^{L\cos\theta}\frac{\mu_0\omega\sigma\sin^3\theta}{2\cos\theta}\,dz = \frac{1}{2}\mu_0\omega\sigma L\sin^3\theta$$

即顶点的磁感应强度大小为 $\frac{1}{2}\mu_0\omega\sigma L\sin^3\theta$　方向与 $\vec{\omega}$ 方向相同