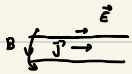
第六周周五作业 4月10日

1. 9.28 *Poynting vector and resistance heating* ∗∗

A longitudinal **E** field inside a wire causes a current via $\mathbf{J}=\sigma\mathbf{E}$. And since the curl of **E** is zero, this same longitudinal **E** component must also exist right outside the surface of the wire. Show that the Poynting vector flux through a cylinder right outside the wire accounts for the *IV* resistance heating.



解: 设导线截面半径为 r 则 $I=\vec{J}\cdot(\pi R^2\hat{n})=\sigma\pi r^2E$

现考虑长度为 L 的导线 $R=\frac{1}{\sigma}\frac{L}{\pi r^2}=\frac{L}{\sigma\pi r^2}$

$P=I^2R=\sigma\pi r^2E^2L$

由安培环路定理 $B\,2\pi r=\mu_0 I$ $B=\frac{\mu_0\sigma rE}{2}$

$\vec{P}=\frac{\vec{E}\times\vec{B}}{\mu_0}=\frac{\sigma rE^2}{2}(-\hat{r})$ $\hat{r}$为沿半方向的单位矢量

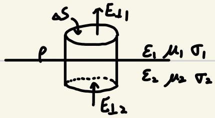
$\Phi_p=|\vec{P}\cdot L\,2\pi r\,\hat{r}|=\sigma\pi r^2LE^2$

则 $\Phi_p=P$ 即通过导线外部圆柱体的 Poynting 矢量的通量是电阻发热的原因

2. 10.14 *Boundary conditions on E and B* ∗∗

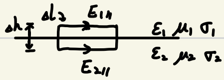
Find the boundary conditions on $E_\parallel$, $E_\perp$, $B_\parallel$, and $B_\perp$ across the interface between two linear dielectrics. Assume that there are no free charges or free currents present.

解：考虑两种不同介质的接触面：



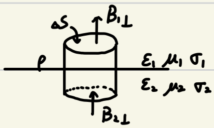
如图 $\Delta S\to 0$ 由高斯定理 $\varepsilon_1 E_{\perp 1}\Delta S - \varepsilon_2 E_{\perp 2}\Delta S = \rho\,\Delta S$

由 $\rho=0$ 得 $\varepsilon_1 E_{1\perp} = \varepsilon_2 E_{2\perp}$ ε_1，ε_2 分别为第一、二种介质的介电常数.



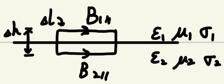
如图由环路定理 $E_{1\parallel}\cdot dl - E_{2\parallel}\,dl = -\frac{d\Phi}{dt} = -\frac{d}{dt}(B\,dl\,dh) = 0$

得 $E_{1\parallel} = E_{2\parallel}$



如图由磁场高斯定理 $B_{1\perp}\Delta S - B_{2\perp}\Delta S = 0$

得 $B_{1\perp} = B_{2\perp}$



如图由安培环路定理 $\frac{B_{1\parallel}}{\mu_1}dl - \frac{B_{2\parallel}}{\mu_2}dl = J\cdot dl$

由 $J=0$ 得 $\frac{B_{1\parallel}}{\mu_1} = \frac{B_{2\parallel}}{\mu_2}$ $\mu_1\mu_2$ 分别为第一、二种介质的磁导率

综上所述 边界条件为 $\varepsilon_1 E_{1\perp} = \varepsilon_2 E_{2\perp}$ $E_{1\parallel} = E_{2\parallel}$

$B_{1\perp} = B_{2\perp}$ $\frac{B_{1\parallel}}{\mu_1} = \frac{B_{2\parallel}}{\mu_2}$

3. 10.18 *Partially filled capacitors* **

Figure 10.33 shows three capacitors of the same area and plate separation. Call the capacitance of the vacuum capacitor C_0. Each of the others is half-filled with a dielectric, with the same dielectric constant κ, but differently disposed, as shown. Find the capacitance of each of these two capacitors. (Neglect edge effects.)

解：设C_1表示中间图中电容大小，C_2表示下图中电容大小.

则 $\frac{1}{C_1} = \frac{1}{2C_0} + \frac{1}{2\kappa C_0}$ 得 $C_1 = \frac{2\kappa}{1+\kappa} C_0$

$C_2 = \frac{C_0}{2} + \frac{\kappa C_0}{2}$ 得 $C_2 = \frac{1+\kappa}{2} C_0$

4\.

10.30 *Polarized hydrogen* **

A hydrogen atom is placed in an electric field E. The proton and the electron cloud are pulled in opposite directions. Assume simplistically (since we are concerned only with a rough result here) that the electron cloud takes the form of a uniform sphere with radius a, with the proton a distance Δz from the center, as shown in Fig. 10.40. Find Δz, and show that your result agrees with Eq. (10.27).

Electron cloud

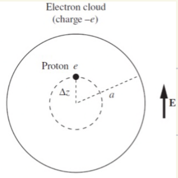


Figure 10.40.

解: 设电子云球体的体电荷密度为ρ 则 $\rho = \frac{e}{\frac{4}{3}\pi a^3} = \frac{3e}{4\pi a^3}$

质子受力平衡得 $eE = \frac{1}{4\pi\varepsilon_0}\frac{e\cdot\rho\frac{4}{3}\pi\Delta z^3}{\Delta z^2}$

得 $\Delta z = \frac{4\pi\varepsilon_0 E a^3}{e}$

$\frac{\Delta z}{a} = \frac{E}{e/4\pi\varepsilon_0 a^2}$ 即结果满足(10.27)

5. 10.42 *Energy density in a dielectric* **

By considering how the introduction of a dielectric changes the energy stored in a capacitor, show that the correct expression for the energy density in a dielectric must be $\epsilon E^2/2$. Then compare the energy stored in the electric field with that stored in the magnetic field in the wave studied in Section 10.15.

解：设在初始电容为 C_0 的电容器内加入介电常数为 ε_r 的介质.

则 $U=\dfrac{Q^2}{2\varepsilon_r C_0}$ $\quad E=\dfrac{Q}{\varepsilon_r C_0 d}$

则 $$\frac{U}{V}=\frac{Q^2}{2\varepsilon_r C_0 S d}=\frac{1}{2}\varepsilon_r\varepsilon_0\frac{Q^2}{\varepsilon_r^2 C_0\frac{\varepsilon_0 S}{d}d^2}=\frac{1}{2}\varepsilon_r\varepsilon_0\left(\frac{Q}{\varepsilon_r\varepsilon_0 d}\right)^2=\frac{1}{2}\varepsilon E^2$$

即电介质内能量密度为 $\frac{1}{2}\varepsilon E^2$

$$U_{总}=\frac{\varepsilon}{2}(E^2+v^2B^2)=\frac{\varepsilon}{2}\left(E^2+\frac{B^2}{\mu_0\varepsilon}\right)=\frac{\varepsilon}{2}(E^2+E^2)=\varepsilon E^2$$

即电介质中电场能量密度是电磁场能量密度的 $\frac{1}{2}$.