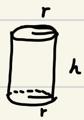
第五周 周五 4月3日作业

1. 7.16 *Energy in a superconducting solenoid* *

A superconducting solenoid designed for whole-body imaging by nuclear magnetic resonance is 0.9 meters in diameter and 2.2 meters long. The field at its center is 3 tesla. Estimate roughly the energy stored in the field of this coil.



解：近似可以认为螺线管能量密度处处相等 则 令 $\rho_u = \frac{B^2}{2\mu_0}$

则 $U = \rho_u \cdot V = \frac{B^2}{2\mu_0} \cdot \pi\left(\frac{d}{2}\right)^2 \cdot h$

代入数据得 $U = 5.01 \times 10^6 \, J$

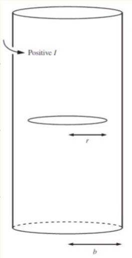
2. 7.27 *Ring in a solenoid* **

An infinite solenoid with radius b has n turns per unit length. The current varies in time according to $I(t) = I_0 \cos \omega t$ (with positive defined as shown in Fig. 7.36). A ring with radius $r < b$ and resistance R is centered on the solenoid's axis, with its plane perpendicular to the axis.

(a) What is the induced current in the ring?

(b) A given little piece of the ring will feel a magnetic force. For what values of t is this force maximum?

(c) What is the effect of the force on the ring? That is, does the force cause the ring to translate, spin, flip over, stretch/shrink, etc.?



解: (a). 螺线管内部磁感应强度大小为 $B=\mu_0 n I(t)$

$\Phi = B \cdot \pi r^2 = \mu_0 n \pi r^2 I_0 \cos(wt)$

由法拉第电磁感应定律 $\varepsilon_1 = -\frac{d\Phi}{dt} = \mu_0 n \pi r^2 I_0 w \sin(wt)$

$I_1 = \frac{\varepsilon_1}{R} = \frac{\mu_0 n \pi w r^2 I_0}{R} \sin(wt)$

(b). 设 F为t时刻环受力大小, 则 $F = |B I_1 \cdot 2\pi r| = \frac{\mu_0^2 \pi^2 n^2 w r^3 I_0^2}{R} |\sin(2wt)|$

当F最大时 $|\sin(2wt)| = 1$ 则 $2wt = \frac{\pi}{2} + k\pi$ $k \in N$ 即 $t = \frac{1}{w}(\frac{\pi}{4} + \frac{k\pi}{2})$ $k \in N$

(c). 在电流 $|I(t)| = |I_0 \cos(wt)|$ 减小阶段, 即 $wt \in [k\pi, \frac{\pi}{2} + k\pi]$ $k \in N$ 阶段, 力会使环有扩张的趋势.

在电流 $|I(t)| = |I_0 \cos(wt)|$ 增大阶段. 即 $wt \in [\frac{\pi}{2} + k\pi, \pi + k\pi]$ $k \in N$ 阶段, 力会使环有收缩趋势

力不会导致环平移.旋转.弯曲.

3. 7.26 *Sliding bar* **

A metal crossbar of mass m slides without friction on two long parallel conducting rails a distance b apart; see Fig. 7.35. A resistor R is connected across the rails at one end; compared with R, the resistance of bar and rails is negligible. There is a uniform field $\mathbf{B}$ perpendicular to the plane of the figure. At time $t = 0$ the crossbar is given a velocity v_0 toward the right. What happens afterward?

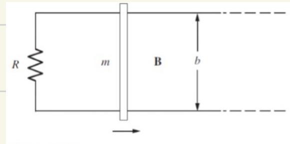


Figure 7.35.

解: 系统中磁场B是不变的. 则只产生动生电动势 大小为 $\varepsilon = Bbv$ 其中 $v = v(t)$

$$I = \frac{\varepsilon}{R} = \frac{Bbv}{R}$$

由牛顿第二定律得 $-BIb = m\frac{dv}{dt}$ 即 $-\frac{B^2b^2}{mR}dt = \frac{dv}{v}$

两侧不定积分得 $-\frac{B^2b^2}{mR}t = \ln v + C$ 当 $t=0$ 时 $v = v_0$ 得 $C = -\ln v_0$

得 $v = v_0 e^{-\frac{B^2b^2}{mR}t}$

(a). 令 $v=0$ 得 $t=\infty$ 即在时间趋向于 $+\infty$ 时. 停止.

(b). $x = \int_0^{+\infty} v\,dt = v_0\int_0^{+\infty} e^{-\frac{B^2b^2}{mR}t} = \frac{mRv_0}{B^2b^2}$

即总共移动了 $\frac{mRv_0}{B^2b^2}$

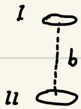
(c). $E_1 = \frac{1}{2}mv_0^2$ 初始系统具有总能量

$E_2 = \int_0^{+\infty} I^2R\,dt = \frac{B^2b^2v_0^2}{R}\int_0^{+\infty} e^{-\frac{2B^2b^2}{mR}t}dt = \frac{1}{2}mv_0^2$ R上消耗的总能量.

即 $E_1 = E_2$ 系统能量守恒 从最初杆的动能 转化为最终R上的内能.

4. 7.35 *M for two rings* **

Derive an approximate formula for the mutual inductance of two circular rings of the same radius a, arranged like wheels on the same axle with their centers a distance b apart. Use an approximation good for $b \gg a$.



解: 如图: 给II环通入一稳恒电流 i

II环在I环处产生的磁感应强度为 $B_{12} = \dfrac{\mu_0 i a^2}{2(a^2+b^2)^{3/2}}$

则 $\Phi_{12} = B_{12} \cdot \pi a^2 = \dfrac{\mu_0 \pi i a^4}{2(a^2+b^2)^{3/2}}$

则 $M_{12} = \dfrac{\Phi_{12}}{i} = \dfrac{\mu_0 \pi a^4}{2(a^2+b^2)^{3/2}}$

则 $b >> a$ 时 $(a^2+b^2)^{3/2} = b^3$ 即 $M_{12} = \dfrac{\mu_0 \pi a^4}{2 b^3}$

5. 7.41 *Opening a switch* **

In the circuit shown in Fig. 7.41 the 10 volt battery has negligible internal resistance. The switch S is closed for several seconds, then opened. Make a graph with the abscissa time in milliseconds, showing the potential of point A with respect to ground, just before and then for 5 milliseconds after the opening of switch S. Show also the variation of the potential at point B in the same period of time.

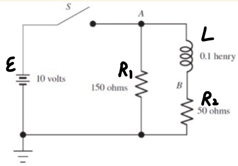


Figure 7.41.

解: A点相对地面的电势即为左侧电阻两端电压.

B点相对地面的电势即为右侧电阻两端电压.

S闭合回路到达稳定状态后 电感 对回路无影响(视为导线)

即在开关断开前的 5ms 内 $U_A = U_B = 10V$

S断开之后 $-L\frac{dI}{dt} = I(R_1+R_2)$ 得 $-\frac{R_1+R_2}{L}dt = \frac{dI}{I}$ 两侧不定积分得 $-\frac{R_1+R_2}{L}t = \ln I + C$ 当 $t=0$ 时 $I_0 = \frac{\varepsilon}{R_2}$ 得 $C = -\ln\frac{\varepsilon}{R_2}$

则 $I = \frac{\varepsilon}{R_2}\cdot e^{-\frac{R_1+R_2}{L}t}$ $(t\geq 0)$

$U_A = IR_1 = \frac{\varepsilon R_1}{R_2}e^{-\frac{R_1+R_2}{L}t}$ $\qquad U_B = IR_2 = \varepsilon e^{-\frac{R_1+R_2}{L}t}$

代入数据 $U_A = \begin{cases} 10, & -5ms < t < 0 \\ 30e^{-2000t}, & 0 \leq t < 5ms \end{cases}$ $\qquad U_B = \begin{cases} 10, & -5ms < t < 0 \\ 10e^{-2000t}, & 0 < t < 5ms \end{cases}$