Review

Rookie-Rounder Djy

June 7, 2021

Name	Parameters	Support	p.d(m).f.	c.d.f.	m.g.f.	**E**	**Var**
Bernoulli D	p	0,1	f(1)=p,f(0)=1-p		$pe^t + 1 - p$	p	p(1-p)
Binomial D	n,p	$x \in N$	$f(x) = \binom{n}{x} p^x (1-p)^{n-x}$		$(pe^t + 1 - p)^n$	np	np(1-p)
Hypergeometric D	N,M,n	[0,n]$\cap N$	$p_X(x) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$			$\frac{nM}{N}$	$\frac{nM}{N}\frac{(N-M)(N-n)}{N(N-1)}$
Possion D	$\lambda > 0$	$x \in N$	$p_X(x) = \frac{e^{-\lambda}\lambda^x}{x!}$		$e^{\lambda(e^t-1)}$	λ	λ
Negative Binomoal D	r,p	$x \in N$	$p_X(x) = \binom{r+x-1}{x} p^r (1-p)^x$		$\left(\frac{p}{1-(1-p)e^t}\right)^r$	$\frac{r(1-p)}{p}$	$\frac{r(1-p)}{p^2}$
Geometry D	p	$x \in N$	$p_X(x) = p(1-p)^x$				
The Normal D	μ, σ^2	$x \in R$	$f(x) = \frac{1}{\sigma\sqrt{2\pi}} exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right)$	$\Phi\left(\frac{x-\mu}{\sigma}\right)$	$exp\left(\mu t + \frac{1}{2}\sigma^2 t^2\right)$	μ	σ^2
The Standard Normal D	0,1	$x \in R$	$f(x) = \frac{1}{\sqrt{2\pi}} exp\left(-\frac{1}{2}x^2\right)$	$\Phi(x)$	$exp\left(\frac{1}{2}t^2\right)$	0	1
Gamma D	α, β	x>0	$f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}$		$\left(\frac{\beta}{\beta - t}\right)^\alpha,\ t < \beta$	$\frac{\alpha}{\beta}$	$\frac{\alpha}{\beta^2}$
Exponential D	β	x>0	$f(x) = \beta e^{-\beta x}$	$F(x) = 1 - e^{\beta x}$	$\frac{\beta}{\beta - t},\ t < \beta$	$\frac{1}{\beta}$	$\frac{1}{\beta^2}$
Beta D	α, β	0<x<1	$f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}$			$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$
Multinomial D	$\mathbf{p} = (p_i)_{i=1}^k$	$\sum_{i=1}^k x_i = n$	$f(\mathbf{x}) = \binom{n}{x_1, x_2, \ldots, x_k} \prod_{i=1}^k p_i^{x_i}$				
The Chi-Square D	m>0	x>0	$f(x) = \Gamma\left(\frac{m}{2}, \frac{1}{2}\right)$			m	2m
The t-D	m>0	$x \in R$	$X = \frac{Z}{(Y/m)^{1/2}},\ Y \sim \chi^2(m),\ Z \sim N(0,1)$				
The F-D	m>0,n>0	x>0	$X = \frac{Y/m}{W/n},\ Y \sim \chi^2(m),\ W \sim \chi^2(n)$				

Important Theorem:

1. $(X_i)_{i=1}^k$ independent follows the Binomial Distribution with n_i and p, then $X = \sum_{i=1}^k X_i$ follows the Binomial Distribution with $n = \sum_{i=1}^k n_i$ and p.
2. $(X_i)_{i=1}^k$ independent follows the Possion Distribution with λ_i, then $X = \sum_{i=1}^k$ follows the Possion Distribution with $\lambda = \sum_{i=1}^k \lambda_k$.

3. $(X_i)_{i=1}^k$ are i.i.d. follows the Geometric Distribution with p, then the sum $X = \sum_{i=1}^k X_i$ follows the Negative Binomial Distribution with k and p.
4. $X \sim N(\mu, \sigma^2)$ and Y=aX+b $a \neq 0$, then $Y \sim N(a\mu + b, a^2\sigma^2)$.
5. $(X_i)_{i=1}^k$ independent follows $N(\mu_i, \sigma_i^2)$, then $X = \sum_{i=1}^k X_i$ follows $N(\sum_{i=1}^k \mu_i, \sum_{i=1}^k \sigma_i^2)$.
6. $(X_i)_{i=1}^k$ independent follows the Gamma Distribution with α_i and β, then $X = \sum_{i=1}^k X_i$ follows the Gamma Distribution with $\sum_{i=1}^k \alpha_i$ and β.
7. $(X_i)_{i=1}^k$ i.i.d. follows the Exponential Distribution with β, then Y=min$\{X_i\}$ follows the Exponential Distribution with $n\beta$.
8. $(X_i)_{i=1}^n$ are random sample from $N(\mu, \sigma^2)$, there are

   (a) the sample mean has

$$\overline{X}_n \sim N(\mu, \sigma^2/n).$$

   (b) Sample mean and sameple variance are independent and

$$\frac{(n-1)S_n^2}{\sigma^2} \sim \chi^2(n-1).$$

   (c) When μ is known we have

$$\frac{\overline{X}_n - \mu}{S_n/\sqrt{n}} \sim t(n-1).$$

   (d) Suppose $(X_i)_{i=1}^m$ and $(Y_j)_{j=1}^n$ are random samples from $N(\mu_X, \sigma_X^2)$ and $N(\mu_Y, \sigma_Y^2)$. For sample variance S_X^2 and S_Y^2 we have

$$\frac{S_X^2/S_Y^2}{\sigma_1^2/\sigma_2^2} \sim F(m-1, n-1).$$

   When $\sigma_X = \sigma_Y = \sigma$ we have

$$\frac{(\overline{X} - \mu_X) - (\overline{Y} - \mu_Y)}{S\sqrt{\frac{1}{m} + \frac{1}{n}}} \sim t(m+n-2),$$

   where S is

$$\frac{(m+n-2)S^2}{\sigma^2} = \frac{(m-1)S_X^2}{\sigma^2} + \frac{(n-1)S_Y^2}{\sigma^2} \sim \chi^2(m+n-2).$$