上海科技大学

2022-2023 学年第一学期期末考试卷

开课单位：物质学院

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考试科目：凝聚态拓扑物理

课程代码：PHYS2122

姓名：

学号：

学院：

班级：

考生须知：

1．请严格遵守考场纪律，禁止任何形式的作弊行为。

2．参加闭卷考试的考生，除携带必要考试用具外，书籍、笔记、掌上电脑和其他电子设备等物品一律按要求放在指定位置。

3．参加开卷考试的考生，可以携带教师指定的材料独立完成考试，但不准相互讨论，不准交换材料。

考试成绩录入表：

题目	1	2	3	4	5	6	7	8	9	10	**总分**
计分											
复核											

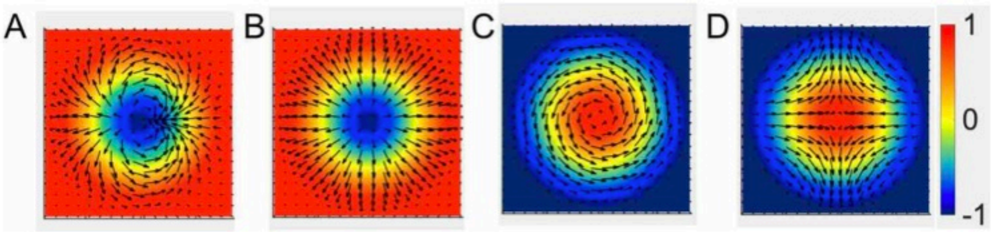
评卷人签名：　　　　**复核人签名：**

日期：　　　　**日期：**

1 Multiple Choice Questions

1. (6) Consider a mapping from a S^2 order parameter space $\vec{n} = (n_1, n_2, n_3)$ of a vector field onto a two-dimensional real-space $\vec{r} = (r_1, r_2)$ with boundaries. Which of the following mappings correspond to a topological index of 1? (colormap indicates n_3 component)

Answer (   )



2. (6) Which of the following properties are from fermion particles?

Answer (   )

A. $[a_i, a_j^\dagger] = \delta_{i,j}$.
B. $\{a_i, a_j^\dagger\} = \delta_{i,j}$.
C. $[a_i, a_j] = 0$.
D. $\{a_i, a_j\} = 0$.

3. (6) Which of the following quantities are naturally quantised:

Answer (   )

A. Magnetic charge q_m.
B. Electric charge q_c.
C. Berry phase γ that is picked up in an adiabatic process.
D. Anomalous Hall conductivity σ_{xy}.
E. Topological winding number in a $S^2 \to S^2$ map.

4. (6) One-band tight-binding model in a two-dimensional square lattice can be described by:

$$\mathcal{H} = -2t \sum_k a_k^\dagger a_k [\cos(k_x a) + \cos(k_y a)] \ , \tag{1}$$

where t is real-valued hopping constant. Evaluate the topological property of this band:

Answer (   )

A. It is topologically trivial, as the kernel Hamiltonian cannot be parametrised in a form of $\sigma \cdot \vec{H}$.
B. It is topologically trivial, as $u_{n,k}(r)$ function is independent of $\vec{k}$.
C. It is topologically nontrivial, as it carries Chern number of $N = 1$.
D. It is topologically nontrivial, as the Hamiltonian describes $S^2 \to S^2$ mapping, which associates integer topological index.
E. It is not sure unless the Berry phase is computed.
F. It is not sure, as it depends on the choice of t.

5. (6) Which of following quantities are mathematically equivalent to **Berry connection**:

Answer (   )

A. Gaussian curvature
B. Vector potential
C. Monopole charge
D. Magnetic field
E. Berry curvature

6. (6) Two-band tight-binding Su-Schrieffer-Heeger (SSH) model in one-dimensional chiral lattice can be described by:

$$\mathcal{H} = \sum_k (a_k^\dagger, b_k^\dagger) \begin{pmatrix} 0, & -t_1 - t_2 e^{-ika} \\ -t_1 - t_2 e^{ika}, & 0 \end{pmatrix} \begin{pmatrix} a_k \\ b_k \end{pmatrix} \quad . \tag{2}$$

where t_1 and t_2 are real-valued hopping constants. Evaluate the topological property of the upper band $|+\rangle$:

Answer (   )

A. It is topologically nontrivial, as the kernel Hamiltonian can be parametrised in a form of $\sigma \cdot \vec{H}$, which immediately puts forward to the existence of effective monopole.
B. It is topologically nontrivial, as $u_{n,k}(r)$ is a function of $\vec{k}$, meaning that Berry connection can be defined everywhere in k-space, leading to a Chern number of 1.
C. It is topologically trivial, as it carries Chern number of $N = 0$.
D. It is topologically trivial, as the Hamiltonian describes $S^2 \to S^1$ mapping, from which topology cannot be defined.
E. It is not sure unless the Berry phase is computed.
F. It is not sure, as it depends on the choice of t_1 and t_2.

7. (6) Which of the following quantities are mathematically equivalent to geometrical **Gaussian curvature**:

Answer (   )

A. Berry connection
B. Vector potential
C. Monopole charge
D. Magnetic field
E. Berry curvature

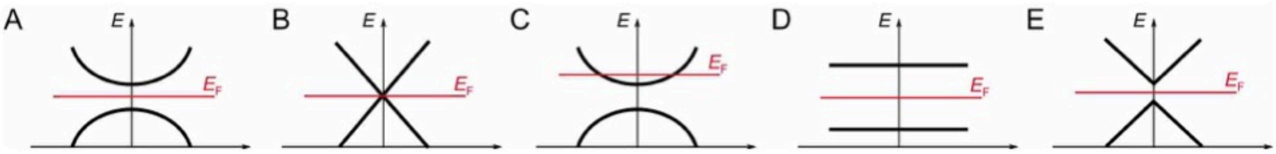
8. (6) Phonons are second-quantisation version of atomic waves. It is then obvious that one can define a set of basis in form of creation/annihilation operators in k-space. This means that at each k, there is a $|\phi_k\rangle$ wave function assigned. Now consider a two-band phonon model in two-dimensional atomic lattice, i.e., two vibration modes, e.g., acoustic mode (band) and optical mode (band). Evaluate the topological property of the bands:

Answer (   )

A. They are topologically trivial, as $\vec{A}(\vec{k}) = i\langle\phi_k|\nabla_k|\phi_k\rangle$ leads to zero.
B. They are topologically nontrivial, as this corresponds to $S^2 \to S^2$ mapping, from which topology can be found to be $N = 1$.
C. It is not sure unless the Berry phase is computed.
D. The topology cannot be defined, as it is a boson system.

9. **(6)** Which of following electronic structures MUST NOT be a topological insulator:

Answer ( )



10. **(6)** Which of the following quantities have the physical interpretation in the unit of a solid angle:

Answer ( )

A. $\iint_{\mathrm{BZ}} dk_x dk_y \vec{H} \cdot (\frac{\partial \vec{H}}{\partial k_y} \times \frac{\partial \vec{H}}{\partial k_x})$
B. $\oint_C \vec{A}(\vec{k}) \cdot d\vec{l}$
C. Monopole charge q_m
D. Anomalous Hall conductivity σ_{xy}
E. $\iint_{\mathrm{BZ}} \vec{B}(\vec{k}) \cdot d\vec{s}$

2 Fill in the Blanks

1. (9) Consider a quantum system with its eigenstate $|n\rangle$, which varies by a parameter $\mathbf{R}$, i.e., $|n(\mathbf{R})\rangle$. In an adiabatically varying-$\mathbf{R}$ process, after a cyclic loop,

the Berry connection is written as ( ).

the Berry curvature is written as ( ).

Therefore the geometric phase that is picked up by the cyclic process is ( ).

2. (9) Quantum Hall effect is an experimentally observable effect in real materials. To perform the experiment, it is usually required the following:

( )

Consequently, the resistivity ρ_{xx} switches between zero and non-zero at different magnetic field. This is due to ( ).

At the same time, the Hall conductivity σ_{xy} is quantised, written as ( ).

3. (9) For a many-body magnetic system, continuous approximation allows the magnetic configuration to be a vector field described by $\mathbf{m}(\mathbf{r})$, where $\mathbf{m}$ is normalised unit vector magnetisation, and $\mathbf{r}$ describes two-dimensional space. Consider the energy density in the form of

$$w = A(\nabla \mathbf{m})^2 + D\mathbf{m} \cdot (\nabla \times \mathbf{m}) - \mathbf{m} \cdot \mathbf{B} \ , \tag{3}$$

The dimensionality of the order parameter space is thus ( ).
The dimensionality of the physical space is thus ( ).
A topological index can take the values of ( ).

4. (13) A quantum system has three degeneracies as its ground state, labeled as $|\phi_n^{(v)}\rangle$, where v labels the degeneracies. An adiabatic evolution $\hat{H}(\mathbf{p})$ in a parameter space of $\mathbf{p}$ perturbs the system's ground state with its local eigenfunction of $|\phi_v(\mathbf{p})\rangle$. The initial state is defined as $|\phi_v(\mathbf{p}_0)\rangle$ at $\mathbf{p}_0$.

If the system pauses at $\mathbf{p}_1$, how many $\mathbf{p}$-dependent eigenvalues are expected? (   )

Will the local gauge transformation be performed? (   )
If yes, the gauge transformation will be(   ).

Their associating local basis at $\mathbf{p}_1$ will be derived as (   ).

The picked-up Berry phase is (   ).

The parameter space of $\mathbf{p}$ can be mapped into another parameter space of $\mathbf{R}$, such that $|\phi_v(\mathbf{p})\rangle = |\phi_v[\mathbf{R}(\mathbf{p})]\rangle = |\phi_v(\mathbf{R})\rangle$. The cyclic path (starts at $\mathbf{R_0}$ and eventually returns to $\mathbf{R}_0$) in $\mathbf{R}$-space, denoted as $\mathcal{R}_L$, encounters two distinct U-matrices in order to diagonalise $\hat{H}(\mathbf{R})$, denoted as $U_1(\mathcal{R}_L)$ and $U_2(\mathcal{R}_L)$.

How many $\mathbf{p}$-dependent eigenvalues are expected during the loop? (   )

Will the local gauge transformation be performed? (   )
If yes, the gauge transformation will be(   ).

Their associating local basis at $\mathbf{R}_0$ will be derived as (   ).

The picked-up Berry phase is (   ).