COMP5048
Visual Analytics

Week 3

Visualisation of Relational Data I:

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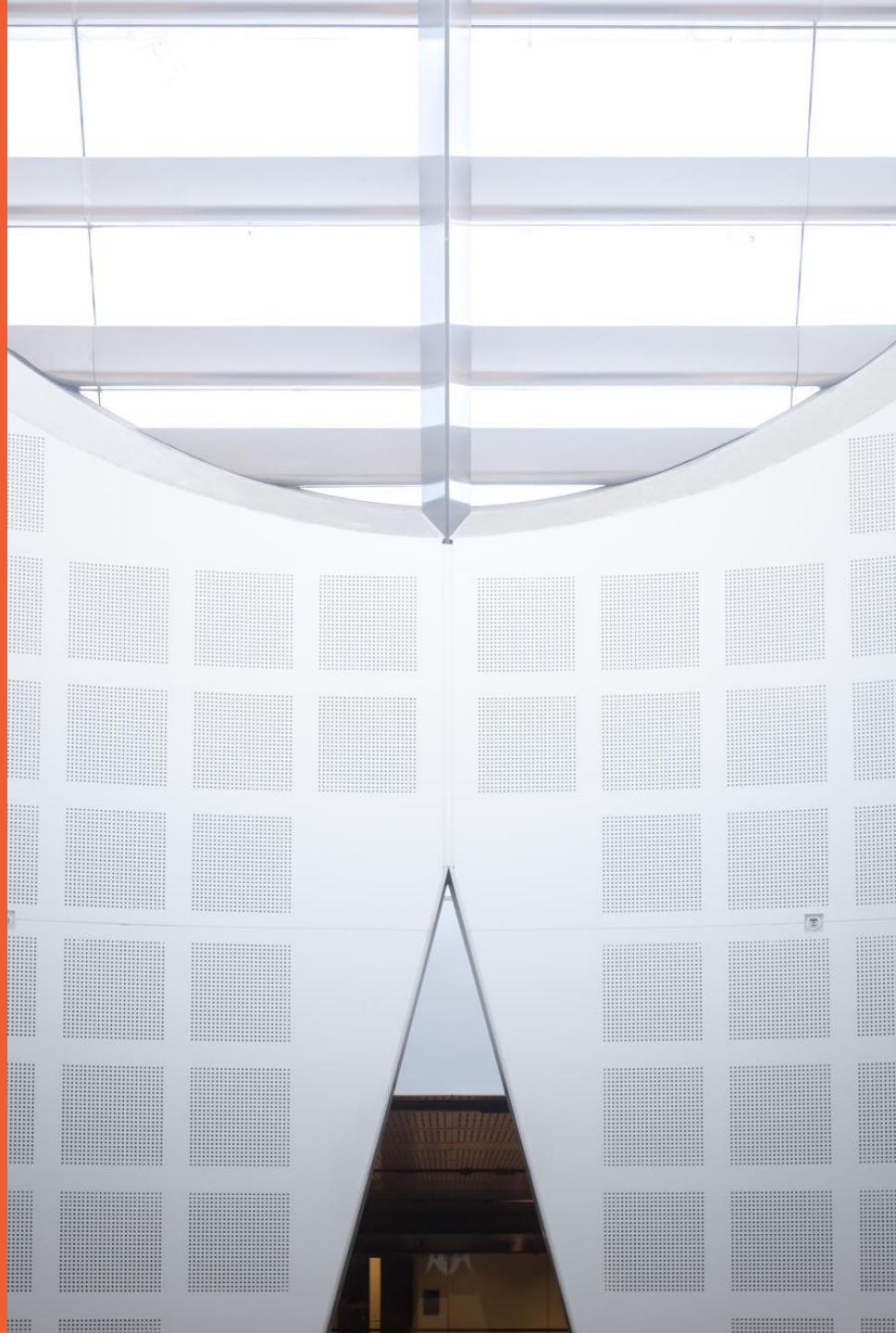
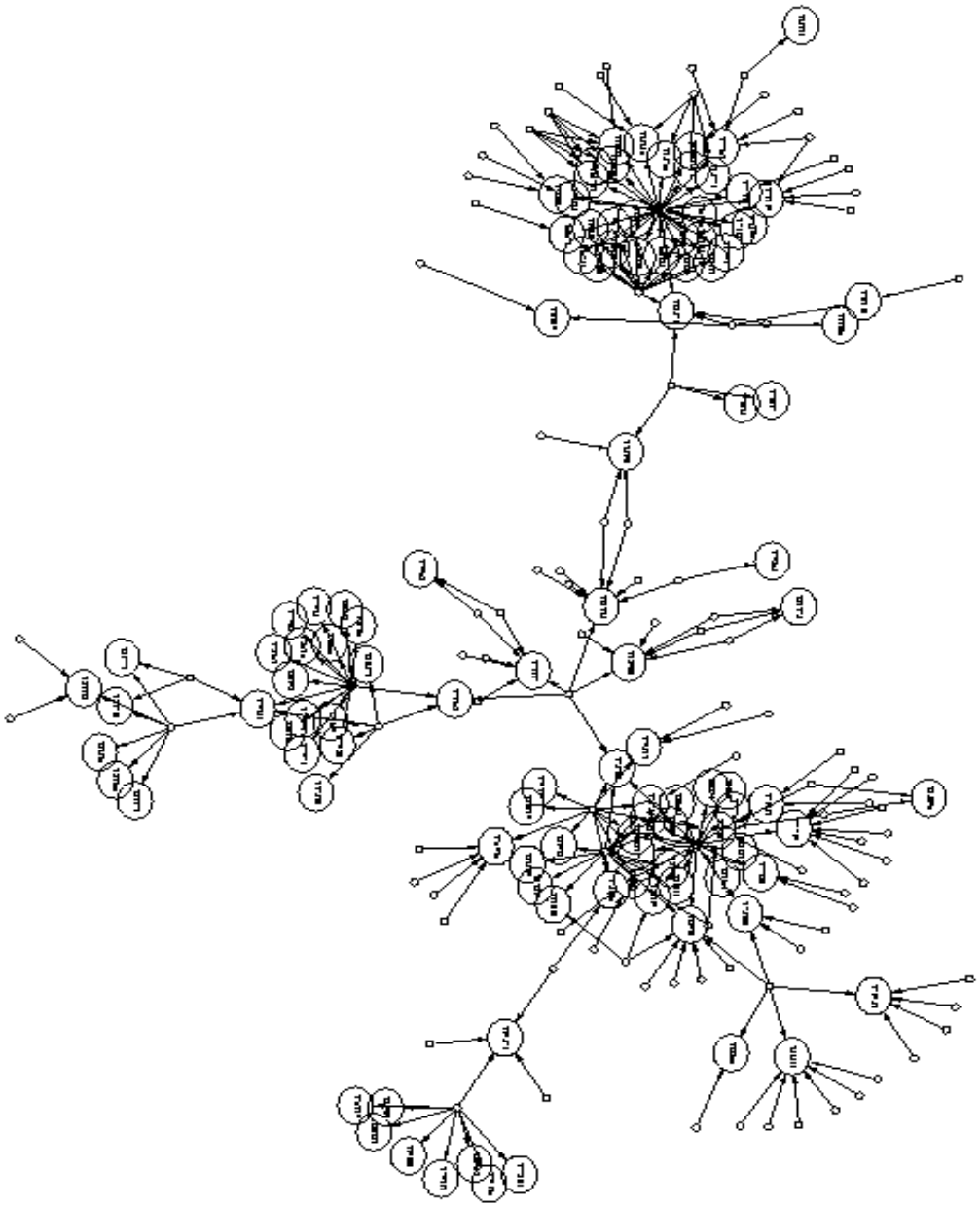
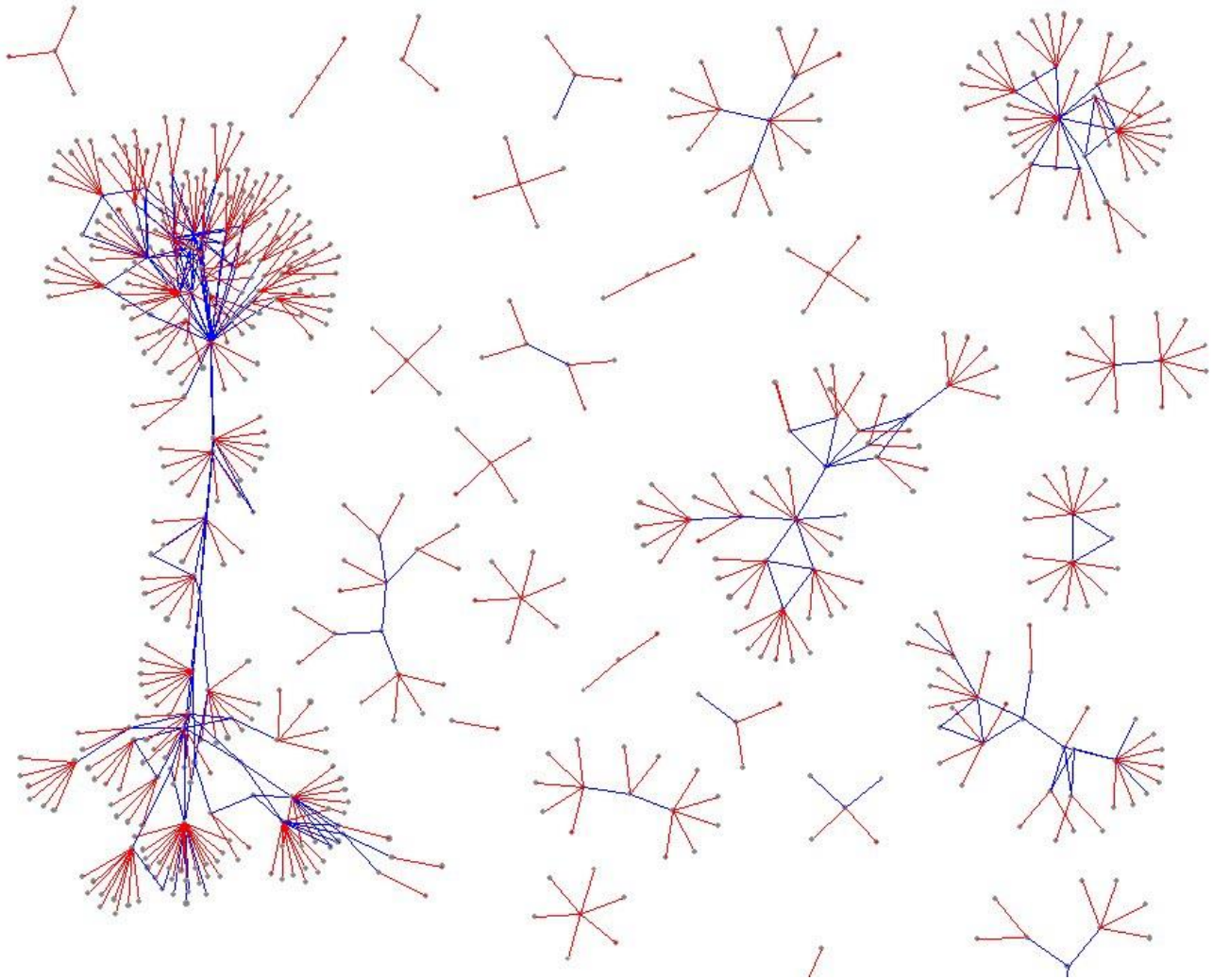
School of Information Technologies

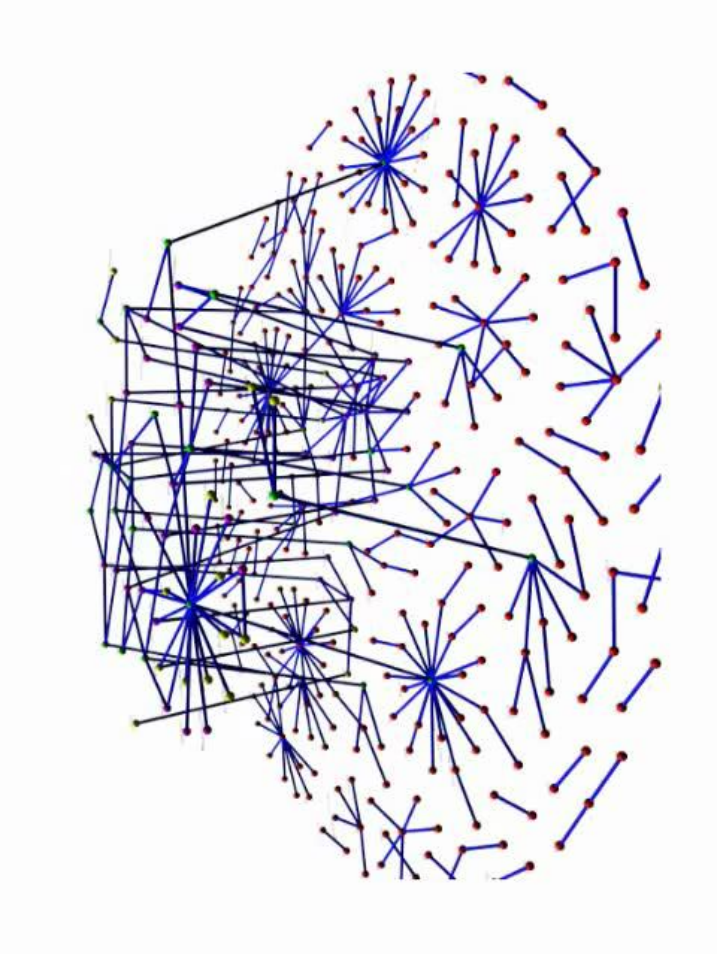
THE UNIVERSITY OF SYDNEY







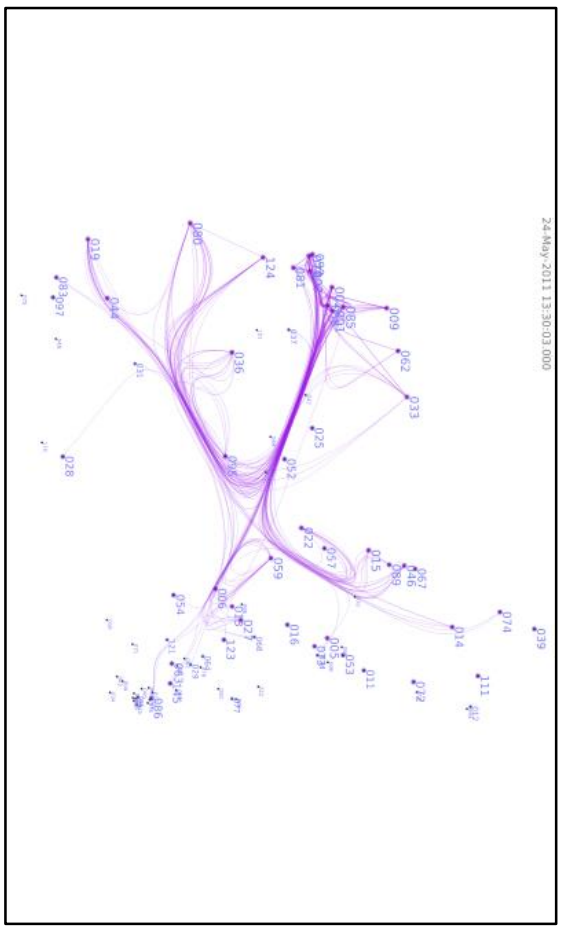
Motifs in a protein-protein interaction network

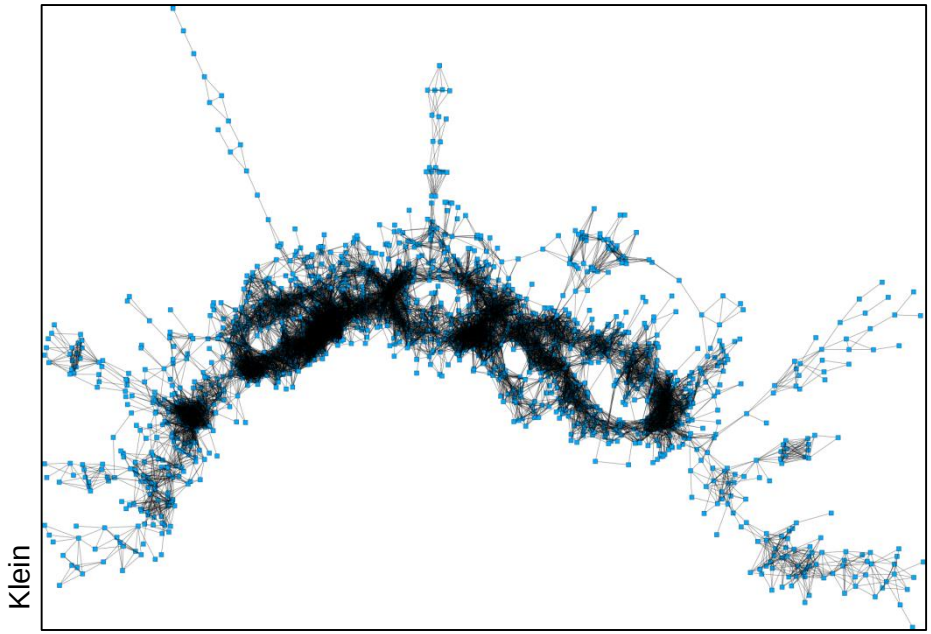


Lanbo Zheng et al.

Klein

Nguyen



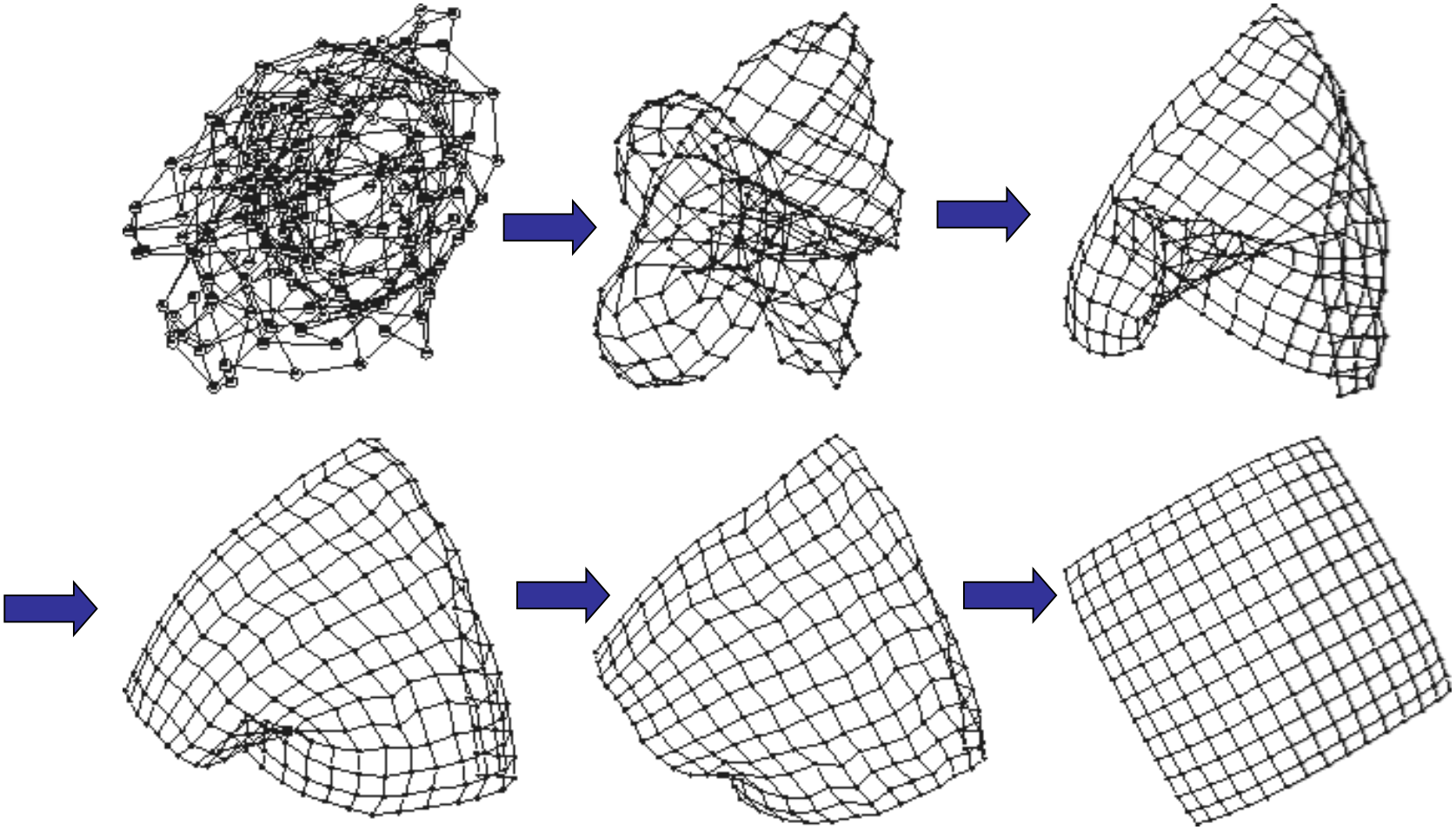

Klein

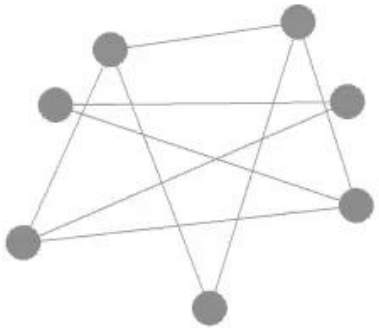
Forces Directed Methods

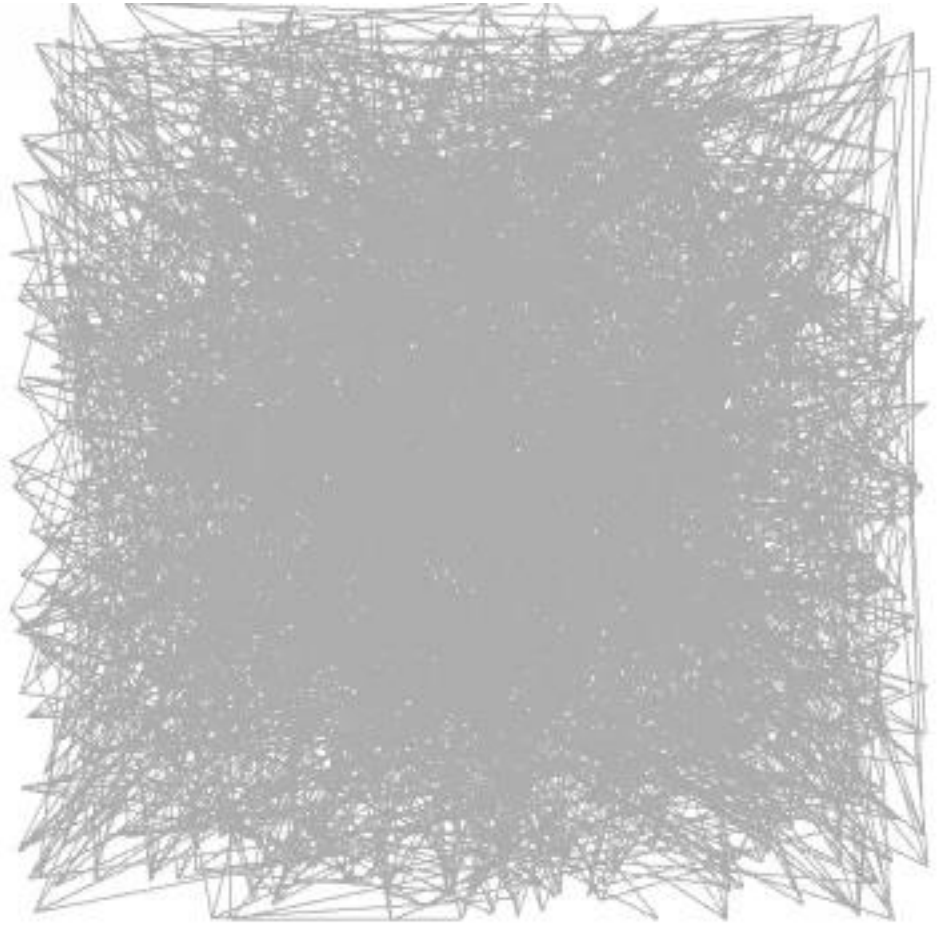
Force-directed graph drawing

- Use a physical analogy to draw graphs
  - View a graph as a system of bodies with forces acting between the bodies.
    - Vertices: particles that exert force on each other
    - Edges: forces between particles

- The algorithm seeks a position of each body with
  - minimal energy
  - sum of the forces on each body is zero







In general, force-directed methods have two parts: model & algorithm

1. ***Model***:
   - Force system
     - The model: a force system defined by vertices & edges
     - The algorithm: a technique for finding an equilibrium configuration, where the sum of the forces on each vertex is zero.

   and/or

   - Energy system
     - The model: maybe defined as an energy system
     - The algorithm: a technique for finding a configuration with locally minimal energy

2. ***Algorithm***:
   - Many different optimization methods

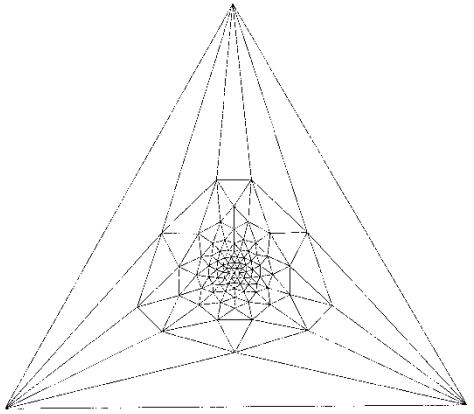
Force directed methods:

1. Barycenter method
2. Spring & electrical force
3. An example using forces: metro maps
4. FADE (Barnes-Hutt)
5. Spectral methods

1. Barycenter Method [Tutte 1960 - 1963]

1. ***Model***
   - Use Hooke's law springs with natural length 0
   - Pin down some vertices

2. ***Algorithm***
   - Solve linear equations



1. ***The Model***

For a graph $\boldsymbol{G} = (\boldsymbol{V}, \boldsymbol{E})$ where each vertex $\boldsymbol{u}$ has a position $\boldsymbol{p_u}. = (\boldsymbol{x_u}, \boldsymbol{y_u})$:

a) Use Hooke's law springs with natural length 0:

- The energy in an edge $(\boldsymbol{u}, \boldsymbol{v})$ (a spring) is
$$\boldsymbol{energy}(\boldsymbol{u}, \boldsymbol{v}) = \boldsymbol{d}(\boldsymbol{p_u}, \boldsymbol{p_v})^2 = (\boldsymbol{x_u} - \boldsymbol{x_v})^2 + (\boldsymbol{y_u} - \boldsymbol{y_v})^2;$$
here $\boldsymbol{d}(\boldsymbol{p_u}, \boldsymbol{p_v})$ is the Euclidean distance between $\boldsymbol{p_u}$ and $\boldsymbol{p_v}$.
- The total energy in the drawing is the sum of the energies in all the edges:
$$\boldsymbol{Energy}(\boldsymbol{p}) = \sum_{(\boldsymbol{u},\boldsymbol{v})\in\boldsymbol{E}} \boldsymbol{energy}(\boldsymbol{u}, \boldsymbol{v})$$

b) Pin down some vertices:

- A subset $\boldsymbol{A}$ of vertices if *fixed*.
- That is, for a vertices $\boldsymbol{a} \in \boldsymbol{A} \subset \boldsymbol{V}$, $\boldsymbol{p_a}$ is chosen *a priori*.
- That is, for $\boldsymbol{a} \in \boldsymbol{A}$, $\boldsymbol{p_a}$ is part of the input.

2. ***Algorithm***
   - Minimize energy by solving linear equations …. →

How to minimize energy:

- Choose a location $\boldsymbol{p(u) = (x_u, y_u)}$ for each $\boldsymbol{u \in V - A}$ to minimize

$$\boldsymbol{Energy(p) = \sum_{(u,v)\in E} d(u,v)^2 \quad = \sum_{(u,v)\in E} (x_u - x_v)^2 + (y_u - y_v)^2}$$

- Note that the minimum is unique, and occurs when

$$\boldsymbol{\frac{\partial\,(energy(p))}{\partial x_u} = 0} \quad \text{and} \quad \boldsymbol{\frac{\partial\,(energy(p))}{\partial y_u} = 0}$$

for each $\boldsymbol{u \in V - A}$.

For each vertex $\boldsymbol{u \in V - A}$, for the $\boldsymbol{x}$-coordinate $\boldsymbol{x_u}$, minimum energy occurs when:

$$\frac{\partial\,(Energy(p))}{\partial x_u} = 0$$

$$\frac{\partial}{\partial x_u}\left(\sum_{(u,v)\in E}(x_u - x_v)^2 + (y_u - y_v)^2\right) = 0$$

$$\sum_{v\in N_u} 2(x_u - x_v) = 0$$

$$\sum_{v\in N_u} x_u \quad - \sum_{v\in N_u} x_v = 0$$

$$deg(u)x_u \quad - \sum_{v\in N_u} x_v \quad = 0$$

Barycentre equations

For each vertex $\boldsymbol{u} \in \boldsymbol{V} - \boldsymbol{A}$, we have an equations in the $\boldsymbol{x}$ direction:

$$\boldsymbol{deg}(\boldsymbol{v})\boldsymbol{x_v} - \sum_{\boldsymbol{u}\in\boldsymbol{N}(\boldsymbol{v})} \boldsymbol{x_u} = 0$$

(and a similar equation in the $\boldsymbol{y}$ direction)

That is:

$$\boldsymbol{x_v} = \frac{\mathbf{1}}{\boldsymbol{deg}(\boldsymbol{v})} \sum_{\boldsymbol{u}\in\boldsymbol{N}(\boldsymbol{v})} \boldsymbol{x_u}$$

barycentre equations

Taking both the $\boldsymbol{x}$ direction and the $\boldsymbol{y}$ direction, the barycentre equations say that

"the position $\boldsymbol{p_v}$ of $\boldsymbol{v}$ is barycentre of the positions $\boldsymbol{p_u}$ of the neighbors $\boldsymbol{u}$ of $\boldsymbol{v}$."

"Pin down" some vertices:- the vertex set $\boldsymbol{V}$ is partitioned into two sets:

a) *fixed* vertex set $\boldsymbol{A}$
   - at least 3 vertices, ie, $|\boldsymbol{A}| \geq \mathbf{3}$.
   - nailed down, pinned down, fixed, unmovable
   - The fixed vertices and their positions may be chosen in different ways; for the moment just assume that they are in general position.

b) *free* vertex set $\boldsymbol{V} - \boldsymbol{A}$

To find a position for each vertex $\boldsymbol{u} \in \boldsymbol{V} - \boldsymbol{A}$,

we must solve two sets of $|\boldsymbol{V} - \boldsymbol{A}|$ *linear* equations in $\mathbf{2}|\boldsymbol{V} - \boldsymbol{A}|$ unknowns:

For all *free* vertices $\boldsymbol{v}$: $\begin{cases} \boldsymbol{x_v} = \frac{\mathbf{1}}{\boldsymbol{deg}(v)} \sum_{\boldsymbol{u} \in \boldsymbol{N}(v)} \boldsymbol{x_u} \\ \boldsymbol{y_v} = \frac{\mathbf{1}}{\boldsymbol{deg}(v)} \sum_{\boldsymbol{u} \in \boldsymbol{N}(v)} \boldsymbol{y_u} \end{cases}$ ← *barycentre equations*

We can write the barycentre equations in matrix form.

Background: graph matrices

» Let $\boldsymbol{M}$ be the adjacency matrix of $\boldsymbol{G} = (\boldsymbol{V}, \boldsymbol{E})$; that is,

$$\boldsymbol{M_{ij}} = \begin{cases} \mathbf{1} \text{ if } (\boldsymbol{i}, \boldsymbol{j}) \in \boldsymbol{E} \\ \mathbf{0} \text{ otherwise} \end{cases}$$

» Let $\boldsymbol{D}$ be the degree matrix of $\boldsymbol{G}$; that is, $\boldsymbol{D}$ is the diagonal matrix of degrees, that is,

$$\boldsymbol{D_{ij}} = \begin{cases} \mathbf{deg}(\boldsymbol{v_i}) \text{ if } \boldsymbol{i} = \boldsymbol{j} \\ \mathbf{0} \quad \text{otherwise} \end{cases}$$

» Then the *Laplacian* $\boldsymbol{L}$ of $\boldsymbol{G}$ is $\mathrm{L} = \boldsymbol{D} - \boldsymbol{M}$, that is,

$$\boldsymbol{L_{ij}} = \begin{cases} \mathbf{deg}(\boldsymbol{v_i}) \text{ if } \boldsymbol{i} = \boldsymbol{j} \\ -\mathbf{1} \quad \text{if } \boldsymbol{i} \neq \boldsymbol{j} \text{ and } (\boldsymbol{i}, \boldsymbol{j}) \in \boldsymbol{E} \\ \mathbf{0} \quad \text{if } \boldsymbol{i} \neq \boldsymbol{j} \text{ and } (\boldsymbol{i}, \boldsymbol{j}) \notin \boldsymbol{E} \end{cases}$$

The barycentre equations (for the x direction):

$$deg(v)x_v \quad - \sum_{u \in N(v) \cap (V-A)} x_u \quad = \quad c_v.$$

In matrix form this is:

barycentre equations

$$L'x = c_X$$

where

- » L' is a matrix formed from the Laplacian by deleting rows and columns corresponding to fixed vertices.
- » x is the column vector indexed by the vertices of $V - A$.
- » c_X is a constant column vector.

And we have similar equations for the y direction.

1. ***Model***:
   - Fix a set $\boldsymbol{A}$ of vertices
   - Place every other vertex at the barycentre of its neighbours; that is, choose the position $(\boldsymbol{x_i}, \boldsymbol{y_i})$ of every free vertex $\boldsymbol{v_i} \in \boldsymbol{V} - \boldsymbol{A}$ to satisfy the barycentre equations

$$\boldsymbol{L'x} = \boldsymbol{c_X},$$
$$\boldsymbol{L'y} = \boldsymbol{c_Y}.$$

2. ***Algorithm***:
   - Solve the barycentre equations, that is, compute $\boldsymbol{L'^{-1}}$.

The matrix $\boldsymbol{L'}$:

- Submatrix or the Laplacian matrix of the input graph
- Using Kirchhoff matrix-tree theorem, the inverse exists if and only if for every vertex in $\boldsymbol{V - A}$ there is a path to a vertex in $\boldsymbol{A}$.
- Weakly diagonally dominant: for every $\boldsymbol{i}$,

$$|\boldsymbol{D_{ii}}| \geq \sum_{\boldsymbol{i \neq j}} |\boldsymbol{D_{ij}}|$$

with

$$|\boldsymbol{D_{ii}}| > \sum_{\boldsymbol{i \neq j}} |\boldsymbol{D_{ij}}|$$

for at least 3 values of $\boldsymbol{i}$.

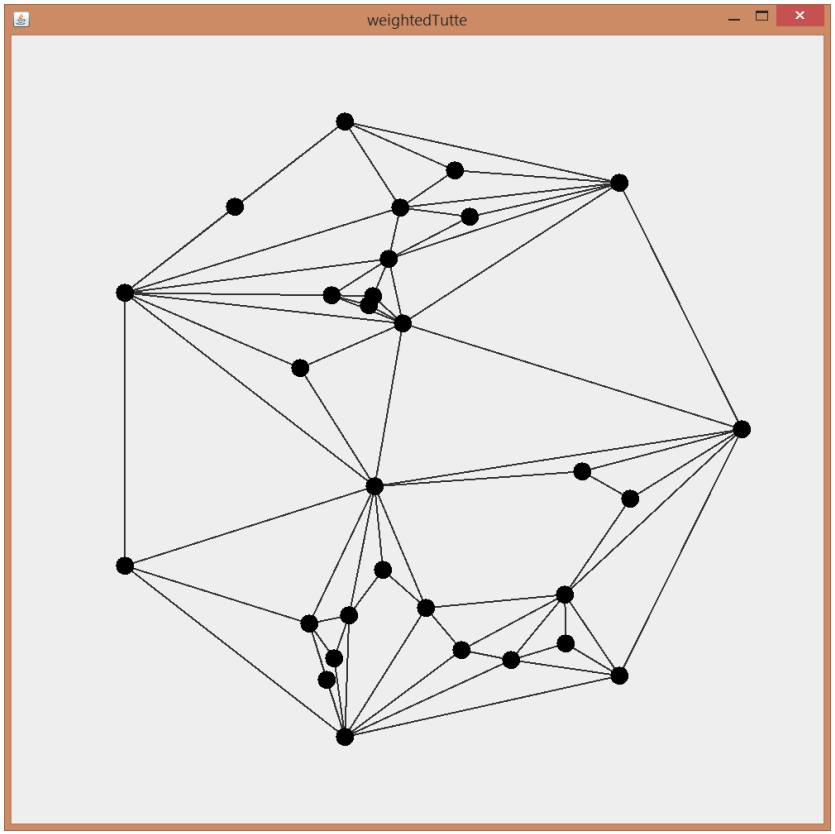
How to solve the barycentre equations:

- Gaussian elimination: $\boldsymbol{O(n^3)}$
- Fast matrix multiplication methods: $\sim\boldsymbol{O(n^{2.5})}$ or less
- For sparse Laplacian matrices (ie, graphs with a linear number of edges, such as planar graphs): Nested dissection methods $\boldsymbol{O(n^{1.5})}$**.**
- Speilman-Teng methods: perhaps $\boldsymbol{O}(\boldsymbol{n}\log\boldsymbol{n})$.

- Gauss-Seidel / Jacobi methods (iterative approximations)
  - » In practice, the Gauss-Seidel / Jacobi methods are very simple and work well because of properties of Laplacian matrices.

Choosing and placing the fixed set $\boldsymbol{A}$**:**

- If $\boldsymbol{G}$ is a planar embedding then we choose $\boldsymbol{A}$ to be an outside face
- Normally we place $\boldsymbol{A}$ at the vertices of a convex polygon



Theorems

Theorem [Tutte 1963] Suppose that $\boldsymbol{G}$ is a triconnected planar graph, $\boldsymbol{f}$ is a face in a planar embedding of $\boldsymbol{G}$, and $\boldsymbol{P}$ is a strictly convex planar drawing of $\boldsymbol{f}$. Then applying the barycentre method with the vertices of $\boldsymbol{f}$ fixed and positioned according to $\boldsymbol{P}$, yields a planar drawing of $\boldsymbol{G}$ in which every face is convex.

Theorem (Informally stated): the barycentre algorithm applied to a graph $\boldsymbol{G}$ displays every dihedral automorphism of $\boldsymbol{G}$ that fixes $\boldsymbol{A}$, as a symmetry of the drawing.

Barycentre method

Advantages

- Elegant and easy to implement
- In practice, the simple Gauss-Seidel / Jacobi iterations converges quickly
- One can use weights on the edges to implement many things:
  - Logical fish-eye lens
  - Constrain chosen paths to be straight lines
  - Cluster nodes together
  - Smooth animation for in-betweening

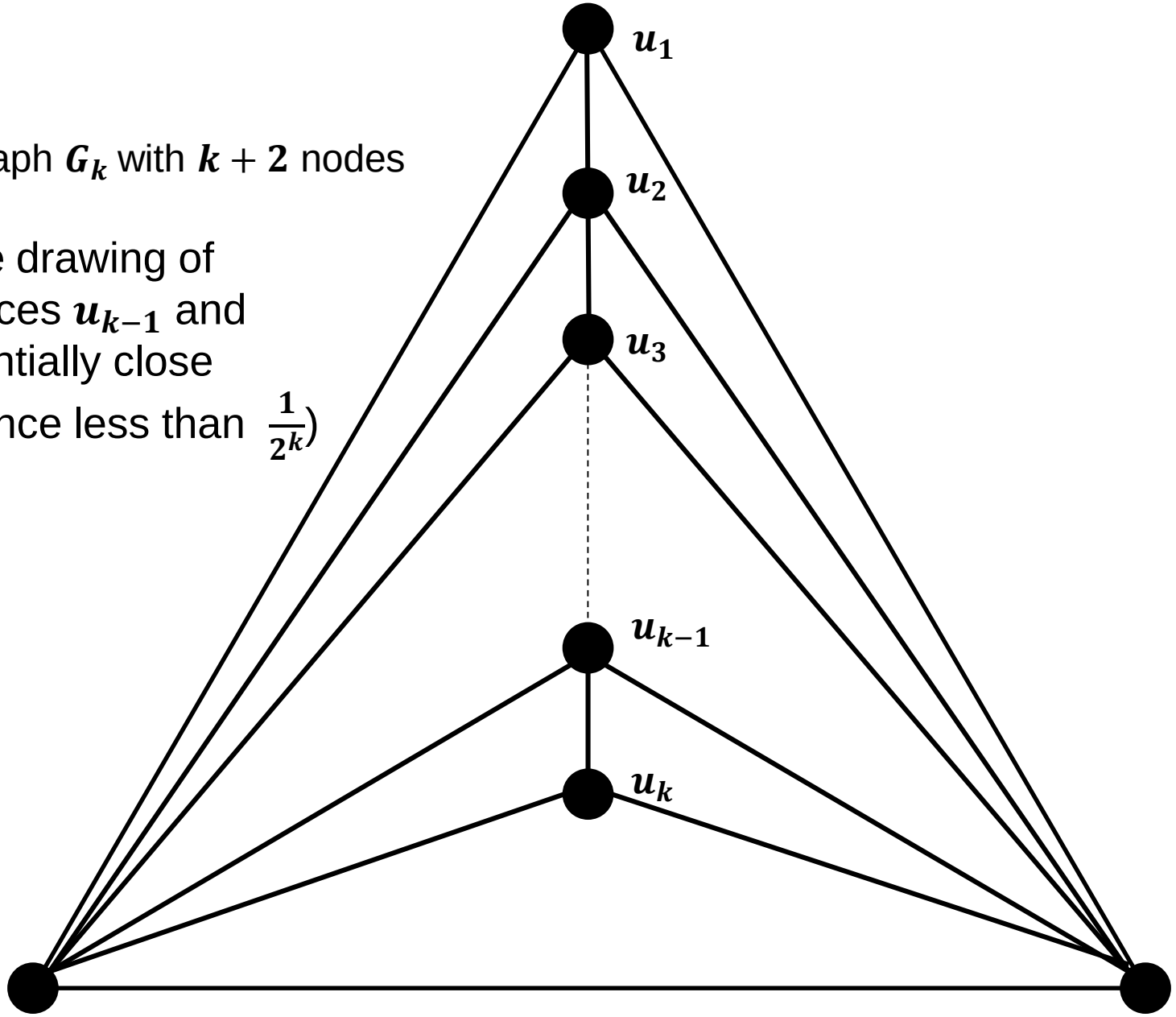
Disadvantages

- Poor vertex resolution

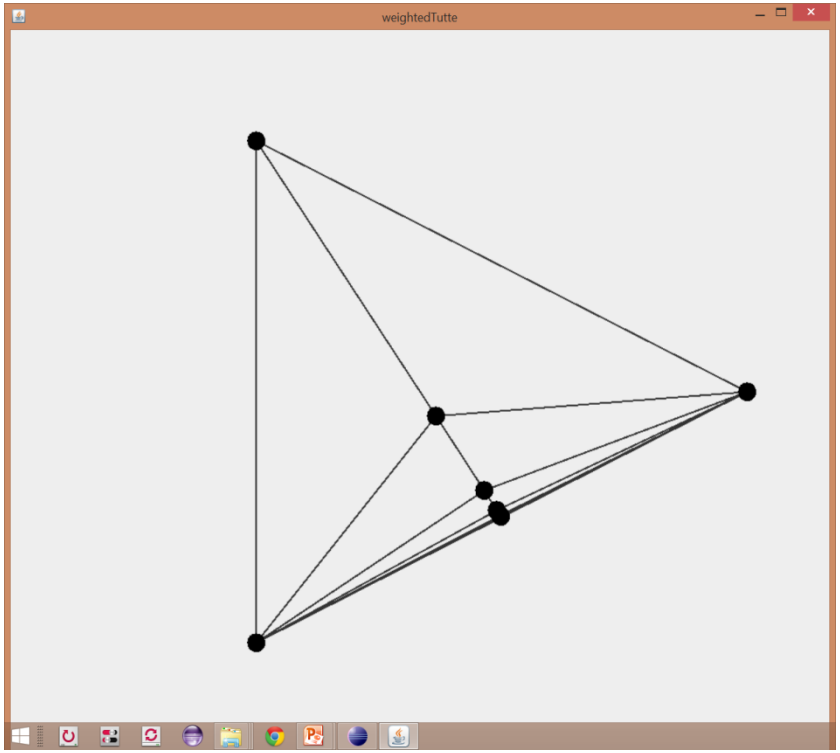
Barycenter Method

Consider this graph G_k with $k + 2$ nodes

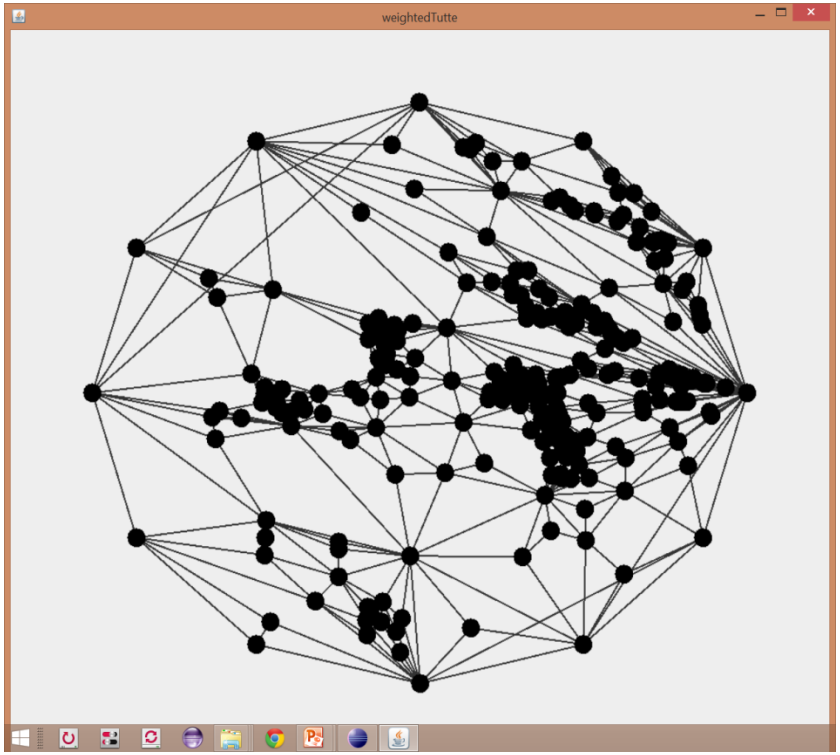
In a barycentre drawing of graph G_k, vertices u_{k-1} and u_k are exponentially close together (distance less than $\frac{1}{2^k}$)



$k = 6$



About 300 vertices



2. Springs & Electrical Forces

Springs and electrical forces

1. ***Model***:
   - Edges are springs
   - Vertices are electrically charged particles that repel each other

2. ***Algorithm***:
   - Many different optimization methods
   - The simplest algorithm is “follow-your-nose”

1. Model

Model edges and vertices with Hooke's Law spring forces plus electrical forces

- Vertex: modelled as an electrically charged particle.
- Edge: modelled as a spring

Force on a particle $\boldsymbol{v}$ **:-**

$$\boldsymbol{F}(\boldsymbol{v}) = \sum_{\boldsymbol{u}\in\boldsymbol{N}(\boldsymbol{v})} \boldsymbol{f}_{\boldsymbol{uv}} + \sum_{\boldsymbol{u}\in\boldsymbol{V},\boldsymbol{u}\neq\boldsymbol{v}} \boldsymbol{g}_{\boldsymbol{uv}}$$

where

$\boldsymbol{N}(\boldsymbol{v})$ is the set of neighbours of $\boldsymbol{v}$;

$\boldsymbol{f}_{\boldsymbol{uv}}$ is the force exerted on $\boldsymbol{v}$ by the (Hooke's law) spring between $\boldsymbol{u}$ and $\boldsymbol{v}$ corresponding to the edge $(\boldsymbol{u}, \boldsymbol{v})$;

$\boldsymbol{g}_{\boldsymbol{uv}}$ is the force exerted on $\boldsymbol{v}$ by the electrical repulsion between $\boldsymbol{u}$ and $\boldsymbol{v}$.

Notes

- The spring force ensure that edges have about the right length.
- The electrical force ensures that vertices are not too close to each other.
- $\boldsymbol{F}(\boldsymbol{v}) = \left(\boldsymbol{F}_{\boldsymbol{x}}(\boldsymbol{v}), \boldsymbol{F}_{\boldsymbol{y}}(\boldsymbol{v})\right)$ is a vector.

The force on a particle $\boldsymbol{v}$ is: -

$$\boldsymbol{F(v)} = \sum_{\boldsymbol{u \in N(v)}} \boldsymbol{\alpha_{uv}(d(p_u, p_v) - \beta_{uv})i_{uv}} \quad + \quad \sum_{\boldsymbol{u \in V, u \neq v}} \frac{\boldsymbol{\gamma_{uv}}}{\boldsymbol{\left(d(p_u, p_v)\right)^2}} \boldsymbol{i_{uv}}$$

where

$\boldsymbol{p_u}$ is the position of vertex $\boldsymbol{u}$,

$\boldsymbol{p_v}$ is the position of vertex $\boldsymbol{v}$,

$\boldsymbol{d(p_u, p_v)}$ is the Euclidean distance between $\boldsymbol{p_u}$ and $\boldsymbol{p_v}$,

$\boldsymbol{i_{uv}}$ is the unit vector in the direction from $\boldsymbol{p_u}$ to $\boldsymbol{p_v}$.

and

$\boldsymbol{\alpha_{uv}}$, $\boldsymbol{\beta_{uv}}$ and $\boldsymbol{\gamma_{uv}}$ are constants.

That is, the force on a particle $\boldsymbol{v}$ is: -

- in the $\boldsymbol{x}$ direction $\boldsymbol{F_x(v)}$ is

$$\sum_{\boldsymbol{u\in N(v)}} \boldsymbol{\alpha_{uv}(d(p_u,p_v)-\beta_{uv})}\left(\frac{\boldsymbol{x_u-x_v}}{\boldsymbol{d(p_u,p_v)}}\right) \quad + \quad \sum_{\boldsymbol{u\in V, u\neq v}} \frac{\boldsymbol{\gamma_{uv}}}{\left(\boldsymbol{d(p_u,p_v)}\right)^2}\left(\frac{\boldsymbol{x_u-x_v}}{\boldsymbol{d(p_u,p_v)}}\right)$$

- in the $\boldsymbol{y}$ direction $\boldsymbol{F_y(v)}$ is

$$\sum_{\boldsymbol{u\in N(v)}} \boldsymbol{\alpha_{uv}(d(p_u,p_v)-\beta_{uv})}\left(\frac{\boldsymbol{y_u-y_v}}{\boldsymbol{d(p_u,p_v)}}\right) \quad + \quad \sum_{\boldsymbol{u\in V, u\neq v}} \frac{\boldsymbol{\gamma_{uv}}}{\left(\boldsymbol{d(p_u,p_v)}\right)^2}\left(\frac{\boldsymbol{y_u-y_v}}{\boldsymbol{d(p_u,p_v)}}\right)$$

This gives $\boldsymbol{2|V|}$ equations.

- Note: The unit vector in the direction from $\boldsymbol{u}$ to $\boldsymbol{v}$ is $\left(\left(\frac{\boldsymbol{x_u-x_v}}{\boldsymbol{d(p_u,p_v)}}\right),\left(\frac{\boldsymbol{y_u-y_v}}{\boldsymbol{d(p_u,p_v)}}\right)\right)$.

$$\boldsymbol{F(v)} = \sum_{\boldsymbol{u \in N(v)}} \boldsymbol{\alpha_{uv}}(\boldsymbol{d(p_u, p_v)} - \boldsymbol{\beta_{uv}})\boldsymbol{i_{uv}} \quad + \quad \sum_{\boldsymbol{u \in V, u \neq v}} \frac{\boldsymbol{\gamma_{uv}}}{\left(\boldsymbol{d(p_u, p_v)}\right)^2} \boldsymbol{i_{uv}}$$

Note:

- $\boldsymbol{\alpha_{uv}}, \boldsymbol{\beta_{uv}}$ and $\boldsymbol{\gamma_{uv}}$ have physical interpretations:
  $\boldsymbol{\alpha_{uv}}$ is the *stiffness* of the spring corresponding to the edge $(\boldsymbol{u}, \boldsymbol{v})$.
  $\boldsymbol{\beta_{uv}}$ is the *natural length* of the spring corresponding to the edge $(\boldsymbol{u}, \boldsymbol{v})$.
  $\boldsymbol{\gamma_{uv}}$ is the *strength* of the electrical repulsion between $\boldsymbol{u}$ and $\boldsymbol{v}$.

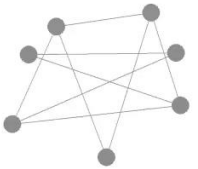
- We can map attributes of the vertices and edges to $\boldsymbol{\alpha_{uv}}, \boldsymbol{\beta_{uv}}$ and $\boldsymbol{\gamma_{uv}}$; for example:
  - $\boldsymbol{\beta_{uv}}$ is the ideal length of the edge $(\boldsymbol{u}, \boldsymbol{v})$, as defined by the application domain.
  - A high value of $\boldsymbol{\alpha_{uv}}$ means that the edge $(\boldsymbol{u}, \boldsymbol{v})$ between $\boldsymbol{u}$ and $\boldsymbol{v}$ is important; a low value of $\boldsymbol{\alpha_{uv}}$ means that this edge is unimportant.

2. Algorithm

We need to compute a position $\boldsymbol{p_v}$ such that $\boldsymbol{F(v) = 0}$ for each $\boldsymbol{v \in V}$, where:

$$\boldsymbol{F(v)} = \sum_{\boldsymbol{u \in N(v)}} \boldsymbol{\alpha_{uv}(d(p_u, p_v) - \beta_{uv})i_{uv}} \quad + \sum_{\boldsymbol{u \in V, u \neq v}} \frac{\boldsymbol{\gamma_{uv}}}{\boldsymbol{\left(d(p_u, p_v)\right)^2}} \boldsymbol{i_{uv}}$$

- This is an energy minimization problem
- The equations are not linear.
- There are many techniques to find an equilibrium configuration (zero force on each node, or locally minimum energy).
- The solutions are not unique
  - That is, there may be locally minimum energy that is not globally minimum.



An algorithm example:

Simple algorithm ("Follow your nose"):

1. $\boldsymbol{p_u}$ = some initial position for each node $\boldsymbol{u}$;
2. Repeat
    - 2.1 $\boldsymbol{F_u}$ **:= 0** for each node $\boldsymbol{u}$;
    - 2.2 For each pair $\boldsymbol{u, v}$ of nodes
        - 2.2.1 calculate the force $\boldsymbol{f_{uv}}$ between $\boldsymbol{u}$ and $\boldsymbol{v}$;
        - 2.2.2 $\boldsymbol{F_u}$ $+=$ $\boldsymbol{f_{uv}}$;
        - 2.2.3 $\boldsymbol{F_v}$ $+=$ $\boldsymbol{f_{uv}}$;
    - 2.3 For each node $\boldsymbol{u}$, $\boldsymbol{p_u}$ **+=** $\varepsilon \boldsymbol{F_u}$;

   Until $\boldsymbol{p_u}$ converges for all $\boldsymbol{u}$;

– Not fast, but allows smooth animation.

Runtime (using the simple follow-your-node algorithm):

- Time to compute the zero-force configuration:-
  - The dominant time is calculating the electrical repulsion forces between all pairs of vertices: $\boldsymbol{O}(|\boldsymbol{V}|^{\boldsymbol{2}})$
  - Total time for an iteration: $\boldsymbol{O}(|\boldsymbol{V}|^{\boldsymbol{2}})$

- This is repeated for each iteration, until convergence or other conditions are met.

- In practice, a few hundred vertices means very slow runtime.

- More complex algorithms are available.

3. Example: using forces

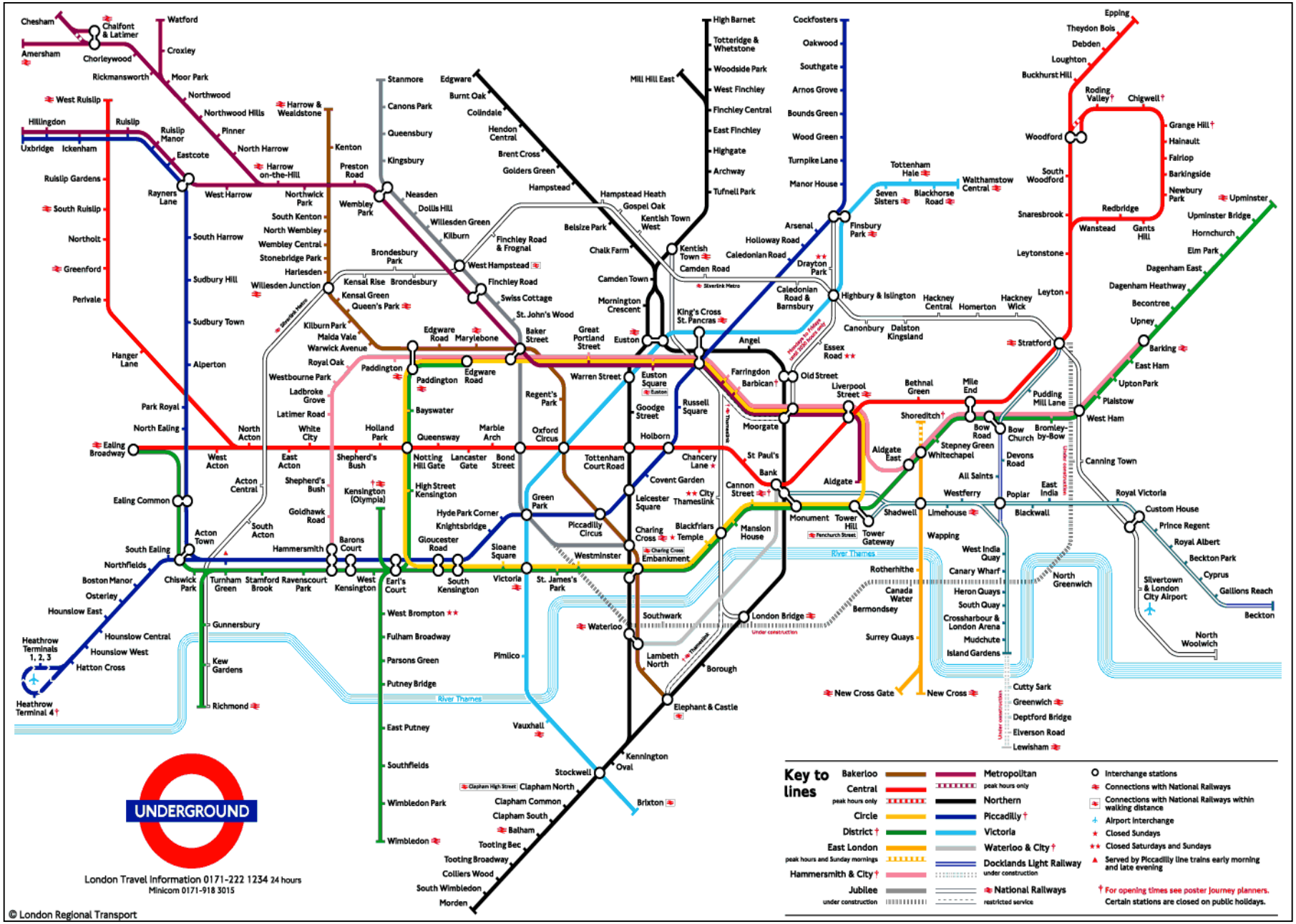
The Metro Map Problem

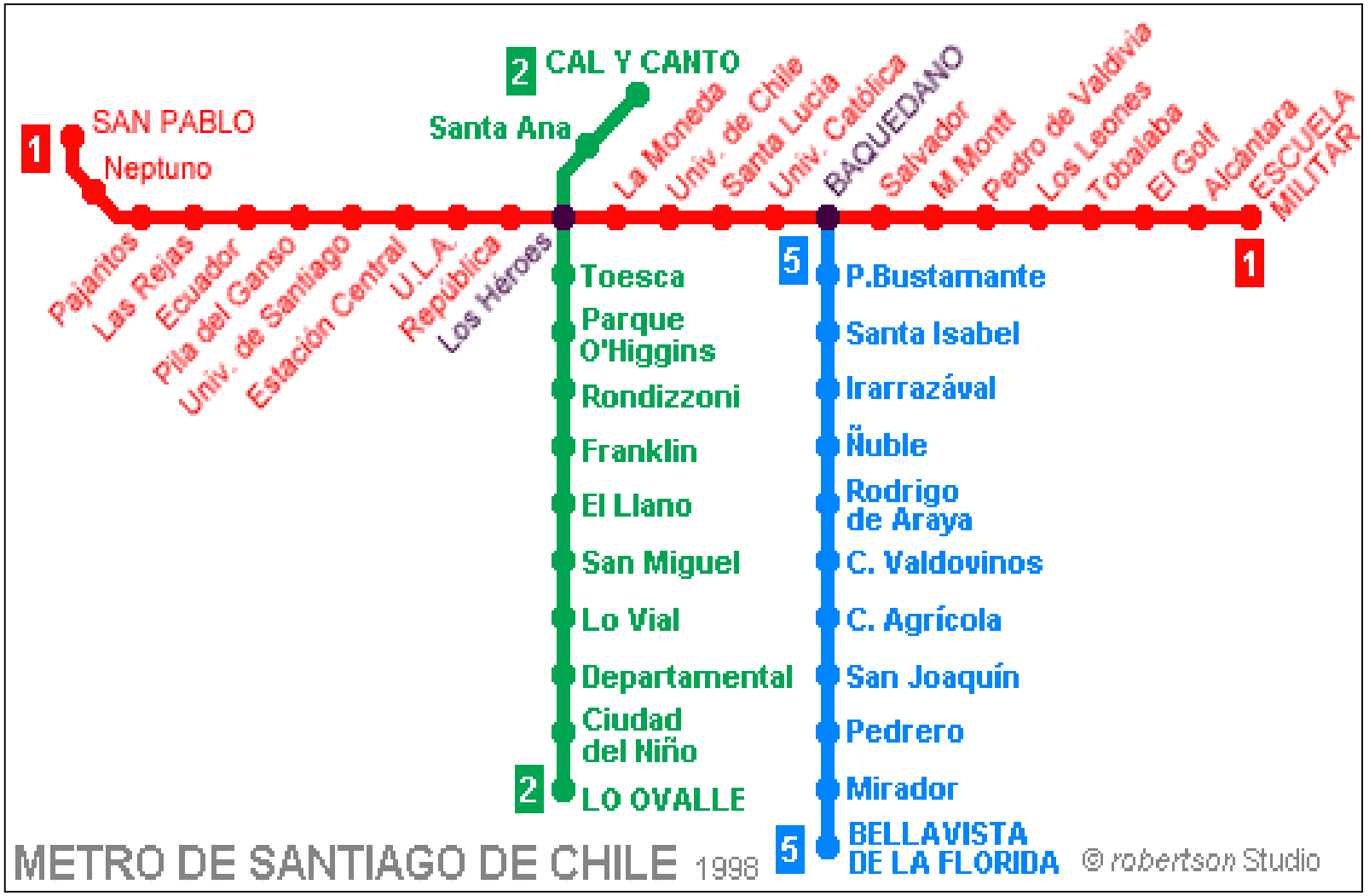
- Existing metro maps, produced manually by professional graphic artists, are ***excellent examples*** of network visualization

- *Can we produce good metro maps* ***automatically?***


REFERENCE
DISTRICT LINE
PICCADILLY LINE
BAKERLOO LINE
MORDEN-EDGWARE LINE
CENTRAL LONDON LINE
METROPOLITAN LINE
NORTHERN CITY LINE
INTERCHANGE STATIONS
ESCALATOR CONNECTION
LONDON UNDERGROUND TRANSPORT

H. Beck, 1931


UNDERGROUND
London Travel Information 0171-222 1234 24 hours
Minicom 0171-918 3015
© London Regional Transport
Key to lines
Bakerloo
Central
peak hours only
Circle
District †
East London
peak hours and Sunday mornings
Hammersmith & City †
Jubilee
under construction
Metropolitan
peak hours only
Northern
Piccadilly †
Victoria
Waterloo & City †
Docklands Light Railway
under construction
National Railways
restricted service
Interchange stations
Connections with National Railways
Connections with National Railways within walking distance
Airport interchange
Closed Sundays
Closed Saturdays and Sundays
Served by Piccadilly line trains early morning and late evening
† For opening times see poster journey planners.
Certain stations are closed on public holidays.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SAN PABLO
Neptuno
Pajaritos
Las Rejas
Ecuador
Pila del Ganso
Univ. de Santiago
Estación Central
U.L.A.
República
Los Héroes
2
CAL Y CANTO
Santa Ana
La Moneda
Univ. de Chile
Santa Lucía
Univ. Católica
BAQUEDANO
Salvador
M.Montt
Pedro de Valdivia
Los Leones
Tobalaba
El Golf
Alcántara
ESCUELA MILITAR
1
Toesca
Parque O'Higgins
Rondizzoni
Franklin
El Llano
San Miguel
Lo Vial
Departamental
Ciudad del Niño
2
LO OVALLE
5
P.Bustamante
Santa Isabel
Irarrazával
Ñuble
Rodrigo de Araya
C. Valdovinos
C. Agrícola
San Joaquín
Pedrero
Mirador
5
BELLAVISTA DE LA FLORIDA
METRO DE SANTIAGO DE CHILE 1998
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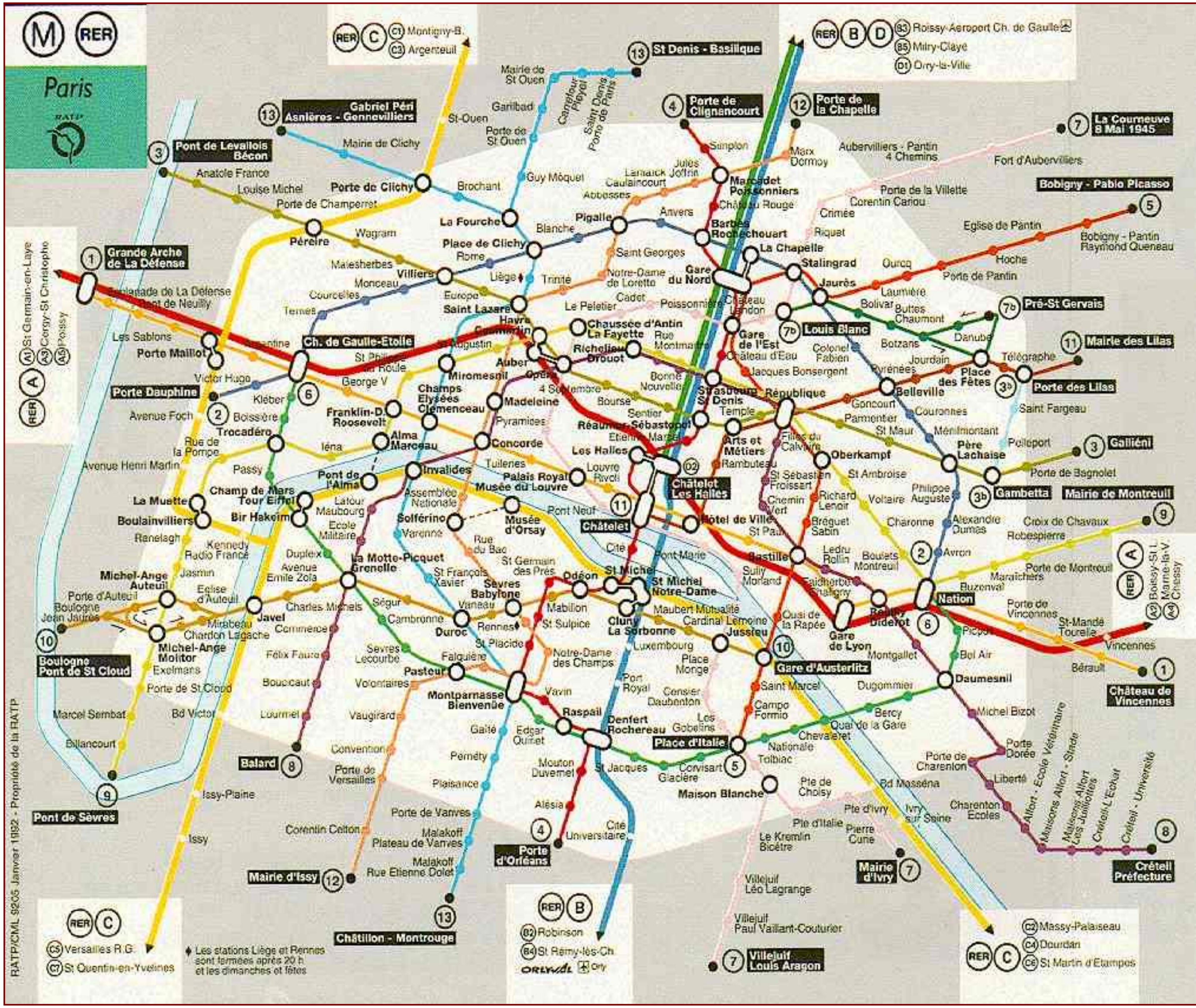
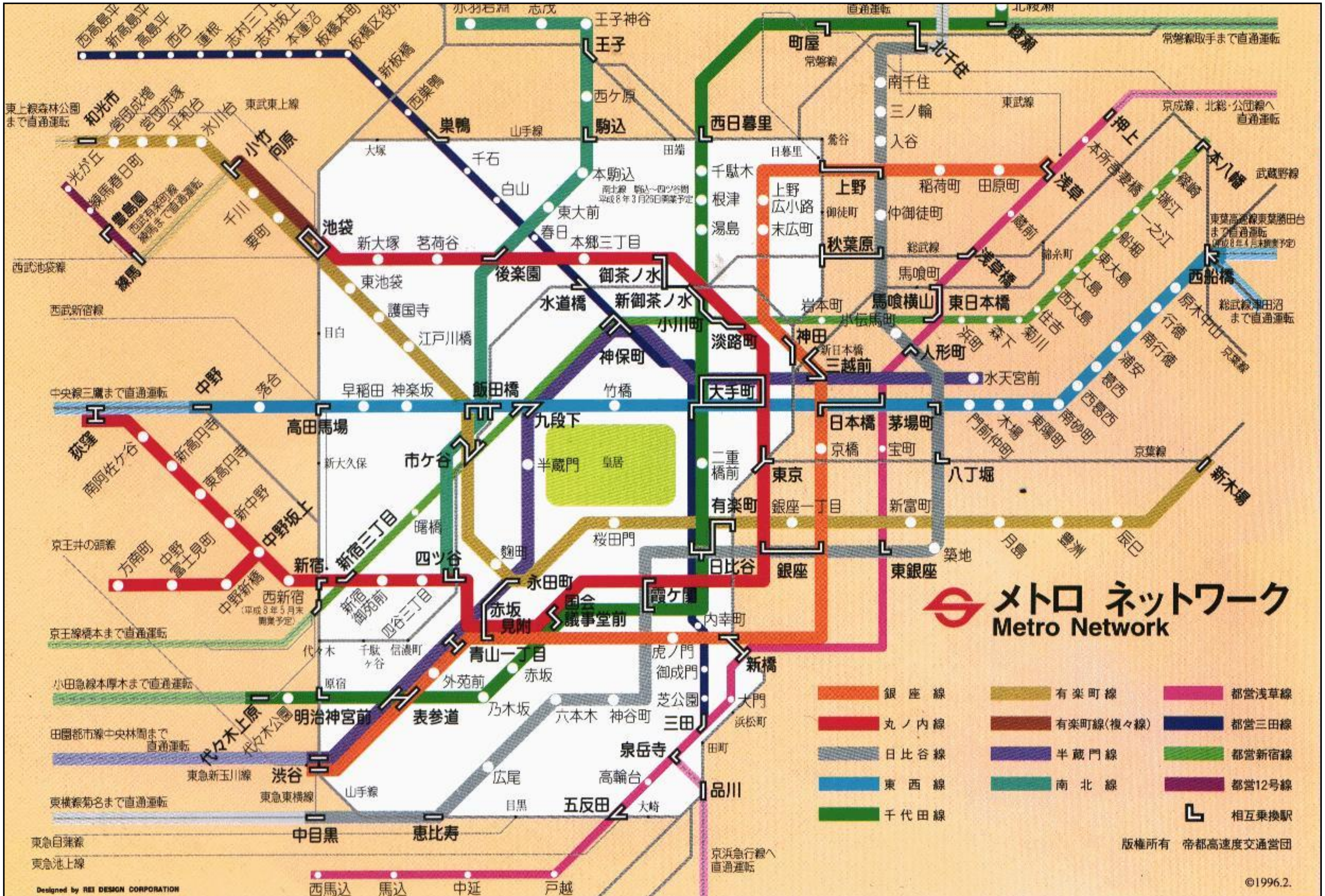

M RER
Paris
RATP
RER C C1 Montigny-B. C3 Argenteuil
13 St Denis - Basilique
RER B D B3 Roissy-Aéroport Ch. de Gaulle B5 Mitry-Claye D1 Orry-la-Ville
13 Gabriel Péri Asnières - Gennevilliers
3 Pont de Levallois Bécon
4 Porte de Clignancourt
12 Porte de la Chapelle
7 La Courneuve 8 Mai 1945
Bobigny - Pablo Picasso 5
1 Grande Arche de La Défense
RER A A1 St Germain-en-Laye A3 Cergy-St Christophe A5 Poissy
Porte Dauphine
7b Pré-St Gervais
11 Mairie des Lilas
3b Porte des Lilas
3 Galliéni
Mairie de Montreuil 9
RER A A2 Boissy-St L. A4 Marne-la-V. Chessy
Château de Vincennes 1
Créteil Préfecture 8
10 Boulogne Pont de St Cloud
Pont de Sèvres 9
Balard 8
Mairie d'Issy 12
13 Châtillon - Montrouge
4 Porte d'Orléans
RER B B2 Robinson B4 St Rémy-lès-Ch. ORLYVAL Orly
7 Villejuif Louis Aragon
Mairie d'Ivry 7
RER C C2 Massy-Palaiseau C4 Dourdan C6 St Martin d'Etampes
RER C C5 Versailles R.G. C7 St Quentin-en-Yvelines
♦ Les stations Liège et Rennes sont fermées après 20 h et les dimanches et fêtes
RATP/CML 9205 Janvier 1992 - Propriété de la RATP
Ch. de Gaulle-Etoile
La Fourche
Place de Clichy
Porte de Clichy
Pigalle
Barbès Rochechouart
Marcadet Poissonniers
Gare du Nord
La Chapelle
Stalingrad
Jaurès
Louis Blanc
Gare de l'Est
Saint Lazare
Havre Caumartin
Chaussée d'Antin La Fayette
Richelieu Drouot
Opéra
Auber
Madeleine
Concorde
Strasbourg St Denis
République
Belleville
Place des Fêtes
Réaumur-Sébastopol
Arts et Métiers
Les Halles
Châtelet Les Halles
Châtelet
Hôtel de Ville
Palais Royal Musée du Louvre
Invalides
Alma Marceau
Franklin-D. Roosevelt
Champs Elysées Clemenceau
Miromesnil
Trocadéro
Porte Maillot
Pont de l'Alma
Champ de Mars Tour Eiffel
Bir Hakeim
La Muette
Boulainvilliers
Michel-Ange Auteuil
Michel-Ange Molitor
Javel
La Motte-Picquet Grenelle
Solférino
Musée d'Orsay
Sèvres Babylone
Duroc
Pasteur
Montparnasse Bienvenüe
Raspail
Denfert Rochereau
Place d'Italie
Maison Blanche
St Michel
St Michel Notre-Dame
Odéon
Cluny La Sorbonne
Jussieu
Gare d'Austerlitz
Gare de Lyon
Bastille
Reuilly Diderot
Nation
Daumesnil
Oberkampf
Père Lachaise
Gambetta
Péreire
Villiers
Wagram
Malesherbes
Monceau
Courcelles
Ternes
Argentine
Les Sablons
Esplanade de La Défense
Pont de Neuilly
Anatole France
Louise Michel
Porte de Champerret
Brochant
Guy Môquet
Mairie de Clichy
St-Ouen
Garibaldi
Mairie de St Ouen
Porte de St Ouen
Carrefour Pleyel
Saint Denis Porte de Paris
Simplon
Jules Joffrin
Lamarck Caulaincourt
Abbesses
Anvers
Blanche
Château Rouge
Marx Dormoy
Aubervilliers - Pantin 4 Chemins
Fort d'Aubervilliers
Porte de la Villette
Corentin Cariou
Crimée
Riquet
Eglise de Pantin
Bobigny - Pantin Raymond Queneau
Hoche
Porte de Pantin
Ourcq
Laumière
Bolivar
Buttes Chaumont
Botzaris
Danube
Pyrénées
Jourdain
Télégraphe
Saint Fargeau
Pelleport
Porte de Bagnolet
Colonel Fabien
Château Landon
Château d'Eau
Jacques Bonsergent
Goncourt
Parmentier
St Maur
Ménilmontant
Couronnes
Temple
Filles du Calvaire
St Ambroise
Voltaire
Philippe Auguste
Alexandre Dumas
Charonne
Croix de Chavaux
Robespierre
Rue St Maur
Avron
Boulets Montreuil
Maraîchers
Porte de Montreuil
Buzenval
Porte de Vincennes
St-Mandé Tourelle
Vincennes
Bérault
Picpus
Bel Air
Montgallet
Michel Bizot
Porte Dorée
Porte de Charenton
Liberté
Charenton Ecoles
Ecole Vétérinaire
Maisons Alfort - Stade
Maisons Alfort Les Juilliottes
Créteil L'Echat
Créteil - Université
Dugommier
Bercy
Quai de la Gare
Chevaleret
Nationale
Tolbiac
Porte de Choisy
Porte d'Ivry
Porte d'Italie
Ivry sur Seine
Pierre Curie
Le Kremlin Bicêtre
Villejuif Léo Lagrange
Villejuif Paul Vaillant-Couturier
Bd Masséna
Corvisart
Glacière
St Jacques
Les Gobelins
Campo Formio
Saint Marcel
Quai de la Rapée
Sully Morland
Pont Marie
Cité
St Paul
Chemin Vert
St Sébastien Froissart
Richard Lenoir
Bréguet Sabin
Ledru Rollin
Faidherbe Chaligny
Rambuteau
Etienne Marcel
Sentier
Bourse
Bonne Nouvelle
Poissonnière
Cadet
Le Peletier
Notre-Dame de Lorette
Saint Georges
Trinité
Liège ♦
Rome
Europe
St Augustin
St Philippe du Roule
George V
Kléber
Victor Hugo
Boissière
Avenue Foch
Iéna
Rue de la Pompe
Avenue Henri Martin
Passy
Ranelagh
Kennedy Radio France
Jasmin
Eglise d'Auteuil
Porte d'Auteuil
Boulogne Jean Jaurès
Mirabeau
Chardon Lagache
Exelmans
Porte de St Cloud
Marcel Sembat
Billancourt
Bd Victor
Issy-Plaine
Issy
Dupleix
Avenue Emile Zola
Charles Michels
Commerce
Félix Faure
Boucicaut
Lourmel
Convention
Porte de Versailles
Vaugirard
Volontaires
Sèvres Lecourbe
Ségur
Cambronne
Ecole Militaire
La Tour Maubourg
Varenne
Assemblée Nationale
Rue du Bac
St Germain des Prés
St François Xavier
Vaneau
Rennes ♦
St Placide
Mabillon
St Sulpice
Notre-Dame des Champs
Vavin
Falguière
Gaîté
Edgar Quinet
Pernety
Plaisance
Mouton Duvernet
Alésia
Porte de Vanves
Malakoff Plateau de Vanves
Malakoff Rue Etienne Dolet
Corentin Celton
Cité Universitaire
Luxembourg
Port Royal
Place Monge
Censier Daubenton
Cardinal Lemoine
Maubert Mutualité
Pont Neuf
Louvre Rivoli
Tuileries
Pyramides
Quatre Septembre
Rue Montmartre
Pont de Sèvres


メトロ ネットワーク
Metro Network
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丸ノ内線
日比谷線
東西線
千代田線
有楽町線
有楽町線(複々線)
半蔵門線
南北線
都営浅草線
都営三田線
都営新宿線
都営12号線
相互乗換駅
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小竹向原
池袋
新大塚
茗荷谷
後楽園
本郷三丁目
御茶ノ水
淡路町
大手町
東京
銀座
赤坂見附
四ツ谷
新宿
西新宿
中野坂上
荻窪
南阿佐ケ谷
新高円寺
東高円寺
新中野
中野新橋
中野富士見町
方南町
王子
王子神谷
西ケ原
駒込
本駒込
東大前
巣鴨
千石
白山
春日
水道橋
新御茶ノ水
小川町
神保町
九段下
半蔵門
飯田橋
市ケ谷
中野
落合
高田馬場
早稲田
神楽坂
竹橋
日本橋
茅場町
門前仲町
木場
東陽町
南砂町
西葛西
葛西
浦安
南行徳
行徳
妙典
原木中山
西船橋
北千住
南千住
三ノ輪
入谷
上野
稲荷町
田原町
浅草
仲御徒町
秋葉原
小伝馬町
人形町
八丁堀
築地
東銀座
日比谷
霞ケ関
神谷町
六本木
広尾
恵比寿
中目黒
町屋
西日暮里
千駄木
根津
湯島
新橋
虎ノ門
溜池山王
国会議事堂前
青山一丁目
表参道
外苑前
明治神宮前
代々木上原
代々木公園
渋谷
乃木坂
赤坂
桜田門
有楽町
銀座一丁目
新富町
月島
豊洲
辰巳
新木場
東池袋
護国寺
江戸川橋
曙橋
新宿三丁目
新宿御苑前
四谷三丁目
麹町
永田町
三越前
水天宮前
押上
本所吾妻橋
浅草橋
東日本橋
馬喰横山
森下
住吉
大島
本八幡
西馬込
馬込
中延
戸越
五反田
高輪台
泉岳寺
三田
芝公園
御成門
内幸町
大門
品川
中央線三鷹まで直通運転
小田急線本厚木まで直通運転
京王線橋本まで直通運転
東武東上線
常磐線取手まで直通運転
京成線、北総・公団線へ直通運転
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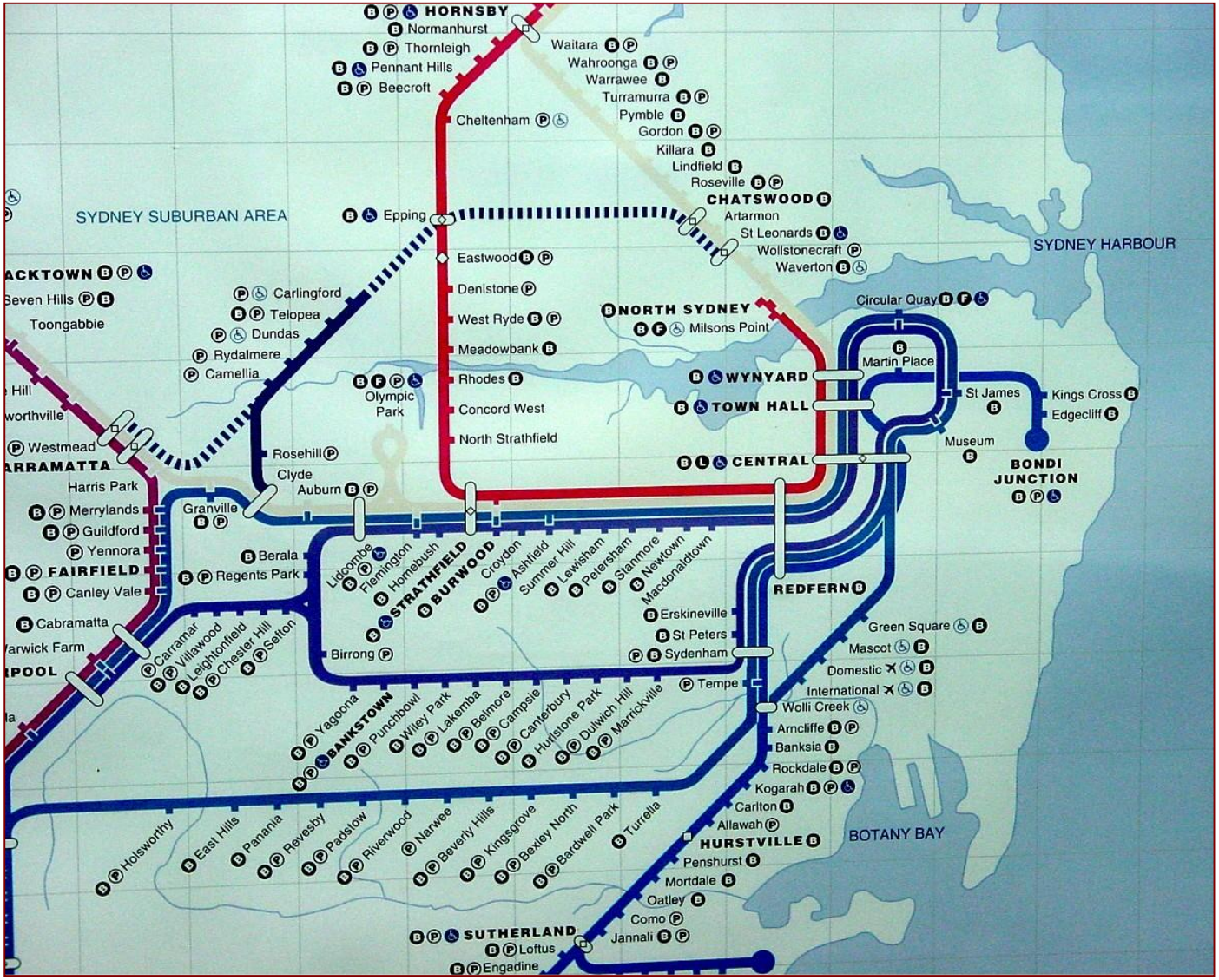
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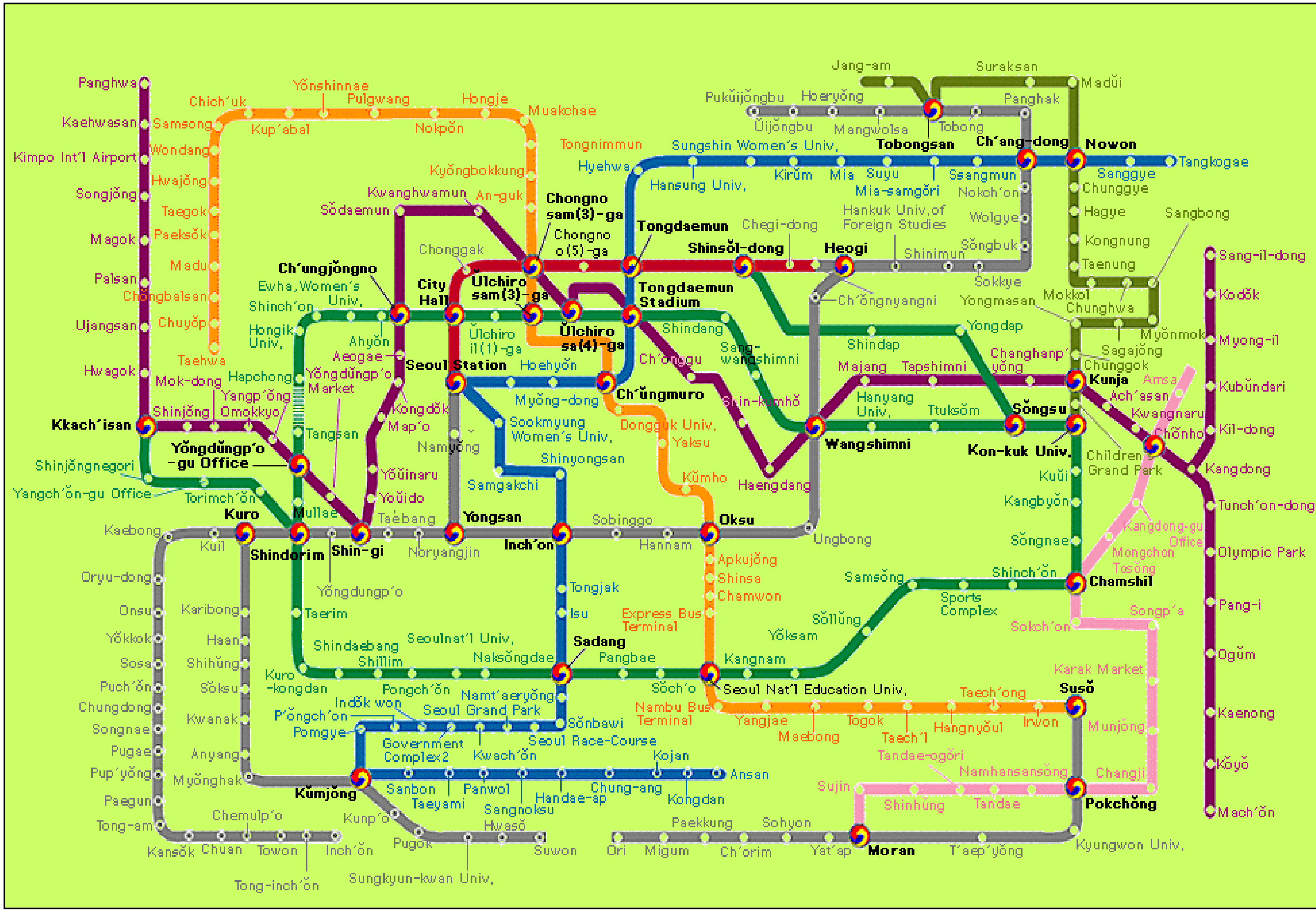
Normanhurst
Thornleigh
Pennant Hills
Beecroft
Cheltenham
Waitara
Wahroonga
Warrawee
Turramurra
Pymble
Gordon
Killara
Lindfield
Roseville
CHATSWOOD
Artarmon
St Leonards
Wollstonecraft
Waverton
SYDNEY HARBOUR
SYDNEY SUBURBAN AREA
Epping
Eastwood
Denistone
West Ryde
Meadowbank
Rhodes
Concord West
North Strathfield
Olympic Park
Carlingford
Telopea
Dundas
Rydalmere
Camellia
Seven Hills
Toongabbie
Westmead
Rosehill
Clyde
Auburn
Granville
Harris Park
Merrylands
Guildford
Yennora
FAIRFIELD
Canley Vale
Cabramatta
Berala
Regents Park
Lidcombe
Flemington
Homebush
STRATHFIELD
BURWOOD
Croydon
Ashfield
Summer Hill
Lewisham
Petersham
Stanmore
Newtown
Macdonaldtown
Carramar
Villawood
Leightonfield
Chester Hill
Sefton
Birrong
Yagoona
BANKSTOWN
Punchbowl
Wiley Park
Lakemba
Belmore
Campsie
Canterbury
Hurlstone Park
Dulwich Hill
Marrickville
Tempe
Holsworthy
East Hills
Panania
Revesby
Padstow
Riverwood
Narwee
Beverly Hills
Kingsgrove
Bexley North
Bardwell Park
Turrella
SUTHERLAND
Loftus
Engadine
NORTH SYDNEY
Milsons Point
Circular Quay
Martin Place
WYNYARD
TOWN HALL
CENTRAL
REDFERN
Erskineville
St Peters
Sydenham
St James
Museum
Kings Cross
Edgecliff
BONDI JUNCTION
Green Square
Mascot
Domestic
International
Wolli Creek
Arncliffe
Banksia
Rockdale
Kogarah
Carlton
Allawah
HURSTVILLE
Penshurst
Mortdale
Oatley
Como
Jannali
BOTANY BAY



Panghwa
Kaehwasan
Kimpo Int'l Airport
Songjŏng
Magok
Palsan
Ujangsan
Hwagok
Kkach'isan
Shinjŏngnegori
Yangch'ŏn-gu Office
Chich'uk
Yŏnshinnae
Pulgwang
Hongje
Muakchae
Samsong
Kup'abal
Nokpŏn
Wondang
Hwajŏng
Taegok
Paeksŏk
Madu
Chŏngbalsan
Chuyŏp
Taehwa
Tongnimmun
Kyŏngbokkung
Kwanghwamun
An-guk
Sŏdaemun
Chongno sam(3)-ga
Chongno o(5)-ga
Chonggak
Ch'ungjŏngno
Ewha Women's Univ.
Shinch'on
Hongik Univ.
Ahyŏn
Aeogae
City Hall
Ŭlchiro sam(3)-ga
Ŭlchiro il(1)-ga
Ŭlchiro sa(4)-ga
Seoul Station
Hoehyŏn
Mok-dong
Hapchong
Yŏngdŭngp'o Market
Shinjŏng
Yangp'yŏng
Omokkyo
Kongdŏk
Map'o
Tangsan
Yŏngdŭngp'o-gu Office
Namyŏng
Yŏŭinaru
Yŏŭido
Torimch'ŏn
Mullae
Kuro
Shindorim
Shin-gil
Taebang
Noryangjin
Yongsan
Kaebong
Kuil
Yŏngdungp'o
Myŏng-dong
Sookmyung Women's Univ.
Shinyongsan
Samgakchi
Ch'ungmuro
Inch'on
Sobinggo
Hannam
Oksu
Tongdaemun
Tongdaemun Stadium
Hyehwa
Hansung Univ.
Sungshin Women's Univ.
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Suyu
Mia-samgŏri
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Tobong
Tobongsan
Panghak
Ch'ang-dong
Nowon
Tangkogae
Sanggye
Chunggye
Hagye
Kongnung
Taenung
Sangbong
Mokkol
Chunghwa
Myŏnmok
Sagajŏng
Yongmasan
Chunggok
Yongdap
Shindap
Majang
Tapshimni
Changhanp'yŏng
Kunja
Amsa
Ach'asan
Kwangnaru
Ch'ŏnho
Hanyang Univ.
Ttuksŏm
Sŏngsu
Kon-kuk Univ.
Children Grand Park
Kuŭi
Kangbyŏn
Sŏngnae
Ungbong
Sang-il-dong
Kodŏk
Myong-il
Kubŭndari
Kil-dong
Kangdong
Tunch'on-dong
Olympic Park
Pang-i
Ogŭm
Kaenong
Kŏyŏ
Mach'ŏn
Kangdong-gu Office
Mongchon Tosŏng
Chamshil
Shinch'ŏn
Sports Complex
Samsŏng
Sŏllŭng
Yŏksam
Songp'a
Sokch'on
Karak Market
Susŏ
Apkujŏng
Shinsa
Chamwon
Express Bus Terminal
Kangnam
Sŏch'o
Seoul Nat'l Education Univ.
Nambu Bus Terminal
Yangjae
Maebong
Togok
Taech'i
Hangnyŏul
Taech'ong
Irwon
Munjŏng
Tandae-ogŏri
Namhansansŏng
Changji
Sujin
Shinhung
Tandae
Pokchŏng
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Yat'ap
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Sohyon
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Indŏk won
P'ŏngch'on
Pomgye
Seoul Grand Park
Government Complex2
Kwach'ŏn
Seoul Race-Course
Sŏnbawi
Kojan
Ansan
Chung-ang
Kongdan
Handae-ap
Sangnoksu
Panwol
Sanbon
Taeyami
Kŭmjŏng
Kunp'o
Pugok
Hwasŏ
Suwon
Sungkyun-kwan Univ.
Karibong
Haan
Shihŭng
Sŏksu
Kwanak
Anyang
Myŏnghak
Oryu-dong
Onsu
Yŏkkok
Sosa
Puch'ŏn
Chungdong
Songnae
Pugae
Pup'yŏng
Paegun
Tong-am
Kansŏk
Chuan
Towon
Inch'ŏn
Chemulp'o
Tong-inch'ŏn


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SAN GIOVANNI
RE DI ROMA
PONTE LUNGO
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COLLI ALBANI
ARCO DI TRAVERTINO
QUADRARO
NUMIDIO QUADRATO
LUCIO SESTIO
GIULIO AGRICOLA
SUBAUGUSTA
CINECITTA'
ANAGNINA
STAZ.TERMINI
CAVOUR
COLOSSEO
CIRCO MASSIMO
PIRAMIDE
OSTIENSE
MAGLIANA
MURATELLA
L.DA VINCI
AEREOPORTO
FIUMICINO
P.GALERIA
GARBATELLA
BASILICA DI S. PAOLO
MARCONI
MAGLIANA
TOR DI VALLE
VITINIA
CASAL BERNOCCHI
ACILIA
OSTIA ANTICA
LIDO CENTRO
STELLA POLARE
CASTEL FUSANO
C.COLOMBO
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EUR FERMI
EUR PALASPORT
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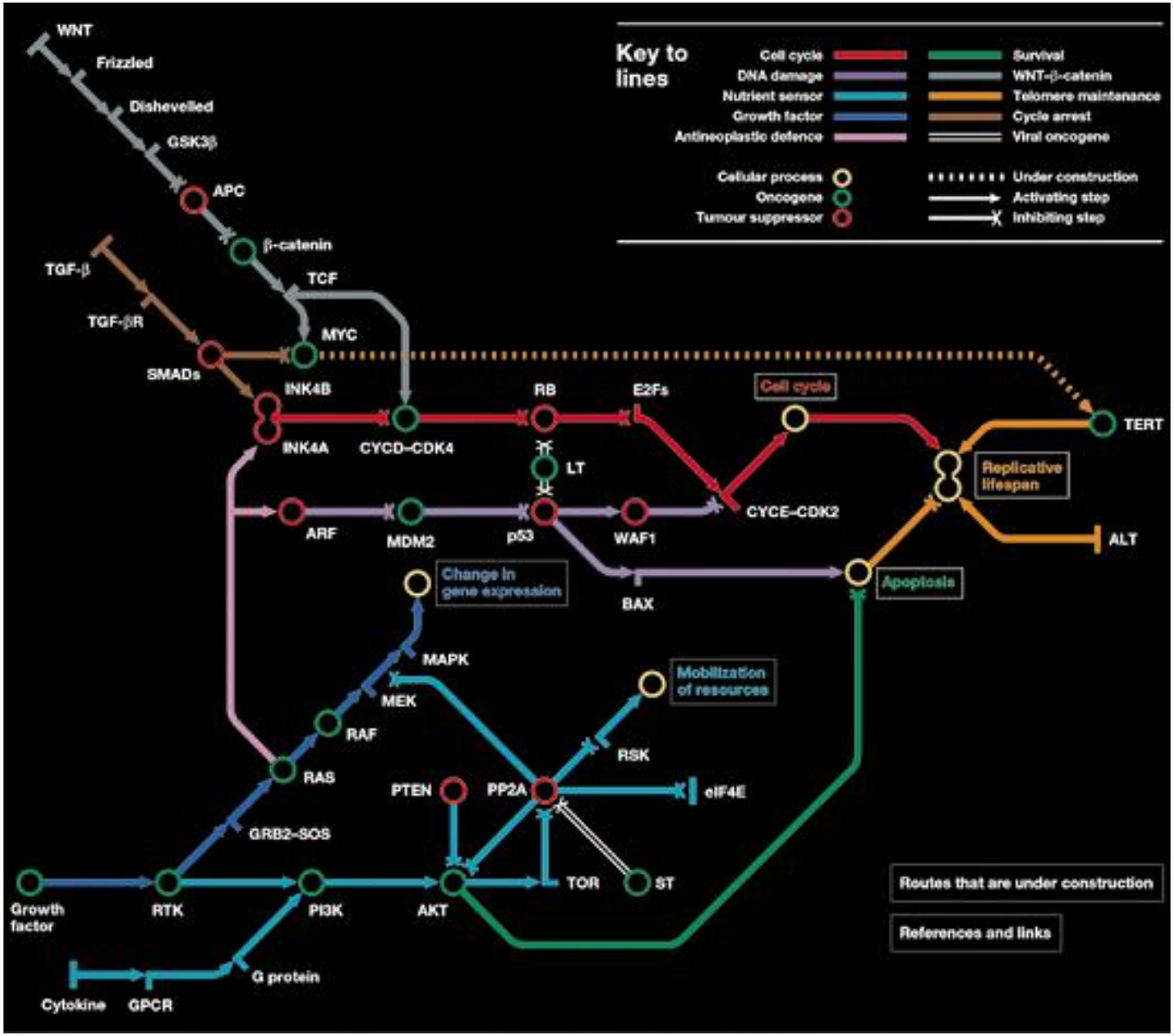
ANAGNINA - OTTAVIANO
METRO LINEA "B"
LAURENTINA - REBIBBIA
FIUMICINO - OSTIENSE
PIRAMIDE - C.COLOMBO (OSTIA)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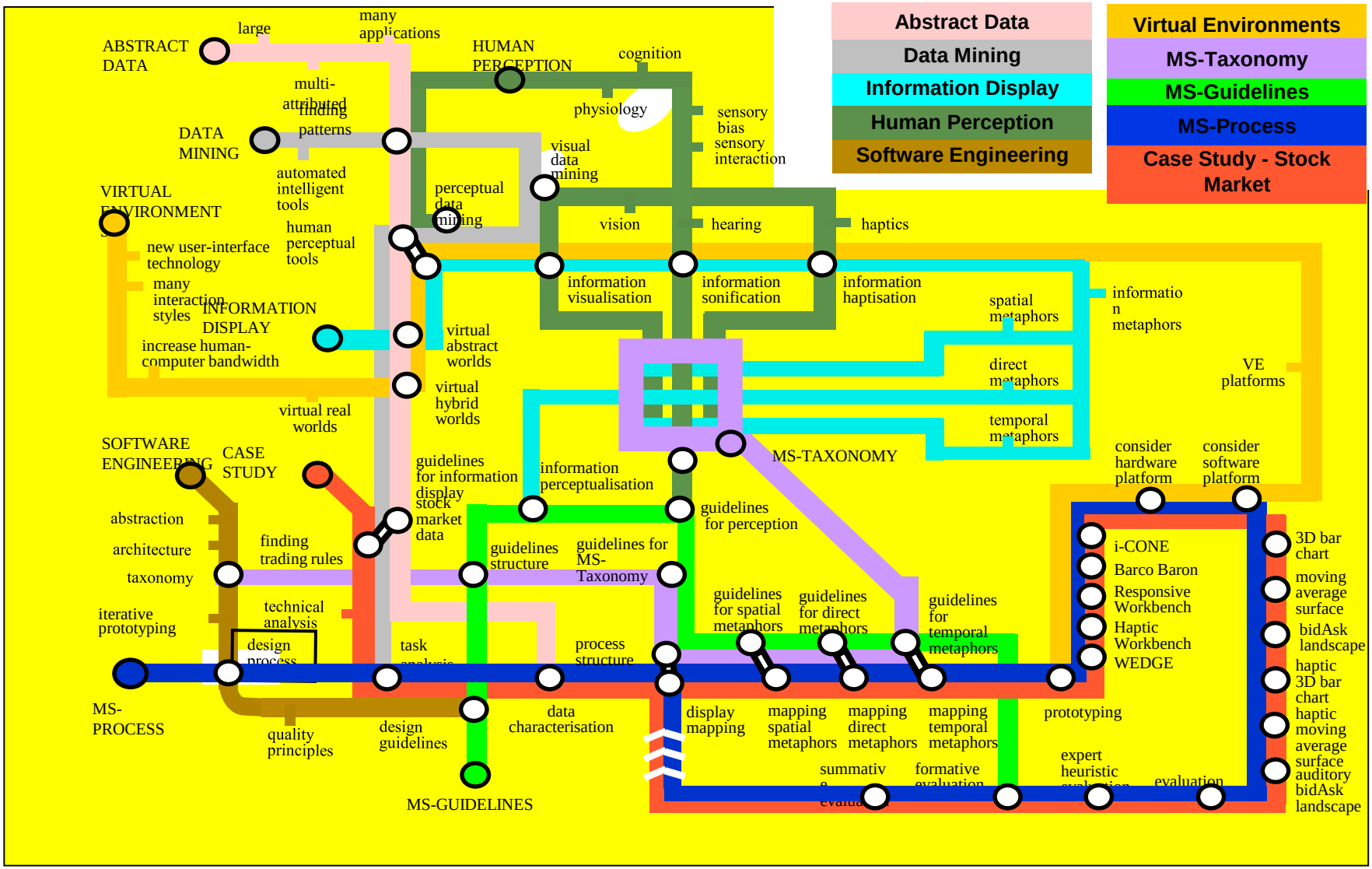
Cell cycle
DNA damage
Nutrient sensor
Growth factor
Antineoplastic defence
Survival
WNT-β-catenin
Telomere maintenance
Cycle arrest
Viral oncogene
Cellular process
Oncogene
Tumour suppressor
Under construction
Activating step
Inhibiting step
WNT
Frizzled
Dishevelled
GSK3β
APC
β-catenin
TCF
TGF-β
TGF-βR
SMADs
MYC
INK4B
INK4A
CYCD–CDK4
RB
E2Fs
Cell cycle
TERT
LT
Replicative lifespan
ALT
ARF
MDM2
p53
WAF1
CYCE–CDK2
Change in gene expression
BAX
Apoptosis
MAPK
MEK
RAF
RAS
Mobilization of resources
RSK
PTEN
PP2A
eIF4E
GRB2–SOS
Growth factor
RTK
PI3K
AKT
TOR
ST
Routes that are under construction
References and links
G protein
Cytokine
GPCR

J. Hallinan



Keith Nesbitt

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2003 OPEN SOURCE ROUTE MAP

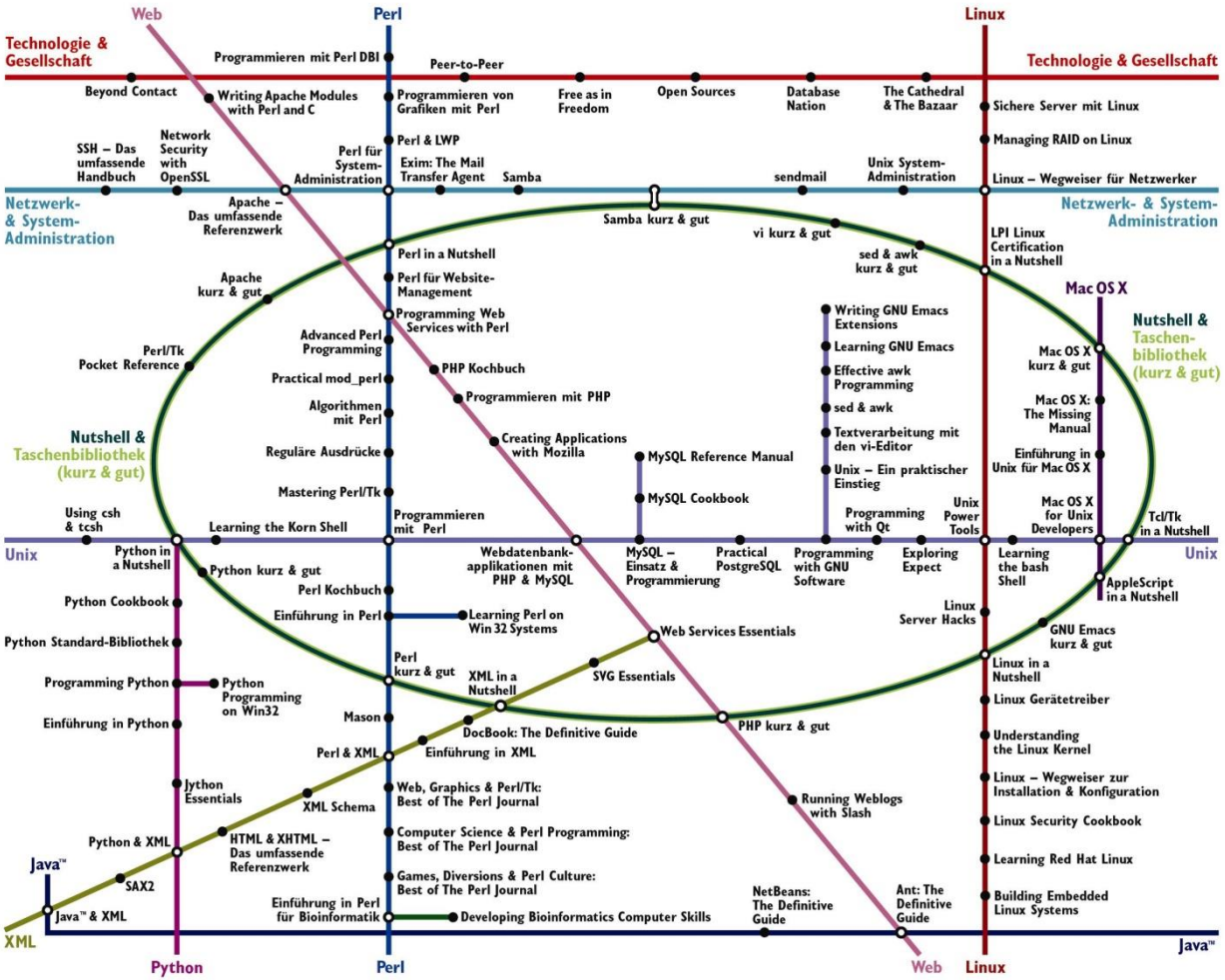
LINUX & KONSORTEN

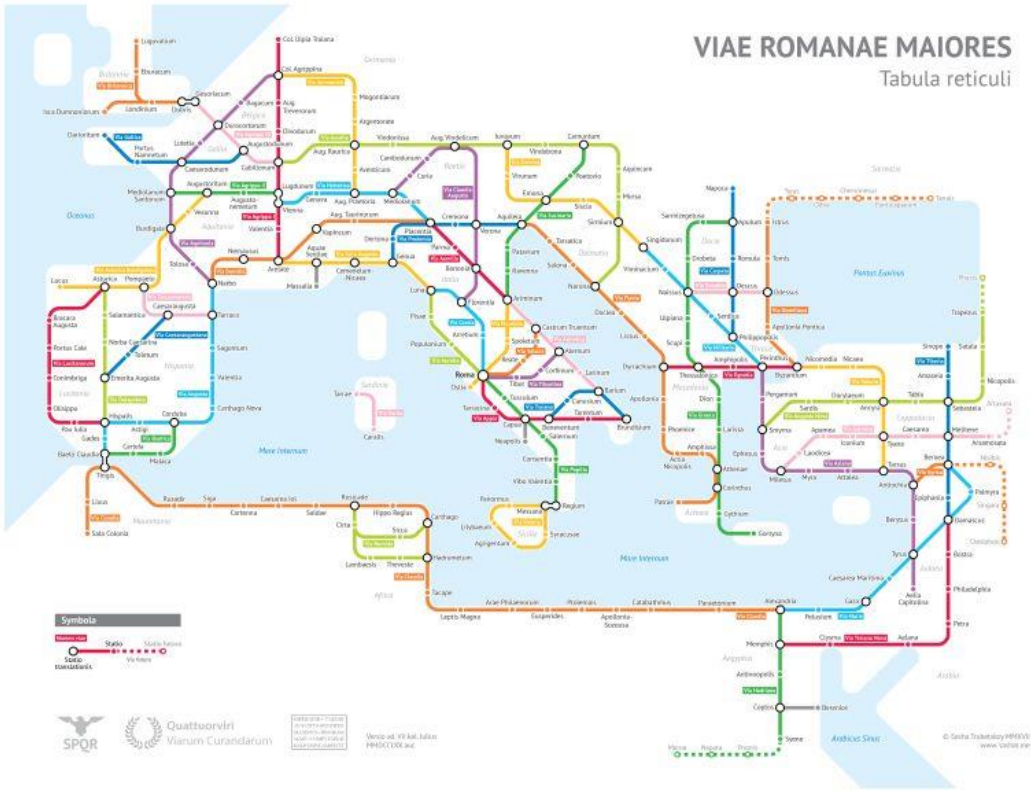
LEGENDE

- Bioinformatik
- Java™
- Linux
- Mac OS X
- Netzwerk- & System-Administration
- Nutshell & Taschenbibliothek (kurz & gut)
- Perl
- Python
- Technologie & Gesellschaft
- Unix
- Web
- XML

www.oreilly.de

Die Grübelei hat ein Ende!
Bücher von O'Reilly




VIAE ROMANAE MAIORES
Tabula reticuli
Symbola
SPQR
Quattuorviri Viarum Curandarum

Scientific Question: Is there an ***E³*** computer algorithm that can produce a layout of a metro map network?

(E^3 = Effective, Efficient, Elegant)

Metro map layout

Model

- Metro stations → steel rings
- Interconnections → magnetized springs
- Vertical/horizontal/45^{o} magnetic field

Algorithm

- Simple follow-your-nose

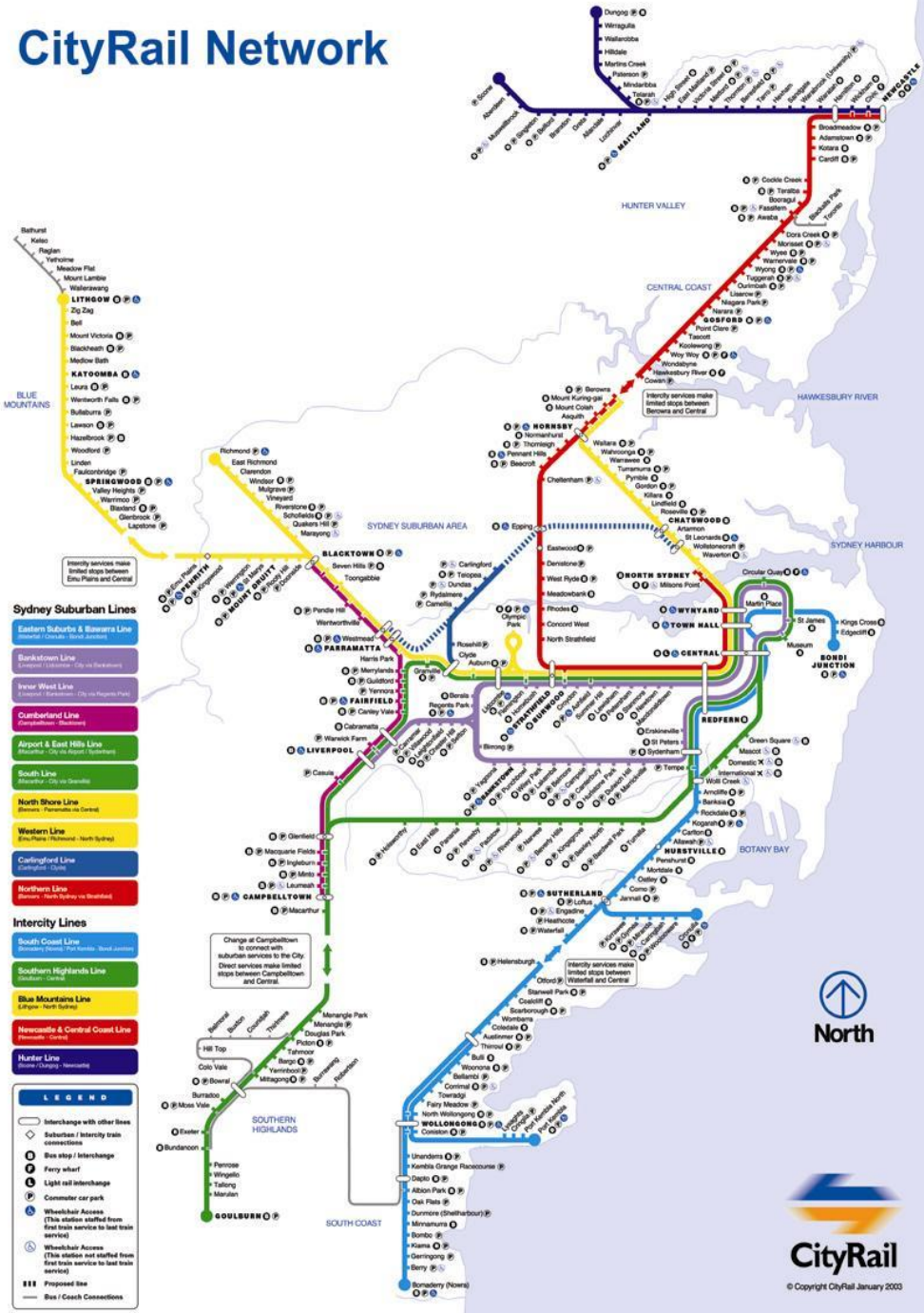
- Plus automatic label placement

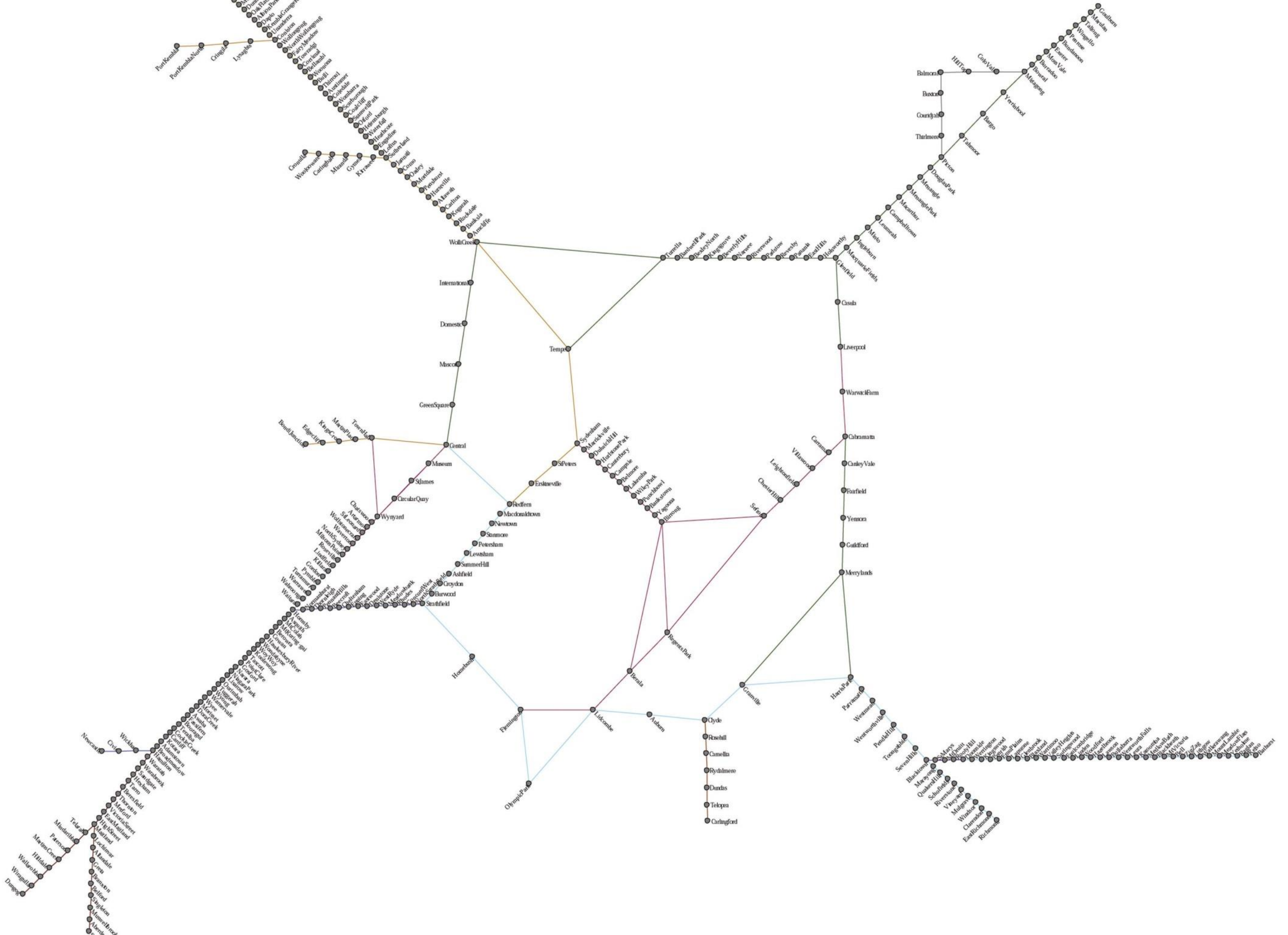
Elegance:

- This spring method is *elegant*

Effectiveness

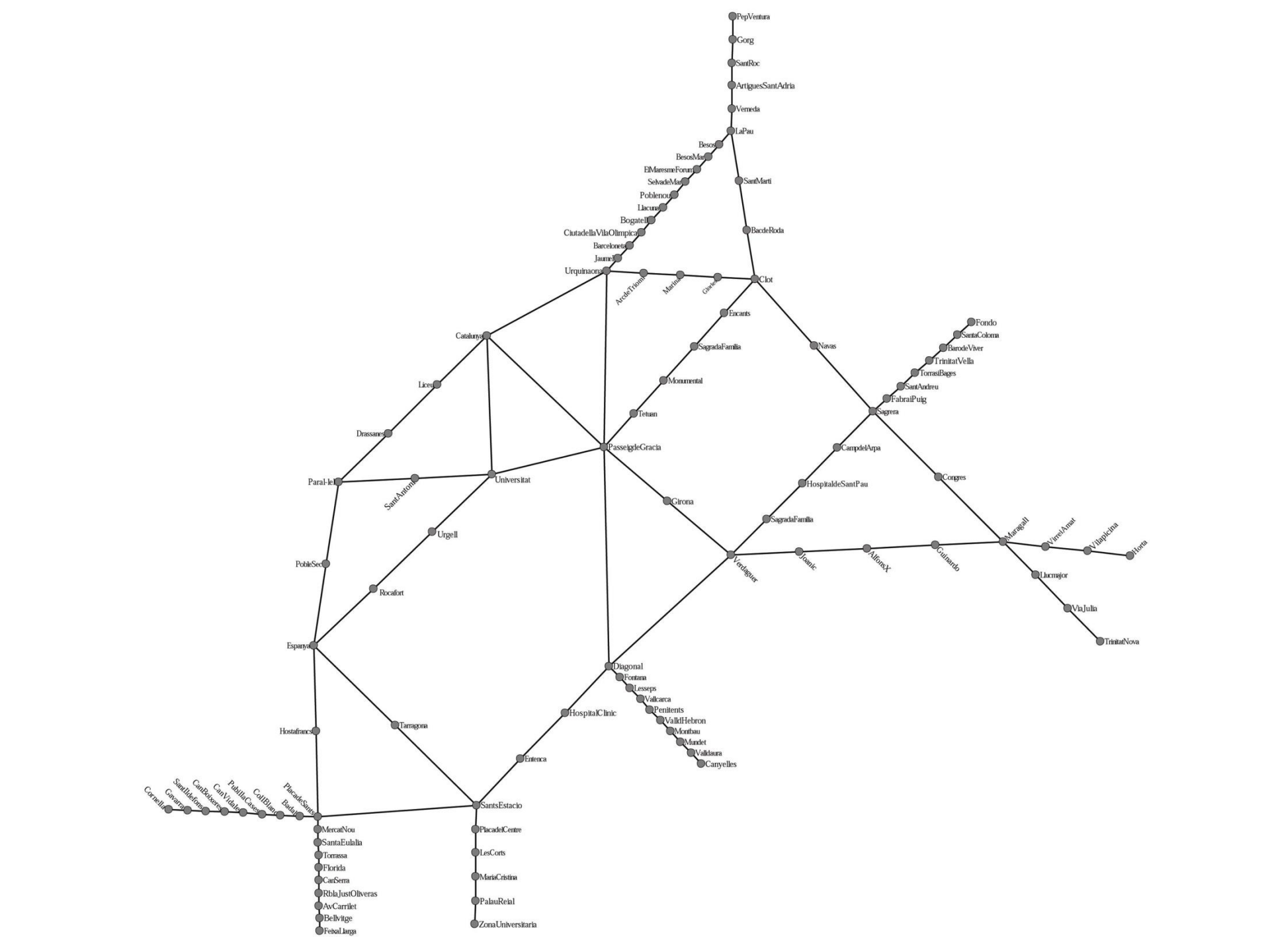
CityRail Network





PortKembla
PortKemblaNorth
Cringila
Lysaghts
WolliCreek
International
Domestic
Mascot
GreenSquare
Central
Tempe
Sydenham
StPeters
Erskineville
Redfern
Macdonaldtown
Newtown
Stanmore
Petersham
Lewisham
SummerHill
Ashfield
Croydon
Burwood
Strathfield
Homebush
Flemington
OlympicPark
Lidcombe
Auburn
Clyde
Rosehill
Camellia
Rydalmere
Dundas
Telopea
Carlingford
Granville
HarrisPark
Parramatta
Westmead
Wentworthville
PendleHill
Toongabbie
SevenHills
Blacktown
Berala
RegentsPark
Sefton
ChesterHill
Leightonfield
Villawood
Carramar
Cabramatta
CanleyVale
Fairfield
Yennora
Guildford
Merrylands
WarwickFarm
Liverpool
Casula
Glenfield
Turrella
Holsworthy
EastHills
Panania
Revesby
Padstow
Riverwood
Narwee
BeverlyHills
Kingsgrove
BexleyNorth
Bardwell Park
Museum
StJames
CircularQuay
Wynyard
TownHall
MartinPlace
KingsCross
Edgecliff
BondiJunction
Balmoral
HillTop
ColoVale
Buxton
Couridjah
Thirlmere
Picton
Tahmoor
Bargo
Yerrinbool
Mittagong
Bowral
Goulburn
Campbelltown
Leumeah
Minto
Ingleburn
MacquarieFields
Newcastle
Civic
Wickham
Dungog
Maitland
Telarah
Richmond
Bathurst
Cronulla
Woolooware
Caringbah
Miranda
Gymea
Kirrawee


TMB
L1 Can Cuiàs
L3 Canyelles
L5 Horta
L11 Trinitat Nova
L4 Trinitat Nova
L3 Zona Universitària
L5 Cornellà
L1 Feixa Llarga
L1 Fondo
L4 La Pau
L2 Pep Ventura
L2 Paral·lel
Parc de Montjuïc
Esplugues de Llobregat
Cornellà de Llobregat
l'Hospitalet de Llobregat
Santa Coloma de Gramenet
Badalona
Sant Adrià de Besòs
Les Planes
Baixador de Vallvidrera
Vallvidrera Superior
Peu del Funicular
Reina Elisenda
Sarrià
Les Tres Torres
La Bonanova
Muntaner
St. Gervasi
Gràcia
Provença
Tibidabo
Pl del Funicular
Av Tibidabo
El Putxet
Pàdua
Pl Molina
Montbau
Mundet
Vall d'Hebron
Valldaura
Penitents
Vallcarca
Lesseps
Fontana
Via Júlia
Vilapicina
Llucmajor
Virrei Amat
Maragall
Alfons X
Guinardó
Congrés
Joanic
Hospital de Sant Pau
Camp de l'Arpa
Sagrera
Fabra i Puig
Trinitat Vella
Torras i Bages
Baró de Viver
Sant Andreu
Santa Coloma
Palau Reial
Maria Cristina
Les Corts
Plaça del Centre
Entença
Hospital Clínic
Diagonal
Verdaguer
Sagrada Família
Encants
Clot
Navas
La Pau
Verneda
Artigues Sant Adrià
Sant Roc
Gorg
Bac de Roda
St. Martí
Besòs
Besòs Mar
El Maresme Fòrum
Selva de Mar
Poblenou
Llacuna
Bogatell
Marina
Glòries
Arc de Triomf
Urquinaona
Jaume I
Ciutadella Vila Olímpica
Barceloneta
Girona
Monumental
Passeig de Gràcia
Tetuan
Universitat
Catalunya
Liceu
Paral·lel
Drassanes
Sant Antoni
Poble Sec
Urgell
Rocafort
Espanya
Tarragona
Sants Estació
Hostafrancs
Mercat Nou
Plaça de Sants
Santa Eulàlia
Badal
Collblanc
Torrassa
Florida
Can Serra
Can Boixeres
Can Vidalet
Pubilla Cases
Gavarra
Sant Ildefons
Cornellà
Rbla.Just Oliveras
Av Carrilet
Bellvitge
Gornal
Ildefons Cerdà
Magòria La Campana
El Prat de Llobregat
Aeroport
Castell de Montjuïc
Mirador
Miramar
Jaume I
St. Sebastià
ATM


PepVentura
Gorg
SantRoc
ArtiguesSantAdria
Verneda
LaPau
Besos
BesosMar
ElMaresmeForum
SelvadeMar
Poblenou
Llacuna
Bogatell
CiutadellaVilaOlimpica
Barceloneta
Jaume I
SantMarti
BacdeRoda
Urquinaona
ArcdeTriomf
Marina
Glories
Clot
Encants
Catalunya
SagradaFamilia
Navas
Fondo
SantaColoma
BarodeViver
TrinitatVella
TorrasiBages
SantAndreu
FabraiPuig
Sagrera
Liceu
Monumental
Tetuan
Drassanes
PasseigdeGracia
CampdelArpa
Congres
Paral-lel
SantAntoni
Universitat
HospitaldeSantPau
Girona
SagradaFamilia
Urgell
Maragall
VirreiAmat
VilapIcina
Horta
PobleSec
Verdaguer
Joanic
AlfonsX
Guinardo
Llucmajor
Rocafort
ViaJulia
TrinitatNova
Espanya
Diagonal
Fontana
Lesseps
Vallcarca
Penitents
ValldHebron
Montbau
Mundet
Valldaura
Canyelles
HospitalClinic
Hostafrancs
Tarragona
Entenca
PlacadeSants
CalBlanc
Badal
CanVidalet
PubillaCases
CanBoixeres
SantIldefons
Cornella
Gavarra
SantsEstacio
MercatNou
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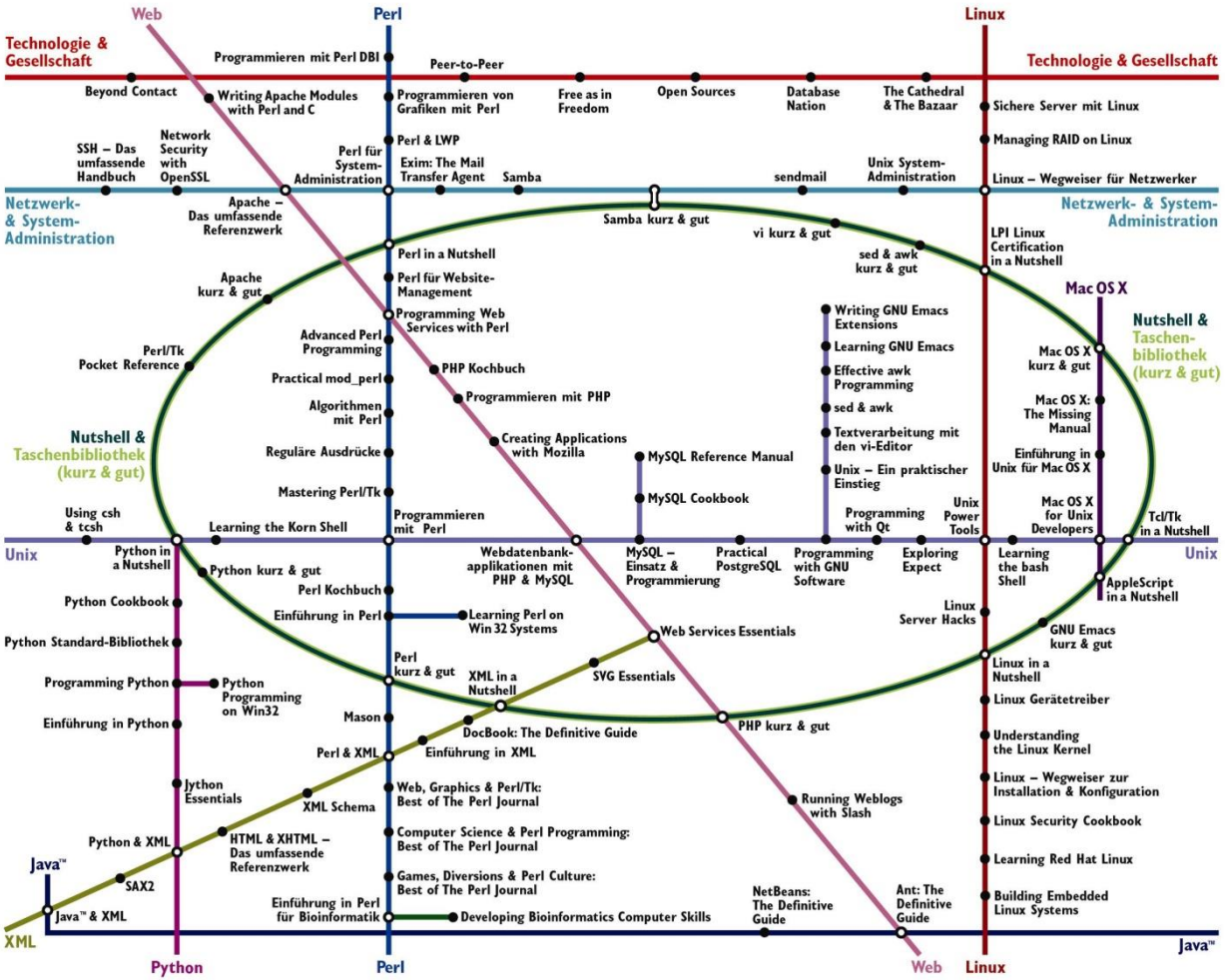

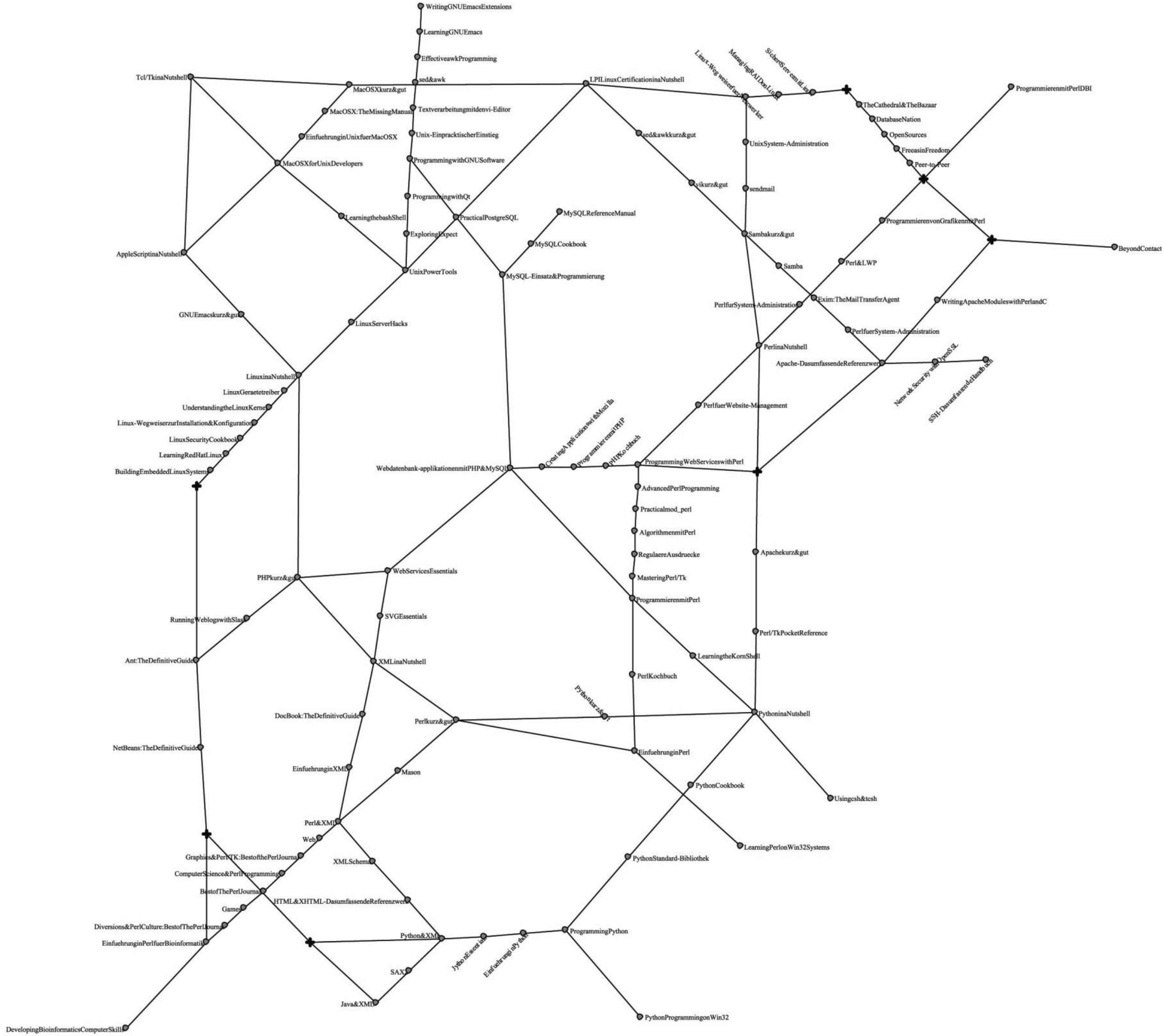
WritingGNUEmacsExtensions
LearningGNUEmacs
EffectiveawkProgramming
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Tcl/TkinaNutshell
MacOSXkurz&gut
MacOSX:TheMissingManual
Textverarbeitungmitdenvi-Editor
EinfuehrunginUnixfuerMacOSX
Unix-EinpracktischerEinstieg
MacOSXforUnixDevelopers
ProgrammingwithGNUSoftware
ProgrammingwithQt
LearningthebashShell
PracticalPostgreSQL
ExploringExpect
AppleScriptinaNutshell
UnixPowerTools
LPILinuxCertificationinaNutshell
sed&awkkurz&gut
vikurz&gut
UnixSystem-Administration
sendmail
Sambakurz&gut
Samba
MySQLReferenceManual
MySQLCookbook
MySQL-Einsatz&Programmierung
TheCathedral&TheBazaar
DatabaseNation
OpenSources
FreeasinFreedom
Peer-to-Peer
ProgrammierenmitPerlDBI
ProgrammierenvonGrafikenmitPerl
BeyondContact
Perl&LWP
PerlfurSystem-Administration
Exim:TheMailTransferAgent
WritingApacheModuleswithPerlandC
PerlfuerSystem-Administration
PerlinaNutshell
Apache-DasumfassendeReferenzwerk
GNUEmacskurz&gut
LinuxServerHacks
LinuxinaNutshell
LinuxGeraetetreiber
UnderstandingtheLinuxKernel
Linux-WegweiserzurInstallation&Konfiguration
LinuxSecurityCookbook
LearningRedHatLinux
BuildingEmbeddedLinuxSystems
PerlfuerWebsite-Management
Webdatenbank-applikationenmitPHP&MySQL
ProgrammingWebServiceswithPerl
AdvancedPerlProgramming
Practicalmod_perl
AlgorithmenmitPerl
RegulaereAusdruecke
MasteringPerl/Tk
ProgrammierenmitPerl
Apachekurz&gut
PHPkurz&gut
WebServicesEssentials
SVGEssentials
RunningWeblogswithSlash
Ant:TheDefinitiveGuide
XMLinaNutshell
Perl/TkPocketReference
LearningtheKornShell
PerlKochbuch
PythoninaNutshell
DocBook:TheDefinitiveGuide
Perlkurz&gut
EinfuehrunginPerl
NetBeans:TheDefinitiveGuide
EinfuehrunginXML
Mason
PythonCookbook
Usingcsh&tcsh
Perl&XML
Web
LearningPerlonWin32Systems
Graphics&PerlTK:BestofthePerlJournal
XMLSchema
PythonStandard-Bibliothek
ComputerScience&PerlProgramming
BestofThePerlJournal
HTML&XHTML-DasumfassendeReferenzwerk
Games
Diversions&PerlCulture:BestofThePerlJournal
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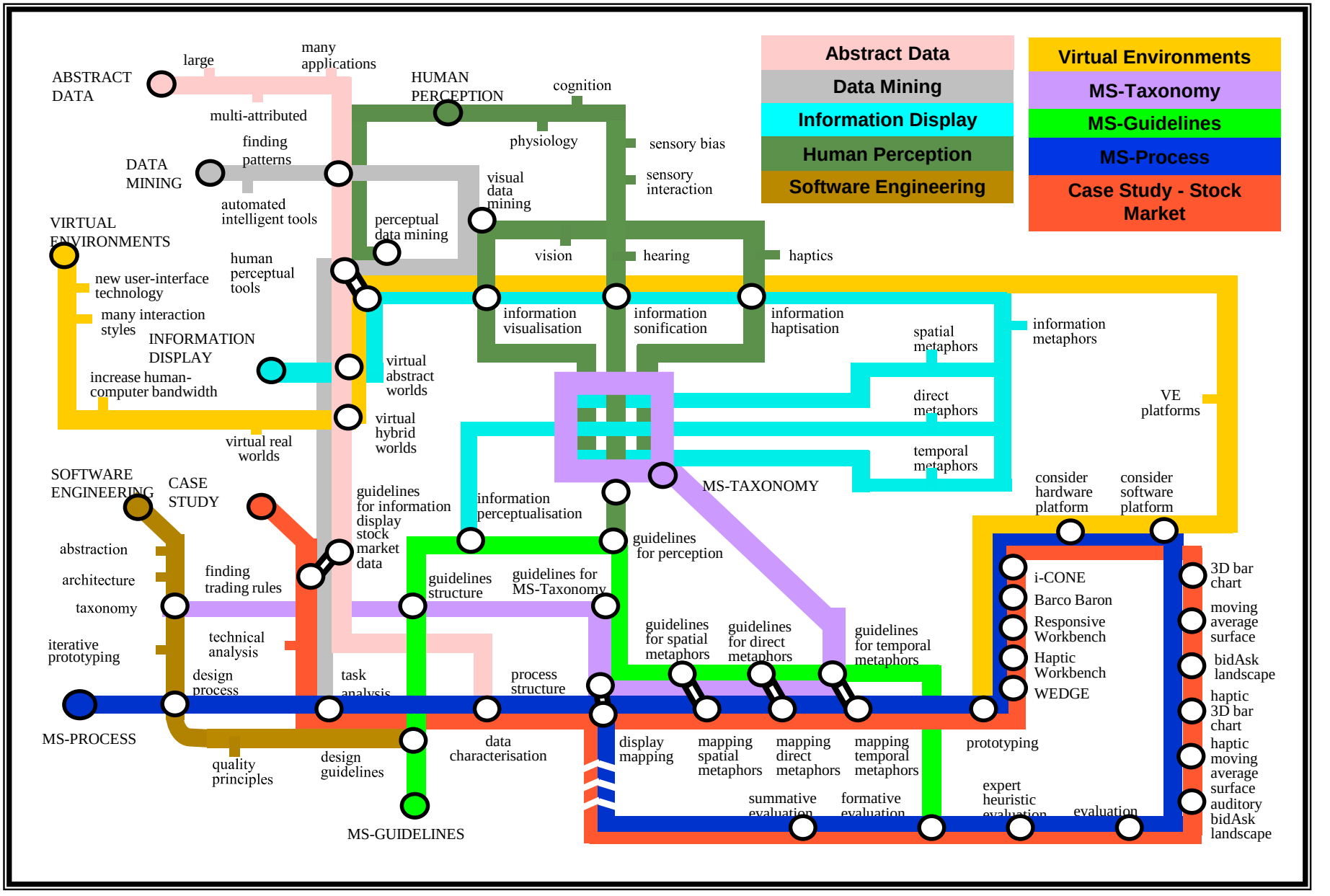
Abstract Data
Data Mining
Information Display
Human Perception
Software Engineering
Virtual Environments
MS-Taxonomy
MS-Guidelines
MS-Process
Case Study - Stock Market
ABSTRACT DATA
large
many applications
multi-attributed
DATA MINING
finding patterns
automated intelligent tools
HUMAN PERCEPTION
cognition
physiology
sensory bias
sensory interaction
visual data mining
perceptual data mining
vision
hearing
haptics
VIRTUAL ENVIRONMENTS
new user-interface technology
many interaction styles
human perceptual tools
INFORMATION DISPLAY
increase human-computer bandwidth
virtual real worlds
virtual abstract worlds
virtual hybrid worlds
information visualisation
information sonification
information haptisation
spatial metaphors
direct metaphors
temporal metaphors
information metaphors
VE platforms
SOFTWARE ENGINEERING
CASE STUDY
abstraction
architecture
taxonomy
iterative prototyping
finding trading rules
technical analysis
design process
task analysis
process structure
MS-PROCESS
quality principles
design guidelines
data characterisation
MS-GUIDELINES
guidelines for information display stock market data
information perceptualisation
guidelines structure
guidelines for MS-Taxonomy
guidelines for perception
MS-TAXONOMY
guidelines for spatial metaphors
guidelines for direct metaphors
guidelines for temporal metaphors
display mapping
mapping spatial metaphors
mapping direct metaphors
mapping temporal metaphors
summative evaluation
formative evaluation
consider hardware platform
consider software platform
i-CONE
Barco Baron
Responsive Workbench
Haptic Workbench
WEDGE
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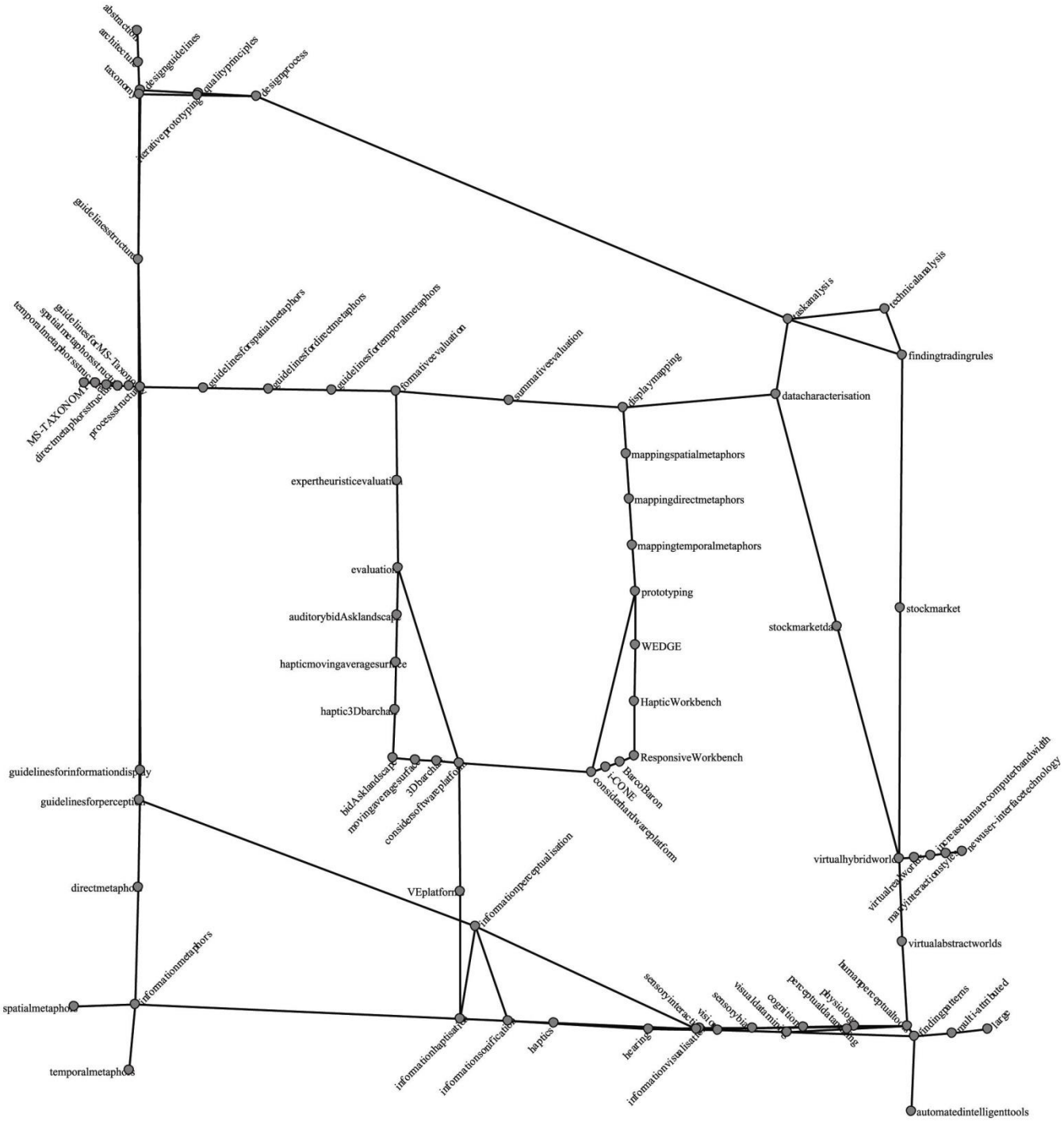
expert heuristic evaluation
evaluation
3D bar chart
moving average surface
bidAsk landscape
haptic 3D bar chart
haptic moving average surface
auditory bidAsk landscape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taxonomy
qualityprinciples
designprocess
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temporalmetaphorsstructure
MS-TAXONOMY
directmetaphorsstructure
processstructure
guidelinesforspatialmetaphors
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guidelinesfortemporalmetaphors
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technicalanalysis
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mappingtemporalmetaphors
evaluation
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movingaveragesurface
3Dbarchart
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i-CONE
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guidelinesforperception
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newuser-interfacetechnology
virtualrealityworlds
manyinteractionstyles
directmetaphors
VEplatform
informationperceptualisation
virtualabstractworlds
informationmetaphors
spatialmetaphors
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perceptualtraining
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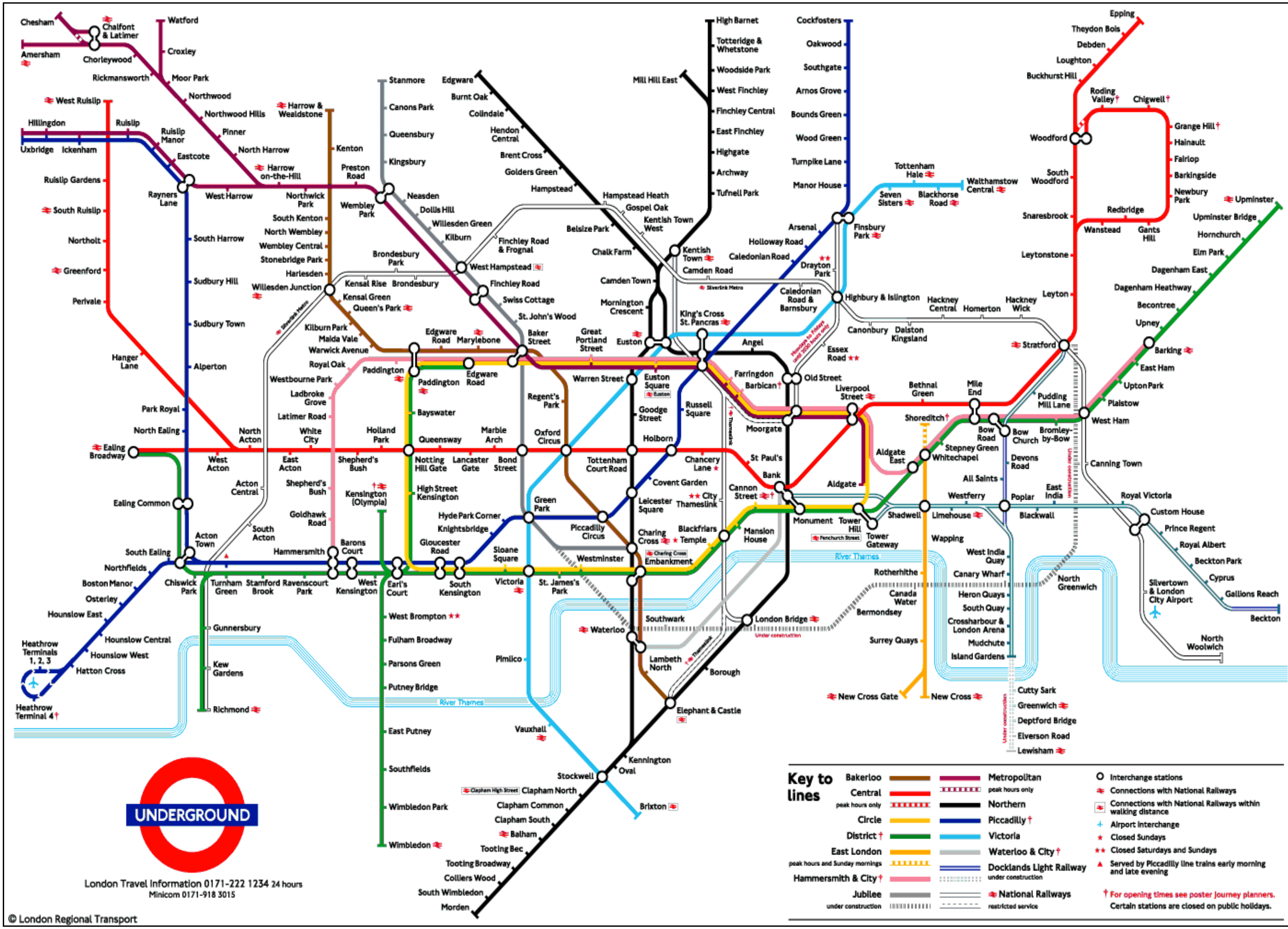
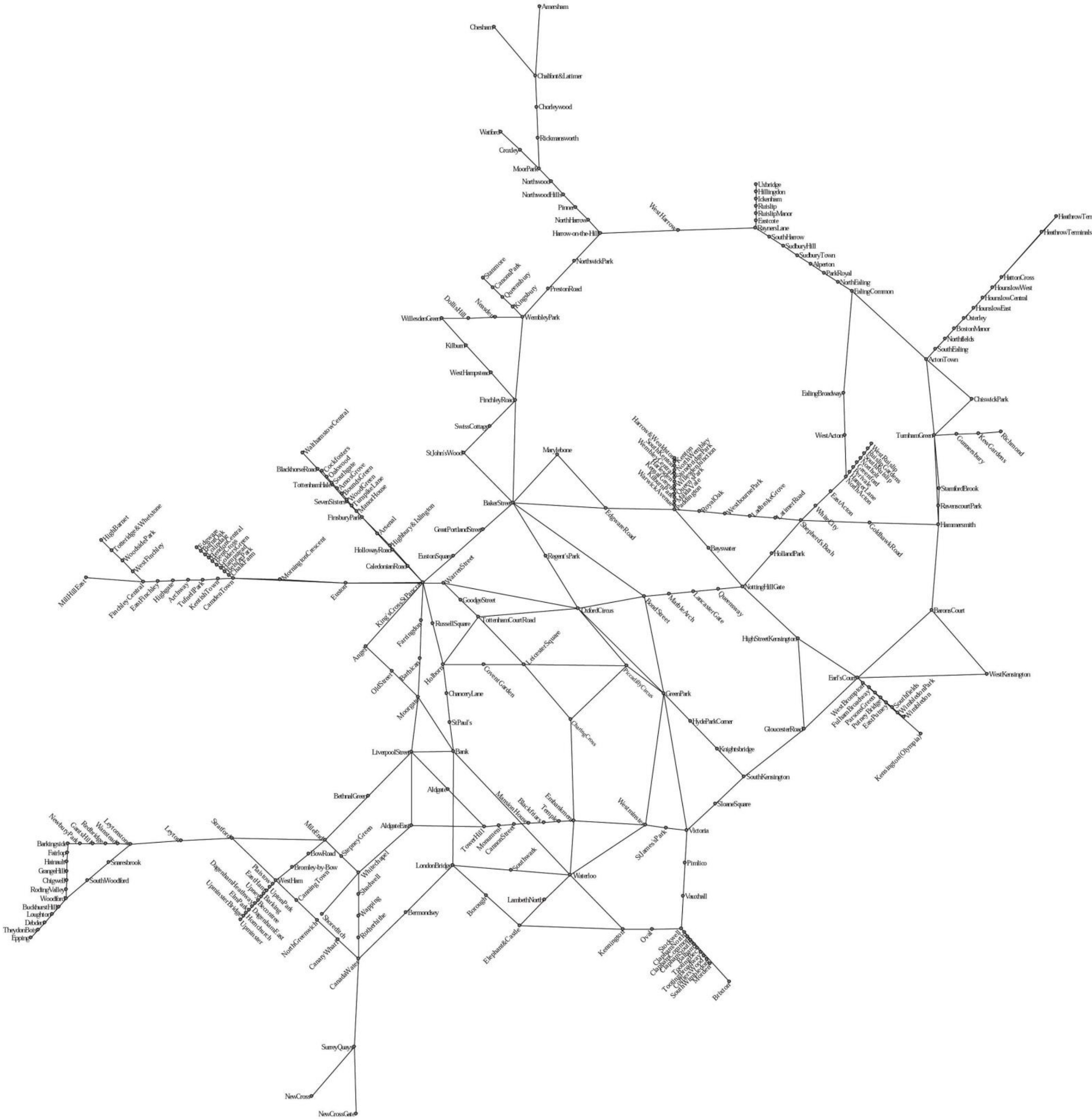
automatedintelligenttools



Chesham
Chalfont & Latimer
Watford
Amersham
Croxley
Chorleywood
Rickmansworth
Moor Park
Northwood
Northwood Hills
Pinner
North Harrow
Harrow-on-the-Hill
Harrow & Wealdstone
West Ruislip
Ruislip
Hillingdon
Ruislip Manor
Uxbridge
Ickenham
Eastcote
Ruislip Gardens
Rayners Lane
West Harrow
South Ruislip
Northolt
Greenford
Perivale
Hanger Lane
South Harrow
Sudbury Hill
Sudbury Town
Alperton
Park Royal
North Ealing
Ealing Broadway
West Acton
North Acton
East Acton
Acton Central
Ealing Common
Acton Town
South Ealing
Northfields
Boston Manor
Osterley
Hounslow East
Hounslow Central
Hounslow West
Hatton Cross
Heathrow Terminals 1, 2, 3
Heathrow Terminal 4
Chiswick Park
Turnham Green
Stamford Brook
Ravenscourt Park
Gunnersbury
Kew Gardens
Richmond
South Acton
Stanmore
Canons Park
Queensbury
Kingsbury
Kenton
Preston Road
Northwick Park
Wembley Park
South Kenton
North Wembley
Wembley Central
Stonebridge Park
Harlesden
Willesden Junction
Kensal Rise
Brondesbury
Brondesbury Park
Kensal Green
Queen's Park
Kilburn Park
Maida Vale
Warwick Avenue
Royal Oak
Westbourne Park
Ladbroke Grove
Latimer Road
White City
Shepherd's Bush
Goldhawk Road
Hammersmith
Barons Court
West Kensington
Kensington (Olympia)
Holland Park
Earl's Court
West Brompton
Fulham Broadway
Parsons Green
Putney Bridge
East Putney
Southfields
Wimbledon Park
Wimbledon
River Thames
Edgware
Burnt Oak
Colindale
Hendon Central
Brent Cross
Golders Green
Hampstead
Neasden
Dollis Hill
Willesden Green
Kilburn
West Hampstead
Finchley Road
Swiss Cottage
St. John's Wood
Finchley Road & Frognal
Belsize Park
Chalk Farm
Camden Town
Mornington Crescent
Hampstead Heath
Gospel Oak
Kentish Town West
Kentish Town
Camden Road
Mill Hill East
High Barnet
Totteridge & Whetstone
Woodside Park
West Finchley
Finchley Central
East Finchley
Highgate
Archway
Tufnell Park
Cockfosters
Oakwood
Southgate
Arnos Grove
Bounds Green
Wood Green
Turnpike Lane
Manor House
Arsenal
Holloway Road
Caledonian Road
Drayton Park
Finsbury Park
Caledonian Road & Barnsbury
Highbury & Islington
Silverlink Metro
Seven Sisters
Tottenham Hale
Blackhorse Road
Walthamstow Central
Hackney Central
Homerton
Hackney Wick
Canonbury
Dalston Kingsland
Essex Road
Mondays to Fridays until 2000 hours only
Edgware Road
Marylebone
Baker Street
Great Portland Street
Euston
King's Cross St. Pancras
Angel
Paddington
Edgware Road
Bayswater
Queensway
Notting Hill Gate
Lancaster Gate
Marble Arch
Bond Street
Regent's Park
Oxford Circus
Warren Street
Euston Square
Goodge Street
Russell Square
Farringdon
Barbican
Old Street
Liverpool Street
Moorgate
Holborn
Tottenham Court Road
Chancery Lane
St Paul's
Bank
Covent Garden
Leicester Square
City Thameslink
High Street Kensington
Hyde Park Corner
Knightsbridge
Gloucester Road
South Kensington
Sloane Square
Green Park
Piccadilly Circus
Charing Cross
Embankment
Blackfriars
Temple
Mansion House
Cannon Street
Monument
Tower Hill
Fenchurch Street
Tower Gateway
Aldgate
Aldgate East
Shoreditch
Whitechapel
Westminster
Victoria
St. James's Park
Waterloo
Southwark
Lambeth North
London Bridge
Borough
Elephant & Castle
Thameslink
Under construction
Pimlico
Vauxhall
Kennington
Oval
Stockwell
Brixton
Clapham North
Clapham High Street
Clapham Common
Clapham South
Balham
Tooting Bec
Tooting Broadway
Colliers Wood
South Wimbledon
Morden
Epping
Theydon Bois
Debden
Loughton
Buckhurst Hill
Roding Valley
Chigwell
Grange Hill
Hainault
Fairlop
Barkingside
Newbury Park
Woodford
South Woodford
Snaresbrook
Redbridge
Wanstead
Gants Hill
Leytonstone
Leyton
Stratford
Bethnal Green
Mile End
Pudding Mill Lane
Bow Road
Bow Church
Bromley-by-Bow
Stepney Green
Devons Road
All Saints
Westferry
Poplar
East India
Blackwall
Limehouse
Shadwell
Wapping
Rotherhithe
Canada Water
Bermondsey
Surrey Quays
New Cross Gate
New Cross
West India Quay
Canary Wharf
Heron Quays
South Quay
Crossharbour & London Arena
Mudchute
Island Gardens
Cutty Sark
Greenwich
Deptford Bridge
Elverson Road
Lewisham
North Greenwich
Canning Town
Royal Victoria
Custom House
Prince Regent
Royal Albert
Beckton Park
Cyprus
Gallions Reach
Beckton
Silvertown & London City Airport
North Woolwich
West Ham
Plaistow
Upton Park
East Ham
Barking
Upney
Becontree
Dagenham Heathway
Dagenham East
Elm Park
Hornchurch
Upminster Bridge
Upminster
UNDERGROUND
London Travel Information 0171-222 1234 24 hours
Minicom 0171-918 3015
© London Regional Transport
Key to lines
Bakerloo
Central
peak hours only
Circle
District
East London
peak hours and Sunday mornings
Hammersmith & City
Jubilee
under construction
Metropolitan
peak hours only
Northern
Piccadilly
Victoria
Waterloo & City
Docklands Light Railway
under construction
National Railways
restricted service
Interchange stations
Connections with National Railways
Connections with National Railways within walking distance
Airport interchange
Closed Sundays
Closed Saturdays and Sundays
Served by Piccadilly line trains early morning and late evening
For opening times see poster journey planners.
Certain stations are closed on public holidays.


Amersham
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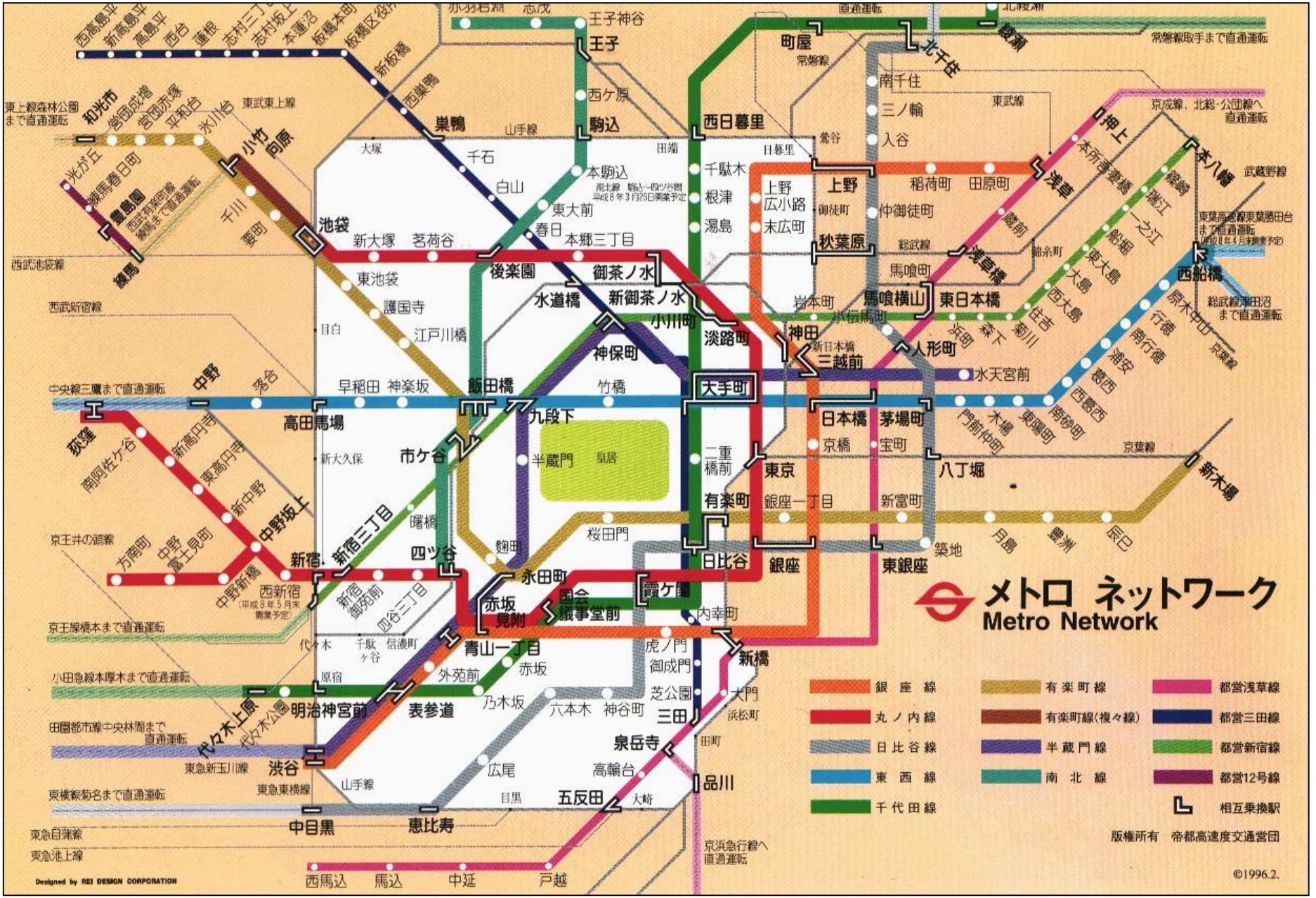
Chalfont&Latimer
Chorleywood
Rickmansworth
Watford
Croxley
MoorPark
Northwood
NorthwoodHills
Pinner
NorthHarrow
Harrow-on-the-Hill
WestHarrow
Uxbridge
Hillingdon
Ickenham
Ruislip
RuislipManor
Eastcote
RaynersLane
SouthHarrow
SudburyHill
SudburyTown
Alperton
ParkRoyal
NorthEaling
EalingCommon
HeathrowTerminal4
HeathrowTerminals1-2-3
HattonCross
HounslowWest
HounslowCentral
HounslowEast
Osterley
BostonManor
Northfields
SouthEaling
ActonTown
ChiswickPark
NorthwickPark
PrestonRoad
Stanmore
CanonsPark
Queensbury
Kingsbury
WembleyPark
Neasden
DollisHill
WillesdenGreen
Kilburn
WestHampstead
FinchleyRoad
SwissCottage
StJohn'sWood
BakerStreet
Marylebone
EdgwareRoad
EalingBroadway
WestActon
TurnhamGreen
Gunnersbury
KewGardens
Richmond
StamfordBrook
RavenscourtPark
Hammersmith
GoldhawkRoad
RoyalOak
WestbournePark
LadbrokeGrove
LatimerRoad
EastActon
WhiteCity
Shepherd'sBush
HollandPark
Bayswater
NottingHillGate
Queensway
LancasterGate
MarbleArch
BondStreet
OxfordCircus
Regent'sPark
GreatPortlandStreet
EustonSquare
WarrenStreet
GoodgeStreet
TottenhamCourtRoad
HighStreetKensington
BaronsCourt
WestKensington
Earl'sCourt
GloucesterRoad
SouthKensington
Knightsbridge
HydeParkCorner
GreenPark
PiccadillyCircus
LeicesterSquare
CoventGarden
CharingCross
Kensington(Olympia)
WalthamstowCentral
BlackhorseRoad
TottenhamHale
SevenSisters
FinsburyPark
Arsenal
Highbury&Islington
HollowayRoad
CaledonianRoad
Cockfosters
Oakwood
Southgate
ArnosGrove
BoundsGreen
WoodGreen
TurnpikeLane
ManorHouse
HighBarnet
Totteridge&Whetstone
WoodsidePark
WestFinchley
MillHillEast
FinchleyCentral
EastFinchley
Highgate
Archway
TufnellPark
KentishTown
CamdenTown
MorningtonCrescent
Euston
King'sCrossStPancras
RussellSquare
Farringdon
Angel
OldStreet
Barbican
Holborn
ChanceryLane
StPaul's
Moorgate
LiverpoolStreet
Bank
Aldgate
AldgateEast
TowerHill
Monument
CannonStreet
MansionHouse
Blackfriars
Temple
Embankment
Westminster
StJames'sPark
Victoria
SloaneSquare
Pimlico
Vauxhall
Waterloo
Southwark
LondonBridge
Borough
LambethNorth
Elephant&Castle
Kennington
Oval
Brixton
BethnalGreen
MileEnd
StepneyGreen
BowRoad
Bromley-by-Bow
WestHam
CanningTown
Whitechapel
Shadwell
Wapping
Rotherhithe
Bermondsey
Shoreditch
NorthGreenwich
CanaryWharf
CanadaWater
SurreyQuays
NewCross
NewCrossGate
Stratford
Leyton
Leytonstone
Wanstead
Redbridge
GantsHill
NewburyPark
Barkingside
Fairlop
Hainault
GrangeHill
Chigwell
RodingValley
Woodford
BuckhurstHill
Loughton
Debden
TheydonBois
Epping
Snaresbrook
SouthWoodford
Plaistow
UptonPark
EastHam
Barking
Upney
Becontree
DagenhamHeathway
DagenhamEast
ElmPark
Hornchurch
UpminsterBridge
Upminster


メトロ ネットワーク
Metro Network
銀座線
丸ノ内線
日比谷線
東西線
千代田線
有楽町線
有楽町線(複々線)
半蔵門線
南北線
都営浅草線
都営三田線
都営新宿線
都営12号線
相互乗換駅
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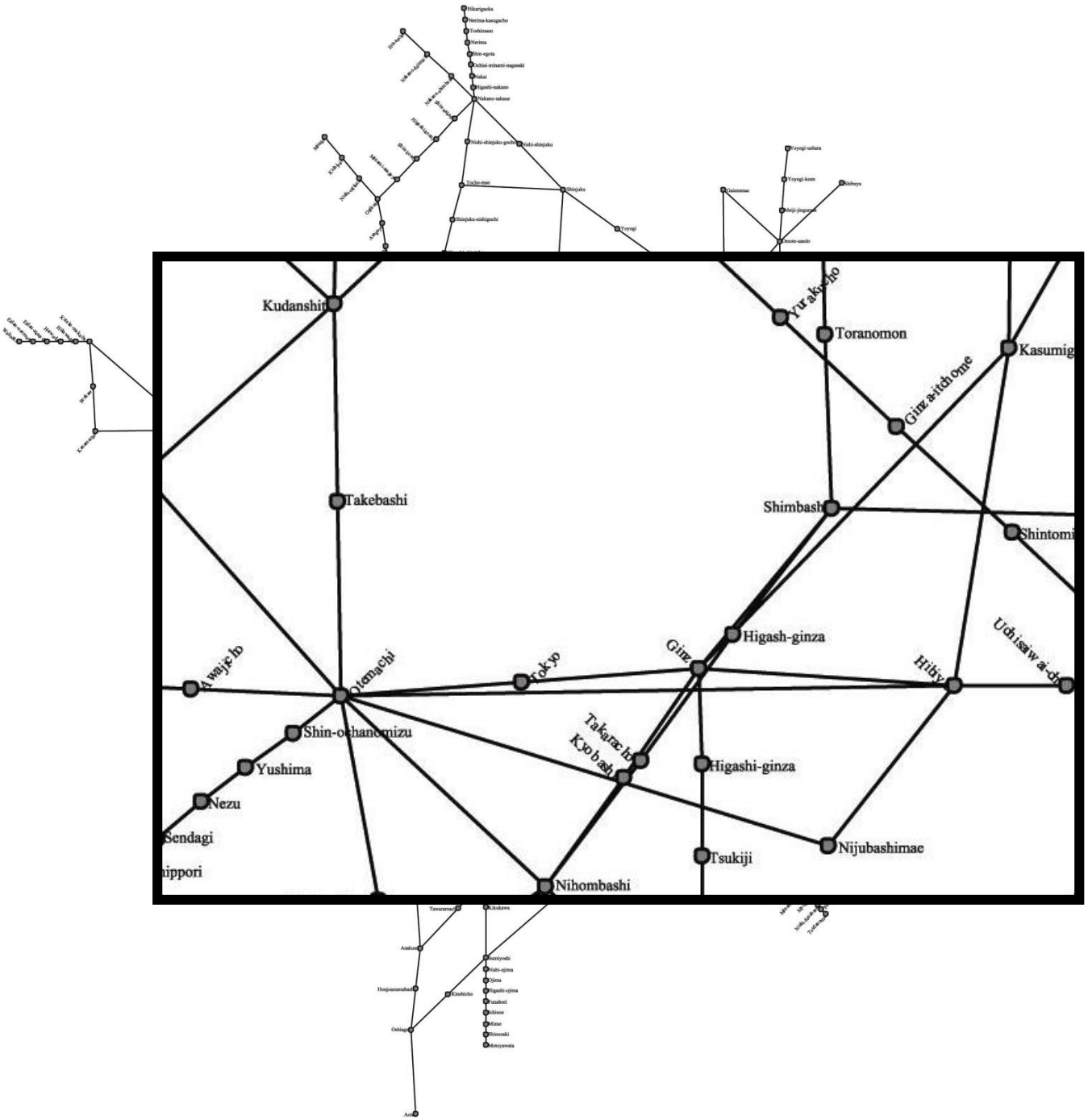
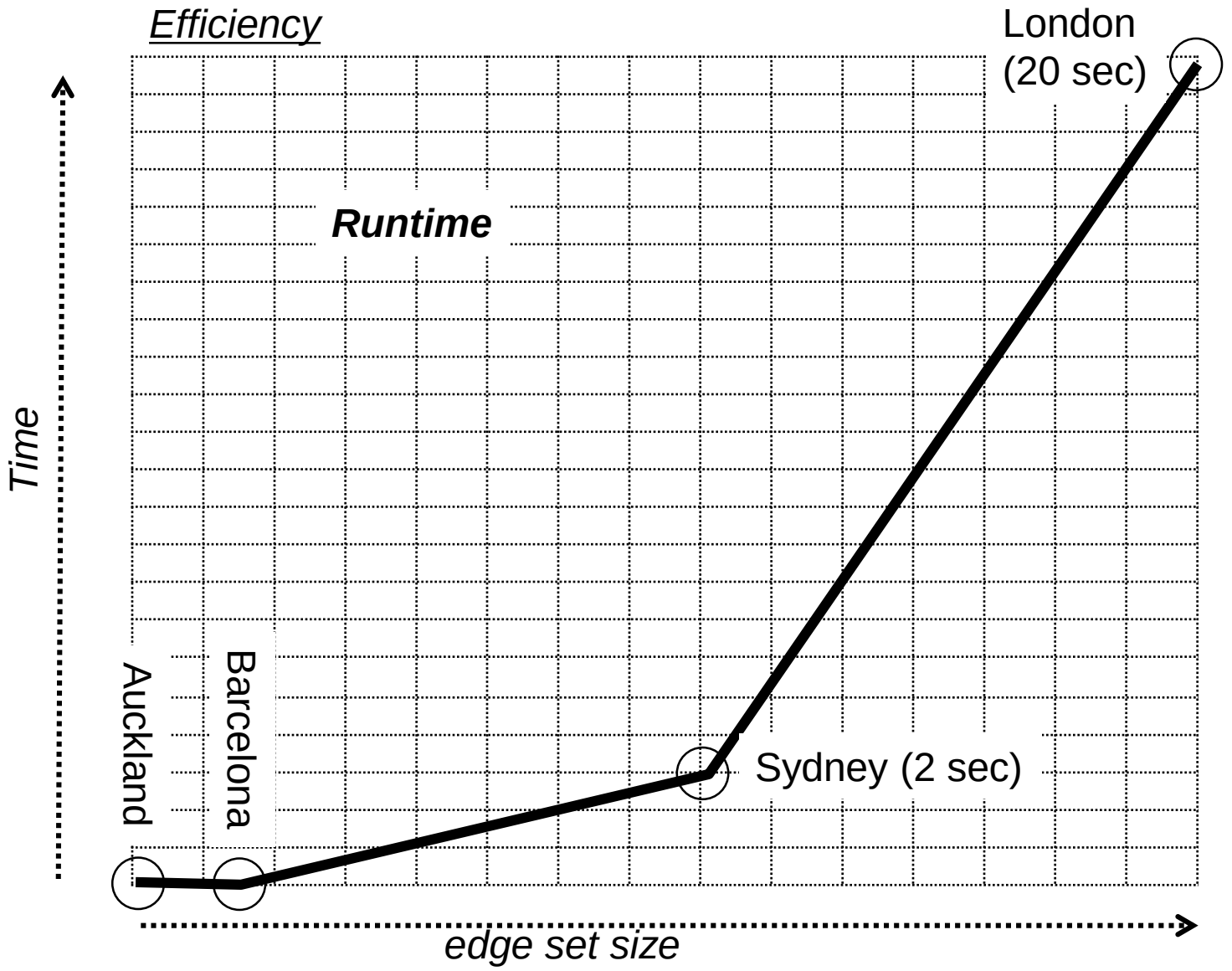
王子
王子神谷
赤羽岩淵
志茂
新板橋
板橋本町
本蓮沼
志村坂上
志村三丁目
西高島平
新高島平
高島平
西台
蓮根
巣鴨
西巣鴨
駒込
西ケ原
大塚
千石
白山
本駒込
東大前
春日
本郷三丁目
池袋
新大塚
茗荷谷
後楽園
東池袋
護国寺
江戸川橋
目白
水道橋
御茶ノ水
新御茶ノ水
小川町
淡路町
神保町
飯田橋
早稲田
神楽坂
竹橋
九段下
高田馬場
中野
落合
中央線三鷹まで直通運転
市ケ谷
半蔵門
皇居
新大久保
大手町
二重橋前
東京
有楽町
銀座一丁目
日比谷
銀座
東銀座
築地
新富町
京橋
日本橋
茅場町
宝町
八丁堀
水天宮前
人形町
三越前
神田
新日本橋
岩本町
馬喰横山
東日本橋
浜町
森下
菊川
住吉
馬喰町
秋葉原
総武線
上野
広小路
御徒町
仲御徒町
稲荷町
田原町
浅草
本所吾妻橋
蔵前
浅草橋
押上
本八幡
千駄木
根津
湯島
上野
日暮里
荒川
入谷
南千住
東武線
北千住
常磐線
常磐線取手まで直通運転
町屋
京成線、北総・公団線へ直通運転
武蔵野線
東葉高速線東葉勝田台まで直通運転
(平成8年4月末開業予定)
総武線津田沼まで直通運転
西船橋
船堀
一之江
瑞江
篠崎
葛西
西大島
東大島
大島
住吉
行徳
南行徳
浦安
原木中山
京葉線
門前仲町
木場
東陽町
南砂町
西葛西
新木場
辰巳
豊洲
月島
高輪台
泉岳寺
三田
芝公園
大門
浜松町
田町
品川
京浜急行線へ直通運転
虎ノ門
御成門
内幸町
新橋
青山一丁目
赤坂
外苑前
乃木坂
六本木
神谷町
原宿
表参道
明治神宮前
代々木上原
代々木公園
代々木
千駄ヶ谷
信濃町
四谷三丁目
新宿御苑前
国会議事堂前
霞ケ関
赤坂見附
永田町
四ツ谷
麹町
桜田門
新宿三丁目
新宿
西新宿
(平成8年5月末開業予定)
中野新橋
中野坂上
曙橋
方南町
中野富士見町
京王井の頭線
京王線橋本まで直通運転
小田急線本厚木まで直通運転
田園都市線中央林間まで直通運転
東急新玉川線
渋谷
東急東横線
東横線菊名まで直通運転
東急目蒲線
東急池上線
中目黒
恵比寿
広尾
目黒
五反田
大崎
山手線
西武池袋線
豊島園
練馬
練馬春日町
光が丘
西武有楽町線練馬まで直通運転
小竹向原
千川
要町
和光市
東上線森林公園まで直通運転
東武東上線
営団成増
営団赤塚
平和台
氷川台
中野
落合
高田馬場
新高円寺
東高円寺
新中野
南阿佐ケ谷
荻窪
中央線三鷹まで直通運転
西武新宿線
西武池袋線
西馬込
馬込
中延
戸越


Kudanshita
Takebashi
Yurakucho
Toranomon
Kasumiga
Ginza-itchome
Shimbashi
Shintomi
Higash-ginza
Ginz
Hibiya
Uchisaiwai-cho
Otemachi
Tokyo
Takarac-ho
Kyobash
Higashi-ginza
Shin-ochanomizu
Yushima
Nezu
Sendagi
Nijubashimae
Tsukiji
Nihombashi

Effectiveness

- The force directed method is OK for the metro map problem
- but not wonderfully effective.

Efficiency


Efficiency
London
(20 sec)
Runtime
Time
Auckland
Barcelona
Sydney (2 sec)
edge set size

This energy based method is not computationally *efficient*.

Conclusion about metro maps:

- Simple force directed methods for metro maps "do not scale"
  - Runtime is too large
  - Some tangles in some maps

Elegance	Yes!
Effectiveness	Maybe
Efficiency	No

Poor performance when data size is large

4. FADE (Barnes-Hutt)

Problem: Simple spring methods are too slow for huge graphs

Computing the forces takes quadratic time

1. $\boldsymbol{p_u}$ = some initial position for each node $\boldsymbol{u}$;
2. Repeat
    - 2.1 $\boldsymbol{F_u}$ **:= 0** for each node $\boldsymbol{u}$;
    - 2.2 For each pair $\boldsymbol{u, v}$ of nodes
        - 2.2.1 calculate the force $\boldsymbol{f_{uv}}$ between $\boldsymbol{u}$ and $\boldsymbol{v}$;
        - 2.2.2 $\boldsymbol{F_u}$ += $\boldsymbol{f_{uv}}$;
        - 2.2.3 $\boldsymbol{F_v}$ += $\boldsymbol{f_{uv}}$;
    - 2.3 For each node $\boldsymbol{u}$, $\boldsymbol{p_u}$ **+=** $\varepsilon \boldsymbol{F_u}$;

   Until $\boldsymbol{p_u}$ converges for all $\boldsymbol{u}$;

Multilevel **approaches**

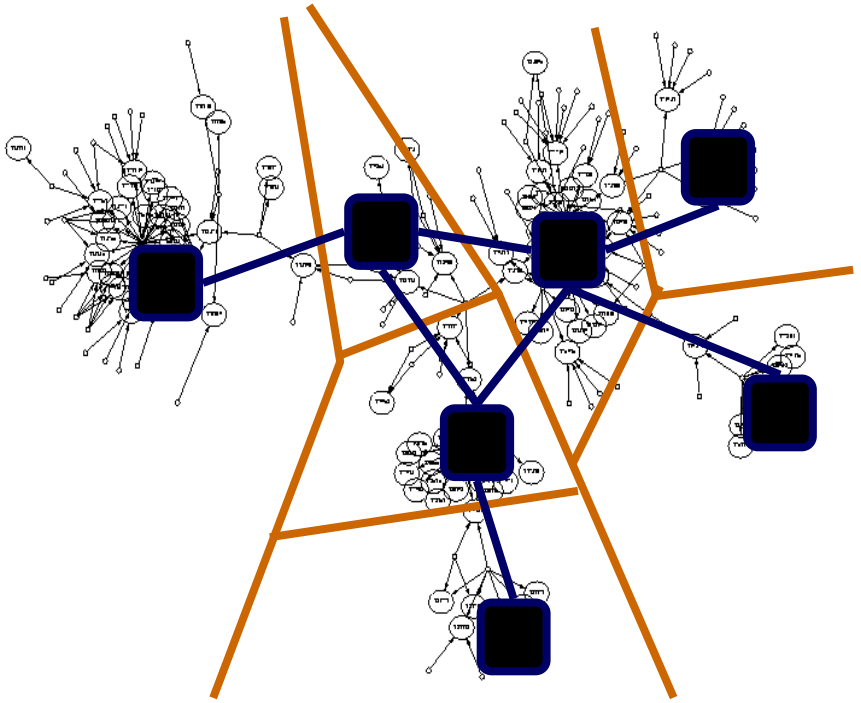
We can use

1. A spring method, and
2. A clustering method to give a cluster hierarchy for the graph

   - The clustering could be geometric or combinatorial

Use divide&conquer on the cluster hierarchy:

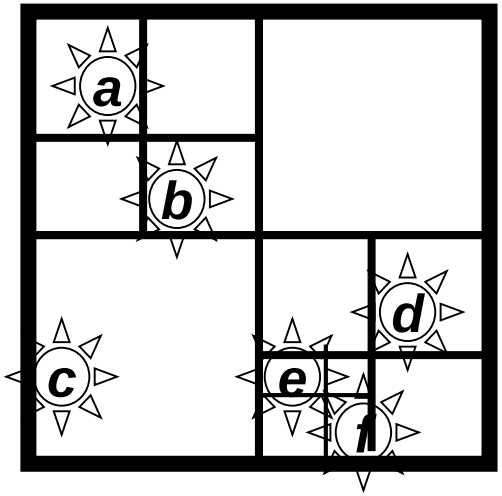
– First apply the spring method at a higher level in the hierarchy, then at a lower level

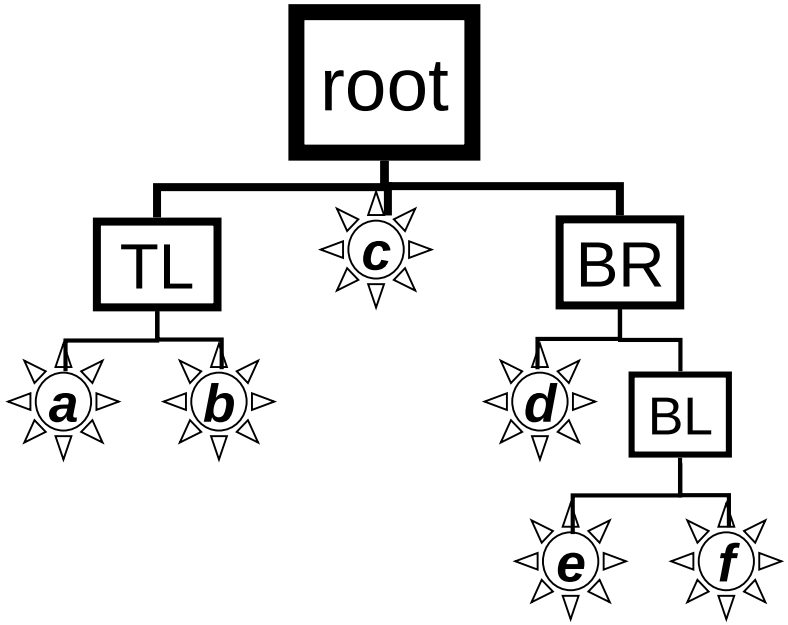


AT&T

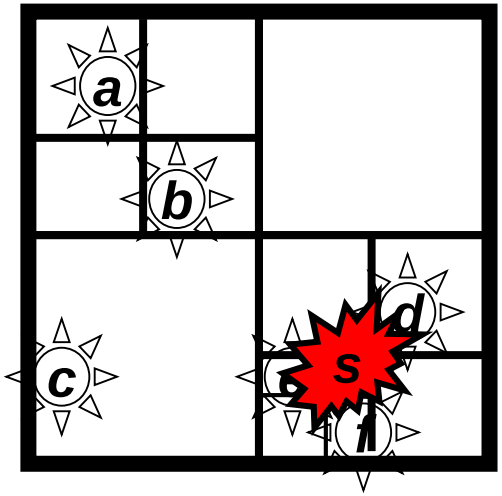
Barnes-Hutt method (1986)

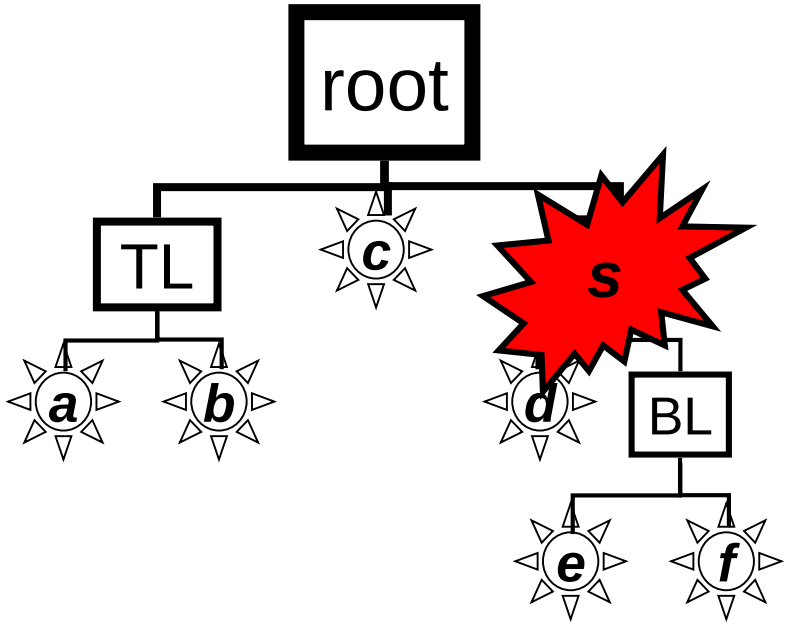
- A method of computing configurations of stars.
  - Use an octtree to cluster the stars
  - Use the forces between the clusters to approximate the forces between individual stars.



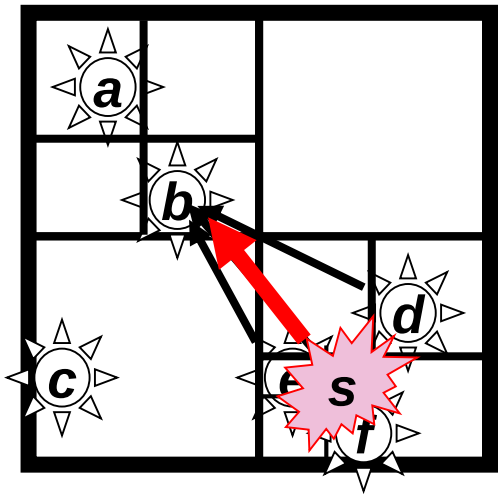


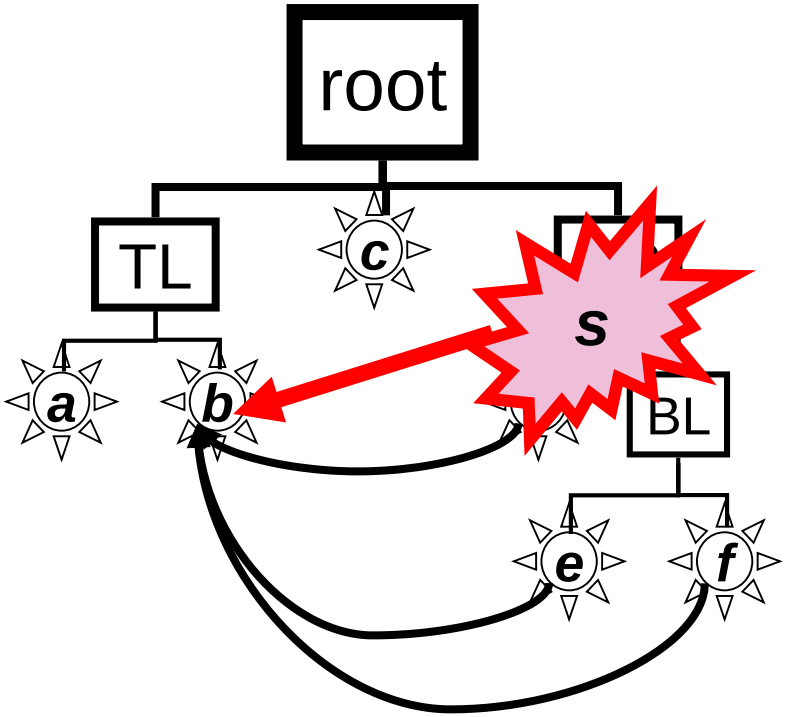
The contents of a subtree of can be approximated by a mass at the centroid.





The force that the subtree s exerts on the star b can approximate the sum of the forces that the nodes in s exert on b.





To compute the force on star $\boldsymbol{x}$, we proceed from the root toward the leaves.

***ComputeForce*(star x; treenode t)**
If the approximation is good
then return the approximation;
else return $\sum_s$ ***ComputeForce***(x, s), where the sum is over all children s of t.

A simple method can be used to determine whether the approximation is good; it depends on the *mass of nodes* and the *distance* between $\boldsymbol{x}$ and $\boldsymbol{s}$.

$$\frac{w(t)}{d(x,t)} < c,$$

where $w(t)$ is the width of t; $d(x, t)$ is the distance between x and t, and c is a constant.

The Barnes-Hutt method is much faster than the usual spring algorithm.

1. $\boldsymbol{p_x}$ = some initial position for each star $\boldsymbol{x}$;
2. Repeat
   - 2.1 Build the quadtree;
   - 2.2 Foreach star $\boldsymbol{x}$
     - **ComputeForce**($\boldsymbol{x}, \boldsymbol{root}$);
   - 2.3 Foreach star $\boldsymbol{x}$: $\boldsymbol{p_x}$ += $\boldsymbol{\varepsilon F_x}$;

   Until $\boldsymbol{p_x}$ converges for all $\boldsymbol{x}$;

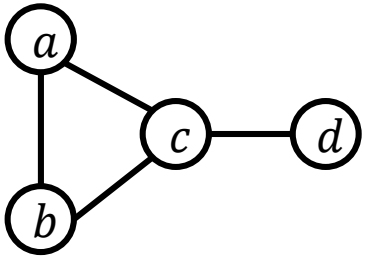
In practice, computing all the forces takes $\boldsymbol{O(n \log n)}$ *time*

FADE method for graph drawing:

1. ***Model***: Use springs and electrical forces

2. ***Algorithm***: Apply Barnes-Hutt to the graph

5. Spectral layout



Adjacency matrix

$\boldsymbol{M} =$

	a	b	c	d
a	0	1	1	0
b	1	0	1	0
c	1	1	0	1
d	0	0	1	0

Degree matrix

$\boldsymbol{D} =$

	a	b	c	d
a	2	0	0	0
b	0	2	0	0
c	0	0	3	0
d	0	0	0	1

Laplacian matrix

$\boldsymbol{L} = \boldsymbol{D} - \boldsymbol{M}$

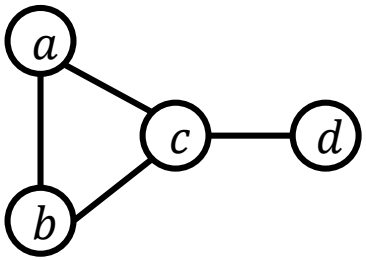
$\boldsymbol{L} =$

	a	b	c	d
a	2	-1	-1	0
b	-1	2	-1	0
c	-1	-1	3	-1
d	0	0	-1	1

The Laplacian is a nice matrix, and its *eigenvalues* are nice:

- All eigenvalues of $\boldsymbol{L}$ are real and non-negative.
- We put eigenvalues in non-decreasing order, so the eigenvalues of Laplacian $\boldsymbol{L}$ are $\boldsymbol{\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_n}$.
- The sequence $\boldsymbol{\lambda_1, \lambda_2, \ldots, \lambda_n}$ is the *spectrum* of $\boldsymbol{L}$ (and the graph $\boldsymbol{G}$).
- The spectrum of a graph defines many of its properties.
- The study of graphs and their spectra is *Spectral Graph Theory*

Note: all rowsums of a Laplacian L are zero.



$\boldsymbol{L} =$

	a	b	c	d
a	2	-1	-1	0
b	-1	2	-1	0
c	-1	-1	3	-1
d	0	0	-1	1

Thus $\boldsymbol{L\widetilde{1} = 0}$ where $\boldsymbol{\widetilde{1} = [1, 1, \ldots, 1]^T}$.

Thus $\boldsymbol{L\widetilde{1} = 0\widetilde{1}}$, that is $\boldsymbol{\lambda = 0}$ is and eigenvalue of $\boldsymbol{L}$ with corresponding eigenvector $\boldsymbol{\widetilde{1}}$.

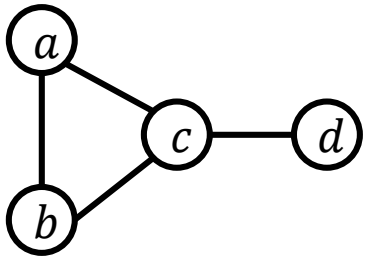
Spectral layout (the simplest form)

2. Algorithm

Input: a graph $\boldsymbol{G} = (\boldsymbol{V}, \boldsymbol{E})$ with $\boldsymbol{n}$ vertices.
Output: a location $\boldsymbol{p}(\boldsymbol{i}) \in \boldsymbol{R^2}$ for each vertex $\boldsymbol{i} \in \boldsymbol{V}$.

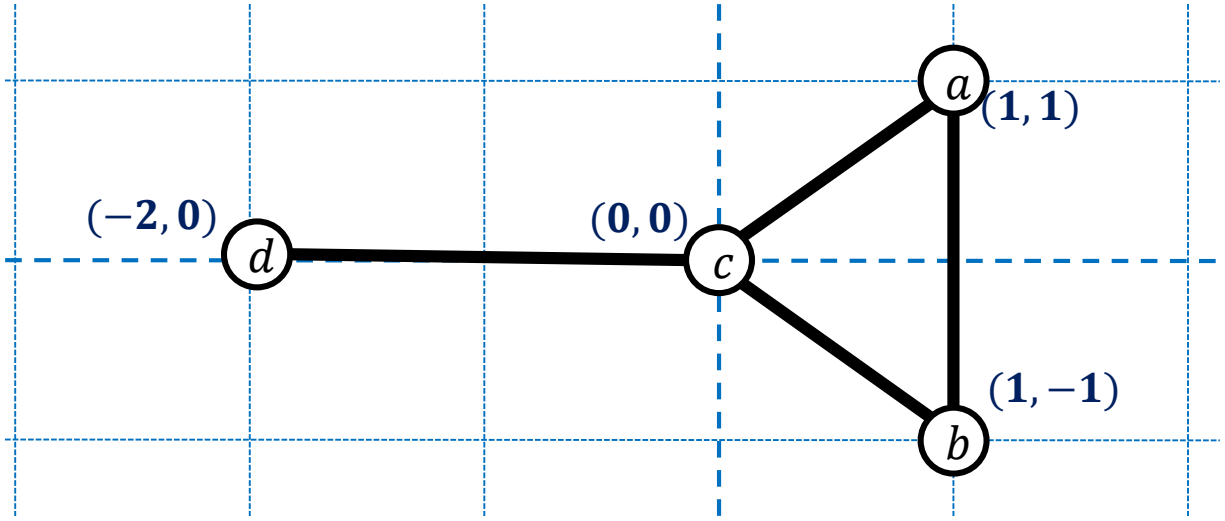
1. Compute the Laplacian $\boldsymbol{L}$ of $\boldsymbol{G}$.
2. Compute the first two nonzero eigenvalues $\boldsymbol{\lambda_2}, \boldsymbol{\lambda_3}$ of $\boldsymbol{L}$
3. Compute their corresponding eigenvectors
   $x_2 = [x_{21}, x_{22}, \dots, x_{2n}]^T$ and $x_3 = [x_{31}, x_{32}, \dots, x_{3n}]^T$
4. For each vertex $\boldsymbol{i} \in \boldsymbol{V}$, $\boldsymbol{p}(\boldsymbol{i}) = (\boldsymbol{x_{2i}}, \boldsymbol{x_{3i}})$.

Example:



$$\boldsymbol{L} = \begin{bmatrix} \mathbf{2} & \mathbf{-1} & \mathbf{-1} & \mathbf{0} \\ \mathbf{-1} & \mathbf{2} & \mathbf{-1} & \mathbf{0} \\ \mathbf{-1} & \mathbf{-1} & \mathbf{3} & \mathbf{-1} \\ \mathbf{0} & \mathbf{0} & \mathbf{-1} & \mathbf{1} \end{bmatrix}$$

- Eigenvalues of $\boldsymbol{L}$ are: $\boldsymbol{\lambda_1 = 0,\ \lambda_2 = 1,\ \lambda_3 = 3,\ \lambda_4 = 4}$.
- Eigenvectors of $\boldsymbol{L}$ are: $\boldsymbol{u_1} = \begin{bmatrix} \mathbf{1} \\ \mathbf{1} \\ \mathbf{1} \\ \mathbf{1} \end{bmatrix}, \boldsymbol{u_2} = \begin{bmatrix} \mathbf{1} \\ \mathbf{1} \\ \mathbf{0} \\ \mathbf{-2} \end{bmatrix}, \boldsymbol{u_3} = \begin{bmatrix} \mathbf{1} \\ \mathbf{-1} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}, \boldsymbol{u_4} = \begin{bmatrix} \mathbf{-1} \\ \mathbf{-1} \\ \mathbf{3} \\ \mathbf{-1} \end{bmatrix}$.
- Thus $\boldsymbol{p(a) = (1,1), p(b) = (1,-1), p(c) = (0,0), p(d) = (-2,0)}$.



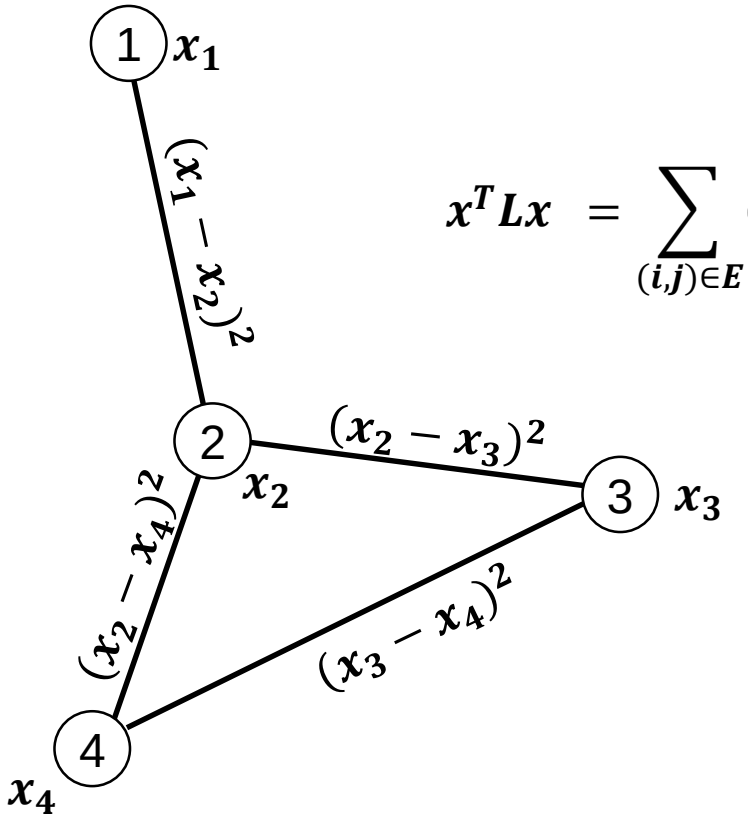
1. The model

- Spectral layout is a force-directed algorithm (sort-of)

Easy Lemma

If $\boldsymbol{L}$ is the Laplacian of a graph $\boldsymbol{G} = (\boldsymbol{V}, \boldsymbol{E})$, and $\boldsymbol{x}$ is a vector, then

$$\boldsymbol{x^T L x} = \sum_{(\boldsymbol{i},\boldsymbol{j}) \in \boldsymbol{E}} \left(\boldsymbol{x_i} - \boldsymbol{x_j}\right)^2.$$



$$\boldsymbol{x} = \begin{bmatrix} \boldsymbol{x_1} \\ \boldsymbol{x_2} \\ \boldsymbol{x_3} \\ \boldsymbol{x_4} \end{bmatrix}$$

$$\boldsymbol{x^T L x} = \sum_{(\boldsymbol{i},\boldsymbol{j}) \in \boldsymbol{E}} \left(\boldsymbol{x_i} - \boldsymbol{x_j}\right)^2$$

A kind of 1-dimensional energy function

Theorem (from the Courant–Fischer–Weyl min-max principle)

Suppose that $\boldsymbol{L}$ is the Laplacian of a graph $\boldsymbol{G} = (\boldsymbol{V}, \boldsymbol{E})$, and the eigenvalues of $\boldsymbol{L}$ are $\boldsymbol{0} = \boldsymbol{\lambda_1} \leq \boldsymbol{\lambda_2} \leq \cdots \leq \boldsymbol{\lambda_n}$, and the eigenvector $\boldsymbol{u_k}$ corresponds to $\boldsymbol{\lambda_k}$. Then

$$\boldsymbol{\lambda_k} = \min_{\boldsymbol{x} \in \boldsymbol{U_k}} \left(\boldsymbol{x^T L x} \right) \quad = \quad \min_{\boldsymbol{x} \in \boldsymbol{U_k}} \sum_{(\boldsymbol{i},\boldsymbol{j}) \in \boldsymbol{E}} \left(\boldsymbol{x_i} - \boldsymbol{x_j} \right)^2$$

where $\boldsymbol{U_k}$ is the set all unit vectors orthogonal to the space spanned by $\{\boldsymbol{u_1}, \boldsymbol{u_2}, \ldots, \boldsymbol{u_{k-1}}\}$. The minimum value is achieved when $\boldsymbol{x} = \boldsymbol{u_k}$.

What all this means :

- Eigenvalues are minimum values of the quadratic function $\boldsymbol{x^T L x}$, under two constraints:
  - Unit sphere constraint
  - Orthogonal to eigenvectors of smaller eigenvalues
- The function $\boldsymbol{x^T L x}$ is a kind of energy
- These minima occur at eigenvectors.

Theorem (from the Courant–Fischer–Weyl min-max principle)

Suppose that $\boldsymbol{L}$ is the Laplacian of a graph $\boldsymbol{G} = (\boldsymbol{V}, \boldsymbol{E})$, and the eigenvalues of $\boldsymbol{L}$ are $\boldsymbol{0} = \boldsymbol{\lambda_1} \leq \boldsymbol{\lambda_2} \leq \cdots \leq \boldsymbol{\lambda_n}$, and the eigenvector $\boldsymbol{u_k}$ corresponds to $\boldsymbol{\lambda_k}$. Then

$$\boldsymbol{\lambda_k} = \min_{\boldsymbol{x} \in \boldsymbol{U_k}} \left(\boldsymbol{x^T L x} \right) \quad = \quad \min_{\boldsymbol{x} \in \boldsymbol{U_k}} \sum_{(\boldsymbol{i},\boldsymbol{j}) \in \boldsymbol{E}} \left(\boldsymbol{x_i} - \boldsymbol{x_j} \right)^2$$

where $\boldsymbol{U_k}$ is the set all unit vectors orthogonal to the space spanned by $\{\boldsymbol{u_1}, \boldsymbol{u_2}, \ldots, \boldsymbol{u_{k-1}}\}$. The minimum value is achieved when $\boldsymbol{x} = \boldsymbol{u_k}$.

What all this means for a Laplacian L:

- Smallest eigenvalue: $\boldsymbol{\lambda_1} = \boldsymbol{0}$ with eigenvector $\boldsymbol{u_1} = [\boldsymbol{1}, \boldsymbol{1}, \ldots, \boldsymbol{1}]^{\boldsymbol{T}}$
- Next smallest eigenvalue: $\boldsymbol{\lambda_2} = \min_{\boldsymbol{x} \in \boldsymbol{U_2}} \sum_{(\boldsymbol{i},\boldsymbol{j}) \in \boldsymbol{E}} \left(\boldsymbol{x_i} - \boldsymbol{x_j} \right)^2$
  - The minimum is over all unit vectors orthogonal to $\boldsymbol{u_1} = [\boldsymbol{1}, \boldsymbol{1}, \ldots, \boldsymbol{1}]^{\boldsymbol{T}}$
  - The minimum is at $\boldsymbol{x} = \boldsymbol{u_2}$, the second eigenvector
- Next smallest eigenvalue: $\boldsymbol{\lambda_3} = \min_{\boldsymbol{x} \in \boldsymbol{U_3}} \sum_{(\boldsymbol{i},\boldsymbol{j}) \in \boldsymbol{E}} \left(\boldsymbol{x_i} - \boldsymbol{x_j} \right)^2$
  - The minimum is over all unit vectors orthogonal to $\boldsymbol{u_1} = [\boldsymbol{1}, \boldsymbol{1}, \ldots, \boldsymbol{1}]^{\boldsymbol{T}}$ and $\boldsymbol{u_2}$
  - The minimum is at $\boldsymbol{x} = \boldsymbol{u_3}$, the third eigenvector

Theorem (from the Courant–Fischer–Weyl min-max principle)

Suppose that $\boldsymbol{L}$ is the Laplacian of a graph $\boldsymbol{G} = (\boldsymbol{V}, \boldsymbol{E})$, and the eigenvalues of $\boldsymbol{L}$ are $\boldsymbol{0} = \boldsymbol{\lambda_1} \leq \boldsymbol{\lambda_2} \leq \cdots \leq \boldsymbol{\lambda_n}$, and the eigenvector $\boldsymbol{u_k}$ corresponds to $\boldsymbol{\lambda_k}$. Then

$$\boldsymbol{\lambda_k} = \min_{\boldsymbol{x} \in \boldsymbol{U_k}} \left(\boldsymbol{x^T L x} \right) \quad = \quad \min_{\boldsymbol{x} \in \boldsymbol{U_k}} \sum_{(\boldsymbol{i},\boldsymbol{j}) \in \boldsymbol{E}} \left(\boldsymbol{x_i} - \boldsymbol{x_j} \right)^2$$

where $\boldsymbol{U_k}$ is the set all unit vectors orthogonal to the space spanned by $\{\boldsymbol{u_1}, \boldsymbol{u_2}, \ldots, \boldsymbol{u_{k-1}}\}$. The minimum value is achieved when $\boldsymbol{x} = \boldsymbol{u_k}$.

What all this means for a Laplacian L:

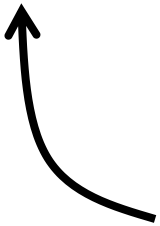
- Second smallest eigenvalue $\boldsymbol{\lambda_2}$ minimizes a kind of energy function
- Third smallest eigenvalue: $\boldsymbol{\lambda_3}$ minimizes a kind of energy function

Theorem (from the Courant–Fischer–Weyl min-max principle)

Suppose that $\boldsymbol{L}$ is the Laplacian of a graph $\boldsymbol{G} = (\boldsymbol{V}, \boldsymbol{E})$, and the eigenvalues of $\boldsymbol{L}$ are $\boldsymbol{0} = \boldsymbol{\lambda_1} \leq \boldsymbol{\lambda_2} \leq \cdots \leq \boldsymbol{\lambda_n}$, and the eigenvector $\boldsymbol{u_k}$ corresponds to $\boldsymbol{\lambda_k}$. Then

$$\boldsymbol{\lambda_k} = \min_{\boldsymbol{x} \in \boldsymbol{U_k}} \left(\boldsymbol{x^T L x} \right) \quad = \quad \min_{\boldsymbol{x} \in \boldsymbol{U_k}} \sum_{(\boldsymbol{i},\boldsymbol{j}) \in \boldsymbol{E}} \left(\boldsymbol{x_i} - \boldsymbol{x_j} \right)^2$$

where $\boldsymbol{U_k}$ is the set all unit vectors orthogonal to the space spanned by $\{\boldsymbol{u_1}, \boldsymbol{u_2}, \ldots, \boldsymbol{u_{k-1}}\}$. The minimum value is achieved when $\boldsymbol{x} = \boldsymbol{u_k}$.



Thus (roughly speaking) the eigenvectors achieve a constrained minimum energy in the edges

Remark:

There is mathematical evidence that spectral layout is good (in theory)

There is some mathematical evidence that spectral layout is good (in theory):

Spectral layout

- Should give faithful clusters
  - The layout gives a relaxation of the ratio-cut problem
- Should give faithful commute distances
  - Random walks are preserved

But in practice the simple spectral layout is not as good as it is in theory.

Spectral layout

Elegance:

- Easy to understand
- Can be coded very quickly using a numerical package, even if you don't understand eigenvalues

Efficiency

- As fast as the best algorithm that you can find to compute the first two nonzero eigenvalues of a Laplacian.
- Scales very well.

Effectiveness

- On real-world graphs it is not so good.
- It works well on mesh-like graphs.

Summary: Energy based methods

Advantages:

- Elegant: Many methods are relatively easy to understand and easy to code
- Good flexibility; supports some constraints
- Heuristic improvements easily added
- Smooth evolution of the drawing into the final configuration helps preserving the user's mental map
- Can be easily extended to 3D
- Often able to display symmetries
- Results are OK

Disadvantages

- High-quality results need high runtime.
- Results are OK but not brilliant.
- Few guarantees on the quality of the drawings produced.
- Difficult to extend to orthogonal & polyline drawings.

Announcement

- Assignments 1 & 2: will be posted on Week 4

- Group assignment for Assignment 2
  - Week 4: Form a group (upto 6 students)
  - Week 5: Final assignment