Part a: Exponential growth model

In this tutorial we will numerically simulate a discrete exponential growth model. In this model the state of the system at t is proportional to its state at (t - 1). The equation for this map is

$x_t = a * x_{t-1}$,

where a is a parameter which controls the rate of the growth.
If a > 0, then the equation describes exponential growth and if a < 1 then the equation describes exponential decay. This model is relevant in many situations e.g. population growth, radioactive decay etc.

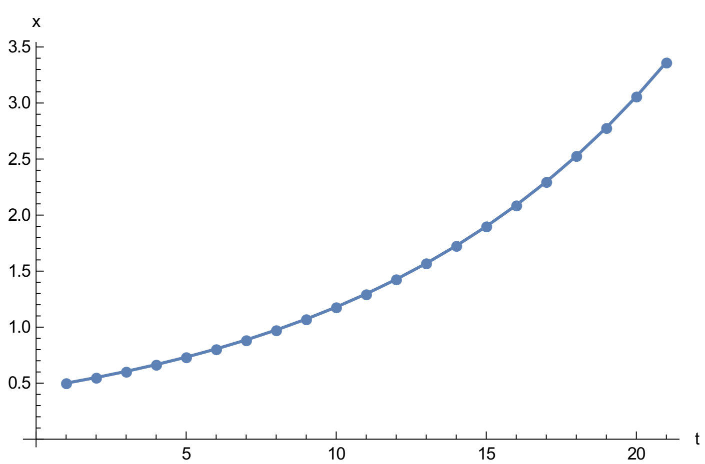
The following code calculates a time series for this model with initial condition $x_0 = 1$, and a = 1.1. The time series is recorded for 20 steps.

```
xresults = {}; (*to store the time series*)
a = 1.1;
(*initialization*)
x = 0.5;
AppendTo[xresults, x];

(*update loop*)
Do[x = a * x; (*update the state value*)
  AppendTo[xresults, x]; (*record the state value*)
  , {20}];
```

Plot the time series

```
ListPlot[xresults, AxesLabel → {"t", "x"}, Joined → True, PlotMarkers → Automatic]
```



Exercise: Try changing the parameters. For example change the value of a to 0.9. What happens when a = 1? Try computing time series with the following update rules

$x_t = 1.1 * x_{t-1} + 1.0$

Part b: Simulating discrete dynamical system with multiple variables

The state in the exponential growth model was given by single real number x. In the following example we consider a map where a state is given by two real numbers (x, y).

The update rules will be given by the following equaitons

$x_t = 0.5\, x_{t-1} + y_{t-1}$

$y_t = -0.5\, x_{t-1} + y_{t-1}$

The initial state will be

$x_0 = 1, \; y_0 = 1$

The following code calculates a time series for this model. The time series is recorded for 20 steps.

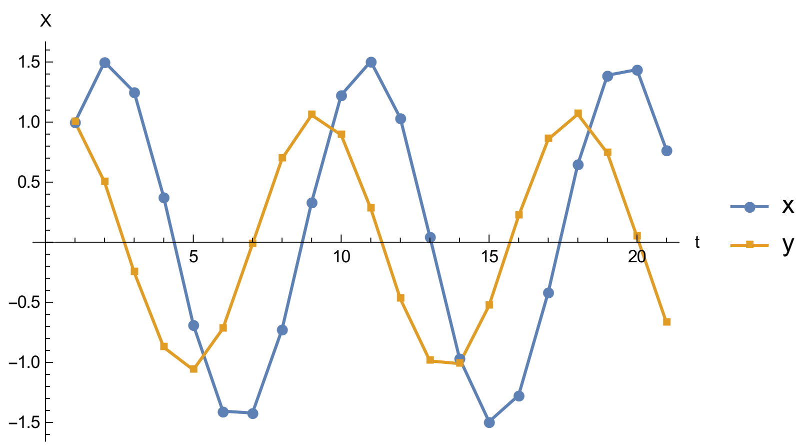
```
results = {};
(*initialize*)
x = 1;
y = 1;

AppendTo[results, {x, y}];

(*update loop*)
Do[xnew = 0.5 * x + y;
 ynew = -0.5 * x + y;
 x = xnew;
 y = ynew;
 AppendTo[results, {x, y}]; (*record the time series data*)
 , {20}]
```

Plot the time series

```
ListPlot[{results[[All, 1]], results[[All, 2]]}, Joined → True,
 PlotMarkers → Automatic, AxesLabel → {"t", "X"}, PlotLegends → {"x", "y"}]
```



The same data also be represented in the phase plot diagram

```
ListPlot[results, Joined → True, PlotMarkers → Automatic,
 Frame → True, PlotRange → {{-2, 2}, {-1.5, 1.5}}, FrameLabel → {"x", "y"}]
```