Homework 2

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5.2

Let s_A and s_B be the Sharpe ratio of portfolios A and B. Let r_A and r_B be the expected returns of these two portfolios, with standard deviation denoted by σ_A and σ_B. Assume that through self financing, portfolio A borrows $(\sigma_B/\sigma_A - 1)$ at risk-free rate r_f to leverage so that its risk is now the same as that of portfolio B.

Show that the excess return of leveraged investment in portfolio A is larger than the expected return of portfolio B if $s_A > s_B$. This shows that the Sharpe ratio measures the efficiency of a portfolio.

5.3

Suppose that three mutual funds (conservative, growth and aggressive) have annual log-returns of 15%, 20% and 30% with volatility of 20%, 30% and 50% respectively. The correlation between any of the 2 funds is 0 and the risk-free rate is 5%.

```
rf <- .05
vol <- c(.20, .30, .50)
r <- c(.15, .20, .30)
expected_return <- .15
Y <- r - rf # excess returns
gamma <- diag(x = 1, nrow = 3)
vol <- diag(vol, nrow = 3)
Sigma <- vol %*% gamma %*% vol
partial_alpha <- as.vector(solve(Sigma) %*% Y )
A <- sum(partial_alpha * Y)/(expected_return - rf)
a.est <- 1/A*solve(Sigma) %*% Y
```

1. What is the min variance portfolio with these 3 mutual funds?
2. Find the optimal portfolio allocation among the 3 mutual funds, if the expected return is set at 15%. Give the associated standard deviation of this portfolio.
3. Compute the Sharpe ratio for the portfolio in A. How does it compare with that in B?

```
## Our risk aversion parameter A is:  7.5

## Our optimal portfolio allocation is:  0.3333333 0.2222222 0.1333333
##  for the conservative, growth and aggressive funds, respectively

## Our min variance is given by:  0.01333333
```

5.10

Let $\boldsymbol{y}$ be the excess returns of risky assets. Let $X = \boldsymbol{\alpha}^T \boldsymbol{y}$ be a portfolio with allocation vector $\boldsymbol{\alpha}$. Denote by $\Sigma = \text{var}(\boldsymbol{y})$ and $\mu = \mathbb{E}(\boldsymbol{y})$. Consider the following decomposition:

$$\boldsymbol{y} = \alpha + \beta X + \epsilon \quad \mathbb{E}(\epsilon) = 0 \quad \text{cov}(\epsilon, X) = 0$$

1. Show that if $\boldsymbol{\alpha} = c\Sigma^{-1}\mu$ then $\alpha = 0$. 2. Conversely if $\alpha = 0$, there exists a constant c such that $\boldsymbol{\alpha} = c\Sigma^{-1}\mu_0$

5.13

Consider the following portfolio optimization problem with a risk-free asset having return r_0:

$$\min \boldsymbol{\alpha}^T \Sigma \boldsymbol{\alpha} \quad \text{such that} \quad \boldsymbol{\alpha}^T \boldsymbol{\mu} + (1 - \boldsymbol{\alpha}^T \mathbf{1}) r_0 = \mu$$

1. The optimal solution is $\boldsymbol{\alpha} = P^{-1}(\mu - r_0)\Sigma^{-1}\mu_0$ where $P = \boldsymbol{\mu}_0^T \Sigma^{-1} \boldsymbol{\mu}_0^T$ is the squared Sharpe ratio, and $\boldsymbol{\mu}_0 = \boldsymbol{\mu} - r_0\mathbf{1}$ is the vector of excess returns. 2. The variance of this portfolio is $\sigma^2 = (\mu - r_0)^2/P$. 3. When $r_0 < \mu$, show that $r_0 + P^{1/2}\sigma = \mu$, namely the optimal allocation for the risky asset $\boldsymbol{\alpha}$ is the tangent portfolio.

5.14

1. Download the monthly data for 8 stocks.

```r
library(quantmod)
start <- as.Date("2001-01-01")
end <- as.Date("2014-12-29")
getSymbols("DGS3MO", src = "FRED", from = start, to = end) # TREASURY
symbols <- c("SPY",
             #"DVMT", # DELL DOES NOT WORK
             "F",  # FORD
             "GE",
             "IBM",
             "INTC", # INTEL
             "JNJ", # JOHNSON
             "MRK", # MERCK
             "MSFT") # MICROSOFT
getSymbols(symbols, from = start, to = end)
closing.prices <- merge.xts(DGS3MO,
                            SPY[,4],
                            F[,4],
                            GE[,4],
                            IBM[,4],
                            INTC[,4],
                            JNJ[,4],
                            MRK[,4],
                            MSFT[,4])
saveRDS(closing.prices, "~/workspace/st790-financial-stats/data/hw2_closingprices.rds")
```

2. Construct the optimal allocation using the monthly data.

```
closing.prices <- readRDS("~/workspace/st790-financial-stats/data/hw2_closingprices.rds")
data <- closing.prices["2001-01-02/2011-12-31"]
data <- na.omit(data)
by_month <- data.frame()
for (i in 1:ncol(data)){
  temp <- data[,i]
  monthly <- log(monthlyReturn(temp)+1)
  by_month <- cbind(by_month, monthly )
  colnames(by_month)[i] <- strsplit(names(temp), "[.]")[[1]][1]
}
# get the excess return
excess_return <- data.frame()
for(j in 3:ncol(by_month)){
  temp <- by_month[,j] - by_month[,1]
  excess_return <- cbind(excess_return, temp)
  colnames(excess_return)[j-2] <- names(temp)
}
# get the alphas
alphas <- c()
betas <- c()
for(k in 1:ncol(excess_return)){
  results <- lsfit(by_month[,2], excess_return[,k])$coefficients
  alphas <- c(alphas, results[1])
  betas <- c(betas, results[2])
}
#
```