# Rebalancing Risk

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# Introduction

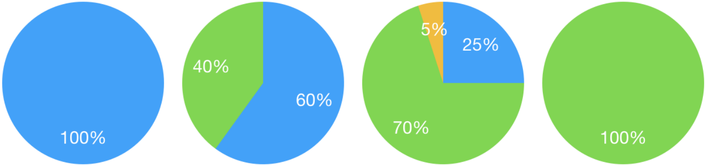



Figure 1: Typical IRA equity-bond mix

# How rebalancing works

- Start with buy-and-hold portfolio
  - 60% allocation to S&P500
  - 40% allocation to 10-year Treasury bond
- Add a short straddle (sell a call and put option)
- Works in a trendless environment, marked by mean reversion

# The problem

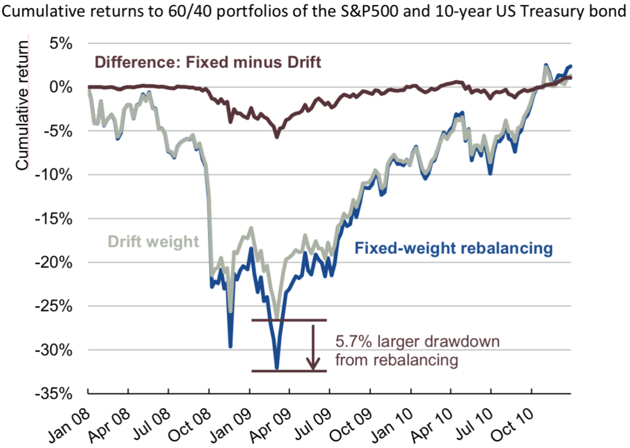



Figure 2: Rebalancing magnified drawdowns. Source: Reuters.

# Fixed-weight rebalance of one asset and cash

If we model a single risky asset $P_t$ as follows:

$$\frac{dP_t}{P_t} = \sigma dW_t$$

$$P_t = P_0 \exp\left(\sigma W_t - \frac{1}{2}\sigma^2 t\right)$$

And we started with wealth $X_t$ and invested a fixed percentage $m$ into our asset:

$$\frac{dX_t}{X_t} = m\frac{dP_t}{P_t}$$

# Fixed-weight rebalance of one asset and cash

Applying Ito's lemma, we will find that:

$$\begin{aligned} X_t &= X_0 \exp\left(m\sigma W_t - \frac{1}{2}m^2\sigma^2 t\right) \\ &= X_0 \left(\frac{P_t}{P_0}\right)^m \exp\left(\frac{1}{2}(m - m^2)\sigma^2 t\right) \\ &\approx cX_0 \left(\frac{P_t}{P_0}\right)^m \end{aligned}$$

# Fixed-weight rebalance of one asset and cash

And a Taylor-series expansion of $X_t$ around $S_t = S_0$:

$$X_t \approx cX_0 \left[ 1 + m(\frac{P_t}{P_0} - 1) + \frac{1}{2} m(m-1)(\frac{P_t}{P_0} - 1)^2 + \text{remainder} \right]$$

When $m < 1$, we can easily see that our quadratic term has a negative coefficient, giving us the negative convexity property in our payoff profile.

# Expanding this into a two asset case

Using the same assumptions as before:

$$
\begin{aligned}
\frac{dS_t}{S_t} &= \sigma_S dW_{S,t} \quad S_t = S_0 \exp\left(\sigma_S W_{S,t} - \frac{1}{2}\sigma_S^2 t\right) \\
\frac{dB_t}{B_t} &= \sigma_B dW_{B,t} \quad B_t = B_0 \exp\left(\sigma_B W_{B,t} - \frac{1}{2}\sigma_B^2 t\right)
\end{aligned}
$$

# Fixed-weight rebalance of two assets

Assuming the return correlation between the assets is $\rho$, our wealth at $t$ is then determined by:

$$\frac{dX_t}{X_t} = m\frac{dS_t}{S_t} + n\frac{dB_t}{B_t}$$

Again, applying Ito's lemma, we arrive at:

$$X_t = X_t \exp\left(m\sigma_S W_{S,t} + n\sigma_B W_{B,t} - \frac{1}{2}(m^2\sigma_S^2 + n^2\sigma_B^2 + 2\rho mn\sigma_S\sigma_B)t\right.$$
$$\approx cX_0\left(\frac{S_t}{S_0}\right)^m\left(\frac{B_t}{B_0}\right)^n$$

# Fixed-weight rebalance of two assets

Carrying out our Taylor series expansion, we'll find:

$$\frac{X_t}{X_0} \approx c\left[1 + m\left(\frac{S_t}{S_0} - 1\right) + n\left(\frac{B_t}{B_0} - 1\right) - \frac{1}{2}mn\left(\frac{S_t}{S_0} - \frac{B_t}{B_0}\right)^2\right]$$

We can see that the fixed-weight rebalance strategy induces negative convexity in the divergence between assets.

# Characteristic return profile

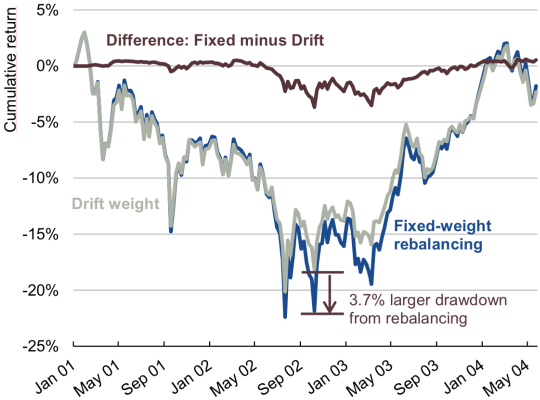
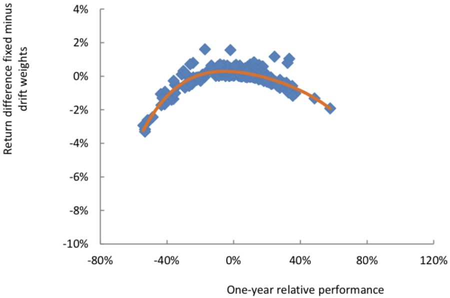
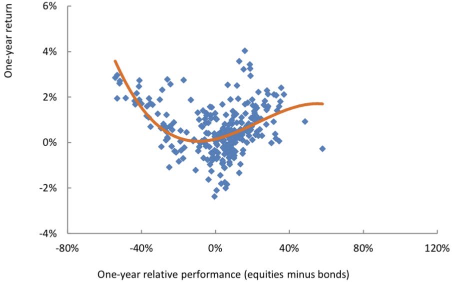



Figure 3: Divergence in stock and bond returns results in negative convexity. Source: Reuters.

# A solution

Leverage opposing return profile of momentum overlay.



# How momentum works

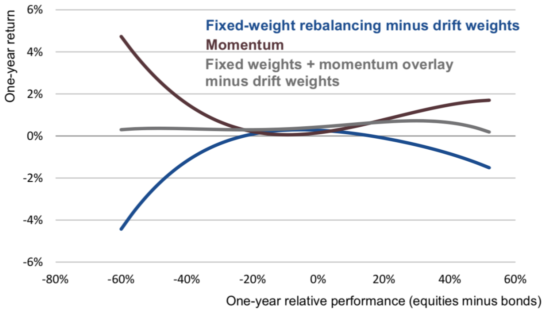
They utilized a proprietary moving-average cross-over system. But in general:


Figure 4: Bullish and bearish signals from Emerson Electric arising from 50-day EMA and 200-day EMA. Source: Stock Charts.

# How momentum works

High frequency momentum overlay reduce risks induced by rebalancing.
The expected profit and loss of the strategy:

$$\mathbb{E}(PL|P_t) = Y_0\psi(t)\left[\left(\log\frac{P_t}{P_0}\right)^2 - \sigma^2 t\right]$$



# Momentum combined with our rebalancing strategy

Letting $Y_0 = kX_0$, a proportion of initial wealth then:

$$\begin{aligned}
\mathbb{E}(PL|P_t) &= cX_0\Big[1 + m(\frac{P_t}{P_0} - 1) + \frac{1}{2}m(m-1)(\frac{P_t}{P_0} - 1)^2\Big] - X_0 \\
&\quad Y_0\psi(t)\Big[\Big(\frac{P_t}{P_0} - 1\Big)^2 - \sigma^2 t + R \\
&\approx X_0\Big[cm\Big(\frac{P_t}{P_0} - 1) + \Big(k\psi(t) + \frac{1}{2}cm(m-1)\Big)\Big(\frac{P_t}{P_0} - 1\Big)^2 + R\Big]
\end{aligned}$$

This is how we can offset the negative convexity from rebalancing. It also produces a distribution of returns that have a thinner left tail, translating to smaller drawdowns.

# Results: tracks the desired long-term allocation

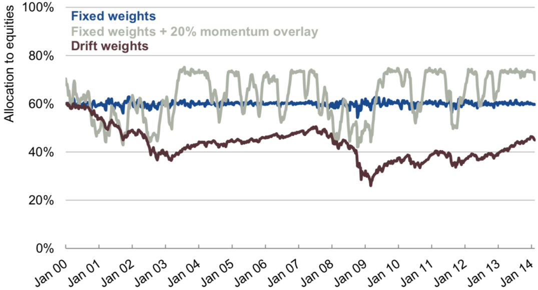



Figure 6: Allocation differences under fixed weight rebalancing, with and without momentum overlay. Source: Man Group.

# Results: improves performance

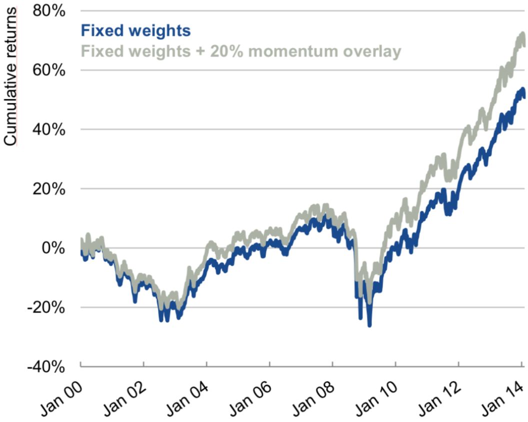



Figure 7: Cumulative returns of a fixed portfolio against one with momentum overlay. Source: Man Group.

# Results: compared to our original

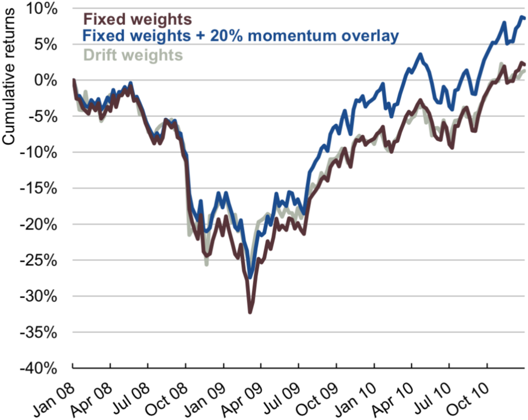



Figure 8: Impact of momentum overlay between 2008 and 2010 period. Source: Man Group.

# Discussion

*"Momentum overlay provides one way to maintain a good rebalancing schedule WITHOUT increasing the drawdown risk."*
*- Granger et al.*

1. In periods of extended divergence, rebalancing continues to allocate to underperforming asset. This active strategy (selling winners and buying losers) is attractive mostly when reversion holds.
2. Rebalancing induces negative convexity, leading to possible large drawdowns.
3. High frequency momentum can be leverage to offset this negative convexity profile.

# Future directions

While the paper utilizes a moving crossover model, there are many other strategies of momentum we can try to implement, including those around absolute time series momentum.

We might even consider pursuing a pure momentum strategy as historical returns (good backtests) have demonstrated its superior performance over time.

We can investigate the inclusion of other risk premia like carry and value outside of just momentum. (Roncalli 2017)

We can assess this alleviation in drawdown behavior in other asset classes. (Moskowitz et al 2012)

# References

Granger, N., Douglas G., Campbell H., Sandy R., and D. Zou. (2014). "Rebalancing risk."

Moskowitz, T.J., Ooi, Y.H., and Pedersen, L.H. (2012). "Time Series Momentum," Journal of Financial Economics, 104(2), pp. 228-250.

Roncalli, T. (2017). "Alternative Risk Premia: What Do We Know?," in Jurczenko, E. Factor Investing and Alternative Risk Premia, ISTE Press – Elsevier.