

Rebalancing Risk

Presenter: Joyce Cahoon

Introduction

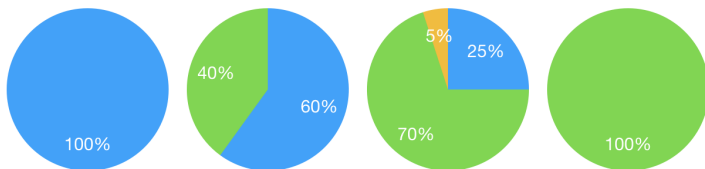


Figure 1: Typical IRA equity-bond mix

How rebalancing works

- Start with buy-and-hold portfolio
 - 60% allocation to S&P500
 - 40% allocation to 10-year Treasury bond
- Add a short straddle (sell a call and put option)
- Works in a trendless environment, marked by mean reversion

The problem

Cumulative returns to 60/40 portfolios of the S&P500 and 10-year US Treasury bond

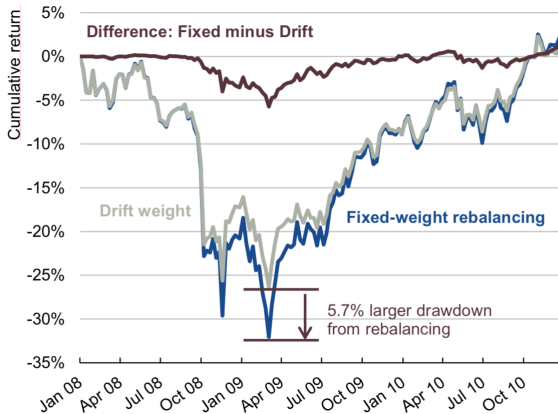


Figure 2: Rebalancing magnified drawdowns. Source: Reuters.

Fixed-weight rebalance of one asset and cash

If we model a single risky asset P_t as follows:

$$\frac{dP_t}{P_t} = \sigma dW_t$$
$$P_t = P_0 \exp\left(\sigma W_t - \frac{1}{2}\sigma^2 t\right)$$

And we started with wealth X_t and invested a fixed percentage m into our asset:

$$\frac{dX_t}{X_t} = m \frac{dP_t}{P_t}$$

Fixed-weight rebalance of one asset and cash

Applying Ito's lemma, we will find that:

$$\begin{aligned}X_t &= X_0 \exp \left(m\sigma W_t - \frac{1}{2}m^2\sigma^2 t \right) \\&= X_0 \left(\frac{P_t}{P_0} \right)^m \exp \left(\frac{1}{2}(m - m^2)\sigma^2 t \right) \\&\approx cX_0 \left(\frac{P_t}{P_0} \right)^m\end{aligned}$$

Fixed-weight rebalance of one asset and cash

And a Taylor-series expansion of X_t around $S_t = S_0$:

$$X_t \approx cX_0 \left[1 + m \left(\frac{P_t}{P_0} - 1 \right) + \frac{1}{2} m(m-1) \left(\frac{P_t}{P_0} - 1 \right)^2 + \text{remainder} \right]$$

When $m < 1$, we can easily see that our quadratic term has a negative coefficient, giving us the negative convexity property in our payoff profile.

Expanding this into a two asset case

Using the same assumptions as before:

$$\begin{aligned}\frac{dS_t}{S_t} &= \sigma_S dW_{S,t} & S_t &= S_0 \exp\left(\sigma_S W_{S,t} - \frac{1}{2}\sigma_S^2 t\right) \\ \frac{dB_t}{B_t} &= \sigma_B dW_{B,t} & B_t &= B_0 \exp\left(\sigma_B W_{B,t} - \frac{1}{2}\sigma_B^2 t\right)\end{aligned}$$

Fixed-weight rebalance of two assets

Assuming the return correlation between the assets is ρ , our wealth at t is then determined by:

$$\frac{dX_t}{X_t} = m \frac{dS_t}{S_t} + n \frac{dB_t}{B_t}$$

Again, applying Ito's lemma, we arrive at:

$$\begin{aligned} X_t &= X_0 \exp(m\sigma_S W_{S,t} + n\sigma_B W_{B,t} - \frac{1}{2}(m^2\sigma_S^2 + n^2\sigma_B^2 + 2\rho mn\sigma_S\sigma_B)t) \\ &\approx cX_0 \left(\frac{S_t}{S_0}\right)^m \left(\frac{B_t}{B_0}\right)^n \end{aligned}$$

Fixed-weight rebalance of two assets

Carrying out our Taylor series expansion, we'll find:

$$\frac{X_t}{X_0} \approx c \left[1 + m \left(\frac{P_t}{P_0} - 1 \right) + n \left(\frac{B_t}{B_0} - 1 \right) + \frac{1}{2} mn \left(\frac{S_t}{S_0} - \frac{B_t}{B_0} \right)^2 \right]$$

Characteristic return profile

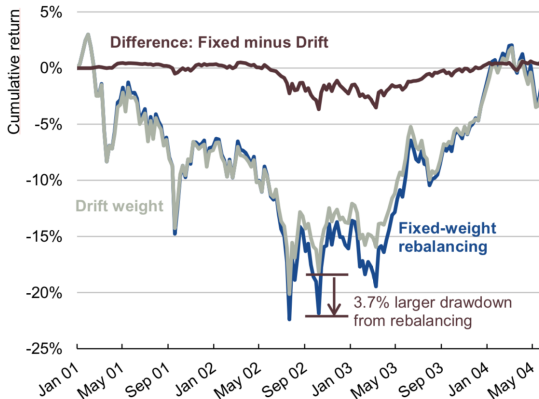
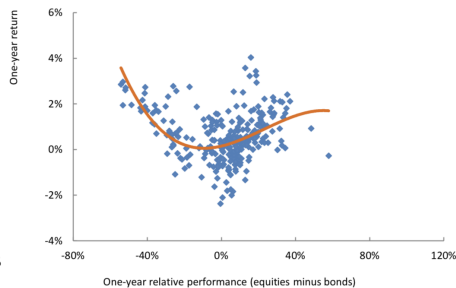
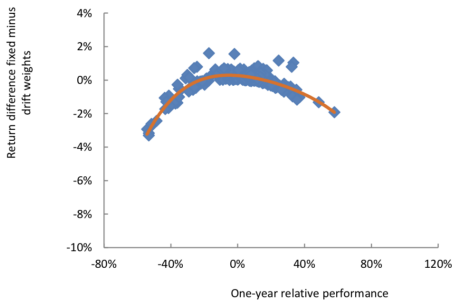


Figure 3: Divergence in stock and bond returns results in negative convexity.
Source: Reuters.

A solution

Leverage opposing return profile of momentum overlay.



How momentum works

They utilized a proprietary moving-average cross-over system. But in general:

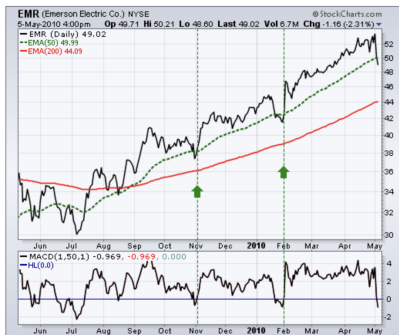
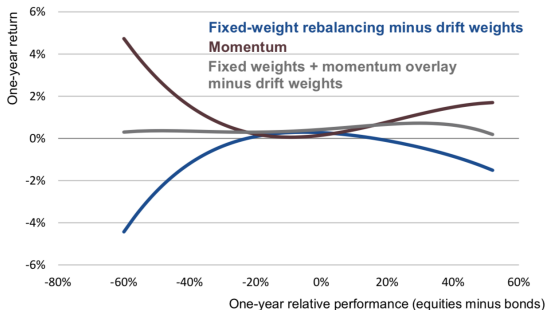


Figure 4: Bullish and bearish signals from Emerson Electric arising from 50-day EMA and 200-day EMA. Source: Stock Charts.

How momentum works

High frequency momentum overlay reduce risks induced by rebalancing.
The expected profit and loss of the strategy:

$$\mathbb{E}(PL|P_t) = Y_0 \psi(t) \left[\log \frac{P_t^2}{P_0} - \sigma^2 t \right]$$



Tracks the desired long-term allocation

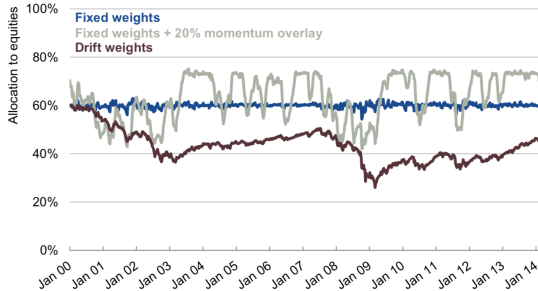


Figure 6: Allocation differences under fixed weight rebalancing, with and without momentum overlay. Source: Man Group.

Improves performance



Figure 7: Cumulative returns of a fixed portfolio against one with momentum overlay. Source: Man Group.

Compared to our original

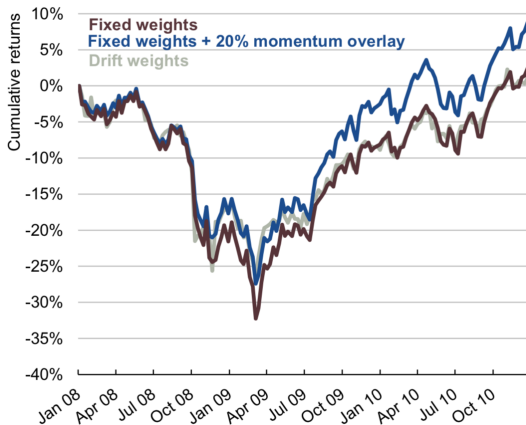


Figure 8: Impact of momentum overlay between 2008 and 2010 period.
Source: Man Group.

Results

“Momentum overlay provides one way to maintain a good rebalancing schedule WITHOUT increasing the drawdown risk.”

Future

Future

- Use of different kinds of momentum overlay

Discussion