Homework 2

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5.2

Let s_A and s_B be the Sharpe ratio of portfolios A and B. Let r_A and r_B be the expected returns of these two portfolios, with standard deviation denoted by σ_A and σ_B. Assume that through self financing, portfolio A borrows $(\sigma_B/\sigma_A - 1)$ at risk-free rate r_f to leverage so that its risk is now the same as that of portfolio B.

Show that the excess return of leveraged investment in portfolio A is larger than the expected return of portfolio B if $s_A > s_B$. This shows that the Sharpe ratio measures the efficiency of a portfolio.

5.3

Suppose that three mutual funds (conservative, growth and aggressive) have annual log-returns of 15%, 20% and 30% with volatility of 20%, 30% and 50% respectively. The correlation between any of the 2 funds is 0 and the risk-free rate is 5%.

```
rf <- .05
vol <- c(.20, .30, .50)
r <- c(.15, .20, .30)
expected_return <- .15
Y <- r - rf # excess returns
gamma <- diag(x = 1, nrow = 3)
vol <- diag(vol, nrow = 3)
Sigma <- vol %*% gamma %*% vol
partial_alpha <- as.vector(solve(Sigma) %*% Y )
A <- sum(partial_alpha * Y)/(expected_return - rf)
a.est <- 1/A*solve(Sigma) %*% Y
```

1. What is the min variance portfolio with these 3 mutual funds?
2. Find the optimal portfolio allocation among the 3 mutual funds, if the expected return is set at 15%. Give the associated standard deviation of this portfolio.
3. Compute the Sharpe ratio for the portfolio in A. How does it compare with that in B?

```
## Our risk aversion parameter A is:  7.5

## Our optimal portfolio allocation is:  0.3333333 0.2222222 0.1333333
##  for the conservative, growth and aggressive funds, respectively

## Our min variance is given by:  0.01333333
```

5.10

Let Y be the excess returns of risky assets. Let $X = a^T Y$ be a portfolio with allocation vector a. Denote by $\Sigma = \text{var}(Y)$ and $\mu = \mathbb{E}(Y)$. Consider the following decomposition:

$$Y = \alpha + \beta X + \epsilon \quad \mathbb{E}(\epsilon) = 0 \quad \text{cov}(\epsilon, X) = 0$$

1. Show that if $a = c\Sigma^{-1}\mu$ then $\alpha = 0$.
2. Conversely if $\alpha = 0$, there exists a constant c such that $a = c\Sigma^{-1}\mu_0$

5.13

Consider the following portfolio optimization problem with a risk-free asset having return r_0:

$$\min \boldsymbol{\alpha}^T \Sigma \boldsymbol{\alpha} \quad \text{such that} \quad \boldsymbol{\alpha}^T \boldsymbol{\mu} + (1 - \boldsymbol{\alpha}^T \mathbf{1}) r_0 = \mu$$

1. The optimal solution is $\boldsymbol{\alpha} = P^{-1}(\mu - r_0)\Sigma^{-1}\mu_0$ where $P = \boldsymbol{\mu}_0^T \Sigma^{-1} \boldsymbol{\mu}_0^T$ is the squared Sharpe ratio, and $\boldsymbol{\mu}_0 = \boldsymbol{\mu} - r_0 \mathbf{1}$ is the vector of excess returns.
2. The variance of this portfolio is $\sigma^2 = (\mu - r_0)^2 / P$.
3. When $r_0 < \mu$, show that $r_0 + P^{1/2}\sigma = \mu$, namely the optimal allocation for the risky asset $\boldsymbol{\alpha}$ is the tangent portfolio.

5.14

1. Download the monthly data for 8 stocks.

```
library(quantmod)
start <- as.Date("2001-01-01")
end <- as.Date("2014-12-29")
getSymbols("DGS3MO", src = "FRED", from = start, to = end) # TREASURY
symbols <- c("SPY",
             #"DVMT", # DELL DOES NOT WORK
             "F",  # FORD
             "GE",
             "IBM",
             "INTC", # INTEL
             "JNJ", # JOHNSON
             "MRK", # MERCK
             "MSFT") # MICROSOFT
getSymbols(symbols, from = start, to = end)
closing.prices <- merge.xts(DGS3MO,
                            SPY[,4],
                            F[,4],
                            GE[,4],
                            IBM[,4],
                            INTC[,4],
                            JNJ[,4],
                            MRK[,4],
                            MSFT[,4])
volume <- merge.xts(DGS3MO,
                            SPY[,5],
                            F[,5],
                            GE[,5],
                            IBM[,5],
                            INTC[,5],
                            JNJ[,5],
                            MRK[,5],
                            MSFT[,5])
saveRDS(closing.prices, "~/workspace/st790-financial-stats/data/hw2_closingprices.rds")
saveRDS(volume, "~/workspace/st790-financial-stats/data/hw2_volume.rds")
```

2. Construct the optimal allocation using the monthly data.

```
closing.prices <- readRDS("~/workspace/st790-financial-stats/data/hw2_closingprices.rds")
data <- closing.prices["2001-01-02/2011-12-31"]
data <- na.omit(data)
by_month <- data.frame()
for (i in 1:ncol(data)){
  temp <- data[,i]
```

```
  monthly <- log(monthlyReturn(temp)+1)
  by_month <- cbind(by_month, monthly )
  colnames(by_month)[i] <- strsplit(names(temp), "[.]")[[1]][1]
}
# get the excess return
excess_return <- data.frame()
for(j in 2:ncol(by_month)){
  temp <- by_month[,j] - by_month[,1] # subtract the 3 mo treas rate
  excess_return <- cbind(excess_return, temp)
  colnames(excess_return)[j-1] <- names(temp)
}
# get the alphas
alphas <- c()
betas <- c()
for(k in 2:ncol(excess_return)){
  results <- lsfit(excess_return[,1], excess_return[,k])$coefficients
  alphas <- c(alphas, results[1])
  betas <- c(betas, results[2])
}
# get the residuals
x <- excess_return$SPY
y <- excess_return[,c(2:8)]
residuals <- resid(lsfit(x, y))
Sigma <- t(residuals) %*% residuals / dim(y)[1]
mreturn <- mean(x)
msigma <- var(x)
T0 <- dim(y)[1]/( 1+mreturn^2/msigma ) * t(alphas) %*% solve(Sigma, alphas) # for testing
# get the optimal allocation
Y <- apply(y, 2, mean)# excess returns
partial_alpha <- as.vector(solve(Sigma) %*% Y )
A <- sum(partial_alpha)/.8 # since invest 20% in riskless asset
a.est <- 1/A*solve(Sigma) %*% Y
rownames(a.est) <- names(y)
#print("This is the optimal allocation:")
knitr::kable(t(a.est))
```

F	GE	IBM	INTC	JNJ	MRK	MSFT
0.0583626	0.1029486	0.1434665	0.0735273	0.2572974	0.071926	0.0924715

3. If allocation is fixed over the next 2 years, compare the performance of the portfolio over next 6-mo, one-yr, two-year, and three-year:

```
data <- closing.prices["2011-12-31/2012-05-31"]
data <- na.omit(data)
by_month <- data.frame()
for (i in 1:ncol(data)){
```

```r
  temp <- data[,i]
  monthly <- log(monthlyReturn(temp)+1)
  by_month <- cbind(by_month, monthly )
  colnames(by_month)[i] <- strsplit(names(temp), "[.]")[[1]][1]
}
# get the excess return
excess_return <- data.frame()
for(j in 2:ncol(by_month)){
  temp <- by_month[,j] - by_month[,1] # subtract the 3 mo treas rate
  excess_return <- cbind(excess_return, temp)
  colnames(excess_return)[j-1] <- names(temp)
}
# get the residuals
x <- excess_return$SPY
y <- excess_return[,c(2:8)]
residuals <- resid(lsfit(x, y))
Sigma <- t(residuals) %*% residuals / dim(y)[1]
# calculate excess returns over 6 month period
y_6 <- apply(y, 2, mean)
# calculate the risk free rate return
rf <- mean(by_month$DGS3MO)
# log return
mu_6 <- rf + sum(as.vector(a.est)*(y_6))
# volatility
std_6 <- sqrt(t(a.est) %*% Sigma %*% a.est)
# Sharpe ratio
s_6 <- (mu_6 - rf)/std_6
# Repeat the above process for one year and two year and three year
```

timeline	return	volatility	sharpe
6 Months	0.0506173	0.0108012	-18.510468
1 Year	0.0181359	0.0130991	-4.444705
2 Years	0.0188856	0.0112804	-2.953154
3 Years	0.0036222	0.0139989	1.634150

4. Create a value weighted portfolio.

F	GE	IBM	INTC	JNJ	MRK	MSFT
0.1032031	0.1238695	0.1062568	0.1470514	0.0860065	0.0616742	0.1719384

Compare the performance:

timeline	return	volatility	sharpe
6 Months	0.0540661	0.0103545	-18.975889
1 Year	0.0168247	0.0138252	-4.306130
2 Years	0.0186687	0.0142250	-2.357101
3 Years	0.0041848	0.0158634	1.477545

5. Create a portfolio with .114 invested in each stock and .2 in the risk-free bond, and compare the performance as before. Note that we only have 7 stocks since DELL is not provided by Yahoo.

timeline	return	volatility	sharpe
6 Months	0.0520906	0.0109909	-18.056892
1 Year	0.0178008	0.0129052	-4.537455
2 Years	0.0187470	0.0122189	-2.737684
3 Years	0.0038139	0.0144713	1.594051