Homework 1

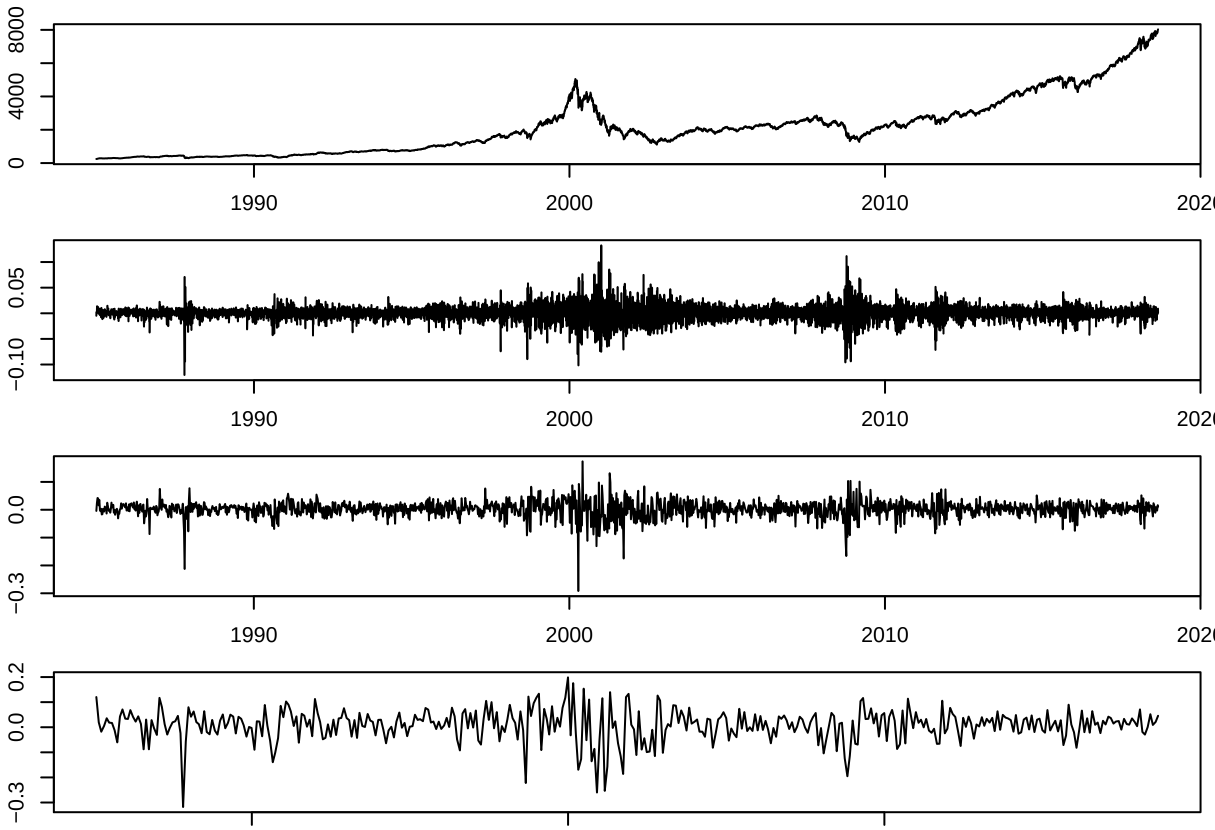
1.1

Download the daily, weekly and monthly prices for the Nasdaq index and the IBM stock from Yahoo. Reproduce figures 1.3-8 using the Nasdaq index and the IBM stock data. For 1.1, using the data from Jan 1985 - Aug 2018.

```r
# Download the data
library(quantmod)
# start <- as.Date("1985-01-01")
# end <- as.Date("2018-08-29")
# getSymbols("^IXIC", src = "yahoo", from = start, to = end)
# getSymbols("IBM", src = "yahoo", from = start, to = end)
# save to avoid reloading
# saveRDS(IXIC, "~/workspace/st790-financial-stats/data/ixic.rds")
# saveRDS(IBM, "~/workspace/st790-financial-stats/data/ibm.rds")
```

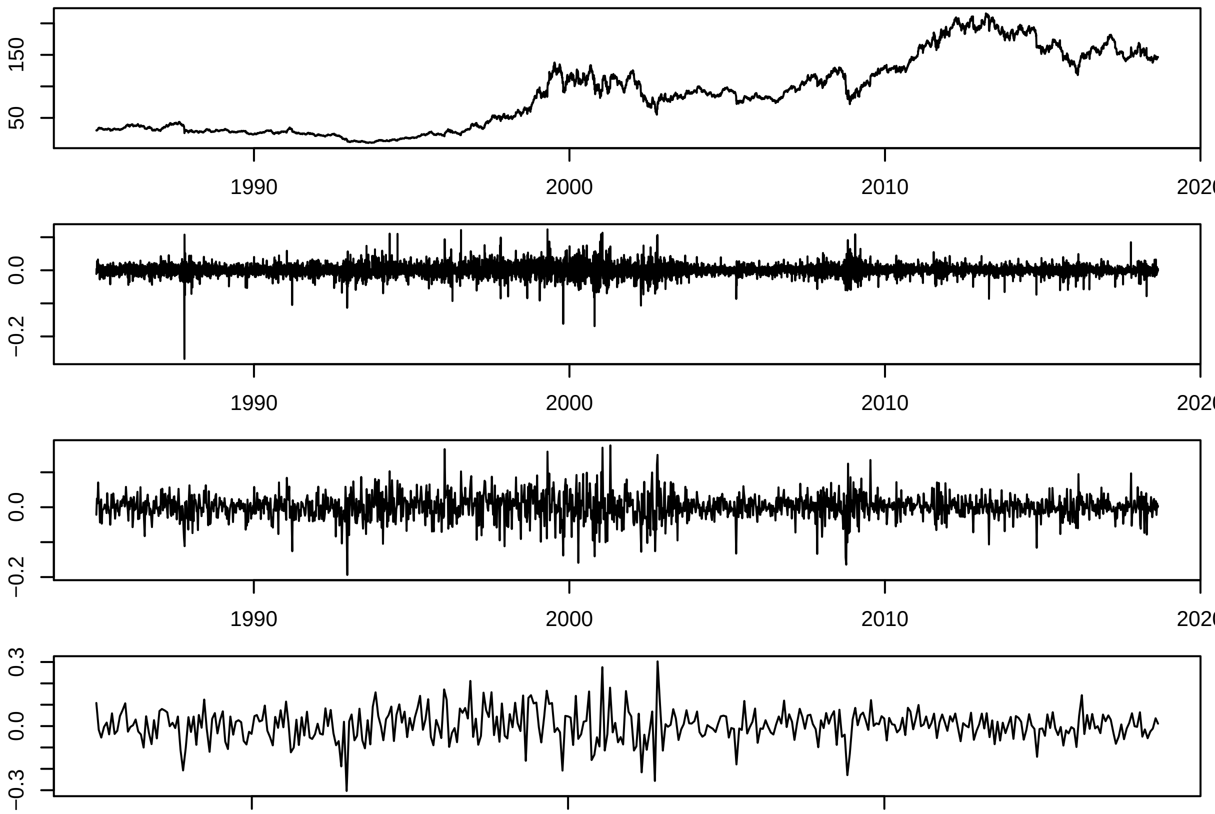
For NASDAQ:

```r
# Read in data
IXIC <- readRDS("~/workspace/st790-financial-stats/data/ixic.rds")
IBM <- readRDS("~/workspace/st790-financial-stats/data/ibm.rds")
# Extract
daily <- log(dailyReturn(IXIC)+1)
weekly <- log(weeklyReturn(IXIC)+1)
monthly <- log(monthlyReturn(IXIC)+1)
# Plot
par(mar=c(1,3,2,1),mfrow=c(4,1))
plot(index(IXIC), as.numeric(IXIC$IXIC.Close), type = "l",
     ylab = "daily price")
plot(index(daily), daily, type = "l",
     ylab="daily log return")
plot(index(weekly), weekly, type = "l",
     ylab="weekly log return")
plot(index(monthly), monthly, type = "l",
     ylab="monthly log return")
```



For IBM:

```r
# Extract
daily <- log(dailyReturn(IBM)+1)
weekly <- log(weeklyReturn(IBM)+1)
monthly <- log(monthlyReturn(IBM)+1)
# Plot
par(mar=c(1,3,2,1),mfrow=c(4,1))
plot(index(IBM), as.numeric(IBM$IBM.Close), type = "l",
     ylab = "daily price")
plot(index(daily), daily, type = "l",
     ylab="daily log return")
plot(index(weekly), weekly, type = "l",
     ylab="weekly log return")
plot(index(monthly), monthly, type = "l",
     ylab="monthly log return")
```



1.3 Consider the following quote from Eugene Fama who was Myron Scholes' thesis adviser: "If the population of prices changes is strictly normal, on the average for any stock… an observation more than 5 standard deviations from the mean should be observed once every 7000 years. In fact such observations seem to occur about once every 3 or 5 years." For $X \sim N(\mu, \sigma^2)$, $P(|X-\mu| > 5\sigma) = 5.733 \times 10^{-7}$, deduce how many observations per year Fama was implicitly assumining to be made. If a year is defined as 252 trading days and daily returns are normal, how many years is it expected to take to get a 5 standard deviation event? How does this answwer to the last question change when the daily returns follow the t-distribution with 4 degrees of freedom?

1.8 According to the efficient market hypothesis, is the return of a portfolio predictable? Is the volatility of a portfolio predictable? State the most appropriate mathematical form of the efficient market hypothesis.

1.9 If the Ljung-Box test is employed to test the efficient market hypothesis, what null hypothesis is to be tested? If the autocorrelation for the first 4 lags of the montly log-returns of the S&P500 is:

$$\hat{\rho}(1) = .2, \quad \hat{\rho}(2) = -0.15, \quad \hat{\rho}(3) = 0.25, \quad \hat{\rho}(4) = 0.12$$

based on the last 5 years of data, is the efficient market hypothesis reasonable?

1.13 Let S_t be the price of an asset at time t. One version of the EMH assumes that the prices of any asset form a martingale process in the sense that:

$$E(S_{t+1}|S_t, S_{t-1}, \ldots) = S_t \quad \forall t$$

To understand the implication of this assumption, we consider the following simple investment strategy. With initial capital C_0 dollars, at the time t we hold α_t dollars in cash and β_t shares of an asset at the price S_t. Hence the value of our investment at time t is $C_t = \alpha_t + \beta_t S_t$. Suppose that our investment is self-financing in the sense that

$$C_{t+1} = \alpha_t + \beta_t S_{t+1} = \alpha_{t+1} + \beta_{t+1} S_{t+1}$$

and our investment strategy is entirely determined by the asset prices up to the time t. Show that if S_t is a martingale process, there exist no strategies such that $C_{t+1} > C_t$ with probability 1.