Supplementary Material for Yuyang's Report

Yuyang Ma[a]

[a] *Department of Industrial and Systems Engineering, Lehigh University, Bethlehem, PA, USA*

Appendix A. Formulating Drone Range

Parameter values used for drone range calculations are given in Table A.1 (Dukkanci et al. (2021)).

Table A.1: Attributes of Small and Large Drones

Notation	**Description**	**Small Drone**	**Large Drone**
δ	Profile Drag Coefficient	0.012	0.012
m_{uav}	UAV Mass (kg)	2.04	10
m_{batt}	Battery Mass (kg)	0.89	5.0
U_{tip}	Tip Speed of the Rotor (m/s)	120	150
s	Rotor Solidity	0.05	0.08
A	Rotor Disk Area (m^2)	0.503	1.0
ω	Blade Angular Velocity (rad/s)	300	250
r	Rotor Radius (meters)	0.4	1.0
k	Correction Factor to Induced Power	0.1	0.15
v_0	Mean Rotor Induced Velocity in Hover (m/s)	4.03	6.0
d_r	Fuselage Drag Ratio	0.6	0.8
B_{mass}	Energy Capacity per Mass of the Battery (J/kg)	540000	540000
θ	Depth of Discharge	0.8	0.8
$max_{payload}$	Max Payload (kg)	2.0	200
ρ	Air Density (kg/m^3)	0.012	
f	Safety factor to reserve energy	1.2	

In paper, we mentioned that we use the following formula to calculate the maximal average speed of the drone:

$$R(v) \triangleq \frac{\Theta}{\frac{\mu_1}{v} + \mu_2 v + \frac{\mu_3}{v^2} + \mu_4 v^2}, \tag{A.1}$$

where $\mu_1, \mu_2, \mu_3, \mu_4$ are the constants related to the parameters of drones. The detailed calculation

*Corresponding author.
Email address: `yuyang.ma@lehigh.edu` (Yuyang Ma)

of these constants are as follows:

$$\Theta = \frac{m_{batt} B_{mass} \theta}{f} \tag{A.2}$$

$$\mu_1 = \frac{\delta}{8} \rho s A r^3 \omega^2 \tag{A.3}$$

$$\mu_2 = \left(\frac{\delta}{8} \rho s A r^3 \omega^2 \right) \times \frac{3}{U_{tip}^2} \tag{A.4}$$

$$\mu_3 = (1+k) \frac{W^{3/2}}{\sqrt{2\rho A}} v_0 \tag{A.5}$$

$$\mu_4 = \frac{1}{2} d_0 \rho s A. \tag{A.6}$$

Notice that the parameter W used in calculation of μ_3 is the total weight of the drone. Package weight ($max_{payload}$) is only taken to account on the forward journey to a gathering point ($W = m_{uav} + m_{batt} + max_{payload}$), not in the return journey from a gathering point ($W = m_{uav} + m_{batt}$). Therefore, the refined calculation of μ_3 is as follows:

$$\mu_3 = (1+k) \frac{1}{2} \left(\frac{(m_{uav} + m_{batt} + max_{payload})^{3/2}}{\sqrt{2\rho A}} + \frac{(m_{uav} + m_{batt})^{3/2}}{\sqrt{2\rho A}} \right) v_0. \tag{A.7}$$

Appendix B. Recap of Benders Decomposition

Benders decomposition was first proposed to solve large-scale linear programming problems, which is also a common technique to solve the two-stage stochastic programming problems as following:

$$\begin{aligned} \min \quad & c^T x + Q(x) \\ \text{subject to} \quad & Ax = b \\ & x \geq 0, \end{aligned}$$

where $Q(x)$ here refers to the second-stage problem, that $Q(x) = \mathbb{E}_{\mathbb{P}}[Q(x, \omega)]$, where $Q(x, \omega) = \min_{y \in \mathbb{R}_+^p} \{q^T y : Wy = h(\omega) - T(\omega)x\}$, and $\mathbb{P}$ is a known distribution of the uncertainty ω. In practice, we want to treat the second-stage problem with discrete probability distribution, that saying, we have overall S different scenarios. Each scenario $s = 1, \ldots, S$ have a probability α_s to occur. To solve such problem, we can use Benders decomposition, where the stochastic programming is separated into two parts: the master problem and sub-problems. For the master problem, the formulation is:

$$
\begin{aligned}
\min \quad & c^T x + q^T y \\
\text{subject to} \quad & Ax = b \\
& x \geq 0,
\end{aligned}
$$

We can observe that, compared to the formulation above, the master problem only retains the constraints for the first stage. The intuition behind Benders decomposition is to find the optimal solution by adding cuts to the master problem until the value of the objective function converges. For a feasible first-stage solution x, $c^T x$ becomes a fixed value, and the remaining part of the objective function is $q^T y$. After the first-stage, one of S potential scenarios reveals. For a certain scenario $s = 1, \ldots, S$, we will have a distinct sub-problem and its dual problem as follows:

$$
\begin{aligned}
\text{(P)} \quad \min \quad & q^T y_s \\
\text{subject to} \quad & W y_s = h_s - T_s x, \\
& y_s \geq 0,
\end{aligned}
\qquad\qquad
\begin{aligned}
\text{(D)} \quad \max \quad & (h_s - T_s x)^T u \\
\text{subject to} \quad & W^T u \leq q,
\end{aligned}
$$

where u is the dual variable. We use $z_s(x)$ to refer to the objective value primal problem with a given x. Since dual problem is derived from the Lagrangian relaxation, it provides a lower bound of the primal problem. Assuming under scenario s, we have the optimal solution u^*, that $(h_i - T_i x)^T u^* \leq \min\ z_s(x)$. In the dual problem, we assume there are n extreme points, $u_j, j = 1, \ldots, n$ and m extreme rays $r_k, k = 1, \ldots, m$. We will discuss the constraints for extreme rays and extreme points, which are also known as feasibility cuts and optimality cuts, separately.

1. For any extreme rays r_k, we want it to be constrained by following:

   $$(h_s - T_s x)^T r_k \leq 0,$$

   if this constraint is violated, which means $(h_s - T_s x)^T r_k > 0$, we just need to scale it with extreme large positive value and add it to the objective function, then the function value goes to ∞. If dual function is unbounded, the primal problem is infeasible, which means that under scenario s, there is no available decision y to make with given x. Therefore, we want to avoid this x by adding the constraint $(h_s - T_s x)^T r_k \leq 0$ to the master problem.

2. For a linear programming problem over a polyhedron, the optimal solution (if any) has to occur on extreme points. According to the weak duality, if we have optimal solution u_j^* for the dual problem, the following inequality

   $$(h_s - T_s x)^T u_j^* \leq z_s(x), \quad \forall s = 1, \ldots, S,$$

holds for every feasible decision y with given x. Therefore, we want to get the tighter lower approximation by keep adding the constraint $(h_s - T_s x)^T u_j^* \leq z_s(x)$ to the master problem, until the dual solution converges to the optimal solution to the primal problem.

References

Dukkanci, O., Kara, B.Y., Bektaş, T., 2021. Minimizing energy and cost in range-limited drone deliveries with speed optimization. Transportation Research Part C: Emerging Technologies 125, 102985.