Drone Delivery System in Post-Disaster Humanitarian Logistics: A Two-Stage Stochastic Programming Approach with Uncertain Wind Conditions

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Introduction

UAVs for Humanitarian Logistics: Exploration by leading companies like DHL[1] and Zipline[2] for medicine deliveries, highlighting the growing interest in drone technology.

Technological Advances: Significant improvements in drone materials, sensing and coordination algorithms, battery life, and regulatory frameworks are propelling UAV adoption.

Facility Location Challenge: The central issue of limited range and payload capacity of drones necessitates strategic planning for the placement of depots or launching sites.

Solution Model: Introduction of a two-stage stochastic programming model focusing on selecting optimal locations for drone launching facilities, and size of drones' fleet to meet spatially distributed affected people' demands efficiently. The detailed formulation will be shown in next section.

[1] DHL 2018.

[2] Dean and Entsie 2020.

Benefits and Challenges of Drone Deliveries

Benefits of Using Drones:

- Improved Accessibility
- Efficiency and Safety
- Real World Applications

Challenges of Drone Deliveries:

- Limited Range and Payload
- Weather Susceptibility
- Computational Complexity

Literature Review

1. **Routing Problem:**
   - ⋆ Murray and Chu (2015)[3] proposed two kinds of routing problems:
     - ◇ Flying Sidekick Traveling Salesman Problem (FSTSP)
     - ◇ Parallel Drone-Scheduling Traveling Salesman Problem
   - ⋆ Murray and Raj (2020)[4] extended the FSTSP with multiple drones during the delivery.
2. **Facility Location Problem:**
   - ◇ Chauhan et al. (2019)[5] proposed a maximal covering facility location model with drones (MCFLPD).
3. **Delivery with Uncertainties:**
   - ◇ Zhu et al. (2022)[6] incorporated demand uncertainty with a two-stage robust optimization model.
   - ◇ Cheng et al. (2024)[7] considered uncertain wind conditions in a drone delivery system.

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[3] Murray and Chu 2015.
[4] Murray and Raj 2020.
[5] Chauhan, Unnikrishnan, and Figliozzi 2019.
[6] Zhu, Boyles, and Unnikrishnan 2022.
[7] Cheng, Adulyasak, and Rousseau 2024.

Review on Dukkanci et al. (2023)[8]

Topic: Using drones to distribute relief supplies in post-disaster scenarios

Objective: Selecting optimal locations for drone launching facilities to minimize unmet demands of affected people

Methodology: Two-stage mixed-integer stochastic programming model

Contributions:

1. Featured an endogenous drone range and time-bound constraints into the model
2. Considered the uncertain road network and demand induced by disasters
3. Proposed a scenario decomposition algorithm (SDA) to solve the problem
4. Conducted a real-world case study to validate the model

Room of Improvements: Fairness issues in distribution, weather uncertainties, unreal assumptions, etc.

[8] Dukkanci, Koberstein, and Kara 2023.

A Sample Graph for Two-Echelon Delivery System

In Dukkanci et al. (2023), they proposed a two-echelon delivery system, where trucks and drones cooperated to distribute the relief items after the disaster. A sample graph of proposed delivery system is illustrated as follows:

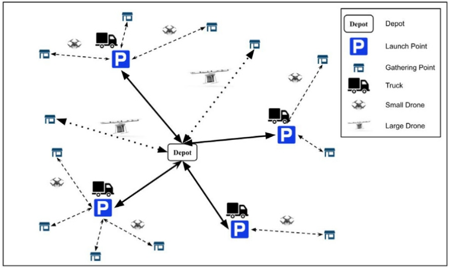

Figure: Two-Echelon Delivery System[9]

[9] Dukkanci, Koberstein, and Kara 2023.

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Problem Statement

In this study, we extend the model proposed by Dukkanci et al. (2023)[10]. The objective is to minimize the total cost of the drone delivery system while ensuring that the unmet demand of affected people is minimized. The detailed statement is listed as follows:

1. Given a set of locations for gathering points I, launch points L and depots D. Sets of available fully-charged drones are: K_{SD} for small drones and K_{LD} for large drones.
2. Decide to open at most p launch points and e depots. Assign each open launch point to one open depot.
3. Disaster (earthquake in our study) happens. Damage percentage of road networks, demands at gathering points, and wind conditions are observed.
4. Determine the size of drone fleets. Assign drones to open facilities (launch points or depots) for serving the gathering points. All deliveries must be finished within T hours.

[10] Dukkanci, Koberstein, and Kara 2023.

Mathematical Notations I

Parameters	**Description**
e	Maximal number for open depots
p	Maximal number for open launch points
Q_T	Capacity of trucks
Q_{SD}	Capacity of small drones
Q_{LD}	Capacity of large drones
c_{SD}	Cost of using small drones
c_{LD}	Cost of using large drones
v_T	Speed of trucks
v_{SD}	Speed of small drones
v_{LD}	Speed of large drones
v^w	Speed of wind
α^w	Direction of wind
T	Length of time period
τ	Preparation time for small drones
w_{dl}	Ground distance between depot d and launch point l after the earthquake
q_i	Demand at gathering point i
π	Penalty per unit of unmet demand

Mathematical Notation II

First-Stage Decision Variables

u_d: 1, if a depot is open at location $d \in D$, 0 otherwise
v_l: 1, if a launch point is open at location $l \in L$, 0 otherwise
w_{dl}: 1, if launch point l is assigned to depot d, 0 otherwise

Second-Stage Decision Variables

g_k: 1, if a drone $k \in \{K_{SD} \cup K_{LD}\}$ is used, 0 otherwise
z_{ilk}: 1, if a small drone $k \in K_{SD}$ is assigned to serve point $i \in I$ from location $l \in \{L \cup D\}$, 0 otherwise
y_{idk}: 1, if a large drone $k \in K_{LD}$ is assigned to serve point $i \in I$ from depot $d \in D$, 0 otherwise
o_i: Unmet demand for gathering point $i \in I$

Two-Stage Stochastic Programming I

First-Stage Problem

Objective: total cost of using drones and the penalty of unmet demand

$$
\begin{aligned}
\min \quad & \mathbb{E}_{\mathbb{P}}\left[Q(u, v, w, \xi)\right] & & \text{(1a)}\\
\text{subject to} \quad & \sum_{d \in D} u_d \leq e & & \text{(1b)}\\
& \sum_{l \in L} v_l \leq p & & \text{(1c)}\\
& \sum_{d \in D} w_{ld} = v_l, & \forall l \in L & \quad \text{(1d)}\\
& w_{ld}, u_d, v_l \in \{0, 1\}. & \forall d \in D, l \in L & \quad \text{(1e)}
\end{aligned}
$$

Notations

The uncertainties in the second-stage are indicated as the parameter $\xi \in \Xi$, where $\xi = \left(\omega, q, v^{wind}, \alpha^{wind}\right)$. $\mathbb{P}$ a known probability distribution of ξ.

Two-Stage Stochastic Programming II

Second-Stage Problem

Objective: minimize the expectation the second stage's cost Q.

$$
\begin{aligned}
Q[u,v,w] := \quad \min \quad & \sum_{i\in I} \pi o_i + \sum_{k\in K_{SD}} c_{SD} g_k + \sum_{k\in K_{LD}} c_{LD} g_k && && \text{(2a)}\\
\text{subject to} \quad & \text{Constraints for open depots, launch points, and using drones} && && \text{(2b)}\\
& \sum_{k\in K_{SD}} \sum_{i\in I} q_i x_{ilk} \le Q_T v_l && \forall l \in L && \text{(2c)}\\
& q_i - \sum_{k\in K_{SD}} \sum_{l\in\{L\cup D\}} Q_{SD} x_{ilk} - \sum_{k\in K_{LD}} \sum_{d\in D} Q_{LD} y_{idk} \le o_i && \forall i \in I && \text{(2d)}\\
& \sum_{d\in D} \frac{\omega_{ld}}{v_T} w_{ld} + \sum_{i\in I} \left(\frac{2d_{li}}{v_{li}} + \tau \right) x_{ilk} \le T && \forall l \in L, k \in K_{SD} && \text{(2e)}\\
& \sum_{i\in I} \left(\frac{2d_{li}}{v_{li}} + \tau \right) x_{ilk} \le T && \forall l \in D, k \in K_{SD} && \text{(2f)}\\
& \frac{2d_{ji}}{v_{ji}} y_{ijk} \le T && \forall j \in D, i \in I, k \in K_{LD} && \text{(2g)}\\
& g_k \ge g_{k+1} && \forall k, k+1 \in K_{SD} && \text{(2h)}\\
& g_k \ge g_{k+1} && \forall k, k+1 \in K_{LD} && \text{(2i)}
\end{aligned}
$$

Maximal Average Speed Calculation for Drones

In this study, we use the following formula proposed by Dukkanci et al.[11] to calculate the maximal average speed for drones to deliver from facility l to gathering point i:

$$R(v) \triangleq \frac{\Theta}{\frac{\mu_1}{v} + \mu_2 v + \frac{\mu_3}{v^2} + \mu_4 v^2}, \tag{3}$$

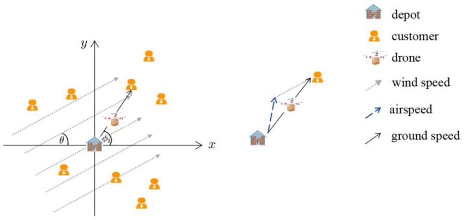
where parameters μ_1, μ_2, μ_3, μ_4, and Θ are constants that depend on the drone model, and the detailed calculation is listed in the supplementary materials.

In practice, we set $R(v) = 2 \times d_{il}$, where d_{il} is the air line distance between facility l and gathering point i, which will not be affected by the disaster.

[11] Dukkanci, Kara, and Bektaș 2021.

Affects of Wind on Delivery Process

As proposed in Cheng et al. (2024)[12], the affect of wind on the delivery process can be illustrated as follows:



Note. The right part is based on vector addition.

Figure: Affects of Wind on Delivery Process

For a drone k is assigned to serve point i from facility l, if maximal average speed calculate by equation (3) $v^k_{max} \leq v^{wind}$, the drone cannot be assigned to this delivery task.

[12]Cheng, Adulyasak, and Rousseau 2024.

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In this study, we use the sample average approximation (SAA) method to formulate the proposed two-stage stochastic programming problem into a deterministic optimization problem.

Sample Average Approximation

$$\min \quad \frac{1}{|S|}\sum_{s\in S}\left(\sum_{i\in I}\pi o_{s,i} + \sum_{k\in K_{SD}} c_{SD} g_{s,k} + \sum_{k\in K_{LD}} c_{LD} g_{s,k}\right) \tag{4a}$$

$$\text{subject to} \quad \text{Constraints (1b) - (1e)} \tag{4b}$$

$$\vdots \tag{4c}$$

$$\sum_{d\in D}\frac{\omega_{sld}}{v_T} w_{ld} + \sum_{i\in I}\left(\frac{2d_{li}}{v_{sli}} + \tau\right) x_{silk} \le T \qquad s\in S, \forall l\in L, k\in K_{SD} \tag{4d}$$

$$\vdots \tag{4e}$$

$$g_{s,k} \ge g_{s,k+1} \qquad \forall s\in S, k, k+1\in K_{SD} \tag{4f}$$

$$g_{s,k} \ge g_{s,k+1} \qquad \forall s\in S, k, k+1\in K_{LD} \tag{4g}$$

Intuition of Scenario Decomposition Algorithm

In stochastic programming, the nonanticipativity constraint requires that the first-stage decision must be independent of the realization of the random variable.

Phase 1: Relaxation of Nonanticipativity

- During the first phase, the nonanticipativity of the first-stage decision variables is relaxed.
- This relaxation enables the decomposition of the problem into individual scenarios.
- Each scenario is treated as a deterministic problem.
- Solving these yields a lower bound for the main problem based on the optimal values from the single-scenario problems.

Phase 2: Fixing First-Stage Variables

- In the second phase, the algorithm fixes the first-stage decision variables using the solutions obtained in the first phase.
- Solves for an upper bound for each scenario.
- The upper bound for the main problem is derived by considering the expected value of the single-scenario solutions.

Repeat the two phases until the upper bound converges to the lower bound, where the relaxed nonanticipativity constraints are satisfied.

Pseudocode of Scenario Decomposition Algorithm

Algorithm 1 Scenario decomposition algorithm

initialize: $UB \leftarrow +\infty$, $LB \leftarrow -\infty$, $R \leftarrow \emptyset$, $x^* \leftarrow 0$, $y^* \leftarrow 0$, $w^* \leftarrow 0$
while $UB > LB$ and $\{0,1\}^{(D+L+DL)} \setminus R \neq \emptyset$ **do**
    for $s = 1$ to S **do**
        solve the *relaxation* of the second stage formulation including constraints to exclude existing solutions
        let $(\bar{x}_s, \bar{y}_s, \bar{w}_s)$ be the optimal solution and lb_s be the optimal objective value
    end for
    $LB \leftarrow \sum_{s=1}^{S} p_s lb_s$, $\hat{R} \leftarrow \bigcup_{s=1}^{S} \{\bar{x}_s, \bar{y}_s, \bar{w}_s\}$, $R \leftarrow R \cup \hat{R}$
    for $\{x, y, w\} \in \hat{R}$ **do**
        $u \leftarrow 0$
        for $s = 1$ to S **do**
            solve the problem formulation given x, y, and w variables; let $f_s(x, y, w)$ be the optimal objective value
            if Problem is infeasible **then**
                set $f_s(x, y, w) \leftarrow \infty$ and $u \leftarrow \infty$
            else
                set $u \leftarrow u + p_s f_s(x, y, w)$
            end if
        end for
        if $UB > u$ **then**
            $UB \leftarrow u$, $x^* \leftarrow x$, $y^* \leftarrow y$ and $w^* \leftarrow w$
        end if
    end for
end while

Intuition of Cluster-Based Heuristic Algorithm

Although the objective function depends on second-stage decision variables, the decisions made in the first-stage, locations for open drone facilities, implicitly affect the objective function for the following reasons:

1. Range limit of drones and time window restrictions affect coverage.
2. Location of drone facilities is critical.
3. First-stage decision variables implicitly affect the objective function.
4. Despite disruptions, aerial distances remain unchanged.

Moreover, we assume that there are more open launch points than depots. Therefore, we prioritize distance between launch points and gathering points over distance between depots and gathering points.

Cluster-Based Heuristic Algorithm (Steps 1-4)

Full steps of the cluster-based heuristic algorithm are listed as follows:

Step 1. Using k-means algorithm to cluster the gathering points into p clusters, where p is the maximal number of open launch points.

Step 2. For each launch point, calculate the total distance between the launch point and all gathering points in a cluster and assign the launch points to the cluster with the smallest total distance.

Step 3. Among all launch points in the same cluster, select the one with the smallest total distance to the gathering points as the open launch point.

Step 4. Using k-means algorithm to cluster all open launch points into e clusters, where e is the maximal number of open depots.

Cluster-Based Heuristic Algorithm (Steps 5-8)

Step 5. For each depot, calculate the total distance between the depot and all open launch points in a cluster and the total distance to the gathering points assigned to those launch points. Assign the depots to the cluster with the smallest total distance.

Step 6. Among all depots in the same cluster, select the one with the smallest total distance to the open launch points as the open depot.

Step 7. The value of the first-stage decision variables u_d and v_l are determined by the open depots and launch points, and the assignment of launch points to depots.

Step 8. Fix the value of the first-stage decision variables and solve the second-stage problem in each scenario. Average the objective value of all scenarios as the optimal solution.

Recap of Benders Decomposition

Using Benders decomposition, the two-stage stochastic programming problem can be decomposed into a master problem and a subproblem. For the master problem, the formulation is:

$$\begin{aligned} \min \quad & c^T x + q^T y \\ \text{subject to} \quad & Ax = b \\ & x \geq 0. \end{aligned}$$

For a feasible first-stage solution x, $c^T x$ becomes a fixed value, and the remaining part of the objective function is $q^T y$. For a certain scenario $s = 1, \ldots, S$, we will have a distinct sub-problem and its dual problem as follows:

$$\begin{aligned} \text{(P)} \quad \min \quad & q^T y_s \\ \text{subject to} \quad & W y_s = h_s - T_s x, \\ & y_s \geq 0, \end{aligned}$$

$$\begin{aligned} \text{(D)} \quad \max \quad & (h_s - T_s x)^T u \\ \text{subject to} \quad & W^T u \leq q. \end{aligned}$$

Benders Cuts

Assuming under scenario s, we have the optimal solution u^*, that $(h_i - T_i x)^T u^* \leq \min\ z_s(x)$. In the dual problem, we assume there are n extreme points, $u_j, j = 1, \ldots, n$ and m extreme rays $r_k, k = 1, \ldots, m$.

For any extreme rays r_k, we want it to be constrained by following:

$$(h_s - T_s x)^T r_k \leq 0,$$

- if $(h_s - T_s x)^T r_k > 0$, the function value goes to ∞;
- **Unbounded** Dual $\Rightarrow$ **Infeasible** Primal;
- Avoid such x by adding the constraint $(h_s - T_s x)^T r_k \leq 0$ to the master problem.

For an optimal solution u_j^* for the dual problem, the following inequality:

$$(h_s - T_s x)^T u_j^* \leq z_s(x), \quad \forall s = 1, \ldots, S,$$

holds for every feasible decision y with given x.

- Get the tighter lower approximation by keep adding the constraint $(h_s - T_s x)^T u_j^* \leq z_s(x)$ to the master problem

Benders decomposition in Mixed-Integer Programming

However, Benders decomposition generally does not work well in mixed-integer programming problems. The main reasons are:

- ⋆ dual variables, which are required to derive both optimality cuts and feasibility cuts, are not well-defined for integer variables;
- ⋆ the performance of Benders decomposition is highly dependent on the quality of the benders cuts generated;

Since Benders decomposition was proposed, several studies has been conducted to address the issues mentioned above. One example will be shown in the next slide.

Benders Decomposition with Integer Variables

For the two-stage SP with binary first-stage decision variables, Laporte and Louveaux 1993 proposed the following cuts:

$$\theta \geq (Q_r(x) - L)\left(\sum_{i \in S_r} x_i - \sum_{i \notin S_r} x_i\right) - (Q_r(x) - L)(|S_r| - 1) + L, \tag{5}$$

where $r = 1 \ldots R$ index feasible first-stage solutions and x refer to the r-th feasible solution. $x_i = 1, \forall i \in S_r$, $x_i = 0, \forall i \notin S_r$. $Q_r(x)$ is the corresponding expected second-stage value. Notation $|S_r|$ refers to the cardinality of the S_r. The L is the lower bound of the second-stage objective function.

However, in a SP model with mixed-integer decision variables in the second stage, this cut is not tight enough to valid the ideal strong duality. Therefore, due to limit of time and complexity of implementation, we do not further consider Benders decomposition in this study.

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Road Distance after Earthquake I

In Erbeyoglu and Bilge (2020)[13], they evaluated four intensity predictive models that can be applied to estimate the damage for all potential earthquakes:

$$I_s = 7.023 + 0.703 M_s - 2.826 \log_{10}(R_{epi}), \tag{6a}$$

$$I_s = 5.002 + 0.75 M_s - 0.0094 R_{epi} - 1.454 \log_{10}(R_{epi}), \tag{6b}$$

$$I_s = 7.494 + 0.744 M_s - 3.377 \log_{10} \sqrt[3]{R_{epi}^3 + h^3} + 0.017 h, \tag{6c}$$

$$I_s = 2.281 + 0.874 M_s - 0.618 \log_{10} \sqrt{1 + \frac{R_{epi}^2}{h^2}} - 0.016 \left(\sqrt{h^2 + R_{epi}^2} - h \right), \tag{6d}$$

where I_s indicates intensity of the earthquake, R_{epi} refers to distance between the epicenter and the facility location, M_s represents the surface-wave magnitude and h is hypocentral depth of the earthquake.

[13] Erbeyoğlu and Bilge 2020.

Road Distance after Earthquake II

The average of the intensity values from the formulas (6a) - (6d) is used as the final intensity value for a certain facility and we can estimate how severe the damage is at each facility location using figure 3:

I_s (MSK)	Definition	Damage States			
		No damage	Slight damage	Medium damage	Heavy damage and collapse
6	VI-Strong	99.5%	0.24%	0.22%	0.04%
7	VII-Very strong	93.83%	2.59%	2.67%	0.91%
8	VIII-Damaging	87.46%	5.31%	4.41%	2.82%
9	IX-Destructive	43.39%	22.75%	18.16%	15.70%
10	X-Devastating	32.51%	19.14%	15.29%	33.06%

Figure: MSK Scale for Earthquake Intensity

We calculate the damage percentages of each depot/launch points and consider their average and increase the road distance by the average.

Performance Evaluation for SDA and Heuristic Algorithm

In this study, we conducted numerical experiments to evaluate the performance of the SDA and heuristic algorithm. We did 2 series of test, each with 12 scenarios. The detailed settings are listed as follows:

- ⋆ maximal open launch points $e \in \{4, 5, 6\}$, maximal number of open depots $p \in \{1, 2\}$,
- ⋆ combination of number of available drones are of 3 bundles: $(K_{SD}, K_{LD}) \in \{(10, 1), (15, 2), (20, 3)\}$.

We solved the SAA formulation with 3 different methods: (1) Gurobi with default settings, (2) scenario decomposition method, and (3) heuristic algorithm. The results are shown in the following slides.

Performance Evaluation Results

Series	p	e	K_{SD}	K_{LD}	SAA	Time (s)	SDA	Time (s)	Heuristic	Time (s)	Gap (%)
1	4	1	10	1	4038.84	8.64	4038.84	116.76	4201.05	8.48	4.02
1	4	1	15	2	3921.66	13.12	3921.66	175.12	4116.05	12.77	4.96
1	4	1	20	3	3851.17	17.27	3850.83	243.13	4031.05	16.99	4.67
1	4	2	10	1	4036.47	9.51	4036.47	130.9	4073.26	8.38	0.91
1	4	2	15	2	3806.17	15.29	3806.17	200.29	3967.86	12.65	4.25
1	4	2	20	3	3686.25	20.46	3686.25	268.38	4022.40	16.85	9.12
1	5	1	10	1	4038.84	9.93	4038.84	135.04	4201.05	8.54	4.02
1	5	1	15	2	3921.66	15.09	3921.66	202.01	4164.75	12.9	6.20
1	5	1	20	3	3851.17	20.21	3850.83	268.94	4031.05	16.96	4.67
1	5	2	10	1	4036.47	9.6	4036.47	127.77	4208.28	8.56	4.26
1	5	2	15	2	3806.17	14.14	3806.17	200.91	3912.02	13.04	2.78
1	5	2	20	3	3686.25	20.98	3686.25	269.32	4072.72	17.41	10.48
1	6	1	10	1	4038.84	9.99	4038.84	134.67	4201.05	8.47	4.02
1	6	1	15	2	3921.66	15.35	3921.66	201.78	4116.05	12.69	4.96
1	6	1	20	3	3851.17	20.55	3850.83	270.05	4031.05	16.97	4.67
1	6	2	10	1	4036.47	9.81	4036.47	135.14	4051.09	8.47	0.36
1	6	2	15	2	3806.17	14.88	3806.17	196.79	3911.91	12.71	2.78
1	6	2	20	3	3686.25	18.9	3686.25	270.72	3837.35	17.25	4.10
Series 1 Average						14.65		197.10		12.78	4.51

Figure: Part of Performance Evaluation Results

Large Scenario Test

From the previous slide, it states that compared to directly using Gurobi with default settings and heuristic algorithm, the SDA takes much longer time to solve the problem. This result indicates the following facts:

1. the structure of the SAA formulation is not complicated;
2. SDA performs bad in large-scale scenarios.

To further verify the conclusion, we conducted a large scenario test. For 3 independent series of test, we increase the number of scenarios considered from 20 to 100. The results are shown in the following:

Test	S	p	e	K_{SD}	K_{LD}	Heuristic	Time (s)	SAA	Time (s)	SDA	Time (s)
7	20	10	4	20	3	4054.92	28.63	4004.85	30.99	4004.85	741.87
8	50	10	4	20	3	3257.99	71.53	3217.14	76.65	3217.14	4945.52
9	100	10	4	20	3	3728.52	147.82	3428.72	158.42	3428.69	16002.26

Figure: Large Scenario Test Results

Value of Considering Weather Uncertainty

Scenario	Our Model	o_s	Original Model	o_s
1	3336.42	7.97	3861.05	9.43
2	3336.42	7.97	3861.05	9.43
3	3336.42	7.97	3861.05	9.43
4	3445.20	8.28	3927.27	9.62
5	3445.20	8.28	3927.27	9.62
6	3336.42	7.97	3861.05	9.43
7	3553.75	8.61	4079.85	10.06
8	3336.42	7.97	3861.05	9.43
9	3336.42	7.97	3861.05	9.43
10	3337.78	7.97	3861.05	9.43
11	3445.20	8.28	3927.27	9.62
12	3564.49	8.64	4079.85	10.06
Average	3400.85	8.15	3914.07	9.58

Figure: Test Results Comparison: Original Model vs. Model w. Weather Uncertainty

Conclusion: the results show that considering weather uncertainty in the model leads to lower objective value and lower average unmet demand.

Impact of Fairness

In this study, we further consider the fairness issue in the problem. To integrate the fairness into the model, we introduce 2 types of equity terms:

Equity Constraint:

$$\sum_{k \in K_{SD}} \sum_{l \in \{L \cup D\}} x_{silk} + \sum_{k \in K_{LD}} \sum_{d \in D} y_{sidk} \geq 1, \quad \forall i \in I, s \in S, \tag{7}$$

The constraint ensures that each gathering point is served by at least one drone.

Penalty Term of Unfairness in Objective Function:

$$\min \quad \frac{1}{|S|} \sum_{s \in S} \left(\sum_{i \in I} \pi o_{s,i} + \sum_{k \in K_{SD}} c_{SD} g_{s,k} + \sum_{k \in K_{LD}} c_{LD} g_{s,k} + \frac{1}{\mu} \left(\max_{i \in I} o_{s,i} - \min_{i \in I} o_{s,i} \right) \right), \tag{8}$$

The extra term penalizes the maximal difference in unmet demand among gathering points for each scenario.

Fairness Test Results

In this test, we set the value of parameters as follows:

1. maximal open launch points $e = 20$, maximal number of open depots $p = 5$, $|K_{SD}| = 30$, $|K_{LD}| = 5$, Time Frame $T = 2.5$ hrs;
2. maximal solution time for Gurobi is 3000 seconds;

Scenario	Equity Constraint		Equity Objective		Equity Combined		SAA	
	Average	Difference	Average	Difference	Average	Difference	Average	Difference
1	3.33	14.96	3.24	6.30	3.33	10.30	3.24	16.96
2	3.33	12.96	3.29	6.30	3.33	10.30	3.24	13.46
3	3.33	14.96	3.25	6.18	3.33	10.30	3.24	13.46
4	3.33	14.96	3.26	6.18	3.33	10.30	3.24	16.96
5	3.33	14.96	3.24	6.18	3.33	10.30	3.24	16.96
6	3.33	14.96	3.28	6.83	3.33	10.30	3.24	16.96
7	3.33	14.96	3.32	6.18	3.33	10.30	3.24	16.96
8	3.33	12.96	3.24	6.18	3.33	10.30	3.24	10.18
9	3.33	11.46	3.24	6.18	3.33	10.30	3.24	16.96
10	3.33	12.96	3.54	6.96	3.33	10.30	3.24	16.96
11	3.33	14.96	3.24	6.30	3.33	10.30	3.24	16.96
12	3.33	14.96	3.24	6.18	3.33	10.30	3.24	12.30
Average	3.33	13.98	3.28	6.35	3.33	10.30	3.24	15.39

Figure: Fairness Test Results

Discussion on Fairness Test Results

- The results show that the fairness constraint and penalty term significantly reduce the maximal difference in unmet demand among gathering points.
- However, both methods have their own advantages and disadvantages:
- **Fairness Constraint:**

  Advantages: Guarantees that each gathering point is served by at least one drone.
  Disadvantages: Break the nice structure, complete recourse, of the stochastic programming model.
- **Penalty Term of Unfairness in Objective Function:**

  Advantages: Keep the complete recourse property of the model; dramatically reduce the maximal difference in unmet demand.
  Disadvantages: Increase the complexity of the model by introducing two auxiliary decision variables, leading to a longer computation time, e.g. both model with modified objective function failed to solve within 3000 seconds in our test.

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Conclusion

In this study, we extended the model proposed by Dukkanci et al. (2023)[14] by considering uncertain wind conditions, integrating the size of fleets into the decision process. We formulated a two-stage stochastic programming model, and proposed to use SDA and heuristic algorithm to solve the problem. Moreover, a series of numerical experiments were conducted, and the results help us to draw the following conclusions:

1. Heuristic algorithm can solve the problem efficiently and effectively. In contrast, SDA can solve the optimal solution at the cost of longer computation time;
2. SDA is not suitable for large-scale scenarios;
3. Information of uncertain wind conditions is valuable in the decision-making process; Considering the affect of weather uncertainty can lead to a better solution with lower objective value and lower average unmet demand;
4. Fairness is an important issue in the optimization of drone-based delivery systems. Despite leading to a harder problem to solve, considering fairness can significantly reduce the difference in unmet demand among gathering points.

[14] Dukkanci, Koberstein, and Kara 2023.

Future Research Directions

Despite the achievements in this study, there are still some aspects that need to be further investigated:

1. Incorporate real-time data and machine learning techniques for enhanced predictive accuracy;
2. Reformulate the problem into robust optimization or distributionally robust optimization models to account for robustness of solution;
3. Considering a more complex drones' operation process;
4. Considering the multi-stop delivery for drones.

Distributionally Robust Optimization

- ◇ True distribution of the random parameters is usually unknown and hard to predict accurately.
- ◇ Inaccurate estimation of $\mathbb{P} \Rightarrow$ biased & sub-optimal.

Distributionally Robust Optimization

Distributionally robust optimization approaches are proposed as follows:

$$(\mathrm{DR}-\mathrm{SP}/\mathrm{DRO}) \quad \min_{x \in X} c^T x + \max_{\hat{\mathbb{P}} \in \mathscr{D}} \mathbf{E}_{\hat{\mathbb{P}}}\left[\mathscr{Q}(x, \xi)\right]. \tag{9}$$

Remark

The purpose of DRO (2) is to minimize the worst-case problem of any possible $\hat{\mathbb{P}}$ in confident set $\mathscr{D}$.

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