Title: TBD

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Abstract

We propsose X, Y, Z

Keywords:

1. Introduction

2. Literature Review

3. Problem Settings

1. Dukkanci et al. (2023)

4. Formulations

We consider this problem as a two-stage stochastic programming problem. In the first-stage, the decision maker is given a set of potential gathering points C, a set of locations for potential depots D, a set of locations for potential launch points L, and two sets of available drones K_S for the small drones, and K_L for large drones. The mathematical notation used in the formulation is given in Table 1.

Table 1: Notation

Sets	
I	Set of gathering points
D	Set of potential depot locations
L	Set of potential depot locations
K_{SD}	Set of small drones
K_{LD}	Set of Large drones
Parameters	
e	Maximal number for open depots
p	Maximal number for open launch points
Q_T	Capacity of Trucks
Q_{SD}	Capacity of small drones
Q_{LD}	Capacity of large drones
c_{SD}	Cost of using small drones
c_{LD}	Cost of using large drones
v_T	Speed of trucks
v_{max}^{SD}	Maximal speed of small drones
v_{max}^{LD}	Maximal speed of large drones
T	Length of time horizon
τ	Preparation time for small drones
ω_{ld}	Ground distance between depot d and launch point l after the earthquake
q_i	Demand at gathering point i
π	Penalty per unit of unmet demand
First-stage decision variables	
u_d	1, if a depot is open at location $d \in D$, 0 otherwise
v_l	1, if a launch point is open at location $l \in L$,0 otherwise
w_{ld}	1, if launch point l is assigned to depot d, 0 otherwise
Second-stage decision variables	
g_k	1, if a drone $k \in \{K_{SD} \cup K_{LD}\}$ is used, 0 otherwise
x_{ilk}	1, if a small drone $k \in K_{SD}$ is assigned to serve point $i \in I$ from location $l \in \{L \cup D\}$, 0 otherwise
y_{idk}	1, if a large drone $k \in K_{LD}$ is assigned to serve point $i \in I$ from depot $d \in D$, 0 otherwise
o_i	Unmet demand for gathering point $i \in I$

The first-stage decision variables are u_d, v_l, and w_{ld}, which represent the decision of opening a depot at location d, opening a launch point at location l, and assigning launch point l to depot d, respectively. The second-stage decision variables are g_k, x_{ilk}, y_{idk}, and o_i, which represent the decision of using drone k, assigning small drone k to serve point i from location l, assigning large drone k to serve point i from depot d, and the unmet demand for gathering point i, respectively. The objective function of the first-stage problem is to minimize the expected cost of the second-stage problem. The first-stage problem is subject to constraints on the number of open depots and launch points, and the assignment of launch points to depots. The second-stage problem is to minimize the expected cost of using drones, the penalty for unmet demand, and the cost of using drones. The second-stage problem is subject to constraints on the assignment of drones to gathering points, the capacity of trucks, the capacity of small drones, the capacity of large drones, the unmet demand, and the assignment of launch points to depots. The complete formulation of the problem is given below.

4.1. Stochastic Program

Our stochastic programming formulation aims to solve the following problem:

$$
\begin{aligned}
\min \quad & \mathbb{E}_{\mathbb{P}}\left[Q(u, v, w, \xi)\right] && && \text{(1a)}\\
\text{subject to} \quad & \sum_{d \in D} u_d \leq e && && \text{(1b)}\\
& \sum_{l \in L} v_l \leq p && && \text{(1c)}\\
& \sum_{d \in D} w_{ld} = v_l, && \forall l \in L && \text{(1d)}\\
& w_{ld} \in \{0,1\}, && \forall d \in D, l \in L && \text{(1e)}\\
& u_d \in \{0,1\}, && \forall d \in D && \text{(1f)}\\
& v_l \in \{0,1\}, && \forall l \in L && \text{(1g)}
\end{aligned}
$$

Our uncertainties in the second-stage decision are indicated as the parameter $\xi \in \Xi$, where $\xi = (\omega, q)$, and the second stage problem is constructed as:

$$Q[u,v,w] := \quad \min \quad \sum_{i \in I} \pi o_i + \sum_{k \in K_{SD}} c_{SD} g_k + \sum_{k \in K_{LD}} c_{LD} g_k \tag{2a}$$

$$\text{subject to} \quad y_{idk} \leq u_d \qquad \forall d \in D, i \in I, k \in \{K_{SD} \cup K_{LD}\} \tag{2b}$$

$$x_{ilk} \leq v_l \qquad \forall l \in L, i \in I, k \in K_{SD} \tag{2c}$$

$$\sum_{k \in K_{SD}} \sum_{i \in I} q_i x_{ilk} \leq Q_T v_l \qquad \forall l \in L \tag{2d}$$

$$q_i - \sum_{k \in K_{SD}} \sum_{l \in \{L \cup D\}} Q_{SD} x_{ilk} - \sum_{k \in K_{LD}} \sum_{d \in D} Q_{LD} y_{idk} \leq o_i \qquad \forall i \in I \tag{2e}$$

$$\sum_{d \in D} \frac{\omega_{ld}}{v_T} w_{ld} + \sum_{i \in I} \left(\frac{2d_{li}}{v_{li}} + \tau \right) x_{ilk} \leq T \qquad \forall l \in L, k \in K_{SD} \tag{2f}$$

$$\sum_{i \in I} \left(\frac{2d_{li}}{v_{li}} + \tau \right) x_{ilk} \leq T \qquad \forall l \in D, k \in K_{SD} \tag{2g}$$

$$\frac{2d_{li}}{v_{li}} y_{idk} \leq T \qquad \forall d \in D, i \in I \tag{2h}$$

$$g_k \in \{0,1\}, \qquad \forall k \in \{K_{SD} \cup K_{LD}\} \tag{2i}$$

$$x_{ilk} \in \{0,1\}, \qquad \forall l \in L, i \in I, k \in K_{SD} \tag{2j}$$

$$y_{idk} \in \{0,1\}, \qquad \forall d \in D, i \in I, k \in K_{LD} \tag{2k}$$

$$o_i \geq 0, \qquad \forall i \in I \tag{2l}$$

5. Computational Results

6. Conclusion

References

Dukkanci, O., Koberstein, A., Kara, B.Y., 2023. Drones for relief logistics under uncertainty after an earthquake. European Journal of Operational Research 310, 117–132.