10
Boosting and Additive Trees

10.1 Boosting Methods

Boosting is one of the most powerful learning ideas introduced in the last twenty years. It was originally designed for classification problems, but as will be seen in this chapter, it can profitably be extended to regression as well. The motivation for boosting was a procedure that combines the outputs of many "weak" classifiers to produce a powerful "committee." From this perspective boosting bears a resemblance to bagging and other committee-based approaches (Section 8.8). However we shall see that the connection is at best superficial and that boosting is fundamentally different.

We begin by describing the most popular boosting algorithm due to Freund and Schapire (1997) called "AdaBoost.M1." Consider a two-class problem, with the output variable coded as $Y \in \{-1, 1\}$. Given a vector of predictor variables X, a classifier $G(X)$ produces a prediction taking one of the two values $\{-1, 1\}$. The error rate on the training sample is

$$\overline{\text{err}} = \frac{1}{N} \sum_{i=1}^{N} I(y_i \neq G(x_i)),$$

and the expected error rate on future predictions is $\text{E}_{XY} I(Y \neq G(X))$.

A weak classifier is one whose error rate is only slightly better than random guessing. The purpose of boosting is to sequentially apply the weak classification algorithm to repeatedly modified versions of the data, thereby producing a sequence of weak classifiers $G_m(x), m = 1, 2, \ldots, M$.

T. Hastie et al., *The Elements of Statistical Learning, Second Edition,*
DOI: 10.1007/b94608_10,

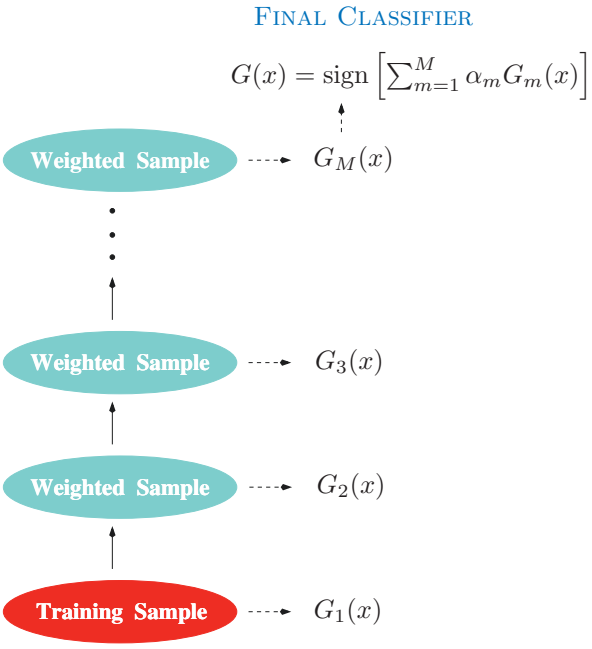

FIGURE 10.1. *Schematic of AdaBoost. Classifiers are trained on weighted versions of the dataset, and then combined to produce a final prediction.*

The predictions from all of them are then combined through a weighted majority vote to produce the final prediction:

$$G(x) = \text{sign}\left(\sum_{m=1}^{M} \alpha_m G_m(x)\right). \tag{10.1}$$

Here $\alpha_1, \alpha_2, \ldots, \alpha_M$ are computed by the boosting algorithm, and weight the contribution of each respective $G_m(x)$. Their effect is to give higher influence to the more accurate classifiers in the sequence. Figure 10.1 shows a schematic of the AdaBoost procedure.

The data modifications at each boosting step consist of applying weights $w_1, w_2, \ldots, w_N$ to each of the training observations (x_i, y_i), $i = 1, 2, \ldots, N$. Initially all of the weights are set to $w_i = 1/N$, so that the first step simply trains the classifier on the data in the usual manner. For each successive iteration $m = 2, 3, \ldots, M$ the observation weights are individually modified and the classification algorithm is reapplied to the weighted observations. At step m, those observations that were misclassified by the classifier $G_{m-1}(x)$ induced at the previous step have their weights increased, whereas the weights are decreased for those that were classified correctly. Thus as iterations proceed, observations that are difficult to classify correctly receive ever-increasing influence. Each successive classifier is thereby forced

Algorithm 10.1 *AdaBoost.M1.*

1. Initialize the observation weights $w_i = 1/N,\ i = 1, 2, \ldots, N$.
2. For $m = 1$ to M:
   (a) Fit a classifier $G_m(x)$ to the training data using weights w_i.
   (b) Compute
   $$\text{err}_m = \frac{\sum_{i=1}^{N} w_i I(y_i \neq G_m(x_i))}{\sum_{i=1}^{N} w_i}.$$
   (c) Compute $\alpha_m = \log((1 - \text{err}_m)/\text{err}_m)$.
   (d) Set $w_i \leftarrow w_i \cdot \exp[\alpha_m \cdot I(y_i \neq G_m(x_i))],\ i = 1, 2, \ldots, N$.
3. Output $G(x) = \text{sign}\left[\sum_{m=1}^{M} \alpha_m G_m(x)\right]$.

to concentrate on those training observations that are missed by previous ones in the sequence.

Algorithm 10.1 shows the details of the AdaBoost.M1 algorithm. The current classifier $G_m(x)$ is induced on the weighted observations at line 2a. The resulting weighted error rate is computed at line 2b. Line 2c calculates the weight α_m given to $G_m(x)$ in producing the final classifier $G(x)$ (line 3). The individual weights of each of the observations are updated for the next iteration at line 2d. Observations misclassified by $G_m(x)$ have their weights scaled by a factor $\exp(\alpha_m)$, increasing their relative influence for inducing the next classifier $G_{m+1}(x)$ in the sequence.

The AdaBoost.M1 algorithm is known as "Discrete AdaBoost" in Friedman et al. (2000), because the base classifier $G_m(x)$ returns a discrete class label. If the base classifier instead returns a real-valued prediction (e.g., a probability mapped to the interval $[-1, 1]$), AdaBoost can be modified appropriately (see "Real AdaBoost" in Friedman et al. (2000)).