诚信应考,考试作弊将带来严重后果！

华南理工大学期末考试

《2020 Exam Calculus B》试卷

注意事项：1. 考前请将密封线内填写清楚；
2. 所有答案请直接答在试卷上(或答题纸上)；
3. 考试形式：闭卷；
4. 本试卷共 8 大题，满分 100 分， 考试时间 120 分钟。

题 号	1	2	3, 4	5, 6	7, 8	总分
得 分						
评卷人						

1. Answer the questions (20):

(1) The series as absolutely convergent, conditionally convergent or divergent series $\sum_{n=1}^{\infty}\frac{(-1)^n}{1+n^2}$

Answer ----------------

(2) Suppose $3x^2z+y^3-xyz^3=0$, find $\partial z/\partial x$

Answer ----------------

(3) Find $div(curl\,\vec{F})$ *and* $grad\,(div\vec{F})$ if $\vec{F}=2xyz\,\vec{i}-3y^2\,\vec{j}+2y^2z\,\vec{k}$

Answer ----------------

(4) Solve differential equation $y''''-4y=0$

Answer ----------------

(4') Does the limit $\lim\limits_{(x,y)\to(0,0)}\frac{xy+x^3}{x^2+y^2}$ exist?

Answer ----------------

(5) Find f such that $\vec{F}=\nabla f$, while $\vec{F}=\left(e^x\cos y+yz\right)\vec{\mathbf{i}}+\left(xz-e^x\sin y\right)\vec{\mathbf{j}}+\left(xy\right)\vec{\mathbf{k}}$

Answer ----------------

2. Evaluate the problems (30):

(1) Test for the convergence or divergence $\sum_{n=1}^{\infty}\left(\frac{n}{3n+2}\right)^{n}$

(2) Find the convergence set for the power series

$$\sum_{n=1}^{\infty}\frac{(3x+1)^{n}}{n\bullet 2^{n}}$$

(3) Find the equation of the plane through $(2,-1,4)$ and perpendicular to both the plane $x+y+z=2$ and $x-y-z=4$

(4) Find the maximum $z = -4x^3y^2$ subject to $x^2 + y^2 = 1$

(5) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \left(4 - x^2 - y^2\right)^{-1/2} dxdy$.

3. (10) Evaluate the flux of $\vec{F}$ across G. Where $\vec{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, G is the surface above x, y plane determined by $z = 1 - x^2 - y^2$ $-\infty < x < +\infty,\ -\infty < y < +\infty,$ and the normal direction upward

4. (10) $\oint_C (e^{3x} + 2y)dx + (x^2 + \sin y)dy$ where C is the rectangle with vertices $(2,1), (6,1), (6,4), (2,4)$

5. (5) Evaluate $\int_C \left(x^2+y^2+z^2\right)ds$;

C is the curve $x=4\cos t,\quad y=4\sin t,\quad z=3t,\quad 0\le t\le 2\pi$

6. (7) Solve differential equation $y''+y=\cot x$

6'. (7) If f is a twice differentiable function and c is a constant number, show that the function

$y(x,t)=\frac{1}{2}\left[f(x-ct)+f(x+ct)\right]$ satisfies $\frac{\partial^2 y}{\partial t^2}=c^2\frac{\partial^2 y}{\partial x^2}$.

7. (8) Evaluate the $\iint\limits_{\partial S} \vec{F} \cdot \vec{n}\, dS$. Where

$\vec{F}(x, y, z) = (x^2 + \cos yz)\vec{i} + (y - e^z)\vec{j} + (z^2 + x^2)\vec{k}$.

S is the solid bounded by $x^2 + y^2 = 4, x + z = 2, z = 0$.

8. (10) Evaluate $\int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_{-\sqrt{9-x^2-z^2}}^{\sqrt{9-x^2-z^2}} (x^2 + y^2 + z^2)^{3/2} dydzdx$