Seat NO. _____ Major _____ School _____ Student ID □□□□□□□□□□ Chinese Name _____

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（DONNOT WRITE YOUR ANSWER IN THIS AREA）

WARNING: MISBEHAVIOR AT EXAM TIME WILL LEAD TO SERIOUS CONSEQUENCE.

SCUT Final Exam

《Calculus II》 Exam Paper B

Notice: **1. Make sure that you have filled the form on the left side of seal line.**
2. Write your answers on the exam paper .
3. This is a close-book exam.
4. The exam with full score of 100 points lasts 120 minutes.

Question No.	1-6	7-17	18	Sum
Score				

一. **Please fill the correct answers in the following blanks.** $(3'\times 6=18')$

1. If $\vec{a}=<-1,2,1>$, $\vec{b}=<2,-1,2>$, then $(\vec{a}-\vec{b})\times(\vec{a}+\vec{b})$ is _____.

2. The directional derivative of $f(x,y,z)=x^2+y^2+z^2$ at the point $(1,-1,2)$ in the direction of $\vec{a}=\sqrt{2}\vec{\mathrm{i}}-\vec{\mathrm{j}}-\vec{\mathrm{k}}$ is _____.

3. Let $z=(1-xy)^y$, $\dfrac{\partial z}{\partial x}$ is _____.

4. The sum of the series $\displaystyle\sum_{k=1}^{+\infty}\frac{2^k}{(2^{k+1}-1)(2^k-1)}$ is _____.

5. The divergence of the vector field $\vec{A}=2e^{xy}\vec{i}+2\cos(xy)\vec{j}+2xz^2\vec{k}$ is_____.

The curl of the vector field $\vec{B}=2(2z-3y)\vec{i}+2(3x-z)\vec{j}+2(y-2x)\vec{k}$ is_____.

6. The minimum distance between $y=x^2$ and $x-y-2=0$ is _____.

二、Finish the following questions. (7-17: $7'\times 11=77'$; 18: $5'\times 1=5'$)

7. Determine the convergence or divergence of the following series.

(1) $\sum_{k=1}^{+\infty}\frac{\arctan k}{1+k^2}$;

(2) $\sum_{n=1}^{+\infty}\frac{n^3}{(2n)!}$

8. Expand the function $f(x)=2\ln\frac{1+x}{1-x}$ into power series of x.

9. Let $\begin{cases}x=t-\ln(1+t)\\ y=t^3+t^2\end{cases}$ and y is a function of x, find $\frac{d^2y}{dx^2}$.

10. Let $z = xf\left(xy\ ,\ \frac{y}{x}\right)$, f has continuous second partial derivatives, find $\frac{\partial z}{\partial y}$, $\frac{\partial^2 z}{\partial y \partial x}$

11. Show that $\lim\limits_{(x,y)\to(0,0)} \frac{xy+y^3}{x^2+y^2}$ does not exist.

12. Compute $\iint\limits_D 2y d\sigma$, D is the region determined by $2a^2 \le x^2+y^2 \le 2ay$ $(a>0)$.

13. Evaluate $\iint_S \sin(y^3)\mathrm{d}A$, where S is the region bounded by $y=\sqrt{x}, y=2$ and $x=0.$

14. Find the area of the part of sphere $x^2+y^2+z^2=a^2$ inside the cylinder $x^2+y^2=ax$ $(a>0).$

15. Evaluate $\int_C (x^2 + y^2 + z^2)ds$, C is the curve $x = 4\cos t, y = 4\sin t, z = 3t, 0 \le t \le 2\pi.$

16. Find the volume of Ω bounded by $z = 4 - x^2 - y^2$ and $x^2 + y^2 = 2x$ in the first octant.

17.Let $\int_L xy^2dx + y\varphi(x)dy$ is independent of path, and $\varphi(x)$ is continuous and derivative, and $\varphi(0)=0$, compute $\int_{(0,0)}^{(1,1)} xy^2dx + y\varphi(x)dy$.

18. Determine whether $(8x^3+18x^2y^2)dx+(12x^3y+12y^5)dy$ is conservative, if yes please find $u(x,y)$ such that $du(x,y)=(8x^3+18x^2y^2)dx+(12x^3y+12y^5)dy$.