Chinese Name_____ Student ID [ ][ ][ ][ ][ ][ ][ ][ ] School _____ Major _____ Sit NO. _____

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WARNING: MISBEHAVIOR AT EXAM TIME WILL LEAD TO SERIOUS CONSEQUENCE.

SCUT Final Exam

2019-2020-2《Calculus II》 Exam Paper A

Notice: **1. Make sure that you have filled the form on the left side of seal line.**
2. Write your answers on the exam paper .
3. This is a close-book exam.
4. The exam with full score of 100 points lasts 120 minutes.

Question No.	1-5	6-12	13-22	Sum
Score				

一. **Answer the questions.** ($3' \times 5 = 15'$)

1. Interchange the integral orders, then $\int_{-1}^{0} dy \int_{-y}^{1} f(x,y)dx + \int_{0}^{1} dy \int_{\sqrt{y}}^{1} f(x,y)dx$.

Answer _____

2. Suppose $ye^{-x} + z\sin x = 0$, find $\partial z/\partial x$

Answer _____

3. Find $\mathrm{div}(\vec{F})$ and $\mathrm{curl}(\vec{F})$ if $\vec{F} = x^2yz\,\vec{i} + 3xyz^3\,\vec{j} + (x^2 - z^2)\,\vec{k}$

Answer _____

4. Find f such that $\vec{F} = \nabla f$, while

$$\vec{F} = \left(45x^4y^2 - 6y^6 + 3\right)\vec{\mathbf{i}} + \left(18x^5y - 12xy^5 + 7\right)\vec{\mathbf{j}}$$

Answer _____

5. Does the limit $\lim\limits_{(x,y)\to(0,0)} \dfrac{xy + \cos x}{xy - \cos x}$ exist?

Answer _____

二、Finish the following questions. (6-11: $6' \times 6 = 36'$; 12: $7' \times 1 = 7'$)

6. Find the equation of the plane through $(6, 2, -1)$ and perpendicular to the line of intersection of planes $4x - 3y + 2z + 5 = 0$ and $3x + 2y - z + 11 = 0$

7. Find the minimize $z = x - \frac{x^3}{8} - \frac{y^2}{3}$ subject to $\frac{x^2}{16} + y^2 = 1$

8. Evaluate $\int_1^2 \int_0^{\sqrt{2x-x^2}} \left(x^2 + y^2\right)^{-1/2} dxdy$.

9. Suppose that a differentiable function $f(x,y)$ satisfies $f(tx,ty)=tf(x,y)$ for all $t>0$. Show that $f(x,y)=x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}$.

10. $\oint_C (x^2+4xy)dx+(2x^2+3y)dy$ where C is the ellipse $9x^2+16y^2=144$ with counter clockwise direction.

11. Evaluate $\int_C (1-y^2)ds$; C is the quarter circle from $(0,-1)$ *to* $(1,0)$ center at the origin

12. Evaluate $\int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_{-\sqrt{9-x^2-z^2}}^{\sqrt{9-x^2-z^2}} (x^2+y^2+z^2)^{3/2} dydzdx$

三、**Please select 6 questions from the following 10 questions** ($7' \times 6 = 42'$)

(请从下面的 10 道题中选择 6 道题目来回答，并把答案写在试卷上)

13. Test for the convergence or divergence $\sum_{n=1}^{\infty} \frac{n}{n5^n + 2}$

14. Find the convergence set for the power series $\sum_{n=0}^{\infty} \frac{(x-1)^n}{(n+1)^2}$

15. Solve differential equation $y'' + y = \sec x$

16. Solve differential equation $y'''' - 2y''' + 5y'' = 0$

17. Let $z = xf\left(xy\ ,\ \frac{y}{x}\right)$, and f has the second-order continuous partial derivatives, find $\frac{\partial z}{\partial y}$, $\frac{\partial^2 z}{\partial y \partial x}$

18. Let $z = f(u, x, y),\quad u = xe^y$, and f has second-order continuous partial derivatives， find $\frac{\partial^2 z}{\partial x \partial y}$.

19. Find $\iiint_{\Omega} z dv$， and Ω is bounded by $x^2 + y^2 = 1$ and $z = 0$, $z = 1$.

20. Find $I = \iiint_{\Omega} \frac{dv}{x^2 + y^2 + z^2}$, Ω is bounded by $z = 1 + \sqrt{1 - x^2 - y^2}$ and $z = 1$.

21. Find $\iint_{\Sigma} xz^2 dydz + \left(x^2 y - z^2\right) dzdx + \left(2xy + y^2 z\right) dxdy$ ， Σ is hemisphere (half a sphere) $z = \sqrt{a^2 - x^2 - y^2}$ with upside direction.

22. Line integral $\int_L xy^2 dx + y\varphi(x) dy$ is independent of path，and $\varphi(x)$ is derivative， $\varphi(0) = 0$ ，find $\int_{(0\ ,\ 0)}^{(1\ ,\ 1)} xy^2 dx + y\varphi(x) dy$ 。

Please write your answers on the exam paper.