

## Quiz 12

1. Let  $f(x, y) = \begin{cases} (x+y)^2 \sin \frac{1}{x^2+y^2}, & x^2+y^2 \neq 0 \\ 0, & x^2+y^2 = 0 \end{cases}$

1) Find  $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ .

2) Determine the continuity of  $\frac{\partial f}{\partial x}$  and  $\frac{\partial f}{\partial y}$  at the origin.

3) Determine the differentiability of  $f(x, y)$  at the origin.

2. Let  $z = xf\left(\frac{y}{x}\right) + yg\left(x, \frac{x}{y}\right)$ ,  $f'', g''$  both exist, try to find  $\frac{\partial^2 z}{\partial x \partial y}$ .

3. If  $z^x = y^z$ , try to find  $dz$ .

4. Assume  $z = f(x, y), x = \varphi(y, z)$ , and  $f, \varphi$  are differentiable. Please find  $\frac{dz}{dx}$ .

5. Find the directional derivative of  $u = e^x \cos(yz)$  at  $(0, 0, 0)$  in the direction of  $\vec{l} = \{2, 1, -2\}$ .

6. Please find the angle between the gradient of  $u = x^2 + y^2 - z^2$  at  $A(a, 0, 0)$  and the gradient of  $u = x^2 + y^2 - z^2$  at  $B(0, a, 0)$ .

7. Let  $\begin{cases} z = x^2 + y^2 \\ x^2 + 2y^2 + 3z^2 = 20 \end{cases}$ , please find  $\frac{dy}{dx}$  and  $\frac{dz}{dx}$ .

8. Please find the equation of the tangent line and the equation of the normal plane of the curve  $x = \sin^2 t, y = \sin t \cos t, z = \cos^2 t$  at  $t = \frac{\pi}{4}$ .

9. Find the equation of the tangent plane of the surface  $ax^2 + by^2 + cz^2 = 1$  ( $abc \neq 0$ ) at the point  $(x_0, y_0, z_0)$ .

10. Find the maximum and minimum values of  $f(x, y) = x^2 - y^2$ ,

here  $(x, y) \in \{(x, y) \mid x^2 + y^2 \leq 4\}$ .