座位号________ 专业________ 学院________ 学号________ 姓名________

（密封线内不答题）

密 封 线

诚信应考,考试作弊将带来严重后果！

华南理工大学期末考试

《20220427 Middle Calculus Exam》试卷

注意事项：1. 考前请将密封线内填写清楚；
2. 所有答案请直接答在试卷上(或答题纸上)；
3. 考试形式：闭卷；
4. 本试卷共 8 大题，满分 100 分， 考试时间 120 分钟。

题 号	1,2	3, 4	5, 6	7, 8	9, 10	总分
得 分						
评卷人						

1. Answer the questions (16):

(1) The series as absolutely convergent, conditionally convergent or divergent series

$$\sum_{n=1}^{\infty}(-1)^{n+1}\sin\frac{\pi}{n}$$

(2) The series as absolutely convergent, conditionally convergent or divergent series

$$\sum_{n=1}^{\infty}(-1)^{n+1}\left(\frac{n}{3n+2}\right)^{n}$$

(3) Find the volume of the tetrahedron with vertices $(-1,2,3)$, $(4,-1,2)$, $(5,6,3)$

(4) Find the distance between each pair of line $\frac{x-3}{1}=\frac{y+2}{1}=\frac{z-1}{2}$, $\frac{x+4}{1}=\frac{y+5}{1}=\frac{z}{2}$

(5) Find ∇f and $D_u f$, where $f = x^3+\tan yz$, $u=(2,0,-4)$

(6) The Maclaurin series of $f(x)=\frac{1}{x^2+x+1}$ at $x=0$

(7) Write a 3 variables function and its limit doesn't exist while (x,y,z) approach to (0,0,0).

(8) Evaluate $\iint_D \sqrt{\frac{x+y}{x-y}}dxdy$, D *is triangle with vertices* $(1,0),(4,0),(4,3)$

2. Evaluate the problems (30):

(1) Show that the curvature of polar curve $r^2=\cos 2\theta$ is directly proportional to r for $r>0$

(2) Find the convergence set for the power series and find the sum

$$\sum_{n=1}^{\infty} n(n+1)(x-1)^n$$

(3) Find the equation of the plane through $(-1,-2,3)$ and perpendicular to both the plane $x-3y+2z=7$ and $2x-2y-z=-3$

(4) If $u(x,t)=\dfrac{f(x-ct)+f(x+ct)}{2}$, Show that $\dfrac{\partial^2 u(x,t)}{\partial t^2}=c^2\dfrac{\partial^2 u(x,t)}{\partial x^2}$

(5) Evaluate $\int_0^1\int_0^{\sqrt{1-x^2}}\left(4-x^2-y^2\right)^{-1/2}dydx$.

3. (5) Find the minimum distance from the origin to the line of intersection of two planes

$x+y+z=8,\quad 2x-y+3z=28$

4. (5) Evaluate $\iint\limits_S\left(x^2+x^4y\right)dA$. Where $S=\left\{(x,y):1\le x^2+y^2\le 4\ \right\}$

5. (5) Switch the order in polar coordinate $\int\limits_{3\pi/4}^{4\pi/3}d\theta\int\limits_{0}^{-5\sec\theta}r^3\sin\theta dr$

6. (7) Find area of region outside the cardioid $r=1+\cos\theta$ and inside the circle $r=\sqrt{3}\sin\theta$

7. (7) Rewrite the iterated integral with the indicated order of integration

$\int_{-1}^{0}\int_{-\sqrt{y+1}}^{\sqrt{y+1}}f(x,y)dxdy,\qquad dydx$

8. (5) Evaluate $\int\limits_{-3}^{3}\int\limits_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}\int\limits_{-\sqrt{9-x^2-z^2}}^{\sqrt{9-x^2-z^2}}(x^2+y^2+z^2)^{3/2}dydzdx$

9 (5) Suppose $x^3e^{y+z}-y\sin(x-z)=0$, find $\partial z/\partial x$

10 (5) Consider the Cobb-Douglas production model for a manufacturing process depending on three input x,y,z with unit costs a,b,c, respectively. Given by

$$P(x,y,z)=k\,x^{\alpha}y^{\beta}z^{\gamma}\quad \alpha>0,\beta>0,\gamma>0,\quad \alpha+\beta+\gamma=1$$

subject to the cost constrain $ax+by+cz=1$. Determine the x,y,z maximize the production