Quiz 13

1. Evaluate the integral $\iint_D |\cos(x+y)|\,\mathrm{d}\sigma$, here D is the region bounded by $y=x, y=0, x=\frac{\pi}{2}$.

2. Evaluate the integral $\iint_D (y-x)^2\,\mathrm{d}\sigma$, D: $y \le R+x, x^2+y^2 \le R^2, y \ge 0 (R>0)$.

3. If $f(x)$ is continuous on $[a,b]$, n is a positive integer, try to prove that

$$\int_a^b \mathrm{d}y \int_a^y (y-x)^{n-1} f(x)\mathrm{d}x = \frac{1}{n}\int_a^b (b-x)^n f(x)\mathrm{d}x .$$

4. Let D be a closed bounded plane region, $f(x,y)$ and $g(x,y)$ are continuous on D, $g(x,y)$ is positive on D, try to prove that: $\exists(\xi,\eta)\in D$, s. t.

$$\iint_D f(x,y)g(x,y)d\sigma = f(\xi,\eta)\iint_D g(x,y)d\sigma .$$

5. Evaluate $\iint_D \mathrm{e}^{\frac{y}{x+y}}\mathrm{d}\sigma$ here D is the region bounded by $y+x=1, y=0, x=0.$.

6. Evaluate $\iiint_S xz\mathrm{d}x\mathrm{d}y\mathrm{d}z$, here S is the solid bounded by $x=y, y=1, z=0$ and $z=x^2$.

7. Evaluate $\iiint_S z^3\mathrm{d}v$, where S is the solid bounded by $z=\sqrt{2-x^2-y^2}$ and $z=x^2+y^2$.

8. Evaluate $\iiint_S (x^3+xy^2)\mathrm{d}v$, where S is the solid bounded by $x^2+(y-1)^2=1$, $z=0, \text{and } z=1$.

9. Evaluate $\iiint_S \sqrt[4]{x^2+y^2+z^2}\mathrm{d}v$, where S is the solid bounded by $x^2+y^2+z^2=z$.

10. Evaluate $\iiint_S \mathrm{e}^{\sqrt{\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}}}\mathrm{d}v$, here S is the solid bounded by $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.

11. The radius of the sphere C is R. And the center of the sphere C *is on the sphere* K: $x^2+y^2+z^2 \le a^2$.

Please find the maximum value of the area of the sphere C inside the sphere K.