

Quiz 13

1. Evaluate the integral $\iint_D |\cos(x+y)| d\sigma$, here D is the region bounded by $y = x, y = 0, x = \frac{\pi}{2}$.

2. Evaluate the integral $\iint_D (y-x)^2 d\sigma$, $D: y \leq R+x, x^2 + y^2 \leq R^2, y \geq 0 (R > 0)$.

3. If $f(x)$ is continuous on $[a, b]$, n is a positive integer, try to prove that

$$\int_a^b dy \int_a^y (y-x)^{n-1} f(x) dx = \frac{1}{n} \int_a^b (b-x)^n f(x) dx.$$

4. Let D be a closed bounded plane region, $f(x, y)$ and $g(x, y)$ are continuous on D , $g(x, y)$ is positive on D , try to prove that: $\exists(\xi, \eta) \in D$, s. t.

$$\iint_D f(x, y) g(x, y) d\sigma = f(\xi, \eta) \iint_D g(x, y) d\sigma.$$

5. Evaluate $\iint_D e^{\frac{y}{x+y}} d\sigma$ here D is the region bounded by $y + x = 1, y = 0, x = 0$.

6. Evaluate $\iiint_S xz dx dy dz$, here S is the solid bounded by $x = y, y = 1, z = 0$ and $z = x^2$.

7. Evaluate $\iiint_S z^3 dv$, where S is the solid bounded by $z = \sqrt{2-x^2-y^2}$ and $z = x^2 + y^2$.

8. Evaluate $\iiint_S (x^3 + xy^2) dv$, where S is the solid bounded by $x^2 + (y-1)^2 = 1$, $z = 0$, and $z = 1$.

9. Evaluate $\iiint_S \sqrt[4]{x^2 + y^2 + z^2} dv$, where S is the solid bounded by $x^2 + y^2 + z^2 = z$.

10. Evaluate $\iiint_S e^{\sqrt{\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}}} dv$, here S is the solid bounded by $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

11. The radius of the sphere C is R . And the center of the sphere C is on the sphere K : $x^2 + y^2 + z^2 \leq a^2$.

Please find the maximum value of the area of the sphere C inside the sphere K .