Quiz 12

1. Let $f(x,y)=\begin{cases}(x+y)^2\sin\dfrac{1}{x^2+y^2}, & x^2+y^2\neq 0\\ 0, & x^2+y^2=0\end{cases}$

   1) Find $\dfrac{\partial f}{\partial x}$, $\dfrac{\partial f}{\partial y}$.

   2) Determine the continuity of $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$ at the origin.

   3) Determine the differentiability of $f(x,y)$ at the origin.

2. Let $z=xf\left(\dfrac{y}{x}\right)+yg(x,\dfrac{x}{y})$, f'',g'' both exist, try to find $\dfrac{\partial^2 z}{\partial x\partial y}$.

3. If $z^x=y^z$, try to find $\mathrm{d}z$.

4. Assume $z=f(x,y),x=\varphi(y,z)$, and f,φ are differentiable. Please find $\dfrac{\mathrm{d}z}{\mathrm{d}x}$.

5. Find the directional derivative of $u=\mathrm{e}^x\cos(yz)$ at $(0,0,0)$ in the direction of $\vec{l}=\{2,1,-2\}$.

6. Please find the angle between the gradient of $u=x^2+y^2-z^2$ at $A(a,0,0)$ and the gradient of $u=x^2+y^2-z^2$ at $B(0,a,0)$.

7. Let $\begin{cases}z=x^2+y^2\\ x^2+2y^2+3z^2=20\end{cases}$, please find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and $\dfrac{\mathrm{d}z}{\mathrm{d}x}$.

8. Please find the equation of the tangent line and the equation of the normal plane of the curve $x=\sin^2 t,y=\sin t\cos t,z=\cos^2 t$ at $t=\dfrac{\pi}{4}$.

9. Find the equation of the tangent plane of the surface $ax^2+by^2+cz^2=1(abc\neq 0)$ at the point (x_0,y_0,z_0).

10. Find the maximum and minimum values of $f(x,y)=x^2-y^2$,

    here $(x,y)\in\{(x,y)\,|\,x^2+y^2\le 4\}$.