Chinese Name______ Student ID □□□□□□□□□□□□ School______ Major______ Sit NO.______

（DONOY WRITE YOUR ANSWER IN THIS AREA）

Sale Line

WARNING: MISBEHAVIOR AT EXAM TIME WILL LEAD TO SERIOUS CONSEQUENCE.

SCUT Final Exam

2021-2022-2《Calculus II》 Middle Test

Notice: **1. Make sure that you have filled the form on the left side of seal line.**

2. Write your answers on the exam paper .

3. This is a close-book exam.

4. The exam with full score of 100 points lasts 90 minutes.

Question No.	1-10	11-15	Sum
Score			

一. **Answer the questions.** ($7'\times 10 = 70'$)

1. Find maximum and minimum values of $u = x - 2y + 2z$ subject to $x^2 + y^2 + z^2 = 1$.

2. Let $f(x, y)=\begin{cases} \dfrac{xy}{\sqrt{x^2+y^2}} & x^2+y^2 \neq 0 \\ 0 & x^2+y^2=0 \end{cases}$

Prove：1）the partial derivatives of $f(x,y)$ exist at (0,0); 2）$f(x,y)$ is not differentiable at $(0, 0)$.

3. Let $z = xf\left(xy, \dfrac{y}{x}\right)$, and f has the second-order continuous partial derivatives, find $\dfrac{\partial z}{\partial y}$, $\dfrac{\partial^2 z}{\partial y \partial x}$.

4. Let $z = f(u, x, y)$, $u = xe^y$, and f has second-order continuous partial derivatives, find $\dfrac{\partial^2 z}{\partial x \partial y}$.

5. Let $z = f(2x - y, y\sin x)$, f has second-order continuous partial derivatives, find $\dfrac{\partial^2 z}{\partial x \partial y}$.

6. Let $z = f(\phi(x) - y, \psi(y) + x)$, and f has second-order continuous partial derivatives, φ, ψ have derivatives, find $\frac{\partial^2 z}{\partial x \partial y}$.

7. Let $G(u\ ,\ v)$ is differentiable, and the equation $G\left(\frac{x}{z}\ ,\ \frac{y}{z}\right) = 0$ implying $z = z(x, y)$, compute

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}.$$

8. Find $\int_{\frac{1}{4}}^{\frac{1}{2}} dx \int_{\frac{1}{2}}^{\sqrt{x}} e^{\frac{x}{y}} dy + \int_{\frac{1}{2}}^{1} dx \int_{x}^{\sqrt{x}} e^{\frac{x}{y}} dy$.

9. Find $\iint\limits_{D} 2xy^2 e^{x^2 y} d\sigma, D$ is bounded by $y = 1, x = \sqrt{y}, x = 0$.

10. Find $\iiint_{\Omega} z dv$, and Ω is bounded by $x^2+y^2=1$ and $z=0$, $z=1$.

二. **Answer the questions.** $(6'\times 5=30')$

11. Find $I=\iiint_{\Omega}\frac{dv}{x^2+y^2+z^2}$, Ω is bounded by $z=1+\sqrt{1-x^2-y^2}$ and $z=1$.

12. Find $\iiint\limits_{\Omega}(x-y-z)^2dv$, Ω is bounded by $x^2+y^2+z^2=1$.

13. Find $\iiint\limits_{\Omega} zdv$, and Ω is determined by $\begin{cases} x^2+y^2+z^2 \le 2 \\ z \ge x^2+y^2 \end{cases}$.

14. Find the volume of Ω which is bounded by $x^2 + y^2 \le a^2$ and planes $z = 0, z = x, (x > 0)$.

15. Find the volume of Ω which is bounded by the closed surface $(x^2 + y^2 + z^2)^2 = a^3 z, (a > 0)$.