Seat NO. ______ Major ______ School ______ Student ID □□□□□□□□□□□□ Chinese Name ______

（DONNOT WRITE YOUR ANSWER IN THIS AREA）

Seal Line　Seal Line　Seal Line

WARNING: MISBEHAVIOR AT EXAM TIME WILL LEAD TO SERIOUS CONSEQUENCE.

SCUT Final Exam

《Calculus II》 Exam Paper A

Notice:
1. **Make sure that you have filled the form on the left side of seal line.**
2. **Write your answers on the exam paper .**
3. **This is a close-book exam.**
4. **The exam with full score of 100 points lasts 120 minutes.**

Question No.	1-6	7-17	18	Sum
Score				

一. **Please fill the correct answers in the following blanks.** $(3'\times 6=18')$

1. Let L be the circle line $x^2+y^2=9$ with anticlockwise, then the line integral

$$\oint_L (2xy-2y)\,dx+\left(x^2-4x\right)dy=\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.$$

2. The divergence of the vector field $\vec{A}=e^{xy}\vec{i}+\cos(xy)\,\vec{j}+xz^2\vec{k}$ is____________________.

The curl of the vector field $\vec{B}=(2z-3y)\vec{i}+(3x-z)\vec{j}+(y-2x)\vec{k}$ is ____________________.

3. If $\vec{a}=\{-1,2,2\},\vec{b}=\{2,-1,2\}$, then $\left(\vec{a}-\vec{b}\right)\times\left(\vec{a}+\vec{b}\right)=$____________________.

4. The area of the region inside the cardioid $r=a(1+\cos\theta)$ is ____________________.

5. The convergence radius of the series $\sum_{n=1}^{+\infty}\frac{n!}{n^n}x^n$ is ____________________.

6. Interchange the integration order, we have $\int_0^2 dy\int_{y^2}^{2y} f(x,y)\,dx=$____________________.

二、Finish the following questions. (7-17: $7' \times 11 = 77'$; 18: $5' \times 1 = 5'$)

7. Find the convergence region and sum function of power series $\sum_{n=0}^{\infty} \frac{x^n}{n+1}$.

8. Expand the function $f(x) = \ln \frac{1+x}{1-x}$ into power series of x.

9. Let $\begin{cases} x = \ln\left(\sin t + \sqrt{1+\sin^2 t}\right) \\ y = \sqrt{1+\sin^2 t} \end{cases}$ and y is a function of x, find $\frac{d^2 y}{dx^2}$.

10. Let $G(u,v)$ be differentiable and $G(u,v)=0$, $u=x^2+yz, v=y^2+xz$ making that z is a differentiable function for x, y, prove that $(2y^2-xz)\frac{\partial z}{\partial x}+(2x^2-yz)\frac{\partial z}{\partial y}=z^2-4xy.$

约 $\frac{\partial G}{\partial x} = 0$

11. Let $f(x\,,\,y)=\begin{cases}\dfrac{xy}{\sqrt{x^2+y^2}} & x^2+y^2\neq 0\\ 0 & x^2+y^2=0\end{cases}$.

Prove that: 1） $f_x(0,0), f_y(0,0)$ exist; 2） $f(x,y)$ is not differentiable at $(0\,,\,0)$.

12. Find $\iint\limits_D |y-x^2|\,dxdy, D:-1\le x\le 1, 0\le y\le 1.$

13. Find the area of the surface $S=\left\{(x,y,z)\middle|x^2+y^2+z^2=a^2\right\}$.

$$\iint_{\Omega} \sqrt{1+f_x^2+f_y^2}$$

14. Find the mass of $\Omega=\left\{(x,y,z)\middle|x^2+y^2+z^2\le a^2\right\}$ with the density $x^2+y^2+z^2$.

15. $\iiint\limits_{\Omega} \sqrt{x^2+y^2}\,\mathrm{d}v$ where Ω is bounded by $x^2+y^2=16, y+z=4, z=0.$

16. Find the minimum distance between the original point (0,0) with the points of the curve

$x^2+xy+y^2+2x-2y-12=0$.

17. Determine whether $(4x^3+9x^2y^2)dx+(6x^3y+6y^5)dy$ is conservative, if yes please find $u(x,y)$ such that $du(x,y)=(4x^3+9x^2y^2)dx+(6x^3y+6y^5)dy$.

18. Find $\int_{\widehat{AB}}[x\varphi(u)+y^2]dx+[y\varphi(u)+2xy]dy$, if $\varphi(u)$ has continuous derivatives, and $u=x^2+y^2$, where AB is any smooth curve in the first quadrant connecting points $A(5,0)$ and $B(3,4)$.

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