

一、 Answer the questions. ($3' \times 6 = 18'$)

1. L is the boundary of circle $x^2 + y^2 = 9$ with counterclockwise direction, then curve integral

$$\oint_L (2xy - 2y)dx + (x^2 - 4x)dy = \underline{\hspace{2cm}}.$$

2. $L: x^2 + y^2 = a^2$ $\oint_L \sqrt{x^2 + y^2} ds = \underline{\hspace{2cm}}.$

3. Let $D: x^2 + y^2 \leq 2x$, write the $\iint_D f(x, y) dx dy$ in polar coordinate,

then $\iint_D f(x, y) dx dy =$ _____

4. If $\vec{a} = \{-1, 2, 2\}, \vec{b} = \{2, -1, 2\}$, then $(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b}) = \underline{\hspace{2cm}}$

5. The equation of plane through the three points $P_1(1, -2, 3), P_2(4, 1, -2), P_3(-2, -3, 0)$ is

6. The divergence of $\vec{A} = e^{xy}\vec{i} + \cos(xy)\vec{j} + xz^2\vec{k} = \underline{\hspace{2cm}}.$

二、 Finish the following questions. (7-16: $7' \times 10 = 70'$; 17-18: $6' \times 2 = 12'$)

7. Please show that $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{\sqrt{n}}$ is conditionally convergent.

8. Expand the function $f(x) = \arctan \frac{1+x}{1-x}$ into power series of x .

9. Line integral $\int_L xy^2 dx + y\varphi(x) dy$ is independent of path, and $\varphi(x)$ is derivative, $\varphi(0) = 0$,

find $\int_{(0,0)}^{(1,1)} xy^2 dx + y\varphi(x) dy$.

10. Find $\int_{\frac{1}{4}}^{\frac{1}{2}} dx \int_{\frac{1}{2}}^{\sqrt{x}} e^{\frac{x}{y}} dy + \int_{\frac{1}{2}}^1 dx \int_x^{\sqrt{x}} e^{\frac{x}{y}} dy$.

11. Find $\iiint_{\Omega} (x^2 + y^2 + z^2) dv$, Ω is bounded by $x^2 + y^2 + z^2 = 1$.

12. Find $\iiint_{\Omega} z dv$, and Ω is bounded by $x^2 + y^2 = 1$ and $z = 0$, $z = 1$.

13. Let $f(u, v)$ is differentiable, $z = z(x, y)$ is determined by $(x+1)z - y^2 = x^2 f(x-z, y)$.
find $dz|_{(0,1)}$.

14. Σ is the surface $z = x^2 + y^2 (z \leq 1)$ with upper side, please calculate

$$I = \iint_{\Sigma} (x-1)^3 dydz + (y-1)^3 dzdx + (z-1) dxdy.$$

15. Find the minimum distance between the original point and the surface $z^2 = x^2 y + 4$.

16. Let $z = f(u, x, y)$, $u = xe^y$, f has the second-order continuous partial derivative, find $\frac{\partial^2 z}{\partial x \partial y}$.

17. Find $\oint_L \frac{(x-1)dy - ydx}{(x-1)^2 + y^2}$, L represents a simple closed curve, including point $(1, 0)$ with counterclockwise direction.

18. Find convergent region and sum function of power series $\sum_{n=0}^{\infty} \frac{x^{2n+2}}{(n+1)(2n+1)}$.