# Interactive rubber-sheeting using dynamic triangulation

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We present a new rubber-sheeting algorithm that is easy to use, computationally efficient, and able to generate accurate and high quality results (Figure 1) [1]. The need of such a rubber-sheeting method arises naturally in the GIS industry, as more and more companies start to provide web-based GIS services for general users.

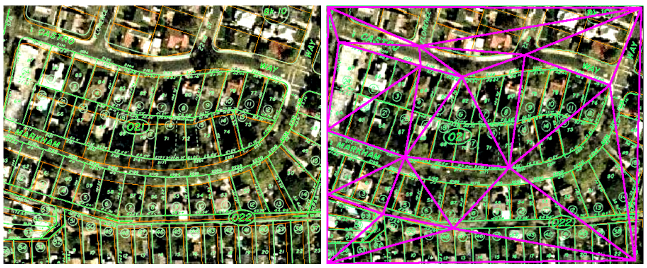

Figure 1: Left: mismatch beteen tax map(green lines) and aerial photo with parcels (yellow lines); Right: result of rubber-sheeting and the dynamic triangulation(purple lines)

Major challenges in traditional rubber-sheeting algorithms are collecting control points and geometric interpolation. Without the ability of interactively viewing the warping result, it is a puzzling task even for a specialist to find a "necessary" and "sufficient" set of control points. For interpolation, piece-wise algorithms such as [White and Griffin 1985] are computationally efficient, but usually have relatively low quality results. Global methods using radial basis functions normally generate smoother warping results [Fogel and Tinney 1996], but suffer from extensive computation and limitations on the number of control points. The concern of both efficiency and quality makes web-based rubber-sheeting application into a dilemma situation.

Our approach has two steps: a piece-wise rubber-sheeting step to interactively find a suitable set of control points, and an optional global warp step to generate smoother results using the specified control point set (Figure 2).

In the first step, we developed a dynamic meshing algorithm based on Delaunay triangulation, which can intuitively decompose the image while a user inserts, deletes or drags a control point. Because of Delaunay triangulation's nice geometric properties, control points closing to each other will be paired up, and the minimum angle of all triangles will be maximized, which helps to reduce oversampling artifacts. To achieve real-time performance, only nearby triangles of a modified control point will be updated. Hence a user can add as many control points as necessary without compromising the performance of the algorithm. The location of control points is not limited as well. To avoid generating flipped triangles by control points moved to "bad" locations (Figure 3), which is a headache for traditional triangulation based piece-wise methods, helping points will be automatically inserted to adjust the triangulation, a technique also used in [Wu and Johnstone 2006] for polygon morphing.

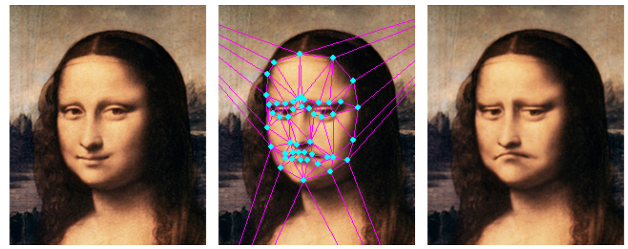

Figure 2: From left to right: original image; interactive result and dynamic triangulation; global warp result

When a sufficient number of control points have been interactively specified with a satisfying result of $C^0$ smoothness, the warping result can be further smoothed by using an optional radial basis global warping method in a second step. Such a hybrid procedure not only makes the selection of control points intuitive and efficient, but also can generate a high quality smooth result.

The algorithm is implemented as a Java Applet and is able to run as a Web-based service.

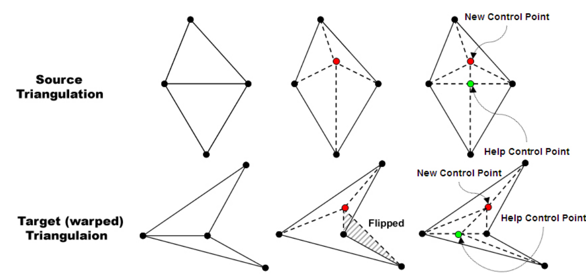


Figure 3: A flipped triangle (shaded) and a helping point (green)

## References

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[1]US Patent Pending