Dynamical Systems II

1 Basic

- FPs, linear stability for non-hyperbolic FPs, invariant subspaces and manifolds

- Lyapunov stability, quasi-asymptotic stability, asymptotic stability
- Lyapunov function (implies LS), strict LF (implies AS)
- La Salle invariance principle: forward invariant, compact subset D , with Lyapunov function V, then all points tend to an invariant subset of $\{\dot{V} = 0\} \cap D$
- Domain of stability: set of points tending to FP
- Bounding function: $\dot{V}$ bounded below away from 0

2 Periodic Orbits

- Poincare index test (2D): all PO have index 1; all sink/source +1, saddle -1
- Dulac's criterion: $\nabla \cdot \phi\dot{\boldsymbol{x}} \neq 0$ in a simply connected domain D, then no PO enclosed in D.
- Gradient criterion: if there exists $\rho > 0$ s.t. $\rho\dot{\boldsymbol{x}} = \nabla\phi$ for single valued function in a simply connected domain, then no PO
- Doubly connected domain has at most one PO
- Poincare-Bendixson Theorem: if forward orbit of a point remains in a compact set which contains no FPs, then have a PO
- Near Hamilton flows: Energy balance method expect PO to be close to a PO of Hamiltonian flow; where δH around one loop of original PO is 0
- Stability of PO: Floquet multipliers: eigenvalues of the matrix for perturbation of flow from a PO (except one copy of 1)
- Floquet exponents

3 Bifurcations

- Extended central manifold to get dynamics on the manifold
- Classification of bifurcations: saddle-node, pitchfork, transcritical
- Structural stability
- Hopf bifurcation
- For maps: period doubling bifurcation

4 Chaos

D-chaos on an invariant set Λ

- SDIC: some δ, any ε, and nearby x, y grow apart
- TT: any open set in λ maps to any other open set with sufficient iteration
- Periodic points dense on λ

G-chaos: horseshoe on domain I