Complex Methods IB

1 Analytic function

- Complex differentiability: same limit definition but quantify δx over all of $\mathbb{C}$
- Analytic (regular/holomorphic) function: f is analytic at z if $\exists$ open neighbourhood of z where f' exists
  (not just differentiable at a point)
- Entire function: analytic throughout $\mathbb{C}$
  (analytic implies infinitely diffable; bounded entire implies constant)
- Complex diffable at $z \implies$ Cauchy-Riemann eqns
  (converse holds if u, v diffable at z)
- $f = u + iv$ analytic $\implies$ $u(x, y)$ and $v(x, y)$ harmonic
- Harmonic conjugates: u, v are harmonic conjugates if they satisfy C-R equations
- Multivalued functions have branch point, introduce branch cut to make single valued and continuous
- Mobius maps
- Conformal map: $f : U \to V$, both U, V open subsets of $\mathbb{C}$
  f is conformal if analytic with non-zero derivative
  $\iff$ angles preserved (i.e. amplitwist)