Statistical Physics II

1 Fundamentals

Microcanonical ensemble

Fixed E

- Entropy $S = k_b \log \Omega(E)$, log of number of microstates
- Entropy of combined system must increase (by counting)
- Temperature

  $$\frac{1}{T} = \frac{\partial S}{\partial E}$$

  Pressure

  $$p = T \frac{\partial S}{\partial V}$$

Canonical Ensemble

Energy E can vary

- Partition function:

$$\begin{aligned}
\beta &= \frac{1}{k_B T} \\
Z &= \sum_n e^{-\beta E_n} \\
S &= k_B \frac{\partial}{\partial T}(T \log Z) \\
\langle E \rangle &= -\frac{\partial}{\partial \beta} \log Z \\
\Delta E^2 &= \frac{\partial^2}{\partial \beta^2} \log Z = k_b T^2 C_V \\
F &= \langle E \rangle - TS = -k_B T \log Z
\end{aligned}$$

Grand canonical ensemble

Both E, N can vary

- Chemical potential

$$\mu = -T\frac{\partial S}{\partial N} = \left(\frac{\partial F}{\partial N}\right)_{T,V}$$

- Grand partition function:

$$\begin{aligned}
\mathcal{Z} &= \sum_n e^{-\beta(E_n - \mu N_n)} \\
\langle N \rangle &= \frac{1}{\beta}\frac{\partial}{\partial \mu}\log Z \\
\Delta N^2 &= \frac{1}{\beta^2}\frac{\partial^2}{\partial \mu^2}\log \mathcal{Z} = k_b T^2 C_V \\
\Phi &= F - \mu N = -p(T,\mu)V
\end{aligned}$$

- First Law

$$\mathrm{d}E = T\,\mathrm{d}S - p\,\mathrm{d}V + \mu\,\mathrm{d}N$$

- Thermodynamic limit: E, N are roughly fixed

2 Classical Gas

- Classical partition function

$$Z_1 = \frac{1}{(2\pi\hbar)^3}\int \mathrm{d}^3p\,\mathrm{d}^3q e^{-\beta H}$$

- Thermal de Broglie wavelenth

$$\lambda = \sqrt{\frac{2\pi\hbar^2}{mk_BT}}$$

- Partition function (for one particle)

$$Z_1 = \frac{V}{\lambda^3}$$

- Recover ideal gas law
- Equipartition of energy: $\frac{1}{2}k_BT$ per degree of freedom
- Sackur-Tetrode equation: energy of an ideal gas
- Maxwell distribution:

$$\frac{4\pi m^3 V}{(2\pi\hbar)^3}\int \mathrm{d}v v^2 e^{-\beta m v^2/2}$$

- Diatomic gas rotation + vibration
- Interacting gas: Mayer f function $1 + f(r_{ij}) = e^{-\beta U(r_{ij})}$,

$$\frac{pV}{Nk_BT} = 1 - \frac{N}{2V}\int d^3 r f(r)$$

  , van der Waals equation

$$k_BT = \left(p + \frac{N^2}{V^2}a\right)\left(\frac{V}{N} - b\right)$$

3 Quantum Gases

- Density of states
- Relativistic gas
- Blackbody radiation, Stefan Boltzmann Law
- Phonons, Debye model
- Bosons, Bose-Einstein Distribution, BE Condensate
- Fermions, Fermi-Dirac Distribution
- Pauli paramagnetism

4 Classical Thermodynamics

- Adiabatic walls: isolate from influences; diathermal walls: not adiabatic
- Zeroth Law: have a notion of temperature
- First Law: energy is a state function
- Second Law: No process is possible whose sole effect is to extract heat from a hot reservoir and convert this entirely to work; no process is possible whose sole effect is transfer of heat from colder to hotter body
- Carnot cycle: adiabatic and isothermal expansion between T_C and T_H, reversible
- Efficiency $\eta = \frac{W}{Q_H} = 1 - \frac{Q_C}{Q_H}$
- Define temperature s.t. for reversible engine
$$\eta = 1 - \frac{T_C}{T_H}$$
- $$\oint \frac{dQ}{T} = 0$$
(inexact differential), thus define entropy
$$S = \int_0^A \frac{dQ}{T}$$
- Third Law

5 Phase Transitions

- Order of phase transition
- Liquid-gas, phase equilibrium, meta-stable
- Critical point, critical exponents
- Ising model, mean field theory