Analysis and Topology IB

1 Uniform Convergence and Uniform Continuity

- Uniform limit of continuous functions is continuous (therefore pointwise limit of cts function discts implies not uniform)
- Uniform limit of bounded functions is bounded
- Uniform limit of integrable functions is integrable (integral of limit = limit of integral)
- If $\sum f_n'$ converges uniformly on $[a, b]$ and $\sum f_n$ converges at one point in $[a, b]$, then $\sum f_n$ converges uniformly, and has derivative $\sum f_n'$.
- Uniformly Cauchy sequences; general principle of uniform convergence
- Weierstrass M-test: series of bounded functions with infinite sum of bounds convergent: series uniformly convergent

Power series

- Power series converges pointwise inside radius of convergence, but not uniformly
- Converges locally uniformly: for any point there is an open set in which uniformly convergent

Uniform continuity

- Any continuous function on a closed bounded interval $[a, b]$ is uniformly continuous (compactness?)
- Continuous implies integrable

2 Metric Spaces

- Axioms; subspaces, product
- Convergence, continuity

- Isometric, Lipschitz, uniformly continuous
- Open/closed ball, open/closed set, neighbourhood: characterisation of continuity
- Homeomorphism, equivalent metric spaces; uniformly/Lipschitz equivalent

3 Completeness, Contraction Mapping Theorem

- Complete space: every Cauchy sequence converges