Linear Algebra

IB

- Vector space, subspace, quotient, sum, direct sum
- Steinitz Exchange Lemma
- Linear independence, span, finite dimensional, basis, dimension
- $\dim(U+V) + \dim(U\cap V) = \dim U + \dim V$
- Linear map, kernel, image, isomorphism, rank-nullity theorem
- Matrices: change of basis, equivalent matrices
- Row rank = col rank = rank of linear map
- Elementary operations, elementary matrices

Duality

• Dual space, dual basis, annihilator, dual space; double dual canonical isomorphism

Bilinear form, matrix representation

Det, trace

- Det is the volume form (alternate and multilinear) up to constant multiple
- Invertible iff non-singular
- Adjugate matrix, inverse

Eigen

- Eigenvalue
- Diagonalisation, simultaneous diagonalisation
- Minimum polynomial

- Cayley-Hamilton
- Multiplicities
- \bullet Generalised Eigenspace Decomposition

Bilinear (specific case)

 $Symmetric,\, quadratic,\, diagonalise$