Magnetism

Jinyuan Wu

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The word *magnetism* covers the response to a magnetic field (paramagnetism, diamagnetism) and the magnetic ground state (ferromagnetism, antiferromagnetism). In everyday language, when we say a material is "magnetic", we mean it has a magnetic ground state – usually ferromagnetism – but so-called non-magnetic materials can still have magnetic response to an external magnetic field.

The ground state is influenced by several mechanisms: exchange interaction, itinerant magnetism, topological magnetism. Above the ground state, we have magnetic excitations like spin waves.

1 Magnetic response of individual electrons

For local electrons, we have Curie's law and Langevin diamagnetism (TODO: ref); for itinerant electrons, we have Pauli paramagnetism, Landau diamagnetism, and more (TODO: ref).

1.1 The magnetic moment

TODO: lecture on the Friday; Hund's rule

1.2 Electromagnetic effective theory

TODO: χ, M

1.3 Paramagnetism of itinerant electrons

$$M = \mu_B(n_\uparrow - n_\downarrow) = \mu_B^2 B\rho(E_\mathrm{F}), \tag{1}$$

1.4 Landau diamagnetism

We can see the magnitude of itinerant paramagnetism usually dominates that of Landau diamagnetism, and therefore most metals are paramagnetic. Transitional metals on the right of the periodic table, like silver, cooper, and gold, however demonstrate diamagnetism. The explanation is nontrivial TODO

Generally all the magnetic responses are rather weak. We need very strong external magnetic field to make these phenomena mechanically significant. With a $\sim 16\,\mathrm{T}$ magnetic field, the diamagnetism of water is obvious enough for us to trap a frog and let it levitate in the air. For superconductors however, since they repulse magnetic field completely (because electromagnetic modes inside a superconductor are gapped), magnetic levitation can be easily observed.

1.5 Van Vleck paramagnetism

2 Some easy-to-explain localized magnetic ground states

In this section we discuss systems in which electrons that contribute most to the magnetic ground state are localized around atoms, and therefore we can talk about magnetism of atoms. In this section we put the problem of domain walls aside: we assume a magnetic order is formed in the thermodynamic limit. TODO: are domain walls classical enough?

A **ferromagnetic** material is a material where the magnetic moments of all atoms are towards the same direction. It breaks the spin rotational symmetry. A **antiferromagnetic** material is a material where the material is broken into several sublattices, each of which are in a ferromagnetic state, but the magnetic momenta among the sublattices cancel each other. It

brakes both the spin rotational symmetry and the translational symmetry. A **ferrimagnetic** material is antiferromagnetic one with a non-zero overall magnetic moment. The states all have *long-range orders*: $\langle \boldsymbol{S_i} \cdot \boldsymbol{S_j} \rangle$ doesn't come to zero even when the distance between the two spins goes to ∞.

2.1 The magnetic dipole-dipole interaction

It can be proved that the dipole-dipole interaction energy is

$$E_{\text{dipole}} = \frac{\mu_0}{4\pi} \left(\frac{\boldsymbol{\mu}_1 \cdot \boldsymbol{\mu}_2}{r^3} - 3 \frac{(\boldsymbol{\mu}_1 \cdot \boldsymbol{r}) \cdot (\boldsymbol{\mu}_2 \cdot \boldsymbol{r})}{r^5} \right). \tag{2}$$

An order of magnitude estimation tells us the energy is $\sim 1 \times 10^{-4}\,\text{eV}$. We know the band energy is $\sim 1\,\text{eV}$, and therefore (2) is too weak.

2.2 Spin-spin interaction caused by exchange interaction

There however exists another interaction channel that gives us a $\boldsymbol{S}_1 \cdot \boldsymbol{S}_2$ Hamiltonian. We know the eigenstates of the 2-electron system

$$H = \sum_{i=1,2} \left(\frac{\boldsymbol{p}_i^2}{2m} - \frac{Ze^2}{r_i} \right) + \frac{e^2}{|\boldsymbol{r}_1 - \boldsymbol{r}_2|} \tag{3}$$

are the **bonding wave function** and the **anti-bonding wave function**: the former is symmetric and the latter is antisymmetric. Now (3) is only about the spatial part. We know Pauli exclusion principle requires the many-body wave function to be antisymmetric, and therefore the bonding wave function of the spatial part comes together with the antisymmetric spin wave function – the **singlet** subspace, and the anti-bonding wave function of the spatial part comes with the symmetric spin wave function – the **triplet** subspace. Thus, the energy difference between the bonding wave function and the anti-bonding wave function can be equivalently attributed to the spin orientation. If ψ^{B} is the ground state, then the spin configuration in the ground state is one-up-one-down.

In this mechanism, what's important is the Coulomb interaction between electrons localized around different atoms. Thus, even if the magnetic momentum of a *single* atom is strong, if the exchange interaction isn't strong enough, the ferromagnetism is still not stable enough. This is (at least qualitatively) demonstrated by experimental results: the Curie temperature seems to be largely independent of the magnetic momentum per atom.

Now we write an effective Hamiltonian between $\boldsymbol{S}_1$ and $\boldsymbol{S}_2$. From $\boldsymbol{S} = \boldsymbol{S}_1 + \boldsymbol{S}_2$, we know

$$2\boldsymbol{S}_1 \cdot \boldsymbol{S}_2 = s(s+1) - s_1(s_1+1) - s_2(s_2+1), \tag{4}$$

and the RHS is $-3/2$ for a singlet and $1/2$ for a triplet, and therefore we find

$$\frac{1}{2} + 2\boldsymbol{S}_1 \cdot \boldsymbol{S}_2 = \pm 1.$$

Now the total energy is $C_{12} \pm J_{12}$, and therefore we get

$$H = C_{12} \pm J_{12} = C_{12} - \frac{1}{2} J_{12} - 2J_{12} \boldsymbol{S}_1 \cdot \boldsymbol{S}_2. \tag{5}$$

So we arrive at the **Heisenberg Hamiltonian**. Some people will replace $2J_{12}$ by J_{12}: this is a frequent notational difference.

2.3 The Heisenberg model

The Heisenberg model is then

$$H = -2 \sum_{\langle \boldsymbol{i}, \boldsymbol{j} \rangle} J_{\boldsymbol{ij}} \boldsymbol{S_i} \boldsymbol{S_j}. \tag{6}$$

At this point we only place magnetic moments on atoms; we ignore all itinerant electrons which have the possibility to be magnetic.

(6) is generally hard to solve. One way to show that it models ferromagnetism is by mean-field theory. We do **Weiss** molecular field model: we assume that (6) is equivalent to a model with no spin-spin interaction, where what is felt by each spin is a mean-field

$$\boldsymbol{H}_{\text{total}} = \boldsymbol{H} + \boldsymbol{H}_M, \quad \boldsymbol{H}_M = \lambda_M \boldsymbol{M}, \tag{7}$$

where $\boldsymbol{H}$ is the external magnetic field (which should be introduced by adding a $\boldsymbol{S} \cdot \boldsymbol{H}$ term in (6)), and we then can obtain the relation between $\boldsymbol{M}$ and $\boldsymbol{H}_{\text{total}}$: TODO

$$\boldsymbol{M} = \left(\frac{N g^2 \mu_B^2 (j+1) j \hat{\boldsymbol{\mu}}}{3 k_{\text{B}} T} H_{\text{total}} \right), \tag{8}$$

and we find

$$\boldsymbol{M} = \chi \boldsymbol{H}, \quad \chi = \frac{C}{T - C\lambda_M}. \tag{9}$$

Obviously, χ diverges when $T \to T_{\text{c}}^+$, where even when we add a very small external $\boldsymbol{H}$, we get a very huge $\boldsymbol{M}$ – an indication that the system becomes ferromagnetic after T passes T_{c}.