Bloch equation and rate equation in two-level systems

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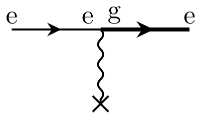
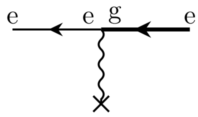
Consider Eq. (5.125) in Steck's lecture notes. The optical Bloch equations can be derived in a diagrammatic way. Basically, we have the following diagrams: for the ρ_{ee} we have (possibly the directions of the lines are wrong; check later)

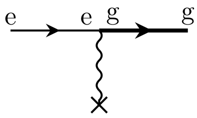


$$\simeq \frac{1}{\partial_t}\frac{\Omega}{2}\rho_{\mathrm{eg}},$$

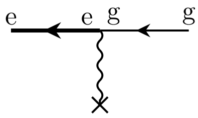


$$\simeq -\frac{1}{\partial_t}\frac{\Omega}{2}\rho_{\mathrm{ge}}, \tag{1}$$

and for the ρ_{eg} part we have

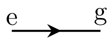


$$\simeq \frac{1}{\partial_t}\frac{\Omega}{2}\rho_{\mathrm{gg}},$$



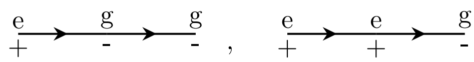
$$\simeq \frac{1}{\partial_t}\frac{\Omega}{2}\rho_{\mathrm{ee}}, \tag{2}$$

and also



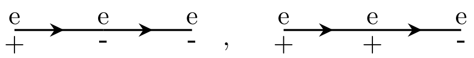
$$\simeq \frac{1}{\partial_t}\Delta\rho_{\mathrm{eg}}. \tag{3}$$

Here we need to note that we are dealing with lesser Green functions, and therefore the free $G^<_{\mathrm{gg}}$ or $G^<_{\mathrm{ee}}$ is just $1/\mathrm{i}\partial_t$; on the other hand, we have (3). This can be illustrated by the following diagrams:



(4)

and we can see that the diagrams for $G^<_{\mathrm{eg}}$, when the external driving field is turned off, contains "ordinary" time-ordered and anti-time-ordered Green functions, and after the summation, the difference between the energies of the e state and the g state enters the self-energy of $G^<_{\mathrm{eg}}$. We can do the same thing for $G^<_{\mathrm{ee}}$ or $G^<_{\mathrm{gg}}$ but this time we just get



(5)

and we find that the contributions of time-ordered and anti-time-ordered Green functions just cancel each other. It should also be noted that Δ itself contains corrections from external fields and is the energy gap minus the external phonon frequency.

Now we turn to how to obtain rate equations – equations where the transitions between e and g are modeled as "scattering" – from the aforementioned optical Bloch equation, or more generally, quantum master equations. The idea is just to first replace $G^<_{\mathrm{eg}}$ by diagrams in terms of $G^<_{\mathrm{ee,gg}}$, and then ignore any frequency dependence in the internal components (similar to COHSEX). It's kind of similar to how we use generalized plasmon-pole model to capture the structure of the RPA dielectric function. Note that until now the theory we have is still a pure-state theory: we just happen to be able to track the dynamics of the system by just looking at the diagonal parts of the single-atom reduced density matrix $G^<_{\mathrm{ee,gg}}/\mathrm{i}$. It's however often further assumed that that's *all we need*, that the non-diagonal components of $G^<$ are not important at all: by taking this step further, we have implicitly assumed that there are strong dephasing factors in the system that keep "observing" the atom into either $|\mathrm{g}\rangle$ or $|\mathrm{e}\rangle$, but forbidding long existence of states like $(|\mathrm{e}\rangle + |\mathrm{g}\rangle)/\sqrt{2}$. Of course, the dephasing factors shouldn't be too strong, or otherwise whenever optical pumping drives the system from $|\mathrm{g}\rangle$ to $|\mathrm{e}\rangle$, ρ_{eg} is almost immediately driven back to zero because of the strong dephasing effect. The consequence is that it's impossible for optical pumping to actually happen; in the same manner, if the atom is already at $|\mathrm{e}\rangle$, it doesn't go down. This is known as quantum Zeno effect, where intense observations freeze the system to the state it's currently in.