Homework 7

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October 25, 2022

Problem 2
Solution

(a) From $j = nev$ we get

$$|\langle \boldsymbol{v} \rangle| = \frac{j}{ne} = \frac{ja^3}{e} = 4.1 \times 10^{-5}\,\mathrm{m/s}.$$

(b) The magnitude of Fermi velocity is $\sim 10 \times 10^8\,\mathrm{m/s}$, so the fraction is $\sim 10^{-13}$.

(c) Since

$$\sigma = \frac{n\tau e^2}{m}, \tag{1}$$

we have

$$\tau = \frac{m}{\rho n e^2} = \frac{m a^3}{\rho e^2} = 8.6 \times 10^{-14}\,\mathrm{s}^{-1}.$$

Problem 3
Solution

(a) The number density, according to (2.76) in A&M, is

$$n = \int_0^{\epsilon_\mathrm{F}} g(\epsilon)\mathrm{d}\epsilon + \left\{ (\mu - \epsilon_\mathrm{F})\, g\left(\epsilon_\mathrm{F}\right) + \frac{\pi^2}{6} \left(k_B T\right)^2 g'\left(\epsilon_\mathrm{F}\right) \right\}, \tag{2}$$

so after adding one electron, we have

$$\frac{1}{V} = \Delta n = \Delta \mu \cdot g(\epsilon_\mathrm{F}),$$

because on the RHS there is only one term that contains μ, and therefore

$$\Delta \mu = \frac{1}{V g(\epsilon_\mathrm{F})}. \tag{3}$$

(b) We have

$$\begin{aligned}
\Delta f &= \Delta \mu \cdot \frac{\mathrm{d}}{\mathrm{d}\mu} \frac{1}{\mathrm{e}^{\beta(\epsilon - \mu)} + 1} \bigg|_{\epsilon = \epsilon_\mathrm{F}} \\
&= \Delta \mu \cdot \frac{\beta}{4} \\
&= \frac{\beta}{4 V g(\epsilon_\mathrm{F})}.
\end{aligned}$$

Since we have (2.65), which is

$$g(\epsilon_\mathrm{F}) = \frac{3n}{2\epsilon_\mathrm{F}},$$

we have

$$\Delta f = \frac{\beta}{4V} \frac{2\epsilon_\mathrm{F}}{3n} = \frac{\epsilon_\mathrm{F}}{6 N k_\mathrm{B} T}. \tag{4}$$

Problem 4

Solution

(a) We choose $T = 298\,\mathrm{K}$. Putting the mass into the formula

$$\lambda = \frac{h}{\sqrt{3mk_{\mathrm{B}}T}}, \tag{5}$$

for an electron, $\lambda = 6.2\times10^{-9}$ m; for a Na atom, $\lambda = 3.1\times10^{-11}$ m; for a ^{4}He, $\lambda = 7.3\times10^{-11}$ m.

(b) When

$$a \sim \frac{h}{\sqrt{3mk_{\mathrm{B}}T}}, \tag{6}$$

or in other words

$$T \sim \frac{h^2}{3mk_{\mathrm{B}}a^2}, \tag{7}$$

the particle is wave-like enough. For an electron it's 1.3×10^5 K. For a Na atom it's 3 K. For ^{4}He it's 18 K. So in ordinary conditions, electrons should always be treated quantum mechanically.

(c) Now the criterion is

$$\left(\frac{k_{\mathrm{B}}T}{P}\right)^{1/3} = \left(\frac{V}{N}\right)^{1/3} = a \sim \frac{h}{\sqrt{3mk_{\mathrm{B}}T}}, \tag{8}$$

and we have

$$T \sim \left(\frac{h^6}{(3mk_{\mathrm{B}})^3}\left(\frac{P}{k_{\mathrm{B}}}\right)^2\right)^{1/5} = 2.3\,\mathrm{K}. \tag{9}$$

This means in ordinary conditions, ^{4}He is still largely classical.

When $T = 3\,\mathrm{K}$, we have $\lambda = 7.3\times10^{-10}$ m, which is much smaller than 1 cm, so ^{4}He atoms in outer space can be treated classically.

(d)