Lattice Boltzmann methods

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We know Boltzmann equation is the first order of the BBGKY hierarchy; since molecular chaos is almost always assumed to truncate the hierarchy, it doesn't capture higher order correlations of density fluctuations; but even Boltzmann equation itself is too much when it comes to simulation. Various continuous formalisms are proposed as the long-range limit of the Boltzmann equation, the most famous being the Navier-Stokes equation, which describes the behavior of Newton fluids and can be derived at the first order of the Chapman-Enskog expansion.

One interesting question is what it means to the Boltzmann equation if a fluid is well described by the N-S equation; we can imagine that the behaviors predicted by the N-S equation, or other equations like the Euler's equation, can be reproduced by the macroscopic behaviors of a much simplified Boltzmann equation with unnecessarily terms dropped; the simplified Boltzmann equation doesn't even need to be a real faithful representation of the microscopic states of the system: we only need it to be simple enough for numerical solving and have the correct macroscopic behaviors.

The Boltzmann collision integral can usually be replaced by something easy to solve, like Bhatnagar Gross and Krook (BGK) model, which is just a fancy name for the relaxation time approximation. If we discretize the Boltzmann equation, we need to discretize both the position space and the velocity space; the position space has to be extensive, so the only simplification we can do is the *restrict the space of possible velocity values*.

It has been shown that a 9-velocity approximation reproduces the incompressible N-S equation [1]. The relation between the viscosity and the relaxation time in the 9-velocity model may deviate from the relation between the viscosity and the real relaxation time, but so what? We're not dealing with shock wave and we care nothing about what really happens at the microscopic level.

By adding more velocity points, we are able to describe

References

[1] Takashi Abe. "Derivation of the lattice Boltzmann method by means of the discrete ordinate method for the Boltzmann equation". In: *Journal of Computational Physics* 131.1 (1997).