Cavity QED

Quantum light-matter interaction to the extreme

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When do all effective theories of light or matter fail?

Maxwell-Langevin equation or atomic quantum master equation

high coherence

?

strong ω dependence,

strong coupling

quantum nature of light

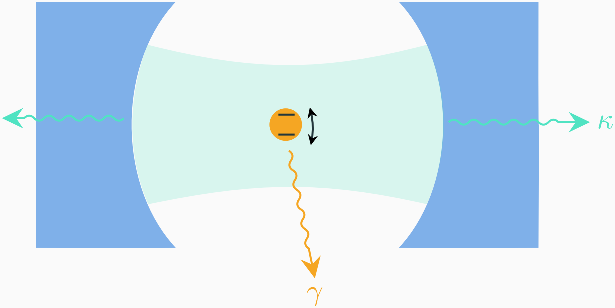
Effective dielectric function,
scattering matrix

Classical light+quantum atom

One scenario: in a cavity.

Cavity and one atom

Cavity quantum electrodynamics (cavity QED)



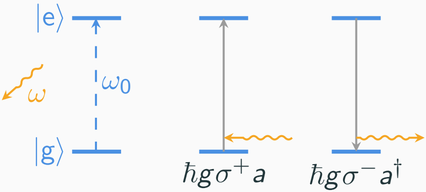
Coupling with the environment

- Cavity leaking κ
- Atomic spontaneous emission rate (outside the cavity) γ
- (Possible non-radiative decay: phonon, etc.)

Strong coupling limit $\kappa, \gamma \ll \boldsymbol{d} \cdot \boldsymbol{E}_{\text{cavity}} \Rightarrow$ cavity QED

Jaynes-Cummings model

- No atom-atom interaction
- Rotating-wave approx.
- Single active photon mode
- No damping at all



$$H^{\text{Jaynes-Cummings}} = \hbar\omega\left(a^\dagger a + \frac{1}{2}\right) + \frac{\hbar\omega_0}{2}\sigma^z + \hbar g(a\sigma^+ + a^\dagger\sigma^-)$$

Possible external field driving $\omega_0 \to \Delta = \omega_0 - \omega_{\text{drive}}$

Quantum Rabi oscillation

Quantum nature of the model

- $|\mathrm{e}\rangle \overset{\text{Spontaneous emission}}{\rightarrow} |\mathrm{g}\rangle$ (but not irreversible)

Dressed state $H^{\text{Jaynes-Cummings}}$ in $\{ |\mathrm{g}, n+1\rangle, |\mathrm{e}, n\rangle\} =$

$$\hbar\omega\left(n+\frac{1}{2}\right) - \frac{\hbar\omega_0}{2} + \begin{pmatrix} \hbar\omega & \hbar g\sqrt{n+1} \\ \hbar g\sqrt{n+1} & \hbar\omega_0 \end{pmatrix}$$

- Oscillation starting with $|\mathrm{e}\rangle$
- Markovian approx. fails
- We have experimental evidence ☛

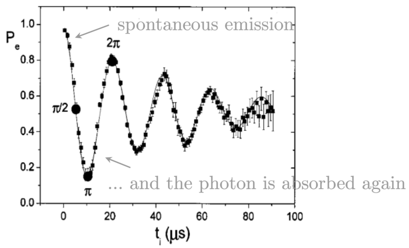


Fig. from S Haroche et al., RMP 73 565 (2001)

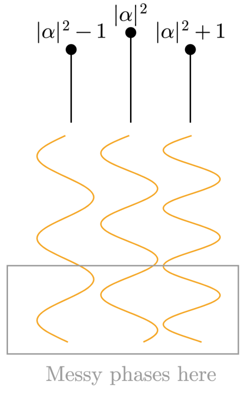
Collapse and revival

Start with $|\mathrm{e},\alpha\rangle$**?** $|\psi(t=0)\rangle = |\mathrm{e}\rangle \otimes \mathrm{e}^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} \underbrace{|\mathrm{e}, n\rangle}_{\leftrightarrow|\mathrm{g},n+1\rangle}$

$$P_\mathrm{e}(t) = \frac{1}{2}\left[1 + \mathrm{e}^{-|\alpha|^2} \sum_{n=0}^{\infty} \frac{|\alpha|^{2n}}{n!} \cos\left(\Omega_n t\right)\right]$$
$$\overset{t\ll 1/g|\alpha|}{=} \frac{1}{2} + \frac{1}{2}\cos(2g|\alpha|t)\,\mathrm{e}^{-\frac{1}{2}g^2t^2}\,.$$

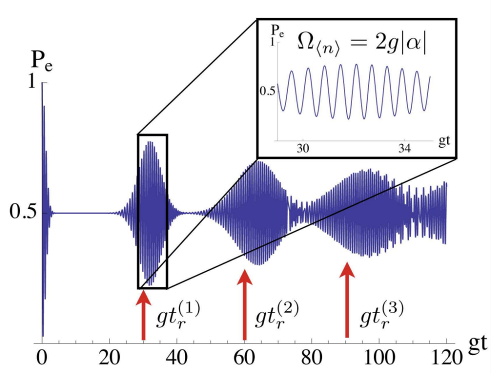
Collapse of P_e **when** $t \ll 1/g|\alpha|$ Because $\phi^{|\mathrm{e},n\rangle}$ not synchronized

This can be simulated by a thermalized state as well; but as $|\psi\rangle$ is not truly incoherent...



Collapse and revival

Revival When the phases of the major components realign again:
$2\pi = (\Omega_{|\alpha|} - \Omega_{|\alpha|^2-1})t$



Revival is a

- Coherent property: not possible in a thermalized state
- "granular" property: see $|\alpha|^2 - 1$

Fig. from arXiv 1111.1143.

Creation of entangled atom pairs

Protocol

1. Move atom 1 (in $|\mathrm{e}\rangle$) into the cavity mode.
2. $|\mathrm{e}_1, 0\rangle \xrightarrow{\frac{1}{2}\Omega_0 t = \frac{\pi}{4}} \frac{1}{\sqrt{2}}(|\mathrm{g}_1, 1\rangle + |\mathrm{e}_1, 0\rangle)$.
3. Move atom 1 out of the light beam. Move atom 2 (in $|\mathrm{g}\rangle$) into the light beam.
4. $\frac{1}{\sqrt{2}}(|\mathrm{g}_1, \underbrace{\mathrm{g}_2, 1}_{\text{coupling happens only here}}\rangle + |\mathrm{e}_1, \mathrm{g}_2, 0\rangle) \xrightarrow{\frac{1}{2}\Omega_0 t = \frac{\pi}{2}} \frac{1}{\sqrt{2}}(|\mathrm{g}_1, \mathrm{e}_2, 0\rangle + |\mathrm{e}_1, \mathrm{g}_2, 0\rangle)$
5. Move all atoms out.

That's how you get an Einstein-Podolsky-Rosen pair.

Cavity and medium

Quantum light-matter interaction to the extreme

"Usual" medium within cavity?

The full theory of quantum optics with $\epsilon(\omega)$ is *hard*!!!

In a theory only about photons: $\epsilon(\omega) \Rightarrow \mathrm{Im}\,\epsilon \neq 0 \Rightarrow$ **non-unitary?**

1. Auxiliary fields needed for frequency dependence
2. $\Rightarrow$ (space-resolved) Input-output formalism
3. $\overset{\text{thermalization}}{\Rightarrow}$ quantum Langevin eq.

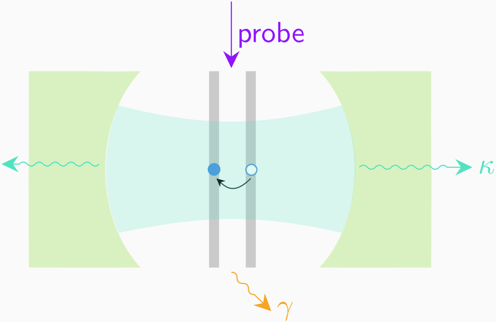
Alternatively we pick up excitations in the medium as auxiliary fields, or "atoms"

See e.g. arXiv quant-ph/0006121, PRL 110, 153602

Cavity QED with exciton

Setup

- One active photon mode
- Probing by the side
- Exciton modes
  ("two-level atom": ✗two excitons in one mode)



The effective theory: one photon mode, several atoms

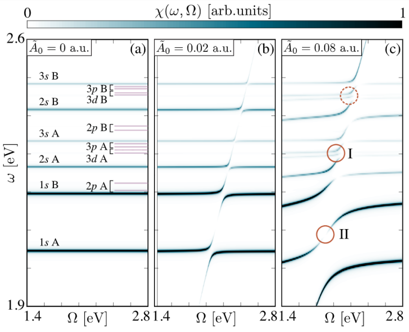
Why this matters $\langle S|\boldsymbol{d}|0\rangle = 0 \not\Rightarrow \langle S_1|\boldsymbol{d}|S_2\rangle = 0$: dark excitons are optically active in this setting!

Cavity QED with excitons as atoms

Exciton polariton is $c_1 \left|\text{exciton}\right\rangle + c_2 \left|\text{photon}\right\rangle$

- Easiest setup
- From the perspective of excitons: virtual photons only

Optical spectrum Ω = cavity mode freq., ω = detection freq.



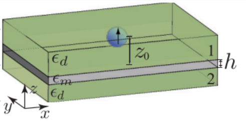
- Dark modes appear (I)
- Quench of optical activity in one band (II)

Fig. from Nano Letters 2019 19 (6), 3473-3479

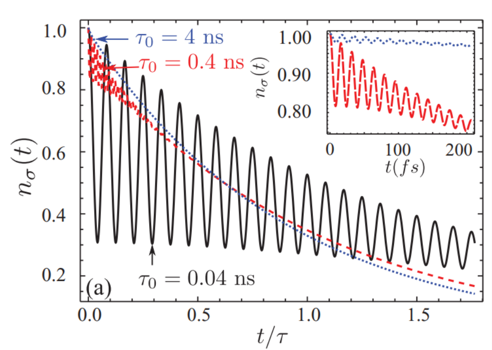
Cavity QED with surface plasmon polaritons (SPPs) as photons

Setup

- Replace “simple” photons by SPPs.
- Atom = exciton on nanoparticle



Rabi oscillation of excited state population



Take home message

- Cavity $\Rightarrow$ highly selective strong light-matter interaction $\Rightarrow$ high coherence $\Rightarrow$ necessity of cavity QED
- Quantum Rabi oscillation, collapse and revival, entanglement
- Cavity QED happens everywhere