Bosonic modes in Fermi liquid

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1 Introduction

The Fermi liquid theory can be justified by diagrammatic resummation:

But interaction channels beside forward scattering that come from Coulomb interaction do not just disappear; they are still a part of the Hamiltonian and will contribute to the specific heat when the system is heated up. Thus, it can be expected that a real condensed matter system that is said to be in a Fermi liquid phase contains more than electron-like quasiparticles.

Characterization of the full spectrum of a system is generally only possible for exactly solvable systems. This report is constrained on bosonic modes in Fermi liquid,[1] or to be specific, on excitations that are oscillation modes of operators with the shape of $c^\dagger_{\boldsymbol{k}+\boldsymbol{q}/2} c_{\boldsymbol{k}-\boldsymbol{q}/2}$. Three-electron behaviors do exist TODO: trion but is beyond the scope of this report.

2 Landau kinetic equation of neutral Fermi liquid

Boltzmann equation can be derived using

The most generalized derivation involves Keldysh field theory and is beyond the scope of this note; we only emphasize here that the derivation depends on gradience expansion and that the quasiparticle picture works well in the system so that the spectral function is approximately

$$A(\boldsymbol{k}, \omega) = \delta(\omega - \operatorname{Re}\Sigma(\boldsymbol{k}, \omega)), \tag{1}$$

where the imaginary part of the self-energy is ignored in the spectral function, but appears in the collision integral on the RHS. The second assumption is by definition satisfied with a Fermi liquid; the first assumption assumes everything in the system changes slowly in the $\boldsymbol{x}_1 - \boldsymbol{x}_2$ direction and imposes a fundamental constraint on the coverage of the quantum Boltzmann equation (Section 5).

It should be noted that the aforementioned procedure does not impose any constraint on $\operatorname{Re}\Sigma$; specifically, it does not dictate that $\operatorname{Re}\Sigma$ cannot have explicit dependence on $G(\boldsymbol{r}, \boldsymbol{r}', t, t')$, and hence $f(\boldsymbol{r}, \boldsymbol{p}, t)$. We can then insert

$$\varepsilon_{\boldsymbol{p}\sigma} = \varepsilon^0_{\boldsymbol{p}\sigma} + \frac{1}{V}\sum_{\boldsymbol{p}',\sigma'} f_{\boldsymbol{p}\boldsymbol{p}'\sigma\sigma'}\,\delta n_{\boldsymbol{p}'\sigma'} \tag{2}$$

into the quantum Boltzmann equation; the resulting equation system is called *Landau equation.* Note that in order to get (2),

In the

When the temperature is non-zero, $\tau \propto 1/T^2$ is finite and zero sound faces strong damping when its frequency is too slow. However, in the low frequency region another bosonic mode is possible: ordinary sound or "first sound", which is a density

It should be noted that the spectrum of first sound is connected to the spectrum of zero sound: In this sense we may say first sound is zero sound with finite temperature correction, but this correction is so severe that the qualitative physical picture is radically changed.

[1] There is a terminological confusion here: the term *Fermi liquid* may refer to a system whose Hamiltonian is exactly in the shape of Fermi liquid energy functional, or it may refer to a system in which the behavior of electron Green function follows the Fermi liquid theory, but may contain other excitations. This note uses the latter definition; thus the phrase "a Fermi liquid" is a shorthand for "a real-world condensed matter system demonstrating Fermi liquid behaviors in its single-electron part".

3 Damping mechanisms

4 Charged Fermi liquid and the plasmon

5 Microscopic bosonic modes beyond the Landau equation

Not all bosonic modes can be obtained by observing oscillation modes of quantum Boltzmann equation, since the latter only works for excitations with a large characteristic length scale in the $\boldsymbol{x}_1 - \boldsymbol{x}_2$ variable, or in other words, when $\boldsymbol{k}$ is small enough. Now consider an exiton in a homogeneous electron liquid; the structure of the exciton follows the same equations as those governing the hydrogen atom, and the characteristic length scale between the electron and the hole is of atomic magnitude, which breaks the "slow variation in $\boldsymbol{x}_1 - \boldsymbol{x}_2$" condition in the gradience expansion step used to derive the quantum Boltzmann equation,[2] and hence cannot be captured appropriately by the latter [1]. We may say zero sound/plasmon is a *hydrodynamic* mode,

The existence of

6 Conclusion

References

[1] David Pines. *Theory of Quantum Liquids: Normal Fermi Liquids*. The "expansion of energy" subsection in Section 1.4 is about the short-range condition. The "charged v.s. neutral system" subsection in Section 3.3 compares plasmon with zero sound. The inability of quantum Boltzmann equation to capture microscopic bound states is discussed on p. 56. CRC Press, 2018.

[2] When an electric field is applied to an exciton, it influences $\boldsymbol{x}_1 - \boldsymbol{x}_2$, not $(\boldsymbol{x}_1 + \boldsymbol{x}_2)/2$; accordingly, it influences $\boldsymbol{k}$, which is now to be understood as the *relative* momentum between the electron and the hole (absence of an electron with momentum $\boldsymbol{k}$ is equivalent to existence of a hole with momentum $-\boldsymbol{k}$). In an exciton $\boldsymbol{k}$ is highly uncertain, which breaks the condition that $\boldsymbol{k}$ is small. Note that since