Boltzmann equation and the like

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1 Boltzmann equation for electrons in a crystal

We define

$$\# \text{ of band } n \text{ electrons in } \Delta\Omega, \Delta V = \frac{\Delta V}{V} \sum_{\boldsymbol{k} \in \Delta\Omega} f_n(\boldsymbol{r}, \boldsymbol{k}, t), = \Delta V \int_{\Delta\Omega} \frac{\mathrm{d}^3 \boldsymbol{k}}{(2\pi)^3} f_n(\boldsymbol{r}, \boldsymbol{k}, t),$$

and therefore when the $\boldsymbol{k}$-grid is dense enough, i.e. when $V \to \infty$, we have

$$\# \text{ of band } n \text{ electrons in } \mathrm{d}^3\boldsymbol{r}\, \mathrm{d}^3\boldsymbol{k} = f_n(\boldsymbol{r}, \boldsymbol{k}, t)\, \mathrm{d}^3\boldsymbol{r}\, \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}. \tag{1}$$

The Boltzmann equation can be derived from the following intuitive notion:

$$\frac{\mathrm{d}}{\mathrm{d}t} \frac{\Delta V}{V} f_n(\boldsymbol{r}, \boldsymbol{k}, t) = \sum_{\text{initial states}} \Gamma_{\text{initial states} \to n\boldsymbol{k}} - \sum_{\text{final states}} \Gamma_{n, \boldsymbol{k} \to \text{final states}},$$

where we have

$$\Gamma_{1 \to 2} = \frac{2\pi}{\hbar} \tag{2}$$