

Fermi liquid

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1 Free electron gas

A free electron gas is described by¹

$$H = \sum_i \frac{\mathbf{p}_i^2}{2m}, \quad (1)$$

and we get a Fermi sphere in the momentum space containing all occupied states, and a Fermi momentum p_F . After we add a periodic potential $V(\mathbf{r})$ to the model, we get **bands**: parabolic bands are displaced periodically according to the \mathbf{G} -grid, and the crossing points of these curves undergo degeneracy breaking, and the spectrum, eventually, becomes several bands confined in the first Brillouin zone. Despite this correction, the number of filled electrons never changes.

For a free electron gas, we have well-known observables like the density of states at E_F (here $\omega = \varepsilon - E_F$)

$$N(\omega = 0) = \frac{m p_F}{\pi^2 \hbar^3}, \quad (2)$$

the electronic specific heat

$$C_v = \underbrace{\frac{\pi^2 k_B^2}{3} N(\omega = 0)}_{\text{Sommerfeld } \gamma} \cdot T, \quad (3)$$

and the magnetic susceptibility

$$\chi = \mu_0 \mu_B^2 \cdot N(\omega = 0). \quad (4)$$

^3He atoms are fermions: we have two electrons, two protons, and only one neutron in one ^3He atom, and therefore there are odd fermions inside and the composite state is a fermion. In a ^3He liquid, since the distance between the atoms is drastically reduced, we can expect the effect of repulsion is much stronger than the case in the gas phase. This is a cold atom model of electrons in solids: we can easily measure the specific heat, etc. of the liquid ^3He .

The C_V - T curve measured looks like TODO: fig 1 As is expected, when $T \rightarrow 0$, we see a $C_V \propto T$ behavior, qualitatively agreeing with (3). The coefficient however is not what is predicted by (2) and (3). Now if we apply pressure to liquid ^3He , C_v/T clearly increases.

For a fermion gas, we can measure the mass according to (3) and (2). Now if we calculate the “mass” of ^3He according to the two equations, we get a pressure-dependent (quite surprising because the density of liquids almost stays the same when we press it) “effective mass” that is never the same as the bare mass of ^3He .

The next question, then, is whether liquid ^3He – a system with known strong interaction – can be equivalently described by something with the effective mass.

2 Assumptions of Fermi liquid theory

2.1 Motivation: scattering changes the distribution of electrons

The general form of Hamiltonian of fermionic systems that is found in condensed matter physics looks like

$$H = \sum_i \frac{\mathbf{p}_i^2}{2m} + \sum_{i,j} V(|\mathbf{r}_i - \mathbf{r}_j|). \quad (5)$$

When we deal with a two-body problem with this dynamics, the collision behavior is completely described by the scattering cross-section. If we extend this picture to a many-body system, we immediately see scattering changes the momentum distribution of particles, because the final states of scattering can obviously be over the Fermi surface, and therefore there is still some distribution above p_F now.

¹For some reason, when talking about Fermi liquid, people like to use \mathbf{p} instead of \mathbf{k} .

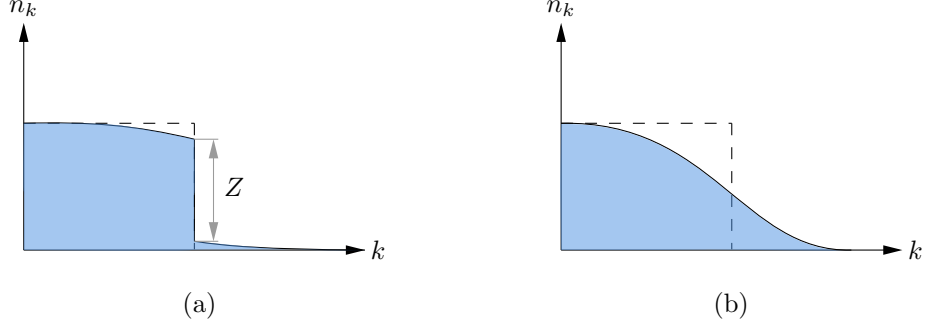


Figure 1: Two types of interaction correction to the electron distribution. (a) The Fermi liquid case: there is still a stepwise change of n with respect to the energy. (b) The non-Fermi liquid case: the change of n is smooth.

2.2 Assumption: correspondence between interactive corrected states and uncorrected states

We assume that we still have “states labeled by \mathbf{p} , n , etc.” in a Fermi liquid, and the ground state and the first several excited states can be totally labeled by $\delta n(\mathbf{p})$, which is $n(\mathbf{p})$ – some sort of “filling” on these states – minus $n_0(\mathbf{p})$ – the Fermi gas filling. We further assume δn is small, so that what we get is Fig. 1(a), instead of Fig. 1(b). This agrees with the observed fact that the $C_v \propto T$ relation is still true, so Fermi liquid should be somehow similar to the Fermi gas. Also, we assume the total number of electrons should change considerably:

$$\delta N = \sum_{\mathbf{p}} \delta n(\mathbf{p}) \ll \sum_{\mathbf{p}} n(\mathbf{p}). \quad (6)$$

2.3 Energy change

We define E to be

$$\delta E = \sum_{\mathbf{p}} \varepsilon(\mathbf{p}) \delta n(\mathbf{p}) + \mathcal{O}(\delta n^2). \quad (7)$$

The chemical potential can be immediately decided:

$$\mu := \left. \frac{\delta E}{\delta n(\mathbf{p})} \right|_{\text{Fermi surface}} = \varepsilon(p_F). \quad (8)$$

$$\delta E = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} \delta n(\mathbf{p}) + \frac{1}{2} \sum_{\mathbf{p}, \mathbf{p}'} f_{\mathbf{p}\mathbf{p}'} \delta n(\mathbf{p}) \delta n(\mathbf{p}') + \mathcal{O}(\delta n^3). \quad (9)$$