Homework 11

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Problem 1 Consider the band structure of InP as having only the electron band and the heavy hole band. Use numerical values from Kittel for the material. a) Where in the band gap is the chemical potential located at 300 K for an ultrapure sample of InP? b) Explain briefly the physical reason why the chemical potential has the value you found in part (a) (i.e., why it changes in the direction it does with T > 0 compared to T = 0).
Solution

(a) In Kittel Section 8.1, we find $E_{\mathrm{g}} = 1.27\,\mathrm{eV}$, and in Section 8.2.5, we find $m_{\mathrm{e}}^* = 0.073m$, $m_{\mathrm{h}} = 0.4m$, so according to

$$\mu = E_{\mathrm{v}} + \frac{1}{2}E_{\mathrm{g}} + \frac{3}{4}k_{\mathrm{B}}T\ln\left(\frac{m_{\mathrm{h}}^*}{m_{\mathrm{e}}^*}\right), \tag{1}$$

we find $\mu - E_{\mathrm{v}} = 0.68\,\mathrm{eV}$, so it's slightly above the middle point between E_{v} and E_{c}.

(b) We know the Fermi-Dirac distribution function for electrons above μ and the Fermi-Dirac distribution function for holes below μ are symmetric to each other. Since m_{h}^* is larger, the valence band is flatter, and near the top of the valence band, the number of states is larger than that near the bottom of the conduction band. So if $\mu - E_{\mathrm{v}} = E_{\mathrm{g}}/2$, the Fermi-Dirac factor in n and p is the same, while the density of state factors are different, and we find $p > n$, which is wrong. So we have to push μ higher to reduce the $f(E)$ factor in p to ensure $n = p$.

Problem 2
Solution

(a) When the hydrogenic model works, the inner details of the donor atom don't matter: the doped material can be seen as a positive charge placed in an undoped material, and the Coulomb field introduced by the former is then screened by the latter. So the spectrum of an electron near the donor atom can be obtained by replacing m by m_{e}^* and ϵ_0 by $\epsilon\epsilon_0$ in the Bohr theory of hydrogen, and we get

$$E_n = \frac{1}{n^2}\frac{\mathrm{e}^4 m_{\mathrm{e}}^*}{2\left(4\pi\epsilon\epsilon_0\hbar\right)^2} = \frac{1}{\epsilon^2}\frac{m_{\mathrm{e}}^*}{m}\frac{1}{n^2}\underbrace{E_0}_{13.6\,\mathrm{eV}}. \tag{2}$$

In Section 8.4.1 we find $\epsilon = 14.55$, and in Section 8.2.5 we find $m_{\mathrm{e}}^* = 0.026m$, so finally we find $E_1 = -1.67 \times 10^{-3}\,\mathrm{eV}$, and therefore the ionization energy is $1.67 \times 10^{-3}\,\mathrm{eV}$.

(b) Similarly we have

$$a = \frac{4\pi\epsilon\epsilon_0\hbar^2}{m_{\mathrm{e}}^* e^2} = \frac{\epsilon m}{m_{\mathrm{e}}^*} \cdot 0.53\,\text{Å}, \tag{3}$$

and for InAs, $a = 29.6\,\mathrm{nm}$. Wikipedia says the lattice constant of InAs is $\sim 6.05\,\text{Å}$, so the radius of the electron donated is much larger than the characteristic length of the crystal structure, which means our approximation is reasonable, or otherwise at some momentum values involved, the hyperbolic approximation of the band is no longer correct, and the details of the crystal structure is visible to the motion of the impurity-bound electron, and the approximation breaks down.

(c) When the distance between impurities is comparable to a, hopping between nearest impurities becomes frequent. So when

$$n \gtrsim \frac{1}{a^3} = 3.86 \times 10^{22}\,\mathrm{m}^{-3},$$

the impurity band becomes visible.

Problem 4

Solution

(a) The intrinsic chemical potential is

$$\mu_{\mathrm{i}} = \frac{1}{2}E_{\mathrm{g}} + \frac{3}{4}k_{\mathrm{B}}T\ln\left(\frac{m_{\mathrm{h}}^*}{e_{\mathrm{e}}^*}\right) = 0.74\,\mathrm{eV}. \tag{4}$$

So

$$n_{\mathrm{i}} = 2\left(\frac{m_{\mathrm{e}}^* k_{\mathrm{B}}T}{2\pi\hbar^2}\right)^{3/2} \mathrm{e}^{-(E_{\mathrm{c}}-\mu_{\mathrm{i}})/k_{\mathrm{B}}T} = 3.48\times 10^{12}\,\mathrm{m}^{-3}. \tag{5}$$

Using Eq. (28.39) in A&M (further assuming $E_{\mathrm{d}} - \mu \gg k_{\mathrm{B}}T$), which is

$$\frac{N_{\mathrm{d}} - N_{\mathrm{a}}}{n_{\mathrm{i}}} = 2\sinh\frac{\mu - \mu_{\mathrm{i}}}{k_{\mathrm{B}}T}, \tag{6}$$

we get $\mu = 0.74\,\mathrm{eV} + 0.54\,\mathrm{eV} = 1.28\,\mathrm{eV}$.

(b) We have (here E_{d} is the donor binding energy)

$$n_{\mathrm{d}} = \frac{N_{\mathrm{d}}}{1/2 + \mathrm{e}^{(E_{\mathrm{c}}-E_{\mathrm{d}}-\mu)/k_{\mathrm{B}}T}} = 1.8\times 10^{14}\,\mathrm{cm}^{-3}. \tag{7}$$

It can be found that most of the donor electrons indeed enter the conduction band, and $N_{\mathrm{d}} - n_{\mathrm{d}} \gg n_{\mathrm{i}}$. The system is therefore in the extrinsic limit.

(c) We have

$$\frac{E_{\mathrm{c}} - \mu}{k_{\mathrm{B}}T} = 4.6, \quad \frac{E_{\mathrm{c}} - E_{\mathrm{d}} - \mu}{k_{\mathrm{B}}T} = 3.3,$$

so the non-degenerate assumption that the above ratios should be very large doesn't hold – but since $\Delta E/k_{\mathrm{B}}T$ appears in the exponential function, in practice it's still good enough.

(d) We have

$$n = \frac{\Delta n + \sqrt{4n_{\mathrm{i}}^2 + \Delta n^2}}{2}, \quad \Delta n = N_{\mathrm{d}} - n_{\mathrm{d}}, \tag{8}$$

and the result is $n = 4.82\times 10^{15}\,\mathrm{cm}^{-3}$.

(e) It can be found that the system is indeed in the extrinsic limit: we have $n = \Delta n$, and the contribution of n_{i} can be ignored.