Reading Note of Topological Insulators by M. Franz and L. Molenkamp

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This is a reading note of [1].

1 SSH model and AKLT chain

We have already done some calculation about the SSH model in Section 15.2 in this note. Here we briefly review the model. The model (spinless version) is Sec. 3.2

$$H = \sum_i (t+\delta t) c^\dagger_{i\mathrm{A}} c_{i\mathrm{B}} + (t-\delta t) c^\dagger_{i+1,\mathrm{A}} c_{i\mathrm{B}} + \text{h.c.} \tag{1}$$ Sec. 3.2, (15)

After a Fourier transformation we have

$$H = \sum_k c^\dagger_{ka} H_{ab} c_{kb}, \quad H(k) = \boldsymbol{d}(k)\cdot\boldsymbol{\sigma}, \tag{2}$$ (16), (17)

where

$$\begin{aligned} d_x(k) &= (t+\delta t) + (t-\delta t)\cos ka, \\ d_y(k) &= (t-\delta t)\sin ka, \\ d_z(k) &= 0. \end{aligned} \tag{3}$$

All two-band models can be rewritten into $\sum_k d_\mu \sigma^\mu$, and here $d_0 = d_3 = 0$. Now we see (3) is a $S^1 \to S^1$ mapping, and it can be classified by the winding number around $\boldsymbol{d} = 0$, which is either 0 or 1 when we adjust t and δt. The phase where the winding number is 0 is a trivial phase, with the same topological properties of the vacuum, so it does not have any edge state, which can be verified explicitly and is illustrated in Figure 1(a). The phase with winding number being 1 is topologically non-trivial, and something must happen at the edge, so we predict there will be an edge state, which can also be explicitly verified and is shown in Figure 1(b).

The existence of a topological phase transition between the two states relies on the fact that the curve of $\boldsymbol{d}$ decided by (3) is S^2. If we can perturb the system with an external Hamiltonian which provides a non-zero d_z, then we can smoothly switch from one phase to another. The problem is under which symmetry constraint we have $d_z = 0$.

It is easy to notice that with half filling, we can regard a pair of ALKT chain, Jordan-Wigner transformation

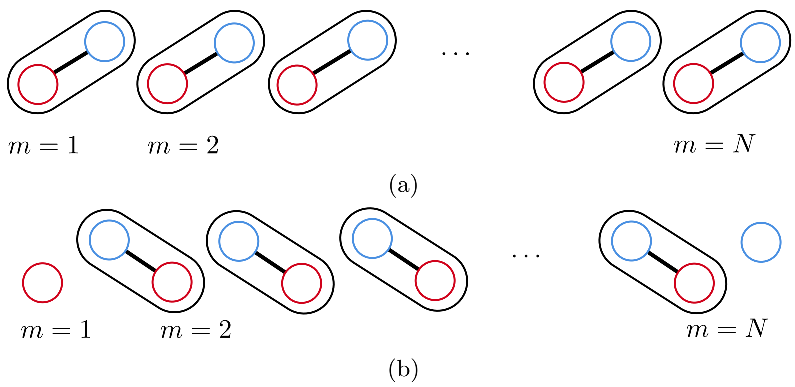

Figure 1: (a)

References

[1] Marcel Franz and Laurens Molenkamp. *Topological insulators.* Elsevier, 2013.