# Midterm

Jinyuan Wu

March 12, 2023

# 1 Problem 1

## 1.1

The electron configuration of Fe is [Ar]$3d^6 4s^2$, and according to Hund's rule, the spins are

$$\underbrace{\boxed{\uparrow\downarrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}}_{\text{3d}}\ \underbrace{\boxed{\uparrow\downarrow}}_{\text{4s}}$$

The electron configuration of $Fe^{2+}$ is [Ar]$3d^6$, and according to Hund's rule, the spins are

$$\underbrace{\boxed{\uparrow\downarrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}}_{\text{3d}}$$

$Fe^{3+}$ is obtained by reducing one electron and the spins are

$$\underbrace{\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}}_{\text{3d}}$$

For iron atoms, the total spin quantum number is $S = 2$, and the total orbital angular momentum quantum number is $L = 2$. Therefore

$$g_J = \frac{3}{2} + \frac{S(S+1) - L(L+1)}{2J(J+1)} = \frac{3}{2}. \tag{1}$$

# 2 Problem 2

## 2.1

A material is **metamagnetic** if when the external magnetic field passes a finite value $H_c$, the magnetic configuration changes all of a sudden. This is a phenomenological term and may be driven by various physical mechanisms. The material $Sr_3Ru_2O_7$ is metamagnetic, because experiments have observed that around $\mu_0 H = 7.9\,\mathrm{T}$, a sharp peak can be seen in magnetic susceptibility [3], and therefore there is indeed a sudden change in the magnetization.

The low-field phase is paramagnetic, and the high-field phase is itinerantly ferromagnetic: the material shows "a rapid change from a paramagnetic state at low fields to a more highly polarized state" [7]. (On the other hand, some other metamagnetic materials undergo an antiferromagnetism-to-ferromagnetism transition; this is not the case for $Sr_3Ru_2O_7$.)

The boundary between the two phases was once thought to be a quantum critical point: above the phase boundary between the low-field phase and the high-field phase, the resistance doesn't have typical Fermi-liquid behaviors [7]; the phase boundary between the two phases is a first-order phrase transition line with a terminating end point, and this critical point is pushed to $T = 0$ when the external magnetic field is pointed towards the $c$ direction, creating a quantum critical point [2]. Further investigations however have found that there are actually *two* peaks in susceptibility near $\mu_0 H = 7.9\,\mathrm{T}$, and this "quantum critical point" is surrounded by two first-order phase transitions [5, 3]. The exotic temperature-resistance curve likely comes from an SDW order on top of the ferromagnetic moment formed between the two aforementioned first-order phase transition, which also gives rise to anisotropic resistance (or in other words, electronic nematic) which isn't induced by the crystal structure and can't be seen away from $\mu_0 H = 7.9\,\mathrm{T}$ [6, 1].

### 2.2

$Sr_3Ru_2O_7$ is an itinerant magnetic material and the metamagnetic transition is likely due to Fermi surface instability. This is explained in [4] with a toy model. The effect or the coupling between the electron magnetic moment and the external magnetic field is modifying the chemical potential for

This also explains why nematic electron fluid is only observed near the so-called quantum critical point $\mu_0 H = 7.9\,\mathrm{T}$: because when the external magnetic field is stronger, the

## 3 Problem 3

### 3.1

## References


[1] Rodolfo Alberto Borzi et al. "Formation of a nematic fluid at high fields in $Sr_3Ru_2O_7$". In: *Science* 315.5809 (2007), pp. 214–217.

[2] SA Grigera et al. "Angular dependence of the magnetic susceptibility in the itinerant metamagnet $Sr_3Ru_2O_7$". In: *Physical Review B* 67.21 (2003), p. 214427.

[3] SA Grigera et al. "Disorder-sensitive phase formation linked to metamagnetic quantum criticality". In: *Science* 306.5699 (2004). Fig. 2 shows the magnetic susceptibility, pp. 1154–1157.

[4] Hae-Young Kee and Yong Baek Kim. "Itinerant metamagnetism induced by electronic nematic order". In: *Physical Review B* 71.18 (2005). The model Hamiltonian is (1), and the Fermi surface configurations and the magnetization curve are displayed in Fig. 2., p. 184402.

[5] Kentaro Kitagawa et al. "Metamagnetic Quantum Criticality Revealed by $^{1}$7O-NMR in the Itinerant Metamagnet $Sr_3Ru_2O_7$". In: *Physical review letters* 95.12 (2005), p. 127001.

[6] C Lester et al. "Field-tunable spin-density-wave phases in $Sr_3Ru_2O_7$". In: *Nature materials* 14.4 (2015), pp. 373–378.

[7] RS Perry et al. "Metamagnetism and critical fluctuations in high quality single crystals of the bilayer ruthenate $Sr_3Ru_2O_7$". In: *Physical Review Letters* 86.12 (2001), p. 2661.