Atomic physics: theories of laser beams and atoms

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1 Overview of atoms

Atomic physics relies on the existence of atoms. The idea that matter consists of atoms was seen in ancient Greek, probably earlier in ancient India. In the West, people usually attribute the idea to Democritus (450 BC), who claimed that there are only atoms and "void". The modern idea of atoms arose to explain the behavior of gases: if we assume the basic degrees of freedom of a gas systems are $\boldsymbol{r}_i$'s and $\boldsymbol{p}_i$'s, then everything works well – except we need to use a constant $\hbar$ to decide the correct entropy of the gas:

$$\mathrm{d}\Omega = \frac{\mathrm{d}^3\boldsymbol{r}\,\mathrm{d}^3\boldsymbol{p}}{h^3}, \quad h = 2\pi\hbar. \tag{1}$$

The origin of $\hbar$ led to the discovery of quantum mechanics.

The quantum nature of atoms is best demonstrated by beam splitters. Beam splitters are widely used in optics: they can be used to make a Mach-Zehnder interferometer, and the observed intensity takes the form of

$$(\cos(\omega t + \phi_1) + \cos(\omega t + \phi_2))^2,$$

and by moving the mirrors, we change $\phi_{1,2}$, and thus peaks and valleys occur in the relation between I and $\phi_{1,2}$.

NMore generally, in Figure 1, we have

$$\begin{aligned} |\boldsymbol{E}_3|^2 + |\boldsymbol{E}_4|^2 &= (t\boldsymbol{E}_1 + r\boldsymbol{E}_2)(t\boldsymbol{E}_1 + r\boldsymbol{E}_2)^* + (t\boldsymbol{E}_2 + r\boldsymbol{E}_1)(t\boldsymbol{E}_2 + r\boldsymbol{E}_1)^* \\ &= (|\boldsymbol{E}_1|^2 + |\boldsymbol{E}_2|^2)(|t|^2 + |r|^2) + (\boldsymbol{E}_1^* \cdot \boldsymbol{E}_2 + \text{h.c.})(t^* r + r^* t). \end{aligned} \tag{2}$$

To ensure energy conservation, we get

$$|t|^2 + |r|^2 = 1, \tag{3}$$

which is expected, and

$$t^* r + r^* t = 0. \tag{4}$$

The last equation means there has to be a phase shift caused by the beam splitter. In the 50 %-50 % case, we have $r = t\mathrm{e}^{\mathrm{i}\phi}$, and solving the above equation, we find $\phi = 90°$. Thus a 50 %-50 % beam splitter introduces a 90° phase difference between the two output beams if one of the input beam is absent, i.e. the corresponding input port is a dark port.

A comment: when we deal with actual light, not just atom beams, polarization also influences the phase shift.

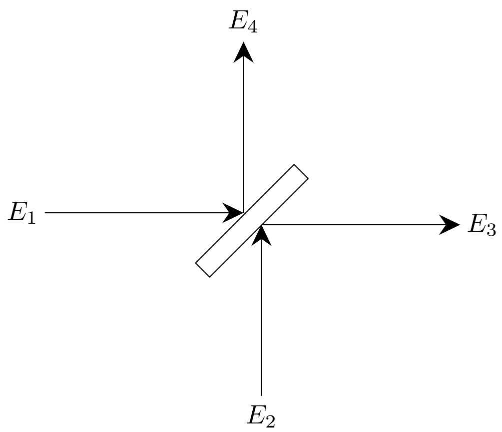

Figure 1: Beam splitter example

2 Blackbody radiation

Blackbody radiation is where atomic physics started. Both the candle and the incandescent light bulb give light when heated up. What makes it interesting is, if we heat up a black object, radiation also occurs: this can be demonstrated by putting a light mill before a piece of heated black object: the vanes rotate in the same way that they rotate when placed before a light bulb. When the temperature isn't too high, we can put a infrared screener between the light mill and the heated black object and we find the rotation rate of the light mill goes down. This means the relation between radiation and temperature of many objects is at least quantitatively modeled by the ideal black body, and therefore studying its behavior is important.

A blackbody can be modeled as a hole on a closed optical cavity: once something goes inside, it's highly unlikely to be reflected back. The possible optical modes between two walls are like

$$\sin\left(\frac{n\pi x}{L}\right) \quad \text{or} \quad \sin\left(\frac{n\pi x}{L}\right),$$

and $n = 0, 1, 2, 3, \ldots$. In a cubic box, we have n_1, n_2, n_3 to label all the modes, and TODO

We will skip the tedious historical attempts to derive the correct blackbody radiation formula, and just jump to the final answer: the energy of each light mode has to be quantized. The first try is to assume

$$E_{\text{mode } \omega} = \hbar\omega n. \tag{5}$$

Thus, for each light mode, we have

$$Z = \sum_{n=0}^{\infty} \mathrm{e}^{-n\hbar\omega/k_{\mathrm{B}}T} = \frac{1}{1 - \mathrm{e}^{-\hbar\omega/k_{\mathrm{B}}T}}, \tag{6}$$

and therefore

$$\langle E \rangle = \frac{1}{Z} \sum_{n=0}^{\infty} n\hbar\omega \mathrm{e}^{-n\hbar\omega/k_{\mathrm{B}}T} = \frac{\hbar\omega}{\mathrm{e}^{\hbar\omega/k_{\mathrm{B}}T} - 1}. \tag{7}$$

The high temperature limit would we

$$\langle E \rangle = \frac{\hbar\omega}{\frac{\hbar\omega}{k_{\mathrm{B}}T} + \frac{1}{2}\left(\frac{\hbar\omega}{k_{\mathrm{B}}T}\right)^2 + \cdots} \to k_{\mathrm{B}}T\left(1 - \frac{1}{2}\frac{\hbar\omega}{k_{\mathrm{B}}T}\right) = k_{\mathrm{B}}T - \frac{1}{2}\hbar\omega.$$

Thus, in order to go back to the classical prediction when $T \to \infty$, the energy of a single mode has to be

$$E = \hbar\omega\left(n + \frac{1}{2}\right), \tag{8}$$

and the correct expected energy is

$$\langle E \rangle = \frac{\hbar\omega}{\frac{\hbar\omega}{k_{\mathrm{B}}T} + \frac{1}{2}\left(\frac{\hbar\omega}{k_{\mathrm{B}}T}\right)^2 + \cdots} \to k_{\mathrm{B}}T\left(1 - \frac{1}{2}\frac{\hbar\omega}{k_{\mathrm{B}}T}\right) = k_{\mathrm{B}}T - \frac{1}{2}\hbar\omega + \frac{1}{2}\hbar\omega. \tag{9}$$

Now the total energy of the cavity can be found: it's

$$U = \int_0^{\infty} \langle E_\omega \rangle \, n(\omega) \, \mathrm{d}\omega = L^3 \tag{10}$$

The energy jet can then be found:

$$J = \frac{1}{4}cU = \underbrace{\frac{\pi^2 k_{\mathrm{B}}^4}{60\hbar^3 c^3}}_{\sigma} T^4. \tag{11}$$

TODO: whether we need to multiply a 2 factor to J; the direction of radiation; the vibrational 1/2 factor; the