Prof. Cosimo Bambi on General Relativity

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This is a note about Prof. Cosimo Bambi's lecture on general relativity from March 25, 2022.

1 Metric tensor as gravitational field

The idea of a metric field as a physical degree of freedom can also be motivated by Newtonian gravity. Consider the well-known fact that *gravitational mass* is the same as *inertial mass*. This means the Lagrangian of a particle in a gravitational potential Φ is

$$L = \frac{1}{2} m \boldsymbol{v}^2 - m\Phi, \tag{1}$$ (5.3)

and this Lagrangian is a low energy approximation of

$$L = -mc\sqrt{c^2 - \boldsymbol{v}^2 + 2\Phi}, \tag{2}$$ (5.5)

so the action is

$$S = -mc \int \mathrm{d}t \, \sqrt{-g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu}, \quad g_{\mu\nu} = \begin{pmatrix} -\left(1 + \frac{2\Phi}{c}\right) & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}. \tag{3}$$ (5.6), (5.7)

So we see that the gravitational potential is absorbed into the metric tensor.

Of course, (2) is just an educated guess, but we will find it leads to the beautiful and well-tested theory of general relativity.

2 Covariant derivative

Sec. 5.2.1 checks the tensor nature of $\nabla_\mu u^\nu$. Sec. 5.2.2 shows what it really does: parallel transport of vectors. (5.25)

Here we briefly summarize the argument used in Sec. 5.2.2. From (5.28) to (5.35), the parallel transport of V^θ and V^r is derived by definition. From (5.36) to (5.40), we calculated the Christoffel symbols using a short cut (compare two forms of the geodesic equations). From (5.41) and (5.43), it is shown that parallel transport can also be calculated using Christoffel symbols, and in (5.44) we show the direct connection between covariant derivative and vector parallel transport.