Squeezing of quantum noise

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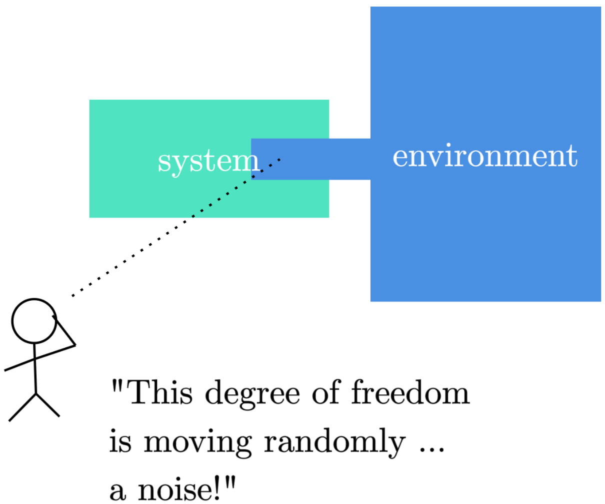
Introduction

Even when the system is isolated from the environment …



classical noise

isolated from the rest of the world

$$H = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 x^2 = \omega\left(n + \frac{1}{2}\right)$$

$$H \neq 0 \;\Rightarrow\; \langle x^2 \rangle, \langle p^2 \rangle \neq 0$$

even at the ground state

quantum noise

There are still uncertainty: *quantum noise*

Visualize quantum noise

$$H=\int \mathrm{d}^3\boldsymbol{r}\left(\frac{1}{2}\epsilon \boldsymbol{E}^2+\frac{1}{2\mu}\boldsymbol{B}^2\right)=\sum_k \omega_k\left(\boxed{a_k^\dagger a_k}+\frac{1}{2}\right)$$

Consider only one mode

$$\boxed{W(x,p)}=\frac{1}{\pi\hbar}\int_{-\infty}^{\infty}\langle x-y|\rho|x+y\rangle e^{-2\mathrm{i}py/\hbar}\mathrm{d}y$$

So that

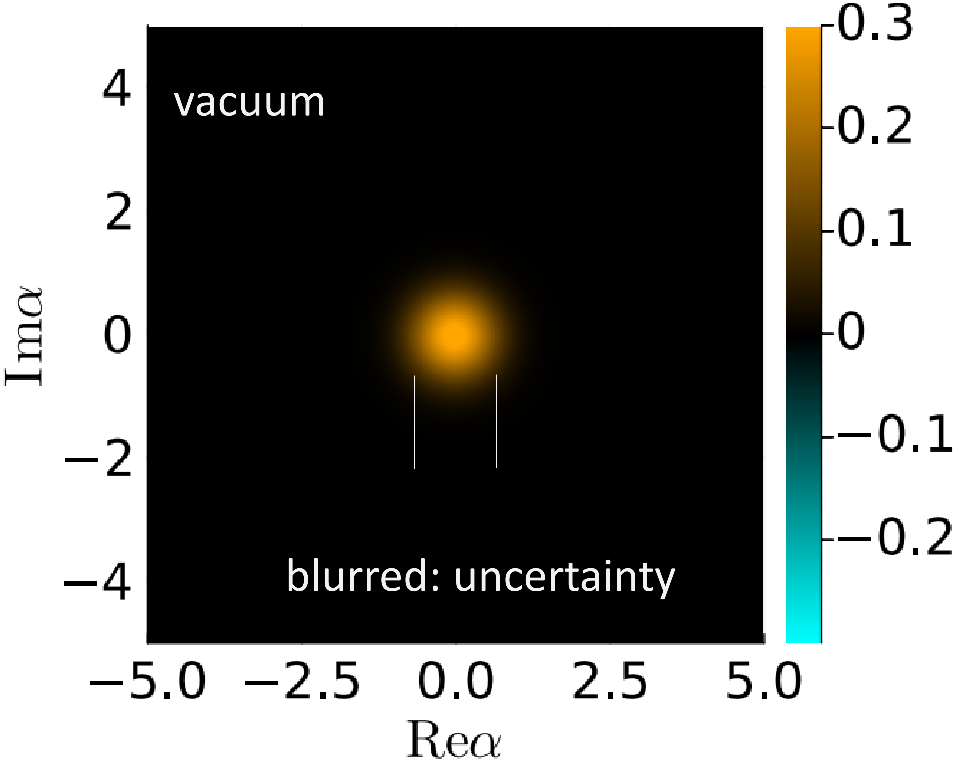
$$a=\frac{1}{\sqrt{2}}(X+\mathrm{i}P),\quad a^\dagger=\frac{1}{\sqrt{2}}(X-\mathrm{i}P).$$

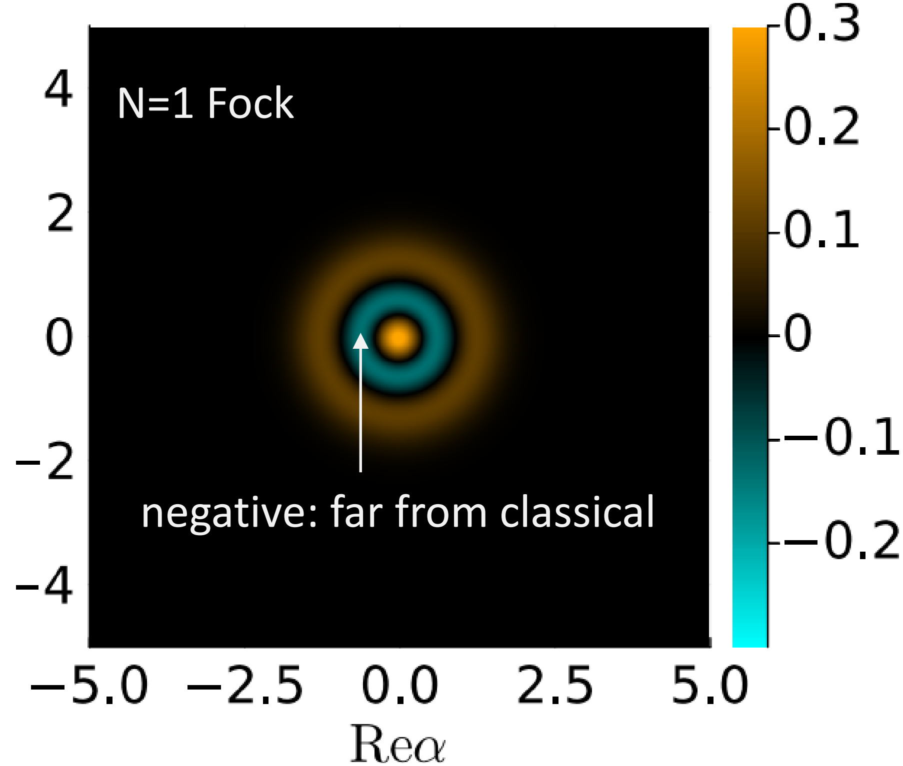
$$\left\langle O\left(a,a^\dagger\right)\right\rangle=\int \mathrm{d}^2\alpha \boxed{W\left(\alpha,\alpha^*\right)}\boxed{O\left(\alpha,\alpha^*\right)}$$

Ordering: $(a^\dagger a+aa^\dagger)/2$

Quasi-probability distribution: **Wigner function**

Visualize quantum noise





To find these Wigner functions: use [QuantumOptics.jl]

Mach-Zehnder interferometer

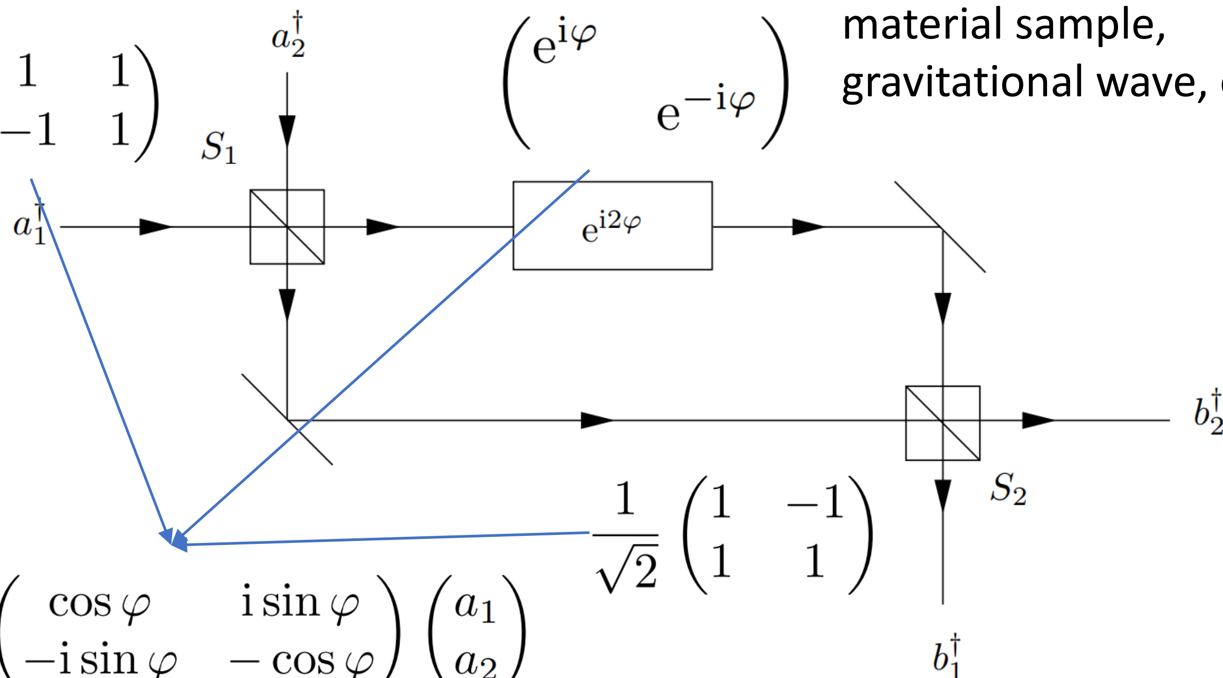
Dark port

Laser input

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} e^{i\varphi} & \\ & e^{-i\varphi} \end{pmatrix}$$

Small angle;
material sample,
gravitational wave, etc.



$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} \cos\varphi & i\sin\varphi \\ -i\sin\varphi & -\cos\varphi \end{pmatrix}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

$$n_{b_2} = b_2^\dagger b_2 = \sin^2\varphi a_1^\dagger a_1 + \cos^2\varphi a_2^\dagger a_2 - i\sin\varphi\cos\varphi(a_1^\dagger a_2 - a_2^\dagger a_1)$$

The origin of quantum noise

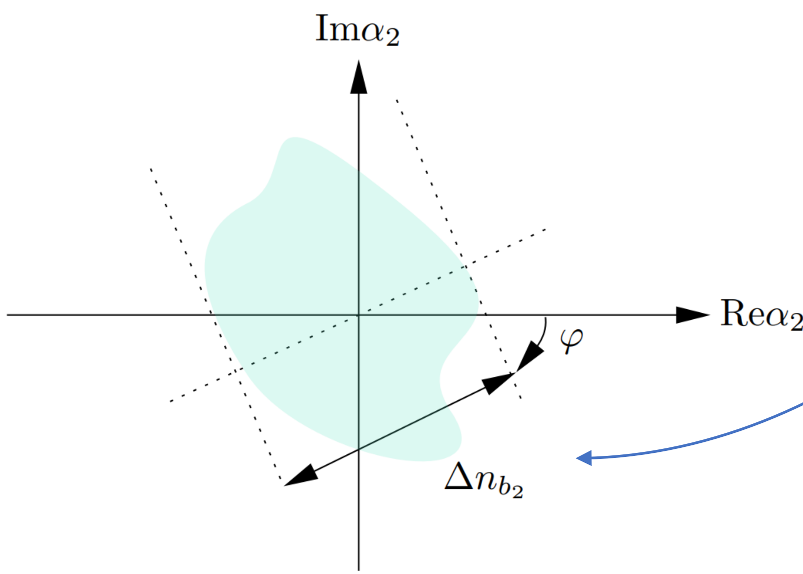
$$n_{b_2} = b_2^\dagger b_2 = \boxed{\sin^2\varphi a_1^\dagger a_1} \boxed{+\cos^2\varphi a_2^\dagger a_2} \boxed{-\mathrm{i}\sin\varphi\cos\varphi(a_1^\dagger a_2 - a_2^\dagger a_1)}$$

suppressed by φ factor

almost zero

a_1 can be treated classically

$$\Delta n_{b_2} \approx \varphi\Delta(\alpha^* a_2 - \alpha a_2^\dagger)$$
$$= 2\varphi|\alpha|\Delta(-\sin\varphi\,\mathrm{Re}\,a_2 + \cos\varphi\,\mathrm{Im}\,a_2)$$



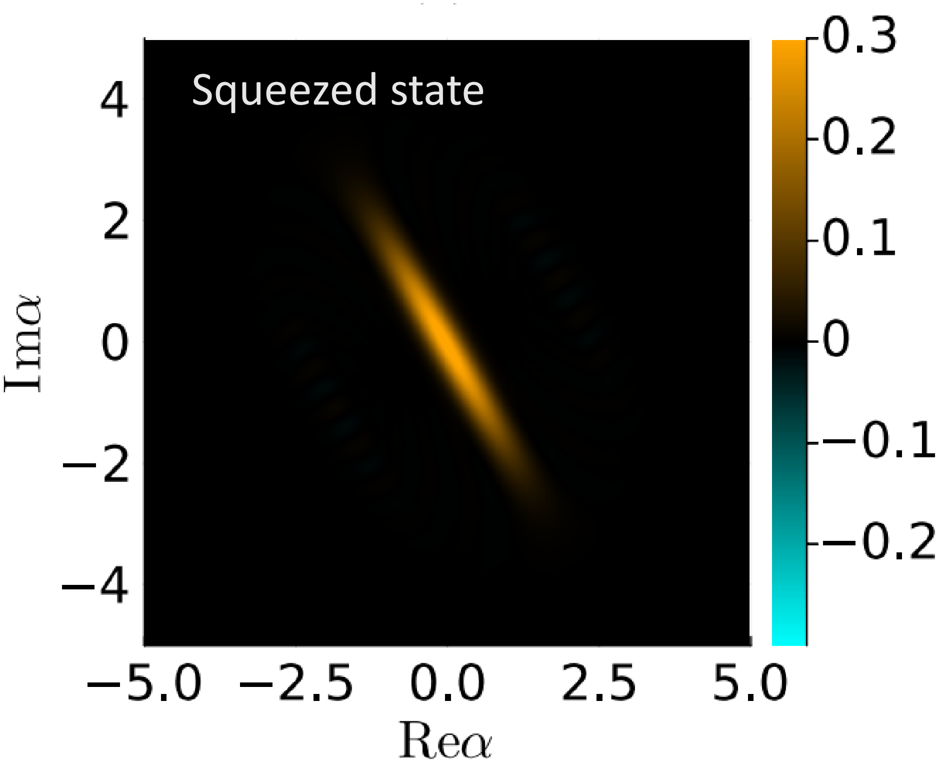
If a_2 is in ordinary vacuum:
standard quantum limit

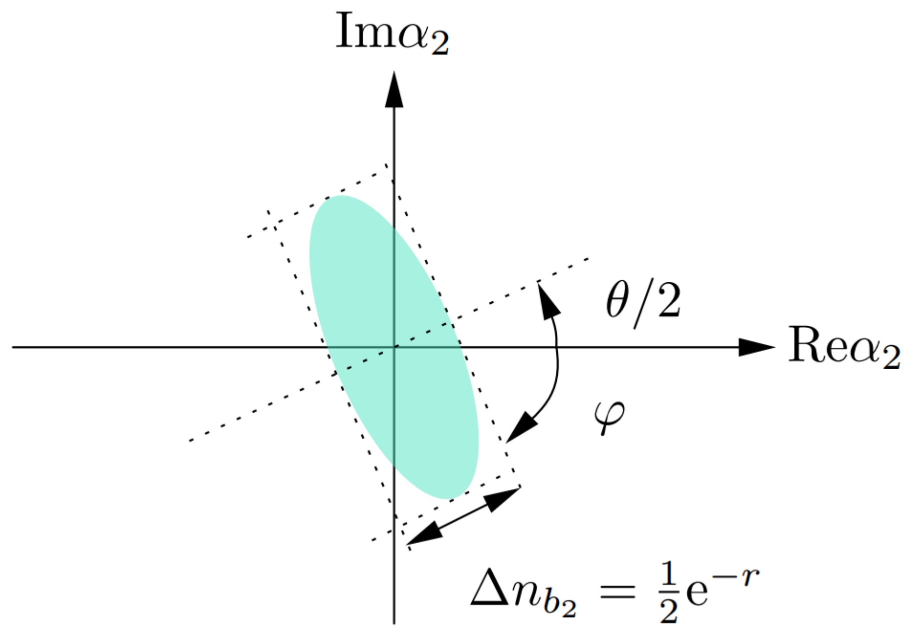
$$\Delta n_{b_2} = \varphi|\alpha|$$

$$\frac{\Delta n_{b_2}}{n_{b_2}} = \frac{1}{|\alpha|\varphi} \sim \frac{1}{\sqrt{N}}$$

Squeezing the quantum noise

To reduce the quantum noise:

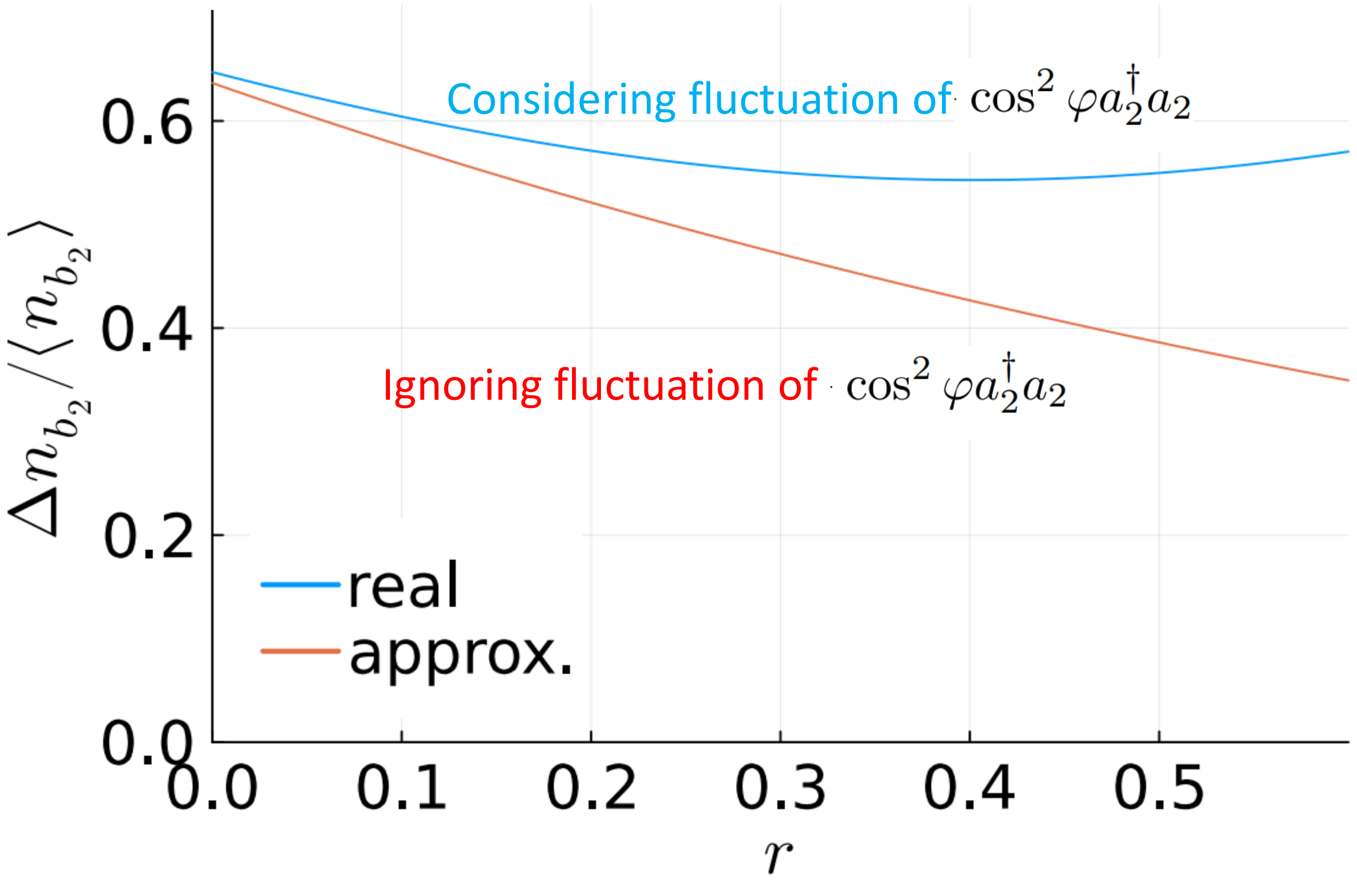




Squeezing the quantum noise

Unfortunately, if we squeeze the vacuum too much …

$\cos^2 \varphi a_2^\dagger a_2$ can no longer be ignored



Discussion

Even better interferometry designs?

Standard quantum limit	Heisenberg limit	"Nonlinear measurement"
$1/\sqrt{N}$	$1/N$	$1/N^{3/2}$