

Bosonic modes in Fermi liquid

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Background

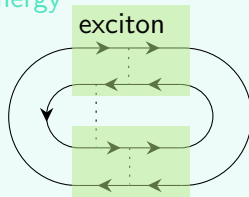
In a Fermi liquid we have ...

- Quasiparticles (electron/hole) with Σ -correction
- Any anything else?

single electron energy



exciton energy



... and more

Question

What to do

Finding modes other than the corrected single electron/hole

Why it's important

Usually not for C_V but for optical response: ϵ , $\chi^{(3)}$, etc.

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Today's topic

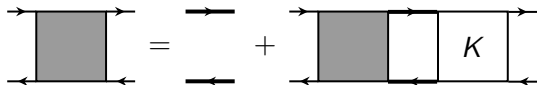
Electron-hole bosonic modes in Fermi liquid (with *some* scattering picked up back, i.e. beyond $\delta E \sim \varepsilon \delta n + f \delta n \delta n$), i.e.

$$|\text{single excitation}\rangle = \sum_{\mathbf{k}_1, \mathbf{k}_2} c_{\mathbf{k}_1 \mathbf{k}_2} \left| \begin{array}{c} \bullet \\ \text{---} \\ \text{---} \end{array} \right\rangle \quad (1)$$

No trion, higher order correlation, or even more exotic spinons, etc.
beyond Fermi liquid

Series calculation

Bethe–Salpeter equation (BSE) is for quantitative calculations.



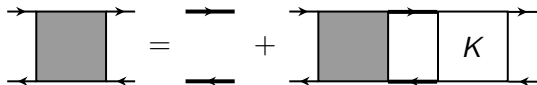
The diagram shows a Feynman diagram equation. On the left is a single shaded rectangular loop with four external lines (two incoming from the left, two outgoing to the right). This is followed by an equals sign, then a plus sign, and then a series of three rectangular loops connected in series. The first loop in the series is shaded, the second is white, and the third is labeled with the letter 'K'. All loops have four external lines. The entire equation is labeled with (2) on the right.

$$\text{Diagram} = \text{Diagram} + \text{Diagram} + K \text{ Diagram} \quad (2)$$

Problem: no picture about “how the electron moves”

Series calculation

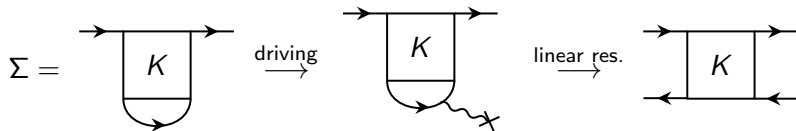
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Linking BSE with single-electron kinetic theory

Linear response of single-electron under external field = BSE


$$(3)$$

simplest single-electron theory: quantum Boltzmann equation (QBE)

What to investigate

Stable oscillation modes of QBE (\Leftrightarrow infinite response to external field \Leftrightarrow bosonic mode): for $n_{\mathbf{p}\sigma\sigma'}(\mathbf{r})$, $\varepsilon_{\mathbf{p}\sigma\sigma'} = \varepsilon[\delta n]$,

$$\frac{\partial n_{\mathbf{p}}}{\partial t} + \underbrace{\frac{\partial \varepsilon_{\mathbf{p}}}{\partial \mathbf{p}} \cdot \frac{\partial n_{\mathbf{p}}}{\partial \mathbf{r}}}_{\text{diffusion}} - \underbrace{\frac{\partial \varepsilon_{\mathbf{p}}}{\partial \mathbf{r}} \cdot \frac{\partial n_{\mathbf{p}}}{\partial \mathbf{p}}}_{\text{force}} + \underbrace{i[\varepsilon_{\mathbf{p}}, n_{\mathbf{p}}]}_{\text{multi-band}} = \underbrace{I_{\text{Fermi golden rule}}}_{\text{collision}} \quad (4)$$

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What to expect

Three types of important bosonic modes:

- Zero sound in uncharged single-band Fermi liquid
- Plasmon in charged single-band Fermi liquid = zero sound + long range interaction
- Exciton in charged multi-band Fermi liquid

Equation governing zero sound

System Single-band Fermi liquid with spin ignored

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Kinetics of uncharged Fermi liquid *Landau equation* = QBE +

$$\varepsilon_{\mathbf{p}}(\mathbf{r}) = \varepsilon_{\mathbf{p}}^0 + \frac{1}{V} \sum_{\mathbf{p}'} f_{\mathbf{p}\mathbf{p}'} \delta n_{\mathbf{p}}(\mathbf{r}) \quad (5)$$

(assumption: $\mathbf{q} \rightarrow 0$ in $c_{\mathbf{p}+\mathbf{q}}^\dagger c_{\mathbf{p}}$, i.e. $\delta n_{\mathbf{p}}(\mathbf{r})$ being smooth in \mathbf{r})

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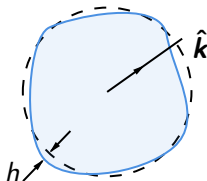
EOM governing zero sound Small disturbance, no collision, :

$$\frac{\partial \delta n_{\mathbf{p}}}{\partial t} + \frac{\partial \varepsilon_{\mathbf{p}}^{\text{static}}}{\partial \mathbf{p}} \cdot \frac{\partial \delta n_{\mathbf{p}}}{\partial \mathbf{r}} - \frac{\partial n_{\mathbf{p}}^{\text{static}}}{\partial \mathbf{p}} \cdot \underbrace{\frac{1}{V} \sum_{\mathbf{p}'} f_{\mathbf{p}\mathbf{p}'} \frac{\partial \delta n_{\mathbf{p}}}{\partial \mathbf{r}}}_{\partial \varepsilon_{\mathbf{p}} / \partial \mathbf{r}} = 0 \quad (6)$$

Fermi surface vibration

Ansatz Disturbance as small as possible ...

$$n_{\mathbf{p}}(\mathbf{r}, t) = e^{i(\mathbf{q} \cdot \mathbf{r} - i\omega t)} \theta(\mu - \varepsilon_{\mathbf{p}}^{\text{stable}} - h(\hat{\mathbf{p}})) \quad (7)$$

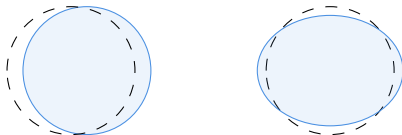


Eigenvalue problem

$$(\omega - \mathbf{q} \cdot \mathbf{v})h(\hat{\mathbf{k}}) = \mathbf{q} \cdot \mathbf{v} \int \frac{d\Omega'}{4\pi} F(\vartheta)h(\hat{\mathbf{k}}'). \quad (8)$$

where \mathbf{v} is single-electron velocity. \Rightarrow zero sound has linear dispersion;
zero sound requires $F \neq 0$

Shape of Fermi surface



Zero sound is not density wave In zero sound $V_{\text{Fermi sea}} = \text{const.} \Rightarrow$
zero sound is not ordinary sound

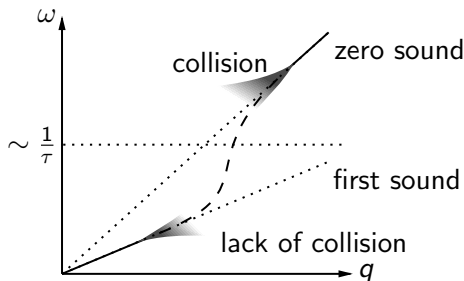
Comparison with ordinary sound

Ordinary sound Fermi liquid theory $\Rightarrow \partial\rho/\partial P \Rightarrow$ another sound mode (“first sound”, ordinary sound, density mode) from hydrodynamics

Relation with zero sound

- First sound appears when $\omega\tau \ll 1$: ordinary hydrodynamics \Leftrightarrow local equilibrium $\Leftrightarrow \tau \ll 1/\omega$
- zero sound appears when $\omega\tau \gg 1$: no collision integral $\Leftrightarrow \tau \gg 1/\omega$

The two are connected: a radical finite- T correction



What happens with long-range interaction

The origin of $f_{pp'}$

$$f_{kk'} = \lim_{q \rightarrow 0} \left[\text{diagram 1} + \text{diagram 2} \right] \quad (9)$$

The equation shows two Feynman diagrams representing the interaction between two particles with momenta k and k' via a long-range interaction with momentum q .
 Diagram 1 (left): Two vertices connected by a horizontal dashed line labeled q . The left vertex has an incoming line from the bottom labeled $k+q$ and an outgoing line to the top labeled k . The right vertex has an incoming line from the bottom labeled $k'+q$ and an outgoing line to the top labeled k' .
 Diagram 2 (right): Two vertices connected by a vertical dashed line labeled $k' - k$. The top vertex has an incoming line from the right labeled k' and an outgoing line to the left labeled k . The bottom vertex has an incoming line from the right labeled $k'+q$ and an outgoing line to the left labeled $k+q$.

Coulomb interaction \Rightarrow first term divergent in \mathbf{k} space \Rightarrow it should be considered in \mathbf{r} space

Landau-Silin eq.

$$\frac{\partial n_{\mathbf{p}}}{\partial t} + \frac{\partial \varepsilon_{\mathbf{p}}}{\partial \mathbf{p}} \cdot \frac{\partial n_{\mathbf{p}}}{\partial \mathbf{r}} - \frac{\partial (\varepsilon_{\mathbf{p}} - e\varphi(\mathbf{r}))}{\partial \mathbf{r}} \cdot \frac{\partial n_{\mathbf{p}}}{\partial \mathbf{p}} + i[\varepsilon_{\mathbf{p}}, n_{\mathbf{p}}] = \underbrace{I_{\text{Fermi golden rule}}}_{\text{collision}} \quad (10)$$

$$\varepsilon_{\mathbf{p}}(\mathbf{r}) = \varepsilon_{\mathbf{p}}^0 + \frac{1}{V} \sum_{\mathbf{p}'} f_{\mathbf{p}\mathbf{p}'} \delta n_{\mathbf{p}'}(\mathbf{r}), \quad \nabla^2 \varphi = e \cdot \frac{1}{V} \sum_{\mathbf{p}} n_{\mathbf{p}}(\mathbf{r}). \quad (11)$$

Fermi liquid, uncharged: zero sound

- Linear, gapless
- From $f_{pp'}$

Fermi liquid, charged: plasmon

- Divergent Hartree term \Rightarrow self-energy correction in real space
- When $\mathbf{q} = 0$: $f_{pp'}$ not important; gapped

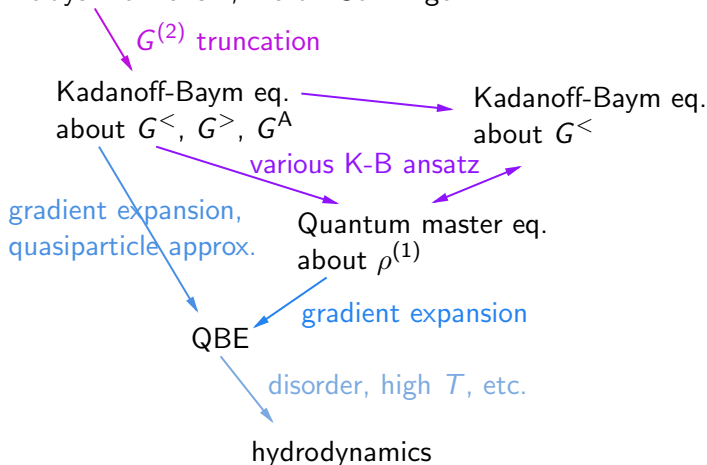
Two bands: exciton

Justifying quantum Boltzmann equation

Is QBE reliable?

Yes! When we intuitively expect it to work –

Keldysh formalism, Martin-Schwinger



Discussion