Bosonic modes in Fermi liquid

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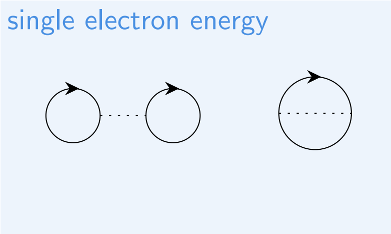
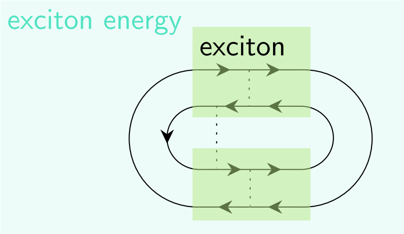
Background

In a Fermi liquid we have ...

- Quasiparticles (electron/hole) with Σ-correction
- Any anything else?





... and more

Question

What to do

Finding modes other than the corrected single electron/hole

Why it's important

Usually not for C_V but for optical response: ϵ, $\chi^{(3)}$, etc.

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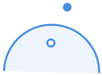
Today's topic

Electron-hole bosonic modes in Fermi liquid (with *some* scattering picked up back, i.e. beyond $\delta E \sim \varepsilon\,\delta n + f\,\delta n\,\delta n$), i.e.

$$|\text{single excitation}\rangle = \sum_{\boldsymbol{k}_1,\boldsymbol{k}_2} c_{\boldsymbol{k}_1\boldsymbol{k}_2} \left| \quad \right\rangle \tag{1}$$



No trion, higher order correlation, or even more exotic spinons, etc. beyond Fermi liquid

Methodology

Series quantitative prediction Bethe-Salpeter eq. (BSE)
Problem: no picture about "how the electron moves"

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Single electron linear response singularity = bosonic modes
$c^\dagger_{\boldsymbol{k}_1} c_{\boldsymbol{k}_2}$ = single-electron distribution = electron-hole pair annihilation operator

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Quantum Boltzmann eq. (QBE) Easiest kinetic theory for single-electron distribution to external field perturbation.
Conditions of QBE

- low (external and inherent) ω
- long wave length
- well-defined quasiparticles; high order correlation not important

Overview

What to investigate

Stable oscillation modes of QBE ($\Leftrightarrow$ infinite response to external field $\Leftrightarrow$ bosonic mode): for $n_{\boldsymbol{p}\sigma\sigma'}(\boldsymbol{r})$, $\varepsilon_{\boldsymbol{p}\sigma\sigma'} = \varepsilon[\delta n]$,

$$\frac{\partial n_{\boldsymbol{p}}}{\partial t} + \underbrace{\frac{\partial \varepsilon_{\boldsymbol{p}}}{\partial \boldsymbol{p}} \cdot \frac{\partial n_{\boldsymbol{p}}}{\partial \boldsymbol{r}}}_{\text{diffusion}} \underbrace{- \frac{\partial \varepsilon_{\boldsymbol{p}}}{\partial \boldsymbol{r}} \cdot \frac{\partial n_{\boldsymbol{p}}}{\partial \boldsymbol{p}}}_{\text{force}} + \underbrace{\mathrm{i}\left[\varepsilon_{\boldsymbol{p}}, n_{\boldsymbol{p}}\right]}_{\text{multi-band}} = \underbrace{I_{\text{Fermi golden rule}}}_{\text{collision}} . \tag{2}$$

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What to expect

Three types of important bosonic modes:

- Zero sound in uncharged single-band Fermi liquid
- Plasmon in charged single-band Fermi liquid = zero sound + long range interaction
- Exciton in charged multi-band Fermi liquid

Equation governing zero sound

System Single-band Fermi liquid with spin ignored

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Kinetics of uncharged Fermi liquid *Landau equation* = QBE +

$$\varepsilon_{\boldsymbol{p}}(\boldsymbol{r}) = \varepsilon_{\boldsymbol{p}}^0 + \frac{1}{V} \sum_{\boldsymbol{p}'} f_{\boldsymbol{p}\boldsymbol{p}'} \, \delta n_{\boldsymbol{p}}(\boldsymbol{r}) \tag{3}$$

(assumption: $\boldsymbol{q} \to 0$ in $c^\dagger_{\boldsymbol{p}+\boldsymbol{q}} c_{\boldsymbol{p}}$, i.e. $\delta n_{\boldsymbol{p}}(\boldsymbol{r})$ being smooth in $\boldsymbol{r}$)

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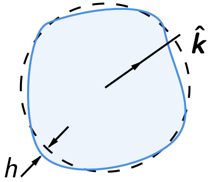
EOM governing zero sound Small disturbance, no collision, :

$$\frac{\partial \, \delta n_{\boldsymbol{p}}}{\partial t} + \frac{\partial \varepsilon^{\text{static}}_{\boldsymbol{p}}}{\partial \boldsymbol{p}} \cdot \frac{\partial \, \delta n_{\boldsymbol{p}}}{\partial \boldsymbol{r}} - \frac{\partial n^{\text{static}}_{\boldsymbol{p}}}{\partial \boldsymbol{p}} \cdot \underbrace{\frac{1}{V} \sum_{\boldsymbol{p}'} f_{\boldsymbol{p}\boldsymbol{p}'} \frac{\partial \, \delta n_{\boldsymbol{p}}}{\partial \boldsymbol{r}}}_{\partial \delta \varepsilon_{\boldsymbol{p}} / \partial \boldsymbol{r}} = 0 \tag{4}$$

Fermi surface vibration

Ansatz Disturbance as small as possible ...

$$n_{\boldsymbol{p}}(\boldsymbol{r}, t) = \mathrm{e}^{\mathrm{i}(\boldsymbol{q}\cdot\boldsymbol{r} - \mathrm{i}\,\omega t)}\, \theta(\mu - \varepsilon_{\boldsymbol{p}}^{\text{stable}} - h(\hat{\boldsymbol{p}})) \tag{5}$$



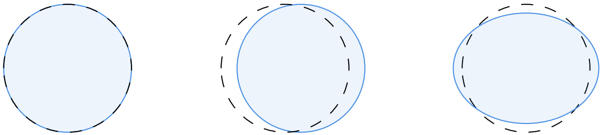
Eigenvalue problem

$$(\omega - \boldsymbol{q}\cdot\boldsymbol{v})h(\hat{\boldsymbol{k}}) = \boldsymbol{q}\cdot\boldsymbol{v} \int \frac{\mathrm{d}\Omega'}{4\pi} F(\vartheta) h(\hat{\boldsymbol{k}}'). \tag{6}$$

where $\boldsymbol{v}$ is single-electron velocity. $\Rightarrow$ zero sound has linear dispersion; zero sound requires $F \neq 0$

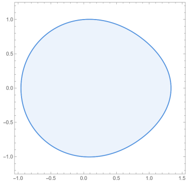
Modes

Shape of Fermi surface



$l = 0$ $\qquad$ $l = 1$ $\qquad$ $l = 2$

Distortion More electrons in $\hat{\boldsymbol{q}}$; less electrons in $-\hat{\boldsymbol{q}}$



Zero sound is not density wave In zero sound $V_{\text{Fermi sea}} = \text{const.} \Rightarrow$ zero sound is not ordinary sound

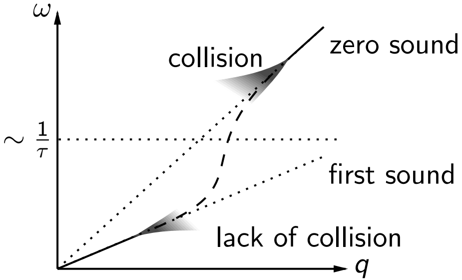
Comparison with ordinary sound

Ordinary sound Fermi liquid theory $\Rightarrow \partial\rho/\partial P \Rightarrow$ another sound mode ("first sound", ordinary sound, density mode) from hydrodynamics

Relation with zero sound

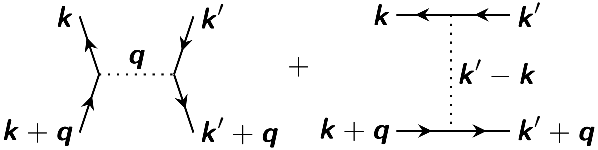
- First sound appears when $\omega\tau \ll 1$: ordinary hydrodynamics $\Leftrightarrow$ local equilibrium $\Leftrightarrow \tau \ll 1/\omega$
- zero sound appears when $\omega\tau \gg 1$: no collision integral $\Leftrightarrow \tau \gg 1/\omega$

The two are connected: a radical finite-T correction



The origin of $f_{\boldsymbol{pp}'}$

$$f_{\boldsymbol{kk}'} = \lim_{\boldsymbol{q}\to 0} \qquad (7)$$



Coulomb interaction $\Rightarrow$ first term divergent in $\boldsymbol{k}$ space $\Rightarrow$ it should be considered in $\boldsymbol{r}$ space

Landau-Silin eq.

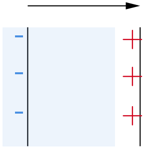
$$\frac{\partial n_{\boldsymbol{p}}}{\partial t} + \frac{\partial \varepsilon_{\boldsymbol{p}}}{\partial \boldsymbol{p}} \cdot \frac{\partial n_{\boldsymbol{p}}}{\partial \boldsymbol{r}} - \frac{\partial(\varepsilon_{\boldsymbol{p}} - e\varphi(\boldsymbol{r}))}{\partial \boldsymbol{r}} \cdot \frac{\partial n_{\boldsymbol{p}}}{\partial \boldsymbol{p}} = \underbrace{I_{\text{Fermi golden rule}}}_{\text{collision}}, \tag{8}$$

$$\varepsilon_{\boldsymbol{p}}(\boldsymbol{r}) = \varepsilon^0_{\boldsymbol{p}} + \frac{1}{V}\sum_{\boldsymbol{p}'} f_{\boldsymbol{pp}'}\,\delta n_{\boldsymbol{p}}(\boldsymbol{r}), \quad \nabla^2\varphi = e \cdot \frac{1}{V}\sum_{\boldsymbol{p}} n_{\boldsymbol{p}}(\boldsymbol{r}). \tag{9}$$

Plasmon mode

Plasmon is gapped When $\boldsymbol{q} \to 0$ we get to the elementary case

$$\boldsymbol{F} = -e\boldsymbol{E}$$



$$m\ddot{\boldsymbol{x}} = -m\omega^2 \boldsymbol{x} = (-e)\boldsymbol{E} = -e \cdot \frac{1}{\epsilon_0} en\boldsymbol{x} \Rightarrow \omega = \sqrt{\frac{ne^2}{\epsilon_0 m}}. \tag{10}$$

Comparison with zero sound When $\boldsymbol{q} \to 0$, $\varphi(\boldsymbol{r}) \Rightarrow$ oscillation: long-range interaction $\Rightarrow$ finite gap

Comparison with first sound $V_{\text{Fermi sea}} = \text{const.}$ in plasmon as well: plasmon is not a density mode

Summary

Fermi liquid, uncharged: zero sound

- Linear, gapless
- From $f_{\boldsymbol{p}\boldsymbol{p}'}$

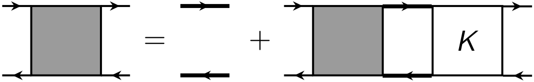
Fermi liquid, charged: plasmon

- Divergent Hartree term $\Rightarrow$ self-energy correction in real space
- When $\boldsymbol{q} = 0$: $f_{\boldsymbol{p}\boldsymbol{p}'}$ not important; gapped

Two bands: exciton

Series calculation

Bethe–Salpeter equation (BSE) is for quantitative calculations.



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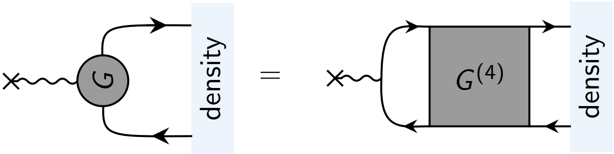
What we need Linear response of single-electron under external field = BSE (simplest single-electron theory: QBE)



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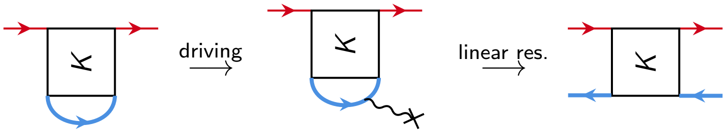
Next step: relation between K and Σ

Linking Σ with K

Linear response of a single self-energy diagram

$\Sigma =$



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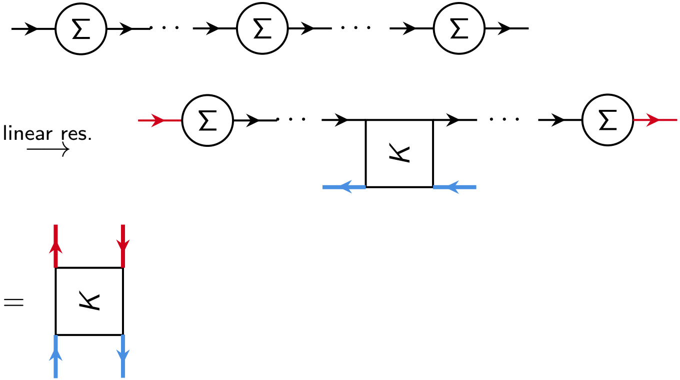
Linking Σ with K

Whole picture



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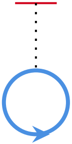
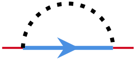
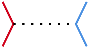
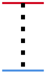
Linking Σ with K

Example: linear response from time-dependent GW = BSE

$\Sigma =$  $+$  ,

$K =$  $+$ 

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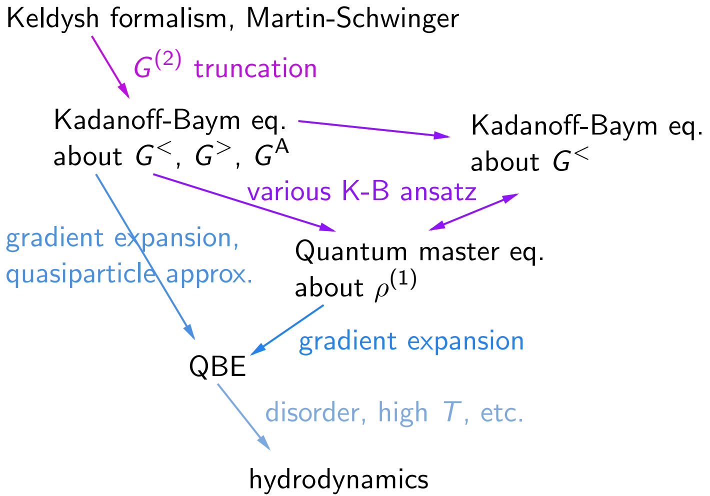
- First term = Electron Hartree term = Electron direct term = Exciton exchange term; $+1$ prefactor;
- Second term = Electron Fock term = Electron exchange term = Exciton direct term; (-1) prefactor.

Is QBE reliable?

Yes! When we intuitively expect it to work –



Discussion