Homework 2

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1 Polarizability, absorption cross-section, stimulated emission, and optical amplification

(a) The EOM of a classical atom described by the Drude-Lorentz model is

$$\ddot{x} + \gamma\dot{x} + \omega_0^2 x = \frac{q}{m}E. \tag{1}$$

For a plane wave external electric field whose phasor is

$$\tilde{E} = \tilde{E}_0 \mathrm{e}^{-\mathrm{i}(\omega t - kz)}, \tag{2}$$

the response of x, in the form of phasor, is

$$\tilde{x} = \frac{\tilde{E}}{-\omega^2 + \omega_0^2 - \mathrm{i}\gamma\omega}, \tag{3}$$

and therefore

$$\tilde{\dot{x}} = -\mathrm{i}\omega \frac{\tilde{E}}{-\omega^2 + \omega_0^2 - \mathrm{i}\gamma\omega}, \tag{4}$$

and the time average power of dissipation is

$$\langle P_{\text{abs}} \rangle = \langle F_{\text{dissipation}} \cdot v \rangle = -\gamma \langle v \cdot v \rangle = \gamma \cdot \frac{1}{2} \operatorname{Re} \tilde{\dot{x}}^* \cdot \tilde{\dot{x}} = -\frac{\gamma}{2} \frac{\omega^2 |\tilde{E}_0|^2}{(\omega^2 - \omega_0^2)^2 + \gamma^2\omega^2}. \tag{5}$$

(b) The absorption cross section can be calculated by

$$\sigma_{\text{abs}} = \frac{|\langle P_{\text{abs}} \rangle|}{\text{input intensity}} = \frac{|\langle P_{\text{abs}} \rangle|}{\frac{1}{\mu_0}\frac{k}{\omega} \underbrace{E^2}_{\frac{1}{2}|\tilde{E}_0|^2}} = \frac{1}{c\epsilon_0} \frac{\omega^2}{(\omega^2 - \omega_0^2)^2 + \gamma^2\omega^2} \gamma. \tag{6}$$