Homework 3

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We need to solve

$$\begin{aligned} u_t &= 4u_{xx} \text{ for } 0 < x < L, t > 0 \\ u(0,t) &= u(L,t) = 0, \\ u(x,0) &= x^2(L-x). \end{aligned} \tag{1}$$

We do Laplace transform with respect to t and get

$$sU - u(t=0) = 4\partial_x^2 U. \tag{2}$$

Since the stimulus $u(t=0)$ is a polynomial, we insert the ansatz

$$U(x,s) = a(s)x^3 + b(s)x^2 + c(s)x + d(s)$$

into the above equation to find a specific solution, and find

$$\begin{aligned} 4(6ax + 2b) - s(ax^3 + bx^2 + cx + d) &= -x^2(L-x), \\ U(x,s) &= -\frac{1}{s}x^3 + \frac{L}{s}x^2 - \frac{24}{s^2}x + \frac{8L}{s^2}. \end{aligned} \tag{3}$$

The inverse Laplace transform therefore tells us

$$u(x,t) = \tag{4}$$