Floquet physics

Periodic driving, formalism, and spetroscopy

Jinyuan Wu

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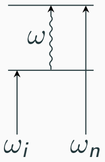
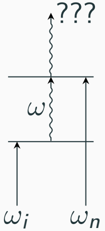
Introduction

Time-dependent perturbation theory, $\cancel{\omega_{\mathrm{eg}} + \omega}$
$\Rightarrow$ Fermi golden rule (finite T or not)



What happens when we consider high order perturbations?



Inherently non-equilibrium The state of photons is a coherent state: $|\Psi\rangle$ far from any eigenstate!

Overview

- From Schrodinger equation to Floquet effective Hamiltonian
- Relation with time-dependent perturbation theory and rotating wave approximation (RWA)?
- Floquet correction to electron band

The Floquet formalism

Quasi-stationary states and quasienergies

Periodically driven Hamiltonian: quasi-eigensystem

Floquet theory $H(t) = H(t+T) \Rightarrow$ for every $|\psi(t)\rangle$,

$$|\psi(t)\rangle = \sum_n |\psi_n(t)\rangle\,, \quad |\psi_n(t)\rangle = \mathrm{e}^{-\mathrm{i}\,\varepsilon_n t/\hbar}\,\underbrace{|\Phi_n(t)\rangle}_{\text{period } T}, \tag{1}$$

$$|\Phi_n(t)\rangle = \underbrace{\sum_m \mathrm{e}^{-\mathrm{i}\,m\omega t}\,|\phi_n^{(m)}\rangle}_{\text{discrete Fourier series}}, \quad \omega = 2\pi/T. \tag{2}$$

Highlights

- $\mathcal{H} \otimes \{m = \ldots, -1, 0, 1, \ldots\}$: **extended Hilbert space**
- $\{\Phi_n(t)\}$: **quasi-stationary states**
- $\{\varepsilon_n\}$: **quasienergies**

Our task How to $\{\phi_n^{(m)}\}$ and $\{\varepsilon_n\}$?

Floquet effective Hamiltonian

Our task A Hamiltonian for $\{\phi_n^{(m)}\}$ and $\{\varepsilon_n\}$?

$$\underbrace{H(t)}_{=:\sum_m \mathrm{e}^{-\mathrm{i}\,m\omega t}\,H^{(m)}} \qquad |\psi_n(t)\rangle = \mathrm{i}\,\hbar\partial_t\,|\psi_n(t)\rangle = \varepsilon_n + \mathrm{i}\,\hbar\partial_t\,|\Phi_n(t)\rangle$$

$$\Rightarrow \sum_{m'} H^{(m-m')}\,|\phi_n^{(m')}\rangle = (\varepsilon_n + m\hbar\omega)\,|\phi_n^{(m)}\rangle$$

Floquet effective Hamiltonian Indeed we have a Hamiltonian!!

Floquet effective Hamiltonian

Our task A Hamiltonian for $\{\phi_n^{(m)}\}$ and $\{\varepsilon_n\}$?

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$$\Rightarrow \boxed{\varepsilon_n\,|\phi_n^{(m)}\rangle = \sum_{m'}(H^{(m-m')} - m\hbar\omega\delta_{mm'})\,|\phi_n^{(m')}\rangle\,.} \tag{3}$$

Floquet effective Hamiltonian Indeed we have a Hamiltonian!!

Floquet effective Hamiltonian

ε_n, $(\cdots, |\phi_n^{(-1)}\rangle, |\phi_n^{(0)}\rangle, |\phi_n^{(1)}\rangle, \cdots)$ are obtained by diagonalizing

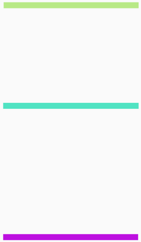
$$
\begin{array}{c}
 \\
m=-2 \\
m=-1 \\
m=0 \\
m=1 \\
 \\
\end{array}
\begin{array}{c}
\begin{array}{cccc} \quad m'=-2 & m'=-1 & m'=0 & m'=1 \end{array} \\
\begin{pmatrix}
 & & \vdots & \vdots & & \\
 & H^{(0)}+2\hbar\omega & H^{(-1)} & H^{(-2)} & H^{(-3)} & \\
\dots & H^{(1)} & H^{(0)}+\hbar\omega & H^{(-1)} & H^{(-2)} & \dots \\
\dots & H^{(2)} & H^{(1)} & H^{(0)} & H^{(-1)} & \dots \\
 & H^{(3)} & H^{(2)} & H^{(1)} & H^{(0)}-\hbar\omega & \\
 & & \vdots & \vdots & &
\end{pmatrix}
\end{array}
$$

- Each "element" is a Hamiltonian on $\mathcal{H}$
- The hole H^{Floquet} is on the extended Hilbert space

Floquet Brillouin zone

Redundancy in $H^{\textbf{Floquet}}$ The number of independent quasienergies is not really multiplied by Floquet subspaces.

$$\mathrm{e}^{-\mathrm{i}(\varepsilon_n - m\hbar\omega)t}\underbrace{\mathrm{e}^{\mathrm{i}\,m\hbar\omega t}\,|\Phi_n(t)\rangle}_{\text{still periodic!}}$$



The number of independent quasi-stationary states $= \dim\mathcal{H}$

So only *one* energy Brillouin zone is needed.

But all $\phi_n^{(m)}$ are all needed to decide $|\Phi_n(t)\rangle$.

Floquet Brillouin zone

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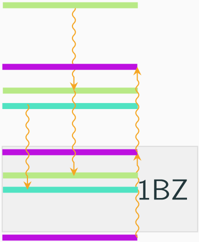

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Floquet formalism in hierarchy of approximations

Other ways to describe periodic driving

- Time-dependent perturbation theory (TD-PT)
- Rotating wave approximation (RWA)

$$
\begin{tikzcd}
\text{Keldysh, etc.} \arrow[r, "T=0"] & \text{TD-PT} \arrow[r, "\text{periodic, no cutoff}"] \arrow[dl] \arrow[d] \arrow[rr, bend right] & \text{Floquet} \arrow[r, "\text{small } \Omega, \Delta"] & \text{RWA} \\
{} & \text{various cutoff schemes} & &
\end{tikzcd}
$$

Floquet formalism in eyes of other formalisms

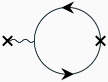
"full" Floquet theory, perturbation theory, and RWA

Floquet formalism v.s. TD-PT

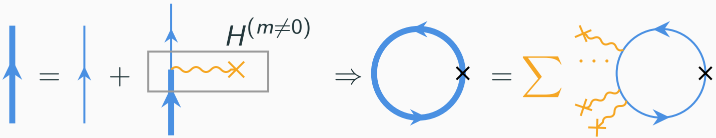
Response from time-dependent perturbation theory = response from $T = 0$ Feynman diagrams.

Example: first-order PT = Lindhard response function

$$\langle \mu^{(1)} \rangle = \mu_{\mathrm{eg}} \underbrace{\Omega}_{\text{Rabi freq.}} \left(\frac{\Omega}{\omega_{\mathrm{eg}} - \omega} \mathrm{e}^{-\mathrm{i}\omega t} + \frac{\Omega}{\omega_{\mathrm{eg}} + \omega} \mathrm{e}^{\mathrm{i}\omega t} + \mathrm{h.c.} \right) =$$

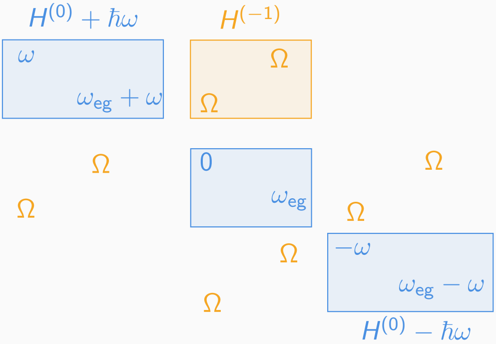


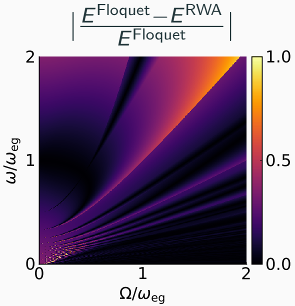
$H^{\textbf{Floquet}}$ = **the non-equilibrium self-energy**



In the full H^{Floquet}, automatically all PT terms are considered!

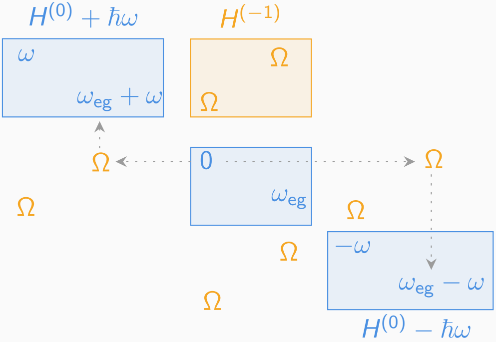
Floquet formalism v.s. RWA

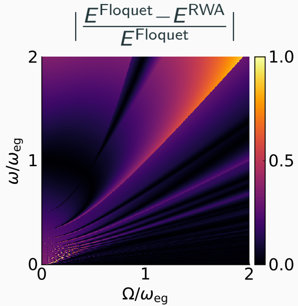




From Floquet to RWA

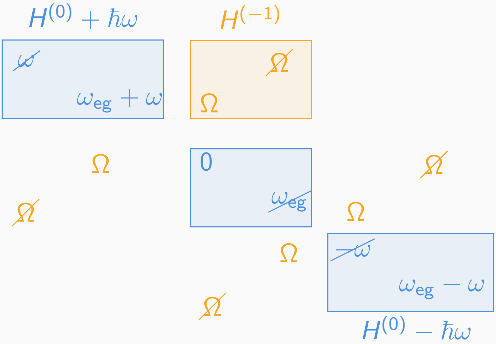
Floquet formalism v.s. RWA

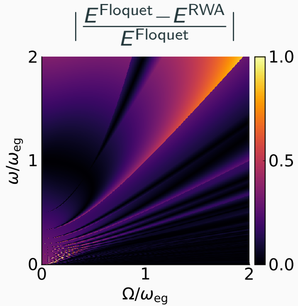




From Floquet to RWA

- Small coupling: only first-order transitions from $|g\rangle$ matters





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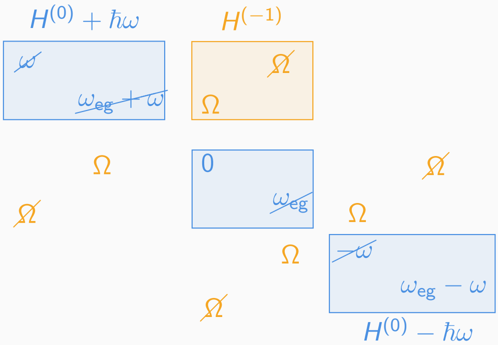
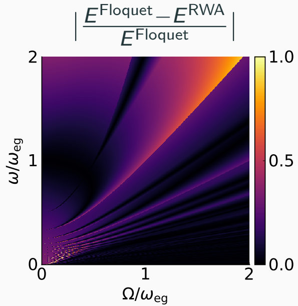


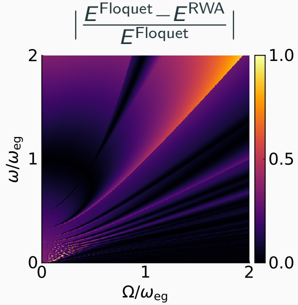


From Floquet to RWA

- Small coupling: only first-order transitions from $|g\rangle$ matters
- Near resonance: only the $m = 1$ $\omega_{eg} - \omega$ state matters

Floquet formalism v.s. RWA

$$H^{\mathrm{RWA}} = \begin{pmatrix} 0 & \Omega \\ \Omega & \omega_{\mathrm{eg}} - \omega \end{pmatrix}.$$



From Floquet to RWA

- Small coupling: only first-order transitions from $|\mathrm{g}\rangle$ matters
- Near resonance: only the $m = 1$ $\omega_{\mathrm{eg}} - \omega$ state matters

Indeed when $\Omega/\omega_{\mathrm{eg}} \lesssim 0.5$, $\omega/\omega_{\mathrm{eg}} \sim 1$, RWA works the best!

Angular-resolved photonemission spetroscopy

Are Floquet states “real”?

ARPES is...

It can be shown that

$$I(\mathbf{k}, \omega) \sim \sum_{n,m} |\phi_n^{(m)}|^2 \delta(\omega - \varepsilon_{n\mathbf{k}}). \tag{4}$$

[1]

[1] Figure from Philipp Rosenzweig et al. "Surface charge-transfer doping a quantum-confined silver monolayer beneath epitaxial graphene". In: *Physical Review B* 105.23 (2022), p. 235428.