# Solid State Physics Homework 1

Jinyuan Wu

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**Solution**

(a) Consider Figure 1. Suppose there are $N$ points in an area $A$ on a certain lattice plane. The volume between $A$ and its counterpart on an adjacent lattice plane is

$$V_{\text{total}} = Ad$$

Now since there are $N$ points in $A$, there are $N$ primitive unit cells between $A$ and its counterpart, so

$$V_{\text{total}} = NV.$$

Thus

$$NV = Ad, \quad \text{areal density} \coloneqq \frac{N}{A} = \frac{d}{V}. \tag{1}$$

(b) To maximize the areal density of lattice points is to maximize $d$.

**Solution** Below I use the side length of the so-called cubes as the length unit.

(a) Based-centered cubic is a Bravais lattice but the name is not on the list of 14. It is not really cubic, because it doesn't have the rotational symmetry around the $x$ and $y$ axes. It's actually a simple tetragonal lattice with $a = 1/\sqrt{2}, c = 1$, and the primitive vectors are

$$\boldsymbol{a}_1 = (1/2, -1/2, 0), \quad \boldsymbol{a}_2 = (1/2, 1/2, 0), \quad \boldsymbol{a}_3 = (0, 0, 1). \tag{2}$$

(b) Similar to the first case, side-centered cubic is also a Bravais lattice but not cubic, because it also doesn't have the rotational symmetry around the $x$ and $y$ axes and therefore is not really "cubic". It's actually a body-centered tetragonal lattice with $a = 1/\sqrt{2}, c = 1$. The primitive vectors are the same with (2).

(c) The edge-centered cubic lattice has the symmetry of a cube and therefore has to be a cubic lattice. By counting the number of lattice points, we find Figure 2 shows one unit cell. No translation symmetry is able to turn one of the points into another, so the unit cell is actually a primitive one. Thus the edge-centered cubic lattice is actually a simple cubic lattice: in each primitive unit cell, there are four points. So the edge-centered cubic lattice is not a Bravais lattice. It's the simple cubic lattice plus the basis shown as black (or red, or blue) in Figure 2.

**Solution** In each conventional cell of the bcc lattice, there are 2 lattice points, and so is the case for hcp. The volume of a conventional cell of the bcc lattice is $a^3$. In the hcp lattice, $c = 2\sqrt{6}a/3$, so the volume is

$$\frac{\sqrt{3}}{2}a^2 \times c = \sqrt{2}a^3.$$

So to keep the density invariant,

$$\sqrt{2}a_{\text{hcp}}^3 = a_{\text{bcc}}^3,$$

and

$$a_{\text{hcp}} = 2^{-1/6}a_{\text{bcc}} = 3.77\,\text{Å}. \tag{3}$$

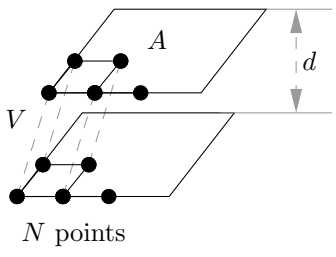


Figure 1: The volume between two areas on adjacent lattice planes

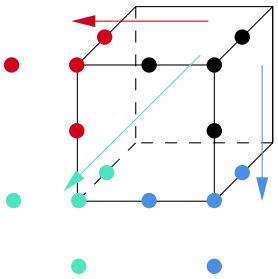

Figure 2: One unit cell of a edge centered cubic lattice

**Solution**

(a) There is only one Ti atom in the cube given, and obviously it's impossible to use a translation symmetry operation to turn one atom into another in the cube. The lattice is a simple cubic lattice, and the primitive lattice vectors are

$$\boldsymbol{a}_1 = (1, 0, 0), \quad \boldsymbol{a}_2 = (0, 1, 0), \quad \boldsymbol{a}_3 = (0, 0, 1). \tag{4}$$

(b) There are five: $1/4 \times 4 = 1$ Sr atom, 1 Ti atom, and $1/2 \times 6 = 3$ O atoms. The coordinates of the Ti atom are $(1/2, 1/2, 1/2)$. The coordinates of the Sr atom are $(0, 0, 0)$. The coordinates of the O atoms are $(1/2, 1/2, 0), (0, 1/2, 1/2), (1/2, 0, 1/2)$.

(c) The nearest neighbors of a Sr atom are 12 Ti atoms. The nearest neighbors of a Ti atom are 6 O atoms. The nearest neighbors of an O atom are 2 Ti atoms.