

Time-dependent adiabatic GW

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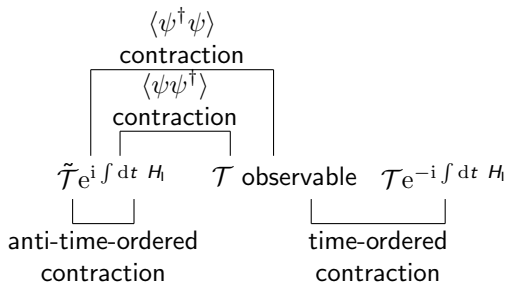
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Motivation

$$\langle A \rangle = \langle S^{-1} \mathcal{T}_t(S A_I(t)) \rangle, \quad S = U(\infty, -\infty) \quad (1)$$

Non-equilibrium state: not pure; contains excited state components;
 $|\Psi_n\rangle$ is excited state $\Rightarrow S |\Psi_n\rangle \neq e^{i\alpha} |\Psi_n\rangle \Rightarrow$ we can't peel the S^{-1} off!!

Solution Four (instead of one) types of propagators: (note S^{-1} is *anti*-time ordered)

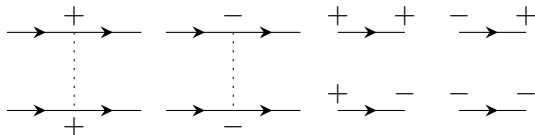


Keldysh formalism

Four types of (fermionic) propagators

$$\begin{aligned} iG^{--} = iG^c &= \langle \mathcal{T} \psi_1 \psi_2^\dagger \rangle, & iG^{++} = iG^a &= \langle \tilde{\mathcal{T}} \psi_1 \psi_2^\dagger \rangle, \\ iG^{+-} = iG^> &= \langle \psi_1 \psi_2^\dagger \rangle, & iG^{-+} = iG^< &= -\langle \psi_2^\dagger \psi_1 \rangle. \end{aligned} \quad (2)$$

Diagrams

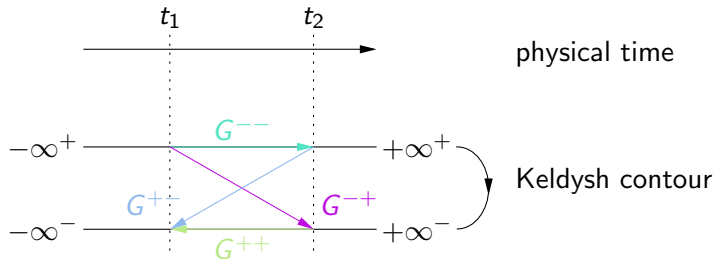


Self-energy

$$G = \begin{pmatrix} G^{--} & G^{-+} \\ G^{+-} & G^{++} \end{pmatrix}, \quad \Sigma = \begin{pmatrix} \Sigma^{--} & \Sigma^{-+} \\ \Sigma^{+-} & \Sigma^{++} \end{pmatrix}, \quad G = G_0 + G_0 \Sigma G. \quad (3)$$

Alternative formulation: Keldysh contour

Keldysh contour The information in the G matrix can be alternatively stored in a time-ordered Green function on *Keldysh contour*



From Keldysh contour to physical contour Lengreth theorem:

$$\begin{aligned}(AB)^{<} &= A^R B^{<} + A^{<} B^A, & (AB)^{>} &= A^R B^{>} + A^{>} B^A, \\ (AB)^R &= A^R B^R, & (AB)^A &= A^A B^A,\end{aligned}\tag{4}$$

where

$$\begin{aligned}A^{>}(t_1, t_2) &= A(t_1^+, t_2^-), & A^{<}(t_1, t_2) &= A(t_1^-, t_2^+), \\ A^R(t_1, t_2) &= \theta(t_1 - t_2)(A^{>} - A^{<}).\end{aligned}\tag{5}$$

Mapping an equation on Keldysh contour to its counterpart on the physical time axis!

Derivation of EOM of $G^{<,>}$ and G^A

Recommended references The following series

Václav Špička, Bedřich Velický, and Anděla Kalvová. “Long and short time quantum dynamics: I. Between Green's functions and transport equations”. In: *Physica E: Low-dimensional Systems and Nanostructures* 29.1-2 (2005), pp. 154–174

Reconstruction of $G^<$

Gradient expansion: from QME to QBE

Example: TODO