Time-dependent adiabatic *GW*

Jinyuan Wu

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Keldysh formalism

Motivation

$$\langle A \rangle = \langle S^{-1} \mathcal{T}_t (S A_\mathrm{I}(t)) \rangle \,, \quad S = U(\infty, -\infty) \tag{1}$$

Non-equilibrium state: not pure; contains excited state components; $|\Psi_n\rangle$ is excited state $\Rightarrow S\,|\Psi_n\rangle \neq \mathrm{e}^{\mathrm{i}\,\alpha}\,|\Psi_n\rangle \Rightarrow$ we can't peel the S^{-1} off!!

Solution Four (instead of one) types of propagators: (note S^{-1} is *anti*-time ordered)

$\langle \psi^\dagger \psi \rangle$
contraction

$\langle \psi \psi^\dagger \rangle$
contraction

$\tilde{\mathcal{T}} \mathrm{e}^{\mathrm{i} \int \mathrm{d}t \; H_\mathrm{I}}$ $\quad \mathcal{T}$ observable $\quad \mathcal{T} \mathrm{e}^{-\mathrm{i} \int \mathrm{d}t \; H_\mathrm{I}}$

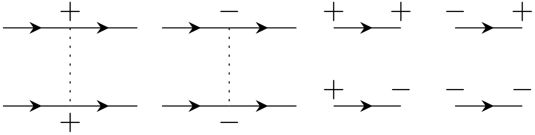
anti-time-ordered contraction $\qquad$ time-ordered contraction

Keldysh formalism

Four types of (fermionic) propagators

$$
\begin{aligned}
\mathrm{i}\, G^{--} = \mathrm{i}\, G^{\mathrm{c}} = \langle \mathcal{T}\, \psi_1 \psi_2^\dagger \rangle\,, &\quad \mathrm{i}\, G^{++} = \mathrm{i}\, G^{\mathrm{a}} = \langle \tilde{\mathcal{T}} \psi_1 \psi_2^\dagger \rangle\,, \\
\mathrm{i}\, G^{+-} = \mathrm{i}\, G^{>} = \langle \psi_1 \psi_2^\dagger \rangle\,, &\quad \mathrm{i}\, G^{-+} = \mathrm{i}\, G^{<} = - \langle \psi_2^\dagger \psi_1 \rangle\,.
\end{aligned} \tag{2}
$$

Diagrams

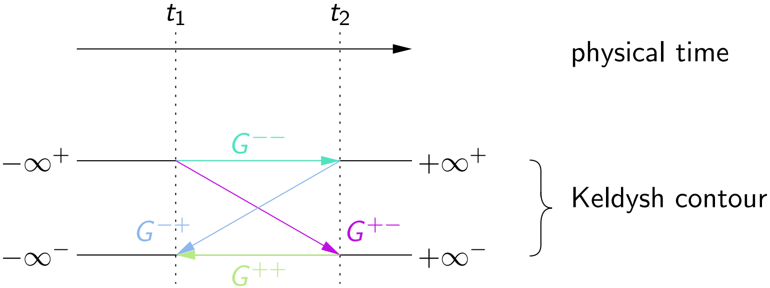


Self-energy

$$
G = \begin{pmatrix} G^{--} & G^{-+} \\ G^{+-} & G^{++} \end{pmatrix}, \quad \Sigma = \begin{pmatrix} \Sigma^{--} & \Sigma^{-+} \\ \Sigma^{+-} & \Sigma^{++} \end{pmatrix}, \quad G = G_0 + G_0 \Sigma G. \tag{3}
$$

Alternative formulation: Keldysh contour

Keldysh contour The information in the G matrix can be alternatively stored in a time-ordered Green function on *Keldysh contour*



Green function EOM

From Keldysh contour to physical contour Lengreth theorem:

$$
\begin{aligned}
&(AB)^< = A^\mathrm{R} B^< + A^< B^\mathrm{A}, \quad (AB)^> = A^\mathrm{R} B^> + A^> B^\mathrm{A}, \\
&(AB)^\mathrm{R} = A^\mathrm{R} B^\mathrm{R}, \quad (AB)^\mathrm{A} = A^\mathrm{A} B^\mathrm{A},
\end{aligned} \tag{4}
$$

$$
\begin{aligned}
&A^>(t_1, t_2) = A(t_1^+, t_2^-), \quad A^<(t_1, t_2) = A(t_1^-, t_2^+), \\
&A^\mathrm{R}(t_1, t_2) = \theta(t_1 - t_2)(A^> - A^<),
\end{aligned} \tag{5}
$$

Václav Špička, Bedřich Velickỳ, and Anděla Kalvová. "Long and short time quantum dynamics: I. Between Green's functions and transport equations". In: *Physica E: Low-dimensional Systems and Nanostructures* 29.1-2 (2005), pp. 154–174