DMRG overview

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1 Introduction

Suppose we are to find the single-particle Green function of a condensed matter system. The standard procedure is to find the irreducible self-energy Σ by Feynman diagram resummation (although there are subtleties concerning the well-definedness and uniqueness of Feynman diagram resummation techniques [1]). In strongly correlated systems, deciding diagrams with most contributions is generally hard, and brutal-force resummation proves intractable.

If, however, it is found that the self-energy (or two-particle vertex, or similar "n-particle self-energy diagrams") is highly local, then in principle, it can be replicated in a *few-body* model: suppose, for example, that $\Sigma_{\boldsymbol{ij}}$ is only important when $|\boldsymbol{i}-\boldsymbol{j}| \leq n$, then we can just choose a patch of sites satisfying the $|\boldsymbol{i}-\boldsymbol{j}| \leq n$ condition, and integrate out the rest of electron modes, and in the resulting few-body model, $\Sigma_{\boldsymbol{ij}}$ is exactly the same as in the original model. The main obstacle now is to decide the parameters in this new few-body model, which can be solved by adopting a self-consistent scheme: $\Sigma_{\boldsymbol{ij}}$, together with the free part of the original model, decides the one-particle Green function, which then can be used to fit the parameters in the few-body model, and then the few-body model can be solved to update the self-energy.

It can be seen that diagrammatically speaking, this idea is also a resummation strategy, though here we pick up diagrams according to their locality, and *all* diagrams, as long as they are local enough and fit in the ansatz of the form of the few-body model, are included when we solve the few-body model. This can also be seen as a mean-field approach, just like many other self-consistent Feynman diagram resummation strategies. Since the parameters in the few-body model may contain time explicitly, we may say what we are doing is a *dynamic* mean-field theory.

This report reviews

2 DMFT for the Hubbard model

We map the

The single-site model can be instantiated by replacing $\mathcal{G}$ by a bath of non-interactive electrons, and we get an Anderson impurity model. TODO: whether this is actually done in real calculation

For Hubbard model, the interaction term in the few-body problem is exactly $Un_\uparrow n_\downarrow$, because it is strictly local and is not corrected when electrons on other sites are integrated out. For models with nearest-neighbor interaction, this is no longer exact, because now integrating out other electron modes means screening of the interaction, and both

This approximation is exact in an infinite lattice [2].

3 Going beyond DMFT: the DΓA

Unfortunately, in some scenarios the original DMFT is bound to fail. This is exemplified in phrase transition, in which long-wavelength behaviors are highly important, and a strictly local Σ cannot be the case. A possible direction of improvement is to assume that Σ is not local, but the 2-particle vertex Γ is local. The spatial-varying self-energy can then be calculated from Γ by TODO

References

[1] O. Gunnarsson et al. "Breakdown of Traditional Many-Body Theories for Correlated Electrons". In: *Phys. Rev. Lett.* 119 (5 Aug. 2017), p. 056402. DOI: 10.1103/PhysRevLett.119.056402. URL: https://link.aps.org/doi/10.1103/PhysRevLett.119.056402.

[2] Antoine Georges and Gabriel Kotliar. "Hubbard model in infinite dimensions". In: *Phys. Rev. B* 45 (12 Mar. 1992), pp. 6479–6483. DOI: 10.1103/PhysRevB.45.6479. URL: https://link.aps.org/doi/10.1103/PhysRevB.45.6479.