Homework 4

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1

The original matrix is

$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \\ 1 & 3 \\ 0 & 1 \end{pmatrix}. \tag{1}$$

Following these steps:

1. Move the third line to the top, and
2. Subtract the second line from the fourth line,

we get the row reduced form

$$\mathbf{A}_{\mathrm{R}} = \begin{pmatrix} 1 & 3 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \tag{2}$$

and applying the same procedure to $\mathbf{I}_{4\times 4}$ we get

$$\mathbf{\Omega}_{\mathrm{R}} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 \end{pmatrix}, \tag{3}$$

such that $\mathbf{\Omega}_{\mathrm{R}}\mathbf{A} = \mathbf{A}_{\mathrm{R}}$.

2

The equations are

$$\begin{aligned} 6x_1 - x_2 + x_3 = 0 \\ x_1 - x_4 + 2x_5 = 0, \\ x_1 - 2x_5 = 0 \end{aligned} \tag{4}$$

which is equivalent to

$$\begin{pmatrix} 6 & -1 & 1 & 0 & 0 \\ 1 & 0 & 0 & -1 & 2 \\ 1 & 0 & 0 & 0 & -2 \end{pmatrix} \mathbf{x} = \mathbf{0}. \tag{5}$$

The row reduced form of the matrix in the LHS is

$$\begin{pmatrix} 1 & 0 & 0 & 0 & -2 \\ 0 & 1 & -1 & 0 & -12 \\ 0 & 0 & 0 & 1 & -4 \end{pmatrix}$$

If we switch the third and the fourth coulomb, we immediate find two independent solutions

$$\begin{pmatrix} 0 \\ -1 \\ 0 \\ -1 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} -2 \\ -12 \\ -4 \\ 0 \\ -1 \end{pmatrix},$$

and if we switch the coulombs back we find a basis of the solution space is

$$\begin{pmatrix} 0 \\ -1 \\ -1 \\ 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} -2 \\ -12 \\ 0 \\ -4 \\ -1 \end{pmatrix}, \tag{6}$$

and the solution space is 2-dimensional, and the general solution looks like

$$x_1 = 2t_2, \quad x_2 = t_1 + 12t_2, \quad x_3 = t_1, \quad x_4 = 4t_2, \quad x_5 = t_2. \tag{7}$$

3

The equation system

$$\begin{aligned} 2x_1 - 3x_2 + x_4 = 1 \\ 3x_1 + x_3 - x_4 = 0 \\ 2x_1 - 3x_2 + 10x_3 = 0 \end{aligned} \tag{8}$$

is equivalent to

$$\begin{pmatrix} 2 & -3 & 0 & 1 \\ 3 & 0 & 1 & -1 \\ 2 & -3 & 10 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}. \tag{9}$$

The reduced matrix of the LHS is

$$\begin{pmatrix} 1 & 0 & 0 & -\frac{3}{10} \\ 0 & 1 & 0 & -\frac{8}{15} \\ 0 & 0 & 1 & -\frac{1}{10} \end{pmatrix},$$

and the general solution of the homogeneous version of the equation is therefore

$$t \begin{pmatrix} \frac{3}{10} \\ \frac{8}{15} \\ \frac{1}{10} \\ 1 \end{pmatrix}.$$

A specific solution of the equation system can be easily found by setting $x_4 = 0$:

$$\begin{pmatrix} \frac{1}{30} \\ -\frac{14}{45} \\ -\frac{1}{10} \end{pmatrix}.$$

So the general solution is

$$x_1 = \frac{3}{10}t + \frac{1}{30}, \quad x_2 = \frac{8}{15}t - \frac{14}{45}, \quad x_3 = \frac{1}{10}t - \frac{1}{10}, \quad x_4 = t. \tag{10}$$

4

Since

$$\mathbf{A} = \begin{pmatrix} -1 & 0 \\ 4 & 4 \end{pmatrix}, \tag{11}$$

we have $\det \mathbf{A} = -4$, and therefore it's not singular. The inverse is given by

$$\mathbf{A}^{-1} = \frac{1}{\det \mathbf{A}} \begin{pmatrix} 4 & -4 \\ 0 & -1 \end{pmatrix}^\top = \begin{pmatrix} -1 & 0 \\ 1 & 1/4 \end{pmatrix}. \tag{12}$$

5

The equation system

$$\begin{aligned} 8x_1 - 4x_2 + 3x_3 &= 0 \\ x_1 + 5x_2 - x_3 &= -5 \\ -2x_1 + 6x_2 + x_3 &= -4 \end{aligned} \tag{13}$$

is equivalent to

$$\underbrace{\begin{pmatrix} 8 & -4 & 3 \\ 1 & 5 & -1 \\ -2 & 6 & 1 \end{pmatrix}}_{\mathbf{A}} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -5 \\ -4 \end{pmatrix}. \tag{14}$$

We have

$$\det \mathbf{A} = 132, \tag{15}$$

and by Cramer's rule we have

$$x_1 = \frac{1}{132} \cdot -66 = -\frac{1}{2}, \quad x_2 = \frac{1}{132} \cdot -114 = -\frac{19}{22}, \quad x_3 = \frac{1}{132} \cdot 24 = \frac{2}{11}. \tag{16}$$

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