Xucheng Wang on Superconducting Phase Fluctuations and the Pseudogap

Jinyuan Wu

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High T_c superconducting has been discovered for several decades, but still with fundamental puzzles like

- Symmetry of order parameter: s-wave or p-wave?
- Mechanism of pairing
- The role of spin fluctuation.

The main topic today is **pseudogap**. When there is no doping, the system is an AF insulator, and with some hole doping we have the superconducting phase, and with higher hole concentration, we get a metal state. When the hole concentration is in the superconducting range but the temperature is risen, we will enter the **pseudogap phase**, where there is no superconductivity but we still have remaining pairing, with a "pseudo" superconducting gap. The region may come from AFM or CDW order competing with SC, and it is also possible that in this region electrons already form pairs, but the pairs do not *condense*. It is also theoretically predicted that pseudogap-like features can arise in BCS theory because of pairing of vortex of the phase θ of the order parameter. Recently, a pseudogap state was reported in strongly disordered conventional superconductors.

Several models, like the Hubbard model, have been confirmed to possess pseudogap-like behaviors. In this presentation we consider a BCS model:

$$H = H_0 + V, V = \int \mathrm{d}^2\boldsymbol{r}\, \Delta(\boldsymbol{r})\psi_\uparrow^\dagger \psi_\downarrow^\dagger + \text{h.c.}, \tag{1}$$

and H_0 is the Hamiltonian of free electrons. The mean field order parameter Δ is disordered, where we assume

$$\langle \Delta(\boldsymbol{r}) \rangle_{\text{dis}} = \langle \Delta^*(\boldsymbol{r}) \rangle_{\text{dis}} = 0, \quad \langle \Delta(\boldsymbol{r}) \Delta^*(\boldsymbol{r}') \rangle_{\text{dis}} = g(\boldsymbol{r} - \boldsymbol{r}'). \tag{2}$$

There are two types of disorder averaging. In **quenched disorder**, the time scale of quenched disorder is much longer than the rest of the system, i.e. the disorder configuration is frozen when the rest of the system evolves, and the free energy is

$$F_{\text{q}} = -\frac{1}{\beta} \langle \ln Z(\beta) \rangle_{\text{dis}}, \tag{3}$$

which means we first calculate physical quantities and *then* calculate the disorder average. On the other hand, we have **annealed disorder**, where the disorder configuration evolves *together* with the rest of the system, and we have

$$F_{\text{a}} = -\frac{1}{\beta} \ln \langle Z(\beta) \rangle_{\text{dis}}. \tag{4}$$

In this setting, we first calculate the partition function after disorder averaging, and then give a definite physical quantity prediction.

We can then repeat the procedure as in Anderson localization and represent the disorder as a self-energy correction. Then the problem is feasible using existing numerical simulation methods. We can also make some approximation when the disorder is not strong. We can evaluate the imaginary part of the self energy explicitly and find its asymptotic behavior when the decoherence length ξ introduced by disorder is small. After replacing the imaginary part of Σ with a function easier to deal with, we can do inverse analytical continuation and get an approximate Σ.

We then will find expected split peak in the spectrum function. We find after introducing ξ, the peaks are broadened with

$$\Delta\omega = \pi\Delta_0 \frac{\xi}{\xi_{\mathrm{BCS}}}, \tag{5}$$

where ξ_{BCS} is the coherence length of the BCS system.

We can then verify whether the above picture works in a 2D Hubbard model. In this model, at half-filling we have SC and CDW together, and the latter suppresses T_{c} to zero, and when doped away from half-filling, CDW is suppressed and SC comes out. Indeed we find pseudogap in the spectrum function.

Note: here we are working in 2D and usually the phase transition is a KT phase transition, and as the temperature goes down, first we get electron pairing and then a KT phase transition, and the region between the two is the pseudogap phase. (??? why after KT phase transition we get SC?)

It should also be noted that the broadening comes from both the disordered Δ and something else, because experimentally, when the temperature is very low, we still have broadened peaks in the spectrum function, which is not the case in (2).