In[29]:= **spectrum** = $\left| \text{Eigenvalues}\left[\dfrac{1}{-\dot{\imath}} \begin{pmatrix} 0 & J_1 + J_2 & 0 & -J_1 - J_2\, e^{-i\,k\,a} \\ -(J_1 + J_2) & 0 & J_1 + J_2\, e^{-i\,k\,a} & 0 \\ 0 & -J_1 - J_2\, e^{i\,k\,a} & 0 & J_1 + J_2 \\ J_1 + J_2\, e^{i\,k\,a} & 0 & -(J_1 + J_2) & 0 \end{pmatrix} \right] \; // \right.$
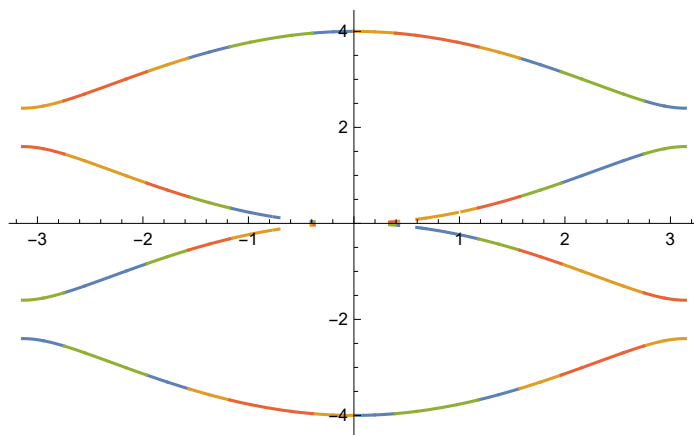
$\left. \text{Simplify} \right| \; //. \; \{J_1 \rightarrow 1.2, \; J_2 \rightarrow 0.8, \; a \rightarrow 1\};$

Plot[spectrum, {k, $-\pi$, π}]

Out[30]=

In[31]:= **spectrum** = $\left| \text{Eigenvalues}\left[\dfrac{1}{-\dot{\imath}} \begin{pmatrix} 0 & J_1 + J_2 & 0 & -J_1 - J_2\, e^{-i\,k\,a} \\ -(J_1 + J_2) & 0 & J_1 + J_2\, e^{-i\,k\,a} & 0 \\ 0 & -J_1 - J_2\, e^{i\,k\,a} & 0 & J_1 + J_2 \\ J_1 + J_2\, e^{i\,k\,a} & 0 & -(J_1 + J_2) & 0 \end{pmatrix} \right] \; // \right.$

$\left. \text{Simplify} \right| \; //. \; \{J_1 \rightarrow 1, \; J_2 \rightarrow 1, \; a \rightarrow 1\};$

Plot[spectrum, {k, $-\pi$, π}, AspectRatio $\rightarrow$ 1 / 2]

Out[32]=
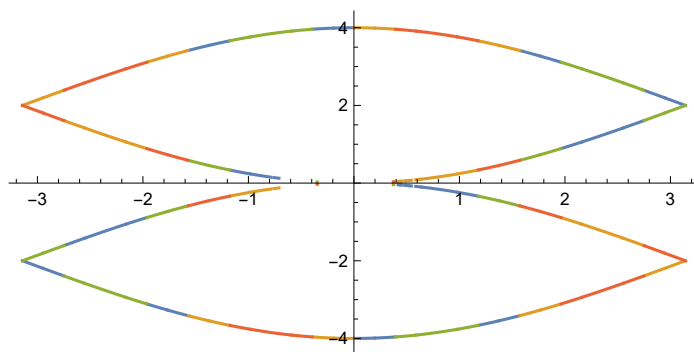
Note: in the following code, the "1/2" factor coming with k is due to the fact that when $J_1 = J_2$, the lattice constant shrinks to one half of the original one; $k + 2\pi$ comes from the corresponding Brillouin zone folding.

In[33]:=
```
Plot[{2 (1 - Cos[k / 2]), 2 (1 - Cos[(k + 2 π) / 2])}, {k, -π, π}, AspectRatio → 1 / 4]
```

Out[33]=