Homework 1

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1 Problem 1: The Beam Splitter

Since $|t|^2 = |r|^2 = 1/2$, we have

$$\begin{pmatrix} E_c \\ E_d \end{pmatrix} = \underbrace{\begin{pmatrix} \mathrm{e}^{\mathrm{i}\phi_{ta}} & \mathrm{e}^{\mathrm{i}\phi_{rb}} \\ \mathrm{e}^{\mathrm{i}\phi_{ra}} & \mathrm{e}^{\mathrm{i}\phi_{tb}} \end{pmatrix}}_{M} \begin{pmatrix} E_a \\ E_b \end{pmatrix}. \tag{1}$$

The unitary condition means

$$MM^\dagger = I, \tag{2}$$

which in turns means

$$\begin{aligned} I &= \frac{1}{2} \begin{pmatrix} \mathrm{e}^{\mathrm{i}\phi_{ta}} & \mathrm{e}^{\mathrm{i}\phi_{rb}} \\ \mathrm{e}^{\mathrm{i}\phi_{ra}} & \mathrm{e}^{\mathrm{i}\phi_{tb}} \end{pmatrix} \begin{pmatrix} \mathrm{e}^{-\mathrm{i}\phi_{ta}} & \mathrm{e}^{-\mathrm{i}\phi_{ra}} \\ \mathrm{e}^{-\mathrm{i}\phi_{rb}} & \mathrm{e}^{-\mathrm{i}\phi_{tb}} \end{pmatrix} \\ &= \frac{1}{2} \begin{pmatrix} 2 & \mathrm{e}^{\mathrm{i}(\phi_{ta}-\phi_{ra})} + \mathrm{e}^{\mathrm{i}(\phi_{rb}-\phi_{tb})} \\ \mathrm{e}^{-\mathrm{i}(\phi_{ta}-\phi_{ra})} + \mathrm{e}^{-\mathrm{i}(\phi_{rb}-\phi_{tb})} & 2 \end{pmatrix}, \end{aligned}$$

and this is equivalent to

$$\mathrm{e}^{\mathrm{i}(\phi_{ta}-\phi_{ra})} + \mathrm{e}^{\mathrm{i}(\phi_{rb}-\phi_{tb})} = 0,$$

or in other words

$$\phi_{ta} - \phi_{ra} = \phi_{rb} - \phi_{tb} + \pi n, \quad n \text{ odd}. \tag{3}$$

2 Problem 2: Interferometers

Solution

3 Correlation function and Other Properties of the Blackbody Field

3.1 Energy at ω; Total Energy

From

$$\boldsymbol{\nabla} \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t},$$

we have

$$\mathrm{i}\boldsymbol{k} \times \boldsymbol{E}_\omega = \mathrm{i}\omega \boldsymbol{B}_\omega,$$

and therefore

$$|\boldsymbol{B}_\omega| = \frac{k}{\omega}|\boldsymbol{E}_\omega| = \frac{1}{c}|\boldsymbol{E}|,$$

so

$$\begin{aligned} u_\omega &= \frac{\epsilon_0}{2}|\boldsymbol{E}|_\omega^2 + \frac{1}{2\mu_0}|\boldsymbol{B}_\omega|^2 \\ &= \frac{\epsilon_0}{2}|\boldsymbol{E}|_\omega^2 + \frac{1}{2\mu_0}\underbrace{\frac{1}{c^2}}_{\mu_0\epsilon_0}|\boldsymbol{E}|_\omega^2 \\ &= \epsilon_0|\boldsymbol{E}|_\omega^2. \end{aligned} \tag{4}$$