

Homework 2

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1

Problem

$$y' - 9y = t, \quad y(0) = 5. \quad (1)$$

Solution After Laplace transformation we get

$$sY(s) - 5 - 9Y(s) = \frac{1}{s^2},$$

$$Y(s) = \frac{5}{s-9} + \frac{1}{s^2(s-9)}.$$

The second term can be decomposed (by multiplying x , x^2 or $(x-9)$ and taking the limit $x \rightarrow 0$ and $x \rightarrow 9$) as

$$\frac{1}{s^2(s-9)} = \frac{1}{81} \frac{1}{s-9} - \frac{1}{81} \frac{1}{s} + \frac{1}{9} \frac{1}{s^2},$$

and

$$Y(s) = \frac{406}{81} \frac{1}{s-9} - \frac{1}{81} \frac{1}{s} - \frac{1}{9} \frac{1}{s^2}.$$

The inverse Laplace transform is

$$y(t) = \frac{406}{81} e^{9t} - \frac{1}{81} + \frac{1}{9} t. \quad (2)$$

2

Problem

$$y'' - 4y' + 4y = f(t); y(0) = -2, y'(0) = 1 \text{ with}$$

$$f(t) = \begin{cases} t & \text{for } 0 \leq t < 3 \\ t+2 & \text{for } t \geq 3 \end{cases}$$

Solution The Laplace transform of the LHS is

$$s^2 Y(s) - sy(0) - y'(0) - 4(sY(s) - y(0)) + 4Y(s) = (s-2)^2 Y(s) + 2s - 9.$$

The RHS is

$$f(t) = t(H(t) - H(t-3)) + (t+2)H(t-3) = tH(t) + 2H(t-3) \xrightarrow{\mathcal{L}} \frac{1}{s^2} + 2 \cdot e^{-3s} \cdot \frac{1}{s}.$$

So the equation is equivalent to

$$(s-2)^2 Y(s) + 2s - 9 = \frac{1}{s^2} + \frac{2e^{-3s}}{s},$$
$$Y(s) = -\frac{2}{s-2} + \frac{5}{(s-2)^2} + \frac{1}{s^2(s-2)^2} + \frac{2}{s(s-2)^2} e^{-3s}. \quad (3)$$