Bosonization in Electronic Systems

Jinyuan Wu

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We have already discussed bosonization in Section. 11.1 in this solid state physics note. Facts like that particle-hole pairs in 1D are much simpler than the case in higher dimensions and that the density-density correlation function have two clear poles, which means we have two bosonic density modes all hint the possibility of bosonization. The first fact is the argument used in this solid state physics note. Here we briefly discuss the second argument. It is well known that the retarded density-density Green function is

$$D^{\mathrm{R}}(\boldsymbol{q}, \omega) = \int \frac{\mathrm{d}^d \boldsymbol{p}}{(2\pi)^d} \frac{n_{\boldsymbol{p}} - n_{\boldsymbol{p}+\boldsymbol{q}}}{\omega - \xi_{\boldsymbol{p}+\boldsymbol{q}} + \xi_{\boldsymbol{p}} + \mathrm{i}0^+}, \tag{1}$$

Fradkin Eq. (6.7)

and when $\boldsymbol{q}$ is small, we have

$$\xi_{\boldsymbol{p}+\boldsymbol{q}} - \xi_{\boldsymbol{p}} \approx \boldsymbol{q} \cdot \boldsymbol{\nabla}_{\boldsymbol{p}} \xi_{\boldsymbol{p}} = \boldsymbol{q} \cdot \boldsymbol{v},$$

and

$$n_{\boldsymbol{p}} - n_{\boldsymbol{p}+\boldsymbol{q}} \approx -\boldsymbol{q} \cdot \boldsymbol{\nabla}_{\boldsymbol{p}} n_{\boldsymbol{p}} = -\boldsymbol{q} \cdot \boldsymbol{\nabla}_{\boldsymbol{p}} \theta(\mu - \epsilon_{\boldsymbol{p}}) \approx \boldsymbol{q} \cdot \boldsymbol{v}_{\mathrm{F}} \delta(-\xi_{\boldsymbol{p}}),$$

and therefore

$$\begin{aligned} D^{\mathrm{R}}(\boldsymbol{q}, \omega) &= \int \frac{\mathrm{d}^d \boldsymbol{p}}{(2\pi)^d} \delta(-\xi_{\boldsymbol{p}}) \frac{\boldsymbol{q} \cdot \boldsymbol{v}_{\mathrm{F}}}{\omega - \boldsymbol{q} \cdot \boldsymbol{v} + \mathrm{i}0^+} \\ &= \int_{\mathrm{FS}} \frac{\mathrm{d}^{d-1} S}{(2\pi)^d} \frac{1}{v_{\mathrm{F}}} \frac{\boldsymbol{q} \cdot \boldsymbol{v}_{\mathrm{F}}}{\omega - \boldsymbol{q} \cdot \boldsymbol{v}_{\mathrm{F}} + \mathrm{i}0^+} \\ &= \int_{\mathrm{FS}} \frac{\mathrm{d}^{d-1} S}{(2\pi)^d} \frac{\boldsymbol{q} \cdot \hat{\boldsymbol{v}}_{\mathrm{F}}}{\omega - \boldsymbol{q} \cdot \boldsymbol{v}_{\mathrm{F}} + \mathrm{i}0^+} \\ &\sim \frac{p_{\mathrm{F}}^{d-1}}{(2\pi)^d} \int_{\mathrm{FS}} \mathrm{d}^{d-1} \Omega \, \frac{\boldsymbol{q} \cdot \hat{\boldsymbol{v}}_{\mathrm{F}}}{\omega - \boldsymbol{q} \cdot \boldsymbol{v}_{\mathrm{F}} + \mathrm{i}0^+}, \end{aligned} \tag{2}$$

Fradkin Eq. (6.10), Wen below Eq. (4.3.1)

where FS means the Fermi surface. For a 1D electron gas we therefore get

$$D^{\mathrm{R}} \sim \frac{q}{2\pi} \left(\frac{1}{\omega - q v_{\mathrm{F}} + \mathrm{i}0^+} - \frac{1}{\omega + q v_{\mathrm{F}} + \mathrm{i}0^+} \right). \tag{3}$$

Fradkin Eq. (6.10), Wen Eq. (4.3.2)

So we get two poles, instead of branch cuts.

This article discusses some details in bosonization, summarizing important facts in famous textbooks and papers. Bosonization can be viewed as one way to exactly solve a system, which we will see in Luttinger liquid and bosonization of spin systems. A more "physical" motivation is to capture the bosonic modes in the system, i.e. density and current fluctuations, or since we usually only deal with low-energy perturbations in condensed matter systems, to capture *sound waves* [1,2]. This is why the fields after bosonization are all density modes: We really do not know what to do other than this. Sometimes, bosonization are known as the *hydrodynamical approach* (for example in Wen's famous textbook), which reveals its physical nature. This name is slightly misleading, as *hydrodynamical approaches* often denote more "kinetic" approaches, which is more about deriving EOMs of expectation values instead of quantum field operators. This physical picture can also be justified using the following line of thinking: one motivation to study the electronic structure of a condensed matter system is to find its electromagnetic response, and A_μ is always coupled to a bilinear of the fermionic fields. What can be revealed directly by electromagnetic responses, therefore, are only bosonic modes (for example see Section 6 here).

Philip Phillips Chapter 10

Wen Section 5.1.3, 5.3.3 and 5.3.4

1 Luttinger liquid from a Hubbard model

Philip Phillips Section 10.1

2 The Luttinger model

References

[1] Sin itiro Tomonaga. Remarks on bloch's method of sound waves applied to many-fermion problems. *Progress of Theoretical Physics*, 5:544–569, 1950.

[2] Norio Kawakami. Tomonaga's Theory for Collective Motion of Fermions: Basic Concept of One-Dimensional Correlated Electrons. *Progress of Theoretical Physics Supplement*, 170:185–197, 05 2007.