Bloch equation and rate equation in two-level systems

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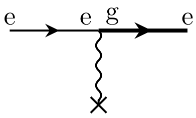
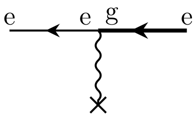
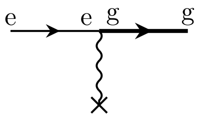
Consider Eq. (5.125) in Steck's lecture notes. The optical Bloch equations can be derived in a diagrammatic way. Basically, we have the following diagrams: for the ρ_{ee} we have (possibly the directions of the lines are wrong; check later)

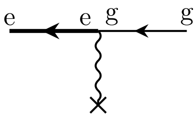


$$\simeq \frac{1}{\partial_t}\frac{\Omega}{2}\rho_{\text{eg}},$$


$$\simeq -\frac{1}{\partial_t}\frac{\Omega}{2}\rho_{\text{ge}}, \tag{1}$$

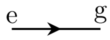
and for the ρ_{eg} part we have



$$\simeq \frac{1}{\partial_t}\frac{\Omega}{2}\rho_{\text{gg}},$$


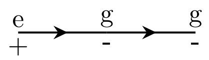
$$\simeq \frac{1}{\partial_t}\frac{\Omega}{2}\rho_{\text{ee}}, \tag{2}$$

and also



$$\simeq \frac{1}{\partial_t}\Delta\rho_{\text{eg}}. \tag{3}$$

Here we need to note that we are dealing with lesser Green functions, and therefore the free $G^<_{\text{gg}}$ or $G^<_{\text{ee}}$ is just $1/\mathrm{i}\partial_t$; on the other hand, we have (3). This can be illustrated by the following diagrams:



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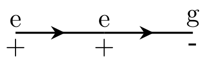
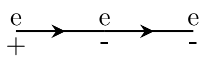


(4)

and we can see that the diagrams for $G^<_{\text{eg}}$, when the external driving field is turned off, contains "ordinary" time-ordered and anti-time-ordered Green functions, and after the summation, the difference between the energies of the e state and the g state enters the self-energy of $G^<_{\text{eg}}$. We can do the same thing for $G^<_{\text{ee}}$ or $G^<_{\text{gg}}$ but this time we just get



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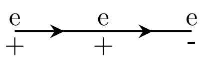

, (5)

and we find that the contributions of time-ordered and anti-time-ordered Green functions just cancel each other. It should also be noted that Δ itself contains corrections from external fields and is the energy gap minus the external phonon frequency.

Now we turn to how to obtain rate equations – equations where the transitions between e and g are modeled as "scattering" – from the aforementioned optical Bloch equation, or more generally, quantum master equations. The idea is just to first replace $G^<_{\text{eg}}$ by diagrams in terms of $G^<_{\text{ee,gg}}$, and then ignore any frequency dependence in the internal components (similar to COHSEX). TODO: possible to do something like GPP?