

# Cavity QED

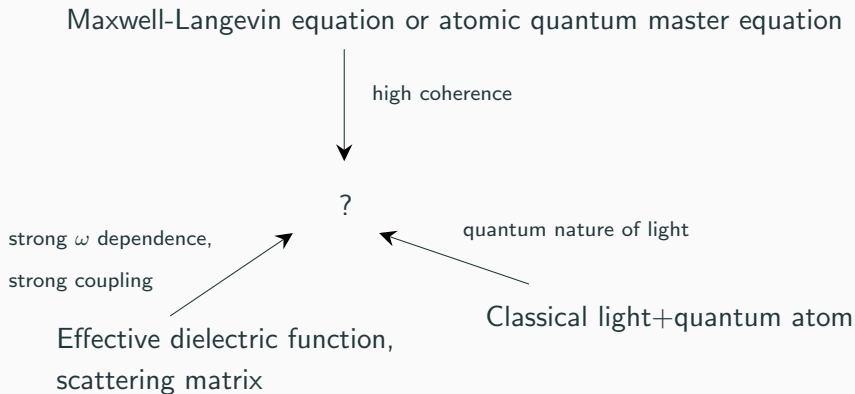
Quantum light-matter interaction to the extreme

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Jinyuan Wu

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# When do all approximate theories fail?

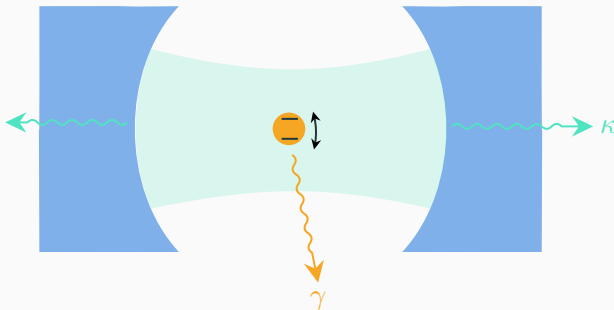


**One scenario: in a cavity.**

# Cavity and one atom

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# High coherence in cavity QED



## Coupling with the environment

- Cavity leaking  $\kappa$
- Atomic spontaneous emission rate (outside the cavity)  $\gamma$
- (Possible non-radiative decay: phonon, etc.)

**Strong coupling limit**  $\kappa, \gamma \ll \mathbf{d} \cdot \mathbf{E}_{\text{cavity}} \Rightarrow$  high coherence

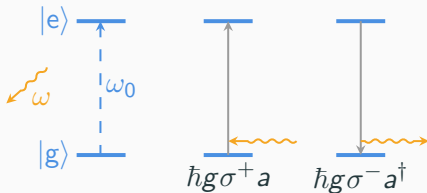
# Frequency selectivity and Jaynes-Cummings model

## Approximations already

- No atom-atom interaction
- No damping at all

## RWA

- Rotating-wave approx.
- Single active photon mode



$$H^{\text{Jaynes-Cummings}} = \hbar\omega \left( a^\dagger a + \frac{1}{2} \right) + \frac{\hbar\omega_0}{2} \sigma^z + \hbar g (a \sigma^+ + a^\dagger \sigma^-)$$

**Possible external field driving**  $\omega_0 \rightarrow \Delta = \omega_0 - \omega_{\text{drive}}$

# Quantum Rabi oscillation

## Quantum nature of the model

- $|e\rangle \xrightarrow{\text{Spontaneous emission}} |g\rangle$  (but not irreversible)

**Dressed state**  $H^{\text{Jaynes-Cummings}}$  in  $\{|g, n+1\rangle, |e, n\rangle\} =$

$$\hbar\omega \left(n + \frac{1}{2}\right) - \frac{\hbar\omega_0}{2} + \begin{pmatrix} \hbar\omega & \hbar g\sqrt{n+1} \\ \hbar g\sqrt{n+1} & \hbar\omega_0 \end{pmatrix}$$

- Oscillation starting with  $|e\rangle$
- Markovian approx. fails
- We have experimental evidence 👉

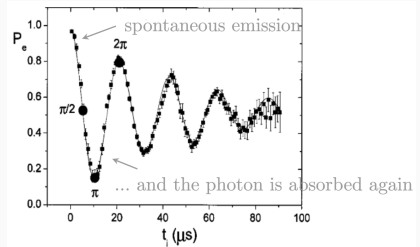


Fig. from S Haroche et al., RMP 73 565 (2001)

# Collapse and revival

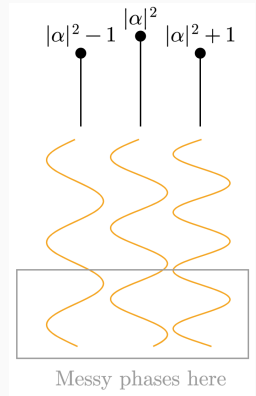
Start with  $|e, \alpha\rangle$ ?  $|e, \alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} \underbrace{|e, n\rangle}_{\leftrightarrow |g, n+1\rangle}$

$$P_e(t) = \frac{1}{2} \left[ 1 + e^{-|\alpha|^2} \sum_{n=0}^{\infty} \frac{|\alpha|^{2n}}{n!} \cos(\Omega_n t) \right]$$

$$t \ll \frac{1}{g|\alpha|} \quad \frac{1}{2} + \frac{1}{2} \cos(2g|\alpha|t) e^{-\frac{1}{2}g^2t^2}.$$

**Collapse of  $P_e$  when  $t \ll 1/g|\alpha|$**  Because  $\phi^{|e,n\rangle}$  not synchronized

This can be simulated by a statistical noise as well; but as  $|\psi\rangle$  is not truly incoherent...



# Collapse and revival

**Revival** When the phases of the major components realign again:

$$2\pi = (\Omega_{|\alpha|} - \Omega_{|\alpha|^2-1})t$$

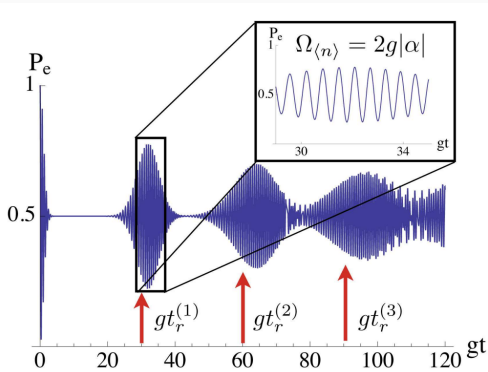


Fig. from arXiv 1111.1143.

Revival is a

- Coherent property:  
not possible with  
statistical noise
- “Granular” property:  
see  $|\alpha|^2 - 1$



# Creation of entangled atom pairs

## Protocol

1. Move atom 1 (in  $|e\rangle$ ) into the cavity mode.
2.  $|e_1, 0\rangle \xrightarrow{\frac{1}{2}\Omega_0 t = \frac{\pi}{4}} \frac{1}{\sqrt{2}}(|g_1, 1\rangle + |e_1, 0\rangle)$ .
3. Move atom 1 out of the light beam. Move atom 2 (in  $|g\rangle$ ) into the light beam.
4.  $\frac{1}{\sqrt{2}}(|g_1, \underbrace{g_2, 1}_{\text{coupling happens only here}}\rangle + |e_1, g_2, 0\rangle) \xrightarrow{\frac{1}{2}\Omega_0 t = \frac{\pi}{2}} \frac{1}{\sqrt{2}}(|g_1, e_2, 0\rangle + |e_1, g_2, 0\rangle)$
5. Move all atoms out.

That's how you get an Einstein-Podolsky-Rosen pair.

# Cavity and medium

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## “Usual” medium within cavity?

The full theory of quantum optics with  $\epsilon(\omega)$  is *hard!!!*

**In a theory only about photons:**  $\epsilon(\omega) \Rightarrow \text{Im } \epsilon \neq 0 \Rightarrow$   
**non-unitary?**

1. Auxiliary fields needed for frequency dependence
2.  $\Rightarrow$  (space-resolved) Input-output formalism
3.  $\xRightarrow{\text{thermalization}}$  quantum Langevin eq.

**Alternatively we pick up excitations in the medium as auxiliary fields, or “atoms”**

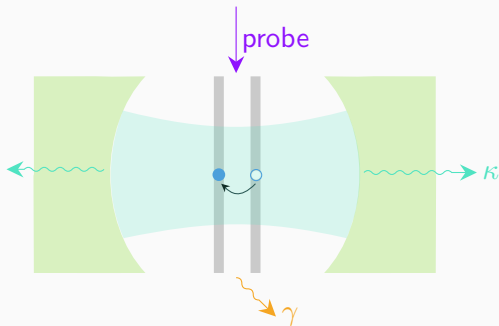
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See e.g. arXiv quant-ph/0006121, PRL 110, 153602

# Cavity QED with exciton

## Setup

- One active photon mode
- Probing by the side
- Exciton modes  
(“two-level atom”: ~~X~~two excitons in one mode)



The effective theory: one photon mode, several atoms

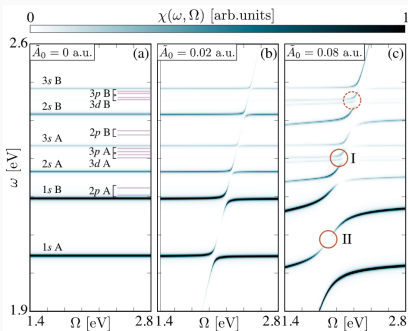
**Why this matters**  $\langle S | \mathbf{d} | 0 \rangle = 0 \not\Rightarrow \langle S_1 | \mathbf{d} | S_2 \rangle = 0$ : dark excitons are optically active in this setting!

# Cavity QED with excitons as atoms

**Exciton polariton** is  $c_1 |\text{exciton}\rangle + c_2 |\text{photon}\rangle$

- Easiest setup
- From the perspective of excitons: virtual photons only

**Optical spectrum**  $\Omega$  = cavity mode freq.,  $\omega$  = detection freq.

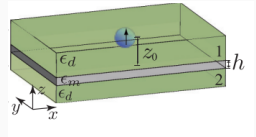


- Dark modes appear (I)
- Quench of optical activity in one band (II)

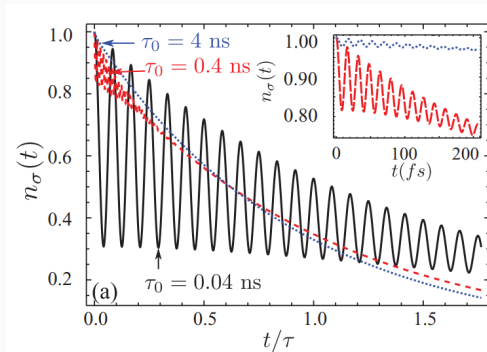
# Cavity QED with surface plasmon polaritons (SPPs) as photons

## Setup

- Replace “simple” photons by SPPs.
- Atom = exciton on nanoparticle



## Rabi oscillation of excited state population



## Take home message

- Cavity  $\Rightarrow$  highly selective strong light-matter interaction  $\Rightarrow$  high coherence  $\Rightarrow$  necessity of cavity QED
- Quantum Rabi oscillation, collapse and revival, entanglement
- Cavity QED happens everywhere