# Technical details in linear SDP bootstrap of $x^4$ anharmonic oscillator

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# Basis of operator space

**The operator space basis**

- All in the form of $x^m p^n$
- Cutoff: $0 \leq m, n \leq 2L$
- In the $M_{ij} = \langle O_i O_j \rangle$ matrix: this means the cutoff is $0 \leq i, j \leq L$

**Indexing of each operator**

- Two digit base-$2L$ number $\overline{mn}$
- Min $= 0$, Max $= (2L+1)^2 - 1$
- Analogy: base-10, $2L = 9$, Min $= 0$, Max $= 99 = (9+1)^2 - 1$

**Implementation**

- `xpopstr_index(x_power, p_power)`: from $(m, n)$ to $\overline{mn}$
- `index_to_xpower(idx)`: from $\overline{mn}$ to $m$
- `index_to_ppower(idx)`: from $\overline{mn}$ to $n$

# Operator algebra

**Operator algebra when formulating the problem**

- $[H, O]$: normal ordered commutator between $x^{m_1} p^{n_1}$ and $x^{m_2} p^{n_2}$
  - Equivalent to commutator between $x^m$ and $p^n$
- $O_i O_j$: normal ordered multiplication of $x^{m_1} p^{n_1}$ and $x^{m_2} p^{n_2}$
  - Equivalent to normal ordering of $p^n x^m$
  - Equivalent to commutator between $x^m$ and $p^n$
- Normal ordered commutator between $x^m$ and $p^n$: McCoy's formula[1]

**Fundamental operations we need**

- $x^l \cdot x^m p^n \cdot p^k$ – implemented as `xpopstr_left_x_right_p_mul`
- $[x^m, p^n]$ – implemented as `xpopstr_comm`

[1] See https://en.wikipedia.org/wiki/Canonical_commutation_relation

# Constructing $M$ matrix

# Imaginary number in linear SDP

Since CSDP doesn't support complex number in the semidefinite constraint, we need to replace 1 by $I_{2\times 2}$ and $\mathrm{i}$ by $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$.