Numerical bootstrap

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What's bootstrap

- A quantum theory = expectations of all Hermitian operators; Hamiltonian/Lagrangian $\Leftrightarrow$ "probability distribution"
- Constraint on the system $\Rightarrow$ relation between different $\langle O \rangle$'s ("**data**"); independent $\langle O \rangle$'s $\Leftrightarrow$ parameters in the model
- Inequality constraint (e.g. positivity of $\langle O^\dagger O \rangle$) $\Rightarrow$ quantum allowed range of $\langle O \rangle$'s
- Solving a class of problems without mentioning explicitly the wave function/path integral: hence the name *bootstrap*

Why we need it

- Because it doesn't fail with strong non-perturbative effects. [1]

[1] See arXiv 2108.11416.

- The most famous example: **conformal bootstrap**
- Constraints: (spinless) two-point function

$$\langle \mathcal{O}(x)\mathcal{O}(y)\rangle = \frac{1}{|x-y|^{2\Delta_{\mathcal{O}}}}, \tag{1}$$

three-point function

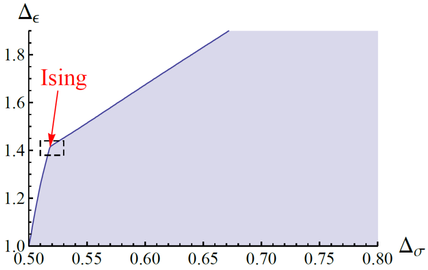
$$\begin{aligned} &\langle \mathcal{A}(x)\mathcal{B}(y)\mathcal{C}(z)\rangle \\ &= \frac{f_{\mathcal{ABC}}}{|x-y|^{\Delta_\mathcal{A}+\Delta_\mathcal{B}-\Delta_\mathcal{C}}|y-z|^{\Delta_\mathcal{B}+\Delta_\mathcal{C}-\Delta_\mathcal{A}}|z-x|^{\Delta_\mathcal{C}+\Delta_\mathcal{A}-\Delta_\mathcal{B}}} \end{aligned} \tag{2}$$

Higher order correlation functions: OPEs.

- Independent parameters: $\{\Delta_{\mathcal{O}}, l_{\mathcal{O}}, f_{\mathcal{ABC}}\}$
- Inequality constraints (self-consistent conditions): determining the range of parameters

Example: conformal bootstrap

- Example of conformal bootstrap: verify whether the critical point of 3D Ising model is a CFT [2]
- Physical picture tells us there are two fields: energy density ϵ, spin field σ
- Below is Fig. 3 in the paper: comparing critical exponents from Monte Carlo simulation of 3D Ising, and the allowed range from conformal bootstrap



[2] 1203.6064