Newton Report

**1. Problem**

Evaulate the error of polynomial interpolation, Numerical Integration using Simpson, Trapezoidal rules and its application.

**2. Description of Work**

1) Using different interpolation point count, evaluate error of functions.

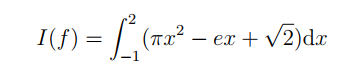
Here , , n = 10, 20

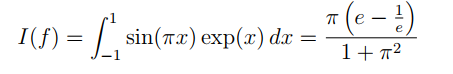
2) Application of Interpolation

Using polynomial interpolation, predict the average yearly temperature for a variety of years.

3) Trapezoid Rule and Simpson Rule

Apply Trapezoid Rule and Simpson Rule to calculate the numerical integral for several functions.





4) Application of Numerial Integration

Apply Numerical Integration to simulate process nameed “carburizing”

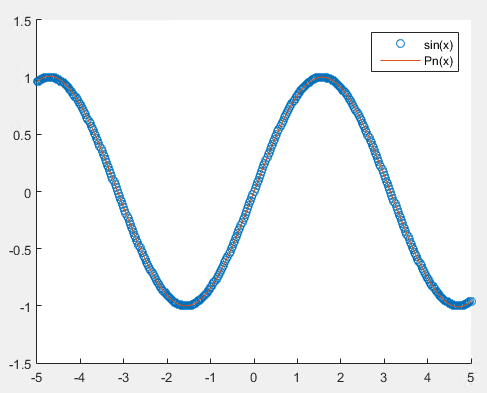
**3. Discussion of test results**

**3.1 Interpolation**

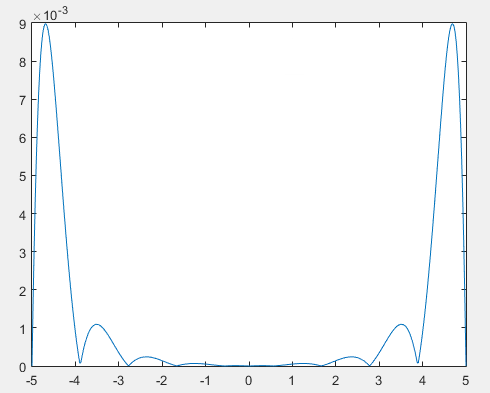
## 3.1.2 Interpolation Result

- The number of nodes: n=10+1, evaluating points: N=400+1.

Interploation Result

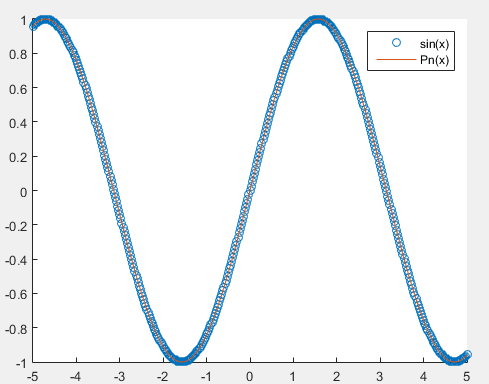


Inerpolation Error

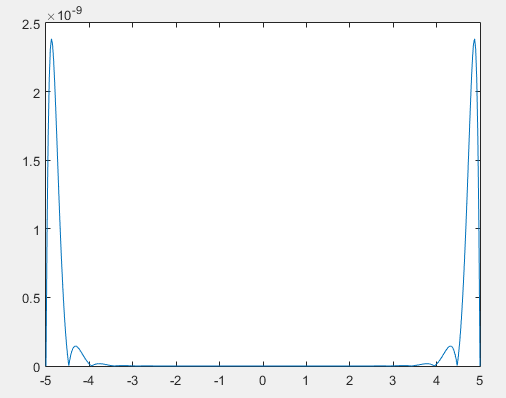


- The number of nodes: n=20+1, evaluating points: N=400+1.

Interploation Result



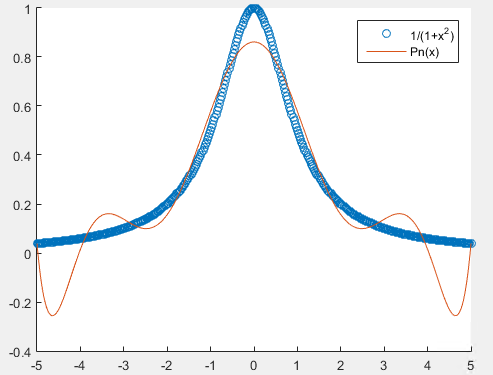
Inerpolation Error



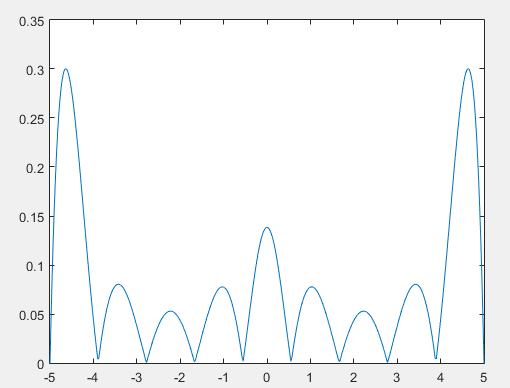
## 3.1.2 Interpolation Result

- The number of nodes: n=10+1, evaluating points: N=400+1.

Interploation Result

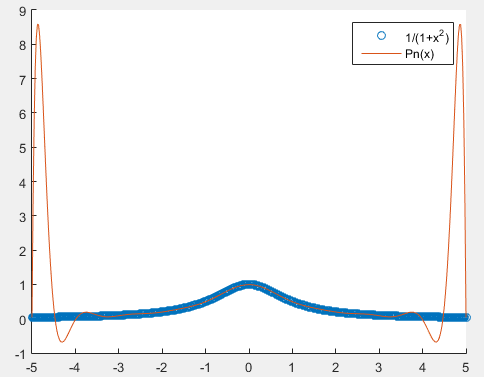


Inerpolation Error

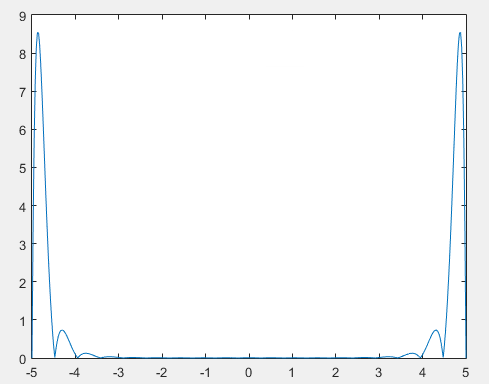


- The number of nodes: n=20+1, evaluating points: N=400+1.

Interploation Result



Inerpolation Error



## 3.1.3 How the value of (n + 1) affects the interpolation error[1][2]

In , the more interpolcation count is, the more accuracy is, but in , when interperation count is increased, accuracy is decreased. It means that larangue polynomial interpolation not always convergence to f(x) when interpolation count is increased.

Because ’s nth derivative value is large when n is increased.

## 3.1.4 Derive the error upper bound

By the interpolation error theorem for equally-spaced nodes, we have

Where ,

So

,

,

**3.2 Application of Interpolation**

xi=[1885, 1895, 1905, 1915, 1925, 1935, 1945, 1955, 1965, 1975, 1985, 1995] ;

fi=[56.70, 56.72, 56.87,56.89, 57.01,57.21, 57.28, 57.18, 57.12, 57.22, 57.65, 57.89] ;

N=200;

-interpolation (prediction of data values inside the interval of data points)

For 1903 1941 1963 1969 1976 1989 1999 years data, predicted average temperature is as follow.

y\_pred = 56.8590 57.2777 57.1198 57.1400 57.2411 57.9739 55.7024

and Real Values are 56.71 57.40 57.25 57.20 56.79 57.69 57.97

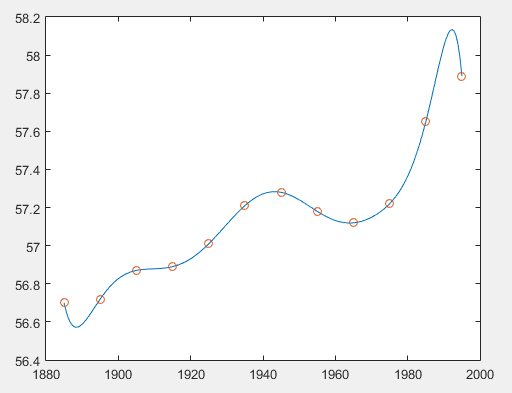
and Interpolation error = 2.2676

- extrapolation (prediction of data values outside the interval of data points).

2009 year predict temperature = 9.5779

2026 year predict temperature = -1056.972

Interpolation polynomial p(x)



Accordingly the error in extrapolation, where *x* is outside [*x*0*;xn*], grows sharply as *x* moves out away from the ending nodes *x*0 and *xn*. So extrapolation may not be reliable, and we should avoid it if possible.

**3.3 Trapezoid Rule and Simpson Rule**

3.3.1 Simpson Rule

True integral = 9.5899959052000963e+00

h = 0.30000000, S = 9.5899959052000945e+00, err = 1.78e-15

h = 0.15000000, S = 9.5899959052000963e+00, err = 0.00e+00, ratio = Inf

h = 0.07500000, S = 9.5899959052000945e+00, err = 1.78e-15, ratio = 0

h = 0.03750000, S = 9.5899959052000963e+00, err = 0.00e+00, ratio = Inf

h = 0.01875000, S = 9.5899959052000998e+00, err = 3.55e-15, ratio = 0

h = 0.00937500, S = 9.5899959052000963e+00, err = 0.00e+00, ratio = Inf

- Due to Simpson’s Rule Theorem,



In above function, , so , so above numerical integration result is resonable.

3.3.2 Trapezoid Rule

True integral = 6.7932618340209472e-01

h = 0.20000000, S = 6.5460005832587065e-01, err = 2.47e-02

h = 0.10000000, S = 6.7316579831292822e-01, err = 6.16e-03, ratio = 4.01373

h = 0.05000000, S = 6.7778740829995288e-01, err = 1.54e-03, ratio = 4.00343

h = 0.02500000, S = 6.7894157219039242e-01, err = 3.85e-04, ratio = 4.00086

h = 0.01250000, S = 6.7923003575925578e-01, err = 9.61e-05, ratio = 4.00021

h = 0.00625000, S = 6.7930214681388801e-01, err = 2.40e-05, ratio = 4.00005

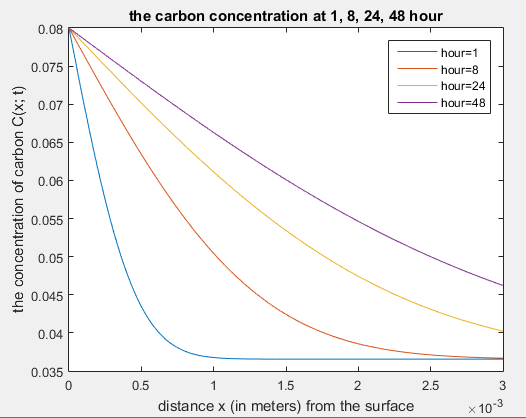
h = 0.00312500, S = 6.7932017427519964e-01, err = 6.01e-06, ratio = 4.00001

h = 0.00156250, S = 6.7932468112162958e-01, err = 1.50e-06, ratio = 4

- Due to Trapezoidal’s Rule Theorem,



**3.4 Application of Numerical Integration**



Above graph says that the more time is passed, the more concentration of carbon to the surface is.