Effective Potential for Massless Scalar Field Scattering Around Charged Black Hole Surrounded by Domain Walls

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I. Introduction

Astrophysics is evolving rapidly with advanced facilities like LIGO and Virgo detecting gravitational waves, and EHT looking at the black hole in the center of Milky Way, driving the search for new physics, as the Standard Model cannot reconcile the discrepancies that arise when general relativity and particle physics intersect. This study investigates the superradiant scattering of massless charged scalar fields, (which can be candidates for hypothetic particles beyond the Standard Model), from charged black holes immersed in the network of domain walls, which are topological defects that may have formed in the early universe. The metric is the key tool to describe the curved space-time in general relativity. To solve the scattering problem, we use the metric of Kiselev spacetime:

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2 d\Omega^2 \tag{1}$$

where $f(r)$ is the metric function containing the information about both the black hole and domain walls network with state parameter $w = -\frac{2}{3}$:

$$f(r) = 1 - \frac{1}{r} + \frac{Q^2}{r^2} - Kr \tag{2}$$

Superradiance is the phenomenon of wave amplification occurring when waves scatter off charged or rotating black holes, resulting from the energy extraction from the black hole. In strong gravity regimes, where standard model scenarios fail, superradiant scattering provides insights into particle behavior and tests general relativity and quantum mechanics in extreme conditions. The test scalar field motion in curved spacetime is described by the Klein-Gordon equation:

$$\left(\nabla^\mu \nabla_\mu - \mu^2\right)\psi = 0 \tag{3}$$

From this, we derive the Schrödinger-like equation for the radial part $\Psi = \Psi_{lm}(r)$ of the test scalar field field $\psi(t, r, \theta, \varphi)$:

$$\frac{d^2\Psi}{dr_*^2} + \left(w^2 - 2w\Phi(r) - V_{eff}(r)\right)\Psi = 0. \tag{4}$$

where $V_{eff}(r)$ is the effective potential for massive charged scalar perturbations:

$$V_{eff}(r) = f(r)\left(\mu^2 + \frac{l(l+1)}{r^2} + \frac{f'(r)}{r}\right) - \Phi(r)^2 \tag{5}$$

where:

- l is the angular number of spherical harmonics,
- $f(r)$ is the function (1) above defining the metric,
- $\Phi(r) = \frac{qQ}{r}$ is the electrostatic potential energy,
- $f'(r)$ denotes derivatives with respect to the radial coordinate r.

In this poster, we focus on the case of a massless scalar field with μ=0 and $l = 0$ for spherical symmetry. Solving $f(r) = 0$ reveals the horizons of the space-time, as discussed in Part II. Analyzing the effective potential provides insights into the quasi-bound states and the feasibility of superradiance.

References

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II. Horizon Structure of the Background

For a given charge Q and domain wall parameter K, the number of real, positive roots of the equation $f(r) = 0$ indicates the number of horizons of the space-time. In our case, up to three distinct horizons can appear:

1. Cauchy Horizon ($\mathbf{r_-}$): The innermost horizon, signifying the limit of predictability within a black hole due to its intersection with past singularities.
2. Event Horizon ($\mathbf{r_+}$): The outer boundary of the black hole's gravitational pull from which nothing can escape, marking the edge of the black hole's crucial influence.
3. Cosmological Horizon ($\mathbf{r_c}$): An outermost horizon that arises due to the repulsive effects attributed to the cosmological expansion, demarcating the boundary of the observable universe.

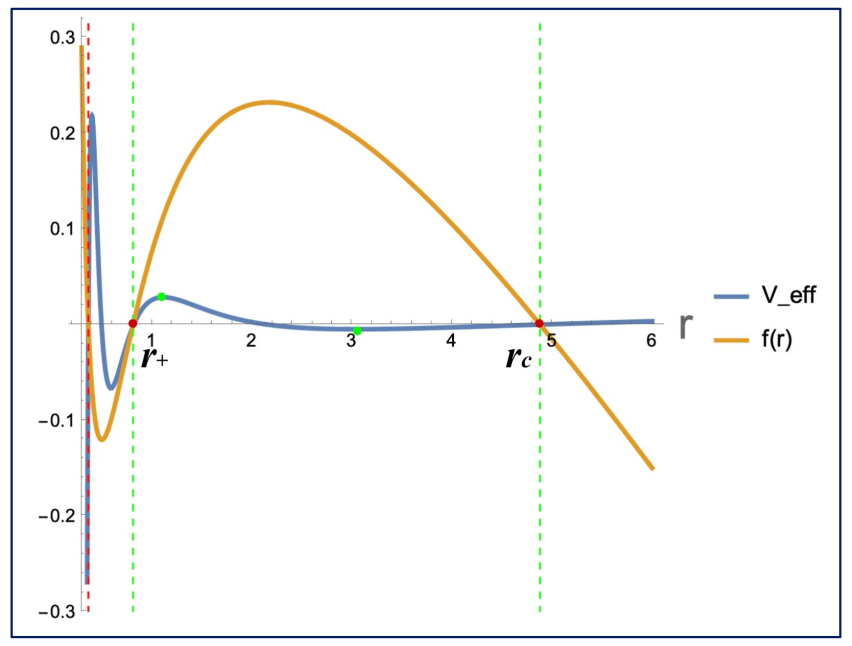

Figure 1: Plot of V_{eff}(r) and $f(r)$

Figure 1 shows a representative plot of the effective potential V_{eff} alongside the metric function $f(r)$. The left red dashed line indicates the location of $\mathbf{r_-}$. The two green dashed lines mark the locations of $\mathbf{r_+}$ and $\mathbf{r_c}$ from left to right. The green dots on V_{eff} represent extrema. **We focus on studying the effective potential in the observable region between the horizons $\mathbf{r_+}$ and $\mathbf{r_c}$.**

Figure 2 shows the number of horizons based on the values of Q^2 and K:

- Blue Region: $\mathbf{r_-}$, $\mathbf{r_+}$, and $\mathbf{r_c}$ all present
- White Region: $\mathbf{r_-}$, $\mathbf{r_+}$, and $\mathbf{r_c}$ all present
- Yellow Region: only $\mathbf{r_-}$ exist

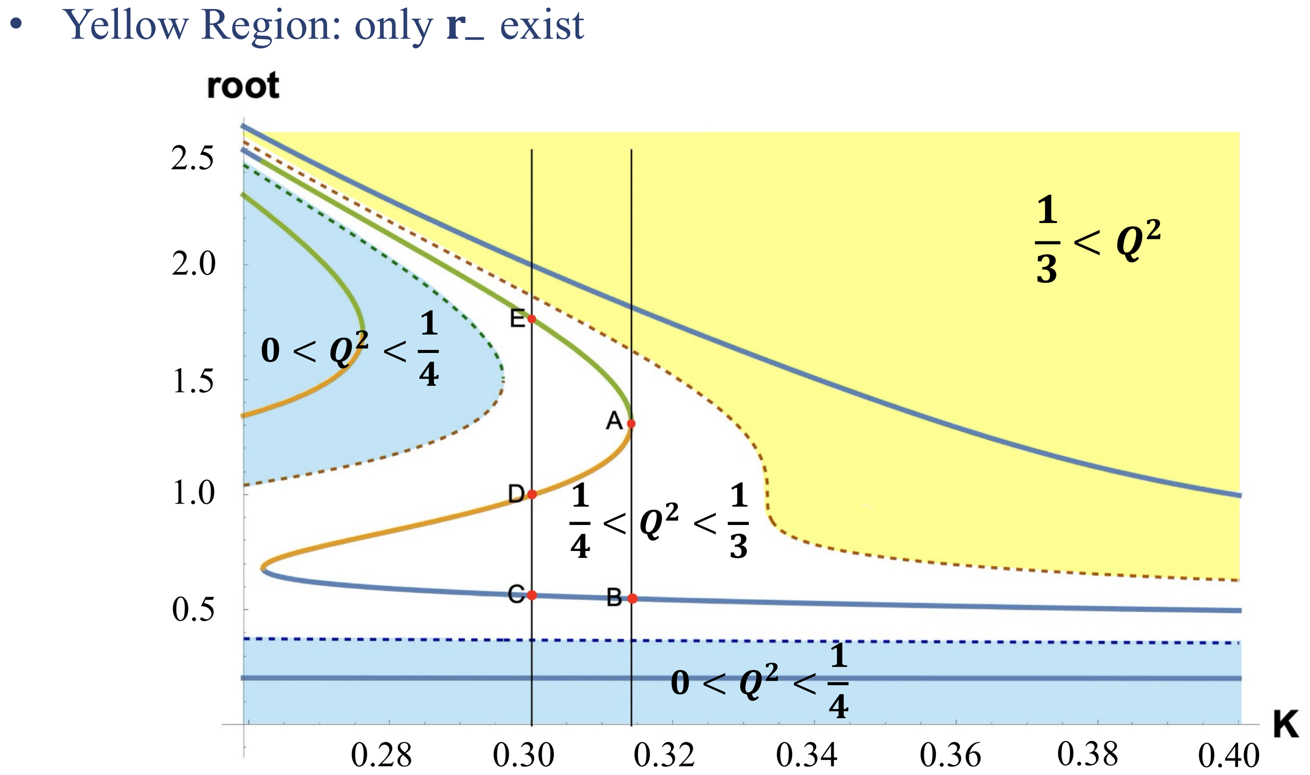

Figure 2: Horizons Configuration of the Space-Time

Signals cannot propagate from $r < r_+$, the event horizon, to an outside observer, nor can they reach inside from $r > r_c$. Thus, r_c represents the outermost limit of the event horizon, beyond which the cosmological expansion exceeds the speed of light. Next, we will analyze the effective potential for the test scalar fields within the white and blue region in Figure 2.

III. V_{eff} for Massless Scalar Fields

$V_{eff}(r)$ in part I determines the dynamics of scalar field perturbations. Depending on the parameters of the model K, w, Q, q^2, μ, $V_{eff}(r)$ can exhibit multiple extrema.

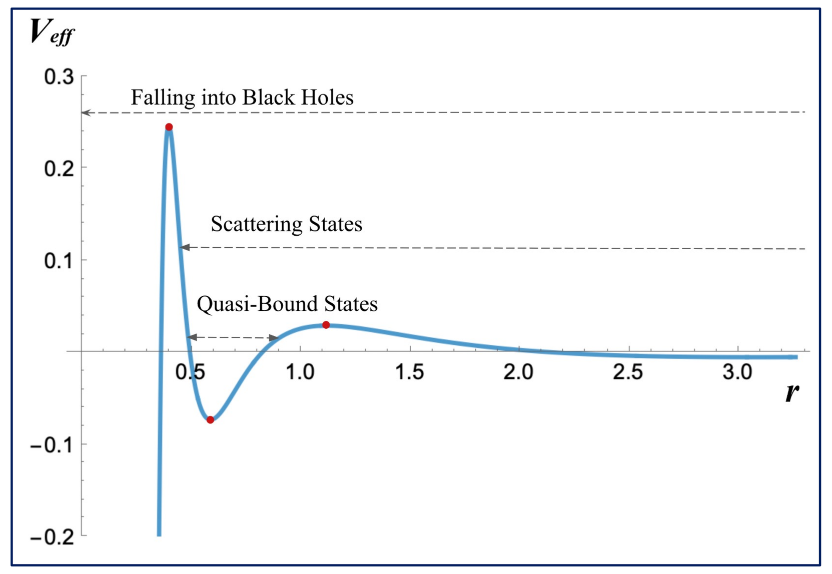

Figure 3: Scattering of Test Scalar Fields on $V_{eff}(r)$

Figure 3 illustrates how the scattering of test scalar fields depends on the behavior of the effective potential. The quasi-bound states occur when incoming waves have energy between the local maximum and minimum of V_{eff}. When the superradiance conditions are met, waves are amplified and trapped in the potential well while extracting energy from the black hole, causing black hole instability.

We use the implicit function theorem to analyze the shape of effective potential $V_{eff}(r)$ which involves multiple variables. This theorem helps us establish relationships between variables and identify extrema by solving for one variable, namely q^2 as a function of others, the steps are as followed:

Step 1: To find the extrema of V_{eff}, solve $\frac{dV}{dr} = 0$.

Step 2: Define $F(r, q^2) = \frac{dV_{eff}}{dr}$ with V_{eff} as a function of r, $Q^2 q^2, K$.

Step 3: Theorem states if $F(r, q^2)$ is continuously differentiable and $\frac{\partial V}{\partial r}$ is non-zero, then locally where $F(r, q^2) = 0$, r can be expressed as a function of q^2.

Step 4: Take derivative of q^2 with respect to r and solve $\frac{\partial q^2}{\partial r} = 0$.

Step 5: Impose constraints on Q^2 and K to ensure f(r) is in the blue and white region on figure 2

We obtain three real and positive roots of $\frac{\partial q^2}{\partial r} = 0$, denoted by r_1 , r_2 , and r_3. The number of roots indicates the number of extrema of q^2 within specific ranges of Q^2 and K as shown in the following table.

K_{dw}	Q^2	q^2 (suppose $q^2(r_1) > q^2(r_3)$)	Extremums on V_{eff}
$0 \le K_{\text{dw}} < K_{\text{dw},1}$	$0 < Q^2 \le Q^2_{R_1}$	$q^2 = q^2(r_1)$	One maximum
		$q^2(r_1) < q^2 < q^2(r_3)$	One maximum and one minimum
		$q^2 = q^2(r_3)$	One maximum and two minimums
		$0 < q^2 < q^2(r_3)$	Two maximums and two minimums
$K_{\text{dw}} = K_{\text{dw},1}$	$Q^2_{R_2} < Q^2 < Q^2_{R_3}$	$q^2 = q^2(r_1)$	Inflection point
		$q^2(r_1) < q^2 < q^2(r_3)$	One maximum and two minimums
		$q^2 = q^2(r_3)$	One maximum and two minimums
		$0 < q^2 < q^2(r_3)$	Two maximums and two minimums
$K_{\text{dw},1} \le K_{\text{dw}} < K_{\text{dw},2}$	$Q^2_{R_2} < Q^2 < Q^2_{R_3}$	$q^2 = q^2(r_1)$	Inflection point
		$q^2(r_1) < q^2 < q^2(r_3)$	One maximum and two minimums
		$q^2 = q^2(r_3)$	One maximum and two minimums
		$0 < q^2 < q^2(r_3)$	Two maximums and two minimums

Table 1: Extremums on V_{eff} for different values of K and Q^2

The number of intersections between $q^2 = c$ (where c is an arbitrary constant) and $q^2(r)$ as a function of r indicates the number of extrema in V_{eff}. For all cases, there is always one maximum and one minimum in the effective potential between the event horizon and the cosmological horizon. **This analysis of the effective potential behavior is crucial for assessing the stability of black holes and the quasi-bound states of the test scalar field.**