Horizon Structure of the Spacetime of Charged Black Hole Surrounded by Domain Walls

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Introduction

Our poster explores black holes in the context of cosmology, focusing on the horizon structure of charged black holes surrounded by domain walls. Domain walls are topological defects of the spacetime that may have formed during the very early stage of our Universe's evolution. They can be observed only through gravitational interaction and can be responsible for the cosmological acceleration. Unlike traditional studies that consider black holes in a vacuum, such as the Schwarzschild black hole, our study presents a more complex and realistic framework by incorporating the effects of the surrounding cosmological media , thus contributing to the pursuit of new physics in cosmological studies. For our charged black hole surrounded by domain walls, the Einstein equations can be solved employing the metric in the form:

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2 d\Omega^2$$

where:

- dt measures time increment,
- dr measures radial increment,
- $r^2 d\Omega^2$ represents the angular distances on a spherical surface at a radius r,
- $f(r)$ contains the information about both the black hole and surrounding matter.

Our case: Domain Walls with $\omega = -\frac{2}{3}$

From the Einstein equations it follows, that in our case the function f(r) can be written in the form:

$$f(r) = 1 - \frac{r_g}{r} + \frac{Q^2}{r^2} - \frac{K}{r^{3\omega+1}}$$

where:

- $r_g = 2GM$ is the gravitational radius of the black hole
- M is the mass of the black hole,
- $\boldsymbol{Q}$ is the charge of the black hole,
- ω is the state parameter of the surrounding cosmological fluid, characterized by barotropic equation of state $\bar{p} = \omega\rho$, with energy density ρ and isotropic pressure p.
- K is a constant related to the curvature of the spacetime with domain walls only.

For domain walls, where the state parameter $\omega = -\frac{2}{3}$, our study imposes dimensionless units to ensure universality of the analysis. Employing the Dominant Energy Condition (DEC), $\rho \geq |p|$, which posits that matter flow velocities cannot surpass light speed, we derive constraints for the parameters of the domain walls network:

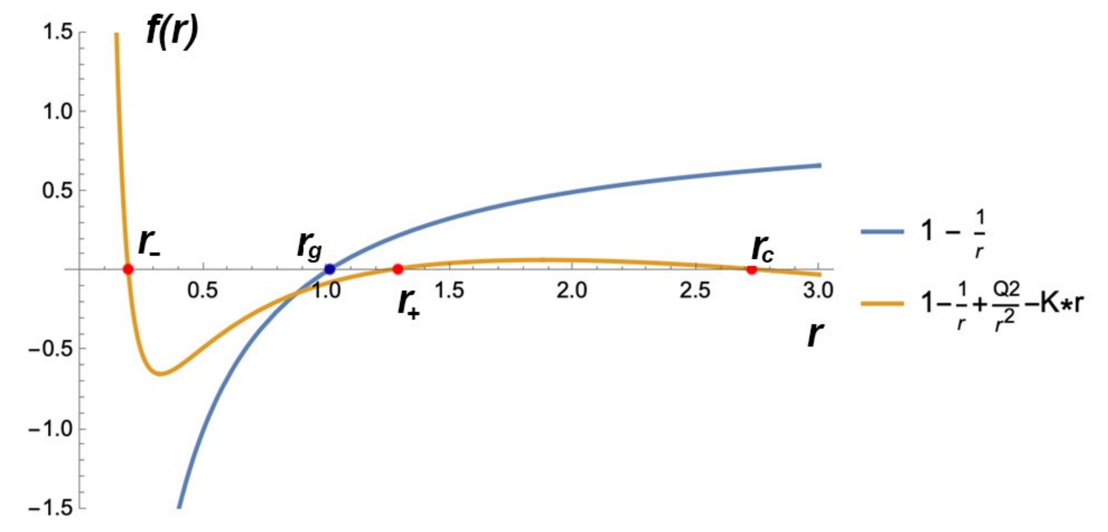
$$-1 \leq \omega < 0, K > 0$$

These parameters are intrinsic to our model, providing a framework for evaluating the system's physical plausibility. Our examination is concentrated on the horizon structure as influenced by the parameters M, $\boldsymbol{Q}$, ω and K. By analyzing the roots of $f(r) = 0$, we categorize the potential horizon configurations.

Horizon Structure of Black Holes

For a given charge Q and domain wall parameter K, the number of real, positive roots of the equation $f(r) = 0$ indicates the number of horizons present. In the model under consideration, up to three distinct horizons can appear:

1. **Cauchy Horizon ($\mathbf{r_-}$)**: The innermost horizon, signifying the limit of predictability within a black hole due to its intersection with past singularities.
2. **Event Horizon ($\mathbf{r_+}$)**: The outer boundary of the black hole's gravitational pull from which nothing can escape, marking the edge of the black hole's crucial influence.
3. **Cosmological Horizon ($\mathbf{r_c}$)**: An outermost horizon that arises due to the repulsive effects attributed to the cosmological expansion, demarcating the boundary of the observable universe.



This plot compares the horizon structure of a Schwarzschild black hole (with $Q = 0$, K $= 0$) with our charged black hole encircled by domain walls. The occurrence of three roots are: r_-, r_+, and r_c, sequentially from the innermost to the outermost, contingent upon the values of Q and K.

Black Hole Horizons and Different Regions of the Spacetime

Setting $\omega = -\frac{2}{3}$ and transitioning everywhere to dimensionless quantities (measured in units of $2GM$), our metric function simplifies to:

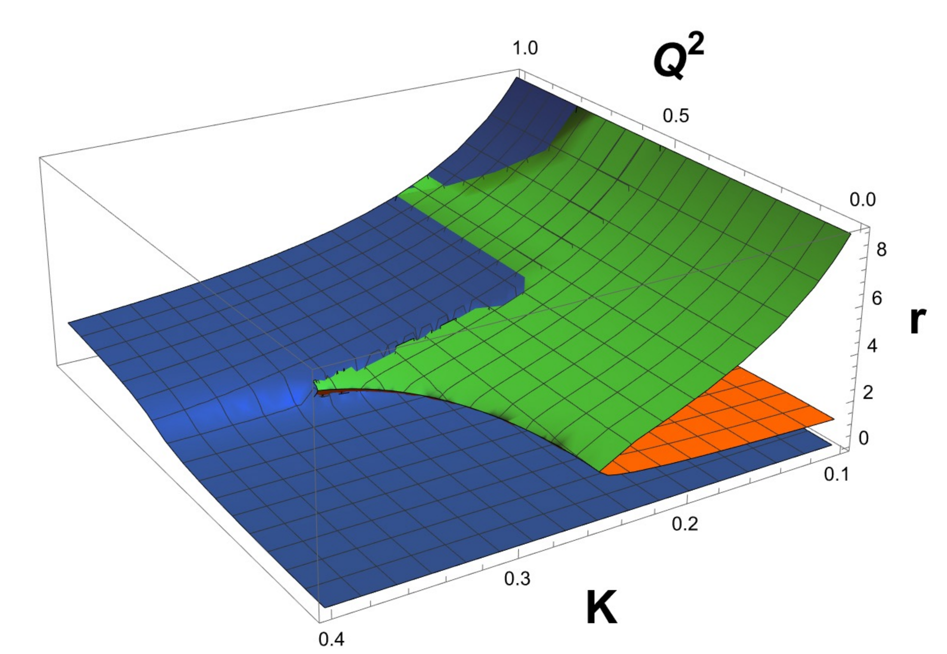
$$f(r) = 1 - \frac{1}{r} + \frac{Q^2}{r^2} - Kr$$

- $f(r) < 0$: indicates essential spacetime instability, characterized by the flipping of signs for g_{tt} and g_{rr},
- $f(r) > 0$: indicates stable spacetime where observers can remain at rest relative to the central mass, permitting the presence of static observers

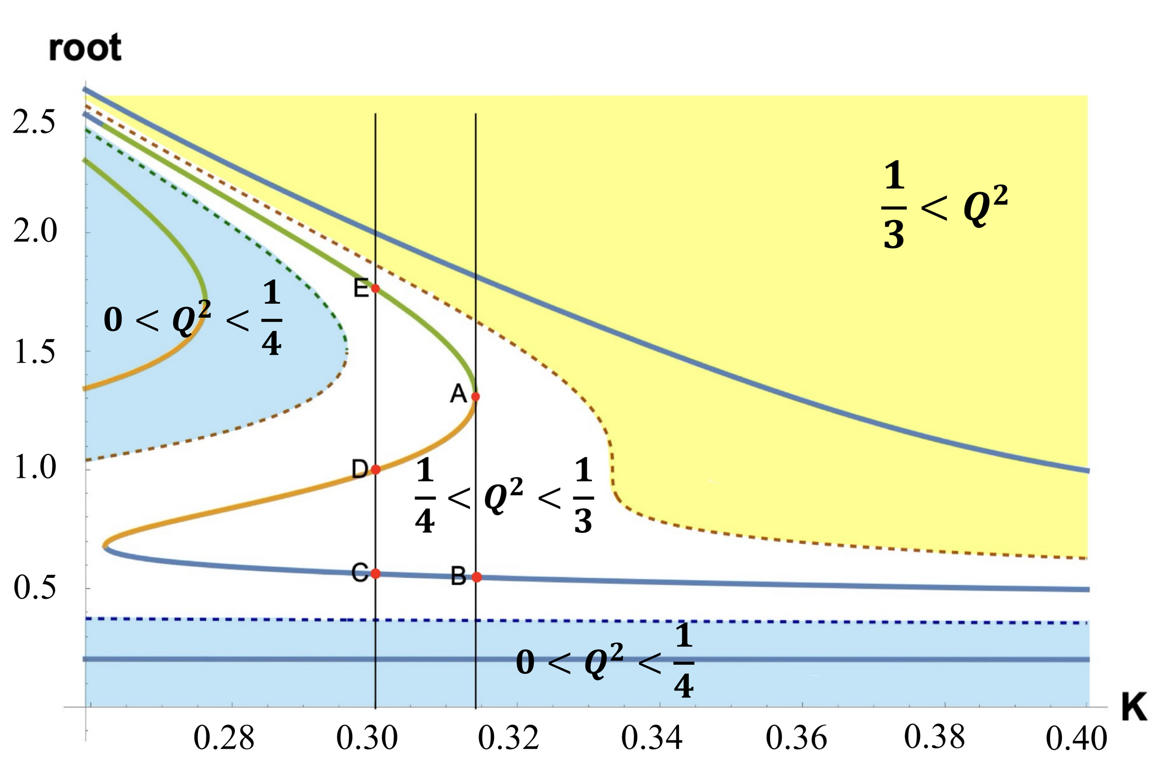
Signals cannot propagate from $r < r_+$, the event horizon, to an outside observer, nor can they reach inside from $r > r_c$. Thus, r_c represents the outermost limit of the event horizon, beyond which the the cosmological expansion exceeds the speed of light.

Roots Analysis

Our analysis maps the number of physically meaningful horizons — real and positive roots of the equation $f(r) = 0$ across varying charges Q and domain wall influences K. The roots correspond to the black hole's Cauchy horizon r_- (the blue curve/surface), event horizon r_+ (the yellow curve/surface), and cosmological horizon r_c (the green curve/surface), ordered from innermost to outermost, respectively. We constrain our analysis to $K > 0$, $Q^2 > 0$ to align with physical expectations the charge and the dominant energy condition. The horizons are sought in the domain $r > 0$ to avoid the physical singularity at at $r = 0$. The 3D plot provides a landscape of horizon existence across the Q^2 and K parameter space.



The 2D plot demonstrates the specific number of horizons for given Q^2 and K values. The regions delineated by dashed lines are determined by the range of Q^2 . The interplay of Q^2 and K within these bounds dictates the root structure of our function $f(r) = 0$.



Region Classifications:

- **Blue Region ($\mathbf{0 < Q^2 < \frac{1}{4}}$)** : The plot shows there are three distinct horizons when $0 < K < \frac{8}{27}$. All three types of horizons (Cauchy, event, and cosmological) are present.
- **White Region ($\mathbf{\frac{1}{4} < Q^2 < \frac{1}{3}}$):** Here, three horizons exist when K is less than the critical value at point $K <$ 0.3141. Beyond this value, only the Cauchy horizon persists.
- **Yellow Region ($\mathbf{Q^2 > \frac{1}{3}}$):** Independent of K, there is only a single horizon, which is the Cauchy horizon.

Discussion and Future Work

Our findings delineate the parameter space where different horizon configurations are possible, enhancing our understanding of these complex spacetime entities.

Building on the insights gained, our next objective is to assess the stability of these black hole systems. By perturbing the spacetime around the horizons and analyzing the resulting dynamics, we can predict stability and potentially identify new physical phenomena. This will deepen our grasp of black hole physics in a cosmological setting and may yield new theoretical predictions to be sought in observational data.

References

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