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Assignment 5

1. a. $((\text{Not}(x22))[4/x22] \rightarrow (\text{Not}(4))$

b. $((\text{Function } x \rightarrow x\ x)(\text{Function } x \rightarrow x\ x))[(10)/x] \rightarrow ((\text{Fun } x \rightarrow x\ x)(\text{Fun } x \rightarrow x\ x))$

c. $(\text{Function } x \rightarrow x+y)[\text{False}/y] \rightarrow \text{Function } x \rightarrow x+\text{False}$

d. $(\text{Let } z = x \text{ In } y)[33/x] \rightarrow \text{Let } z = 33 \text{ In } y$

2. a. Ring:

$$\frac{e_1 \Rightarrow v_1,\ e_2 \Rightarrow v_2,\ e_3 \Rightarrow v_3,\ \ldots\ e_n \Rightarrow v_n,\ n \geq 1}{\text{Ring } e_1 @ e_2 \ldots @ e_n \Rightarrow \text{Ring } v_1 @ v_2 \ldots @ v_n}$$

Left:

$$\frac{e \Rightarrow \text{Ring } v_1 @ v_2 @ v_3 \ldots @ v_n,\ n \geq 1}{\text{Left}(e) \Rightarrow \text{Ring } v_2 @ v_3 @ v_4 \ldots @ v_n @ v_1}$$

Right:

$$\frac{e \Rightarrow \text{Ring } v_1 @ v_2 \ldots @ v_n,\ n \geq 1}{\text{Right}(e) \Rightarrow \text{Ring } v_n @ v_1 \ldots @ v_{n-1}}$$

Add:

$$\frac{e_1 \Rightarrow \text{Ring } v_1 @ \ldots @ v_n \quad e_2 \Rightarrow v_{n+1},\ n \geq 1}{\text{Add } e_1\ e_2 \Rightarrow \text{Ring } v_{n+1} @ v_1 \ldots @ v_n}$$

Drop:

$$\frac{e \Rightarrow \text{Ring } v_1 @ \ldots @ v_n,\ n \geq 2}{\text{Drop}(e) \Rightarrow \text{Ring } v_2 @ \ldots @ v_n}$$

Get:

$$\frac{e \Rightarrow \text{Ring } v_1 @ \ldots @ v_n,\ n \geq 1}{\text{Get}(e) \Rightarrow v_1}$$

2 b.

$$\dfrac{\dfrac{\dfrac{\dfrac{\overline{0 \Rightarrow 0} \quad \overline{1 \Rightarrow 1}}{0+1 \Rightarrow 1} \quad , \overline{2 \Rightarrow 2} \quad \overline{3 \Rightarrow 3}}{\text{Ring } (0+1)@2@3 \Rightarrow \text{Ring } 1@2@3}}{\text{Left } (\text{Ring } 1@2@3) \Rightarrow \text{Ring } 2@3@1}}{\text{Drop } (\text{Left } (\text{Ring } 1@2@3)) \Rightarrow \text{Ring } 3@1}$$

2. c. Directly no b/c no way to directy encode empty Lists with Rings I.E. Ne explict empty Ring defintion

However, we could encode with the following:

Empty : Ring (0) * this encoding has the first element of the Ring be the number of elements in the list

Cons : Fun a → Fun b ⇒ If Get b = 0 Then Ring 1@a Else [b is list a is element]
Let c = Get b + 1 in Add Add (Drop b) a) c

is Empty : Fun Lst → If Get lst = 0 Then True Else False

head : Fun lst → If Get lst=0 Then (1 0) Else Get (Drop (lst))
(list is empty)

Tail : Fun lst → If Get lst = 0 Then Ring(0) Else
(return empty list)

If Get Lst = 1 Then Rigg(0)
Else Let c = Get lst − 1 in
Add (Drop (Drop lst)) c

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3 Cheap y= (Fun code ->
              Let repl = Fun self -> Fun C -> Fun arg ->
                 If C=10 Then
                      (1,0)
                 Else
                      code (Self self (C+1)) arg
           In repl repl 0)
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4 a.

Stash:
$$\frac{\langle e, s_1\rangle \Rightarrow \langle v, s_2\rangle}{\langle \text{Stash} e, s_1\rangle \Rightarrow \langle \_, \{v\}\rangle}$$

s denotes global value Store

- denotes silent output

unstash:
$$\frac{s \neq \{\}}{\langle \text{unstash}, s\rangle \Rightarrow \langle v, s\rangle}$$

plus:
$$\frac{\langle e_1, s_1\rangle \Rightarrow \langle v_1, s_2\rangle, \quad \langle e_2, s_2\rangle \Rightarrow \langle v_2, s_3\rangle}{\langle e_1 + e_2, s_1\rangle \Rightarrow \langle v_1 + v_2, s_3\rangle}$$

b. (Let _ = Stash 5 In Unstash) + Unstash ⇒ 10

$$\frac{\dfrac{\dfrac{\dfrac{}{\langle 5, \{\}\rangle \Rightarrow \langle 5, \{\}\rangle}}{\langle \text{Stash } 5, \{\}\rangle \Rightarrow \langle \_, \{5\}\rangle}, \quad \dfrac{}{\langle \text{unstash}, \{5\} \Rightarrow \langle 5, \{5\}\rangle}}{\langle \text{Let } \_ = \text{Stash } 5 \text{ In Unstash}, \{\}\rangle \Rightarrow \langle 5, \{5\}\rangle}, \quad \dfrac{}{\langle \text{unstash}, \{5\}\rangle \Rightarrow \langle 5, \{5\}\rangle}}{\langle (\text{Let } \_ = \text{Stash } 5 \text{ In unstash}) + \text{unstash}, \{5\}\rangle \Rightarrow \langle 10, \{5\}\rangle}$$