Methods to Improve the Volatility Forecasting in Exchange Rate Market

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Introduction

Volatility is a measure of uncertainty or risk in financial markets. Mathematically, it is defined as the absolute log return of an asset class (difference between the logarithmic price at two consecutive time intervals). Over the past 20 years, researchers and financial institutions have been trying to study the pattern of volatility and design models which can predict its movement accurately. Modeling and forecasting volatility in financial markets provides valuable insights into market dynamics, risk assessment, asset pricing, option trading, and portfolio management. It helps market participants make informed decisions, manage risks, and optimize investment strategies. One thing to note is that different asset classes have various pricing factors and therefore demonstrate very different volatility patterns in high frequency trading. Out of all, the exchange rate market is the largest and most liquid financial market in the world, and it has been quite volatile over the past one year due to the under-performance of global economy. In this project, we will use the high-frequency trading data in the exchange rate market to construct and assess volatility forecasting models.

Research objectives

- **Objective 1:** To explore different models and methods to forecast the volatility of the exchange rate market (dollar pairs) and tune the best-fit univariate model.
- **Objective 2:** To further investigate the problems in multivariate context as co-volatlity modelling.

Statistical Theory

Volatility Stylized Fact:

Volatility is clustered and persistent. A large/small price change is often followed by another large/small price change. Such volatility clustering can be seem in the left subplot of Figure 1 (high volatility clusters indicated by red circles). Besides, the serial correlations are persistent as they decay very slowly in time lag k. The right subplot of Figure 1 below shows that the autocorrelation of the daily volatility of EURUSD rate is still significantly positive even at lag $k = 30$.

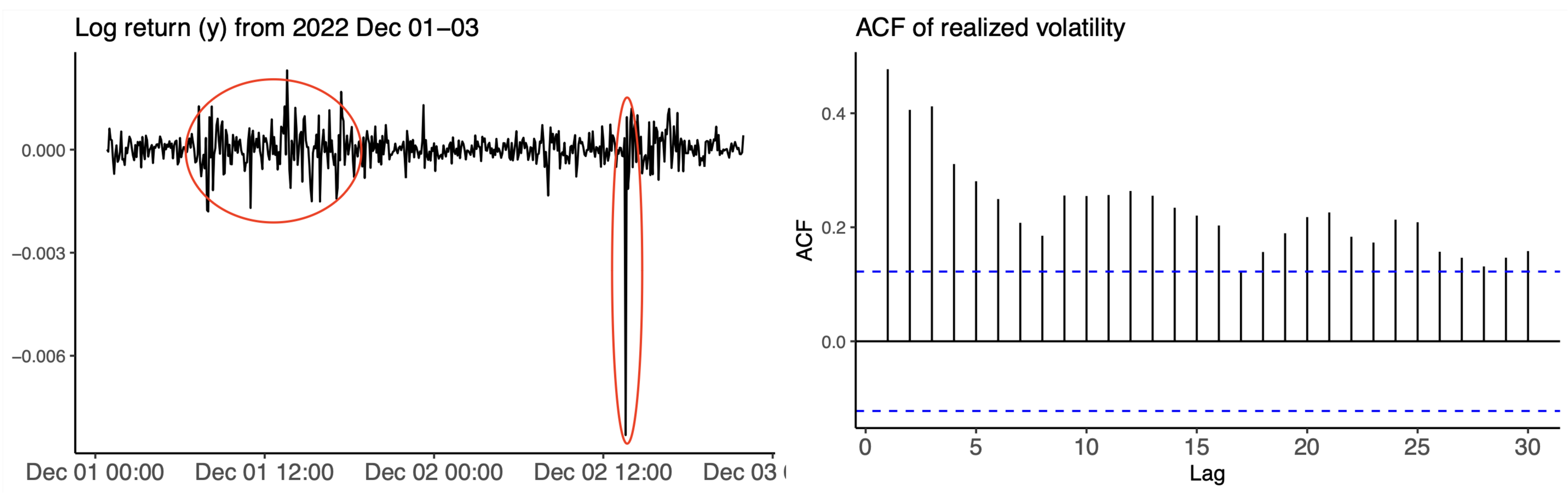

Figure 1. Log return and ACF plots demonstrating volatility clustering and persistence

Asymptotic distribution of quadratic and bipower variation:

Let $Y_{j,i}$ denotes the logarithmic price of the currency at unit time j on day i, and let the discretely sampled δ-period log returns between $Y_{j-1,i}$ and $Y_{j,i}$ be denoted by $y_{j,i}$. Then we can define the daily realized variance RV_i as below: (note that daily realized volatility is simply $\sqrt{RV_i}$)

$$RV_i = \sum_{j=1}^{[1/\delta]} (Y_{j+1,i} - Y_{j,i})^2 = \sum_{j=1}^{[1/\delta]} y_{j,i}^2$$

where $\delta = 288$ is used throughout this whole project (trading data of 5 min-interval). Then, it follows directly by the theory of quadratic variation [Barndorff-Nielsen and Shephard, 2002] that for $\delta \to 0$

$$RV_i \xrightarrow{\mathbb{P}} \int_0^t \sigma_s^2 ds + \sum_{j=0}^{N_i} (\Delta J_j)^2 \tag{1}$$

where σ^2 represents the diffusive components (continuous volatility), ΔJ represents the jump components (discontinuous volatility), N_i is the number of jumps occur on day i and $t = 1$ unit day. Following the idea of separating the diffusive and jump components of the quadratic variation, the idea of standardized realized bipower variation is defined as below [Barndorff-Nielsen and Shephard, 2003]

$$BV_i = \mu_1^{-2} \sum_{j=1}^{[1/\delta]} |y_{j-1,i}||y_{j,i}| \xrightarrow{\mathbb{P}} \int_i^{i+1} \sigma_s^2 ds \quad \text{where} \quad \mu_1 = \sqrt{\frac{2}{\pi}} \tag{2}$$

Therefore, by combining the result of Equation 1 and 2, it can be derived that as $\sigma \to 0$ [Barndorff-Nielsen and Shephard, 2003],

$$RV_i - \mu_1^2 BV_i \xrightarrow{\mathbb{P}} \sum_{j=0}^{N_i} (\Delta J_j)^2 \tag{3}$$

The three asymptotic distributions above, together with the persistence volatility, later serve as the key intuition behind our model design and construction.

Models and Techniques

HAR model

We would start with the basic HAR-RV model first proposed by Corsi [2009]. It is a linear model considers just three volatility components corresponding to time horizons of one day (1d), one week (1w), and one month (1m). The weekly and monthly volatility is taken into account due to the persistence of volatility. The HAR-RV model is defined as

$$\sqrt{RV_{t+1}} = \phi_0 + \phi^{(d)}\sqrt{RV_t} + \phi^{(w)}\sqrt{RV_{t:t-4}} + \phi^{(m)}\sqrt{RV_{t:t-22}} + \epsilon_0 \tag{4}$$

This linear model can be fitted easily with the realized volatility data. We would then further modify this model aiming to make it fit better to the chosen market data.

Semi-variance model

The idea of realized up and down semivariance measures separates the total realized variation into two components associated with the positive and negative high-frequency returns [Bollerslev et al., 2020], and the two components are defined as below:

$$RV_t^+ = \sum_{j=1}^{[1/\delta]} y_{j,i}^2 \mathbb{I}_{\{y_{j,i} \geq 0\}}, \; RV_t^- = \sum_{j=1}^{[1/\delta]} y_{j,i}^2 \mathbb{I}_{\{y_{j,i} < 0\}}$$

This is based on the intuition that up and downside risks are not treated equally by investors. Then, simple modifications can be made to the HAR model - the RV_t component of HAR-RV model can be split to RV_t^+ and RV_t^-, and $\phi^{(d+)}$ and $\phi^{(d-)}$ are estimated instead, which give rise to Semi-variance HAR (SHAR) model. This technique can be applied in all linear models that involves the RV_t component.

Models and Techniques (continued)

Continuous-Jump Separation

Inspired by the asymptotic distributions of quadratic and bipower variation, it is possible that one could build (S)HAR models for the diffusive and jump parts of RV_t separately, and combine the two forecasting results together to obtain the final forecast for volatility. We have employed three ways to separate the diffusive/continuous (C) and jump/discontinuous (J) components:

- A constant threshold α is set. $C_t = \sum_j y_{j,t}^2 \mathbb{I}_{\{|y_{j,t}| \leq \alpha\}}$ and $J_t = \sum_j y_{j,t}^2 \mathbb{I}_{\{|y_{j,t}| > \alpha\}}$.
- Asymptotic distribution results are used. $C_t = BV_t$ and $J_t = \max(RV_t - BV_t, 0)$.
- Ratio jump test statistic Z (details seen in the next section) is applied. $C_t = RV_t \mathbb{I}_{\{Z \leq \Phi_{0.99}\}} + BV_t \mathbb{I}_{\{Z_t > \Phi_{0.99}\}}$ and $J_t = (RV_t - BV_t)\mathbb{I}_{\{Z_t > \Phi_{0.99}\}}$

We then apply the above three separation methods to split the two components for separate forecasting. We have noted that jump process is not as persistent as the diffusive parts, and therefore besides the HAR model, we also tried the naive estimate $\sqrt{J_{t+1}} = \sqrt{J_t}$ and one-day linear model $\sqrt{J_{t+1}} = \beta_0 + \beta^{(d)}\sqrt{J_t}$ for the jump forecasting.

Forecast Aggregation

It has been found in the electricity forecasting studies that combining members of the original set of forecasts (experts) can lead to a significant improvement on the forecasting accuracy [Gaillard and Goude, 2015]. At each time t, the aggregation forecaster/rule would compute the weights of each expert prediction with knowledge of the past observations and of the expert advice. Then the aggregation forecast is formed by the inner products of the experts predictions and the corresponding weights. In this project, we would apply 7 developed aggregation rules from the `opera` package in R to examine whether this method is also useful under the price volatility forecast context.

Preliminary Results and discussion

Adjusted Ratio Jump test

To gain a better understanding of our trading data, we first perform the adjusted ratio jump test developed by Barndorff-Nielsen and Shephard [2006]. The ratio statistic Z_t (which converges consistently to a standard normal distribution) is used to test whether 'jump' process exists on the trading day t. We have performed the test for all 257 trading days from 2022 May to 2023 Apr. **The rejection rate at 95% CI is 0.2568 and is 0.1478 for 99% CI.** The test results suggests that over the past one year, there are quite a number of trading days where jumps being observed. We take the date July 13 2022 as an example. One could observe a great price jump around 12.30pm on that day from the left subplot. This is followed by the release of US CPI data (inflation data). This is correctly reflected on the right subplot showing a rejection of the null hypothesis on Jul 13, supporting that jump existed on that day. **It is found that large jumps identified by the test often corresponds to market news announcements.**

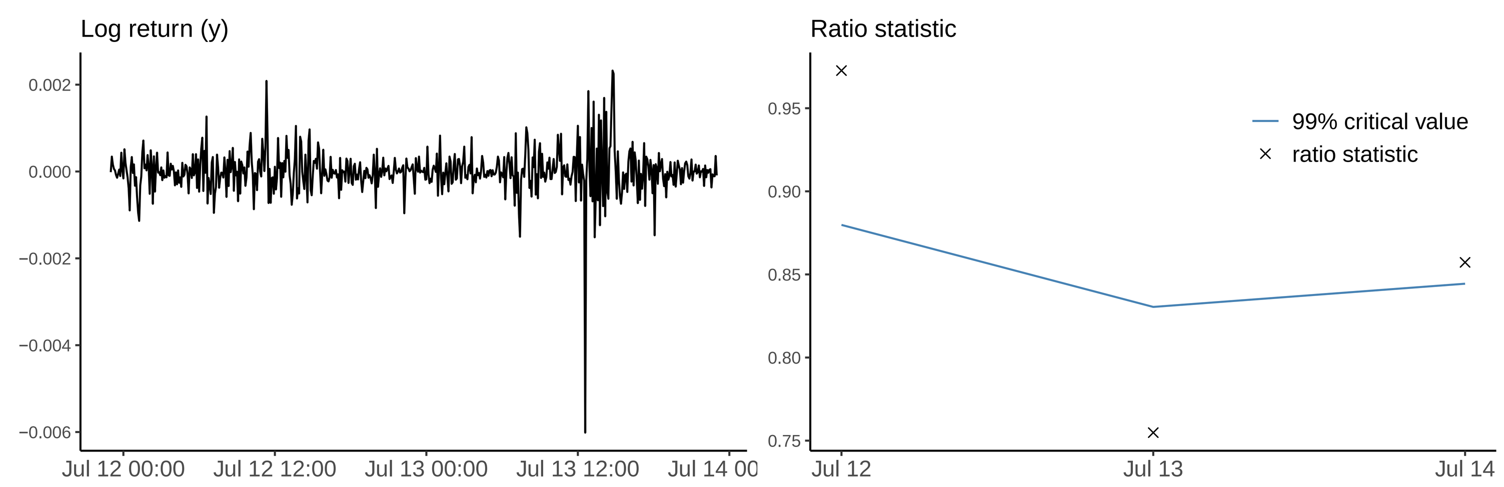

Figure 2. Log return of EURUSD and its corresponding daily jump test result from Jul 12-14

Univariate model performance

We have constructed and fit a total of 18 individual models. For each model, we first fit with the entire data set and perform the in-sample forecast on all 257 trading days. Then, we use a rolling window to get the one-day out-of-sample forecast on the last two months of the data set. Both the in-sample and out-of-sample MSE are computed. With the forecasting errors, we also perform the Diebold-Mariano (DM) test for predictive accuracy. The original HAR-RV model is set as the base case, and 17 DM tests are performed to see if the modified model is more accurate than the base model. Indicated by the MSE value and the DM test statistics, we found the **4 models which use the 2nd and 3rd continuous-jump separation method significantly outperformed the original HAR model, with C modelled by HAR and J modelled by HAR or one-day linear forecast**.

Aggregated Forecast

Figure 3 below shows the weights assigned to the four outperforming experts with the Bernstein Online Aggregation (BOA) rule. We notice from the plot that the first two models are generally assigned with greater weights than the bottom two models. So far we have not found clear sign that forecast aggregation has significant improvement on the individual expert volatility forecasts.

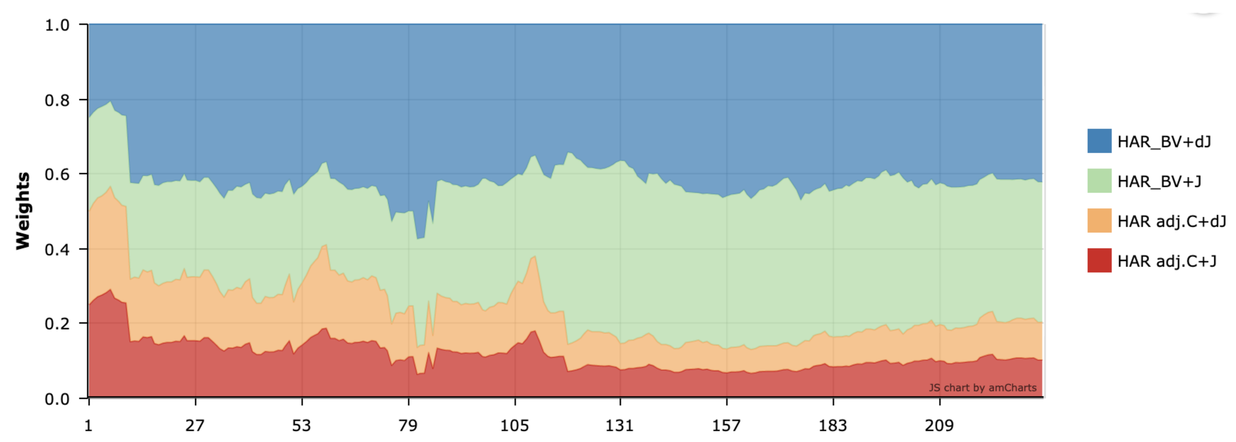

Figure 3. BOA weight plots of different univariate experts

Research to be continued

So far we have only forecasting on a point univariate case. We wish to further derive the results of **probabilistic forecasting** which would be more useful in applications. Beyond that, we would like to fully explore **co-volatility modelling and forecasting on a multivariate case**.

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