22F CS-513 C HW 1

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1 Problem 1

1. By question we can get that:

$$P(J) = 0.2 \quad (1)$$
$$P(S) = 0.3 \quad (2)$$
$$P(J) \cap P(S) = 0.08 \quad (3)$$

By Conditional Probability:

$$P(J|S) = \frac{P(J) \cap P(S)}{P(S)} = \frac{0.08}{0.3} = \frac{4}{15}$$

So when Susan was at the bank last Monday then the probability that Jerry was there too is $\frac{4}{15}$.

2. By Bayes Rule, we know the probability that Jerry was at the bank but Susan not last Friday:

$$P(J|S^C) = \frac{P(S^C|J)P(J)}{P(S^C)}$$

We can get that;

$$P(S^C) = 1 - P(S) = 0.7$$
$$P(S^C|J) = 1 - P(S|J) = \frac{P(J) \cap P(S)}{P(J)} = 1 - \frac{0.08}{0.2} = 1 - 0.4 = \frac{3}{5}$$

Hence, we can calculate that:

$$P(J|S^C) = \frac{0.6 \times 0.2}{0.7} = \frac{6}{35}$$

So the probability that Jerry was at the bank but Susan not last Friday is $\frac{6}{35}$.

3. When at least one of them was at the bank, the probability that both of them were there is:

$$P = P(J \cap S|S \cup J) \quad (4)$$
$$= \frac{P(J \cap S) \cap P(J \cup S)}{P(J \cup S)} \quad (5)$$
$$= \frac{P(J \cap S)}{P(J \cup S)} \quad (6)$$

$$P(J \cup S) = P(J) + P(S) - P(J \cap S) = 0.2 + 0.3 - 0.08 = 0.42$$

$$P = \frac{P(J \cap S)}{P(J \cup S)} = \frac{0.08}{0.42} = \frac{4}{21}$$

So the probability is $\frac{4}{21}$.

2 Problem 2

1. We can see

$$P(H) = 0.8 \quad (7)$$
$$P(S) = 0.9 \quad (8)$$
$$P(H \cup S) = 0.91 \quad (9)$$

Then we can calculate:

$$P(H \cap S) = P(H) + P(S) - P(H \cup S) = 0.8 + 0.9 - 0.91 = 0.79$$

The probability that only Harold gets a "B" is:

$$P(H \cap S^C) = P(H) - P(H \cap S) = 0.8 - 0.79 = 0.01$$

2. The probability that only Sharon gets a "B" is:

$$P(S \cap H^C) = P(S) - P(H \cap S) = 0.9 - 0.79 = 0.11$$

3. The probability that both won't get a "B" is:

$$P = 1 - P(H \cup S) = 1 - 0.91 = 0.09$$

3 Problem 3

$$P(J)P(S) = 0.2 \times 0.3 = 0.06 \neq 0.08 = P(J) \cap P(S)$$

These events are not independent.

4 Problem 4

1. P(the sum is 6| the second die shows 5)=P(the first die shows 1) $\neq$ P(the sum is 6)
   They are not independent.

2. P(the sum is 7| the first die shows 5)=P(the first die shows 2) $\neq$ P(the sum is 7)
   They are not independent.

5 Problem 5

1. Let A be finding oil, TX be drilling in TX, AK be drilling in AK , NJ be drilling in NJ.

$$P(TX \cap B) = 0.3$$

$$P(AF \cap B) = 0.2$$

$$P(NJ \cap B) = 0.1$$

Since the oil company is considering drilling in either TX, AK and NJ. So

$$P(A) = P(TX \cap B) + P(AF \cap B) + P(NJ \cap B) = 0.6$$

So the probability of finding oil is 0.6.

2. Let B be finding oil, A be drilled in TX. Then we want to compute $P(A|B)$.

$$P(B) = 0.6 \tag{10}$$
$$P(A \cap B) = 0.3 \tag{11}$$

$$P(A|B) = P(A \cap B)/P(B) = 0.3/0.6 = 0.5$$

Hence, the probability is 0.5.

6 Problem 6

1. The probability that a passenger did not survive is

$$P = 1490/2201 \approx 0.6770$$

2. The probability that a passenger was staying in the first class is

$$P = 325/2201 = 0.14766$$

3. Given that a passenger survived, the probability that the passenger was staying in the first class is:
P(1st class | survived) = 203/711 $\approx$ 0.2855.

4. Since P(1st class | survived) = 203/711 $\neq$ 325/2201 = P(1st class), we know that they are not independent.

5. Given that a passenger survived, the probability that the passenger was staying in the first class and the passenger was a child is:
P(1st class $\cap$ child | survived) = 6/711

6. Given that a passenger survived, the probability that the passenger was an adult is:
P(adult | survived) = 654/711

7. If they are independent, then
P(1st class $\cap$ adult | survived) = P(adult | survived) P(1st class | survived)
P(1st class $\cap$ adult | survived)= 197/711 $\approx$0.2771
P(adult | survived) P(1st class | survived)=654/711 $\times$ 203/711 $\approx$ 0.2626 $\neq$ 0.2771
So they are not independent.