**MSIS2510: Prescriptive Analytics**

# Dynamic Airline Revenue Optimization

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**June 2024**

## Abstract

This report showcases how mathematical optimization, using the Gurobi Python API, can enhance airline revenues through an optimal seat pricing strategy. We formulate the Yield Management Problem as a three-period stochastic programming model, addressing demand uncertainty and pricing adjustments. Our implementation, "Model Building in Mathematical Programming," demonstrates significant potential for maximizing airline profits.

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## Introduction

The airline industry relies heavily on ticket sales, which constitute 60% of its revenue. However, airlines face the unique challenge of perishable inventory - once a flight departs, any unsold seats represent lost revenue that cannot be recovered. To address this, airlines employ yield management strategies, which allow for dynamic price adjustments in real-time based on demand fluctuations. This approach aims to fill as many seats as possible while maximizing revenue.

## Problem Statement

The project aims to develop an advanced pricing optimization system for an airline to maximize expected revenue from ticket sales while meeting demand commitments for different seat classes. Specifically, the project focuses on the following objectives:

- Optimize Initial Price Levels and Seat Allocations
- Determine Provisional Number of Planes and Future Pricing
- Maximize Yield under Various Scenarios
- Utilize Historical and Actual Demand Data

### Flight and Pricing Data

For this project, the optimization model will focus on flights scheduled to depart in three weeks. The airline may require up to six planes, each costing £50,000 to hire. Plane configuration and price options to select from are as follows:

- 37 First Class seats
- 38 Business Class seats
- 47 Economy Class seats

| Class | Option 1 | Option 2 | Option 3 | |
|---|---|---|---|---|
| First | £ 1,200 | £ 1,000 | £ 950 | Period 1 |
| Business | £ 900 | £ 800 | £ 600 | |
| Economy | £ 500 | £ 300 | £ 200 | |
| First | £ 1,400 | £ 1,300 | £ 1,150 | Period 2 |
| Business | £ 1,100 | £ 900 | £ 750 | |
| Economy | £ 700 | £ 400 | £ 350 | |
| First | £ 1,500 | £ 900 | £ 850 | Period 3 |
| Business | £ 820 | £ 800 | £ 500 | |
| Economy | £ 480 | £ 470 | £ 450 | |

### Demand and Forecast Data

Demand is uncertain but will be affected by price. The probabilities of the scenarios in each period are:

Scenario 1: 10%  Scenario 2: 70%  Scenario 3: 20%

The forecasted demand levels in terms of seats are shown in the following tables:

| Period 1 | | | | |
|---|---|---|---|---|
| Class | Option 1 | Option 2 | Option 3 | |
| First | 10 | 15 | 20 | Scenario 1 |
| Business | 20 | 25 | 35 | |
| Economy | 45 | 55 | 60 | |
| First | 20 | 25 | 35 | Scenario 2 |
| Business | 40 | 42 | 45 | |
| Economy | 50 | 52 | 63 | |
| First | 45 | 50 | 60 | Scenario 3 |
| Business | 45 | 46 | 47 | |
| Economy | 55 | 56 | 64 | |

| Period 2 | | | | |
|---|---|---|---|---|
| Class | Option 1 | Option 2 | Option 3 | |
| First | 20 | 25 | 35 | Scenario 1 |
| Business | 42 | 45 | 46 | |
| Economy | 50 | 52 | 60 | |
| First | 10 | 40 | 50 | Scenario 2 |
| Business | 50 | 60 | 80 | |
| Economy | 60 | 65 | 90 | |
| First | 50 | 55 | 80 | Scenario 3 |
| Business | 20 | 30 | 50 | |
| Economy | 10 | 40 | 60 | |

| Period 3 | | | | |
|---|---|---|---|---|
| Class | Option 1 | Option 2 | Option 3 | |
| First | 30 | 35 | 40 | Scenario 1 |
| Business | 40 | 50 | 55 | |
| Economy | 50 | 60 | 80 | |
| First | 30 | 40 | 60 | Scenario 2 |
| Business | 10 | 40 | 45 | |
| Economy | 50 | 60 | 70 | |
| First | 50 | 70 | 80 | Scenario 3 |
| Business | 40 | 45 | 60 | |
| Economy | 60 | 65 | 70 | |

**Actual Demand Data**

We use actual demand from period 1 to adjust pricing strategy in period 2 and so on. It turned out that demand in each period was as shown in the table besides:

| Class | Option 1 | Option 2 | Option 3 | |
|---|---|---|---|---|
| First | 25 | 30 | 40 | Period 1 |
| Business | 50 | 40 | 45 | |
| Economy | 50 | 53 | 65 | |
| First | 22 | 45 | 50 | Period 2 |
| Business | 45 | 55 | 75 | |
| Economy | 50 | 60 | 80 | |
| First | 45 | 60 | 75 | Period 3 |
| Business | 20 | 40 | 50 | |
| Economy | 55 | 60 | 75 | |

## Model Formulation

### Sets and Indices

1. $i,j,k \in$ Scenarios: Indices and set of scenarios.
2. $h \in$ Options: Index and set of price options.
3. $c \in$ Class: Index and set of seats categories.

### Parameters

1. $\mathrm{Cap}_c \in N$: Capacity per plane for class $c$.
2. $\mathrm{Cost} \in R^+$: Cost to hire a plane.
3. $\mathrm{Prob}_i \in [0,1]$: Probability of scenario $i$.
4. $\mathrm{f1}_{i,c,h} \in R^+$: Forecast demand in period 1 for class $c$ under price option $h$ and scenario $i$.
5. $\mathrm{f2}_{i,j,c,h} \in R^+$: Forecast demand in period 2 for class $c$ under price option $h$ if scenario $i$ holds in period 1, and scenario $j$ in period 2.
6. $\mathrm{f3}_{i,j,k,c,h} \in R^+$: Forecast demand in period 3 for class $c$ under price option $h$ if scenario $i$ holds in period 1, scenario $j$ in period 2, and scenario $k$ in period 3.
7. $\mathrm{price1}_{c,h} \in R^+$: Price of option $h$ chosen for class $c$ in period 1.
8. $\mathrm{price2}_{i,c,h} \in R^+$: Price of option $h$ chosen for class $c$ in period 2 as a result of scenario $i$ in period 1.
9. $\mathrm{price3}_{i,j,c,h} \in R^+$: Price of option $h$ chosen for class $c$ in period 3 as a result of scenario $i$ in period 1, and scenario $j$ in period 2.

### Decision Variables

1. $p1_{c,h} \in \{0,1\}$: Binary variable is equal to one if price of option $h$ is chosen for class $c$ in period 1.
2. $p2_{i,c,h} \in \{0,1\}$: This binary variable is equal to one if price of option $h$ is chosen for class $c$ in period as a result of scenario $i$ in period 1.
3. $p3_{i,j,c,h} \in \{0,1\}$: This binary variable is equal to one if price of option $h$ is chosen for class $c$ in period 3 as a result of scenario $i$ in period 1 and scenario $j$ in period 2.
4. $s1_{i,c,h} \in R^+$: Number of tickets to be sold in period 1 for class $c$ under price option $h$ and scenario $i$.
5. $s2_{i,j,c,h} \in R^+$: Number of tickets to be sold in period 2 for class $c$ under price option $h$ if scenario $i$ holds in period 1, and scenario $j$ in period 2.
6. $s3_{i,j,k,c,h} \in R^+$: Number of tickets to be sold in period 3 for class $c$ under price option $h$ if scenario $i$ in period 1, scenario $j$ in period 2, and scenario $k$ in period 3.
7. $n \in N$: Number of planes to fly.

**Objective Function**

Maximize Expected Profit: $\sum$ (Revenue−Cost)

$$\text{Profit} = \Big(\sum_{i\in \text{Scenarios}} \sum_{c\in \text{Class}} \sum_{h\in \text{Options}} \text{prob}_i * \text{price1}_{c,h} * p1_{c,h} * s1_{i,c,h} +$$

$$\sum_{i\in \text{Scenarios}} \sum_{j\in \text{Scenarios}} \sum_{c\in \text{Class}} \sum_{h\in \text{Options}} \text{prob}_i * \text{prob}_j * \text{price2}_{c,h} * p2_{i,c,h} * s2_{i,j,c,h} +$$

$$\sum_{i\in \text{Scenarios}} \sum_{j\in \text{Scenarios}} \sum_{k\in \text{Scenarios}} \sum_{c\in \text{Class}} \sum_{h\in \text{Options}} \text{prob}_i * \text{prob}_j * \text{prob}_k * \text{price3}_{c,h} * p2_{i,j,c,h} * s3_{i,j,k,c,h}\Big) - \text{cost} * \text{n}$$

**Constraints**

**Price Option Constraint**: Only one price option must be chosen for each class in each period

1. $\sum_{h\in \text{Options}} p1_{c,h} = 1$ $\forall\, c \in \text{Class}$
2. $\sum_{h\in \text{Options}} p2_{i,c,h} = 1$ $\forall\, c \in \text{Class},\ i \in \text{Scenarios}$
3. $\sum_{h\in \text{Options}} p3_{i,j,c,h} = 1$ $\forall\, c \in \text{Class},\ i, j \in \text{Scenarios}$

**Sales Constraint**: Sales cannot exceed forecasted demand for any period.

4. $s1_{i,c,h} \le \text{f1}_{i,c,h} * p1_{c,h}$ $\forall\, i \in \text{Scenarios},\ c \in \text{Class},\ h \in \text{Options}$
5. $s2_{i,j,c,h} \le \text{f2}_{j,c,h} * p2_{i,c,h}$ $\forall\, i, j \in \text{Scenarios},\ c \in \text{Class},\ h \in \text{Options}$
6. $s3_{i,j,k,c,h} \le \text{f3}_{k,c,h} * p3_{i,j,c,h}$ $\forall\, i, j, k \in \text{Scenarios},\ c \in \text{Class},\ h \in \text{Options}$

**Capacity Constraint**: Total seats sold must not exceed plane capacity for each class.

7. $\sum_{h\in \text{Options}} s1_{i,c,h} + \sum_{h\in \text{Options}} s2_{i,j,c,h} + \sum_{h\in \text{Options}} s3_{i,j,k,c,h} \le \text{cap}_c * n$ $\forall\, i, j, k \in \text{Scenarios},\ c \in \text{Class}$

**Plane Limit Constraint**: Up to six planes can be hired.

8. $n \le 6$

__Gurobi Optimization__

Step 1: Week-1 Price Optimization (Start of Week 1): Leverage demand forecasts to maximize profit while ensuring demand and capacity constraints are met for each week.

Step 2: Week-2 Price Optimization (Start of Week 2): Replace forecasted demand for Week 1 with actual demand and fix pricing options for Week 1 based on the optimal prices obtained from Step 1

Step 3: Week-3 Price Optimization (Start of Week 3): Replace forecasted demand for Week 1 and Week 2 with actual demand and fix pricing options for Week 1 and Week 2 based on the optimal prices obtained from previous steps.

Step 4: Final Model (Before Flight Take-off): Utilize actual demand data and optimal prices obtained from the three-step pricing optimization process to maximize profit.

__Results__

- Week 1: The expected total profit is £166,189 and the no. of planes booked is 3.
- Week 2: The expected total profit at the beginning of week 2 is £170,792.
- Week 3: The expected total profit in week 3 is £173,680.

❖ The price options selected for each week is as follows:

| Week 1 prices: | | |
|---|---|---|
| Class | Option | Price |
| First | option 1 | £1,200 |
| Business | option 2 | £ 900 |
| Economy | option 3 | £ 500 |

| Week 2 prices: | | |
|---|---|---|
| Class | Option | Price |
| First | option 1 | £1,150 |
| Business | option 2 | £1,100 |
| Economy | option 3 | £ 700 |

| Week 3 prices: | | |
|---|---|---|
| Class | Option | Price |
| First | option 1 | £1,500 |
| Business | option 2 | £ 800 |
| Economy | option 3 | £ 480 |

❖ **Final Results:** The actual total profit is £184,030 and number of planes used: 3.0

| Week 1 | | | |
|---|---|---|---|
| Class | Seats Sold | Price | Revenue |
| First | 25 | £ 1,200 | £30,000 |
| Business | 50 | £ 900 | £45,000 |
| Economy | 50 | £ 500 | £25,000 |

| Week 2 | | | |
|---|---|---|---|
| Class | Seats Sold | Price | Revenue |
| First | 41 | £ 1,150 | £47,150 |
| Business | 45 | £ 1,100 | £49,500 |
| Economy | 50 | £ 700 | £35,000 |

| Week 3 | | | |
|---|---|---|---|
| Class | Seats Sold | Price | Revenue |
| First | 45 | £ 1,500 | £ 67,500 |
| Business | 19 | £ 800 | £ 15,200 |
| Economy | 41 | £ 480 | £ 19,680 |

## Impact

Implementing advanced demand forecasting enables airlines to offer competitive prices, optimize seat occupancy, and maximize revenue. This approach streamlines operations, reducing costs and enhancing efficiency. By adjusting pricing based on seasonal trends, airlines can further boost revenue potential and improve customer satisfaction, fostering loyalty and long-term success.

## Real World examples

- ❖ Southwest : Implemented dynamic pricing, resulting in a 15% increase in revenue and a 10% reduction in operational costs within one year
- ❖ Delta: Utilizes advanced yield management systems, leading to higher customer satisfaction and loyalty, thereby sustaining a competitive advantage

## Other Use Cases

- ❖ Hotel Revenue Management
- ❖ Healthcare Resource Management
- ❖ E-Commerce Dynamic Pricing
- ❖ Concert Ticket Pricing Optimization
- ❖ Car Rental Pricing Optimization

## Conclusion

In conclusion, Dynamic Revenue Optimization stands as a vital strategy across industries, offering multifaceted benefits. From driving revenue growth to enhancing operational efficiency and improving customer satisfaction, its impact is profound and far-reaching. By continuously adapting pricing strategies to meet evolving market demands, businesses can not only maximize their revenue potential but also ensure long-term competitiveness in today's dynamic marketplace. Embracing Dynamic Revenue Optimization as a fundamental business practice is crucial for organizations seeking sustainable growth and success in the modern business landscape.