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OVERVIEW

- Airlines industry makes 60% of the revenue from selling tickets.
- They have fixed number of seats and once flight departs, the opportunity of selling seats is lost.
- Yield management allows airlines to adjust prices in real-time to match demand, ensuring they fill as many seats as possible while maximizing revenue.
- Demand for airline seats fluctuate based on booking time, customer segment, competitive pricing

Core idea - to sell the right product to the right customer at the right time for the right price.



PROBLEM STATEMENT

The project is to develop an advanced pricing optimization system for an airline that maximizes expected revenue from ticket sales while meeting demand commitments for different seat classes. Specifically, the project aims to:

- Optimize initial price levels and seat allocations for current period
- Determine provisional number of planes and future pricing
- Maximize yield while meeting demand commitments under various scenarios
- Utilize historical demand forecasts and actual demand data to iteratively adjust pricing strategies for subsequent periods



FLIGHT AND PRICING DETAILS

The flights will depart in three weeks and require up to six planes, each costing £50,000 to hire. Each plane has the following:

- > 37 First Class
- > 38 Business Class
- > 47 Economy Class

The airline needs to decide on an initial price for each of these seats, and will then have the opportunity to update the price after one week and two weeks. Pricing option available are:

	Option 1	Option 2	Option 3	-
First	£1200	£1000	£950	Period 1
Business	£900	£800	£600	
Economy	£500	£300	£200	
First	£1400	£1300	£1150	Period 2
Business	£1100	£900	£750	
Economy	£700	£400	£350	
First	£1500	£900	£850	Period 3
Business	£820	£800	£500	
Economy	£480	£470	£450	





DEMAND AND FORECAST

- Demand is uncertain and influenced by price.
- The probabilities of the three scenarios in each period are as follows:

Scenario 1	0.1
Scenario 2	0.7
Scenario 3	0.2

• Forecasted demand levels for each class and each scenario per period:

Scenario 1

Class	Price Option 1	Price Option 2	Price Option 3
First	10	15	20
Business	20	25	35
Economy	45	55	60

Period 1:

Scenario 2

Class	Price Option 1	Price Option 2	Price Option 3
First	20	25	35
Business	40	42	45
Economy	50	52	63

Scenario 3

Class	Price Option 1	Price Option 2	Price Option 3	
First	45	50	60	
Business	45	46	47	
Economy	55	56	64	



DEMAND AND FORECAST

Scenario 1

Class	Price Option 1	Price Option 2	Price Option 3
First	20	25	35
Business	42	45	46
Economy	50	52	60

Scenario 1

Class	Price Option 1	Price Option 2	Price Option 3
First	30	35	40
Business	40	50	55
Economy	50	60	80

Period 2:

Scenario 2

Class	Price Option 1	Price Option 2	Price Option 3	
First	10	40	50	
Business	50	60	80	
Economy	60	65	90	

Period 3:

Scenario 2

Class	Price Option 1	Price Option 2	Price Option 3	
First	30	40	60	
Business	10	45	40	
Economy	50	60	70	

Scenario 3

Class	Price Option 1	Price Option 2	Price Option 3
First	50	55	80
Business	20	30	50
Economy	10	40	60

Scenario 3

Class	Price Option 1	Price Option 2	Price Option 3
First	50	70	80
Business	40	45	40
Economy	60	65	70

We use actual demand from period 1 to adjust pricing strategy in period 2 and so on



ACTUAL DEMAND

The actual demand for each period:

	Price option 1	Price option 2	Price option 3	
First	25	30	40	Period 1
Business	50	40	45	
Economy	50	53	65	
First	22	45	50	Period 2
Business	45	55	75	
Economy	50	60	80	
First	45	60	75	Period 3
Business	20	40	50	
Economy	55	60	75	

Sets and indices

- $i,j,k \in Scenarios$
- $h \in Options$
- $c \in \mathsf{Class}$

Parameters

- Cap $\in N$
- Cost $\in R^+$
- $\operatorname{Prob}_{i} \in [0,1]$

Indices and set of scenarios.

Index and set of price options.

Index and set of seats categories.

Capacity per plane for class c.

Cost to hire a plane.

Probability of scenario i.





Parameters

•
$$\mathsf{f1}_{i,c,h} \subseteq R^+$$

• $f2_{i,j,c,\hbar} \in R^+$

•
$$f3_{i,j,k,c,\hbar} \in R^+$$

- price1 $\in R^+$
- price2_{i,c,h} $\in R^+$
- price3 $_{i,j,c,\hbar} \in R^+$

Forecast demand in period 1 for class c under price option h and scenario i.

Forecast demand in period 2 for class c under price option h if scenario i holds in period 1, and scenario j in period 2.

Forecast demand in period 3 for class c under price option h if scenario i holds in period 1, scenario j in period 2, and scenario k in period 3.

Price of option h chosen for class c in period 1.

Price of option h chosen for class c in period 2 as a result of scenario i in period 1.

Price of option h chosen for class c in period 3 as a result of scenario i in period 1, and scenario j in period 2.



Decision Variables

- $p1_{c,h} \subseteq \{0,1\}$ period 1.
- $p2_{i,c,h} \in \{0,1\}$ period 2
- $p3_{i,j,c,\hbar} \in \{0,1\}$
- $\bullet \quad s\mathbf{1}_{i,c,h} \subseteq R^+$
- $s2_{i,j,c,h} \in R^+$
- $s3_{i,j,k,c,h} \in R^+$
- \bullet $n \in N$

This binary variable is equal to one if price of option \hbar is chosen for class c in

This binary variable is equal to one if price of option h is chosen for class c in as a result of scenario i in period 1.

This binary variable is equal to one if price of option h is chosen for class c in period 3 as a result of scenario i in period 1 and scenario j in period 2.

Number of tickets to be sold in period 1 for class c under price option h and scenario i.

Number of tickets to be sold in period 2 for class c under price option \hbar if scenario i holds in period 1, and scenario j in period 2.

Number of tickets to be sold in period 3 for class c under price option h if scenario j in period 1, scenario j in period 2, and scenario k in period 3.

Number of planes to fly.

ZZ IHS E

MODEL FORMULATION

Constraints for period 1, 2 and 3

• Price Option Constraint: Only one price option must be chosen for each class in each period

$$\begin{array}{ll} \sum\limits_{\hbar \in \text{Options}} p\mathbf{1}_{c,\hbar} = 1 & \forall \ c \in \text{Class} \,, \\ \\ \sum\limits_{\hbar \in \text{Options}} p\mathbf{2}_{i,c,\hbar} = 1 & \forall \ c \in \text{Class}, \ i \in \text{Scenarios} \,, \\ \\ \\ \sum\limits_{\hbar \in \text{Options}} p\mathbf{3}_{i,j,c,\hbar} = 1 & \forall \ c \in \text{Class}, \ i,j \in \text{Scenarios} \,. \end{array}$$

• Sales Constraint: Sales cannot exceed forecasted demand for any period.

$$s\mathbf{1}_{i,c,\hbar} \leq \mathbf{f}\mathbf{1}_{i,c,\hbar} * p\mathbf{1}_{c,\hbar} \qquad \forall i \in \text{Scenarios}, c \in \text{Class}, \hbar \in \text{Options}$$

$$s\mathbf{2}_{i,j,c,\hbar} \leq \mathbf{f}\mathbf{2}_{j,c,\hbar} * p\mathbf{2}_{i,c,\hbar} \qquad \forall i,j \in \text{Scenarios}, c \in \text{Class}, \hbar \in \text{Options}$$

$$s\mathbf{3}_{i,j,k,c,\hbar} \leq \mathbf{f}\mathbf{3}_{k,c,\hbar} * p\mathbf{3}_{i,j,c,\hbar} \qquad \forall i,j,k \in \text{Scenarios}, c \in \text{Class}, \hbar \in \text{Options}$$



Constraints

• Capacity Constraint: Total seats sold must not exceed plane capacity for each class.

$$\sum_{h \in \text{Options}} s1_{i,c,h} + \sum_{h \in \text{Options}} s2_{i,j,c,h} + \sum_{h \in \text{Options}} s3_{i,j,k,c,h} \le \text{cap}_{c}^{*} n \quad \forall i,j,k \in \text{Scenarios}, c \in \text{Class}$$

• Plane Limit Constraint: Up to six planes can be hired.



Objective Function

Maximize Expected Profit: Σ (Revenue-Cost)

Where:

Revenue - Revenue from ticket sales in period 1, 2 and 3. Cost - Cost of hiring planes.





GUROBI OPTIMIZATION

Step 1: Week-1 Price Optimization (Start of Week 1)

- > Leverage demand forecasts to maximize profit while ensuring demand and capacity constraints are met for each week.
- > Utilize the Gurobi optimization engine to find the optimal pricing solution at the beginning of Week 1.

Step 2: Week-2 Price Optimization (Start of Week 2)

- > Replace forecasted demand for Week 1 with actual demand and fix pricing options for Week 1 based on the optimal prices obtained from Step 1.
- > Invoke the Gurobi optimization engine again to maximize profit and find the optimal pricing solution at the beginning of Week 2.

Step 3: Week-3 Price Optimization (Start of Week 3)

- > Replace forecasted demand for Week 1 and Week 2 with actual demand and fix pricing options for Week 1 and Week 2 based on the optimal prices obtained from previous steps.
- > Re-run the Gurobi optimization engine to maximize profit and find the optimal pricing solution at the beginning of Week 3.

Step 4: Final Model (Before Flight Take-off)

- Utilize actual demand data and optimal prices obtained from the three-step pricing optimization process to maximize profit.
- Find actual profit generated.



- The expected total profit is: £ 166,189
- Number of planes to book: 3.0
- Week 1 Prices:

Class	Option	Price
First	Option 1	£1,200
Business	Option 1	£900
Economy	Option 1	£500



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Provisional Prices:

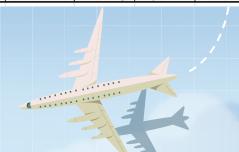
Week 2:

Scenario	Class	Option	Price
scenario 1	First	Option 3	£1,150
scenario 1	Business	Option 1	£1,100
scenario 1	Economy	Option 1	£700
scenario 2	First	Option 3	£1,150
scenario 2	Business	Option 1	£1,100
scenario 2	Economy	Option 1	£700
scenario 3	First	Option 2	£1,300
scenario 3	Business	Option 1	£1,100
scenario 3	Economy	Option 1	£700

Week 3:

Scenario 1	Scenario 2	Class	Option	Price
Scenario 1	Scenario 1	First	Option 1	£1,500
Scenario 1	Scenario 1	Business	Option 2	£800
Scenario 1	Scenario 1	Economy	Option 1	£480
Scenario 1	Scenario 2	First	Option 1	£1,500
Scenario 1	Scenario 2	Business	Option 2	008£
Scenario 1	Scenario 2	Economy	Option 1	£480
Scenario 1	Scenario 3	First	Option 1	£1,500
Scenario 1	Scenario 3	Business	Option 2	008£
Scenario 1	Scenario 3	Economy	Option 3	£450
Scenario 2	Scenario 1	First	Option 1	£1,500
Scenario 2	Scenario 1	Business	Option 2	£800
Scenario 2	Scenario 1	Economy	Option 1	£480
Scenario 2	Scenario 2	First	Option 1	£1,500
Scenario 2	Scenario 2	Business	Option 2	008£

Scenario 1	Scenario 2	Class	Option	Price
Scenario 2	Scenario 2	Economy	Option 1	£480
Scenario 2	Scenario 3	First	Option 1	£1,500
Scenario 2	Scenario 3	Business	Option 2	008£
Scenario 2	Scenario 3	Economy	Option 3	£450
Scenario 3	Scenario 1	First	Option 1	£1,500
Scenario 3	Scenario 1	Business	Option 2	008£
Scenario 3	Scenario 1	Economy	Option 1	£480
Scenario 3	Scenario 2	First	Option 1	£1,500
Scenario 3	Scenario 2	Business	Option 2	008£
Scenario 3	Scenario 2	Economy	Option 1	£480
Scenario 3	Scenario 3	First	Option 1	£1,500
Scenario 3	Scenario 3	Business	Option 2	008£
Scenario 3	Scenario 3	Economy	Option 3	£450





- The expected total profit at the beginning of week 2 is: £ 170,792
- Week 2 Prices:

Class	Option	Price
First	Option 3	£1,150
Business	Option 1	£1,100
Economy	Option 1	£700

```
# Fix price options of week 1
for c,h in ch:
    p1ch[c,h].lb = opt_p1ch[c,h]
```

```
# use hindsight demand of week 1
for i,c,h in ich:
    fcst1[i,c,h] = 0
    fcst1[i,c,h] = demand1[c,h]*opt_p1ch[c,h]
```





Provisional Prices for Week 3

				Vic
Scenario 1	Scenario 2	Class	Option	Price
Scenario 1	Scenario 1	First	Option 1	£1,500
Scenario 1	Scenario 1	Business	Option 2	£800
Scenario 1	Scenario 1	Economy	Option 1	£480
Scenario 1	Scenario 2	First	Option 1	£1,500
Scenario 1	Scenario 2	Business	Option 2	£800
Scenario 1	Scenario 2	Economy	Option 1	£480
Scenario 1	Scenario 3	First	Option 1	£1,500
Scenario 1	Scenario 3	Business	Option 2	£80 <mark>0</mark>
Scenario 1	Scenario 3	Economy	Option 3	£450

Scenario 1	Scenario 2	Class	Option	Price
Scenario 2	Scenario 1	First	Option 1	£1,500
Scenario 2	Scenario 1	Business	Option 2	£800
Scenario 2	Scenario 1	Economy	Option 1	£480
Scenario 2	Scenario 2	First	Option 1	£1,500
Scenario 2	Scenario 2	Business	Option 2	£800
Scenario 2	Scenario 2	Economy	Option 1	£480
Scenario 2	Scenario 3	First	Option 1	£1,500
Scenario 2	Scenario 3	Business	Option 2	£800
Scenario 2	Scenario 3	Economy	Option 3	£450

Scenario 1	Scenario 2	Class	Option	Price
Scenario 3	Scenario 1	First	Option 1	£1,500
Scenario 3	Scenario 1	Business	Option 2	£800
Scenario 3	Scenario 1	Economy	Option 1	£480
Scenario 3	Scenario 2	First	Option 1	£1,500
Scenario 3	Scenario 2	Business	Option 2	£800
Scenario 3	Scenario 2	Economy	Option 1	£480
Scenario 3	Scenario 3	First	Option 1	£1,500
Scenario 3	Scenario 3	Business	Option 2	£800
Scenario 3	Scenario 3	Economy	Option 3	£450



- The expected total profit in week 3 is: £ 173,680
- Week 3 prices:

Class	Option	Price
First	Option 1	£1,500
Business	Option 2	£800
Economy	Option 1	£480

```
# Fix price options of week 2
for i,c,h in ich:
    p2ich[i,c,h].lb = opt_p2ich[i,c,h]
```

```
# use hindsight demand of week 2
for j,c,h in jch:
   fcst2[j,c,h] = 0
   fcst2[j,c,h] = demand2[c,h]*opt_p2ich[j,c,h]
```





FINAL RESULTS - BEFORE TAKE OFF

- We ran the model using the actual demand of weeks 1, 2, and 3
- The actual total profit is: £184,030
- Number of planes used: 3.0
- Seats Sold and Revenue

Week 1

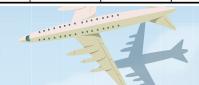
Class	Seats Sold	Price per Seat	Revenue
First Class	25	£1,200	£30,000
Business	50	£900	£45,000
Economy	50	£500	£25,000

Week 2

Class	Seats Sold	Price per Seat	Revenue
First Class	41	£1,150	£47,150
Business	45	£1,100	£49,500
Economy	50	£700	£35,000

Week 3

Class	Seats Sold	Price per Seat	Revenue
First Class	45	£1,500	£67,500
Business	19	£800	£15,200
Economy	41	£480	£19,680





IMPACT

- Demand Forecasting ensures that they offer competitive prices while maximizing revenue by filling more seats at optimal prices
- Capacity Management reduces per-seat operational costs
- Operational Efficiency achieve faster response times, better resource utilization, and lower administrative costs
- Customer Satisfaction leads to higher satisfaction and loyalty
- Seasonal Adjustments Optimizing pricing and seat allocation based on seasonal travel trends to maximize revenue



REAL WORLD EXAMPLE

Southwest'

Implemented dynamic pricing, resulting in a 15% increase in revenue and a 10% reduction in operational costs within one year



Utilizes advanced yield management systems, leading to higher customer satisfaction and loyalty, thereby sustaining a competitive advantage





OTHER USE CASES



Hotel Revenue Management



Healthcare Resource
Management



E-commerce Dynamic Pricing



Concert Ticket
Pricing Optimization



Car Rental Pricing
Optimization





In conclusion, Dynamic Revenue Optimization is essential strategy for any industry. It drives revenue growth, enhance operational efficiency, improve customer satisfaction, and ensure long-term competitiveness

