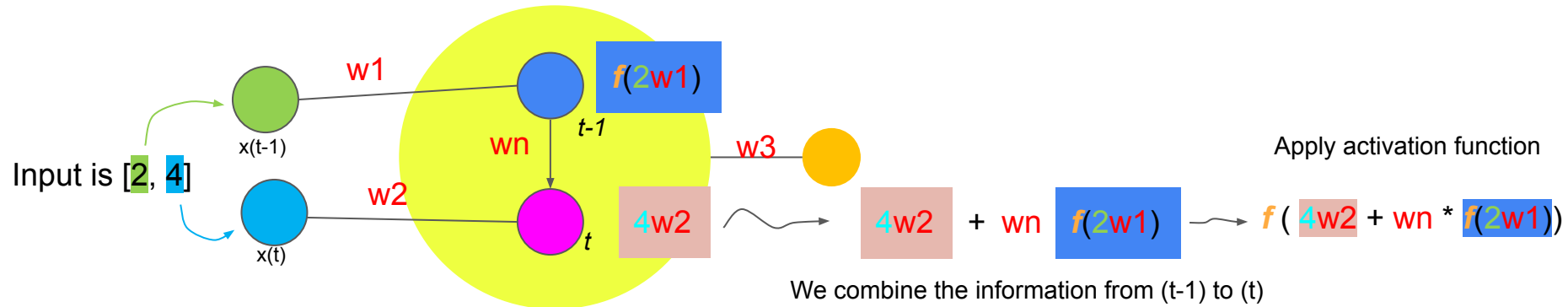


LSTM (Long short-term memory)

Difference between RNN and LSTM

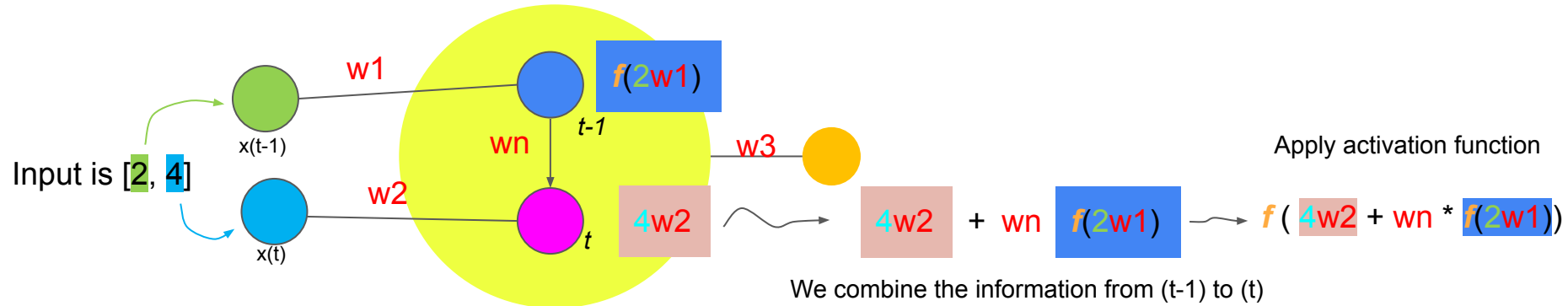
The difference between Simple RNN and LSTM

For RNN, within one neuron, all the intermediate neuron values are updated time step by time step (e.g., from $(t-1)$ to (t)):

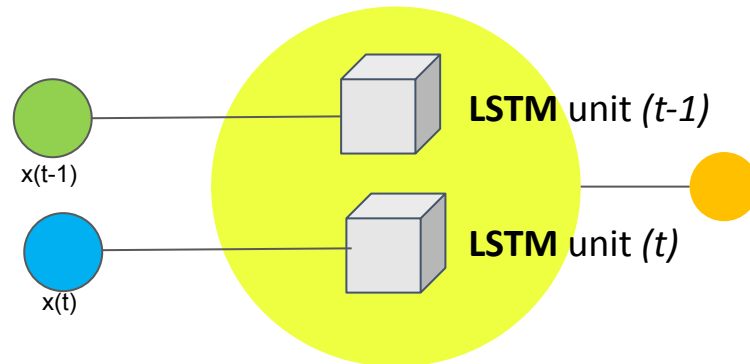


The difference between Simple RNN and LSTM

For RNN, within one neuron, all the intermediate neuron values are updated time step by time step (e.g., from $(t-1)$ to (t)):



For LSTM, instead of an intermediate neuron, we have LSTM unit to go from one time step to the next, and we therefore avoid the use of w_n , which is causing the gradient vanishing/explosion issue in RNN



How LSTM works: concept

Long term
memory (t-1):
 $z_l(t-1)$

Each LSTM unit must have three inputs:

- input (e.g., output from last time step)
- short term memory (updated from last step)
- long term memory (updated from last step)

Short term memory (t-1) $\sim z_s(t-1)$

Input $\sim x_1(t)$

Long term
memory (t-1):
 $z_l(t-1)$

Each LSTM unit must have three inputs:

- input (e.g., output from last time step)
- short term memory (updated from last step)
- long term memory (updated from last step)

$$W_{s1} * z_s(t-1) + \\ W_{i1} * x_1(t)$$

Combine short term
memory and input together

Short term memory (t-1) $\sim z_s(t-1)$

Input $\sim x_1(t)$

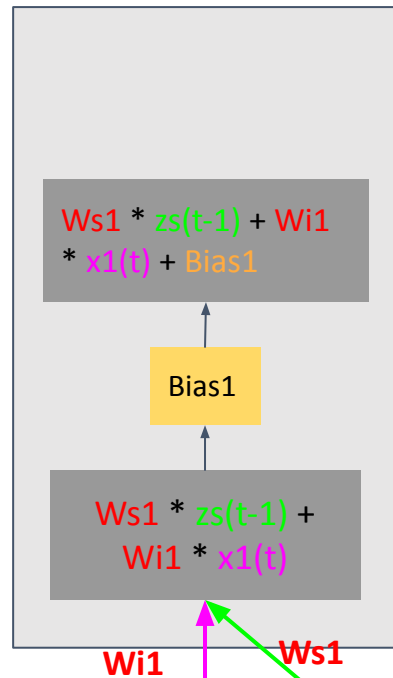
W_{i1}

W_{s1}

Long term
memory (t-1):
 $z_l(t-1)$

Each LSTM unit must have three inputs:

- input (e.g., output from last time step)
- short term memory (updated from last step)
- long term memory (updated from last step)



Add bias to the
combined value

Short term memory (t-1) ~ $z_s(t-1)$

Input ~ $x_1(t)$

Long term
memory (t-1):
 $z_l(t-1)$

Each LSTM unit must have three inputs:

- input (e.g., output from last time step)
- short term memory (updated from last step)
- long term memory (updated from last step)

After **Sigmoid**, a
value between 0%
(0.0) and 100%
(1.0) will be
produced

A sigmoid
function f

Apply the Sigmoid
function, the output
of the Sigmoid
function $f(\dots)$ will be a
percentage between
0.0% and 100.0%

$$W_{s1} * z_s(t-1) + W_{i1} * x_1(t) + Bias_1$$

Bias1

$$W_{s1} * z_s(t-1) + W_{i1} * x_1(t)$$

W_{i1}

W_{s1}

Short term memory (t-1) $\sim z_s(t-1)$

Input $\sim x_1(t)$

Long term
memory (t-1):
 $z_l(t-1)$

Each LSTM unit must have three inputs:

- input (e.g., output from last time step)
- short term memory (updated from last step)
- long term memory (updated from last step)

Long term memory (from t-1) to
be remembered:
 $z_l(t-1) = z_l(t-1) * f(Ws1 * z_s(t-1) +$
 $Wi1 * x1(t) + Bias1)$

This represents the long
term memory (from last
time step) to be
remembered

After **Sigmoid**, a
value between 0%
(0.0) and 100%
(1.0) will be
produced

A sigmoid
function f

$Ws1 * z_s(t-1) + Wi1$
 $* x1(t) + Bias1$

Bias1

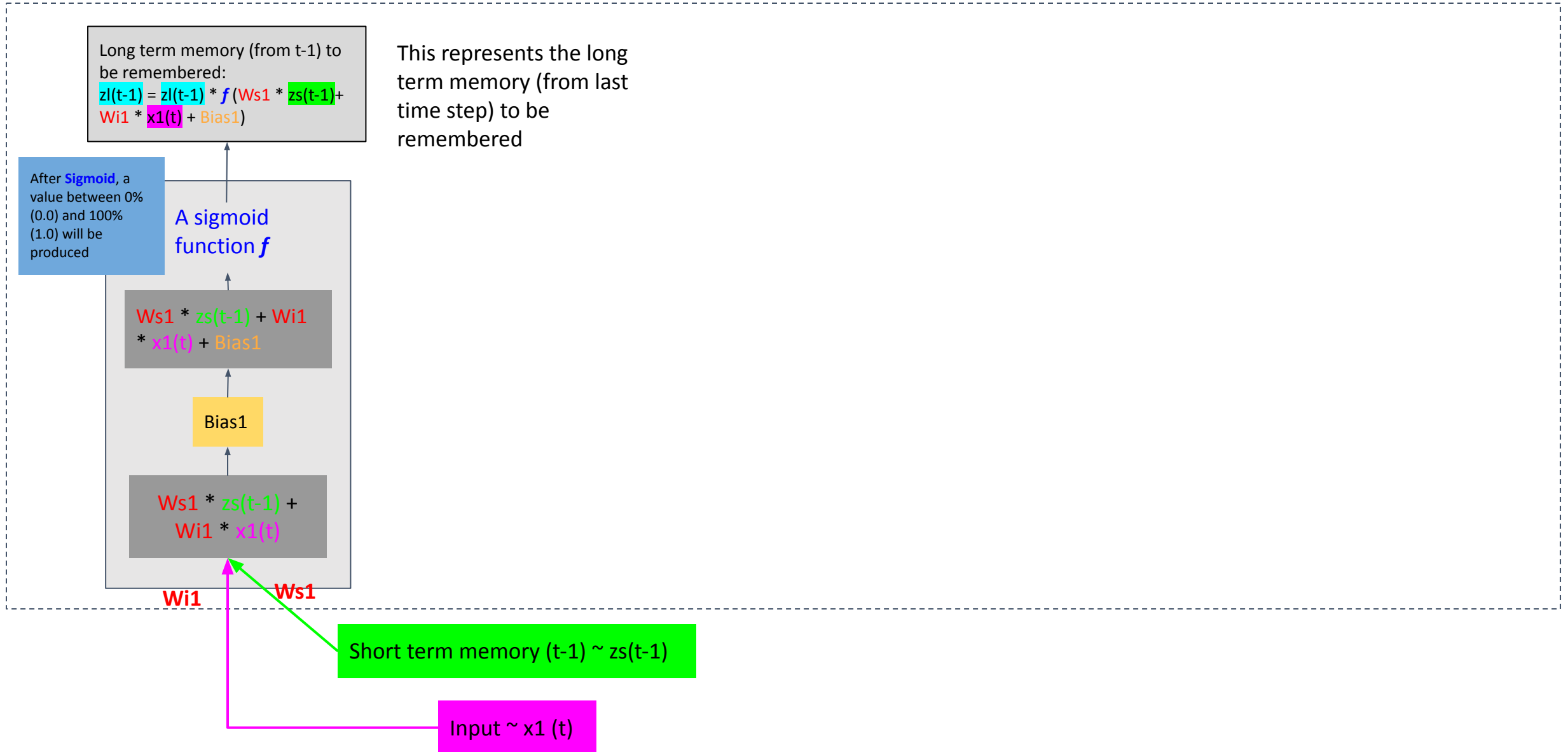
$Ws1 * z_s(t-1) +$
 $Wi1 * x1(t)$

$Wi1$

$Ws1$

Short term memory (t-1) $\sim z_s(t-1)$

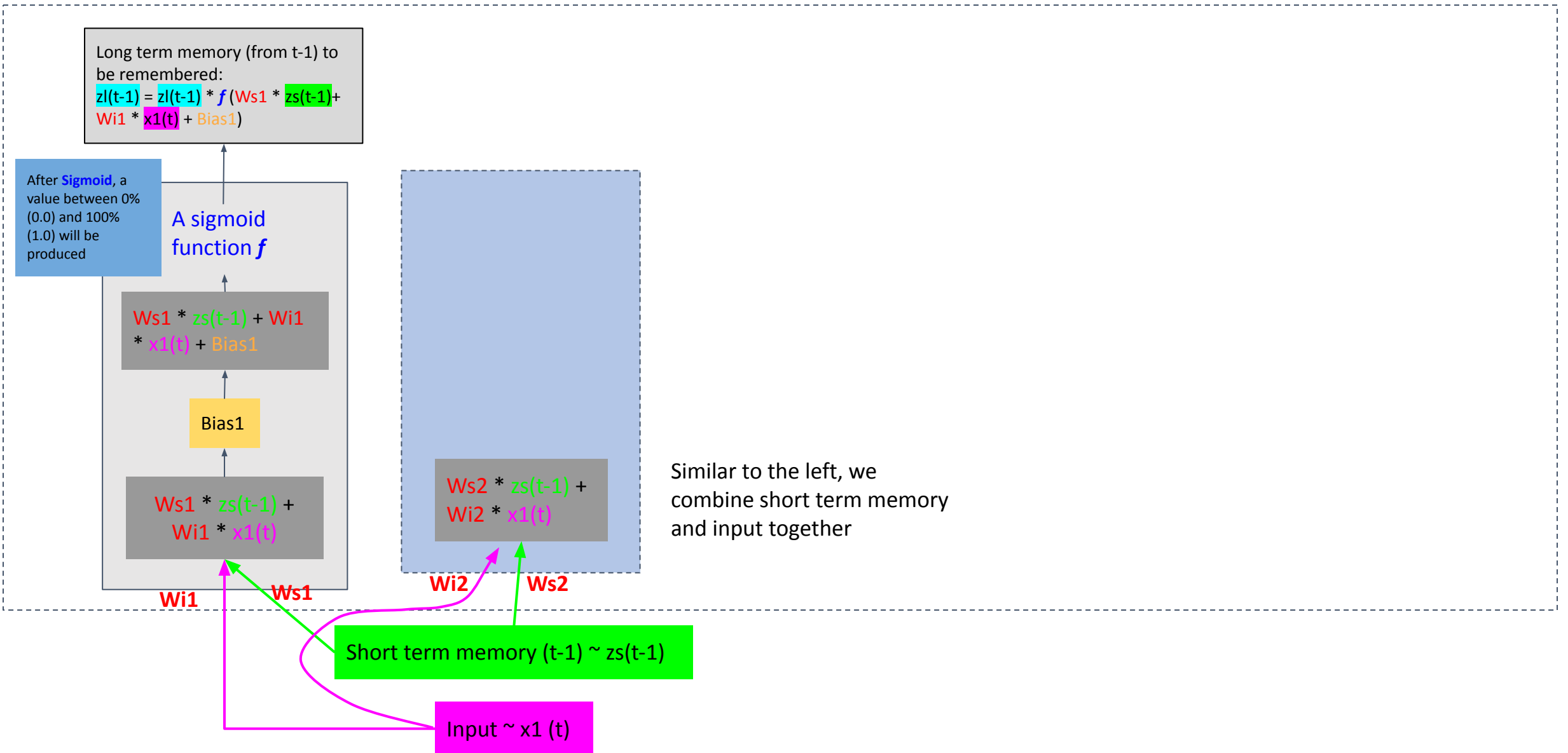
Input $\sim x1(t)$



Long term
memory (t-1):
 $z_l(t-1)$

Each LSTM unit must have three inputs:

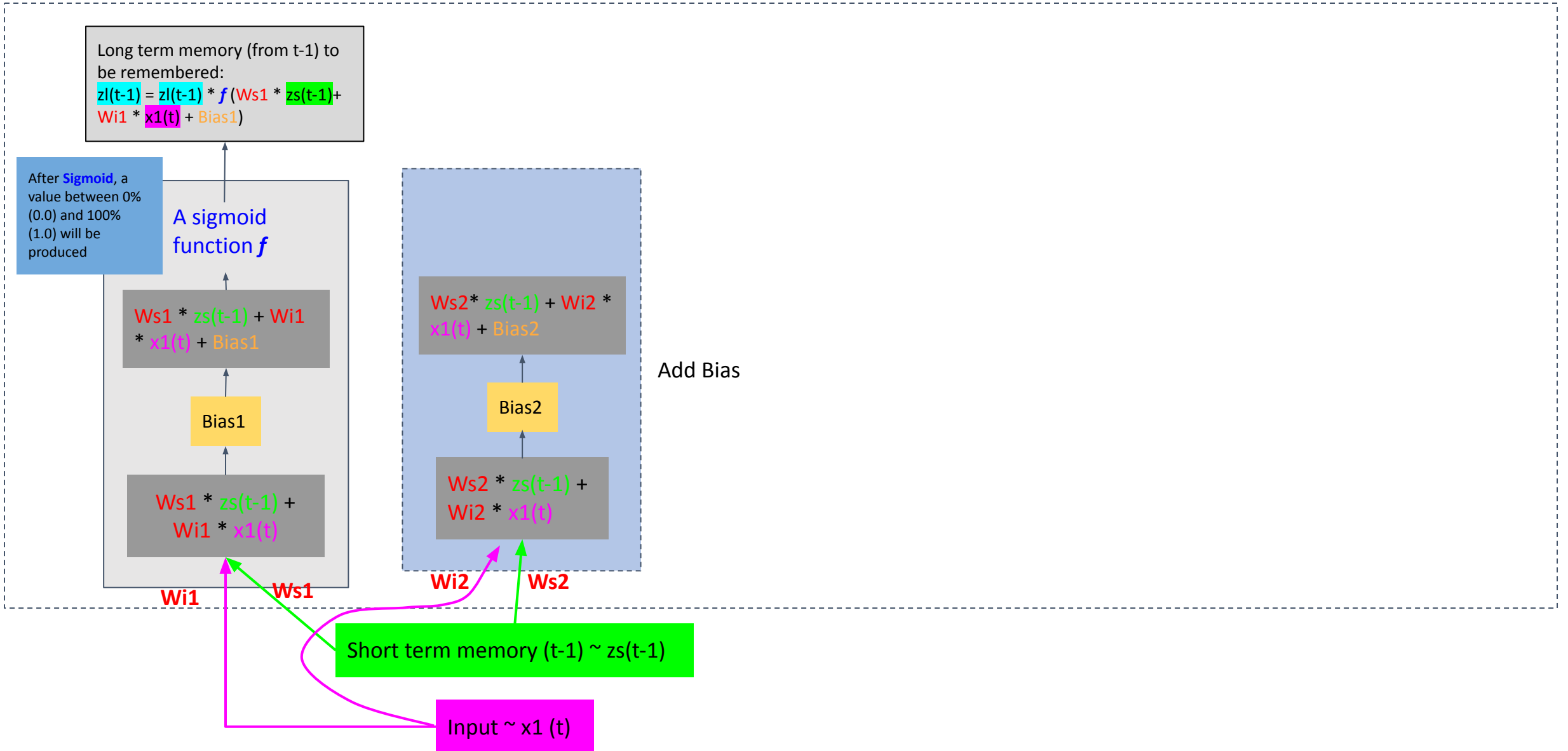
- input (e.g., output from last time step)
- short term memory (updated from last step)
- long term memory (updated from last step)



Long term
memory (t-1):
 $z_l(t-1)$

Each LSTM unit must have three inputs:

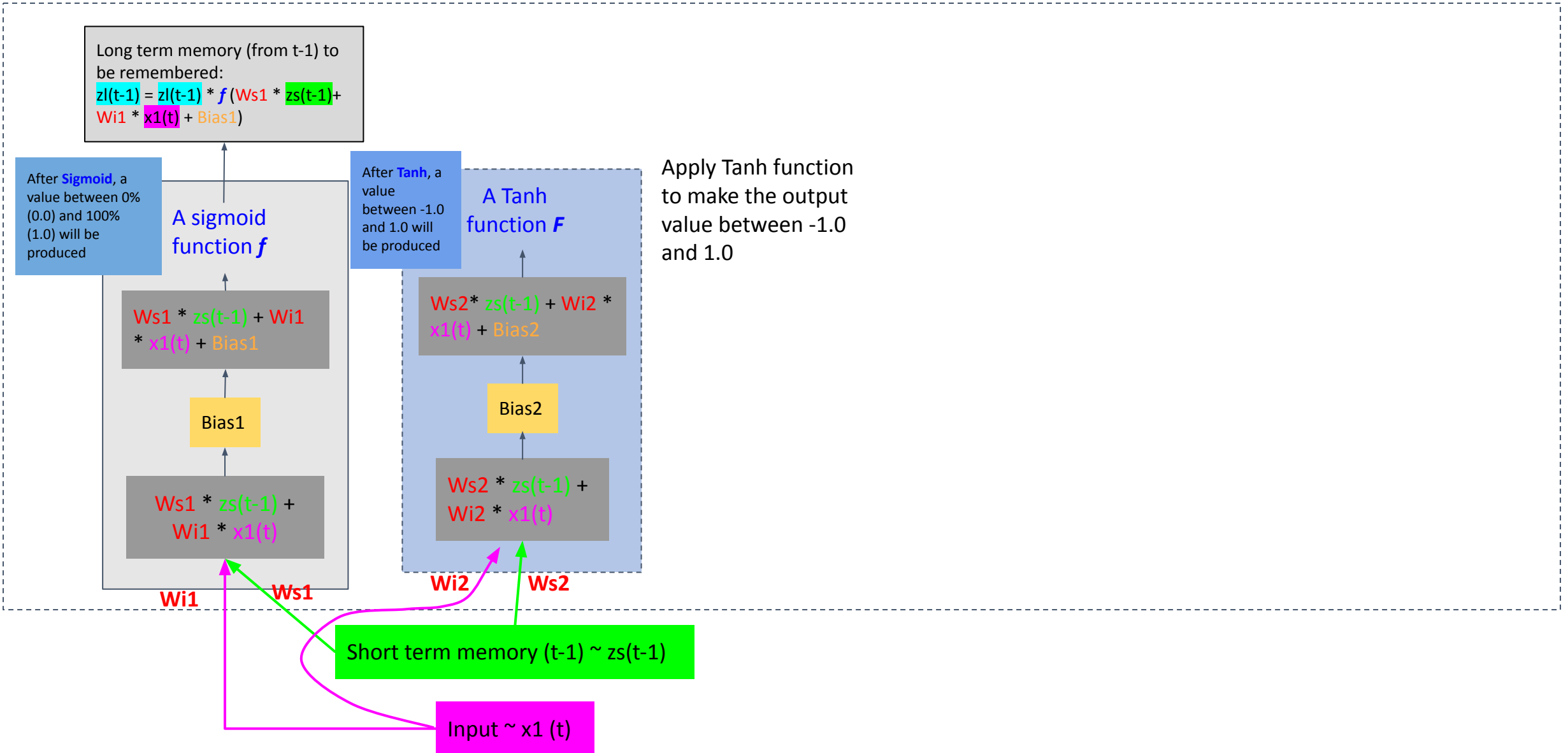
- input (e.g., output from last time step)
- short term memory (updated from last step)
- long term memory (updated from last step)



Long term
memory (t-1):
 $z_l(t-1)$

Each LSTM unit must have three inputs:

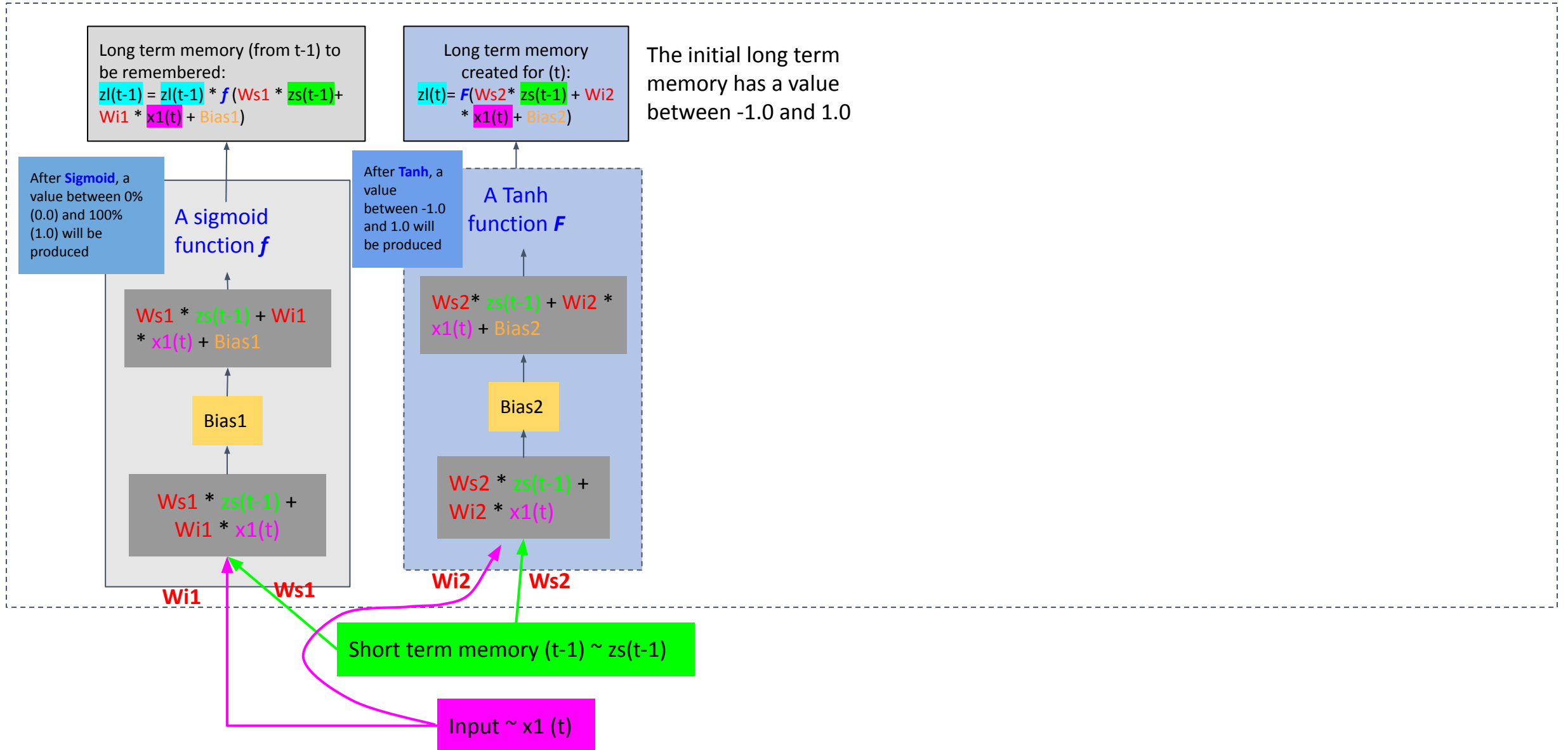
- input (e.g., output from last time step)
- short term memory (updated from last step)
- long term memory (updated from last step)



Long term
memory (t-1):
 $z_l(t-1)$

Each LSTM unit must have three inputs:

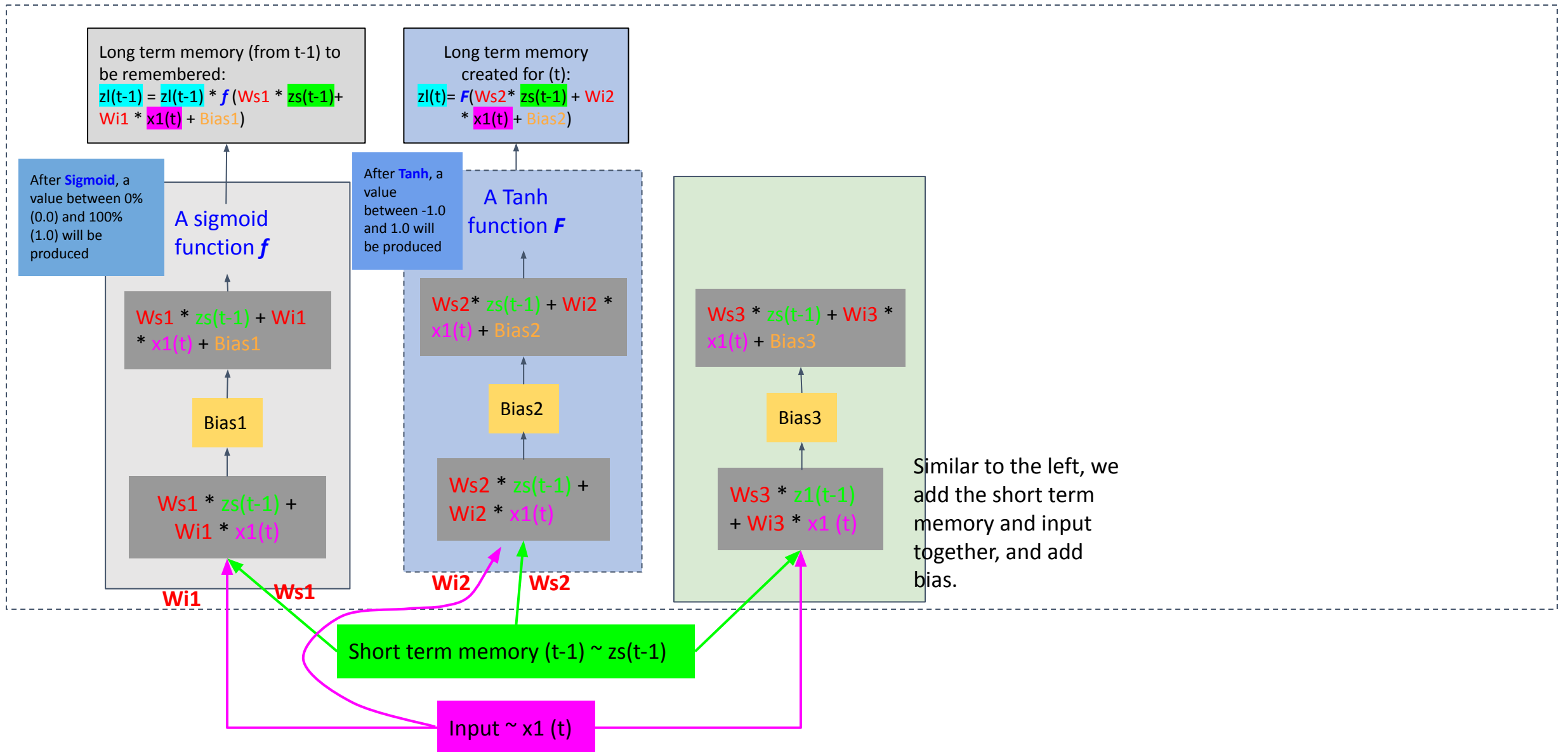
- input (e.g., output from last time step)
- short term memory (updated from last step)
- long term memory (updated from last step)



Long term
memory (t-1):
 $z_l(t-1)$

Each LSTM unit must have three inputs:

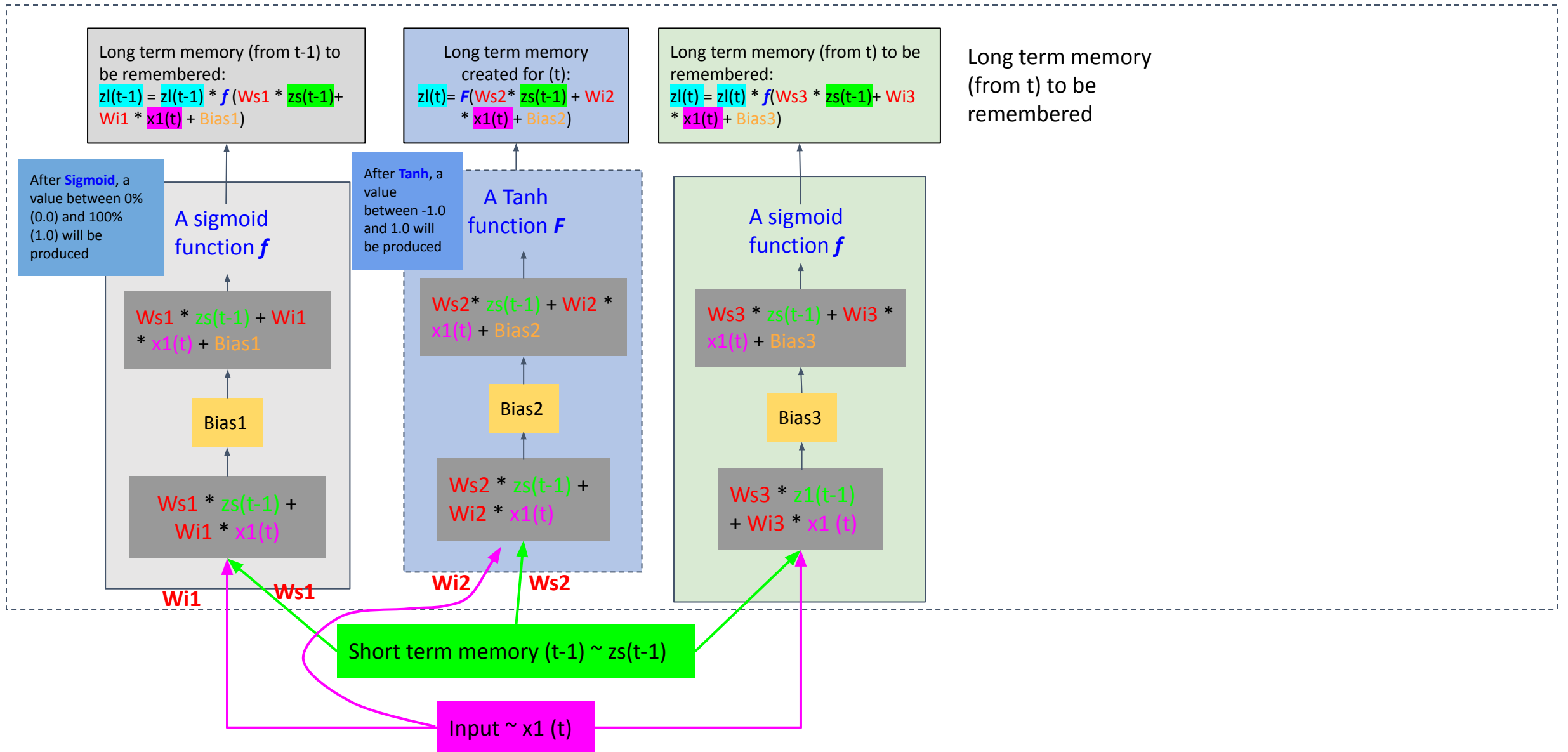
- input (e.g., output from last time step)
- short term memory (updated from last step)
- long term memory (updated from last step)



Long term
memory (t-1):
 $z_l(t-1)$

Each LSTM unit must have three inputs:

- input (e.g., output from last time step)
- short term memory (updated from last step)
- long term memory (updated from last step)

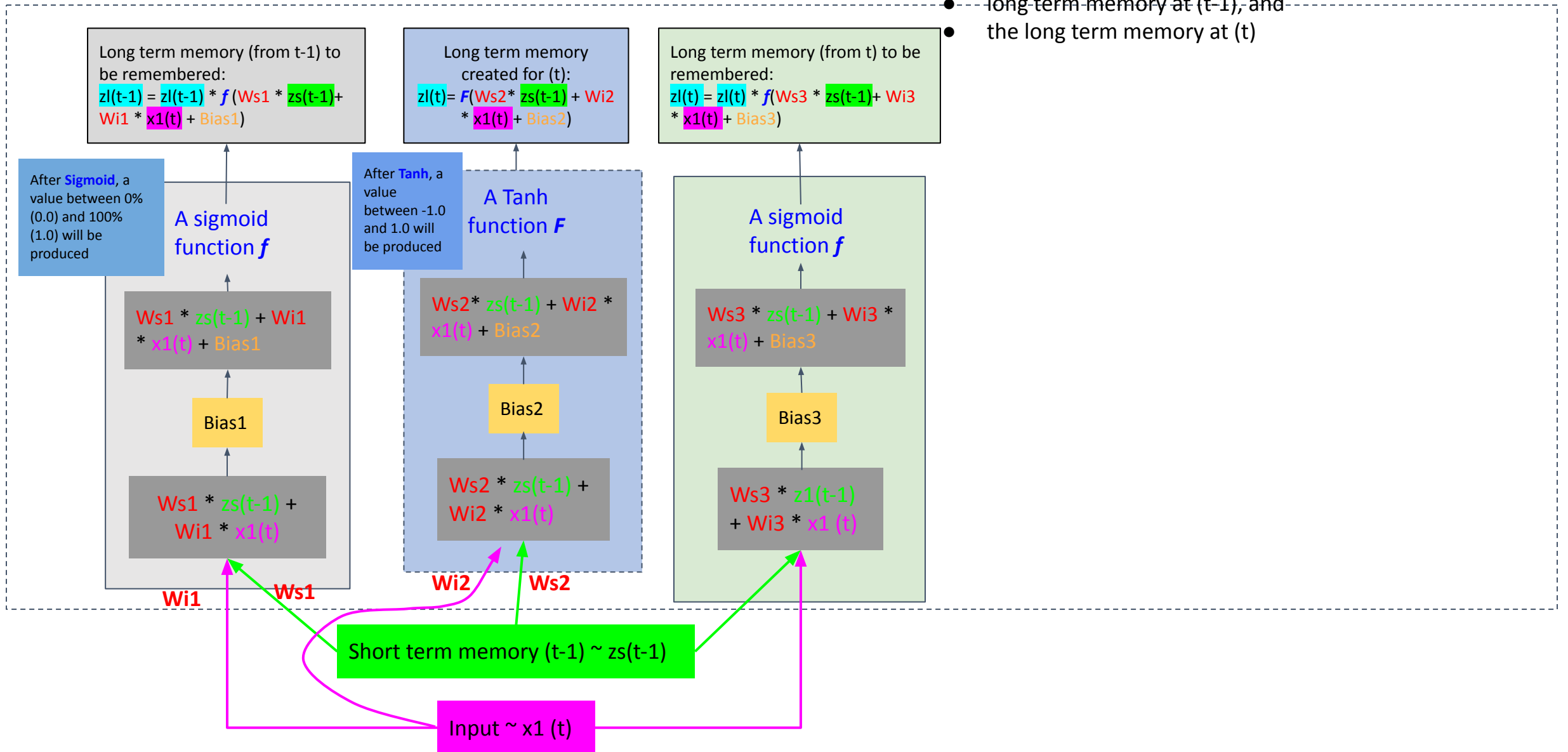


Long term
memory (t-1):
 $z_l(t-1)$

New long term
memory at (t):
 $z_l(t-1) + z(t)$

Each LSTM unit must have three inputs:

- input (e.g., output from last time step)
 - short term memory (updated from last step)
 - long term memory (updated from last step)
- The new long term memory updated at (t) is the sum of
- long-term-memory at (t-1), and
 - the long term memory at (t)



Long term
memory (t-1):
 $z_l(t-1)$

New long term
memory at (t):
 $z_l(t-1) + z(t)$

Each LSTM unit must have three inputs:

- input (e.g., output from last time step)
- short term memory (updated from last step)
- long term memory (updated from last step)

Long term memory (from t-1) to
be remembered:
 $z_l(t-1) = z_l(t-1) * f(Ws1 * z_s(t-1) +$
 $Wi1 * x1(t) + Bias1)$

Long term memory
created for (t):
 $z_l(t) = F(Ws2 * z_s(t-1) + Wi2$
 $* x1(t) + Bias2)$

Long term memory (from t) to be
remembered:
 $z_l(t) = z_l(t) * f(Ws3 * z_s(t-1) + Wi3$
 $* x1(t) + Bias3)$

After **Sigmoid**, a
value between 0%
(0.0) and 100%
(1.0) will be
produced

A sigmoid
function f

$$Ws1 * z_s(t-1) + Wi1 * x1(t) + Bias1$$

Bias1

$$Ws1 * z_s(t-1) +$$
$$Wi1 * x1(t)$$

$Wi1$

$Ws1$

After **Tanh**, a
value
between -1.0
and 1.0 will
be produced

A Tanh
function F

$$Ws2 * z_s(t-1) + Wi2 * x1(t) + Bias2$$

Bias2

$$Ws2 * z_s(t-1) +$$
$$Wi2 * x1(t)$$

$Wi2$

$Ws2$

A sigmoid
function f

$$Ws3 * z_s(t-1) + Wi3 * x1(t) + Bias3$$

Bias3

$$Ws3 * z1(t-1)$$
$$+ Wi3 * x1(t)$$

$Wi3$

$Ws3$

Short term memory (t):
 $z_s(t) = F[z_l(t-1) + z_l(t)]$

A Tanh
function F

After updated the long term
memory, let's look at the short term
memory:

We apply the Tanh function to the
newly updated memory, and get the short term memory at
(t)

Short term memory (t-1) ~ $z_s(t-1)$

Input ~ $x1(t)$

Long term
memory (t-1):
 $z_l(t-1)$

New long term
memory at (t):
 $z_l(t-1) + z(t)$

Each LSTM unit must have three inputs:

- input (e.g., output from last time step)
- short term memory (updated from last step)
- long term memory (updated from last step)

Long term memory (from t-1) to
be remembered:
 $z_l(t-1) = z_l(t-1) * f(Ws1 * z_s(t-1) +$
 $Wi1 * x1(t) + Bias1)$

Long term memory
created for (t):
 $z_l(t) = F(Ws2 * z_s(t-1) + Wi2$
 $* x1(t) + Bias2)$

Long term memory (from t) to be
remembered:
 $z_l(t) = z_l(t) * f(Ws3 * z_s(t-1) + Wi3$
 $* x1(t) + Bias3)$

Then let's look at how much
calculated short term memory we
want to keep, similar to the left, we
combine input (t) and short term
memory from (t-1), and add bias.

After **Sigmoid**, a
value between 0%
(0.0) and 100%
(1.0) will be
produced

A sigmoid
function f

$$Ws1 * z_s(t-1) + Wi1 * x1(t) + Bias1$$

Bias1

$$Ws1 * z_s(t-1) + Wi1 * x1(t)$$

$Wi1$

$Ws1$

After **Tanh**, a
value
between -1.0
and 1.0 will
be produced

A Tanh
function F

$$Ws2 * z_s(t-1) + Wi2 * x1(t) + Bias2$$

Bias2

$$Ws2 * z_s(t-1) + Wi2 * x1(t)$$

$Wi2$

$Ws2$

A sigmoid
function f

$$Ws3 * z_s(t-1) + Wi3 * x1(t) + Bias3$$

Bias3

$$Ws3 * z_s(t-1) + Wi3 * x1(t)$$

$Wi3$

$Ws3$

Short term memory (t):
 $z_s(t) = F[z_l(t-1) + z_l(t)]$

A Tanh
function F

$$Ws4 * z_s(t-1) + Wi4 * x1(t) + Bias4$$

Bias4

$$Ws4 * z_s(t-1) + Wi4 * x1(t)$$

$Wi4$

$Ws4$

Short term memory (t-1) $\sim z_s(t-1)$

Input $\sim x1(t)$

Long term
memory (t-1):
 $z_l(t-1)$

New long term
memory at (t):
 $z_l(t-1) + z(t)$

Each LSTM unit must have three inputs:

- input (e.g., output from last time step)
- short term memory (updated from last step)
- long term memory (updated from last step)

Long term memory (from t-1) to
be remembered:
 $z_l(t-1) = z_l(t-1) * f(Ws1 * z_s(t-1) +$
 $Wi1 * x1(t) + Bias1)$

Long term memory
created for (t):
 $z_l(t) = F(Ws2 * z_s(t-1) + Wi2$
 $* x1(t) + Bias2)$

Long term memory (from t) to be
remembered:
 $z_l(t) = z_l(t) * f(Ws3 * z_s(t-1) + Wi3$
 $* x1(t) + Bias3)$

Apply a sigmoid function to
calculate the percentage of the
calculated short term memory to be
kept ...

After **Sigmoid**, a
value between 0%
(0.0) and 100%
(1.0) will be
produced

A sigmoid
function f

$$Ws1 * z_s(t-1) + Wi1 * x1(t) + Bias1$$

Bias1

$$Ws1 * z_s(t-1) + Wi1 * x1(t)$$

$Wi1$

$Ws1$

After **Tanh**, a
value
between -1.0
and 1.0 will
be produced

A Tanh
function F

$$Ws2 * z_s(t-1) + Wi2 * x1(t) + Bias2$$

Bias2

$$Ws2 * z_s(t-1) + Wi2 * x1(t)$$

$Wi2$

$Ws2$

A sigmoid
function f

$$Ws3 * z_s(t-1) + Wi3 * x1(t) + Bias3$$

Bias3

$$Ws3 * z_s(t-1) + Wi3 * x1(t)$$

$Wi3$

$Ws3$

Short term memory (t):
 $z_s(t) = F[z_l(t-1) + z_l(t)]$

A Tanh
function F

A sigmoid
function f

$$Ws4 * z_s(t-1) + Wi4 * x1(t) + Bias4$$

Bias4

$$Ws4 * z_s(t-1) + Wi4 * x1(t)$$

$Wi4$

$Ws4$

Short term memory (t-1) ~ $z_s(t-1)$

Input ~ $x1(t)$

Long term
memory (t-1):
 $z_l(t-1)$

New long term
memory at (t):
 $z_l(t-1) + z(t)$

Each LSTM unit must have three inputs:

- input (e.g., output from last time step)
- short term memory (updated from last step)
- long term memory (updated from last step)

Long term memory (from t-1) to
be remembered:
 $z_l(t-1) = z_l(t-1) * f(Ws1 * z_s(t-1) +$
 $Wi1 * x1(t) + Bias1)$

Long term memory
created for (t):
 $z_l(t) = F(Ws2 * z_s(t-1) + Wi2$
 $* x1(t) + Bias2)$

Long term memory (from t) to be
remembered:
 $z_l(t) = z_l(t) * f(Ws3 * z_s(t-1) + Wi3$
 $* x1(t) + Bias3)$

The output the updated short term
memory at (t)

After **Sigmoid**, a
value between 0%
(0.0) and 100%
(1.0) will be
produced

A sigmoid
function f

$$Ws1 * z_s(t-1) + Wi1 * x1(t) + Bias1$$

Bias1

$$Ws1 * z_s(t-1) + Wi1 * x1(t)$$

$Wi1$

$Ws1$

After **Tanh**, a
value
between -1.0
and 1.0 will
be produced

A Tanh
function F

$$Ws2 * z_s(t-1) + Wi2 * x1(t) + Bias2$$

Bias2

$$Ws2 * z_s(t-1) + Wi2 * x1(t)$$

$Wi2$

$Ws2$

A sigmoid
function f

$$Ws3 * z_s(t-1) + Wi3 * x1(t) + Bias3$$

Bias3

$$Ws3 * z_s(t-1) + Wi3 * x1(t)$$

$Wi3$

$Ws3$

Short term memory (t):
 $z_s(t) = F[z_l(t-1) + z_l(t)]$

A Tanh
function F

A sigmoid
function f

$$Ws4 * z_s(t-1) + Wi4 * x1(t) + Bias4$$

Bias4

$$Ws4 * z_s(t-1) + Wi4 * x1(t)$$

$Wi4$

$Ws4$

Short term memory (t-1) ~ $z_s(t-1)$

Input ~ $x1(t)$

New short term memory (t) ~
 $z_s(t)$

Long term
memory (t-1):
 $z_l(t-1)$

New long term
memory at (t):
 $z_l(t-1) + z(t)$

Each LSTM unit must have three inputs:

- input (e.g., output from last time step)
- short term memory (updated from last step)
- long term memory (updated from last step)

Long term memory (from t-1) to
be remembered:
 $z_l(t-1) = z_l(t-1) * f(Ws1 * z_s(t-1) +$
 $Wi1 * x1(t) + Bias1)$

Long term memory
created for (t):
 $z_l(t) = F(Ws2 * z_s(t-1) + Wi2$
 $* x1(t) + Bias2)$

Long term memory (from t) to be
remembered:
 $z_l(t) = z_l(t) * f(Ws3 * z_s(t-1) + Wi3$
 $* x1(t) + Bias3)$

After Sigmoid, a
value between 0%
(0.0) and 100%
(1.0) will be
produced

A sigmoid
function f

After Tanh, a
value
between -1.0
and 1.0 will
be produced

A Tanh
function F

A sigmoid
function f

A Tanh
function F

A sigmoid
function f

$$Ws1 * z_s(t-1) + Wi1 * x1(t) + Bias1$$

Bias1

$$Ws1 * z_s(t-1) + Wi1 * x1(t)$$

$Wi1$

$Ws1$

$$Ws2 * z_s(t-1) + Wi2 * x1(t) + Bias2$$

Bias2

$$Ws2 * z_s(t-1) + Wi2 * x1(t)$$

$Wi2$

$Ws2$

$$Ws3 * z_s(t-1) + Wi3 * x1(t) + Bias3$$

Bias3

$$Ws3 * z_s(t-1) + Wi3 * x1(t)$$

$Wi3$

$Ws3$

Short term memory (t):
 $z_s(t) = F[z_l(t-1) + z_l(t)]$

$$Ws4 * z_s(t-1) + Wi4 * x1(t) + Bias4$$

Bias4

$$Ws4 * z_s(t-1) + Wi4 * x1(t)$$

$Wi4$

$Ws4$

Short term memory (t-1) ~ $z_s(t-1)$

Input ~ $x1(t)$

New short term memory (t) ~ $z_s(t)$

This is called "forget gate",
which controls:
the percentage of long term
memory (t-1) (e.g., $z_l(t-1)$)
to be remembered

Long term memory (t-1):
 $z_l(t-1)$

New long term memory at (t):
 $z_l(t-1) + z(t)$

Each LSTM unit must have three inputs:

- input (e.g., output from last time step)
- short term memory (updated from last step)
- long term memory (updated from last step)

This is called "input gate", which controls:
The long term memory updated at (t) (e.g., $z_l(t)$) to be remembered

Long term memory (from t-1) to be remembered:
 $z_l(t-1) = z_l(t-1) * f(Ws1 * z_s(t-1) + Wi1 * x1(t) + Bias1)$

After Sigmoid, a value between 0% (0.0) and 100% (1.0) will be produced

A sigmoid function f

$$Ws1 * z_s(t-1) + Wi1 * x1(t) + Bias1$$

Bias1

$$Ws1 * z_s(t-1) + Wi1 * x1(t)$$

$Wi1$

$Ws1$

Long term memory created for (t):
 $z_l(t) = F(Ws2 * z_s(t-1) + Wi2 * x1(t) + Bias2)$

After Tanh, a value between -1.0 and 1.0 will be produced

A Tanh function F

$$Ws2 * z_s(t-1) + Wi2 * x1(t) + Bias2$$

Bias2

$$Ws2 * z_s(t-1) + Wi2 * x1(t)$$

$Wi2$

$Ws2$

Long term memory (from t) to be remembered:
 $z_l(t) = z_l(t) * f(Ws3 * z_s(t-1) + Wi3 * x1(t) + Bias3)$

A sigmoid function f

$$Ws3 * z_s(t-1) + Wi3 * x1(t) + Bias3$$

Bias3

$$Ws3 * z_s(t-1) + Wi3 * x1(t)$$

$Wi3$

$Ws3$

Short term memory (t):
 $z_s(t) = F[z_l(t-1) + z_l(t)]$

A Tanh function F

A sigmoid function f

$$Ws4 * z_s(t-1) + Wi4 * x1(t) + Bias4$$

Bias4

$$Ws4 * z_s(t-1) + Wi4 * x1(t)$$

$Wi4$

$Ws4$

Short term memory (t-1) $\sim z_s(t-1)$

Input $\sim x1(t)$

New short term memory (t) $\sim z_s(t)$

Long term
memory (t-1):
 $z_l(t-1)$

New long term
memory at (t):
 $z_l(t-1) + z(t)$

Each LSTM unit must have three inputs:

- input (e.g., output from last time step)
- short term memory (updated from last step)
- long term memory (updated from last step)

Long term memory (from t-1) to
be remembered:
 $z_l(t-1) = z_l(t-1) * f(Ws1 * z_s(t-1) +$
 $Wi1 * x1(t) + Bias1)$

Long term memory
created for (t):
 $z_l(t) = F(Ws2 * z_s(t-1) + Wi2$
 $* x1(t) + Bias2)$

Long term memory (from t) to be
remembered:
 $z_l(t) = z_l(t) * f(Ws3 * z_s(t-1) + Wi3$
 $* x1(t) + Bias3)$

This is called “output gate”,
which controls:
The short term memory created
at (t) (e.g., $z_s(t)$) to be
remembered

After Sigmoid, a
value between 0%
(0.0) and 100%
(1.0) will be
produced

A sigmoid
function f

$$Ws1 * z_s(t-1) + Wi1 * x1(t) + Bias1$$

Bias1

$$Ws1 * z_s(t-1) +$$

$$Wi1 * x1(t)$$

$Wi1$

$Ws1$

After Tanh, a
value
between -1.0
and 1.0 will
be produced

A Tanh
function F

$$Ws2 * z_s(t-1) + Wi2 * x1(t) + Bias2$$

Bias2

$$Ws2 * z_s(t-1) +$$

$$Wi2 * x1(t)$$

$Wi2$

$Ws2$

A sigmoid
function f

$$Ws3 * z_s(t-1) + Wi3 * x1(t) + Bias3$$

Bias3

$$Ws3 * z1(t-1)$$

$$+ Wi3 * x1(t)$$

$Ws3$

$Wi3$

Short term memory (t):
 $z_s(t) = F[z_l(t-1) + z_l(t)]$

A Tanh
function F

A sigmoid
function f

$$Ws4 * z_s(t-1) + Wi4 * x1(t) + Bias4$$

Bias4

$$Ws4 * z1(t-1)$$

$$+ Wi4 * x1(t)$$

$Wi4$

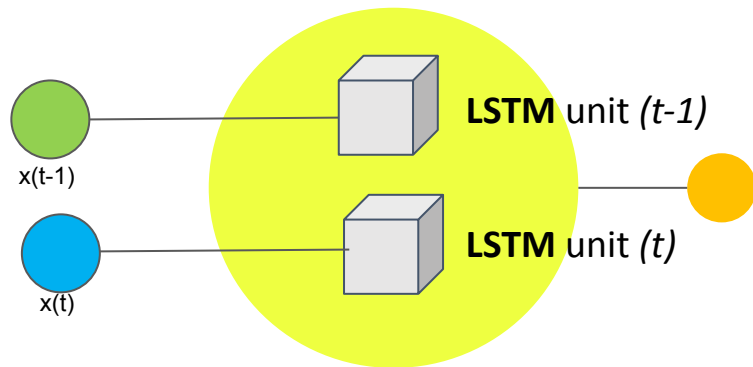
$Ws4$

Short term memory (t-1) ~ $z_s(t-1)$

Input ~ $x1(t)$

New short term memory (t) ~
 $z_s(t)$

From the above we understand that the workflow in the LSTM unit:



By "**Forget gate**":

- Step 1: Calculating the Long term memory from (t-1) to be kept
- Step 2: Creating the Long term memory at (t)

By "**Input gate**":

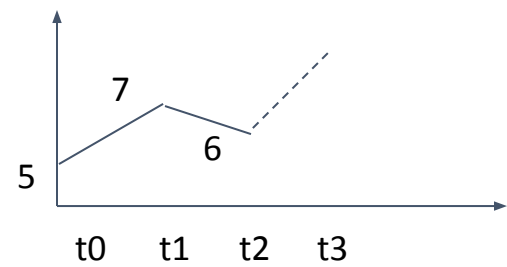
- Step 3: Calculating the Long term memory from (t) to be kept
- Step 4: Updating the Long term memory at (t)

By "**Output gate**":

- Step 5: Creating the Short term memory at (t)
- Step 6: Calculating the Short term memory from (t) to be kept
- Step 7: Updating the Short term memory at (t)

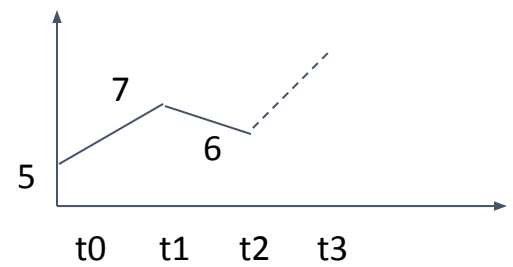
How LSTM works: a real case

How it works in a real data time series

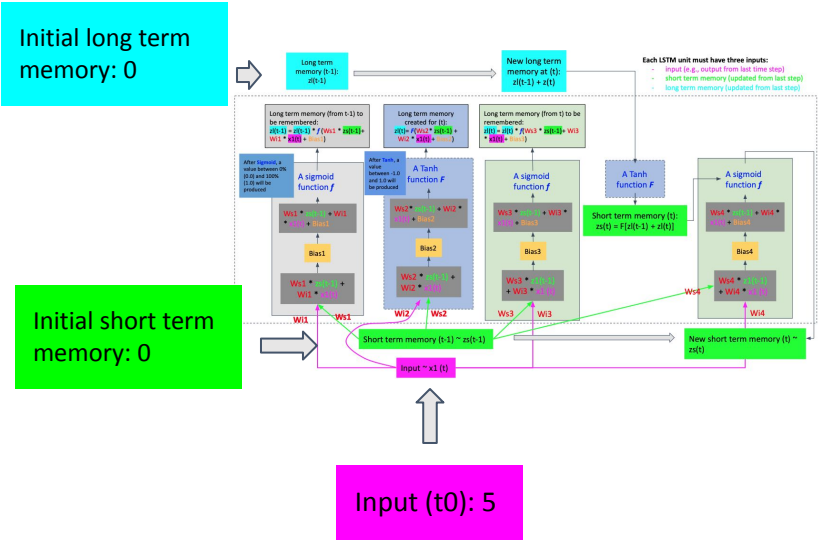


Assuming we have a time series from t_0 to t_2 , and we would like to predict the value at t_3

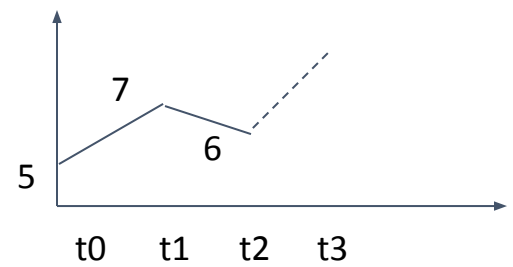
How it works in a real data time series



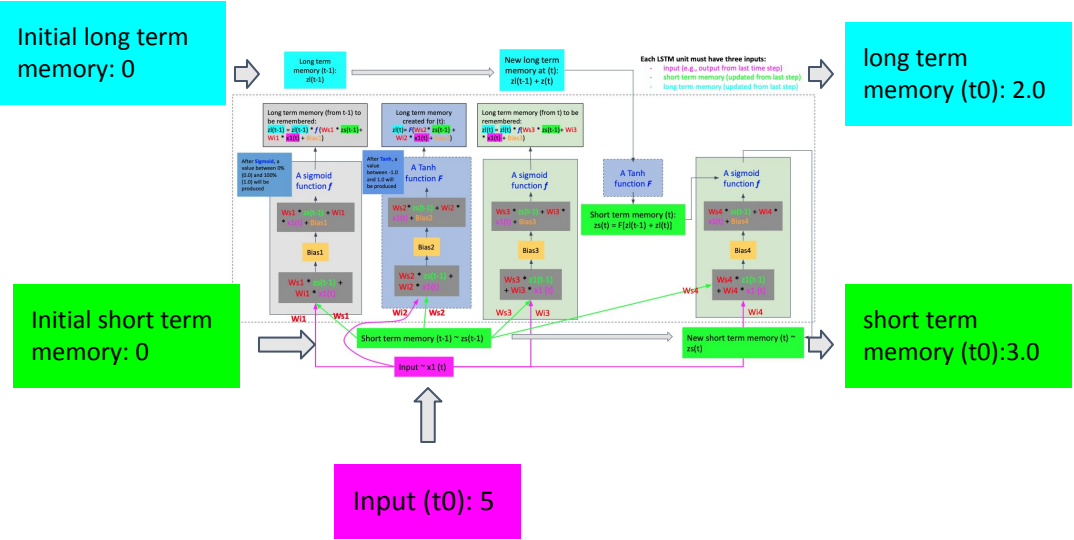
Assuming we have a time series from t0 to t2, and we would like to predict the value at t3



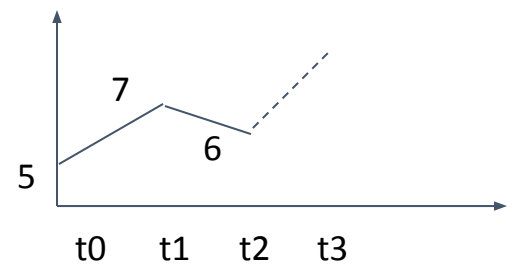
How it works in a real data time series



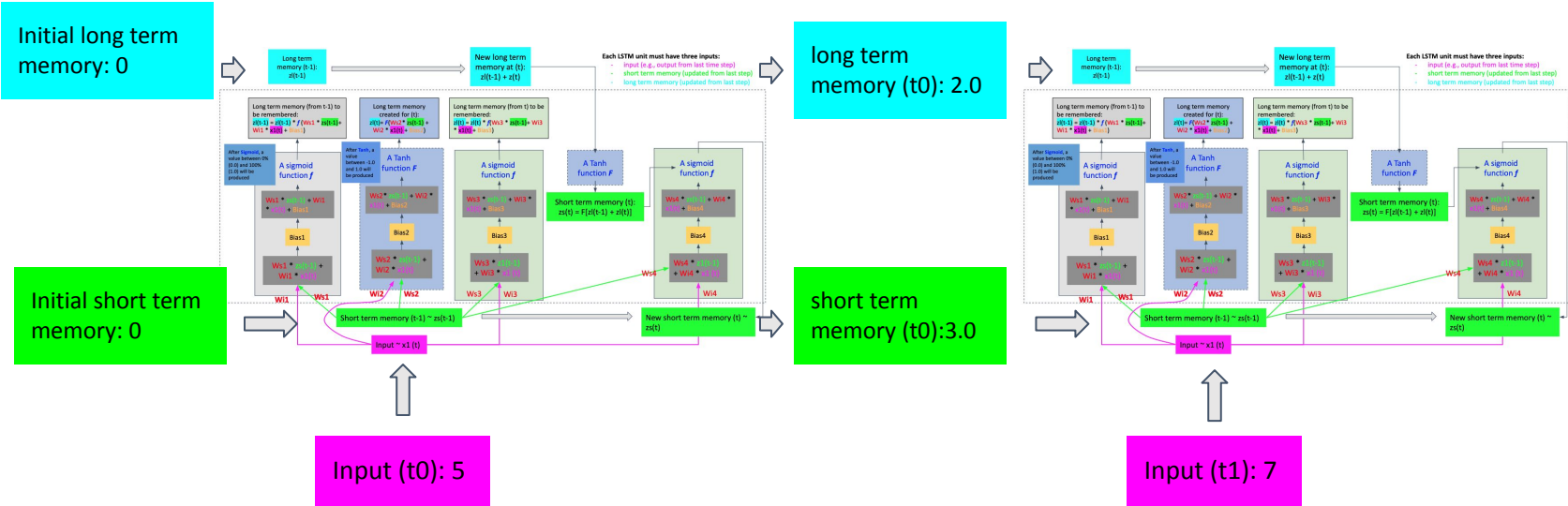
Assuming we have a time series from t0 to t2, and we would like to predict the value at t3



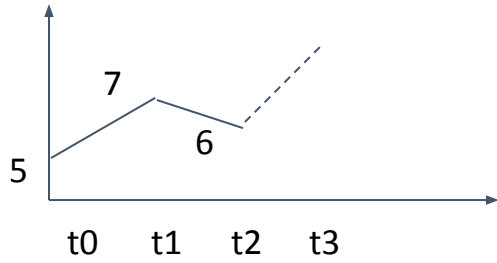
How it works in a real data time series



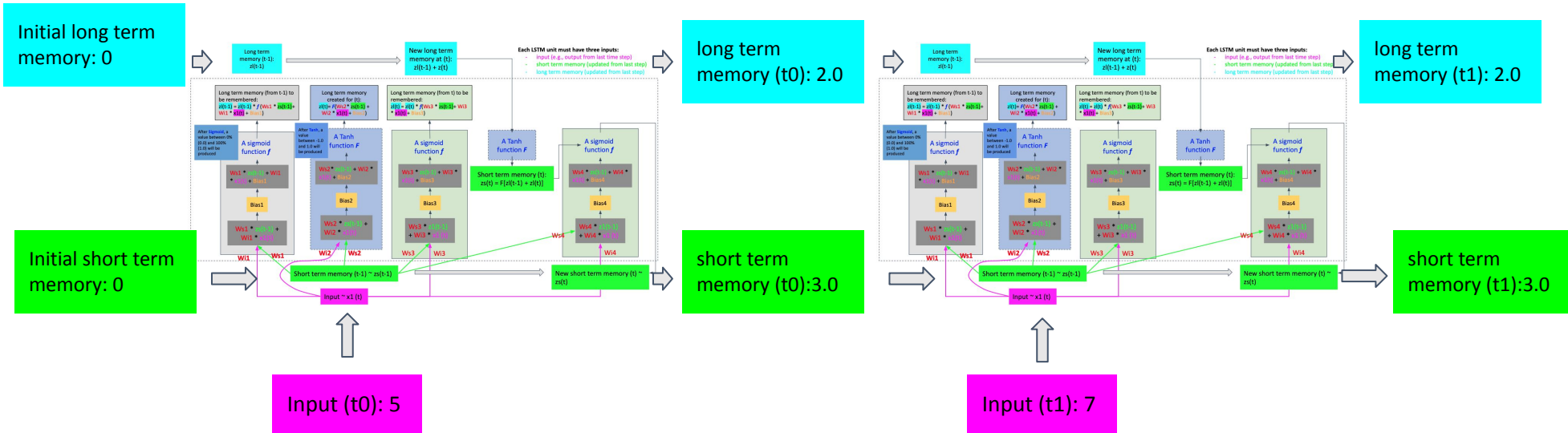
Assuming we have a time series from t0 to t2, and we would like to predict the value at t3



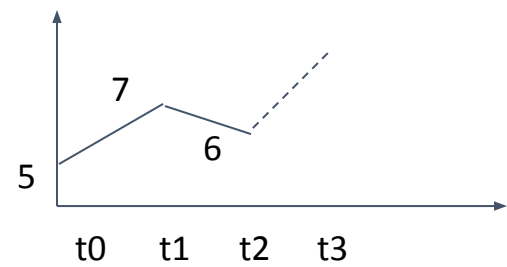
How it works in a real data time series



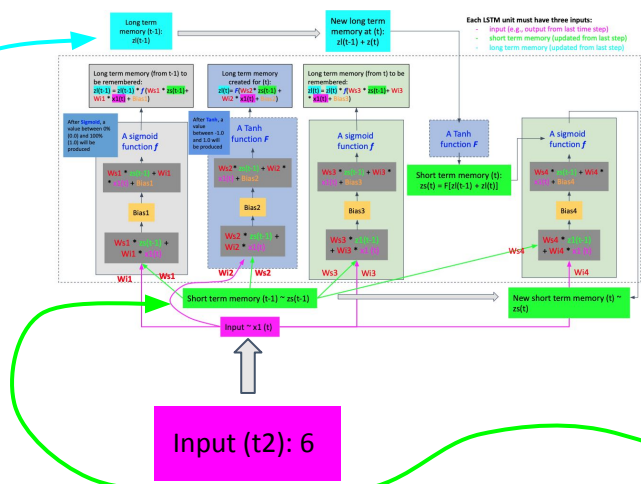
Assuming we have a time series from t_0 to t_2 , and we would like to predict the value at t_3



How it works in a real data time series



Assuming we have a time series from t_0 to t_2 , and we would like to predict the value at t_3



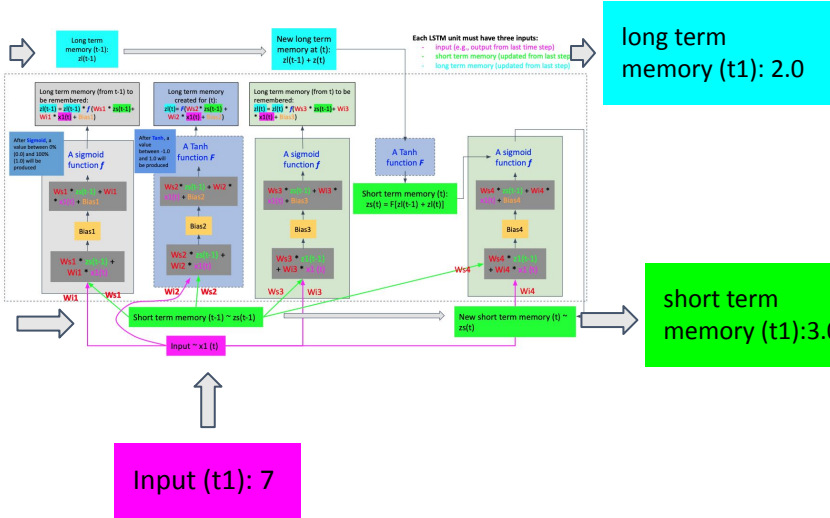
Initial long term memory: 0

Initial short term memory: 0

Input (t_0): 5

long term memory (t_0): 2.0

short term memory (t_0): 3.0

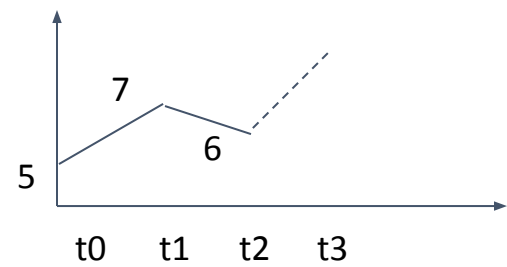


long term memory (t_1): 2.0

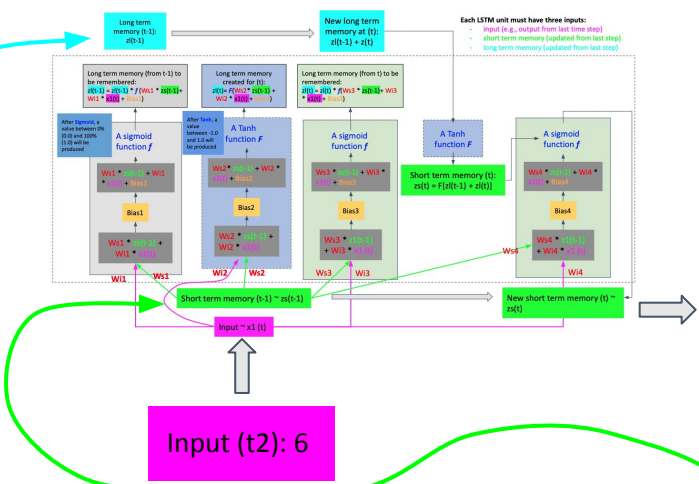
short term memory (t_1): 3.0

Input (t_1): 7

How it works in a real data time series



Assuming we have a time series from t_0 to t_2 , and we would like to predict the value at t_3



Short term memory (t_2) is the prediction at (t_3)

Initial long term memory: 0

Initial short term memory: 0

Input (t_0): 5

long term memory (t_0): 2.0

short term memory (t_0): 3.0

long term memory (t_1): 2.0

short term memory (t_1): 3.0

Input (t_1): 7