Appendix

Here we provide all the terms involved in the between- or within- study variance estimator derived from each working model or terms used in the expectation of the estimator under the CHE model.

Quandratic forms Q_E and Q_1

Q_E is the weighted residual sum of squares

$$\begin{aligned} Q_E &= \mathbf{r}'\mathbf{W}\mathbf{r} \\ &= \mathbf{T}' \underbrace{\left[\mathbf{I} - \mathbf{W}\mathbf{X}\left(\mathbf{X}'\mathbf{W}\mathbf{X}\right)^{-1}\mathbf{X}'\right]\mathbf{W}\left[\mathbf{I} - \mathbf{X}\left(\mathbf{X}'\mathbf{W}\mathbf{X}\right)^{-1}\mathbf{X}'\mathbf{W}\right]}_{\mathbf{\Lambda}_E}\mathbf{T}, \end{aligned}$$

Q_1 is another quadratic form

$$\begin{aligned} Q_1 &= \mathbf{r}'\mathbf{J}\mathbf{r} \\ &= \mathbf{T}' \underbrace{\left[\mathbf{I} - \mathbf{W}\mathbf{X}\left(\mathbf{X}'\mathbf{W}\mathbf{X}\right)^{-1}\mathbf{X}'\right]\mathbf{J}\left[\mathbf{I} - \mathbf{X}\left(\mathbf{X}'\mathbf{W}\mathbf{X}\right)^{-1}\mathbf{X}'\mathbf{W}\right]}_{\mathbf{\Lambda}_1}\mathbf{T}, \end{aligned}$$

Terms involves design matrices, weight matrices, and study sizes

$$\begin{aligned} A_1 &= \sum_{j=1}^{J} k_j^2 - 2\mathrm{tr}\left[\ddot{\mathbf{M}}\sum_{j=1}^{J} k_j \ddot{w}_j \mathbf{X}_j'\mathbf{J}_j\mathbf{X}_j\right] + \mathrm{tr}\left[\ddot{\mathbf{M}}\left(\sum_{j=1}^{J}\mathbf{X}_j'\mathbf{J}_j\mathbf{X}_j\right)\ddot{\mathbf{M}}\left(\sum_{j=1}^{J}\ddot{w}_j^2\mathbf{X}_j'\mathbf{J}_j\mathbf{X}_j\right)\right] \\ A_2 &= \sum_{j=1}^{J} k_j \dot{w}_j - \mathrm{tr}\left(\dot{\mathbf{M}}\sum_{j=1}^{J}\dot{w}_j^2\mathbf{X}_j'\mathbf{J}_j\mathbf{X}_j\right) \\ B_1 &= \sum_{j=1}^{J} k_j - 2\mathrm{tr}\left[\ddot{\mathbf{M}}\sum_{j=1}^{J}\ddot{w}_j\mathbf{X}_j'\mathbf{J}_j\mathbf{X}_j\right] + \mathrm{tr}\left[\ddot{\mathbf{M}}\left(\sum_{j=1}^{J}\mathbf{X}_j'\mathbf{J}_j\mathbf{X}_j\right)\ddot{\mathbf{M}}\left(\sum_{j=1}^{J}\ddot{w}_j^2\mathbf{X}_j'\mathbf{X}_j\right)\right] \\ B_2 &= \sum_{j=1}^{J} k_j \ddot{w}_j - \mathrm{tr}\left(\ddot{\mathbf{M}}\sum_{j=1}^{J}\ddot{w}_j^2\mathbf{X}_j'\mathbf{X}_j\right) \end{aligned}$$

$$
\begin{aligned}
C_1 &= \sum_{j=1}^{J} \frac{k_j}{\ddot{w}_j} - \mathrm{tr}\left[\ddot{\mathbf{M}} \sum_{j=1}^{J} \mathbf{X}_j' \mathbf{J}_j \mathbf{X}_j\right] \\
C_2 &= K - p \\
D &= \rho \ \mathrm{tr}\left[\dot{\mathbf{M}} \sum_{j=1}^{J} \frac{\dot{w}_j}{k_j} \left(\mathbf{X}_j' \mathbf{J}_j \mathbf{X}_j - \mathbf{X}_j' \mathbf{X}_j\right)\right] + \mathrm{tr}\left(\dot{\mathbf{M}} \sum_{j=1}^{J} \frac{\dot{w}_j}{k_j} \mathbf{X}_j' \mathbf{X}_j\right) \\
E &= \sum_{j=1}^{J} k_j \dot{w}_j - \mathrm{tr}\left(\dot{\mathbf{M}} \sum_{j=1}^{J} \dot{w}_j^2 \mathbf{X}_j' \mathbf{X}_j\right) \\
F &= \sum_{j=1}^{J} \frac{k_j^2}{\ddot{w}_j} - 2\mathrm{tr}\left[\ddot{\mathbf{M}} \sum_{j=1}^{J} k_j \mathbf{X}_j' \mathbf{J}_j \mathbf{X}_j\right] + \mathrm{tr}\left[\ddot{\mathbf{M}} \left(\sum_{j=1}^{J} \mathbf{X}_j' \mathbf{J}_j \mathbf{X}_j\right) \ddot{\mathbf{M}} \left(\sum_{j=1}^{J} \ddot{w}_j \mathbf{X}_j \mathbf{J}_j \mathbf{X}_j\right)\right] \\
G &= \mathrm{tr}\left[\ddot{\mathbf{M}} \sum_{j=1}^{J} \ddot{w}_j \mathbf{X}_j' \mathbf{J}_j \mathbf{X}_j\right] - p \\
H &= \sum_{j=1}^{J} k_j \breve{w}_j - \mathrm{tr}\left(\breve{\mathbf{M}} \sum_{j=1}^{J} \breve{w}_j^2 \mathbf{X}_j' \mathbf{X}_j\right) \\
L &= \sum_{j=1}^{J} k_j \breve{w}_j - \mathrm{tr}\left(\mathbf{M} \sum_{j=1}^{J} \breve{w}_j^2 \mathbf{X}_j' \mathbf{J}_j \mathbf{X}_j\right) \\
O &= \ \mathrm{tr}\left[\mathbf{M} \sum_{j=1}^{J} \breve{w}_j \left(\mathbf{X}_j' \mathbf{J}_j \mathbf{X}_j - \mathbf{X}_j' \mathbf{X}_j\right)\right] \\
R &= \sum_{j=1}^{J} \bar{w}_j - \mathrm{tr}\left(\bar{\mathbf{M}} \sum_{j=1}^{J} \bar{w}_j^2 \mathbf{x}_j' \mathbf{x}_j\right) \\
S &= J - p \\
U &= \sum_{j=1}^{J} \frac{\bar{w}_j}{k_j} - \mathrm{tr}\left(\bar{\mathbf{M}} \sum_{j=1}^{J} \frac{\bar{w}_j^2}{k_j} \mathbf{x}_j' \mathbf{x}_j\right)
\end{aligned}
$$