### A. (Partial) "University" Database Definition

```
student( student_id, firstname, lastname, date_of_birth, userid )
course( course_code, name, calendar_desc, credits)
section( course_code, section_number, term, instructor_id )
enrolment( student_id, course_code, section_number, term, grade )
instructor( id, firstname, lastname, peng, department_id, salary )
department( id, name, faculty_id, budget )
faculty( id, name )
```

| Table Name | Foreign Key(s) |
|---|---|
| section | course_code → course.course_code<br>instructor_id → instructor.id |
| enrolment | student_id → student.student_id<br>course_code, section_number, term → section.course_code, section_number, term |
| instructor | department_id → department.id |
| department | faculty_id → faculty.id |

### B. Relational Algebra

| Symbol | Meaning | Symbol | Meaning | Symbol | Meaning |
|---|---|---|---|---|---|
| $\sigma$ | Selection | $\Pi$ | Projection | $\cup$ | Union |
| $\times$ | Cartesian Product | $-$ | Minus (Difference) | $\rho$ | Rename |
| $\cap$ | Intersection | $\leftarrow$ | Assignment | $\bowtie$ | Join |
| ⟕ | Left Outer Join | ⟖ | Right Outer Join | ⟗ | Full Outer Join |

### C. Armstrong's Axioms & Derived Rules

- **Reflexivity**: If $\alpha$ is a set of attributes and $\beta$ is contained within $\alpha$, then $\alpha \to \beta$ holds.
- **Augmentation**: If $\alpha \to \beta$ holds and $\gamma$ is a set of attributes, then $\gamma\alpha \to \gamma\beta$ holds.
- **Transitivity**: If $\alpha \to \beta$ holds and $\beta \to \gamma$ holds, then $\alpha \to \gamma$ holds.
- **Union**: If $\alpha \to \beta$ holds and $\alpha \to \gamma$ holds, then $\alpha \to \beta\gamma$ holds.
- **Decomposition**: If $\alpha \to \beta\gamma$ holds, then $\alpha \to \beta$ holds and $\alpha \to \gamma$ holds (reverse of previous rule).
- **Pseudotransitivity**: If $\alpha \to \beta$ holds and $\gamma\beta \to \delta$ holds, then $\alpha\gamma \to \delta$ holds.

### D. Query Transformation Rules

1. Conjunctive Selection: $\sigma_{\theta_1 \wedge \theta_2}(E) = \sigma_{\theta_1}(\sigma_{\theta_2}(E))$
2. Selection Commutes: $\sigma_{\theta_1}(\sigma_{\theta_2}(E)) = \sigma_{\theta_2}(\sigma_{\theta_1}(E))$
3. Projection Redundancy Elimination: $\Pi_{L_1}(\Pi_{L_2}(\Pi_{L_3}(E))) = \Pi_{L_1}(E)$
4. Commuting Selection and Projection: $\Pi_L(\sigma_\theta(E)) = \sigma_\theta(\Pi_L(E))$
5. Selection Combination
   (a) $\sigma_\theta(E_1 \times E_2) = E_1 \bowtie_\theta E_2$
   (b) $\sigma_{\theta_1}(E_1 \bowtie_{\theta_2} E_2) = E_1 \bowtie_{\theta_1 \wedge \theta_2} E_2$
6. Theta Joins Commute: $E_1 \bowtie_\theta E_2 = E_2 \bowtie_\theta E_1$
7. Natural Join Associates:
   (a) $(E_1 \bowtie E_2) \bowtie E_3 = E_1 \bowtie (E_2 \bowtie E_3)$.
   (b) $(E_1 \bowtie_{\theta_1} E_2) \bowtie_{\theta_2 \wedge \theta_3} E_3 = E_1 \bowtie_{\theta_1 \wedge \theta_3} (E_2 \bowtie_{\theta_2} E_3)$.
   (c) $(E_1 \times E_2) \times E_3 = E_1 \times (E_2 \times E_3)$.
8. Selection Distribution:
   (a) $\sigma_{\theta_0}(E_1 \bowtie_\theta E_2) = (\sigma_{\theta_0}(E_1)) \bowtie_\theta E_2$
   (b) $\sigma_{\theta_1 \wedge \theta_2}(E_1 \bowtie_\theta E_2) = (\sigma_{\theta_1}(E_1)) \bowtie_\theta (\sigma_{\theta_2}(E_2))$
9. Projection Distribution:
   (a) $\Pi_{L_1 \cup L_2}(E_1 \bowtie_\theta E_2) = (\Pi_{L_1}(E_1)) \bowtie_\theta (\Pi_{L_2}(E_2))$
   (b) $\Pi_{L_1 \cup L_2}(E_1 \bowtie_\theta E_2) = \Pi_{L_1 \cup L_2}((\Pi_{L_1 \cup L_3}(E1)) \bowtie_\theta (\Pi_{L_2 \cup L_4}(E_2)))$
10. Set Operations Commute:
    (a) $E_1 \cup E_2 = E_2 \cup E_1$
    (b) $E_1 \cap E_2 = E_2 \cap E_1$
11. Set Operations Associate:
    (a) $(E_1 \cup E_2) \cup E_3 = E_1 \cup (E_2 \cup E_3)$
    (b) $(E_1 \cap E_2) \cap E_3 = E_1 \cap (E_2 \cap E_3)$
12. Selection Distribution 2:
    (a) $\sigma_\theta(E_1 \cup E_2) = \sigma_\theta(E_1) \cup \sigma_\theta(E_2)$
    (b) $\sigma_\theta(E_1 \cap E_2) = \sigma_\theta(E_1) \cap \sigma_\theta(E_2)$
    (c) $\sigma_\theta(E_1 - E_2) = \sigma_\theta(E_1) - \sigma_\theta(E_2)$
13. Projection Distribution 2: $\Pi_L(E_1 \cup E_2 = (\Pi_L(E_1)) \cup (\Pi_L(E_2))$