

Lecture 33 — More Advanced Queueing Theory

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New considerations—or complications—arise.

We'll talk about two settings, one that I love and one that I hate: food halls and Service Ontario.

No points for guessing which is which.

At Service Ontario, every staff member can help you with any task.

But at the food hall, the Gelato place cannot provide tacos.

Are tacos and gelato interchangeable? Maybe...



Maintenance, Planned and Unplanned

Services have downtime, both planned and unplanned.

If the pizza oven breaks, no more pizza today.

Usually we do not think about unplanned downtime in service capacity design.

New services can appear: what if a Banh Mi place opens?



Restaurant opening is rarely a surprise...

Interchangeability of services is a spectrum:

- Full; Starving? Tacos, pizza, shawarma? Just feed me now!
- Partial; Just a bit hungry? Want tacos, would have shawarma.
- None; I came for tacos and want nothing else.

All the food hall examples are about food—at least the possibility of interchangeability exists.

Dietitian: To be healthy you should eat fruit and vegetables.

Me: So I am allowed to consume only that.

Dietitian:



But: I came for a drivers' license; a new health card is not a possible substitute.

Too Long



Three Kinds of Problem

Balking: don't get in the line at all.

Reneging: get in line, get frustrated, leave.

Loss: No capacity, request to enqueue refused ($M/M/k/k$ system).

In both cases where I choose to leave, there is an implicit or explicit calculation—what would I calculate?

How Long Now?

IF THERE ARE X PEOPLE IN LINE ALREADY

r

$$A = \pi r^2$$

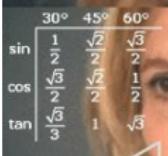


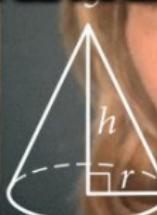
Image 5

$$\int \sin x dx = -\cos x + C$$

$$\int \frac{dx}{x} = \ln|x| + C$$

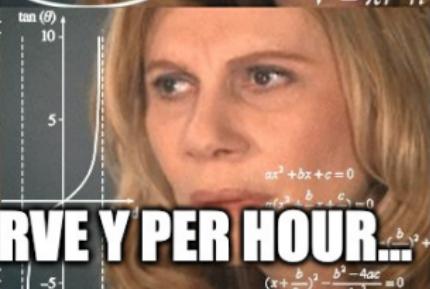
$$\int \tan x dx = -\ln|\cos x| + C$$

$$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + C$$



$$r$$

$$V = \pi r^2 h$$



AND THEY CAN SERVE Y PER HOUR...

Compare the calculated value with my willingness to wait...

One possibility: my estimate of service time or queue length wrong.

What if people can join the line ahead of me?



Priorities for queueing may exist: some can go to the front of the line!

Example: people with mobility restrictions at Service Ontario.

This makes my wait longer and might make me give up waiting.

Priorities open up new questions, such as:

- How much, if any, does giving priority to one group over another help the group being given priority?
- How much, if any, does giving priority to one group disadvantage the group not being given priority?

Priorities open up new questions, such as:

- Can the priority system incentivize people to choose things that are less popular?
- Recognizing that if everyone has priority, nobody has priority, how many requests can have priority before all benefit is lost?

Laboratory Study with a Mouse



- Doo Lee / CC BY 4.0

https://commons.wikimedia.org/wiki/File:Steamboat_Willie_Enters_the_Public_Domain.jpg

Yes. That mouse.

We're going to discuss the Disney FastPass and FastPass+ systems.

The full video: <https://www.youtube.com/watch?v=9yjZpBq1XBE>

It has a lot of Disney history, but let's try not to get distracted.

Simulation is required because of the complexity here.

Some reasons why...

Every customer (requester) is an independent agent, which implies:

- They have different times of arrival at and departure from the park. Most arrive early in the day
- They have different preferences of what they want to do while there
- They have different willingness to wait for the things they want to do
- They may or may not be willing to come back another day

The park has opening and closing times which implies:

- Requests cannot be submitted before opening time
- Requests cannot be submitted after closing time
- Requests submitted too close to closing time may not be served before closing

- The park has different services that each have their own service rate and any of them could be down for maintenance (independently of any others)
- The services have a fixed maximum capacity: you cannot make more seats on the rides or run them faster to get more people through quicker

Is this model relevant to software queueing?

Trip to Service Ontario should be arrive, get service, leave...

And ideally not return for quite a while!

At the food hall I'll eventually get full...



I go to the mouse park and want to do as much as I can do in the day.

Does it matter what the people ahead of me in the line do before or after?

Could we argue that after a ride a guest leaves and is replaced?

The simulation has many different user types. These determine:

- 1 How long they stay
- 2 When they balk
- 3 What they want to do

The third point covers preferences for rides and non-rides.

Our three systems:

- 1 No Priority
- 2 FastPass
- 3 FastPass+



Priority queues? Ratio of 4:1, 20:1, even 100:1.

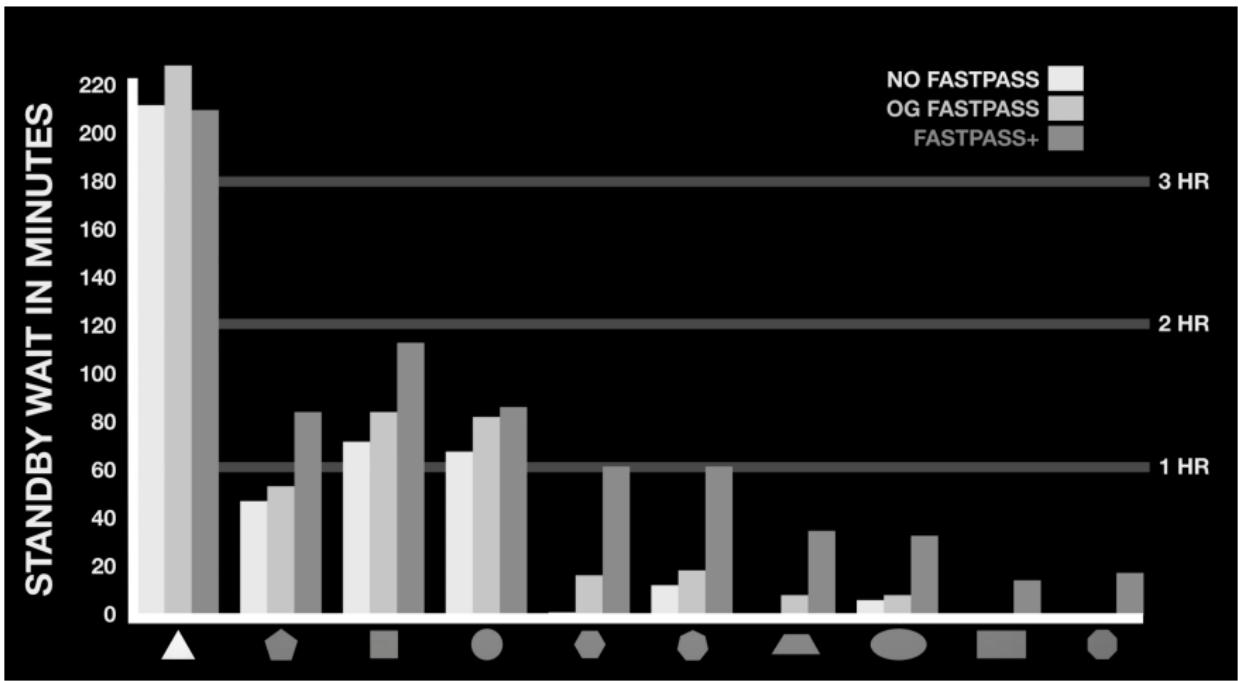
Did you want to play around with the simulation yourself?

<https://github.com/TouringPlans/shapeland>

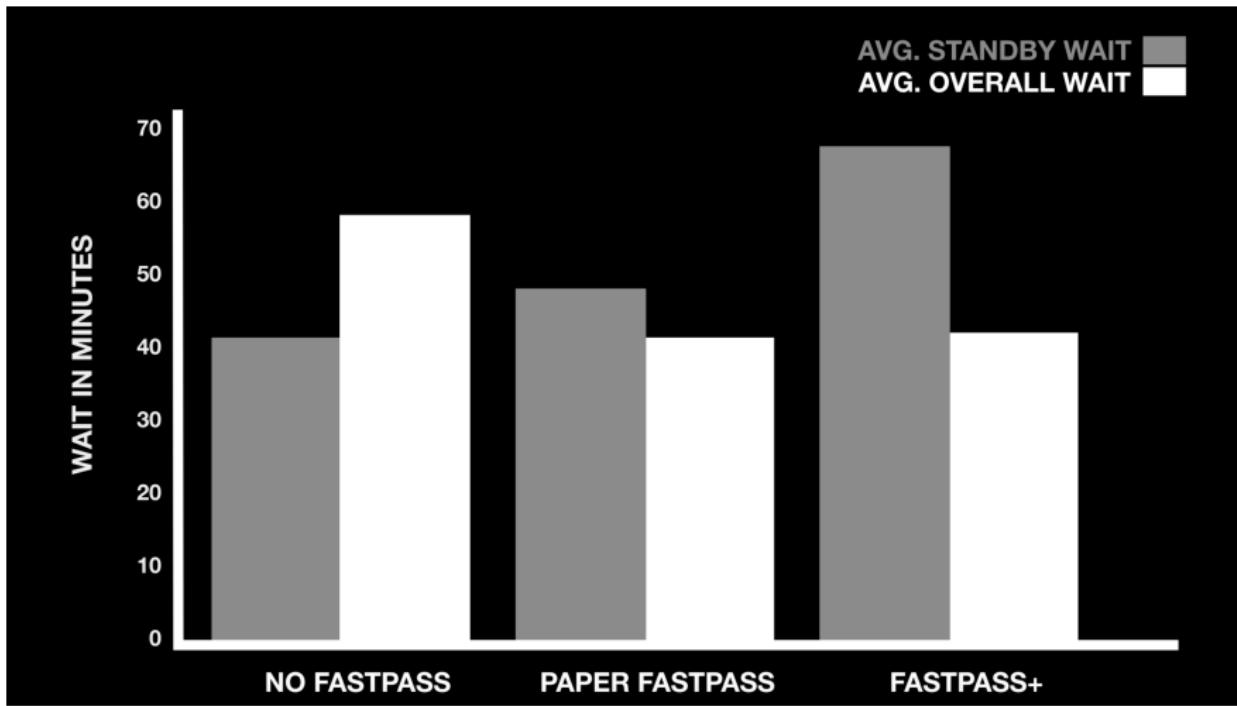
It's Python, but it's not super complex.

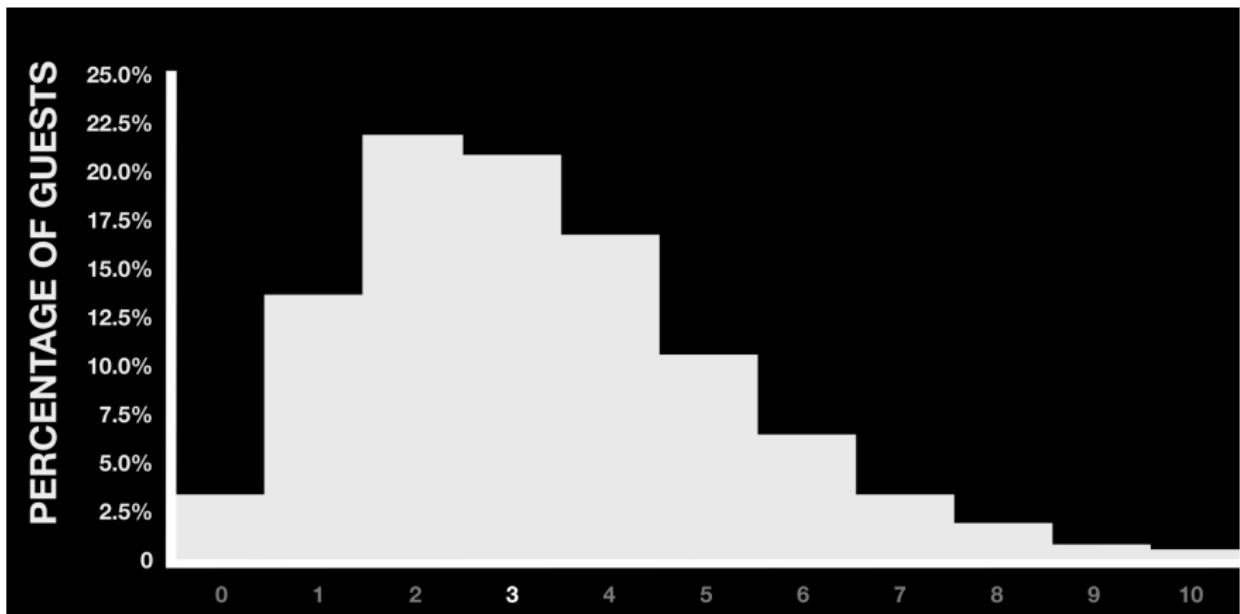
- 1** Standby Waits
- 2** Overall Waits
- 3** Average Number of Rides Experienced

Standby Waits



Overall Waits





Priority passes don't affect wait times for most popular things...
But do encourage more usage of less-popular things.

Is that improvement?



Simulation doesn't account for downtime during the day.

In the simple model of handling that, send people out of the queue.

More complex: what if people get a FastPass(+) as compensation?

The knowledge factor results in huge inequality!



Interesting lesson: complexity allows exploitation.