

1. (35 pts) An exhibition is going on in a town today from $t = 0$ (9 am) to $t = 720$ (6 pm), and celebrities will attend!! There are n celebrities C_1, \dots, C_n and each celebrity C_j will attend during a time interval $I_j : [\ell_j, u_j]$ where in $0 \leq \ell_j < u_j \leq 720$. Note: the ends of the interval are *inclusive*. An advertising company for high-end luxury goods wants to broadcast its ad on a large screen television in the exhibition, multiple times during the day. In particular, each celebrity must see the ad but the company also wants to minimize the number of times the ad must be shown.

For example:

Celebrity	$[\ell, u]$
1	$[2, 53]$
2	$[5, 50]$
3	$[5, 98]$
4	$[102, 150]$
5	$[120, 186]$
6	$[85, 190]$

Then, if the ad is shown at times $t_1 = 50$ and $t_2 = 150$, then all 6 of the celebrities will see the ad.

- (a) Greedy algorithm \mathcal{A} selects a time instance when the maximum number of celebrities are present simultaneously. An ad is scheduled at this time and the celebrities covered by this ad are then removed from further consideration. The algorithm \mathcal{A} is then applied recursively to the remaining celebrities.

Give an example where this algorithm shows more ads than is necessary.

- (b) Let C_j represent the celebrity who *leaves* first and let $[\ell_j, u_j]$ be the time interval for C_j . Suppose we have some solution t_1, t_2, \dots, t_k for the ad times that cover all celebrities. Let t_1 be the earliest ad time.

Prove the following facts for the earliest scheduled at t_1 . For each part, your proof must clearly spell out the argument. Overly long explanations or proofs by examples will receive no credit. You may ask the course staff for help with the wordings of your proofs.

S1. Prove $t_1 \leq u_j$. (Three sentences. Hint: proof by contradiction.)

S2. If $t_1 \leq \ell_j$, then t_1 can be deleted, and the remaining ads still form a valid solution. (Five sentences. Hint: suppose deleting t_1 leaves a celebrity uncovered, when should that celebrity have arrived and left? Prove a contradiction.)

- S3. If $t_1 < u_j$, then t_1 can be modified to be equal to u_j , while still remaining a valid solution. (Three sentences. Hint: suppose setting $t_1 := u_j$ leaves a celebrity uncovered, then when should that celebrity have arrived and left? Prove a contradiction.)
- (c) Use the insight from (1b) to design a greedy algorithm that is optimal.
- (i) Write pseudocode for your algorithm.
 - (ii) Prove that your algorithm is correct and give its running time complexity.
 - (iii) Demonstrate the solution your algorithm yields when applied to the $n = 6$ example above.