Theorem 1. *If $\mathcal{G} = (G, \mathcal{L})$ is the minimizing right resolving presentation of an irreducible sofic shift X and X is and N-step shift of finite type, then $\mathsf{X}_G \cong \mathsf{X}_{\mathcal{G}}$.*

Proof. Let x, y be walks in X_G. If $\mathcal{L}_\infty(x) = \mathcal{L}_\infty(y)$, then for any i, the paths $x_{[i-n,i-1]}$ and $y_{[i-n,i-1]}$ present the same word. Because that word is of length N, the word is synchronizing for $\mathcal{G}$ (from 3.4.17), so those paths end at the same vertex. Since $\mathcal{L}(x_{[i]}) = \mathcal{L}(y_{[i]})$, $\mathcal{G}$ is right resolving, and $x_{[i-n,i-1]}$ and $y_{[i-n,i-1]}$ end at the same vertex, then $x_{[i]} = y_{[i]}$ and hence $x = y$, so $\mathcal{L}_\infty$ is injective. By definition, $\mathcal{L}_\infty$ is surjective. Therefore, $\mathcal{L}_\infty$ is bijective and a conjugacy from X_G to $\mathsf{X}_{\mathcal{G}}$. $\square$

Definition 2. *A graph G is a finite set of verticies $\mathcal{V} = \mathcal{V}(G)$ and a finite set of edges $\mathcal{E} = \mathcal{E}(G)$ with each $e \in \mathcal{E}$ starting at a vertex $i(e) \in \mathcal{V}$ and terminating at a vertex $t(e) \in \mathcal{V}$. Note that two edges can start at terminate at the same vertex.*

Definition 3. *Let G and H be graphs. A graph isomorphism from G to H is a bijective pair of maps $\partial\Phi : \mathcal{V}(G) \to \mathcal{V}(H)$ and $\Phi : \mathcal{E}(G) \to \mathcal{E}(H)$ such that $i(\Phi(e)) = \partial\Phi(i(e))$ and $t(\Phi(e)) = \partial\Phi(t(e))$ for all $e \in \mathcal{E}(G)$. If there exists a graph isomorphism between G and H, then G and H are graph isomorphic and is denoted $G \cong H$.*

Definition 4. *Let $\mathcal{G} = (G, \mathcal{L}_G)$ and $\mathcal{H} = (H, \mathcal{L}_H)$ be labeled graphs. A label-graph isomorphism is a graph isomorphism $(\partial\Phi, \Phi) : G \to H$ such that $\mathcal{L}_H(\Phi(e)) = \mathcal{L}_G(e)$ for all $e \in \mathcal{E}(G)$, which is denoted $(\partial\Phi, \Phi) : \mathcal{G} \to \mathcal{H}$. If there exists a label-graph isomorphism between $\mathcal{G}$ and $\mathcal{H}$, then $\mathcal{G}$ and $\mathcal{H}$ are label-graph isomorphic (or just isomorphic) and is denoted $\mathcal{G} \cong \mathcal{H}$.*

Theorem 5. *If $(\partial\Phi, \Phi) : G \to H$ is a graph isomorphism from G to H, then $(\partial\Phi^{-1}, \Phi^{-1}) : H \to G$ is a graph isomorphism from H to G.*

Proof. For an edge $e_G \in \mathcal{E}(G)$, we have

$$\begin{aligned} \partial\Phi(i(e)) &= i(\Phi(e)) \\ \partial\Phi^{-1}(\partial\Phi(i(e))) &= \partial\Phi^{-1}(i(\Phi(e))) \\ i(e) &= \partial\Phi^{-1}(i(\Phi(e))) \end{aligned}$$

Hence, for an edge $e_H \in \mathcal{E}(H)$, $\Phi^{-1}(e_H) \in \mathcal{E}(G)$ so

$$\begin{aligned} i(\Phi^{-1}(e_H)) &= \partial\Phi^{-1}(i(\Phi(\Phi^{-1}(e_H)))) \\ &= \partial\Phi^{-1}(i(e_H)) \end{aligned}$$

A similar argument shows that $t(\Phi^{-1}(e_H)) = \partial\Phi^{-1}(t(e))$.

$\square$

Theorem 6. *Let $\mathcal{G} = (G, \mathcal{L}_G)$ and $\mathcal{H} = (H, \mathcal{L}_H)$ be labeled graphs. If $\mathcal{G} \cong \mathcal{H}$, then $\mathsf{X}_{\mathcal{G}} = \mathsf{X}_{\mathcal{H}}$.*

Proof. For $x \in \mathsf{X}_{\mathcal{G}}$, there exists a $y \in \mathsf{X}_G$ such that $x_i = \mathcal{L}_G(y_i) = \mathcal{L}_H(\Phi(y_i))$. Note that for all $i \in \mathbb{Z}$,

$$\begin{aligned} t(y_i) &= i(y_{i+1}) \\ \partial\Phi(t(y_i)) &= \partial\Phi(i(y_{i+1})) \\ t(\Phi(y_i)) &= i(\Phi(y_{i+1})) \end{aligned}$$

so $\Phi_\infty(y) \in \mathsf{X}_H$ and therefore $x = (\mathcal{L}_H \circ \Phi)_\infty(y) \in \mathsf{X}_{\mathcal{H}}$.

$\square$