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A Novel Pseudo Nearest Neighbor Classification Method Using Local Harmonic Mean Distance

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Abstract: In the realm of machine learning, the KNN classification algorithm is widely recognized as a straightforward and efficient method for classification. However, its sensitivity to the K value is too high when handling small samples or outliers, which seriously affects its classification performance. The purpose of this article is to introduce a new KNN-based classifier named the Novel Pseudo Nearest Neighbor Classification Method Using Local Harmonic Mean Distance (LMPHNN). LMPHNN is the first to introduce harmonic mean as a measure of distance to improve classification performance based on LMPNN rules and Harmonic mean distance (HMD). For our proposed LMPHNN classifier, we will begin by seeking the k nearest neighbors for every class and produce k distinct local vectors as prototypes. Next, we create pseudo nearest neighbors (PNNs) based on the local mean for each class by comparing the HMD of the sample with the first k group, which is distinct from the initial k group. The classification is ultimately determined by calculating the Euclidean distance between the query sample and PNNs, which is based on the local mean of these categories. To evaluate the effectiveness of the suggested classifier, extensive experiments are conducted on numerous real UCI datasets and combined datasets by employing this approach in addition to seven KNN-based classifiers. We use metrics such as precision, recall, accuracy, and F1 as evaluation criteria. According to the experimental results, LMPHNN achieves an average precision of 97% on all datasets, surpassing other methods by an impressive margin of 14%. The average recall was 80%, a 12% improvement over other methods. The average accuracy of the proposed classifier exceeds that of other classifiers, demonstrating a 5% enhancement. It also achieves a higher average F1 value, 13% higher than other methods. In summary, other classifiers are not as good as the proposed classifier in terms of performance, and LMPHNN demonstrates lower sensitivity when dealing with small sample sizes.

Keywords: k nearest neighbor; local mean vector; pseudo nearest neighbor; Harmonic mean distance

1 Introduction

Classification algorithms are a crucial technique in machine learning. It can automatically determine which data belongs to a category based on the input data, and correctly divide different instances into their respective classifications. Many fields, including text classification and pattern recognition, frequently employ classification algorithms. It possesses the ability to measure and forecast objects effectively, resulting in great usefulness. Currently, the popular classification algorithms in machine learning consist of decision trees, Bayesian classification, support vector machines, neural networks, k nearest neighbor(KNN) algorithms, and so on. The KNN rule [1] is considered a traditional nonparametric classification algorithm [2–4]. Within the realm of pattern classification, Fix and Hodges [5] were the first to introduce the KNN rule, which is known to be one of the simplest nonparametric

techniques. The main idea behind the KNN algorithm is to locate the KNNs of the query sample and assign the most representative category to the query sample by majority voting. This algorithm combines classification ideas with simple implementation. KNN is renowned for its excellent performance in the Bayesian sense when the training sample size N and neighborhood size K approach infinity, with the condition that the limit of K/N tends to zero [6]. Despite having numerous advantages, KNN classifiers, there are still some major disadvantages, as follows [7, 8]. When dealing with a limited amount of data, particularly when outliers [9] are present, the expected Bayesian asymptotic performance is challenging for the algorithm to attain, KNN ignores the geometric distribution of neighbors and is not always the most suitable when measuring the distance of the query sample from the neighbor, and a suitable distance metric is not found. To mitigate the impact of outliers, Mitani and Hamamoto [10] propose a dependable local mean-based k -nearest neighbor algorithm (LMKNN), in which the local mean vector (LMV) of KNNs of every single class is used to categorize patterns of queries [3, 10]. LMKNN [3] has been in existence since its initial introduction, LMKNN has been effectively utilized in group-based classification [11], discriminant analysis [12], and distance metric learning [13]. Later, To address the issue resulting from selecting KNNs of the identical state, measures need to be taken, Zeng et al. [14] proposed a pseudo-nearest neighbor (PNN) rule using distance-weighted local learning. PNN classifiers are a method of generating pseudo nearest neighbor samples by first finding KNNs samples in each class and then using the distance between different neighbors, instead of using the original nearest neighbor samples. Classification is done by querying the category of the nearest pseudo neighbor. From the extensive experimental results reported in [3, 14], LMKNN and PNN classifiers show significant resistance and superior performance in classification performance. A new PNN classifier is introduced, which utilizes the local mean as a basis (LMPNN) [15]. The LMPNN merges the fundamental concepts of the LMKNN and PNN rules. The LMPNN classifier achieves good classification performance by merging the distance with assigned weights between the query sample and the LMV in each category to obtain the pseudo distance. Although these algorithms are well done for outliers in [3, 10, 14, 16], their sensitivity to small sample sizes remains in terms of classification performance due to sample noise and imprecision [2, 17].

From the spatial geometric distribution of neighbor locations, the nearest centroid neighbor (NCN) was successfully derived to make neighbors as symmetrical as possible around the query sample position in the event of limited sample size [13]. Subsequently, in the KNN-based pattern classification [18], the first proposal introduces the k -nearest centroid neighbor (KNCN) rule, which builds upon the NCN concept. The main goal of the KNCN rule is to offer a justification for the similarity based on distance and the arrangement in space of k neighbors in the training dataset. Its excellent performance has been demonstrated in numerous experimental studies, especially when dealing with a small sample size. [18–20]. In any case, KNCN overestimates the importance of some centroid neighbors, which can cause classification performance [19–21] to become unreliable, and it is inappropriate for KNCN to assume that k -centroid neighbors have the same weight when making classification decisions. Subsequently, an extended KNCN method was proposed—the LMKNCN classifier [4], the query pattern is assigned to the class label that has the nearest local centroid mean vector, and this scheme comprehensively considers the adjacent relationship and distribution of K neighbors in space, but also takes into account the more dependable local averages within distinct categories when making classification decisions. For the parameter k , it is less sensitive to classification performance. Although the performance of the classifier has surpassed KNCN, it is still at a sufficient level. Nevertheless, it is crucial to acknowledge that it is only appropriate for specific problems. In KNN-based classification, the Euclidean distance is the conventional measure [8, 22, 23] used, which assumes that the data follows a Gaussian isotropic distribution. Conversely, when the size of the neighborhood K is large, the assumption that the community has an isotropic distribution is frequently invalid. Hence, the size of the neighborhood is highly dependent on the community's scale, denoted as K . To determine the sample's distance from the training sample, you need to look for a suitable distance metric, the MLM-KHNN [24] classifier first introduced the HMD as a distance metric, which is different from the traditional Euclidean distance, the HMD metric considers the distance between the local mean under different k values and the query

variable x , reducing the dependence and sensitivity to k , LMKHNCN [25] based on this idea also introduced HMD as a distance metric into LMKNCN. Experiments show that the classifier exhibits good results, and since the results obtained using the HMD measure perform well, we use this measure in our proposed method.

Considering the dialogue on appeal, to address the issue of k 's sensitivity in the KNN-based classification algorithm and enhance classification performance, we present an LMPHNN classifier. This classifier derives from the LMPNN classifier and employs HMD as a distance metric in addition to the local mean. To decrease sensitivity to k , the initial step of our suggested classifier involves the use of KNNs from every class to create local means. Then, it utilizes the local mean from the top r ($r \leq k$) group in each class to generate local PNNs from the HMD of the query sample. In contrast to the LMPNN classifier, which utilizes Euclidean distance, our proposed approach employs the harmonic mean as a distance metric. While assigning weights to neighbors with different distances from the test point, harmonic means and local means are used to balance the gap between different neighbors, that is, while giving higher weights to closer neighbors, the importance of farther neighbors is considered, and harmonic means are used based on LMPNN to further reduce the sensitivity to K . To assess the accuracy of LMPHNN's classification, we conducted experiments using 9 machine learning datasets obtained from UCI and Kaggle. Experimental findings demonstrate that the suggested approach has a minimal classification error, which affirms its capability as a reliable and efficient classifier.

The remaining part of our article has been structured in the following manner: In the second section, there is a concise summary of the relevant classifiers and their underlying motivations that have been suggested, which have had an impact on our research. In the third section, we introduce our proposed LMPHNN method and discuss its application in various characteristic factors. In the fourth section, we perform a large number of experimental comparisons between the real UCI dataset and the Kaggle dataset, validate our method with other competing KNN-based classifiers, and demonstrate the superiority of our method over these competing KNN-based classifiers. Section 5 of our discussion delved into the necessary complexity for computing a rival classifier, and we provided an overall summary of the entire article in Section 6.

2 Structure

One of the top nonparametric classifiers in text classification algorithms is the KNN algorithm. There have been numerous KNN-based optimization algorithms developed in the past few years. Examples include algorithms such as LMKNN, which is founded upon the KNN algorithm using local mean, and LMPNN, which is built upon pseudo nearest neighbor using the local mean classifier. In this section, we will examine the impact of LMKNN and LMPNN classifiers on our work, as well as the concept of utilizing harmonic means as a distance metric in this article.

2.1 The LMKNN rule

Mitani and Hamamoto [3] introduced LMKNN as a solution to counteract the impact of outliers present in the training set (TS), and experiments have proved that LMKNN can serve outliers well in small-sample training. LMKNN relies on Euclidean distance as a means to determine classification decisions. To be more specific, for every category within the training set, we identify the KNNs to the query point x , and subsequently calculate the LMV of these k neighbors. Finally, the classification judgment is made according to the Euclidean distance between the query point x and these LMVs. In general classification problems, it is supposed a training set $TS = \{x_n \in \mathbb{R}^d\}_{n=1}^N$ is the TS eigenspace given d dimension, where N represents the overall count of training samples, and $y_n \in \{c_1, c_2, \dots, c_M\}$ for the class label of x_n . $TSc_i = \{x_j^i \in \mathbb{R}^m\}_{j=1}^{N_i}$ represents a subset of the class c_i , N_i denotes the overall quantity of training examples for the class. In LMKNN rules, the query pattern x can be determined using these steps.

(1) Retrieve the c_1 nearest neighbors from the set associated with each class $k(k \leq N_i)$ of the query sample, and look for the query pattern x .

Let $T_{\omega_i}^k(x) = \{x_j^i \in \mathbb{R}^d\}_{j=1}^k$ be the set of KNNs of x in class c_1 using the Euclidean distance metric, and the Euclidean distance is utilized for calculating the distance from x to y .e. Eq(1)

$$d(x, x_j^i) = \sqrt{(x - x_j^i)^T (x - x_j^i)} \quad (1)$$

(2) Determine the LMV $\text{lm}_{c_i}^k$ of the local values obtained from the set $T_{ik}^{NN}(x)$ of class c_i :

$$\text{lm}_{c_i}^k = \frac{1}{k} \sum_{j=1}^k x_j^i \quad (2)$$

(3) Apply equation(1) to determine the distance $d(x, \text{lm}_{c_i}^k)$ separating the test point x and the LMV $\text{lm}_{c_i}^k$ of class c_i .

(4) If point x represents the test point that has the least distance to the LMV of the class c_1 compared to other classes, it is classified as belonging to the class c_1 .

$$c = \arg \min_{c_i} d(x, \text{lm}_{c_i}^k) \quad (3)$$

It is important to note that when $k = 1$, the LMKNN and 1-NN classifiers yield the same classification result. However, in LMKNN, the concept of K is distinct from that of KNN. KNN selects k nearest neighbors from the entire training dataset, while LMKNN utilizes the local mean vectors of k nearest neighbors within every single category. The objective of LMKNN is to identify the category that is most similar to the local area of the query sample. As a result, incorporating local mean vectors helps to effectively mitigate the impact of outliers, particularly when dealing with limited sample sizes.

2.2 The LMPNN rule

To enhance the classification performance of the LMKNN classifier, further improvements are needed, Gou [15] et al. proposed a pseudo-nearest neighbor classifier (LMPNN) based on the local mean. This classifier's design draws inspiration from both the PNN[14] and the LMKNN[3]. Experimental results indicate that the classifier exhibits strong classification performance. To begin with the LMPNN classifier, the initial step involves identifying the set of test points as well as the KNNs from each class and then finding the local mean of the neighbors of r group ($r \leq k$) in each set, and then the sum of the weighted distances is computed by assigning weights to each local mean to form the PNN of the local mean. The classification label is then determined by comparing the distance sizes of various local mean pseudo-nearest neighbors. Based on the observation of the extensive experimental results in the range [4, 22], it can be concluded that the LMPNN classifier is more stable in dealing with outliers and has better classification performance than the traditional KNN classifier.

When provided with a query point x and a training set $TS = \{x_n \in \mathbb{R}^d\}_{n=1}^N$, $TS_{c_i}(x) = \{x_j^i \in \mathbb{R}^d\}_{j=1}^{N_i}$ is a subset of TS in class c_i , where d is the characteristic dimension, N and N_i are the sample sizes of TS and $TS_{c_i}^k$, separately. To determine the class label of x in the LMPNN rule, the following steps are followed:

(1) Find the KNNs of the test point x from TS_{c_i} of each class c_1 in the training set TS , such as $TS_{c_i}^k(x) = \{x_j^i \in \mathbb{R}^d\}_{j=1}^k$, And the KNNs $x_1^i, x_2^i \dots x_k^i$ are order blue in ascending order according to their Euclidean distance from the query point x .

(2) Calculate the LMV \bar{x}_j^i of the first $j(j \leq k)$ nearest neighbors of the test point x in class c_i

$$\bar{x}_j^i = \frac{1}{j} \sum_{r=1}^j x_r^i \quad (4)$$

Let $\bar{TS}_{c_i}^k(x) = \{\bar{x}_j^i \in \mathbb{R}^d\}_{j=1}^k$ represent the set of k LMVs corresponding to KNNs in class c_i , $d(x, \bar{x}_1^i), d(x, \bar{x}_2^i), \dots, d(x, \bar{x}_k^i)$ is the corresponding Euclidean distance from \bar{x}_j^i to x in the set $\bar{TS}_{c_i}^k(x)$

(3) Give varying weights to the k LMVs of each class, the weight \bar{W}_j^i of the j th LMV \bar{x}_j^i obtained from class c_i is determined as:

$$\bar{W}_j^i = \frac{1}{j} \quad j = 1, \dots, k \quad (5)$$

(4) Determine the local mean-based PNN of the test point x for each class, Let \bar{x}_i^{PNN} represent the local mean-based PNN of x in the class c_i . The distance $d(x, \bar{x}_i^{PNN})$ from x to \bar{x}_i^{PNN} is calculated as follows:

$$d(x, \bar{x}_i^{PNN}) = (\bar{W}_1^i \times d(x, \bar{x}_1^i) + \bar{W}_2^i \times d(x, \bar{x}_2^i) + \dots + \bar{W}_k^i \times d(x, \bar{x}_k^i)) \quad (6)$$

(5) According to the local mean of Equation (6), class c_i is assigned to the test point x , which is where the pseudo nearest neighbor is situated:

$$c = \arg \min_{c_i} d(x, \bar{x}_i^{PNN}) \quad (7)$$

2.3 Harmonic Mean Distance

The KNN classifier uses Euclidean distance as the metric and assumes data follows a Gaussian isotropic distribution. Nonetheless, when the number of chosen neighbors k is large, the assumption of a Gaussian isotropic distribution is frequently unsuitable. Consequently, the sensitivity of KNN to the number of neighbors increases significantly. The MLM-KHNN [24] classifier first utilizes HMD as a similarity measure. The main emphasis is on calculating the reliability of local means within each category, thus prioritizing more dependable methods across different categories. Unlike the LMKNN classifier, this method reduces the error rate by reducing the sensitivity to the parameter k , rather than using Euclidean distances to unify the k values of all k means in each class like the LMKNN classifier. The reason for incorporating the HMD metric into our proposed method is due to its strong performance. To calculate the HMD metric, one must sum the harmonic mean of the Euclidean distances between a particular data point and another set of data points. In this paper, we use HMD to generate local mean pseudo nearest neighbors of LMPNN. When the traditional LMPNN generates the PNNs of the local mean, the sum of the weights of the top $j(j \leq k)$ local mean and the Euclidean distance of the query sample is replaced by the HMD, and the sum of the local mean weights of the top $j(j \leq k)$ group and the harmonic mean of the query sample in every single class are solved, and the local pseudo nearest neighbor is obtained, and In the end, the test sample is categorized as the point with the lowest local average. For instance, if x represents a query sample, it is supposed $\bar{TS}_{c_i}^k(x) = \{\bar{x}_j^i \in \mathbb{R}^d\}_{j=1}^k$ represents a vector in the set that corresponds to the first nearest $j(j \leq k)$ LMV in the class c_i , $\bar{x}_j^i = \frac{1}{j} \sum_{r=1}^j x_r^i$ represents the LMV of the test point x and the nearest neighbors of the first $j(j \leq k)$ in the i th class.

Afterwards, $\text{HMD}(x, \{\bar{x}_j^i\}_{j=1}^j)$ from the test point x and set $\bar{TS}_{c_i}^k(x)$ is calculated as:

$$\text{HMD}(x, \{\bar{x}_j^i\}_{j=1}^j) = \frac{j}{\sum_{r=1}^j \frac{1}{d(x, \{\bar{x}_j^i\}_{j=1}^j)}} \quad (8)$$

3 The presented LMPHNN classifier

3.1 The motivation

Statistical pattern recognition researchers have noted that the LMKHNN algorithm's performance is greatly influenced by the selection of a single neighborhood size k for each class, as well as the decision to use a uniform k value across all classes in the rule. However, it is well-known that LMKHNN exhibits robustness to outliers, primarily due to its utilization of the HMD. In contrast, the concept of LMPNN has shown the ability to overcome the influence of outliers, thus leading to improved classification performance.

Taking inspiration from the robustness of LMKHNN and the effectiveness of LMPNN, we propose a novel classification method that aims to combine the strengths of both approaches. This innovative approach is expected to enhance classification performance by mitigating the sensitivity to k values while simultaneously addressing the challenge of outliers.

LMPNN is a PNN classifier based on LMVs, which reduces the effect of noise on the classification results and improves the classification performance by being more accurate in selecting PNNs. Local subclasses are generated by the KNNs of each class. Hence, the selection of parameter k holds significant importance in producing LMVs that accurately portray their respective classes.

Nonetheless, the selection of parameter k in the LMPNN rule presents two main challenges that may lead to misclassifications.

First, selecting an appropriate parameter k plays a crucial role in generating a LMV that effectively encapsulates the characteristics of its corresponding class. Selecting a small value for k might result in the PNN not being accurate enough, thus affecting the classification performance. Selecting a large value for k might result in the PNN containing too much noise, thus affecting the classification performance. Second, if the selected features are not representative enough, then even if the appropriate k value is chosen, it may lead to misclassification. Utilizing the same k value for all classes, as in the LMPNN rule, is not a reasonable approach.

We introduce a novel classifier termed the LMPHNN rule, leveraging local means to effectively address this issue. The approach involves the computation of the KNNs for each class, resulting in k distinct LMVs. Notably, these LMVs vary in their distances from the query sample, and the ones that have shorter distances are considered more appropriate for representing their respective classes for classification purposes. Our emphasis lies in identifying these closer local subclasses, allowing for varying k values across different classes.

To achieve this goal, we propose using HMD, evaluating the similarity between each class's set of multi-LMVs and the query sample x . Subsequently, we classify the sample into the class exhibiting the smallest HMD. In comparison to LMPNN and other KNN-based classifiers, our LMPHNN classifier has a reduced classification error rate and a decreased sensitivity to changes in the neighborhood size k .

3.2 The LMPHNN classification rule

The LMPHNN is presented in this subsection as an extension of the LMPNN rule. Although the method we propose has a formal resemblance to the LMPNN rule, it is fundamentally different from it in terms of the scheme for finding PNNs. In the LMPNN rule, the LMVs of the first KNNs in each category are first computed. Then, the PNNs are determined by using the k LMVs of these categories. The LMPHNN method, on the other hand, introduces the HMD between multiple LMVs in each category and the query sample x is measured to determine the similarity between them. Ultimately, the samples are categorized based on the categories with pseudo-nearest HMDs.

Let $T = \{x_n \in \mathbb{R}^d\}_{n=1}^N$ consist of a set of M classes for training purposes $\omega_1, \dots, \omega_M$ and $T_{\omega_i} = \{x_j^i \in \mathbb{R}^d\}_{j=1}^{N_i}$ be a class subset of T from the class ω_i , where d represents the feature dimension, N and

N_i represent the sample numbers of T and T_{ω_i} , respectively. In the proposed LMPHNN rule, the class label of a query point x is determined through the following steps:

(1) Find the KNNs from T_{ω_i} for each class in ω_i or the test point x in the training set T , referred to as $T_{\omega_i}^k(x) = \{x_j^i \in \mathbb{R}^d\}_{j=1}^k$, and the KNNs $x_1^i, x_2^i, \dots, x_k^i$ are sorted in ascending order based on their corresponding Euclidean distances to x .

(2) Calculate the LMV \bar{x}_j^i of the first j nearest neighbors of a test point x from class ω_i .

$$\bar{x}_j^i(x) = \frac{1}{j} \sum_{l=1}^j x_l^i \quad (9)$$

Let $\bar{T}_{\omega_i}^k(x) = \{\bar{x}_j^i \in \mathbb{R}^d\}_{j=1}^k$ denote the set of the k LMVs corresponding to the KNNs in the class ω_i , and $d(x, \bar{x}_1^i), d(x, \bar{x}_2^i), \dots, d(x, \bar{x}_k^i)$ represent the Euclidean distances to x for each of them.

$HMD(x, \bar{x}_j^k)$ is the HMD between x and the k multi-LMVs \bar{x}_j^i

$$HMD(x, \bar{x}_j^k) = \frac{k}{\sum_{i=1}^k \frac{1}{d(x, \bar{x}_j^i)}} \quad (10)$$

Note that the LMV \bar{x}_1^i for the first nearest neighbor, denoted as x_1^i , is identical to this neighbor.

(3) Allocate varying weights to the k LMVs in each class, following a similar approach as PNN.

Specifically, the weight \bar{W}_j^i of the j -th LMV \bar{x}_j^i from the class ω_i is calculated as:

$$\bar{W}_j^i = \frac{1}{j} \quad j = 1, \dots, k \quad (11)$$

(4) Discover the LMPNN for the sample point x within every class. Denote \bar{x}_i^{PNN} as the LMPNN for x from class ω_i , and let ω_i represent the class label of ω_i . The harmonic distance

$HMD(x, \bar{x}_i^{PNN})$ between x and \bar{x}_i^{PNN} is computed as follows:

$$HMD(x, \bar{x}_i^{PNN}) = \bar{W}_1^i \times HMD(x, \bar{x}_1^i) + \bar{W}_2^i \times HMD(x, \bar{x}_2^i) + \dots + \bar{W}_k^i \times HMD(x, \bar{x}_k^i) \quad (12)$$

(5) Classify the test point x as belonging to class c based on the LMPNN distance metric, which is the HMD to the nearest neighbors, as defined by Eq. (12), of all the classes.

$$c = \arg \min_{\omega_i} HMD(x, \bar{x}_i^{PNN}) \quad (13)$$

3.3 The LMPHNN algorithm

The steps to implement the LMPHNN classifier are as follows:

Requirements:

x : a query pattern

$T = \{x_n \in \mathbb{R}^d\}_{n=1}^N$: a training set

N_1, \dots, N_M : the amount of training samples for each of the M classes.

$\omega_1, \dots, \omega_M$: M class labels.

$T_{\omega_i} = \{x_j^i \in \mathbb{R}^d\}_{j=1}^{N_i}$: a training subset from the class ω_i

k : the neighborhood size.

M : the number of classes in T

Ensure: Predict the label assigned to a query pattern for classification purposes based on its closest LMPNN among classes.

Step 1: Compute the distances from class ω_i to x for each of the training samples.

for $j=1$ **to** N_i **do**

$$d(x, x_j^i) = \sqrt{(x - x_j^i)^T (x - x_j^i)}$$

end for

Step 2: Find the KNN of x in T_{ω_i} , sorted in ascending order based on their distances $d(x, x_j^i)$, say

$$T_{\omega_i}^k(x) = \{x_j^i \in \mathbb{R}^d\}_{j=1}^k$$

Step 3: Using $T_{\omega_i}^k(x)$, Compute the LMV \bar{x}_j^i of x 's first j nearest neighbors, and then calculate the HMD $HMD(x, \bar{x}_j^i)$ between x and k multi-LMVs \bar{x}_j^i .

for $j=1$ **to** k **do**

$$\bar{x}_j^i(x) = \frac{1}{j} \sum_{l=1}^j x_l^i$$

$$d(x, \bar{x}_j^i) = \sqrt{(x - \bar{x}_j^i)^T (x - \bar{x}_j^i)}$$

$$HMD(x, \bar{x}_j^i) = \frac{k}{\sum_{l=1}^k \frac{1}{d(x, \bar{x}_l^i)}}$$

end for

Subsequently, establish $\bar{T}_{\omega_i}^k(x) = \{\bar{x}_j^i \in \mathbb{R}^d\}_{j=1}^k$ and $\bar{D}_{\omega_i}^k = \{HMD(x, \bar{x}_1^i), \dots, HMD(x, \bar{x}_k^i)\}$

Step 4: Allocate the weight \bar{W}_j^i to the j -th LMV \bar{x}_j^i in the set $T_{\omega_i}^k(x)$.

for $j=1$ **to** k **do**

$$\bar{W}_j^i = \frac{1}{j} \quad j = 1, \dots, k$$

end for

Subsequently, set $\bar{W}_{\omega_i}^k = \{\bar{W}_1^i, \dots, \bar{W}_k^i\}$

Step 5: Using $\bar{W}_{\omega_i}^k$ and $\bar{D}_{\omega_i}^k$, finding the LMPNN \bar{x}_i^{PNN} that satisfies the given constraint

$$HMD(x, \bar{x}_i^{PNN}) = \bar{W}_1^i \times HMD(x, \bar{x}_1^i) + \bar{W}_2^i \times HMD(x, \bar{x}_2^i) + \dots + \bar{W}_k^i \times HMD(x, \bar{x}_k^i)$$

Step 6: Assign the class c of the LMPNN with the nearest HMD to x .

$$c = \arg \min_{\omega_i} HMD(x, \bar{x}_i^{PNN})$$

Note that when $k = 1$, it is categorized into LMPNN rules and PNN rules, which exhibit comparable classification performance to the 1-NN classifier. Section 3.2 provides a detailed explanation of the pseudo-code used for the LMPHNN classifier.

3.4 Difference between LMPHNN and LMPNN

In pattern classification, computational complexity plays a crucial role in designing efficient classifiers. To elucidate the advantages of the proposed LMPHNN classifier, in this section, we will examine and compare the computational complexity of the LMPNN and LMPHNN classifiers. Specifically, our focus is on assessing the computational complexity associated with online computations during the classification phase.

In the following discussion, the following notation is utilized by us: n for the overall quantity of training examples, n_1 for the number of training examples belonging to the class ω_i , d for the feature space is characterized by the number of dimensions it has, and c for the number of classes.

For the LMPNN classifier:

In the stage of classification, the initial step involves searching for the KNN in each class by using Euclidean distance as the determining factor. The computational complexity of this operation can be described as $O(n_1d + n_2d + \dots + n_cd)$, which can also be abbreviated as $O(nd)$. Moreover, the $O(n_1k + n_2k + \dots + n_ck)$ comparisons remain unchanged, which are equal to $O(nk)$.

Subsequently, we compute k LMVs corresponding to the KNN per class and determine the distances between these categorical k LMVs and the query pattern.

The step's computational complexity can be described as $O(2ckd)$ is required. Next, we assign weight w_j to the LMVs per class and find the j -th LMPNN for each class. This step requires computational operations on the order of $O(3ck)$.

The final step involves determining the class of the query pattern using a certain approach, denoted as $O(c)$. Consequently, the computational complexity of LMPNN is $O(nd + nk + 2ckd + 3ck + c)$ in all its steps.

For the proposed LMPHNN classifier:

There are four steps involved in the classification process of the proposed LMPHNN classifier. The initial step is similar to the LMPNN rule, where LMPNN initially computes the distances between the query pattern and all the training samples of each class with a complexity of $O(nd + nk)$.

The LMPHNN method acquires k multilocalized mean vectors during the second step, which requires the use of $O(2ckd)$.

For each class, the D is computed between the query x and k multi-LMVs in the third step. This calculation involves $O(cdk)$ multiplications and $O(cdk + ck)$ sum operations, as demonstrated in Eq(9).

Fourth step, the weight W_j is designated to the j -th LMV per class, and the LMPNN for each class is determined. This step also demands $O(3ck)$.

In conclusion, the proposed method determines the class with the lowest HMD to the given query and assigns the query sample to that class through comparisons, which $O(c)$ represents computational complexity. Therefore, the overall computational complexity of the LMPHNN rule can be summarized as $O(nd + nk + 4cdk + 4ck + c)$.

4 Experiments

To confirm the classification performance of the proposed LMPHNN, we conduct a comparison between LMPHNN and other competing classifiers: KNN[1], LMKNN[3], KNCN[18], LMPNN[15], LMKHNN[24], LMKNCN[4] and PNN[14]. Extensive experiments on 9 real UCI and Kaggle datasets were conducted to study the error rate, which is a highly effective metric in the domain of pattern classification. In addition, we verify the effectiveness of the proposed approach on various real datasets by assessing additional metrics including accuracy, recall, precision, and F1.

All the experiments of the study were conducted on @2.30GHz 11th generation Intel Core i7-11800H CPU. The Windows 10 operating system is running on a system memory of 16gb, with a 64-bit architecture. The experimental platform was Matlab R2020a.

4.1 Datasets

In this subsection, we offer a summary of the datasets that we used in our experiments. A total of nine datasets from UCI and Kaggle machine learning datasets are selected, which are sourced from real-world scenarios. These 9 real-world datasets differ significantly in terms of sample size, attributes, and categories.

Its main features include sample size, attributes, classes and test sets are displayed in the table provided below.

Table 1: Datasets from UCI and Kaggle Used in Experiments

	Samples	Attributes	Classes	Testing set
Predictive Maintenance(PM)	10000	12	2	5000
Wine	178	13	3	79
Sleep health and lifestyle(SL)	378	12	3	189
Titanic	889	8	3	444
Airline Passenger Satisfaction(APS)	981	23	3	490
Bank Marketing(BM)	4878	17	2	2439
Breast Cancer(BC)	4024	16	2	2012
Telecom Churn(TC)	2666	20	2	1333
Milk Quality Prediction(MQP)	1059	8	3	529

4.2 Experiments on real UCI, Kaggle datasets

To demonstrate and objectively emphasise the classification performance of the proposed LMPHNN method, the LMPHNN is experimentally compared with KNN, LMKNN, LMKNCN, KNCN, LMPNN, and PNN on nine real UCI, Kaggle datasets through error rates.

For classifying query samples, the LMKNN rule uses LMVs in each class. To achieve improved classification performance, PNN and LMPNN rules are successfully developed and built upon the LMKNN rule. The PNN rule initially seeks to acquire PNNs within each class and subsequently assigns the labels of the nearest PNNs to the query samples, while the LMPNN rule is a synthesis of the concepts from both the LMKNN and the PNN approaches. KNCN is a classification algorithm whose rule uses each

class of the KNN samples to classify the query samples. LMKNCN is an improved version of the KNCN algorithm that computes the LMV of the nearest neighbor samples. LMKHNN is a measure of local similarity based on LMKNN that uses HMD.

Table 2: Accuracy of each model

Accuracy	LMPHN N	PNN	LMPN N	LMKN N	LMKHNN N	KNN	KNCN	LMKNCN
PM	0.5294	0.5208	0.5266	0.5241	0.5278	0.5066	0.5184	0.5209
Wine	0.7890	0.7191	0.7341	0.7553	0.7665	0.6704	0.7079	0.7054
SL	0.8507	0.7808	0.8333	0.8363	0.8130	0.8315	0.7945	0.8459
Titanic	0.7585	0.6236	0.7370	0.7055	0.7447	0.6929	0.6987	0.7067
APS	0.6009	0.4299	0.5748	0.5755	0.5957	0.5741	0.5571	0.5506
BM	0.7732	0.6281	0.7580	0.7463	0.7676	0.7413	0.7217	0.7252
BC	0.8857	0.8258	0.8703	0.8576	0.8781	0.8710	0.8442	0.8462
TC	0.8649	0.1620	0.8456	0.8250	0.8571	0.8080	0.8097	0.8110
MQP	0.9966	0.9401	0.9945	0.9836	0.9943	0.9861	0.9910	0.9962
	0.7832	0.6256	0.7638	0.7566	0.7716	0.7424	0.7381	0.7453 0.7408

Table 3: Recall of each model

Recall	LMPHNN	PNN	LMPNN	LMKNN	LMKHNN	KNN	KNCN	LMKNCN
PM	0.5655	0.5572	0.5629	0.5593	0.5636	0.3632	0.5959	0.6062
Wine	0.8084	0.7440	0.7579	0.7759	0.7909	0.6179	0.5659	0.4976
SL	0.8658	0.8003	0.8512	0.8502	0.8315	0.8083	0.7675	0.8151
Titanic	0.7815	0.6531	0.7625	0.7319	0.7707	0.6369	0.2219	0.2315
APS	0.6344	0.4640	0.6105	0.6082	0.6294	0.3915	0.0102	0.0163
BM	0.7972	0.6639	0.7827	0.7708	0.7913	0.6638	0.6765	0.7009
BC	0.8981	0.8433	0.8846	0.8741	0.8915	0.6875	0.9067	0.9193
TC	0.8820	0.2532	0.8624	0.8466	0.8751	0.6981	0.2864	0.2509
MQP	0.9971	0.9468	0.9952	0.9845	0.9950	0.9802	0.9781	0.9907

Table 4: Precision of each model

Precision	LMPHN N	PNN	LMPN N	LMKNN	LMKHNN	KNN	KNCN	LMKNCN N
PM	0.9310	0.9299	0.9303	0.9316	0.9310	0.5111	0.5992	0.5996
Wine	0.9742	0.9618	0.9659	0.9703	0.9665	0.6464	0.8632	0.9658
SL	0.9816	0.9712	0.9771	0.9827	0.9766	0.7921	0.8866	0.9290
Titanic	0.9687	0.9448	0.9636	0.9586	0.9642	0.6290	0.3584	0.3606
APS	0.9401	0.8990	0.9331	0.9372	0.9386	0.4219	0.0748	0.0920
BM	0.9681	0.9531	0.9670	0.9662	0.9687	0.7813	0.6989	0.6948
BC	0.9861	0.9788	0.9833	0.9803	0.9846	0.7444	0.9091	0.9011
TC	0.9801	0.9049	0.9801	0.9726	0.9793	0.5793	0.3253	0.3139
MQP	0.9996	0.9928	0.9994	0.9992	0.9994	0.9847	1.0000	1.0000

Table 5: F1 of each model

F1	LMPHN N	PNN	LMPNN	LMKNN	LMKHN N	KNN	KNCN	LMKNC N
PM	0.6923	0.6849	0.6899	0.6877	0.6909	0.4196	0.5975	0.6029
Wine	0.8820	0.8366	0.8466	0.8603	0.8678	0.6297	0.6833	0.6564
SL	0.9193	0.8765	0.9090	0.9108	0.8969	0.7996	0.8226	0.8682
Titanic	0.8627	0.7681	0.8486	0.8272	0.8537	0.6242	0.2738	0.2816
APS	0.7507	0.6013	0.7300	0.7305	0.7466	0.4049	0.0179	0.0276
BM	0.8720	0.7716	0.8623	0.8546	0.8685	0.7092	0.6875	0.6978
BC	0.9394	0.9046	0.9306	0.9233	0.9351	0.7040	0.9079	0.9101
TC	0.9275	0.2789	0.9163	0.9039	0.9230	0.6297	0.3046	0.2786
MQP	0.9983	0.9691	0.9973	0.9917	0.9972	0.9820	0.9889	0.9953

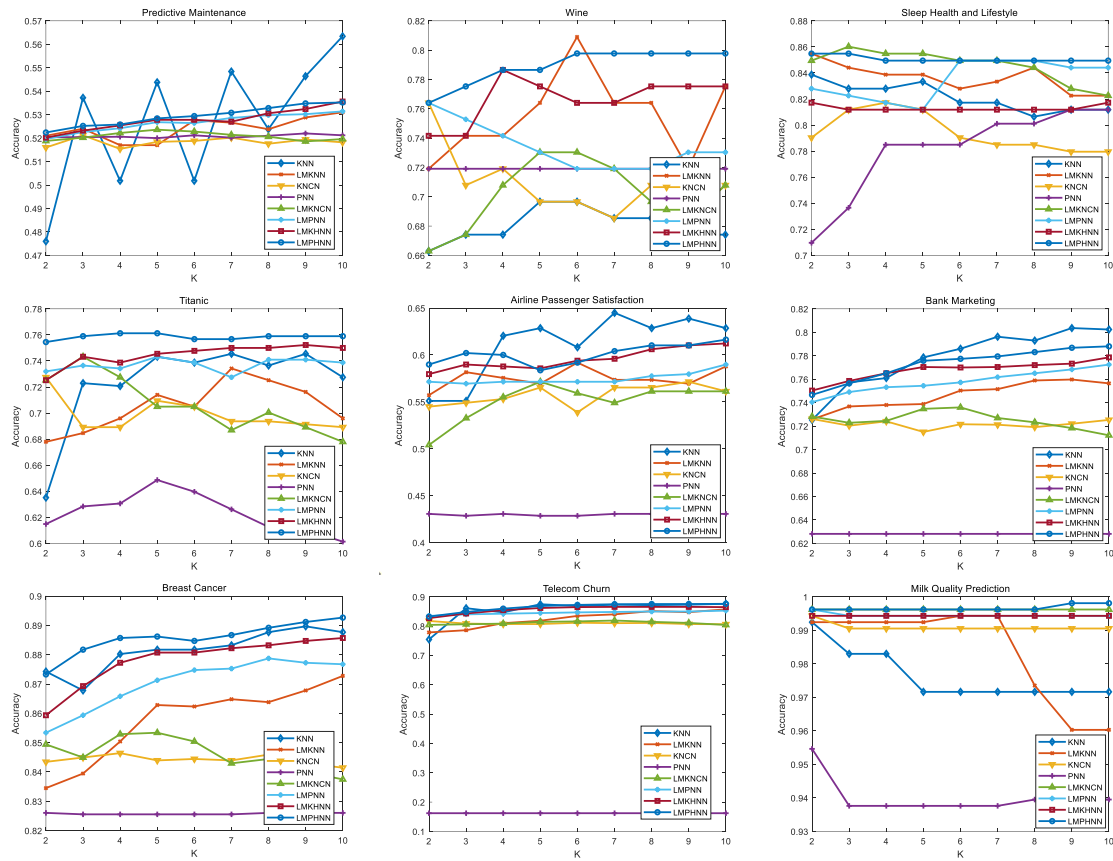


Figure 1: Accuracy curve of each model on the datasets

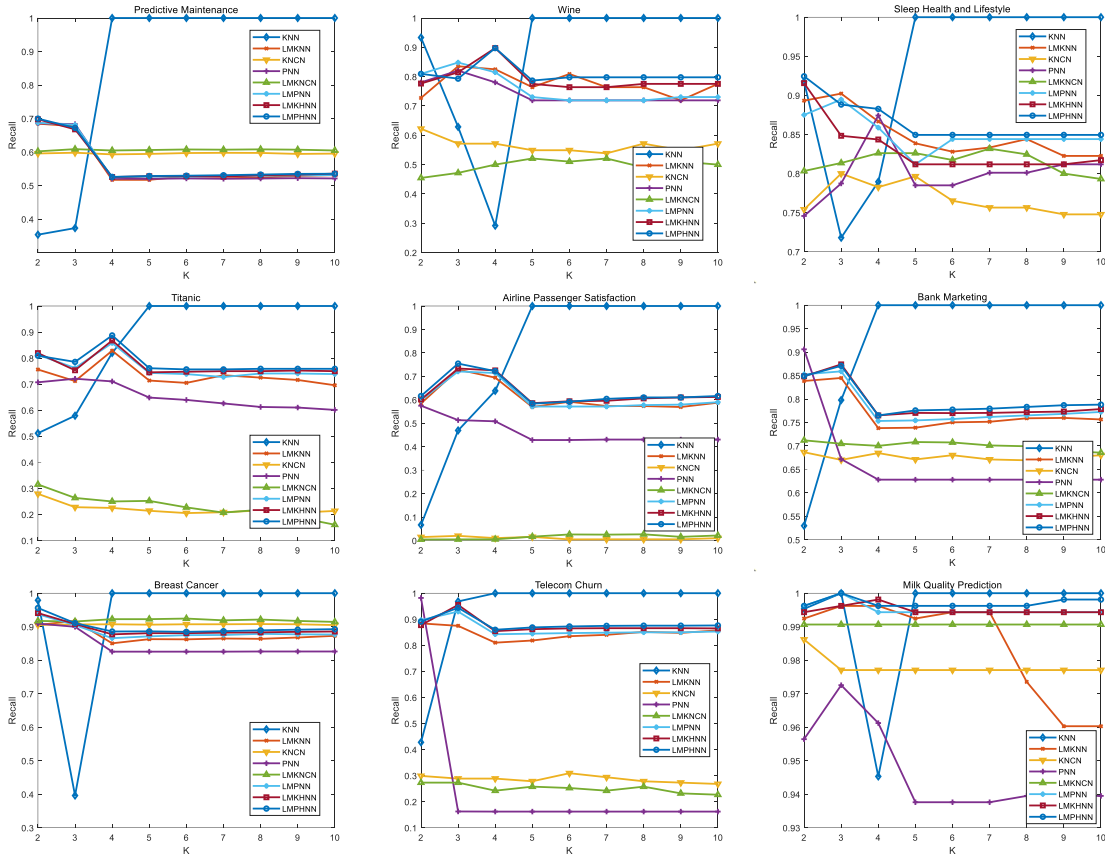
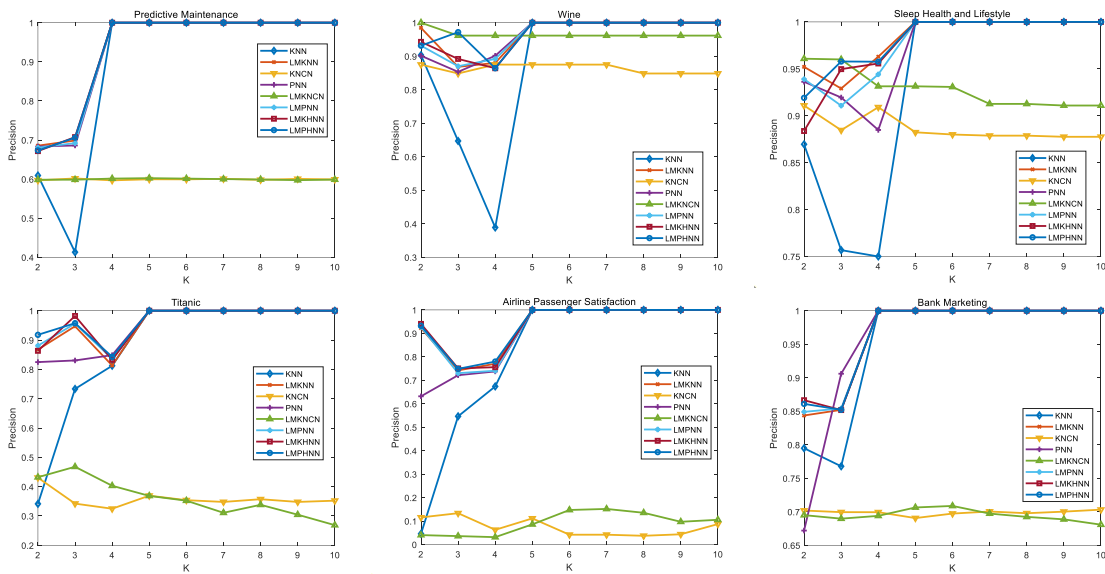


Figure 2: Recall accuracy curve of each model on the datasets



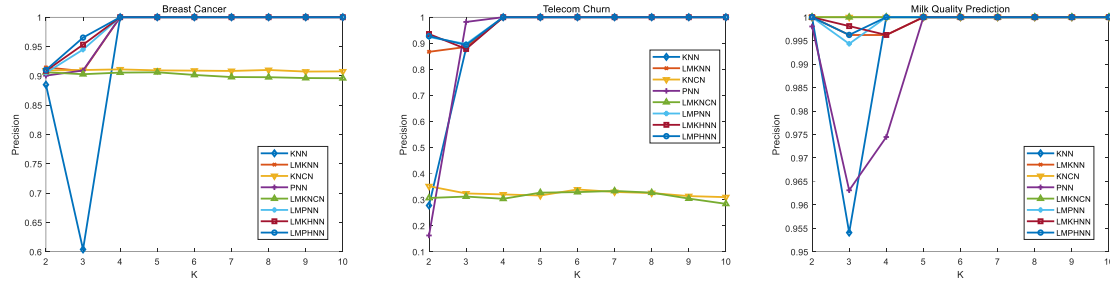


Figure 3: Precision accuracy curve of each model on the datasets

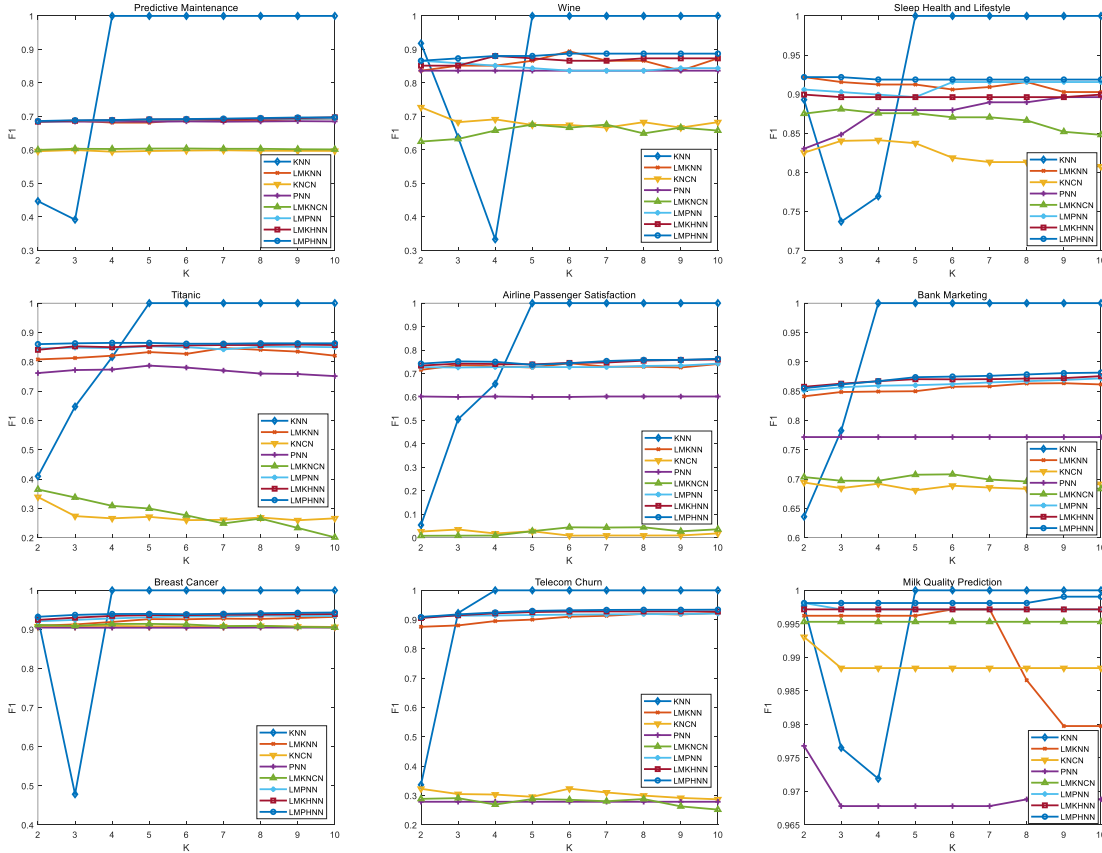


Figure 4: F1 Accuracy curve of each model on the datasets

The test set is formed by selecting the remaining samples from each real dataset after randomly choosing the training samples. The ratio of training and test sets is 1:1. We performed a total of nine experiments on each dataset, resulting in nine different training and test sets that were utilized to evaluate performance. In Step 1, we chose the neighborhood parameter k in a range from 2 to 10 for these experiments.

Table 2 displays the experimental findings, it is evident that the proposed LMPHNN classifier consistently outperforms all nine advanced KNN-based methods in terms of classification accuracy across nearly all nine real datasets.

In particular, by introducing the concept of HMD similarity, the LMPHNN classifier focuses on more dependable local means within various classes by coordinating the means, which results in a decreased sensitivity of the proposed method to the parameter k as it more accurately captures the features and similarities between the data and considerably decreases the standard LMPNN rule's error rate, which

improves the accuracy and performance of the classification. This part will be proved in 4.3.

For LMKNN, LMKHNN and LMKNCN classifiers, although they have quite high precision or recall on a few datasets, the proposed LMPHNN classifiers are still able to outperform them on most datasets.

In terms of performance, the suggested LMPHNN outperforms the other five methods, according to the results obtained. This is a good indication that our LMPHNN method has better robustness than KNN, LMKNN, LMKNCN, KNCN, LMPNN and PNN.

4.3 Effect of HMD on K-value sensitivity

To determine the impact of introducing the HMD on the sensitivity of the k value, we conducted an experiment aimed at reducing the sensitivity of the k value to the performance of the algorithm. In this study, we selected three algorithms related to our approach: the LMPNN, the LMKNN, and the LMKHNN, and compared them with our proposed new approach, the LMPHNN. We carried out experiments on a total of four datasets.

To measure the impact of the algorithm's performance on the k values, it is necessary to quantify their sensitivity, and we chose to use the evaluation's standard deviation. Specifically, for each k value, we calculate the differences between all metric values (including accuracy, recall, precision, and F1) and their respective means. Subsequently, we compute the standard deviations of these metric differences. This analysis allows us to assess the stability and sensitivity of the model's performance across different K values. The formula is as follows.

σ : standard deviation

k : the neighborhood size.

x_i : represents the value of indicator i. The indicators are accuracy, recall, precision and F1

\bar{x} : represents the mean of the indicators, which are accuracy, recall, precision and F1

$$\sigma = \sqrt{\frac{1}{k} \sum_{i=2}^k (x_i - \bar{x})^2} \quad (14)$$

Based on the experiments described above, we conducted a study in the range of K values from 2 to 10. In calculating the standard deviation, we set the value of K to 10, thus obtaining the standard deviation of the indicators with k in the range from 2 to 10.

A smaller standard deviation indicates that the algorithm's results vary to a lesser extent for different values of k, which means that the algorithm is less sensitive to changes in k values. This metric helps us to assess the stability and robustness of the algorithm after the introduction of the HMD, as well as its impact on the sensitivity to k values.

Table 6 Standard deviation of accuracy for each model

Accuracy	LMPHNN	LMPNN	LMKNN	LMKHNN
Wine	0.0123	0.0159	0.0285	0.0157
SL	0.0024	0.0134	0.0108	0.0024
Titanic	0.0021	0.0047	0.0176	0.0079
BC	0.0054	0.0084	0.0124	0.0081

Table 7 Standard deviation of recall for each model

Recall	LMPHNN	LMPNN	LMKNN	LMKHNN
Wine	0.0339	0.0510	0.0404	0.0426
SL	0.0270	0.0232	0.0302	0.0347

Titanic	0.0410	0.0412	0.0379	0.0406
BC	0.0218	0.0212	0.0208	0.0193

Table 8 Standard deviation of precision for each model

Precision	LMPHNN	LMPNN	LMKNN	LMKHNN
Wine	0.0473	0.0534	0.0548	0.0540
SL	0.0298	0.0355	0.0274	0.0404
Titanic	0.0521	0.0589	0.0666	0.0628
BC	0.0291	0.0326	0.0369	0.0306

Table 9 Standard deviation of F1 for each model

F1	LMPHNN	LMPNN	LMKNN	LMKHNN
Wine	0.0077	0.0105	0.0184	0.0101
SL	0.0014	0.0080	0.0064	0.0014
Titanic	0.0013	0.0031	0.0121	0.0053
BC	0.0030	0.0048	0.0072	0.0046

Based on the results in the above table, it can be clearly seen that LMPHNN performs well in terms of overall performance. This not only provides solid support for our new approach but also further validates the positive impact of the introduction of HMD in producing a significant reduction in the sensitivity of the classifier to k-value changes. This finding provides strong evidence that our algorithm is able to achieve better performance under different k-values while maintaining stability and robustness, which is of great practical significance for solving real-world problems in related fields.

5 Conclusions

This paper presents a newly developed technique referred to as the pseudo-nearest neighbor rule based on the local mean of the harmonic mean distance(LMPHNN). Our inspiration for LMPHNN comes from the LMPNN and HMD rules. LMPHNN serves as an extension of LMPNN with the primary objective of addressing outliers to enhance classification performance. Within the framework of LMPHNN, we employ the HMD as a similarity metric and leverage LMVs from the LMPNN rule for classification based on the respective LMVs of each nearest neighbor within each class. Subsequently, the assigned class label to the query pattern is determined based on the class label of the PNN that is nearest, as determined by the local mean. To judge the classification performance of this approach, extensive experiments were conducted across nine real UCI and Kaggle datasets. Comparative analyses were performed against well-known classifiers including KNN, LMKNN, LMKNCN, KNCN, LMPHNN, and PNN. The consistent experimental outcomes underscore the effectiveness and stability of the LMPHNN method, resulting in favorable classification performance. Furthermore, an area for future research lies in the development of an adaptive LMPNN method that incorporates the harmonic mean distance for pattern classification.

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Availability of Data and Materials: The data that support the findings of this study are available from the corresponding author, Z. Lu, upon reasonable request.

Conflicts of Interest: The authors declare that they have no conflicts of interest to report regarding the present study.

References

- [1] T. Cover and P. Hart, "Nearest neighbor pattern classification," *IEEE Trans. Inf. Theory*, vol. 13, no. 1, pp. 21–27, Jan. 1967.
- [2] Z. Ma, Z. Liu, C. Luo, and L. Song, "Evidential classification of incomplete instance based on K-nearest centroid neighbor," *J. Intell. Fuzzy Syst.*, vol. 41, no. 6, pp. 7101–7115, Jan. 2021.
- [3] Y. Mitani and Y. Hamamoto, "A local mean-based nonparametric classifier," *Pattern Recognit. Lett.*, vol. 27, no. 10, pp. 1151–1159, Jul. 2006.
- [4] J. Gou, Z. Yi, L. Du, and T. Xiong, "A Local Mean-Based k-Nearest Centroid Neighbor Classifier," *Comput. J.*, vol. 55, no. 9, pp. 1058–1071, Sep. 2012.
- [5] E. Fix and J. L. Hodges, "Discriminatory Analysis. Nonparametric Discrimination: Consistency Properties," *Int. Stat. Rev. Rev. Int. Stat.*, vol. 57, no. 3, pp. 238–247, 1989.
- [6] T. Wagner, "Convergence of the nearest neighbor rule," *IEEE Trans. Inf. Theory*, vol. 17, no. 5, pp. 566–571, Sep. 1971.
- [7] X. Wu *et al.*, "Top 10 algorithms in data mining," *Knowl. Inf. Syst.*, vol. 14, no. 1, pp. 1–37, Jan. 2008.
- [8] J. Gou, L. Du, Y. Zhang, and T. Xiong, "A New Distance-weighted k -nearest Neighbor Classifier," *J Inf Comput Sci*, vol. 9, Nov. 2011.
- [9] K. Fukunaga, *Introduction to Statistical Pattern Recognition*. Elsevier, 2013.
- [10] Y. Mitani and Y. Hamamoto, "Classifier design based on the use of nearest neighbor samples," in *Proceedings 15th International Conference on Pattern Recognition. ICPR-2000*, Barcelona, Spain: IEEE Comput. Soc, 2000.
- [11] N. A. Samsudin and A. P. Bradley, "Nearest neighbour group-based classification," *Pattern Recognit.*, vol. 43, no. 10, pp. 3458–3467, Oct. 2010.
- [12] J. Yang, L. Zhang, J. Yang, and D. Zhang, "From classifiers to discriminators: A nearest neighbor rule induced discriminant analysis," *Pattern Recognit.*, vol. 44, no. 7, pp. 1387–1402, Jul. 2011.
- [13] J. Chai, H. Liu, B. Chen, and Z. Bao, "Large margin nearest local mean classifier," *Signal Process.*, vol. 90, no. 1, pp. 236–248, Jan. 2010.
- [14] Y. Zeng, Y. Yang, and L. Zhao, "Pseudo nearest neighbor rule for pattern classification," *Expert Syst. Appl.*, vol. 36, no. 2, Part 2, pp. 3587–3595, Mar. 2009.
- [15] J. Gou, Y. Zhan, Y. Rao, X. Shen, X. Wang, and W. He, "Improved pseudo nearest neighbor classification," *Knowl.-Based Syst.*, vol. 70, pp. 361–375, Nov. 2014.
- [16] Y. Zeng, Y. Yang, and L. Zhao, "Nonparametric classification based on local mean and class statistics," *Expert Syst. Appl.*, vol. 36, no. 4, pp. 8443–8448, May 2009.
- [17] Y. Xu, Q. Zhu, Z. Fan, M. Qiu, Y. Chen, and H. Liu, "Coarse to fine K nearest neighbor classifier," *Pattern Recognit. Lett.*, vol. 34, no. 9, pp. 980–986, Jul. 2013.
- [18] J. S. Sánchez, F. Pla, and F. J. Ferri, "On the use of neighbourhood-based non-parametric classifiers1This work has partially been supported by projects P1B96-13 (Fundació Caixa-Castelló) and AGF95-0712-C03-01 and TIC95-676-C02-01 (Spanish CICYT).1," *Pattern Recognit. Lett.*, vol. 18, no. 11, pp. 1179–1186, Nov. 1997.
- [19] J. S. Sánchez, F. Pla, and F. J. Ferri, "Improving the k-NCN classification rule through heuristic modifications1This work has partially been supported by projects P1B96-13 (Fundació Caixa-Castelló), and AGF95-0712-C03-01 and TIC95-676-C02-01 (Spanish CICYT).1," *Pattern Recognit. Lett.*, vol. 19, no. 13, pp. 1165–1170, Nov. 1998.
- [20] J. S. Sánchez, F. Pla, and F. J. Ferri, "Prototype selection for the nearest neighbour rule through proximity graphs," *Pattern Recognit. Lett.*, vol. 18, no. 6, pp. 507–513, Jun. 1997.
- [21] S. Grabowski, "Limiting the set of neighbors for the k-NCN decision rule: greater speed with preserved classification accuracy," in *Proceedings of the International Conference Modern Problems of Radio Engineering, Telecommunications and Computer Science, 2004*.
- [22] J. Yu, Q. Tian, J. Amores, and N. Sebe, "Toward Robust Distance Metric Analysis for Similarity Estimation," in *2006 IEEE Computer Society Conference on Computer Vision and Pattern Recognition (CVPR'06)*, Jun. 2006, pp. 316–322.

- [23] J. Wang, P. Neskovic, and L. N. Cooper, "Improving nearest neighbor rule with a simple adaptive distance measure," *Pattern Recognit. Lett.*, vol. 28, no. 2, pp. 207–213, Jan. 2007.
- [24] Z. Pan, Y. Wang, and W. Ku, "A new k-harmonic nearest neighbor classifier based on the multi-local means," *Expert Syst. Appl.*, vol. 67, pp. 115–125, Jan. 2017.
- [25] S. Mehta, X. Shen, J. Gou, and D. Niu, "A New Nearest Centroid Neighbor Classifier Based on K Local Means Using Harmonic Mean Distance," *Information*, vol. 9, no. 9, Art. no. 9, Sep. 2018.
- [26] Y. Yang, P. Zheng, F. Zeng, P. Xin, G. He, and K. Liao, "Metal Corrosion Rate Prediction of Small Samples Using an Ensemble Technique," *Comput. Model. Eng. Sci.*, vol. 134, no. 1, pp. 267–291, 2022.