

Algorithm Library

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头文件

DEBUG 头

```
1  #include <bits/stdc++.h>
2  using namespace std;
3  using i64=long long;
4  using i128=__int128;
5
6  namespace DBG
7  {
8      template <class T>
9      void _dbg(const char *f,T t) { cerr<<f<<'\n'; }
10
11     template <class A,class... B>
12     void _dbg(const char *f,A a,B... b)
13     {
14         while (*f!=',') cerr<<*f++;
15         cerr<<'\n';
16         _dbg(f+1,b...);
17     }
18
19     template <class T>
20     ostream& operator << (ostream& os,const vector<T> &v)
21     {
22         os<<"[ ";
23         for (const auto &x:v) os<<x<<", ";
24         os<<"]";
25         return os;
26     }
27
28     #define dbg(...) _dbg(#__VA_ARGS__, __VA_ARGS__)
29 }
30
31 using namespace DBG;
```

__int128 输出流

```
1  ostream &operator << (ostream &os,i128 n)
2  {
3      string s;
4      bool neg=n<0;
5      if (neg) n=-n;
6      while (n)
7      {
8          s+='0'+n%10;
9          n/=10;
10     }
11     if (neg) s+='-';
12     reverse(s.begin(),s.end());
13     if (s.empty()) s+='0';
14     return os<<s;
15 }
```

常用数学函数

```
1  i64 ceilDiv(i64 n,i64 m)
2  {
3      if (n>=0) return (n+m-1)/m;
4      else return n/m;
5  }
6
7  i64 floorDiv(i64 n,i64 m)
8  {
9      if (n>=0) return n/m;
10     else return (n-m+1)/m;
11 }
12
13 i128 gcd(i128 a,i128 b)
14 {
```

```

15     return b?gcd(b,a%b):a;
16 }

```

数学

欧拉筛

```

1  vector<int> minp,primes;
2
3  void sieve(int n)
4  {
5      minp.assign(n+1,0);
6      primes.clear();
7      for (int i=2;i<=n;i++)
8      {
9          if (!minp[i])
10             {
11                 minp[i]=i;
12                 primes.push_back(i);
13             }
14             for (auto p:primes)
15             {
16                 if (i*p>n) break;
17                 minp[i*p]=p;
18                 if (p==minp[i]) break;
19             }
20     }
21 }

```

取模类 (MInt)

```

1  template <class T>
2  constexpr T power(T a,i64 b)
3  {
4      T res=1;
5      for (;b>=1,a*=a)
6          if (b&1) res*=a;
7      return res;
8  }
9
10 template <int P>
11 struct MInt
12 {
13     int x;
14     constexpr MInt():x{} {}
15     constexpr MInt(i64 x):x{norm(x%getMod())} {}
16
17     static int Mod;
18     constexpr static int getMod()
19     {
20         if (P>0) return P;
21         else return Mod;
22     }
23
24     constexpr static void setMod(int Mod_) { Mod=Mod_; }
25
26     constexpr int norm(int x) const
27     {
28         if (x<0) x+=getMod();
29         if (x>=getMod()) x-=getMod();
30         return x;
31     }
32
33     constexpr int val() const { return x; }
34
35     explicit constexpr operator int () const { return x; }
36
37     constexpr MInt operator - () const
38     {

```

```

39     MInt res;
40     res.x=norm(getMod()-x);
41     return res;
42 }
43
44 constexpr MInt inv() const
45 {
46     assert(x!=0);
47     return power(*this,getMod()-2);
48 }
49
50 constexpr MInt &operator *= (MInt rhs) &
51 {
52     x=1ll*x*rhs.x%getMod();
53     return *this;
54 }
55
56 constexpr MInt &operator += (MInt rhs) &
57 {
58     x=norm(x+rhs.x);
59     return *this;
60 }
61
62 constexpr MInt &operator -= (MInt rhs) &
63 {
64     x=norm(x-rhs.x);
65     return *this;
66 }
67
68 constexpr MInt &operator /= (MInt rhs) &
69 {
70     return *this*=rhs.inv();
71 }
72
73 friend constexpr MInt operator * (MInt lhs,MInt rhs)
74 {
75     MInt res=lhs;
76     res*=rhs;
77     return res;
78 }
79
80 friend constexpr MInt operator + (MInt lhs,MInt rhs)
81 {
82     MInt res=lhs;
83     res+=rhs;
84     return res;
85 }
86
87 friend constexpr MInt operator - (MInt lhs,MInt rhs)
88 {
89     MInt res=lhs;
90     res-=rhs;
91     return res;
92 }
93
94 friend constexpr MInt operator / (MInt lhs,MInt rhs)
95 {
96     MInt res=lhs;
97     res/=rhs;
98     return res;
99 }
100
101 friend constexpr istream &operator >> (istream &is,MInt &a)
102 {
103     i64 v;
104     is>>v;
105     a=MInt(v);
106     return is;
107 }
108
109 friend constexpr ostream &operator << (ostream &os,const MInt &a) { return os<<a.val(); }

```

```

110
111     friend constexpr bool operator == (MInt lhs, MInt rhs) { return lhs.val() == rhs.val(); }
112
113     friend constexpr bool operator != (MInt lhs, MInt rhs) { return lhs.val() != rhs.val(); }
114 };
115
116 template<>
117 int MInt<0>::Mod=1;
118
119 template<int V, int P>
120 constexpr MInt<P> CInv=MInt<P>(V).inv();

```

组合数

```

1  struct Comb
2  {
3      int n;
4      vector<Z> _fac, _inv, _finv;
5
6      Comb():n{0}, _fac{1}, _inv{0}, _finv{1}{}
7      Comb(int n):Comb() { init(n); }
8
9      void init(int m)
10     {
11         m=min(m, Z::getMod()-1);
12         if (m<=n) return;
13         _fac.resize(m+1);
14         _inv.resize(m+1);
15         _finv.resize(m+1);
16
17         for (int i=n+1; i<=m; i++)
18             _fac[i]=_fac[i-1]*i;
19         _finv[m]=_fac[m].inv();
20         for (int i=m; i>n; i--)
21         {
22             _finv[i-1]=_finv[i]*i;
23             _inv[i]=_finv[i]*_fac[i-1];
24         }
25         n=m;
26     }
27
28     Z fac(int m)
29     {
30         if (m>n) init(m<<1);
31         return _fac[m];
32     }
33
34     Z finv(int m)
35     {
36         if (m>n) init(m<<1);
37         return _finv[m];
38     }
39
40     Z inv(int m)
41     {
42         if (m>n) init(m<<1);
43         return _inv[m];
44     }
45
46     Z binom(int n, int m)
47     {
48         if (n<m || m<0) return 0;
49         return fac(n)*finv(m)*finv(n-m);
50     }
51 } comb;

```

多项式

```

1  vector<int> rev;
2  vector<Z> roots{0,1};

```

```

3
4 void dft(vector<Z> &a)
5 {
6     int n=a.size();
7     if (int(rev.size())!=n)
8     {
9         int k=__builtin_ctz(n)-1;
10        rev.resize(n);
11        for (int i=0;i<n;i++)
12            rev[i]=rev[i>>1]>>1|(i&1)<<k;
13    }
14    for (int i=0;i<n;i++)
15        if (rev[i]<i)
16            swap(a[i],a[rev[i]]);
17    if (int(roots.size())<n)
18    {
19        int k=__builtin_ctz(roots.size());
20        roots.resize(n);
21        while ((1<<k)<n)
22        {
23            Z e=power(Z(3),(P-1)>>(k+1));
24            for (int i=1<<(k-1);i<(1<<k);i++)
25            {
26                roots[i<<1]=roots[i];
27                roots[i<<1|1]=roots[i]*e;
28            }
29            k++;
30        }
31    }
32    for (int k=1;k<n;k<<=1)
33        for (int i=0;i<n;i+=k*2)
34            for (int j=0;j<k;j++)
35            {
36                Z u=a[i+j],v=a[i+j+k]*roots[j+k];
37                a[i+j]=u+v;
38                a[i+j+k]=u-v;
39            }
40 }
41
42 void idft(vector<Z> &a)
43 {
44     int n=a.size();
45     reverse(a.begin()+1,a.end());
46     dft(a);
47     Z inv=(1-P)/n;
48     for (int i=0;i<n;i++) a[i]*=inv;
49 }
50
51 struct Poly
52 {
53     vector<Z> a;
54
55     Poly(){}
56     explicit Poly(int size,function<Z(int)>f=[](int) { return 0; }):a(size)
57     { for (int i=0;i<size;i++) a[i]=f(i); }
58     Poly(const vector<Z> &a):a(a){}
59     Poly(const initializer_list<Z> &a):a(a){}
60
61     int size() const { return a.size(); }
62
63     void resize(int n) { a.resize(n); }
64
65     Z operator [] (int idx) const
66     {
67         if (idx<size()) return a[idx];
68         else return 0;
69     }
70
71     Z &operator [] (int idx) { return a[idx]; }
72
73     Poly mulxk(int k) const

```

```

74 {
75     auto b=a;
76     b.insert(b.begin(),k,0);
77     return Poly(b);
78 }
79
80 Poly modxk(int k) const
81 {
82     k=min(k,size());
83     return Poly(vector<Z>(a.begin(),a.begin()+k));
84 }
85
86 Poly divxk(int k) const
87 {
88     if (size()<=k) return Poly();
89     return Poly(vector<Z>(a.begin()+k,a.end()));
90 }
91
92 friend Poly operator + (const Poly &a,const Poly &b)
93 {
94     vector<Z> res(max(a.size(),b.size()));
95     for (int i=0;i<int(res.size());i++)
96         res[i]=a[i]+b[i];
97     return Poly(res);
98 }
99
100 friend Poly operator - (const Poly &a,const Poly &b)
101 {
102     vector<Z> res(max(a.size(),b.size()));
103     for (int i=0;i<int(res.size());i++)
104         res[i]=a[i]-b[i];
105     return Poly(res);
106 }
107
108 friend Poly operator - (const Poly &a)
109 {
110     vector<Z> res(a.size());
111     for (int i=0;i<int(res.size());i++)
112         res[i]=-a[i];
113     return Poly(res);
114 }
115
116 friend Poly operator * (Poly a,Poly b)
117 {
118     if (!a.size()||!b.size()) return Poly();
119     if (a.size()<b.size()) swap(a,b);
120     if (b.size()<128)
121     {
122         Poly c(a.size()+b.size()-1);
123         for (int i=0;i<a.size();i++)
124             for (int j=0;j<b.size();j++)
125                 c[i+j]+=a[i]*b[j];
126         return c;
127     }
128     int sz=1,tot=a.size()+b.size()-1;
129     while (sz<tot) sz<<=1;
130     a.a.resize(sz);
131     b.a.resize(sz);
132     dft(a.a);
133     dft(b.a);
134     for (int i=0;i<sz;i++)
135         a.a[i]=a[i]*b[i];
136     idft(a.a);
137     a.a.resize(tot);
138     return a;
139 }
140
141 friend Poly operator * (Z a,Poly b)
142 {
143     for (int i=0;i<int(b.size());i++) b[i]*=a;
144     return b;

```



```

145     }
146
147     friend Poly operator * (Poly a,Z b)
148     {
149         for (int i=0;i<int(a.size());i++) a[i]*=b;
150         return a;
151     }
152
153     Poly &operator += (Poly b) { return (*this)=(*this)+b; }
154     Poly &operator -= (Poly b) { return (*this)=(*this)-b; }
155     Poly &operator *= (Poly b) { return (*this)=(*this)*b; }
156     Poly &operator *= (Z b) { return (*this)=(*this)*b; }
157
158     Poly deriv() const
159     {
160         if (a.empty()) return Poly();
161         vector<Z> res(size()-1);
162         for (int i=0;i<size()-1;i++)
163             res[i]=(i+1)*a[i+1];
164         return Poly(res);
165     }
166
167     Poly integr() const
168     {
169         vector<Z> res(size()+1);
170         for (int i=0;i<size();i++)
171             res[i+1]=a[i]/(i+1);
172         return Poly(res);
173     }
174
175     Poly inv(int m) const
176     {
177         Poly x{a[0].inv()};
178         int k=1;
179         while (k<m)
180         {
181             k<<=1;
182             x=(x*(Poly{2}-modxk(k)*x)).modxk(k);
183         }
184         return x.modxk(m);
185     }
186
187     Poly ln(int m) const { return (deriv()*inv(m)).integr().modxk(m); }
188
189     Poly exp(int m) const
190     {
191         Poly x{1};
192         int k=1;
193         while (k<m)
194         {
195             k<<=1;
196             x=(x*(Poly{1}-x.ln(k)+modxk(k))).modxk(k);
197         }
198         return x.modxk(m);
199     }
200
201     Poly pow(int k,int m) const
202     {
203         int i=0;
204         while (i<size()&&a[i].val()==0) i++;
205         if (i==size()||1ll*i*k>=m) return Poly(vector<Z>(m));
206         Z v=a[i];
207         auto f=divxk(i)*v.inv();
208         return (f.ln(m-i*k)*k).exp(m-i*k).mulxk(i*k)*power(v,k);
209     }
210
211     Poly sqrt(int m) const
212     {
213         Poly x{1};
214         int k=1;
215         while (k<m)

```

```

216     {
217         k<=1;
218         x=(x+(modxk(k)*x.inv(k)).modxk(k))*((P+1)/2);
219     }
220     return x.modxk(m);
221 }
222 Poly mult(Poly b) const
223 {
224     if (b.size()==0) return Poly();
225     int n=b.size();
226     reverse(b.a.begin(),b.a.end());
227     return ((*this)*b).divxk(n-1);
228 }
229
230 vector<Z> eval(vector<Z> x) const
231 {
232     if (size()==0) return vector<Z>(x.size(),0);
233     const int n=max(int(x.size()),size());
234     vector<Poly> q(n<<2);
235     vector<Z> ans(x.size());
236     x.resize(n);
237     function<void(int,int,int)> build=[&](int p,int l,int r)
238     {
239         if (r-l==1) q[p]=Poly{1,-x[l]};
240         else
241         {
242             int m=(l+r)>>1;
243             build(p<<1,l,m);
244             build(p<<1|1,m,r);
245             q[p]=q[p<<1]*q[p<<1|1];
246         }
247     };
248     function<void(int,int,int,const Poly&)> work=[&](int p,int l,int r,const Poly &num)
249     {
250         if (r-l==1)
251         {
252             if (l<int(ans.size())) ans[l]=num[0];
253         }
254         else
255         {
256             int m=(l+r)>>1;
257             work(p<<1,l,m,num.mult(q[p<<1|1]).modxk(m-l));
258             work(p<<1|1,m,r,num.mult(q[p<<1]).modxk(r-m));
259         }
260     };
261     build(1,0,n);
262     work(1,0,n,mult(q[1].inv(n)));
263     return ans;
264 }
265 };

```

原根表

	prime	r	k	g
1	3	1	1	2
2	5	1	2	2
3	17	1	4	3
4	97	3	5	5
5	193	3	6	5
6	257	1	8	3
7	7681	15	9	17
8	12289	3	12	11
9	40961	5	13	3
10	65537	1	16	3
11	786433	3	18	10
12	5767169	11	19	3
13	7340033	7	20	3
14	23068673	11	21	3
15	104857601	25	22	3
16	167772161	5	25	3
17	469762049	7	26	3

19	1004535809	479	21	3
20	2013265921	15	27	31
21	2281701377	17	27	3
22	3221225473	3	30	5
23	75161927681	35	31	3
24	77309411329	9	33	7
25	206158430209	3	36	22
26	2061584302081	15	37	7
27	2748779069441	5	39	3
28	6597069766657	3	41	5
29	39582418599937	9	42	5
30	79164837199873	9	43	5
31	263882790666241	15	44	7
32	1231453023109121	35	45	3
33	1337006139375617	19	46	3
34	3799912185593857	27	47	5
35	4222124650659841	15	48	19
36	7881299347898369	7	50	6
37	31525197391593473	7	52	3
38	180143985094819841	5	55	6
39	1945555039024054273	27	56	5
40	4179340454199820289	29	57	3

线性基

```

1  struct LB
2  {
3      static constexpr int L=60;
4      array<i64,L+1> a{};
5
6      LB(){}
7
8      LB(const vector<i64> &v) { init(v); }
9
10     bool insert(i64 t)
11     {
12         for (int i=L;i>=0;i--)
13             if (t&(1ll<<i))
14             {
15                 if (!a[i])
16                 {
17                     a[i]=t;
18                     return 1;
19                 }
20                 t^=a[i];
21             }
22         return 0;
23     }
24
25     void init(const vector<i64> &v) { for (auto x:v) insert(x); }
26
27     bool check(i64 t)
28     {
29         for (int i=L;i>=0;i--)
30             if (t&(1ll<<i))
31                 if (!a[i]) return 0;
32             else t^=a[i];
33         return 1;
34     }
35
36     i64 QueryMax()
37     {
38         i64 res=0;
39         for (int i=L;i>=0;i--)
40             res=max(res,res^a[i]);
41         return res;
42     }
43
44     i64 QueryMin()
45     {
46         for (int i=0;i<=L;i++)

```

```

47         if (a[i]) return a[i];
48     return 0;
49 }
50
51 i64 QueryKth(int k)
52 {
53     i64 res=0;
54     int cnt=0;
55     array<i64,L+1> tmp{};
56     for (int i=0;i<=L;i++)
57     {
58         for (int j=i-1;j>=0;j--)
59             if (a[i]&(1ll<<j)) a[i]^=a[j];
60         if (a[i]) tmp[cnt++]=a[i];
61     }
62     if (k>=(1ll<<cnt)) return -1;
63     for (int i=0;i<cnt;i++)
64         if (k&(1ll<<i)) res^=tmp[i];
65     return res;
66 }
67 };

```

min-plus 卷积

$\mathcal{O}(n \log n)$, 但要求 b 是凸的。

```

1  template <class T>
2  vector<T> min_plus_convolution(const vector<T> &a,const vector<T> &b)
3  {
4      int n=a.size(),m=b.size();
5      vector<T> c(n+m-1);
6
7      function<void(int,int,int,int)> solve=[&](int l,int r,int ql,int qr)
8      {
9          if (l>r) return;
10         int mid=(l+r)>>1;
11         while (ql+m<=l) ++ql;
12         while (qr>r) --qr;
13         int qmid=-1;
14         c[mid]=inf;
15         for (int i=ql;i<=qr;i++)
16         {
17             if (a[i]+b[mid-i]-i<c[mid])
18             {
19                 c[mid]=a[i]+b[mid-i];
20                 qmid=i;
21             }
22             else if (mid-i>=0&&mid-i<m) qmid=i;
23         }
24         solve(l,mid-1,ql,mid);
25         solve(mid+1,r,qmid,qr);
26     };
27
28     solve(0,n+m-2,0,n-1);
29     return c;
30 }

```

数据结构

并查集（启发式合并 + 带撤销）

```

1  struct DSU
2  {
3      int n=0;
4      vector<int> fa,siz;
5      stack<int> s;
6
7      DSU(int n) { init(n); }
8

```

```

9     void init(int n)
10    {
11        fa.resize(n);
12        iota(fa.begin(), fa.end(), 0);
13        siz.assign(n, 1);
14        while (!s.empty()) s.pop();
15    }
16
17    int get(int x) { return fa[x]==x?x:get(fa[x]); }
18
19    void merge(int x, int y)
20    {
21        x=get(x), y=get(y);
22        if (x==y) return;
23        if (siz[x]<siz[y]) swap(x, y);
24        s.push(y), fa[y]=x, siz[x]+=siz[y];
25    }
26
27    void undo()
28    {
29        if (s.empty()) return;
30        int y=s.top();
31        s.pop();
32        siz[fa[y]]-=siz[y];
33        fa[y]=y;
34    }
35
36    void back(int t=0) { while (s.size()>t) undo(); }
37 };

```

状压 RMQ

```

1  template <class T, class Cmp=less<T>>
2  struct RMQ
3  {
4      const Cmp cmp=Cmp();
5      static constexpr unsigned B=64;
6      using u64=unsigned long long;
7      int n;
8      vector<vector<T>> a;
9      vector<T> pre, suf, ini;
10     vector<u64> stk;
11
12     RMQ() {}
13     RMQ(const vector<T> &v) { init(v); }
14
15     void init(const vector<T> &v)
16     {
17         n=v.size();
18         pre=suf=ini=v;
19         stk.resize(n);
20         if (!n) return;
21         const int M=(n-1)/B+1;
22         const int lg=__lg(M);
23         a.assign(lg+1, vector<T>(M));
24         for (int i=0; i<M; i++)
25         {
26             a[0][i]=v[i*B];
27             for (int j=1; j<B&& i*B+j<n; j++)
28                 a[0][i]=min(a[0][i], v[i*B+j], cmp);
29         }
30         for (int i=1; i<n; i++)
31             if (i%B) pre[i]=min(pre[i], pre[i-1], cmp);
32         for (int i=n-2; i>=0; i--)
33             if (i%B!=B-1) suf[i]=min(suf[i], suf[i+1], cmp);
34         for (int j=0; j<lg; j++)
35             for (int i=0; i+(2<<j)<=M; i++)
36                 a[j+1][i]=min(a[j][i], a[j][i+(1<<j)], cmp);
37         for (int i=0; i<M; i++)
38         {
39             const int l=i*B;

```

```

40         const int r=min(1U*n,l+B);
41         u64 s=0;
42         for (int j=l;j<r;j++)
43         {
44             while (s&&cmp(v[j],v[__lg(s)+l])) s^=1ULL<<__lg(s);
45             s|=1ULL<<(j-l);
46             stk[j]=s;
47         }
48     }
49 }
50
51 //查询区间 [l,r) 的 RMQ
52 T operator()(int l,int r)
53 {
54     if (l/B!=(r-1)/B)
55     {
56         T ans=min(suf[l],pre[r-1],cmp);
57         l=l/B+1,r=r/B;
58         if (l<r)
59         {
60             int k=__lg(r-l);
61             ans=min({ans,a[k][l],a[k][r-(1<<k)]},cmp);
62         }
63         return ans;
64     }
65     else
66     {
67         int x=B*(l/B);
68         return ini[__builtin_ctzll(stk[r-1]>>(l-x))+1];
69     }
70 }
71 };

```

ST 表

```

1  template <class T>
2  struct ST
3  {
4      int n;
5      vector<vector<T>>> a;
6
7      ST() {}
8      ST(const vector<T> &v) { init(v); }
9
10     void init(const vector<T> &v)
11     {
12         n=v.size();
13         if (!n) return;
14         const int lg=__lg(n);
15         a.assign(lg+1,vector<T>(n));
16         a[0]=v;
17         for (int j=0;j<lg;j++)
18             for (int i=0;i+(2<<j)<=n;i++)
19                 a[j+1][i]=__gcd(a[j][i],a[j][i+(1<<j)]);
20     }
21
22     T operator()(int l,int r)
23     {
24         int k=__lg(r-l);
25         return __gcd(a[k][l],a[k][r-(1<<k)]);
26     }
27 };

```

树状数组

```

1  template <class T>
2  struct BIT
3  {
4      int n;
5      vector<T> a;

```

```

6
7 BIT(int n_=0) { init(n_); }
8
9 void init(int n_)
10 {
11     n=n_;
12     a.assign(n,T{});
13 }
14
15 void add(int x,const T &v)
16 {
17     for (int i=x+1;i<=n;i+=i&-i)
18         a[i-1]=a[i-1]+v;
19 }
20
21 //查询区间 [0,x)
22 T sum(int x)
23 {
24     T ans{};
25     for (int i=x;i>0;i-=i&-i)
26         ans=ans+a[i-1];
27     return ans;
28 }
29
30 //查询区间 [l,r)
31 T rangeSum(int l,int r) { return sum(r)-sum(l); }
32
33 int select(const T &k)
34 {
35     int x=0;
36     T cur{};
37     for (int i=1<<__lg(n);i>=1)
38     {
39         if (x+i<=n&&cur+a[x+i-1]<=k)
40         {
41             x+=i;
42             cur=cur+a[x-1];
43         }
44     }
45     return x;
46 }
47 };

```

线段树

```

1 template <class Info,class Tag>
2 struct SGT
3 {
4     int n;
5     vector<Info> info;
6     vector<Tag> tag;
7
8     SGT():n(0) {}
9     SGT(int n_,Info v_=Info()) { init(n_,v_); }
10
11     template <class T>
12     SGT(vector<T> init_) { init(init_); }
13
14     void init(int n_,Info v_=Info()) { init(vector(n_,v_)); }
15
16     template <class T>
17     void init(vector<T> init_)
18     {
19         n=init_.size();
20         info.assign(4<<__lg(n),Info());
21         tag.assign(4<<__lg(n),Tag());
22         function<void(int,int,int)> build=[&](int p,int l,int r)
23         {
24             if (r-l==1)
25             {
26                 info[p]=init_[l];

```

```

27         return;
28     }
29     int m=(l+r)>>1;
30     build(p<<1,l,m);
31     build(p<<1|1,m,r);
32     pushup(p);
33 };
34 build(1,0,n);
35 }
36
37 void pushup(int p) { info[p]=info[p<<1]+info[p<<1|1]; }
38
39 void apply(int p,const Tag &v)
40 {
41     info[p].apply(v);
42     tag[p].apply(v);
43 }
44
45 void pushdown(int p)
46 {
47     apply(p<<1,tag[p]);
48     apply(p<<1|1,tag[p]);
49     tag[p]=Tag();
50 }
51
52 void modify(int p,int l,int r,int x,const Info &v)
53 {
54     if (r-l==1)
55     {
56         info[p]=v;
57         return;
58     }
59     int m=(l+r)>>1;
60     pushdown(p);
61     if (x<m) modify(p<<1,l,m,x,v);
62     else modify(p<<1|1,m,r,x,v);
63     pushup(p);
64 }
65
66 //O(log n) 单点修改
67 void modify(int p,const Info &v) { modify(1,0,n,p,v); }
68
69 Info rangeQuery(int p,int l,int r,int x,int y)
70 {
71     if (l>=y||r<=x) return Info();
72     if (l>=x&&r<=y) return info[p];
73     int m=(l+r)>>1;
74     pushdown(p);
75     return rangeQuery(p<<1,l,m,x,y)+rangeQuery(p<<1|1,m,r,x,y);
76 }
77
78 //O(log n) 区间查询 [l,r)
79 Info rangeQuery(int l,int r) { rangeQuery(1,0,n,l,r); }
80
81 void rangeApply(int p,int l,int r,int x,int y,const Tag &v)
82 {
83     if (l>=y||r<=x) return;
84     if (l>=x&&r<=y)
85     {
86         apply(p,v);
87         return;
88     }
89     int m=(l+r)>>1;
90     pushdown(p);
91     rangeApply(p<<1,l,m,x,y,v);
92     rangeApply(p<<1|1,m,r,x,y,v);
93     pushup(p);
94 }
95
96 //O(log n) 区间操作 [l,r)
97 void rangeApply(int l,int r,const Tag &v) { rangeApply(1,0,n,l,r,v); }

```



```

98
99 //O(log n) 区间 [l,r) 内查找第一个合法位置
100 template <class F>
101 int findFirst(int p,int l,int r,int x,int y,F pred)
102 {
103     if (l>=y||r<=x||!pred(info[p])) return -1;
104     if (r-l==1) return l;
105     int m=(l+r)>>1;
106     pushdown(p);
107     int res=findFirst(p<<1,l,m,x,y,pred);
108     if (res==-1) res=findFirst(p<<1|1,m,r,x,y,pred);
109     return res;
110 }
111
112 template <class F>
113 int findFirst(int l,int r,F pred) { return findFirst(1,0,n,l,r,pred); }
114
115 template <class F>
116 int findLast(int p,int l,int r,int x,int y,F pred)
117 {
118     if (l>=y||r<=x||!pred(info[p])) return -1;
119     if (r-l==1) return l;
120     int m=(l+r)>>1;
121     pushdown(p);
122     int res=findFirst(p<<1|1,m,r,x,y,pred);
123     if (res==-1) res=findFirst(p<<1,l,m,x,y,pred);
124     return res;
125 }
126
127 template <class F>
128 int findLast(int l,int r,F pred) { return findLast(1,0,n,l,r,pred); }
129 };
130
131 //这里默认乘法优先 (x*a+b)*c+d=x*(a*c)+(b*c+d)
132 struct Tag
133 {
134     i64 a=1,b=0;
135     void apply(Tag t)
136     {
137         a*=t.a;
138         b=b*t.a+t.b;
139     }
140 };
141
142 struct Info
143 {
144     i64 x=0,l=0,r=0;
145     void apply(Tag t)
146     {
147         int len=r-l+1;
148         x=x*t.a+len*t.b;
149     }
150 };
151
152 Info operator + (Info a,Info b)
153 {
154     return {a.x+b.x,min(a.l,b.l),max(a.r,b.r)};
155 }

```

字符串

字符串哈希（随机模数）

```

1 bool isPrime(int n)
2 {
3     if (n<=1) return 0;
4     for (int i=2;i*i<=n;i++)
5         if (n%i==0) return 0;
6     return 1;
7 }

```

```

8
9  int findPrime(int n)
10 {
11     while (!isPrime(n)) n++;
12     return n;
13 }
14
15 mt19937 rng(time(0));
16 const int P=findPrime(rng()%9000000000+1000000000);
17 struct StrHash
18 {
19     int n;
20     vector<int> h,p;
21
22     StrHash(const string &s){ init(s); }
23
24     void init(const string &s)
25     {
26         n=s.size();
27         h.resize(n+1);
28         p.resize(n+1);
29         p[0]=1;
30         for (int i=0;i<n;i++) h[i+1]=(10ll*h[i]+s[i]-'a')%P;
31         for (int i=0;i<n;i++) p[i+1]=10ll*p[i]%P;
32     }
33
34     //查询 [l,r) 的区间哈希
35     int get(int l,int r) { return (h[r]+1ll*(P-h[l])*p[r-l])%P; }
36 };

```

KMP

```

1  vector<int> KMP(const string &s)
2  {
3      int now=0;
4      vector<int> pre(s.size(),0);
5      for (int i=1;i<s.size();i++)
6      {
7          while (now&&s[i]!=s[now]) now=pre[now-1];
8          if (s[i]==s[now]) now++;
9          pre[i]=now;
10     }
11     return pre;
12 }

```

Z 函数

```

1  vector<int> zFunction(string s)
2  {
3      int n=s.size();
4      vector<int> z(n);
5      z[0]=n;
6      for (int i=1,j=1;i<n;i++)
7      {
8          z[i]=max(0,min(j+z[j]-i,z[i-j]));
9          while (i+z[i]<n&&s[z[i]]==s[i+z[i]]) z[i]++;
10         if (i+z[i]>j+z[j]) j=i;
11     }
12     return z;
13 }

```

AC 自动机

```

1  struct ACAM
2  {
3      static constexpr int ALPHABET=26;
4      struct Node
5      {
6          int len;

```

```

7         int link;
8         array<int,ALPHABET> next;
9         Node():len{0},link{0},next{}{}
10    };
11
12    vector<Node> t;
13
14    ACAM() { init(); }
15
16    void init()
17    {
18        t.assign(2,Node());
19        t[0].next.fill(1);
20        t[0].len=-1;
21    }
22
23    int newNode()
24    {
25        t.emplace_back();
26        return t.size()-1;
27    }
28
29    int add(const string &a)
30    {
31        int p=1;
32        for (auto c:a)
33        {
34            int x=c-'a';
35            if (t[p].next[x]==0)
36            {
37                t[p].next[x]=newNode();
38                t[t[p].next[x]].len=t[p].len+1;
39            }
40            p=t[p].next[x];
41        }
42        return p;
43    }
44
45    void work()
46    {
47        queue<int> q;
48        q.push(1);
49        while (!q.empty())
50        {
51            int x=q.front();
52            q.pop();
53            for (int i=0;i<ALPHABET;i++)
54            {
55                if (t[x].next[i]==0) t[x].next[i]=t[t[x].link].next[i];
56                else
57                {
58                    t[t[x].next[i]].link=t[t[x].link].next[i];
59                    q.push(t[x].next[i]);
60                }
61            }
62        }
63    }
64
65    int next(int p,int x) { return t[p].next[x]; }
66
67    int link(int p) { return t[p].link; }
68
69    int size() { return t.size(); }
70 };

```

后缀数组

```

1 struct SA
2 {
3     int n;
4     vector<int> sa,rk,lc;

```

```

5  SA(const string &s)
6  {
7      n=s.length();
8      sa.resize(n);
9      rk.resize(n);
10     lc.resize(n-1);
11     iota(sa.begin(),sa.end(),0);
12     sort(sa.begin(),sa.end(),[&](int a,int b){ return s[a]<s[b]; });
13     rk[sa[0]]=0;
14     for (int i=1;i<n;i++) rk[sa[i]]=rk[sa[i-1]]+(s[sa[i]]!=s[sa[i-1]]);
15     int k=1;
16     vector<int> tmp,cnt(n);
17     tmp.reserve(n);
18     while (rk[sa[n-1]]<n-1)
19     {
20         tmp.clear();
21         for (int i=0;i<k;i++) tmp.push_back(n-k+i);
22         for (auto i:sa)
23             if (i>=k) tmp.push_back(i-k);
24         fill(cnt.begin(),cnt.end(),0);
25         for (int i=0;i<n;i++) cnt[rk[i]]++;
26         for (int i=1;i<n;i++) cnt[i]+=cnt[i-1];
27         for (int i=n-1;i>=0;i--) sa[--cnt[rk[tmp[i]]]]=tmp[i];
28         swap(rk,tmp);
29         rk[sa[0]]=0;
30         for (int i=1;i<n;i++)
31             rk[sa[i]]=rk[sa[i-1]]+(tmp[sa[i-1]]<tmp[sa[i]]||sa[i-1]+k==n||tmp[sa[i-1]+k]<tmp[sa[i]+k]);
32         k<<=1;
33     }
34     for (int i=0,j=0;i<n;i++)
35     {
36         if (rk[i]==0) j=0;
37         else
38         {
39             for (j--j>0;i+j<n&&sa[rk[i]-1]+j<n&&s[i+j]==s[sa[rk[i]-1]+j]); j++;
40             lc[rk[i]-1]=j;
41             //lc[i]:lcp(sa[i],sa[i+1]),lcp(sa[i],sa[j])=min{lc[i...j-1]}
42         }
43     }
44 };

```

(广义) 后缀自动机

```

1  struct SAM
2  {
3      static constexpr int ALPHABET=26;
4      struct Node
5      {
6          int len;
7          int link;
8          array<int,ALPHABET> next;
9          Node():len{},link{},next{} {}
10     };
11
12     vector<Node> t;
13
14     SAM() { init(); }
15
16     void init()
17     {
18         t.assign(2,Node());
19         t[0].next.fill(1);
20         t[0].len=-1;
21     }
22
23     int newNode()
24     {
25         t.emplace_back();
26         return t.size()-1;
27     }
28

```

```

29  int extend(int lst,int c)
30  {
31      if (t[lst].next[c]&& t[t[lst].next[c]].len==t[lst].len+1)
32          return t[lst].next[c];
33      int p=lst,np=newNode(),flag=0;
34      t[np].len=t[p].len+1;
35      while (!t[p].next[c])
36      {
37          t[p].next[c]=np;
38          p=t[p].link;
39      }
40      if (!p)
41      {
42          t[np].link=1;
43          return np;
44      }
45      int q=t[p].next[c];
46      if (t[q].len==t[p].len+1)
47      {
48          t[np].link=q;
49          return np;
50      }
51      if (p==lst) flag=1,np=0,t.pop_back();
52      int nq=newNode();
53      t[nq].link=t[q].link;
54      t[nq].next=t[q].next;
55      t[nq].len=t[p].len+1;
56      t[q].link=t[np].link=nq;
57      while (p&& t[p].next[c]==q)
58      {
59          t[p].next[c]=nq;
60          p=t[p].link;
61      }
62      return flag?nq:np;
63  }
64
65  int add(const string &a)
66  {
67      int p=1;
68      for (auto c:a) p=extend(p,c-'a');
69      return p;
70  }
71
72  int next(int p,int x) { return t[p].next[x]; }
73
74  int link(int p) { return t[p].link; }
75
76  int len(int p) { return t[p].len; }
77
78  int size() { return t.size(); }
79  };

```

Manacher

```

1  vector<int> manacher(vector<int> s)
2  {
3      vector<int> t{0};
4      for (auto c:s)
5      {
6          t.push_back(c);
7          t.push_back(0);
8      }
9      int n=t.size();
10     vector<int> r(n);
11     for (int i=0,j=0;i<n;i++)
12     {
13         if (j*2-i>=0&&j+r[j]>i) r[i]=min(r[j*2-i],j+r[j]-i);
14         while (i-r[i]>=0&&i+r[i]<n&&t[i-r[i]]==t[i+r[i]]) r[i]++;
15         if (i+r[i]>j+r[j]) j=i;
16     }
17     return r;

```

```
18 }
```

回文自动机

```
1 struct PAM
2 {
3     static constexpr int ALPHABET_SIZE=28;
4     struct Node
5     {
6         int len,link,cnt;
7         array<int,ALPHABET_SIZE> next;
8         Node():len{},link{},cnt{},next{}{}
9     };
10    vector<Node> t;
11    int suff;
12    string s;
13
14    PAM() { init(); }
15
16    void init()
17    {
18        t.assign(2,Node());
19        t[0].len=-1;
20        suff=1;
21        s.clear();
22    }
23
24    int newNode()
25    {
26        t.emplace_back();
27        return t.size()-1;
28    }
29
30    bool add(char c,char offset='a')
31    {
32        int pos=s.size();
33        s+=c;
34        int let=c-offset;
35        int cur=suff,curlen=0;
36        while (1)
37        {
38            curlen=t[cur].len;
39            if (pos-curlen-1>=0&&s[pos-curlen-1]==s[pos]) break;
40            cur=t[cur].link;
41        }
42        if (t[cur].next[let])
43        {
44            suff=t[cur].next[let];
45            return 0;
46        }
47        int num=newNode();
48        suff=num;
49        t[num].len=t[cur].len+2;
50        t[cur].next[let]=num;
51        if (t[num].len==1)
52        {
53            t[num].link=t[num].cnt=1;
54            return 1;
55        }
56        while (1)
57        {
58            cur=t[cur].link;
59            curlen=t[cur].len;
60            if (pos-curlen-1>=0&&s[pos-curlen-1]==s[pos])
61            {
62                t[num].link=t[cur].next[let];
63                break;
64            }
65        }
66        t[num].cnt=t[t[num].link].cnt+1;
67        return 1;
68    }
69 }
```

```

68     }
69 };

```

图论

强连通分量

```

1  struct SCC
2  {
3      int n,cur,cnt;
4      vector<vector<int>> adj;
5      vector<int> stk,dfn,low,bel;
6
7      SCC() {}
8      SCC(int n) { init(n); }
9
10     void init(int n)
11     {
12         this->n=n;
13         adj.assign(n,{});
14         stk.clear();
15         dfn.assign(n,-1);
16         low.resize(n);
17         bel.assign(n,-1);
18         cur=cnt=0;
19     }
20
21     void add(int u,int v) { adj[u].push_back(v); }
22
23     void dfs(int x)
24     {
25         dfn[x]=low[x]=cur++;
26         stk.push_back(x);
27         for (auto y:adj[x])
28         {
29             if (dfn[y]==-1)
30             {
31                 dfs(y);
32                 low[x]=min(low[x],low[y]);
33             }
34             else if (bel[y]==-1) low[x]=min(low[x],dfn[y]);
35         }
36         if (dfn[x]==low[x])
37         {
38             int y;
39             do
40             {
41                 y=stk.back();
42                 bel[y]=cnt;
43                 stk.pop_back();
44             } while (y!=x);
45             cnt++;
46         }
47     }
48
49     vector<int> work()
50     {
51         for (int i=0;i<n;i++)
52             if (dfn[i]==-1) dfs(i);
53         return bel;
54     }
55
56     struct Graph
57     {
58         int n;
59         vector<pair<int,int>> edges;
60         vector<int> siz,cnte;
61     };
62
63     Graph compress()

```

```

64     {
65         Graph G;
66         G.n=cnt;
67         G.siz.resize(cnt);
68         G.cnte.resize(cnt);
69         for (int i=0;i<n;i++)
70         {
71             G.siz[bel[i]]++;
72             for (auto j:adj[i])
73                 if (bel[i]!=bel[j])
74                     G.edges.emplace_back(bel[j],bel[i]);
75         }
76         return G;
77     };
78 };

```

边双连通分量

```

1  struct EBCC
2  {
3      int n;
4      vector<vector<int>> adj;
5      vector<int> stk,dfn,low,bel;
6      int cur,cnt;
7
8      EBCC() {}
9      EBCC(int n) { init(n); }
10
11     void init(int n)
12     {
13         this->n=n;
14         adj.assign(n,{});
15         dfn.assign(n,-1);
16         low.resize(n);
17         bel.assign(n,-1);
18         stk.clear();
19         cur=cnt=0;
20     }
21
22     void add(int u,int v)
23     {
24         adj[u].push_back(v);
25         adj[v].push_back(u);
26     }
27
28     void dfs(int x,int p)
29     {
30         dfn[x]=low[x]=cur++;
31         stk.push_back(x);
32         for (auto y:adj[x])
33         {
34             if (y==p) continue;
35             if (dfn[y]==-1)
36             {
37                 dfs(y,x);
38                 low[x]=min(low[x],low[y]);
39             }
40             else if (bel[y]==-1&&dfn[y]<dfn[x]) low[x]=min(low[x],dfn[y]);
41         }
42         if (dfn[x]==low[x])
43         {
44             int y;
45             do
46             {
47                 y=stk.back();
48                 bel[y]=cnt;
49                 stk.pop_back();
50             } while (y!=x);
51             cnt++;
52         }
53     }

```



```

54
55     vector<int> work()
56     {
57         dfs(0,-1);
58         return bel;
59     }
60
61     struct Graph
62     {
63         int n;
64         vector<pair<int,int>> edges;
65         vector<int> siz,cnte;
66     };
67
68     Graph compress()
69     {
70         Graph G;
71         G.n=cnt;
72         G.siz.resize(cnt);
73         G.cnte.resize(cnt);
74         for (int i=0;i<n;i++)
75         {
76             G.siz[bel[i]]++;
77             for (auto j:adj[i])
78             {
79                 if (bel[i]<bel[j]) G.edges.emplace_back(bel[i],bel[j]);
80                 else if (i<j) G.cnte[bel[i]]++;
81             }
82         }
83         return G;
84     };
85 };

```

轻重链剖分

```

1     struct HLD
2     {
3         int n;
4         vector<int> siz,top,dep,pa,in,out,seq;
5         vector<vector<int>> adj;
6         int cur;
7
8         HLD(){}
9         HLD(int n) { init(n); }
10
11        void init(int n)
12        {
13            this->n=n;
14            siz.resize(n);
15            top.resize(n);
16            dep.resize(n);
17            pa.resize(n);
18            in.resize(n);
19            out.resize(n);
20            seq.resize(n);
21            cur=0;
22            adj.assign(n,{});
23        }
24
25        void addEdge(int u,int v)
26        {
27            adj[u].push_back(v);
28            adj[v].push_back(u);
29        }
30
31        void work(int rt=0)
32        {
33            top[rt]=rt;
34            dep[rt]=0;
35            pa[rt]=-1;
36            dfs1(rt);

```

```

37     dfs2(rt);
38 }
39
40 void dfs1(int u)
41 {
42     if (pa[u] != -1) adj[u].erase(find(adj[u].begin(), adj[u].end(), pa[u]));
43     siz[u] = 1;
44     for (auto &v: adj[u])
45     {
46         pa[v] = u;
47         dep[v] = dep[u] + 1;
48         dfs1(v);
49         siz[u] += siz[v];
50         if (siz[v] > siz[adj[u][0]])
51             swap(v, adj[u][0]);
52     }
53 }
54
55 void dfs2(int u)
56 {
57     in[u] = cur++;
58     seq[in[u]] = u;
59     for (auto v: adj[u])
60     {
61         top[v] = (v == adj[u][0]) ? top[u] : v;
62         dfs2(v);
63     }
64     out[u] = cur;
65 }
66
67 int lca(int u, int v)
68 {
69     while (top[u] != top[v])
70     {
71         if (dep[top[u]] > dep[top[v]]) u = pa[top[u]];
72         else v = pa[top[v]];
73     }
74     return dep[u] < dep[v] ? u : v;
75 }
76
77 int dist(int u, int v) { return dep[u] + dep[v] - (dep[lca(u, v)] << 1); }
78
79 int jump(int u, int k)
80 {
81     if (dep[u] < k) return -1;
82     int d = dep[u] - k;
83     while (dep[top[u]] > d) u = pa[top[u]];
84     return seq[in[u] - dep[u] + d];
85 }
86
87 bool isAncestor(int u, int v) { return in[u] <= in[v] && in[v] < out[u]; }
88
89 int rootedParent(int u, int v) // u -> root, v -> point
90 {
91     if (u == v) return u;
92     if (!isAncestor(v, u)) return pa[v];
93     auto it = upper_bound(adj[v].begin(), adj[v].end(), u, [&](int x, int y) { return in[x] < in[y]; }) - 1;
94     return *it;
95 }
96
97 int rootedSize(int u, int v) // same as rootedParent
98 {
99     if (u == v) return n;
100    if (!isAncestor(v, u)) return siz[v];
101    return n - siz[rootedParent(u, v)];
102 }
103
104 int rootedLca(int a, int b, int c) { return lca(a, b) ^ lca(b, c) ^ lca(c, a); }
105 };

```

2-SAT

```
1  struct TwoSat
2  {
3      int n;
4      vector<vector<int>> e;
5      vector<bool> ans;
6
7      TwoSat(int n):n(n),e(n<<1),ans(n){}
8
9      void addClause(int u,bool f,int v,bool g)
10     {
11         e[u*2+!f].push_back(v*2+g);
12         e[v*2+!g].push_back(u*2+f);
13     }
14
15     bool satisfiable()
16     {
17         vector<int> id(n*2,-1),dfn(n*2,-1),low(n*2,-1),stk;
18         int now=0,cnt=0;
19         function<void(int)> tarjan=[&](int u)
20         {
21             stk.push_back(u);
22             dfn[u]=low[u]=now++;
23             for (auto v:e[u])
24             {
25                 if (dfn[v]==-1)
26                 {
27                     tarjan(v);
28                     low[u]=min(low[u],low[v]);
29                 }
30                 else if (id[v]==-1)
31                     low[u]=min(low[u],dfn[v]);
32             }
33             if (dfn[u]==low[u])
34             {
35                 int v;
36                 do
37                 {
38                     v=stk.back();
39                     stk.pop_back();
40                     id[v]=cnt;
41                 } while (v!=u);
42                 cnt++;
43             }
44         };
45         for (int i=0;i<n*2;i++)
46             if (dfn[i]==-1)
47                 tarjan(i);
48         for (int i=0;i<n;i++)
49         {
50             if (id[i*2]==id[i*2+1]) return 0;
51             ans[i]=id[i*2]>id[i*2+1];
52         }
53         return 1;
54     }
55     vector<bool> answer() { return ans; }
56 };
```

计算几何

EPS

```
1  const double eps=1e-8;
2  int sgn(double x)
3  {
4      if (fabs(x)<eps) return 0;
5      if (x>0) return 1;
6      return -1;
7  }
```

Point

```
1  template <class T>
2  struct Point
3  {
4      T x,y;
5      Point(T x_=0,T y_=0):x(x_),y(y_) {}
6
7      Point &operator += (Point p) &
8      {
9          x+=p.x;
10         y+=p.y;
11         return *this;
12     }
13
14     Point &operator -= (Point p) &
15     {
16         x-=p.x;
17         y-=p.y;
18         return *this;
19     }
20
21     Point &operator *= (T v) &
22     {
23         x*=v;
24         y*=v;
25         return *this;
26     }
27
28     Point operator - () const { return Point(-x,-y); }
29
30     friend Point operator + (Point a,Point b) { return a+=b; }
31     friend Point operator - (Point a,Point b) { return a-=b; }
32     friend Point operator * (Point a,T b) { return a*=b; }
33     friend Point operator * (T a,Point b) { return b*=a; }
34
35     friend bool operator == (Point a,Point b) { return a.x==b.x&& a.y==b.y; }
36
37     friend istream &operator >> (istream &is,Point &p) { return is>>p.x>>p.y; }
38
39     friend ostream &operator << (ostream &os,Point p) { return os<<'('<<p.x<<','<<p.y<<')'; }
40 };
41
42 template <class T>
43 T dot(Point<T> a,Point<T> b) { return a.x*b.x+a.y*b.y; }
44
45 template <class T>
46 T cross(Point<T> a,Point<T> b) { return a.x*b.y-a.y*b.x; }
47
48 template <class T>
49 T square(Point<T> p) { return dot(p,p); }
50
51 template <class T>
52 double length(Point<T> p) { return sqrt(double(square(p))); }
53
54 long double length(Point<long double> p) { return sqrt(square(p)); }
```

Line

```
1  template <class T>
2  struct Line
3  {
4      Point<T> a,b;
5      Line(Point<T> a_=Point<T>(),Point<T> b_=Point<T>()):a(a_),b(b_) {}
6  };
```

距离

```
1  template <class T>
2  double dis_PP(Point<T> a,Point<T> b) { return length(a-b); }
```

```

3
4 template <class T>
5 double dis_PL(Point<T> a,Line<T> l) { return fabs(cross(a-l.a,a-l.b))/dis_PP(l.a,l.b); }
6
7 template <class T>
8 double dis_PS(Point<T> a,Line<T> l)
9 {
10     if (dot(a-l.a,l.b-l.a)<0) return dis_PP(a,l.a);
11     if (dot(a-l.b,l.a-l.b)<0) return dis_PP(a,l.b);
12     return dis_PL(a,l);
13 }

```

点绕中心旋转

```

1 template <class T>
2 Point<T> rotate(Point<T> a,double alpha)
3 { return Point<T>(a.x*cos(alpha)-a.y*sin(alpha),a.x*sin(alpha)+a.y*cos(alpha)); }

```

关于线的对称点

```

1 template <class T>
2 Point<T> lineRoot(Point<T> a,Line<T> l)
3 {
4     Point<T> v=l.b-l.a;
5     return l.a+v*(dot(a-l.a,v)/dot(v,v));
6 }
7
8 template <class T>
9 Point<T> symmetry_PL(Point<T> a,Line<T> l) { return a+(lineRoot(a,l)-a)*2; }

```

位置关系判断

```

1 template <class T>
2 bool pointOnSegment(Point<T> a,Line<T> l)
3 { return (sgn(cross(a-l.a,a-l.b))==0)&&(sgn(dot(a-l.a,a-l.b))<=0); }
4
5 template <class T>
6 bool lineCrossLine(Line<T> a,Line<T> b)
7 {
8     double f1=cross(b.a-a.a,a.b-a.a),f2=cross(b.b-a.a,a.b-a.a);
9     double g1=cross(a.a-b.a,b.b-b.a),g2=cross(a.b-b.a,b.b-b.a);
10    return ((f1<0)^(f2<0))&&((g1<0)^(g2<0));
11 }
12
13 template <class T>
14 bool pointOnLineLeft(Point<T> a,Line<T> l) { return cross(l.b-l.a,a-l.a)>0; }
15
16 //适用任意多边形,0(n)
17 template <class T>
18 bool pointInPolygon(Point<T> a,const vector<Point<T>> &p)
19 {
20     int n=p.size();
21     for (int i=0;i<n;i++)
22         if (pointOnSegment(a,Line<T>(p[i],p[(i+1)%n])))
23             return 1;
24     bool t=0;
25     for (int i=0;i<n;i++)
26     {
27         Point<T> u=p[i],v=p[(i+1)%n];
28         if (u.x<a.x&&v.x>a.x&&pointOnLineLeft(a,Line<T>(v,u))) t^=1;
29         if (u.x>a.x&&v.x<a.x&&pointOnLineLeft(a,Line<T>(u,v))) t^=1;
30     }
31     return t;
32 }
33
34 //适用凸多边形,0(log n)
35 template <class T>
36 bool pointInPolygon_(Point<T> a,const vector<Point<T>> &p)
37 {

```

```

38     int n=p.size();
39     if (cross(a-p[0],p[1]-p[0])<0||cross(a-p[0],p[n-1]-p[0])>0) return 0;
40     if (pointOnSegment(a,Line<T>(p[0],p[1]))||pointOnSegment(a,Line<T>(p[n-1],p[0]))) return 1;
41     int l=1,r=n-1;
42     while (l+1<r)
43     {
44         int mid=(l+r)>>1;
45         if (cross(a-p[l],p[mid]-p[l])<0) l=mid;
46         else r=mid;
47     }
48     if (cross(a-p[l],p[r]-p[l])>0) return 0;
49     if (pointOnSegment(a,Line<T>(p[l],p[r]))) return 1;
50     return 1;
51 }

```

线段交点

```

1 //小心平行
2 template <class T>
3 Point<T> lineIntersection(Line<T> a,Line<T> b)
4 {
5     Point<T> u=a.a-b.a,v=a.b-a.a,w=b.b-b.a;
6     double t=cross(u,w)/cross(w,v);
7     return a.a+t*v;
8 }

```

过定点做圆的切线

```

1 template <class T>
2 vector<Line<T>> tan_PC(Point<T> a,Point<T> c,T r)
3 {
4     Point<T> v=c-a;
5     vector<Line<T>> res;
6     int dis=dis_PP(a,c);
7     if (sgn(dis-r)==0) res.push_back(rotate(v,acos(-1)/2));
8     else if (dis>r)
9     {
10         double alpha=asin(r/dis);
11         res.push_back(rotate(v,alpha));
12         res.push_back(rotate(v,-alpha));
13     }
14     return res;
15 }

```

两圆交点

```

1 template <class T>
2 vector<Point<T>> circleIntersection(Point<T> c1,T r1,Point<T> c2,T r2)
3 {
4     auto get=[&](Point<T> c,T r,double alpha)->Point<T>
5     { return Point<T>(c.x+cos(alpha)*r,c.y+sin(alpha)*r); };
6
7     auto angle=[&](Point<T> a)->double { return atan2(a.x,a.y); };
8
9     vector<Point<T>> res;
10    double d=dis_PP(c1,c2);
11    if (sgn(d)==0) return res;
12    if (sgn(r1+r2-d)<0) return res;
13    if (sgn(fabs(r1-r2)-d)>0) return res;
14    double alpha=angle(c2-c1);
15    double beta=acos((r1*r1-r2*r2+d*d)/(r1*d*2));
16    Point<T> p1=get(c1,r1,alpha-beta),p2=get(c1,r1,alpha+beta);
17    res.push_back(p1);
18    if (p1!=p2) res.push_back(p2);
19    return res;
20 }

```

多边形面积

```
1 template <class T>
2 double polygonArea(const vector<Point<T>> &p)
3 {
4     int n=p.size();
5     double res=0;
6     for (int i=1;i<n-1;i++) res+=cross(p[i]-p[0],p[i+1]-p[0]);
7     return fabs(res/2);
8 }
```

自适应辛普森法

```
1 //注意边界函数值不能小于 eps
2 double f(double x) { return pow(x,0.5); }
3 double calc(double l,double r)
4 {
5     double mid=(l+r)/2.0;
6     return (r-l)*(f(l)+f(r)+f(mid)*4.0)/6.0;
7 }
8 double simpson(double l,double r,double lst)
9 {
10     double mid=(l+r)/2.0;
11     double fl=calc(l,mid),fr=calc(mid,r);
12     if (sgn(fl+fr-lst)==0) return fl+fr;
13     else return simpson(l,mid,fl)+simpson(mid,r,fr);
14 }
```

静态凸包

```
1 template <class T>
2 vector<Point<T>> getHull(vector<Point<T>> p)
3 {
4     vector<Point<T>> h,l;
5     sort(p.begin(),p.end(),[&](auto a,auto b)
6     {
7         if (a.x!=b.x) return a.x<b.x;
8         else return a.y<b.y;
9     });
10    p.erase(unique(p.begin(),p.end()),p.end());
11    if (p.size()<=1) return p;
12    for (auto a:p)
13    {
14        while (h.size()>1&&sgn(cross(a-h.back(),a-h[h.size()-2]))<=0) h.pop_back();
15        while (l.size()>1&&sgn(cross(a-l.back(),a-l[l.size()-2]))>=0) l.pop_back();
16        l.push_back(a);
17        h.push_back(a);
18    }
19    l.pop_back();
20    reverse(h.begin(),h.end());
21    h.pop_back();
22    l.insert(l.end(),h.begin(),h.end());
23    return l;
24 }
```

旋转卡壳求直径

```
1 template <class T>
2 double getDiameter(vector<Point<T>> p)
3 {
4     double res=0;
5     if (p.size()==2) return dis_PP(p[0],p[1]);
6     int n=p.size();
7     p.push_back(p.front());
8     int j=2;
9     for (int i=0;i<n;i++)
10    {
11        while (sgn(cross(p[i+1]-p[i],p[j]-p[i])-cross(p[i+1]-p[i],p[j+1]-p[i]))<0)
12            j=(j+1)%n;
13        res=max({res,dis_PP(p[i],p[j]),dis_PP(p[i+1],p[j])});
14    }
```

```
14     }  
15     return res;  
16 }
```