

Algorithm Library

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常用文件

DEBUG 头

```
1  #include <bits/stdc++.h>
2  using namespace std;
3  using i64=long long;
4  using i128=__int128;
5
6  namespace DBG
7  {
8      template <class T>
9      void _dbg(const char *f,T t) { cerr<<f<<'\n'; }
10
11     template <class A,class... B>
12     void _dbg(const char *f,A a,B... b)
13     {
14         while (*f!=',') cerr<<*f++;
15         cerr<<'\n';
16         _dbg(f+1,b...);
17     }
18
19     template <class T>
20     ostream& operator << (ostream& os,const vector<T> &v)
21     {
22         os<<"[ ";
23         for (const auto &x:v) os<<x<<", ";
24         os<<"]";
25         return os;
26     }
27
28     #define dbg(...) _dbg(#__VA_ARGS__, __VA_ARGS__)
29 }
30
31 using namespace DBG;
```

__int128 输出流

```
1  ostream &operator << (ostream &os,i128 n)
2  {
3      string s;
4      bool neg=n<0;
5      if (neg) n=-n;
6      while (n)
7      {
8          s+='0'+n%10;
9          n/=10;
10     }
11     if (neg) s+='-';
12     reverse(s.begin(),s.end());
13     if (s.empty()) s+='0';
14     return os<<s;
15 }
```

常用数学函数

```
1  i64 ceilDiv(i64 n,i64 m)
2  {
3      if (n>=0) return (n+m-1)/m;
4      else return n/m;
5  }
6
7  i64 floorDiv(i64 n,i64 m)
8  {
9      if (n>=0) return n/m;
10     else return (n-m+1)/m;
11 }
12
13 i128 gcd(i128 a,i128 b)
14 {
```

```

15     return b?gcd(b,a%b):a;
16 }

```

纳秒级随机种子

```

1 mt19937_64 rng(chrono::steady_clock::now().time_since_epoch().count());

```

Linux 对拍

记得先 `chmod 777 check.sh`.

```

1 for ((i=0;i<100;i++))
2 do
3     ./A_Generator > A.in
4     ./A < A.in > A.out
5     ./A_Good < A.in > A.ans
6
7     if diff A.out A.ans;
8     then
9         echo "AC"
10    else
11        echo "WA"
12        exit 1
13    fi
14 done

```

数学

欧拉筛

```

1 vector<int> minp,primes;
2
3 void sieve(int n)
4 {
5     minp.assign(n+1,0);
6     primes.clear();
7     for (int i=2;i<=n;i++)
8     {
9         if (!minp[i])
10        {
11            minp[i]=i;
12            primes.push_back(i);
13        }
14        for (auto p:primes)
15        {
16            if (i*p>n) break;
17            minp[i*p]=p;
18            if (p==minp[i]) break;
19        }
20    }
21 }

```

取模类 (MInt)

```

1 template <class T>
2 constexpr T power(T a,i64 b)
3 {
4     T res=1;
5     for (;b>>=1,a*=a)
6         if (b&1) res*=a;
7     return res;
8 }
9
10 template <int P>
11 struct MInt
12 {
13     int x;
14     constexpr MInt():x{} {}

```

```

15 constexpr MInt(i64 x):x{norm(x%getMod())} {}
16
17 static int Mod;
18 constexpr static int getMod()
19 {
20     if (P>0) return P;
21     else return Mod;
22 }
23
24 constexpr static void setMod(int Mod_) { Mod=Mod_; }
25
26 constexpr int norm(int x) const
27 {
28     if (x<0) x+=getMod();
29     if (x>=getMod()) x-=getMod();
30     return x;
31 }
32
33 constexpr int val() const { return x; }
34
35 explicit constexpr operator int () const { return x; }
36
37 constexpr MInt operator - () const
38 {
39     MInt res;
40     res.x=norm(getMod()-x);
41     return res;
42 }
43
44 constexpr MInt inv() const
45 {
46     assert(x!=0);
47     return power(*this,getMod()-2);
48 }
49
50 constexpr MInt &operator *= (MInt rhs) &
51 {
52     x=1ll*x*rhs.x%getMod();
53     return *this;
54 }
55
56 constexpr MInt &operator += (MInt rhs) &
57 {
58     x=norm(x+rhs.x);
59     return *this;
60 }
61
62 constexpr MInt &operator -= (MInt rhs) &
63 {
64     x=norm(x-rhs.x);
65     return *this;
66 }
67
68 constexpr MInt &operator /= (MInt rhs) &
69 {
70     return *this*=rhs.inv();
71 }
72
73 friend constexpr MInt operator * (MInt lhs,MInt rhs)
74 {
75     MInt res=lhs;
76     res*=rhs;
77     return res;
78 }
79
80 friend constexpr MInt operator + (MInt lhs,MInt rhs)
81 {
82     MInt res=lhs;
83     res+=rhs;
84     return res;
85 }

```

```

86
87     friend constexpr MInt operator - (MInt lhs, MInt rhs)
88     {
89         MInt res=lhs;
90         res-=rhs;
91         return res;
92     }
93
94     friend constexpr MInt operator / (MInt lhs, MInt rhs)
95     {
96         MInt res=lhs;
97         res/=rhs;
98         return res;
99     }
100
101     friend constexpr istream &operator >> (istream &is, MInt &a)
102     {
103         i64 v;
104         is>>v;
105         a=MInt(v);
106         return is;
107     }
108
109     friend constexpr ostream &operator << (ostream &os, const MInt &a) { return os<<a.val(); }
110
111     friend constexpr bool operator == (MInt lhs, MInt rhs) { return lhs.val()==rhs.val(); }
112
113     friend constexpr bool operator != (MInt lhs, MInt rhs) { return lhs.val()!=rhs.val(); }
114 };
115
116 template<>
117 int MInt<0>::Mod=1;
118
119 template<int V, int P>
120 constexpr MInt<P> CInv=MInt<P>(V).inv();

```

组合数

```

1  struct Comb
2  {
3      int n;
4      vector<Z> _fac, _inv, _finv;
5
6      Comb():n{0}, _fac{1}, _inv{0}, _finv{1}{}
7      Comb(int n):Comb() { init(n); }
8
9      void init(int m)
10     {
11         m=min(m, Z::getMod()-1);
12         if (m<=n) return;
13         _fac.resize(m+1);
14         _inv.resize(m+1);
15         _finv.resize(m+1);
16
17         for (int i=n+1; i<=m; i++)
18             _fac[i]=_fac[i-1]*i;
19         _finv[m]=_fac[m].inv();
20         for (int i=m; i>n; i--)
21         {
22             _finv[i-1]=_finv[i]*i;
23             _inv[i]=_finv[i]*_fac[i-1];
24         }
25         n=m;
26     }
27
28     Z fac(int m)
29     {
30         if (m>n) init(m<<1);
31         return _fac[m];
32     }
33

```

```

34     Z finv(int m)
35     {
36         if (m>n) init(m<<1);
37         return _finv[m];
38     }
39
40     Z inv(int m)
41     {
42         if (m>n) init(m<<1);
43         return _inv[m];
44     }
45
46     Z binom(int n,int m)
47     {
48         if (n<m||m<0) return 0;
49         return fac(n)*finv(m)*finv(n-m);
50     }
51 } comb;

```

多项式

```

1  vector<int> rev;
2  vector<Z> roots{0,1};
3
4  void dft(vector<Z> &a)
5  {
6      int n=a.size();
7      if (int(rev.size())!=n)
8      {
9          int k=__builtin_ctz(n)-1;
10         rev.resize(n);
11         for (int i=0;i<n;i++)
12             rev[i]=rev[i>>1]>>1|(i&1)<<k;
13     }
14     for (int i=0;i<n;i++)
15         if (rev[i]<i)
16             swap(a[i],a[rev[i]]);
17     if (int(roots.size())<n)
18     {
19         int k=__builtin_ctz(roots.size());
20         roots.resize(n);
21         while ((1<<k)<n)
22         {
23             Z e=power(Z(3),(P-1)>>(k+1));
24             for (int i=1<<(k-1);i<(1<<k);i++)
25             {
26                 roots[i<<1]=roots[i];
27                 roots[i<<1|1]=roots[i]*e;
28             }
29             k++;
30         }
31     }
32     for (int k=1;k<n;k<<=1)
33         for (int i=0;i<n;i+=k*2)
34             for (int j=0;j<k;j++)
35             {
36                 Z u=a[i+j],v=a[i+j+k]*roots[j+k];
37                 a[i+j]=u+v;
38                 a[i+j+k]=u-v;
39             }
40 }
41
42 void idft(vector<Z> &a)
43 {
44     int n=a.size();
45     reverse(a.begin()+1,a.end());
46     dft(a);
47     Z inv=(1-P)/n;
48     for (int i=0;i<n;i++) a[i]*=inv;
49 }
50

```



```

51 struct Poly
52 {
53     vector<Z> a;
54
55     Poly(){}
56     explicit Poly(int size,function<Z(int)>f=[](int) { return 0; }):a(size)
57     { for (int i=0;i<size;i++) a[i]=f(i); }
58     Poly(const vector<Z> &a):a(a){}
59     Poly(const initializer_list<Z> &a):a(a){}
60
61     int size() const { return a.size(); }
62
63     void resize(int n) { a.resize(n); }
64
65     Z operator [] (int idx) const
66     {
67         if (idx<size()) return a[idx];
68         else return 0;
69     }
70
71     Z &operator [] (int idx) { return a[idx]; }
72
73     Poly mulxk(int k) const
74     {
75         auto b=a;
76         b.insert(b.begin(),k,0);
77         return Poly(b);
78     }
79
80     Poly modxk(int k) const
81     {
82         k=min(k,size());
83         return Poly(vector<Z>(a.begin(),a.begin()+k));
84     }
85
86     Poly divxk(int k) const
87     {
88         if (size()<=k) return Poly();
89         return Poly(vector<Z>(a.begin()+k,a.end()));
90     }
91
92     friend Poly operator + (const Poly &a,const Poly &b)
93     {
94         vector<Z> res(max(a.size(),b.size()));
95         for (int i=0;i<int(res.size());i++)
96             res[i]=a[i]+b[i];
97         return Poly(res);
98     }
99
100    friend Poly operator - (const Poly &a,const Poly &b)
101    {
102        vector<Z> res(max(a.size(),b.size()));
103        for (int i=0;i<int(res.size());i++)
104            res[i]=a[i]-b[i];
105        return Poly(res);
106    }
107
108    friend Poly operator - (const Poly &a)
109    {
110        vector<Z> res(a.size());
111        for (int i=0;i<int(res.size());i++)
112            res[i]=-a[i];
113        return Poly(res);
114    }
115
116    friend Poly operator * (Poly a,Poly b)
117    {
118        if (!a.size()||!b.size()) return Poly();
119        if (a.size()<b.size()) swap(a,b);
120        if (b.size()<128)
121        {

```

```

122         Poly c(a.size()+b.size()-1);
123         for (int i=0;i<a.size();i++)
124             for (int j=0;j<b.size();j++)
125                 c[i+j]+=a[i]*b[j];
126         return c;
127     }
128     int sz=1,tot=a.size()+b.size()-1;
129     while (sz<tot) sz<<=1;
130     a.a.resize(sz);
131     b.a.resize(sz);
132     dft(a.a);
133     dft(b.a);
134     for (int i=0;i<sz;i++)
135         a.a[i]=a[i]*b[i];
136     idft(a.a);
137     a.resize(tot);
138     return a;
139 }
140
141 friend Poly operator * (Z a,Poly b)
142 {
143     for (int i=0;i<int(b.size());i++) b[i]*=a;
144     return b;
145 }
146
147 friend Poly operator * (Poly a,Z b)
148 {
149     for (int i=0;i<int(a.size());i++) a[i]*=b;
150     return a;
151 }
152
153 Poly &operator += (Poly b) { return (*this)=(*this)+b; }
154 Poly &operator -= (Poly b) { return (*this)=(*this)-b; }
155 Poly &operator *= (Poly b) { return (*this)=(*this)*b; }
156 Poly &operator *= (Z b) { return (*this)=(*this)*b; }
157
158 Poly deriv() const
159 {
160     if (a.empty()) return Poly();
161     vector<Z> res(size()-1);
162     for (int i=0;i<size()-1;i++)
163         res[i]=(i+1)*a[i+1];
164     return Poly(res);
165 }
166
167 Poly integr() const
168 {
169     vector<Z> res(size()+1);
170     for (int i=0;i<size();i++)
171         res[i+1]=a[i]/(i+1);
172     return Poly(res);
173 }
174
175 Poly inv(int m) const
176 {
177     Poly x{a[0].inv()};
178     int k=1;
179     while (k<m)
180     {
181         k<<=1;
182         x=(x*(Poly{2}-modxk(k)*x)).modxk(k);
183     }
184     return x.modxk(m);
185 }
186
187 Poly ln(int m) const { return (deriv()*inv(m)).integr().modxk(m); }
188
189 Poly exp(int m) const
190 {
191     Poly x{1};
192     int k=1;

```

```

193     while (k<m)
194     {
195         k<=1;
196         x=(x*(Poly{1}-x.ln(k)+modxk(k)).modxk(k);
197     }
198     return x.modxk(m);
199 }
200
201 Poly pow(int k,int m) const
202 {
203     int i=0;
204     while (i<size()&&a[i].val()==0) i++;
205     if (i==size()||1ll*i*k>=m) return Poly(vector<Z>(m));
206     Z v=a[i];
207     auto f=divxk(i)*v.inv();
208     return (f.ln(m-i*k)*k).exp(m-i*k).mulxk(i*k)*power(v,k);
209 }
210
211 Poly sqrt(int m) const
212 {
213     Poly x{1};
214     int k=1;
215     while (k<m)
216     {
217         k<=1;
218         x=(x+(modxk(k)*x.inv(k)).modxk(k))*((P+1)/2);
219     }
220     return x.modxk(m);
221 }
222 Poly mult(Poly b) const
223 {
224     if (b.size()==0) return Poly();
225     int n=b.size();
226     reverse(b.a.begin(),b.a.end());
227     return ((*this)*b).divxk(n-1);
228 }
229
230 vector<Z> eval(vector<Z> x) const
231 {
232     if (size()==0) return vector<Z>(x.size(),0);
233     const int n=max(int(x.size()),size());
234     vector<Poly> q(n<2);
235     vector<Z> ans(x.size());
236     x.resize(n);
237     function<void(int,int,int)> build=[&](int p,int l,int r)
238     {
239         if (r-l==1) q[p]=Poly{1,-x[l]};
240         else
241         {
242             int m=(l+r)>>1;
243             build(p<<1,l,m);
244             build(p<<1|1,m,r);
245             q[p]=q[p<<1]*q[p<<1|1];
246         }
247     };
248     function<void(int,int,int,const Poly&> work=[&](int p,int l,int r,const Poly &num)
249     {
250         if (r-l==1)
251         {
252             if (l<int(ans.size())) ans[l]=num[0];
253         }
254         else
255         {
256             int m=(l+r)>>1;
257             work(p<<1,l,m,num.mult(q[p<<1|1]).modxk(m-l));
258             work(p<<1|1,m,r,num.mult(q[p<<1]).modxk(r-m));
259         }
260     };
261     build(1,0,n);
262     work(1,0,n,mult(q[1].inv(n)));
263     return ans;

```

```

264     }
265 };

```

原根表

	prime	r	k	g
1	3	1	1	2
2	5	1	2	2
3	17	1	4	3
4	97	3	5	5
5	193	3	6	5
6	257	1	8	3
7	7681	15	9	17
8	12289	3	12	11
9	40961	5	13	3
10	65537	1	16	3
11	786433	3	18	10
12	5767169	11	19	3
13	7340033	7	20	3
14	23068673	11	21	3
15	104857601	25	22	3
16	167772161	5	25	3
17	469762049	7	26	3
18	1004535809	479	21	3
19	2013265921	15	27	31
20	2281701377	17	27	3
21	3221225473	3	30	5
22	75161927681	35	31	3
23	77309411329	9	33	7
24	206158430209	3	36	22
25	2061584302081	15	37	7
26	2748779069441	5	39	3
27	6597069766657	3	41	5
28	39582418599937	9	42	5
29	79164837199873	9	43	5
30	263882790666241	15	44	7
31	1231453023109121	35	45	3
32	1337006139375617	19	46	3
33	3799912185593857	27	47	5
34	4222124650659841	15	48	19
35	7881299347898369	7	50	6
36	31525197391593473	7	52	3
37	180143985094819841	5	55	6
38	1945555039024054273	27	56	5
39	4179340454199820289	29	57	3
40				

线性基

```

1  struct LB
2  {
3      static constexpr int L=60;
4      array<i64,L+1> a{};
5
6      LB(){}
7
8      LB(const vector<i64> &v) { init(v); }
9
10     bool insert(i64 t)
11     {
12         for (int i=L;i>=0;i--)
13             if (t&(1ll<<i))
14             {
15                 if (!a[i])
16                 {
17                     a[i]=t;
18                     return 1;
19                 }
20                 else t^=a[i];
21             }
22     return 0;

```

```

23     }
24
25     void init(const vector<i64> &v) { for (auto x:v) insert(x); }
26
27     bool check(i64 t)
28     {
29         for (int i=L;i>=0;i--)
30             if (t&(1ll<<i))
31                 if (!a[i]) return 0;
32                 else t^=a[i];
33         return 1;
34     }
35
36     i64 QueryMax()
37     {
38         i64 res=0;
39         for (int i=L;i>=0;i--)
40             res=max(res,res^a[i]);
41         return res;
42     }
43
44     i64 QueryMin()
45     {
46         for (int i=0;i<=L;i++)
47             if (a[i]) return a[i];
48         return 0;
49     }
50
51     i64 QueryKth(int k)
52     {
53         i64 res=0;
54         int cnt=0;
55         array<i64,L+1> tmp{};
56         for (int i=0;i<=L;i++)
57         {
58             for (int j=i-1;j>=0;j--)
59                 if (a[i]&(1ll<<j)) a[i]^=a[j];
60             if (a[i]) tmp[cnt++]=a[i];
61         }
62         if (k>=(1ll<<cnt)) return -1;
63         for (int i=0;i<cnt;i++)
64             if (k&(1ll<<i)) res^=tmp[i];
65         return res;
66     }
67 };

```

min-plus 卷积

$\mathcal{O}(n \log n)$, 但要求 b 是凸的。

```

1     template <class T>
2     vector<T> min_plus_convolution(const vector<T> &a,const vector<T> &b)
3     {
4         int n=a.size(),m=b.size();
5         vector<T> c(n+m-1);
6
7         function<void(int,int,int,int)> solve=[&](int l,int r,int ql,int qr)
8         {
9             if (l>r) return;
10            int mid=(l+r)>>1;
11            while (ql+m<=l) ++ql;
12            while (qr>r) --qr;
13            int qmid=-1;
14            c[mid]=inf;
15            for (int i=ql;i<=qr;i++)
16            {
17                if (a[i]+b[mid-i]-i<c[mid])
18                {
19                    c[mid]=a[i]+b[mid-i];
20                    qmid=i;
21                }

```

```

22         else if (mid-i>=0&&mid-i<m) qmid=i;
23     }
24     solve(l,mid-1,ql,mid);
25     solve(mid+1,r,qmid,qr);
26 };
27
28 solve(0,n+m-2,0,n-1);
29 return c;
30 }

```

模意义分数还原

分别是求：分子不大于 A 时分子最大的分数；分子分母最大值最小的分数。

```

1 pair<int,int> restore(int q,int A)
2 {
3     int x=q,y=P,a=1,b=0;
4     while (x>A)
5     {
6         swap(x,y);
7         swap(a,b);
8         a-=x/y*b;
9         x%=y;
10    }
11    return make_pair(x,a);
12 }
13
14 pair<int,int> restore(int x)
15 {
16     vector<int> a;
17     int p=P;
18     Z inv=Z(x).inv();
19     while (x)
20     {
21         a.push_back(x);
22         swap(x,p);
23         x%=p;
24     }
25     pair<int,int> res{P,P};
26     for (auto ca:a)
27     {
28         int cb=(Z(ca)*inv).x;
29         ca=min(ca,P-ca);
30         cb=min(cb,P-cb);
31         if (max(res.first,res.second)>max(ca,cb))
32             res={ca,cb};
33     }
34     return res;
35 }

```

Exgcd

```

1 void exgcd(i64 a,i64 b,i64 &x,i64 &y)
2 {
3     if (!b)
4     {
5         x=1; y=0;
6         return;
7     }
8     exgcd(b,a%b,x,y);
9     swap(x,y);
10    y-=a/b*x;
11    return;
12 }

```

二元一次不定方程

给定不定方程 $ax + by = c$ 。

若该方程无整数解，输出 -1 。

若该方程有整数解，且有正整数解，则输出其正整数解的数量，所有正整数解中 x 的最小值，所有正整数解中 y 的最小值，所有正整数解中 x 的最大值，以及所有正整数解中 y 的最大值。

若方程有整数解，但没有正整数解，输出所有整数解中 x 的最小正整数值， y 的最小正整数值。

```
1 void exgcd(i64 a,i64 b,i64 &x,i64 &y)
2 {
3     if (!b)
4     {
5         x=1; y=0;
6         return;
7     }
8     exgcd(b,a%b,x,y);
9     swap(x,y);
10    y-=a/b*x;
11    return;
12 }
13
14 i64 ceilDiv(i64 n,i64 m)
15 {
16     if (n>=0) return (n+m-1)/m;
17     else return n/m;
18 }
19
20 i64 floorDiv(i64 n,i64 m)
21 {
22     if (n>=0) return n/m;
23     else return (n-m+1)/m;
24 }
25
26 void R()
27 {
28     i64 a,b,c,x,y,t;
29     cin>>a>>b>>c;
30     t=__gcd(a,b);
31     if (c%t)
32     {
33         cout<<"-1\n";
34         return;
35     }
36     exgcd(a,b,x,y);
37     x*=c/t,y*=c/t;
38     i64 l=ceilDiv(1ll-x,b/t),r=floorDiv(y-1ll,a/t);
39     if (l>r) cout<<x+l*b/t<<' '<<y-r*a/t<<' \n';
40     else cout<<r-1ll<<' '<<x+l*b/t<<' '<<y-r*a/t<<' '<<x+r*b/t<<' '<<y-l*a/t<<' \n';
41     return;
42 }
```

行列式求值

$\mathcal{O}(n^3)$

```
1 Z det(vector<vector<Z>> a)
2 {
3     int n=a.size(),fl=1;
4     Z res=1;
5     for (int i=0;i<n;i++)
6     {
7         for (int j=i+1;j<n;j++)
8         {
9             while (a[i][j].x)
10            {
11                int d=a[j][i].x/a[i][j].x;
12                for (int k=i;k<n;k++)
13                    a[j][k]-=a[i][k]*d;
14                swap(a[i],a[j]);
15                fl=-fl;
16            }
17            swap(a[i],a[j]);
```

```

18         fl=-fl;
19     }
20 }
21 for (int i=0;i<n;i++) res*=a[i][i];
22 res*=fl;
23 return res;
24 }

```

数据结构

并查集（启发式合并 + 带撤销）

```

1 struct DSU
2 {
3     int n=0;
4     vector<int> fa,siz;
5     stack<int> s;
6
7     DSU(int n) { init(n); }
8
9     void init(int n)
10    {
11        fa.resize(n);
12        iota(fa.begin(),fa.end(),0);
13        siz.assign(n,1);
14        while (!s.empty()) s.pop();
15    }
16
17    int get(int x) { return fa[x]==x?x:get(fa[x]); }
18
19    void merge(int x,int y)
20    {
21        x=get(x),y=get(y);
22        if (x==y) return;
23        if (siz[x]<siz[y]) swap(x,y);
24        s.push(y),fa[y]=x,siz[x]+=siz[y];
25    }
26
27    void undo()
28    {
29        if (s.empty()) return;
30        int y=s.top();
31        s.pop();
32        siz[fa[y]]-=siz[y];
33        fa[y]=y;
34    }
35
36    void back(int t=0) { while (s.size()>t) undo(); }
37 };

```

状压 RMQ

```

1 template <class T,class Cmp=less<T>>
2 struct RMQ
3 {
4     const Cmp cmp=Cmp();
5     static constexpr unsigned B=64;
6     using u64=unsigned long long;
7     int n;
8     vector<vector<T>> a;
9     vector<T> pre,suf,ini;
10    vector<u64> stk;
11
12    RMQ() {}
13    RMQ(const vector<T> &v) { init(v); }
14
15    void init(const vector<T> &v)
16    {
17        n=v.size();

```



```

18     pre=suf=ini=v;
19     stk.resize(n);
20     if (!n) return;
21     const int M=(n-1)/B+1;
22     const int lg=__lg(M);
23     a.assign(lg+1,vector<T>(M));
24     for (int i=0;i<M;i++)
25     {
26         a[0][i]=v[i*B];
27         for (int j=1;j<B&& i*B+j<n;j++)
28             a[0][i]=min(a[0][i],v[i*B+j],cmp);
29     }
30     for (int i=1;i<n;i++)
31         if (i%B) pre[i]=min(pre[i],pre[i-1],cmp);
32     for (int i=n-2;i>=0;i--)
33         if (i%B!=B-1) suf[i]=min(suf[i],suf[i+1],cmp);
34     for (int j=0;j<lg;j++)
35         for (int i=0;i+(2<<j)<=M;i++)
36             a[j+1][i]=min(a[j][i],a[j][i+(1<<j)],cmp);
37     for (int i=0;i<M;i++)
38     {
39         const int l=i*B;
40         const int r=min(1U*n,l+B);
41         u64 s=0;
42         for (int j=l;j<r;j++)
43         {
44             while (s&&cmp(v[j],v[__lg(s)+l])) s^=1ULL<<__lg(s);
45             s|=1ULL<<(j-l);
46             stk[j]=s;
47         }
48     }
49 }
50
51 //查询区间 [l,r) 的 RMQ
52 T operator()(int l,int r)
53 {
54     if (l/B!=(r-1)/B)
55     {
56         T ans=min(suf[l],pre[r-1],cmp);
57         l=l/B+1,r=r/B;
58         if (l<r)
59         {
60             int k=__lg(r-l);
61             ans=min({ans,a[k][l],a[k][r-(1<<k)]},cmp);
62         }
63         return ans;
64     }
65     else
66     {
67         int x=B*(l/B);
68         return ini[__builtin_ctzll(stk[r-1]>>(l-x))+1];
69     }
70 }
71 };

```

ST 表

```

1  template <class T>
2  struct ST
3  {
4      int n;
5      vector<vector<T>> a;
6
7      ST() {}
8      ST(const vector<T> &v) { init(v); }
9
10     void init(const vector<T> &v)
11     {
12         n=v.size();
13         if (!n) return;
14         const int lg=__lg(n);

```

```

15     a.assign(lg+1,vector<T>(n));
16     a[0]=v;
17     for (int j=0;j<lg;j++)
18         for (int i=0;i+(2<<j)<=n;i++)
19             a[j+1][i]=__gcd(a[j][i],a[j][i+(1<<j)]);
20 }
21
22 T operator()(int l,int r)
23 {
24     int k=__lg(r-l);
25     return __gcd(a[k][l],a[k][r-(1<<k)]);
26 }
27 };

```

树状数组

```

1  template <class T>
2  struct BIT
3  {
4      int n;
5      vector<T> a;
6
7      BIT(int n_=0) { init(n_); }
8
9      void init(int n_)
10     {
11         n=n_;
12         a.assign(n,T{});
13     }
14
15     void add(int x,const T &v)
16     {
17         for (int i=x+1;i<=n;i+=i&-i)
18             a[i-1]=a[i-1]+v;
19     }
20
21     //查询区间 [0,x)
22     T sum(int x)
23     {
24         T ans{};
25         for (int i=x;i>0;i-=i&-i)
26             ans=ans+a[i-1];
27         return ans;
28     }
29
30     //查询区间 [l,r)
31     T rangeSum(int l,int r) { return sum(r)-sum(l); }
32
33     int select(const T &k)
34     {
35         int x=0;
36         T cur{};
37         for (int i=1<<__lg(n);i;i>>=1)
38         {
39             if (x+i<=n&&cur+a[x+i-1]<=k)
40             {
41                 x+=i;
42                 cur=cur+a[x-1];
43             }
44         }
45         return x;
46     }
47 };

```

线段树

```

1  template <class Info,class Tag>
2  struct SGT
3  {
4      int n;

```

```

5     vector<Info> info;
6     vector<Tag> tag;
7
8     SGT():n(0) {}
9     SGT(int n_,Info v_=Info()) { init(n_,v_); }
10
11     template <class T>
12     SGT(vector<T> init_) { init(init_); }
13
14     void init(int n_,Info v_=Info()) { init(vector(n_,v_)); }
15
16     template <class T>
17     void init(vector<T> init_)
18     {
19         n=init_.size();
20         info.assign(4<<__lg(n),Info());
21         tag.assign(4<<__lg(n),Tag());
22         function<void(int,int,int)> build=[&](int p,int l,int r)
23         {
24             if (r-l==1)
25             {
26                 info[p]=init_[l];
27                 return;
28             }
29             int m=(l+r)>>1;
30             build(p<<1,l,m);
31             build(p<<1|1,m,r);
32             pushup(p);
33         };
34         build(1,0,n);
35     }
36
37     void pushup(int p) { info[p]=info[p<<1]+info[p<<1|1]; }
38
39     void apply(int p,const Tag &v)
40     {
41         info[p].apply(v);
42         tag[p].apply(v);
43     }
44
45     void pushdown(int p)
46     {
47         apply(p<<1,tag[p]);
48         apply(p<<1|1,tag[p]);
49         tag[p]=Tag();
50     }
51
52     void modify(int p,int l,int r,int x,const Info &v)
53     {
54         if (r-l==1)
55         {
56             info[p]=v;
57             return;
58         }
59         int m=(l+r)>>1;
60         pushdown(p);
61         if (x<m) modify(p<<1,l,m,x,v);
62         else modify(p<<1|1,m,r,x,v);
63         pushup(p);
64     }
65
66     //O(log n) 单点修改
67     void modify(int p,const Info &v) { modify(1,0,n,p,v); }
68
69     Info rangeQuery(int p,int l,int r,int x,int y)
70     {
71         if (l>=y||r<=x) return Info();
72         if (l>=x&&r<=y) return info[p];
73         int m=(l+r)>>1;
74         pushdown(p);
75         return rangeQuery(p<<1,l,m,x,y)+rangeQuery(p<<1|1,m,r,x,y);

```

```

76     }
77
78     //O(log n) 区间查询 [l,r)
79     Info rangeQuery(int l,int r) { rangeQuery(1,0,n,l,r); }
80
81     void rangeApply(int p,int l,int r,int x,int y,const Tag &v)
82     {
83         if (l>y||r<=x) return;
84         if (l>=x&&r<=y)
85         {
86             apply(p,v);
87             return;
88         }
89         int m=(l+r)>>1;
90         pushdown(p);
91         rangeApply(p<<1,l,m,x,y,v);
92         rangeApply(p<<1|1,m,r,x,y,v);
93         pushup(p);
94     }
95
96     //O(log n) 区间操作 [l,r)
97     void rangeApply(int l,int r,const Tag &v) { rangeApply(1,0,n,l,r,v); }
98
99     //O(log n) 区间 [l,r) 内查找第一个合法位置
100    template <class F>
101    int findFirst(int p,int l,int r,int x,int y,F pred)
102    {
103        if (l>y||r<=x||!pred(info[p])) return -1;
104        if (r-l==1) return l;
105        int m=(l+r)>>1;
106        pushdown(p);
107        int res=findFirst(p<<1,l,m,x,y,pred);
108        if (res==-1) res=findFirst(p<<1|1,m,r,x,y,pred);
109        return res;
110    }
111
112    template <class F>
113    int findFirst(int l,int r,F pred) { return findFirst(1,0,n,l,r,pred); }
114
115    template <class F>
116    int findLast(int p,int l,int r,int x,int y,F pred)
117    {
118        if (l>y||r<=x||!pred(info[p])) return -1;
119        if (r-l==1) return l;
120        int m=(l+r)>>1;
121        pushdown(p);
122        int res=findFirst(p<<1|1,m,r,x,y,pred);
123        if (res==-1) res=findFirst(p<<1,l,m,x,y,pred);
124        return res;
125    }
126
127    template <class F>
128    int findLast(int l,int r,F pred) { return findLast(1,0,n,l,r,pred); }
129 };
130
131 //这里默认乘法优先 (x*a+b)*c+d=x*(a*c)+(b*c+d)
132 struct Tag
133 {
134     i64 a=1,b=0;
135     void apply(Tag t)
136     {
137         a*=t.a;
138         b=b*t.a+t.b;
139     }
140 };
141
142 struct Info
143 {
144     i64 x=0,l=0,r=0;
145     void apply(Tag t)
146     {

```

```

147         int len=r-l+1;
148         x=x*t.a+len*t.b;
149     }
150 };
151
152 Info operator + (Info a,Info b)
153 {
154     return {a.x+b.x,min(a.l,b.l),max(a.r,b.r)};
155 }

```

字符串

字符串哈希（随机模数）

```

1  bool isPrime(int n)
2  {
3      if (n<=1) return 0;
4      for (int i=2;i*i<=n;i++)
5          if (n%i==0) return 0;
6      return 1;
7  }
8
9  int findPrime(int n)
10 {
11     while (!isPrime(n)) n++;
12     return n;
13 }
14
15 mt19937 rng(time(0));
16 const int P=findPrime(rng()%9000000000+1000000000);
17 struct StrHash
18 {
19     int n;
20     vector<int> h,p;
21
22     StrHash(const string &s){ init(s); }
23
24     void init(const string &s)
25     {
26         n=s.size();
27         h.resize(n+1);
28         p.resize(n+1);
29         p[0]=1;
30         for (int i=0;i<n;i++) h[i+1]=(10ll*h[i]+s[i]-'a')%P;
31         for (int i=0;i<n;i++) p[i+1]=10ll*p[i]%P;
32     }
33
34     //查询 [l,r) 的区间哈希
35     int get(int l,int r) { return (h[r]+1ll*(P-h[l])*p[r-l])%P; }
36 };

```

KMP

```

1  vector<int> KMP(const string &s)
2  {
3      int now=0;
4      vector<int> pre(s.size(),0);
5      for (int i=1;i<s.size();i++)
6      {
7          while (now&&s[i]!=s[now]) now=pre[now-1];
8          if (s[i]==s[now]) now++;
9          pre[i]=now;
10     }
11     return pre;
12 }

```

Z函数

```
1 vector<int> zFunction(string s)
2 {
3     int n=s.size();
4     vector<int> z(n);
5     z[0]=n;
6     for (int i=1,j=1;i<n;i++)
7     {
8         z[i]=max(0,min(j+z[j]-i,z[i-j]));
9         while (i+z[i]<n&& s[z[i]]==s[i+z[i]]) z[i]++;
10        if (i+z[i]>j+z[j]) j=i;
11    }
12    return z;
13 }
```

AC 自动机

```
1 struct ACAM
2 {
3     static constexpr int ALPHABET=26;
4     struct Node
5     {
6         int len;
7         int link;
8         array<int,ALPHABET> next;
9         Node():len{0},link{0},next{}{}
10    };
11
12    vector<Node> t;
13
14    ACAM() { init(); }
15
16    void init()
17    {
18        t.assign(2,Node());
19        t[0].next.fill(1);
20        t[0].len=-1;
21    }
22
23    int newNode()
24    {
25        t.emplace_back();
26        return t.size()-1;
27    }
28
29    int add(const string &a)
30    {
31        int p=1;
32        for (auto c:a)
33        {
34            int x=c-'a';
35            if (t[p].next[x]==0)
36            {
37                t[p].next[x]=newNode();
38                t[t[p].next[x]].len=t[p].len+1;
39            }
40            p=t[p].next[x];
41        }
42        return p;
43    }
44
45    void work()
46    {
47        queue<int> q;
48        q.push(1);
49        while (!q.empty())
50        {
51            int x=q.front();
52            q.pop();
53            for (int i=0;i<ALPHABET;i++)
```

```

54         {
55             if (t[x].next[i]==0) t[x].next[i]=t[t[x].link].next[i];
56             else
57             {
58                 t[t[x].next[i]].link=t[t[x].link].next[i];
59                 q.push(t[x].next[i]);
60             }
61         }
62     }
63 }
64
65 int next(int p,int x) { return t[p].next[x]; }
66
67 int link(int p) { return t[p].link; }
68
69 int size() { return t.size(); }
70 };

```

后缀数组

```

1  struct SA
2  {
3      int n;
4      vector<int> sa,rk,lc;
5      SA(const string &s)
6      {
7          n=s.length();
8          sa.resize(n);
9          rk.resize(n);
10         lc.resize(n-1);
11         iota(sa.begin(),sa.end(),0);
12         sort(sa.begin(),sa.end(),[&](int a,int b){ return s[a]<s[b]; });
13         rk[sa[0]]=0;
14         for (int i=1;i<n;i++) rk[sa[i]]=rk[sa[i-1]]+(s[sa[i]]!=s[sa[i-1]]);
15         int k=1;
16         vector<int> tmp,cnt(n);
17         tmp.reserve(n);
18         while (rk[sa[n-1]]<n-1)
19         {
20             tmp.clear();
21             for (int i=0;i<k;i++) tmp.push_back(n-k+i);
22             for (auto i:sa)
23                 if (i>=k) tmp.push_back(i-k);
24             fill(cnt.begin(),cnt.end(),0);
25             for (int i=0;i<n;i++) cnt[rk[i]]++;
26             for (int i=1;i<n;i++) cnt[i]+=cnt[i-1];
27             for (int i=n-1;i>=0;i--) sa[--cnt[rk[tmp[i]]]]=tmp[i];
28             swap(rk,tmp);
29             rk[sa[0]]=0;
30             for (int i=1;i<n;i++)
31                 rk[sa[i]]=rk[sa[i-1]]+(tmp[sa[i-1]]<tmp[sa[i]] || sa[i-1]+k==n || tmp[sa[i-1]+k]<tmp[sa[i]+k]);
32             k<<=1;
33         }
34         for (int i=0,j=0;i<n;i++)
35         {
36             if (rk[i]==0) j=0;
37             else
38             {
39                 for (j--j>0;i+j<n&&sa[rk[i]-1]+j<n&&s[i+j]==s[sa[rk[i]-1]+j]); j++;
40                 lc[rk[i]-1]=j;
41             } //lc[i]:lcp(sa[i],sa[i+1]),lcp(sa[i],sa[j])=min{lc[i...j-1]}
42         }
43     }
44 };

```

(广义) 后缀自动机

```

1  struct SAM
2  {
3      static constexpr int ALPHABET=26;

```

```

4  struct Node
5  {
6      int len;
7      int link;
8      array<int,ALPHABET> next;
9      Node():len{},link{},next{} {}
10 };
11
12 vector<Node> t;
13
14 SAM() { init(); }
15
16 void init()
17 {
18     t.assign(2,Node());
19     t[0].next.fill(1);
20     t[0].len=-1;
21 }
22
23 int newNode()
24 {
25     t.emplace_back();
26     return t.size()-1;
27 }
28
29 int extend(int lst,int c)
30 {
31     if (t[lst].next[c]&&t[t[lst].next[c]].len==t[lst].len+1)
32         return t[lst].next[c];
33     int p=lst,np=newNode(),flag=0;
34     t[np].len=t[p].len+1;
35     while (!t[p].next[c])
36     {
37         t[p].next[c]=np;
38         p=t[p].link;
39     }
40     if (!p)
41     {
42         t[np].link=1;
43         return np;
44     }
45     int q=t[p].next[c];
46     if (t[q].len==t[p].len+1)
47     {
48         t[np].link=q;
49         return np;
50     }
51     if (p==lst) flag=1,np=0,t.pop_back();
52     int nq=newNode();
53     t[nq].link=t[q].link;
54     t[nq].next=t[q].next;
55     t[nq].len=t[p].len+1;
56     t[q].link=t[np].link=nq;
57     while (p&&t[p].next[c]==q)
58     {
59         t[p].next[c]=nq;
60         p=t[p].link;
61     }
62     return flag?nq:np;
63 }
64
65 int add(const string &a)
66 {
67     int p=1;
68     for (auto c:a) p=extend(p,c-'a');
69     return p;
70 }
71
72 int next(int p,int x) { return t[p].next[x]; }
73
74 int link(int p) { return t[p].link; }

```



```

75     int len(int p) { return t[p].len; }
76
77     int size() { return t.size(); }
78
79 };

```

Manacher

```

1  vector<int> manacher(vector<int> s)
2  {
3      vector<int> t{0};
4      for (auto c:s)
5      {
6          t.push_back(c);
7          t.push_back(0);
8      }
9      int n=t.size();
10     vector<int> r(n);
11     for (int i=0,j=0;i<n;i++)
12     {
13         if (j*2-i>=0&&j+r[j]>i) r[i]=min(r[j*2-i],j+r[j]-i);
14         while (i-r[i]>=0&&i+r[i]<n&&t[i-r[i]]==t[i+r[i]]) r[i]++;
15         if (i+r[i]>j+r[j]) j=i;
16     }
17     return r;
18 }

```

回文自动机

```

1  struct PAM
2  {
3      static constexpr int ALPHABET_SIZE=28;
4      struct Node
5      {
6          int len,link,cnt;
7          array<int,ALPHABET_SIZE> next;
8          Node():len{},link{},cnt{},next{}{}
9      };
10     vector<Node> t;
11     int suff;
12     string s;
13
14     PAM() { init(); }
15
16     void init()
17     {
18         t.assign(2,Node());
19         t[0].len=-1;
20         suff=1;
21         s.clear();
22     }
23
24     int newNode()
25     {
26         t.emplace_back();
27         return t.size()-1;
28     }
29
30     bool add(char c,char offset='a')
31     {
32         int pos=s.size();
33         s+=c;
34         int let=c-offset;
35         int cur=suff,curlen=0;
36         while (1)
37         {
38             curlen=t[cur].len;
39             if (pos-curlen-1>=0&&s[pos-curlen-1]==s[pos]) break;
40             cur=t[cur].link;
41         }

```

```

42     if (t[cur].next[let])
43     {
44         suff=t[cur].next[let];
45         return 0;
46     }
47     int num=newNode();
48     suff=num;
49     t[num].len=t[cur].len+2;
50     t[cur].next[let]=num;
51     if (t[num].len==1)
52     {
53         t[num].link=t[num].cnt=1;
54         return 1;
55     }
56     while (1)
57     {
58         cur=t[cur].link;
59         curlen=t[cur].len;
60         if (pos-curlen-1>=0&&s[pos-curlen-1]==s[pos])
61         {
62             t[num].link=t[cur].next[let];
63             break;
64         }
65     }
66     t[num].cnt=t[t[num].link].cnt+1;
67     return 1;
68 }
69 };

```

图论

Dijkstra

注意设定合适的 inf。

```

1  vector<i64> dijk(const vector<vector<pair<int,i64>>> &adj,int s)
2  {
3      int n=adj.size();
4      using pa=pair<i64,int>;
5      vector<i64> d(n,inf);
6      vector<int> ed(n);
7      priority_queue<pa,vector<pa>,greater<pa>> q;
8      q.push({0,s}); d[s]=0;
9      while (!q.empty())
10     {
11         int u=q.top().second;
12         q.pop();
13         ed[u]=1;
14         for (auto [v,w]:adj[u])
15             if (d[u]+w<d[v])
16             {
17                 d[v]=d[u]+w;
18                 q.push({d[v],v});
19             }
20         while (!q.empty()&&ed[q.top().second]) q.pop();
21     }
22     return d;
23 }

```

SPFA

注意设定合适的 inf。

```

1  vector<i64> spfa(const vector<vector<pair<int,i64>>> &adj,int s)
2  {
3      int n=adj.size();
4      assert(n);
5      queue<int> q;
6      vector<int> len(n),ed(n);

```

```

7     vector<i64> d(n,inf);
8     q.push(s); d[s]=0;
9     while (!q.empty())
10    {
11        int u=q.front();
12        q.pop();
13        ed[u]=0;
14        for (auto [v,w]:adj[u])
15            if (d[u]+w<d[v])
16            {
17                d[v]=d[u]+w;
18                len[v]=len[u]+1;
19                if (len[v]>n) return {};
20                if (!ed[v]) ed[v]=1,q.push(v);
21            }
22    }
23    return d;
24 }

```

Johnson

```

1     vector<vector<i64>> dijk(const vector<vector<pair<int,i64>>> &adj)
2     {
3         vector<vector<i64>> res;
4         for (int i=0;i<adj.size();i++)
5             res.push_back(dijk(adj,i));
6         return res;
7     }
8
9     vector<i64> spfa(const vector<vector<pair<int,i64>>> &adj)
10    {
11        int n=adj.size();
12        assert(n);
13        queue<int> q;
14        vector<int> len(n),ed(n,1);
15        vector<i64> d(n);
16        for (int i=0;i<n;i++) q.push(i);
17        while (!q.empty())
18        {
19            int u=q.front();
20            q.pop();
21            ed[u]=0;
22            for (auto [v,w]:adj[u])
23                if (d[u]+w<d[v])
24                {
25                    d[v]=d[u]+w;
26                    len[v]=len[u]+1;
27                    if (len[v]>n) return {};
28                    if (!ed[v]) ed[v]=1,q.push(v);
29                }
30        }
31        return d;
32    }
33
34    vector<vector<i64>> john(vector<vector<pair<int,i64>>> adj)
35    {
36        int n=adj.size();
37        assert(n);
38        auto h=spfa(adj);
39        if (!h.size()) return {};
40        for (int u=0;u<n;u++)
41            for (auto &[v,w]:adj[u])
42                w+=h[u]-h[v];
43        auto res=dijk(adj);
44        for (int u=0;u<n;u++)
45            for (int v=0;v<n;v++)
46                if (res[u][v]!=inf)
47                    res[u][v]-=h[u]-h[v];
48        return res;
49    }

```

强连通分量

```
1 struct SCC
2 {
3     int n,cur,cnt;
4     vector<vector<int>> adj;
5     vector<int> stk,dfn,low,bel;
6
7     SCC() {}
8     SCC(int n) { init(n); }
9
10    void init(int n)
11    {
12        this->n=n;
13        adj.assign(n,{});
14        stk.clear();
15        dfn.assign(n,-1);
16        low.resize(n);
17        bel.assign(n,-1);
18        cur=cnt=0;
19    }
20
21    void add(int u,int v) { adj[u].push_back(v); }
22
23    void dfs(int x)
24    {
25        dfn[x]=low[x]=cur++;
26        stk.push_back(x);
27        for (auto y:adj[x])
28        {
29            if (dfn[y]==-1)
30            {
31                dfs(y);
32                low[x]=min(low[x],low[y]);
33            }
34            else if (bel[y]==-1) low[x]=min(low[x],dfn[y]);
35        }
36        if (dfn[x]==low[x])
37        {
38            int y;
39            do
40            {
41                y=stk.back();
42                bel[y]=cnt;
43                stk.pop_back();
44            } while (y!=x);
45            cnt++;
46        }
47    }
48
49    vector<int> work()
50    {
51        for (int i=0;i<n;i++)
52            if (dfn[i]==-1) dfs(i);
53        return bel;
54    }
55
56    struct Graph
57    {
58        int n;
59        vector<pair<int,int>> edges;
60        vector<int> siz,cnte;
61    };
62
63    Graph compress()
64    {
65        Graph G;
66        G.n=cnt;
67        G.siz.resize(cnt);
68        G.cnte.resize(cnt);
69        for (int i=0;i<n;i++)
```

```

70     {
71         G.siz[bel[i]]++;
72         for (auto j:adj[i])
73             if (bel[i]!=bel[j])
74                 G.edges.emplace_back(bel[j],bel[i]);
75     }
76     return G;
77 };
78 };

```

边双连通分量

```

1  struct EBCC
2  {
3      int n;
4      vector<vector<int>> adj;
5      vector<int> stk,dfn,low,bel;
6      int cur,cnt;
7
8      EBCC() {}
9      EBCC(int n) { init(n); }
10
11     void init(int n)
12     {
13         this->n=n;
14         adj.assign(n,{});
15         dfn.assign(n,-1);
16         low.resize(n);
17         bel.assign(n,-1);
18         stk.clear();
19         cur=cnt=0;
20     }
21
22     void add(int u,int v)
23     {
24         adj[u].push_back(v);
25         adj[v].push_back(u);
26     }
27
28     void dfs(int x,int p)
29     {
30         dfn[x]=low[x]=cur++;
31         stk.push_back(x);
32         for (auto y:adj[x])
33         {
34             if (y==p) continue;
35             if (dfn[y]==-1)
36             {
37                 dfs(y,x);
38                 low[x]=min(low[x],low[y]);
39             }
40             else if (bel[y]==-1&&dfn[y]<dfn[x]) low[x]=min(low[x],dfn[y]);
41         }
42         if (dfn[x]==low[x])
43         {
44             int y;
45             do
46             {
47                 y=stk.back();
48                 bel[y]=cnt;
49                 stk.pop_back();
50             } while (y!=x);
51             cnt++;
52         }
53     }
54
55     vector<int> work()
56     {
57         dfs(0,-1);
58         return bel;
59     }

```

```

60
61 struct Graph
62 {
63     int n;
64     vector<pair<int,int>> edges;
65     vector<int> siz,cnt;
66 };
67
68 Graph compress()
69 {
70     Graph G;
71     G.n=cnt;
72     G.siz.resize(cnt);
73     G.cnt.resize(cnt);
74     for (int i=0;i<n;i++)
75     {
76         G.siz[bel[i]]++;
77         for (auto j:adj[i])
78         {
79             if (bel[i]<bel[j]) G.edges.emplace_back(bel[i],bel[j]);
80             else if (i<j) G.cnt[bel[i]]++;
81         }
82     }
83     return G;
84 };
85 };

```

轻重链剖分

```

1 struct HLD
2 {
3     int n;
4     vector<int> siz,top,dep,pa,in,out,seq;
5     vector<vector<int>> adj;
6     int cur;
7
8     HLD(){}
9     HLD(int n) { init(n); }
10
11 void init(int n)
12 {
13     this->n=n;
14     siz.resize(n);
15     top.resize(n);
16     dep.resize(n);
17     pa.resize(n);
18     in.resize(n);
19     out.resize(n);
20     seq.resize(n);
21     cur=0;
22     adj.assign(n,{});
23 }
24
25 void addEdge(int u,int v)
26 {
27     adj[u].push_back(v);
28     adj[v].push_back(u);
29 }
30
31 void work(int rt=0)
32 {
33     top[rt]=rt;
34     dep[rt]=0;
35     pa[rt]=-1;
36     dfs1(rt);
37     dfs2(rt);
38 }
39
40 void dfs1(int u)
41 {
42     if (pa[u]!=-1) adj[u].erase(find(adj[u].begin(),adj[u].end(),pa[u]));

```

```

43     siz[u]=1;
44     for (auto &v:adj[u])
45     {
46         pa[v]=u;
47         dep[v]=dep[u]+1;
48         dfs1(v);
49         siz[u]+=siz[v];
50         if (siz[v]>siz[adj[u][0]])
51             swap(v,adj[u][0]);
52     }
53 }
54
55 void dfs2(int u)
56 {
57     in[u]=cur++;
58     seq[in[u]]=u;
59     for (auto v:adj[u])
60     {
61         top[v]=(v==adj[u][0])?top[u]:v;
62         dfs2(v);
63     }
64     out[u]=cur;
65 }
66
67 int lca(int u,int v)
68 {
69     while (top[u]!=top[v])
70     {
71         if (dep[top[u]]>dep[top[v]]) u=pa[top[u]];
72         else v=pa[top[v]];
73     }
74     return dep[u]<dep[v]?u:v;
75 }
76
77 int dist(int u,int v) { return dep[u]+dep[v]-(dep[lca(u,v)]<<1); }
78
79 int jump(int u,int k)
80 {
81     if (dep[u]<k) return -1;
82     int d=dep[u]-k;
83     while (dep[top[u]]>d) u=pa[top[u]];
84     return seq[in[u]-dep[u]+d];
85 }
86
87 bool isAncestor(int u,int v) { return in[u]<=in[v]&&in[v]<out[u]; }
88
89 int rootedParent(int u,int v)//u->root,v->point
90 {
91     if (u==v) return u;
92     if (!isAncestor(v,u)) return pa[v];
93     auto it=upper_bound(adj[v].begin(),adj[v].end(),u,[&](int x,int y){ return in[x]<in[y]; })-1;
94     return *it;
95 }
96
97 int rootedSize(int u,int v)//same as rootedParent
98 {
99     if (u==v) return n;
100    if (!isAncestor(v,u)) return siz[v];
101    return n-siz[rootedParent(u,v)];
102 }
103
104 int rootedLca(int a,int b,int c) { return lca(a,b)^lca(b,c)^lca(c,a); }
105 };

```

2-SAT

```

1 struct TwoSat
2 {
3     int n;
4     vector<vector<int>>> e;
5     vector<bool> ans;

```

```

6
7 TwoSat(int n):n(n),e(n<<1),ans(n){}
8
9 void addClause(int u,bool f,int v,bool g)
10 {
11     e[u*2+!f].push_back(v*2+g);
12     e[v*2+!g].push_back(u*2+f);
13 }
14
15 bool satisfiable()
16 {
17     vector<int> id(n*2,-1),dfn(n*2,-1),low(n*2,-1),stk;
18     int now=0,cnt=0;
19     function<void(int)> tarjan=[&](int u)
20     {
21         stk.push_back(u);
22         dfn[u]=low[u]=now++;
23         for (auto v:e[u])
24         {
25             if (dfn[v]==-1)
26             {
27                 tarjan(v);
28                 low[u]=min(low[u],low[v]);
29             }
30             else if (id[v]==-1)
31                 low[u]=min(low[u],dfn[v]);
32         }
33         if (dfn[u]==low[u])
34         {
35             int v;
36             do
37             {
38                 v=stk.back();
39                 stk.pop_back();
40                 id[v]=cnt;
41             } while (v!=u);
42             cnt++;
43         }
44     };
45     for (int i=0;i<n*2;i++)
46         if (dfn[i]==-1)
47             tarjan(i);
48     for (int i=0;i<n;i++)
49     {
50         if (id[i*2]==id[i*2+1]) return 0;
51         ans[i]=id[i*2]>id[i*2+1];
52     }
53     return 1;
54 }
55 vector<bool> answer() { return ans; }
56 };

```

最大流

```

1 template <class T>
2 struct MaxFlow
3 {
4     struct _Edge
5     {
6         int to;
7         T cap;
8         _Edge(int to,T cap):to(to),cap(cap){}
9     };
10
11     int n;
12     vector<_Edge> e;
13     vector<vector<int>> g;
14     vector<int> cur,h;
15
16     MaxFlow(){}
17     MaxFlow(int n) { init(n); }

```



```

18
19 void init(int n)
20 {
21     this->n=n;
22     e.clear();
23     g.assign(n,{});
24     cur.resize(n);
25     h.resize(n);
26 }
27
28 bool bfs(int s,int t)
29 {
30     h.assign(n,-1);
31     queue<int> que;
32     h[s]=0;
33     que.push(s);
34     while (!que.empty())
35     {
36         const int u=que.front();
37         que.pop();
38         for (int i:g[u])
39         {
40             auto [v,c]=e[i];
41             if (c>0&&h[v]==-1)
42             {
43                 h[v]=h[u]+1;
44                 if (v==t) return 1;
45                 que.push(v);
46             }
47         }
48     }
49     return 0;
50 }
51
52 T dfs(int u,int t,T f)
53 {
54     if (u==t) return f;
55     auto r=f;
56     for (int &i=cur[u];i<int(g[u].size());i++)
57     {
58         const int j=g[u][i];
59         auto [v,c]=e[j];
60         if (c>0&&h[v]==h[u]+1)
61         {
62             auto a=dfs(v,t,min(r,c));
63             e[j].cap-=a;
64             e[j^1].cap+=a;
65             r-=a;
66             if (r==0) return f;
67         }
68     }
69     return f-r;
70 }
71
72 void addEdge(int u,int v,T c)
73 {
74     g[u].push_back(e.size());
75     e.emplace_back(v,c);
76     g[v].push_back(e.size());
77     e.emplace_back(u,0);
78 }
79
80 T flow(int s,int t)
81 {
82     T ans=0;
83     while (bfs(s,t))
84     {
85         cur.assign(n,0);
86         ans+=dfs(s,t,numeric_limits<T>::max());
87     }
88     return ans;

```

```

89     }
90
91     vector<bool> minCut()
92     {
93         vector<bool> c(n);
94         for (int i=0;i<n;i++) c[i]=(h[i]!=-1);
95         return c;
96     }
97
98     struct Edge
99     {
100         int from;
101         int to;
102         T cap;
103         T flow;
104     };
105
106     vector<Edge> edges()
107     {
108         vector<Edge> a;
109         for (int i=0;i<e.size();i+=2)
110         {
111             Edge x;
112             x.from=e[i+1].to;
113             x.to=e[i].to;
114             x.cap=e[i].cap+e[i+1].cap;
115             x.flow=e[i+1].cap;
116             a.push_back(x);
117         }
118         return a;
119     }
120 };

```

最小费用最大流

```

1  template <class T>
2  struct MinCostFlow
3  {
4      struct _Edge
5      {
6          int to;
7          T cap;
8          T cost;
9          _Edge(int to,T cap,T cost):to(to),cap(cap),cost(cost){}
10
11      };
12
13      int n;
14      vector<_Edge> e;
15      vector<vector<int>> g;
16      vector<T> h,dis;
17      vector<int> pre;
18
19      bool john(int s,int t)
20      {
21          dis.assign(n,numeric_limits<T>::max());
22          pre.assign(n,-1);
23          priority_queue<pair<T,int>,vector<pair<T,int>>,greater<pair<T,int>>> q;
24          dis[s]=0;
25          q.emplace(0,s);
26          while (!q.empty())
27          {
28              T d=q.top().first;
29              int u=q.top().second;
30              q.pop();
31              if (dis[u]!=d) continue;
32              for (int i:g[u])
33              {
34                  int v=e[i].to;
35                  T cap=e[i].cap;
36                  T cost=e[i].cost;

```

```

37         if (cap>0&&dis[v]>d+h[u]-h[v]+cost)
38         {
39             dis[v]=d+h[u]-h[v]+cost;
40             pre[v]=i;
41             q.emplace(dis[v],v);
42         }
43     }
44 }
45 return dis[t]!=numeric_limits<T>::max();
46 }
47
48 MinCostFlow(){}
49 MinCostFlow(int n) { init(n); }
50
51 void init(int n_)
52 {
53     n=n_;
54     e.clear();
55     g.assign(n,{});
56 }
57
58 void addEdge(int u,int v,T cap,T cost)
59 {
60     g[u].push_back(e.size());
61     e.emplace_back(v,cap,cost);
62     g[v].push_back(e.size());
63     e.emplace_back(u,0,-cost);
64 }
65
66 pair<T,T> flow(int s,int t)
67 {
68     T flow=0;
69     T cost=0;
70     h.assign(n,0);
71     while (john(s,t))
72     {
73         for (int i=0;i<n;i++) h[i]+=dis[i];
74         T aug=numeric_limits<int>::max();
75         for (int i=t;i!=s;i=e[pre[i]^1].to)
76             aug=min(aug,e[pre[i]].cap);
77         for (int i=t;i!=s;i=e[pre[i]^1].to)
78         {
79             e[pre[i]].cap-=aug;
80             e[pre[i]^1].cap+=aug;
81         }
82         flow+=aug;
83         cost+=aug*h[t];
84     }
85     return make_pair(flow,cost);
86 }
87
88 struct Edge
89 {
90     int from;
91     int to;
92     T cap;
93     T cost;
94     T flow;
95 };
96
97 vector<Edge> edges()
98 {
99     vector<Edge> a;
100     for (int i=0;i<e.size();i+=2)
101     {
102         Edge x;
103         x.from=e[i+1].to;
104         x.to=e[i].to;
105         x.cap=e[i].cap+e[i+1].cap;
106         x.cost=e[i].cost;
107         x.flow=e[i+1].cap;

```

```

108         a.push_back(x);
109     }
110     return a;
111 }
112 };

```

计算几何

EPS

```

1  const double eps=1e-8;
2  int sgn(double x)
3  {
4      if (fabs(x)<eps) return 0;
5      if (x>0) return 1;
6      return -1;
7  }

```

Point

```

1  template <class T>
2  struct Point
3  {
4      T x,y;
5      Point(T x_=0,T y_=0):x(x_),y(y_) {}
6
7      Point &operator += (Point p) &
8      {
9          x+=p.x;
10         y+=p.y;
11         return *this;
12     }
13
14     Point &operator -= (Point p) &
15     {
16         x-=p.x;
17         y-=p.y;
18         return *this;
19     }
20
21     Point &operator *= (T v) &
22     {
23         x*=v;
24         y*=v;
25         return *this;
26     }
27
28     Point operator - () const { return Point(-x,-y); }
29
30     friend Point operator + (Point a,Point b) { return a+=b; }
31     friend Point operator - (Point a,Point b) { return a-=b; }
32     friend Point operator * (Point a,T b) { return a*=b; }
33     friend Point operator * (T a,Point b) { return b*=a; }
34
35     friend bool operator == (Point a,Point b) { return a.x==b.x&& a.y==b.y; }
36
37     friend istream &operator >> (istream &is,Point &p) { return is>>p.x>>p.y; }
38
39     friend ostream &operator << (ostream &os,Point p) { return os<<'('<<p.x<<','<<p.y<<')'; }
40 };
41
42 template <class T>
43 int sgn(const Point<T> &a) { return a.y>0||(a.y==0&&a.x>0)?1:-1; }
44
45 template <class T>
46 T dot(Point<T> a,Point<T> b) { return a.x*b.x+a.y*b.y; }
47
48 template <class T>
49 T cross(Point<T> a,Point<T> b) { return a.x*b.y-a.y*b.x; }

```

```

50
51 template <class T>
52 T square(Point<T> p) { return dot(p,p); }
53
54 template <class T>
55 double length(Point<T> p) { return sqrt(double(square(p))); }
56
57 long double length(Point<long double> p) { return sqrt(square(p)); }

```

Line

```

1 template <class T>
2 struct Line
3 {
4     Point<T> a,b;
5     Line(Point<T> a_=Point<T>(),Point<T> b_=Point<T>()):a(a_),b(b_) {}
6 };

```

距离

```

1 template <class T>
2 double dis_PP(Point<T> a,Point<T> b) { return length(a-b); }
3
4 template <class T>
5 double dis_PL(Point<T> a,Line<T> l) { return fabs(cross(a-l.a,a-l.b))/dis_PP(l.a,l.b); }
6
7 template <class T>
8 double dis_PS(Point<T> a,Line<T> l)
9 {
10     if (dot(a-l.a,l.b-l.a)<0) return dis_PP(a,l.a);
11     if (dot(a-l.b,l.a-l.b)<0) return dis_PP(a,l.b);
12     return dis_PL(a,l);
13 }

```

点绕中心旋转

```

1 template <class T>
2 Point<T> rotate(Point<T> a,double alpha)
3 { return Point<T>(a.x*cos(alpha)-a.y*sin(alpha),a.x*sin(alpha)+a.y*cos(alpha)); }

```

关于线的对称点

```

1 template <class T>
2 Point<T> lineRoot(Point<T> a,Line<T> l)
3 {
4     Point<T> v=l.b-l.a;
5     return l.a+v*(dot(a-l.a,v)/dot(v,v));
6 }
7
8 template <class T>
9 Point<T> symmetry_PL(Point<T> a,Line<T> l) { return a+(lineRoot(a,l)-a)*2; }

```

位置关系判断

```

1 template <class T>
2 bool pointOnSegment(Point<T> a,Line<T> l)
3 { return (sgn(cross(a-l.a,a-l.b))==0)&&(sgn(dot(a-l.a,a-l.b))<=0); }
4
5 template <class T>
6 bool lineCrossLine(Line<T> a,Line<T> b)
7 {
8     double f1=cross(b.a-a.a,a.b-a.a),f2=cross(b.b-a.a,a.b-a.a);
9     double g1=cross(a.a-b.a,b.b-b.a),g2=cross(a.b-b.a,b.b-b.a);
10    return ((f1<0)^(f2<0))&&((g1<0)^(g2<0));
11 }
12
13 template <class T>
14 bool pointOnLineLeft(Point<T> a,Line<T> l) { return cross(l.b-l.a,a-l.a)>0; }

```

```

15 //适用任意多边形,  $O(n)$ 
16 template <class T>
17 bool pointInPolygon(Point<T> a, const vector<Point<T>> &p)
18 {
19     int n=p.size();
20     for (int i=0; i<n; i++)
21         if (pointOnSegment(a, Line<T>(p[i], p[(i+1)%n])))
22             return 1;
23     bool t=0;
24     for (int i=0; i<n; i++)
25     {
26         Point<T> u=p[i], v=p[(i+1)%n];
27         if (u.x<a.x&&v.x>=a.x&&pointOnLineLeft(a, Line<T>(v, u))) t^=1;
28         if (u.x>=a.x&&v.x<a.x&&pointOnLineLeft(a, Line<T>(u, v))) t^=1;
29     }
30     return t;
31 }
32
33 //适用凸多边形,  $O(\log n)$ 
34 template <class T>
35 bool pointInPolygon_(Point<T> a, const vector<Point<T>> &p)
36 {
37     int n=p.size();
38     if (cross(a-p[0], p[1]-p[0])<0 || cross(a-p[0], p[n-1]-p[0])>0) return 0;
39     if (pointOnSegment(a, Line<T>(p[0], p[1])) || pointOnSegment(a, Line<T>(p[n-1], p[0]))) return 1;
40     int l=1, r=n-1;
41     while (l+1<r)
42     {
43         int mid=(l+r)>>1;
44         if (cross(a-p[l], p[mid]-p[l])<0) l=mid;
45         else r=mid;
46     }
47     if (cross(a-p[l], p[r]-p[l])>0) return 0;
48     if (pointOnSegment(a, Line<T>(p[l], p[r]))) return 1;
49     return 1;
50 }
51

```

线段交点

```

1 //小心平行
2 template <class T>
3 Point<T> lineIntersection(Line<T> a, Line<T> b)
4 {
5     Point<T> u=a.b-a, v=a.b-a, w=b.b-b.a;
6     double t=cross(u, w)/cross(w, v);
7     return a.a+t*v;
8 }

```

过定点做圆的切线

```

1 template <class T>
2 vector<Line<T>> tan_PC(Point<T> a, Point<T> c, T r)
3 {
4     Point<T> v=c-a;
5     vector<Line<T>> res;
6     int dis=dis_PP(a, c);
7     if (sgn(dis-r)==0) res.push_back(rotate(v, acos(-1)/2));
8     else if (dis>r)
9     {
10         double alpha=asin(r/dis);
11         res.push_back(rotate(v, alpha));
12         res.push_back(rotate(v, -alpha));
13     }
14     return res;
15 }

```

两圆交点

```
1 template <class T>
2 vector<Point<T>> circleIntersection(Point<T> c1,T r1,Point<T> c2,T r2)
3 {
4     auto get=[&](Point<T> c,T r,double alpha)->Point<T>
5     { return Point<T>(c.x+cos(alpha)*r,c.y+sin(alpha)*r); };
6
7     auto angle=[&](Point<T> a)->double { return atan2(a.x,a.y); };
8
9     vector<Point<T>> res;
10    double d=dis_PP(c1,c2);
11    if (sgn(d)==0) return res;
12    if (sgn(r1+r2-d)<0) return res;
13    if (sgn(fabs(r1-r2)-d)>0) return res;
14    double alpha=angle(c2-c1);
15    double beta=acos((r1*r1-r2*r2+d*d)/(r1*d*2));
16    Point<T> p1=get(c1,r1,alpha-beta),p2=get(c1,r1,alpha+beta);
17    res.push_back(p1);
18    if (p1!=p2) res.push_back(p2);
19    return res;
20 }
```

多边形面积

```
1 template <class T>
2 double polygonArea(const vector<Point<T>> &p)
3 {
4     int n=p.size();
5     double res=0;
6     for (int i=1;i<n-1;i++) res+=cross(p[i]-p[0],p[i+1]-p[0]);
7     return fabs(res/2);
8 }
```

自适应辛普森法

```
1 //注意边界函数值不能小于 eps
2 double f(double x) { return pow(x,0.5); }
3 double calc(double l,double r)
4 {
5     double mid=(l+r)/2.0;
6     return (r-l)*(f(l)+f(r)+f(mid)*4.0)/6.0;
7 }
8 double simpson(double l,double r,double lst)
9 {
10    double mid=(l+r)/2.0;
11    double fl=calc(l,mid),fr=calc(mid,r);
12    if (sgn(fl+fr-lst)==0) return fl+fr;
13    else return simpson(l,mid,fl)+simpson(mid,r,fr);
14 }
```

静态凸包

```
1 template <class T>
2 vector<Point<T>> getHull(vector<Point<T>> p)
3 {
4     vector<Point<T>> h,l;
5     sort(p.begin(),p.end(),[&](auto a,auto b)
6     {
7         if (a.x!=b.x) return a.x<b.x;
8         else return a.y<b.y;
9     });
10    p.erase(unique(p.begin(),p.end()),p.end());
11    if (p.size()<=1) return p;
12    for (auto a:p)
13    {
14        while (h.size()>1&&sgn(cross(a-h.back(),a-h[h.size()-2]))<=0) h.pop_back();
15        while (l.size()>1&&sgn(cross(a-l.back(),a-l[l.size()-2]))>=0) l.pop_back();
16        l.push_back(a);
17        h.push_back(a);
18    }
```

```

18     }
19     l.pop_back();
20     reverse(h.begin(),h.end());
21     h.pop_back();
22     l.insert(l.end(),h.begin(),h.end());
23     return l;
24 }

```

旋转卡壳求直径

```

1  template <class T>
2  double getDiameter(vector<Point<T>> p)
3  {
4      double res=0;
5      if (p.size()==2) return dis_PP(p[0],p[1]);
6      int n=p.size();
7      p.push_back(p.front());
8      int j=2;
9      for (int i=0;i<n;i++)
10     {
11         while (sgn(cross(p[i+1]-p[i],p[j]-p[i])-cross(p[i+1]-p[i],p[j+1]-p[i]))<0)
12             j=(j+1)%n;
13         res=max({res,dis_PP(p[i],p[j]),dis_PP(p[i+1],p[j])});
14     }
15     return res;
16 }

```

半平面交

```

1  template <class T>
2  vector<Point<T>> hp(vector<Line<T>> lines)
3  {
4      sort(lines.begin(),lines.end(),[&](auto l1,auto l2)
5      {
6          auto d1=l1.b-l1.a;
7          auto d2=l2.b-l2.a;
8
9          if (sgn(d1)!=sgn(d2)) return sgn(d1)==1;
10         return cross(d1,d2)>0;
11     });
12
13     deque<Line<T>> ls;
14     deque<Point<T>> ps;
15     for (auto l:lines)
16     {
17         if (ls.empty())
18         {
19             ls.push_back(l);
20             continue;
21         }
22         while (!ps.empty()&&!pointOnLineLeft(ps.back(),l))
23         {
24             ps.pop_back();
25             ls.pop_back();
26         }
27         while (!ps.empty()&&!pointOnLineLeft(ps[0],l))
28         {
29             ps.pop_front();
30             ls.pop_front();
31         }
32         if (cross(l.b-l.a,ls.back().b-ls.back().a)==0)
33         {
34             if (dot(l.b-l.a,ls.back().b-ls.back().a)>0)
35             {
36                 if (!pointOnLineLeft(ls.back().a,l))
37                 {
38                     assert(ls.size()==1);
39                     ls[0]=l;
40                 }
41                 continue;

```



```

42         }
43         return {};
44     }
45     ps.push_back(lineIntersection(ls.back(),l));
46     ls.push_back(l);
47 }
48 while (!ps.empty() && !pointOnLineLeft(ps.back(),ls[0]))
49 {
50     ps.pop_back();
51     ls.pop_back();
52 }
53 if (ls.size() <= 2) return {};
54 ps.push_back(lineIntersection(ls[0],ls.back()));
55 return vector(ps.begin(),ps.end());
56 }

```