

Algorithm Library

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Contents

头文件	2
DEBUG 头	2
__int128 输出流	2
常用数学函数	2
数学	3
欧拉筛	3
取模类 (MInt)	3
组合数	5
多项式	5
线性基	9
原根表	10
数据结构	11
并查集 (启发式合并 + 带撤销)	11
状压 RMQ	11
树状数组	13
线段树	13
字符串	16
字符串哈希 (随机模数)	16
KMP	16
Z 函数	16
AC 自动机	17
后缀数组	18
(广义) 后缀自动机	18
Manacher	19
回文自动机	20
图论	21
强连通分量	21
边双连通分量	22
轻重链剖分	23
2-SAT	25
计算几何	26
EPS	26
Point	26
Line	27
距离	27
点绕中心旋转	27
关于线的对称点	27
位置关系判断	27
线段交点	28
过定点做圆的切线	28
两圆交点	28
多边形面积	29
自适应辛普森法	29
静态凸包	29
旋转卡壳求直径	30

头文件

DEBUG 头

```
1  #include <bits/stdc++.h>
2  using namespace std;
3  using i64=long long;
4  using i128=__int128;
5
6  namespace DBG
7  {
8      template <class T>
9      void _dbg(const char *f,T t) { cerr<<f<<'\n'; }
10
11     template <class A,class... B>
12     void _dbg(const char *f,A a,B... b)
13     {
14         while (*f!=',') cerr<<*f++;
15         cerr<<'\n';
16         _dbg(f+1,b...);
17     }
18
19     template <class T>
20     ostream& operator << (ostream& os,const vector<T> &v)
21     {
22         os<<"[ ";
23         for (const auto &x:v) os<<x<<", ";
24         os<<"]";
25         return os;
26     }
27
28     #define dbg(...) _dbg(#__VA_ARGS__, __VA_ARGS__)
29 }
30
31 using namespace DBG;
```

__int128 输出流

```
1  ostream &operator << (ostream &os,i128 n)
2  {
3      string s;
4      bool neg=n<0;
5      if (neg) n=-n;
6      while (n)
7      {
8          s+='0'+n%10;
9          n/=10;
10     }
11     if (neg) s+='-';
12     reverse(s.begin(),s.end());
13     if (s.empty()) s+='0';
14     return os<<s;
15 }
```

常用数学函数

```
1  i64 ceilDiv(i64 n,i64 m)
2  {
3      if (n>=0) return (n+m-1)/m;
4      else return n/m;
5  }
6
7  i64 floorDiv(i64 n,i64 m)
8  {
9      if (n>=0) return n/m;
10     else return (n-m+1)/m;
11 }
12
13 i128 gcd(i128 a,i128 b)
14 {
```

```

15     return b?gcd(b,a%b):a;
16 }

```

数学

欧拉筛

```

1  vector<int> minp,primes;
2
3  void sieve(int n)
4  {
5      minp.assign(n+1,0);
6      primes.clear();
7      for (int i=2;i<=n;i++)
8      {
9          if (!minp[i])
10             {
11                 minp[i]=i;
12                 primes.push_back(i);
13             }
14             for (auto p:primes)
15             {
16                 if (i*p>n) break;
17                 minp[i*p]=p;
18                 if (p==minp[i]) break;
19             }
20     }
21 }

```

取模类 (MInt)

```

1  template <class T>
2  constexpr T power(T a,i64 b)
3  {
4      T res=1;
5      for (;b>=1,a*=a)
6          if (b&1) res*=a;
7      return res;
8  }
9
10 template <int P>
11 struct MInt
12 {
13     int x;
14     constexpr MInt():x{} {}
15     constexpr MInt(i64 x):x{norm(x%getMod())} {}
16
17     static int Mod;
18     constexpr static int getMod()
19     {
20         if (P>0) return P;
21         else return Mod;
22     }
23
24     constexpr static void setMod(int Mod_) { Mod=Mod_; }
25
26     constexpr int norm(int x) const
27     {
28         if (x<0) x+=getMod();
29         if (x>=getMod()) x-=getMod();
30         return x;
31     }
32
33     constexpr int val() const { return x; }
34
35     explicit constexpr operator int () const { return x; }
36
37     constexpr MInt operator - () const
38     {

```

```

39     MInt res;
40     res.x=norm(getMod()-x);
41     return res;
42 }
43
44 constexpr MInt inv() const
45 {
46     assert(x!=0);
47     return power(*this,getMod()-2);
48 }
49
50 constexpr MInt &operator *= (MInt rhs) &
51 {
52     x=1ll*x*rhs.x%getMod();
53     return *this;
54 }
55
56 constexpr MInt &operator += (MInt rhs) &
57 {
58     x=norm(x+rhs.x);
59     return *this;
60 }
61
62 constexpr MInt &operator -= (MInt rhs) &
63 {
64     x=norm(x-rhs.x);
65     return *this;
66 }
67
68 constexpr MInt &operator /= (MInt rhs) &
69 {
70     return *this*=rhs.inv();
71 }
72
73 friend constexpr MInt operator * (MInt lhs,MInt rhs)
74 {
75     MInt res=lhs;
76     res*=rhs;
77     return res;
78 }
79
80 friend constexpr MInt operator + (MInt lhs,MInt rhs)
81 {
82     MInt res=lhs;
83     res+=rhs;
84     return res;
85 }
86
87 friend constexpr MInt operator - (MInt lhs,MInt rhs)
88 {
89     MInt res=lhs;
90     res-=rhs;
91     return res;
92 }
93
94 friend constexpr MInt operator / (MInt lhs,MInt rhs)
95 {
96     MInt res=lhs;
97     res/=rhs;
98     return res;
99 }
100
101 friend constexpr istream &operator >> (istream &is,MInt &a)
102 {
103     i64 v;
104     is>>v;
105     a=MInt(v);
106     return is;
107 }
108
109 friend constexpr ostream &operator << (ostream &os,const MInt &a) { return os<<a.val(); }

```

```

110
111     friend constexpr bool operator == (MInt lhs, MInt rhs) { return lhs.val() == rhs.val(); }
112
113     friend constexpr bool operator != (MInt lhs, MInt rhs) { return lhs.val() != rhs.val(); }
114 };
115
116 template<>
117 int MInt<0>::Mod=1;
118
119 template<int V, int P>
120 constexpr MInt<P> CInv=MInt<P>(V).inv();

```

组合数

```

1  struct Comb
2  {
3      int n;
4      vector<Z> _fac, _inv, _finv;
5
6      Comb():n{0}, _fac{1}, _inv{0}, _finv{1}{}
7      Comb(int n):Comb() { init(n); }
8
9      void init(int m)
10     {
11         m=min(m, Z::getMod()-1);
12         if (m<=n) return;
13         _fac.resize(m+1);
14         _inv.resize(m+1);
15         _finv.resize(m+1);
16
17         for (int i=n+1; i<=m; i++)
18             _fac[i]=_fac[i-1]*i;
19         _finv[m]=_fac[m].inv();
20         for (int i=m; i>n; i--)
21         {
22             _finv[i-1]=_finv[i]*i;
23             _inv[i]=_finv[i]*_fac[i-1];
24         }
25         n=m;
26     }
27
28     Z fac(int m)
29     {
30         if (m>n) init(m<<1);
31         return _fac[m];
32     }
33
34     Z finv(int m)
35     {
36         if (m>n) init(m<<1);
37         return _finv[m];
38     }
39
40     Z inv(int m)
41     {
42         if (m>n) init(m<<1);
43         return _inv[m];
44     }
45
46     Z binom(int n, int m)
47     {
48         if (n<m || m<0) return 0;
49         return fac(n)*finv(m)*finv(n-m);
50     }
51 } comb;

```

多项式

```

1  vector<int> rev;
2  vector<Z> roots{0,1};

```

```

3
4 void dft(vector<Z> &a)
5 {
6     int n=a.size();
7     if (int(rev.size())!=n)
8     {
9         int k=__builtin_ctz(n)-1;
10        rev.resize(n);
11        for (int i=0;i<n;i++)
12            rev[i]=rev[i>>1]>>1|(i&1)<<k;
13    }
14    for (int i=0;i<n;i++)
15        if (rev[i]<i)
16            swap(a[i],a[rev[i]]);
17    if (int(roots.size())<n)
18    {
19        int k=__builtin_ctz(roots.size());
20        roots.resize(n);
21        while ((1<<k)<n)
22        {
23            Z e=power(Z(3),(P-1)>>(k+1));
24            for (int i=1<<(k-1);i<(1<<k);i++)
25            {
26                roots[i<<1]=roots[i];
27                roots[i<<1|1]=roots[i]*e;
28            }
29            k++;
30        }
31    }
32    for (int k=1;k<n;k<<=1)
33        for (int i=0;i<n;i+=k*2)
34            for (int j=0;j<k;j++)
35            {
36                Z u=a[i+j],v=a[i+j+k]*roots[j+k];
37                a[i+j]=u+v;
38                a[i+j+k]=u-v;
39            }
40 }
41
42 void idft(vector<Z> &a)
43 {
44     int n=a.size();
45     reverse(a.begin()+1,a.end());
46     dft(a);
47     Z inv=(1-P)/n;
48     for (int i=0;i<n;i++) a[i]*=inv;
49 }
50
51 struct Poly
52 {
53     vector<Z> a;
54
55     Poly(){}
56     explicit Poly(int size,function<Z(int)>f=[](int) { return 0; }):a(size)
57     { for (int i=0;i<size;i++) a[i]=f(i); }
58     Poly(const vector<Z> &a):a(a){}
59     Poly(const initializer_list<Z> &a):a(a){}
60
61     int size() const { return a.size(); }
62
63     void resize(int n) { a.resize(n); }
64
65     Z operator [] (int idx) const
66     {
67         if (idx<size()) return a[idx];
68         else return 0;
69     }
70
71     Z &operator [] (int idx) { return a[idx]; }
72
73     Poly mulxk(int k) const

```

```

74 {
75     auto b=a;
76     b.insert(b.begin(),k,0);
77     return Poly(b);
78 }
79
80 Poly modxk(int k) const
81 {
82     k=min(k,size());
83     return Poly(vector<Z>(a.begin(),a.begin()+k));
84 }
85
86 Poly divxk(int k) const
87 {
88     if (size()<=k) return Poly();
89     return Poly(vector<Z>(a.begin()+k,a.end()));
90 }
91
92 friend Poly operator + (const Poly &a,const Poly &b)
93 {
94     vector<Z> res(max(a.size(),b.size()));
95     for (int i=0;i<int(res.size());i++)
96         res[i]=a[i]+b[i];
97     return Poly(res);
98 }
99
100 friend Poly operator - (const Poly &a,const Poly &b)
101 {
102     vector<Z> res(max(a.size(),b.size()));
103     for (int i=0;i<int(res.size());i++)
104         res[i]=a[i]-b[i];
105     return Poly(res);
106 }
107
108 friend Poly operator - (const Poly &a)
109 {
110     vector<Z> res(a.size());
111     for (int i=0;i<int(res.size());i++)
112         res[i]=-a[i];
113     return Poly(res);
114 }
115
116 friend Poly operator * (Poly a,Poly b)
117 {
118     if (!a.size()||!b.size()) return Poly();
119     if (a.size()<b.size()) swap(a,b);
120     if (b.size()<128)
121     {
122         Poly c(a.size()+b.size()-1);
123         for (int i=0;i<a.size();i++)
124             for (int j=0;j<b.size();j++)
125                 c[i+j]+=a[i]*b[j];
126         return c;
127     }
128     int sz=1,tot=a.size()+b.size()-1;
129     while (sz<tot) sz<<=1;
130     a.a.resize(sz);
131     b.a.resize(sz);
132     dft(a.a);
133     dft(b.a);
134     for (int i=0;i<sz;i++)
135         a.a[i]=a[i]*b[i];
136     idft(a.a);
137     a.a.resize(tot);
138     return a;
139 }
140
141 friend Poly operator * (Z a,Poly b)
142 {
143     for (int i=0;i<int(b.size());i++) b[i]*=a;
144     return b;

```



```

145     }
146
147     friend Poly operator * (Poly a,Z b)
148     {
149         for (int i=0;i<int(a.size());i++) a[i]*=b;
150         return a;
151     }
152
153     Poly &operator += (Poly b) { return (*this)=(*this)+b; }
154     Poly &operator -= (Poly b) { return (*this)=(*this)-b; }
155     Poly &operator *= (Poly b) { return (*this)=(*this)*b; }
156     Poly &operator *= (Z b) { return (*this)=(*this)*b; }
157
158     Poly deriv() const
159     {
160         if (a.empty()) return Poly();
161         vector<Z> res(size()-1);
162         for (int i=0;i<size()-1;i++)
163             res[i]=(i+1)*a[i+1];
164         return Poly(res);
165     }
166
167     Poly integr() const
168     {
169         vector<Z> res(size()+1);
170         for (int i=0;i<size();i++)
171             res[i+1]=a[i]/(i+1);
172         return Poly(res);
173     }
174
175     Poly inv(int m) const
176     {
177         Poly x{a[0].inv()};
178         int k=1;
179         while (k<m)
180         {
181             k<<=1;
182             x=(x*(Poly{2}-modxk(k)*x)).modxk(k);
183         }
184         return x.modxk(m);
185     }
186
187     Poly ln(int m) const { return (deriv()*inv(m)).integr().modxk(m); }
188
189     Poly exp(int m) const
190     {
191         Poly x{1};
192         int k=1;
193         while (k<m)
194         {
195             k<<=1;
196             x=(x*(Poly{1}-x.ln(k)+modxk(k))).modxk(k);
197         }
198         return x.modxk(m);
199     }
200
201     Poly pow(int k,int m) const
202     {
203         int i=0;
204         while (i<size()&&a[i].val()==0) i++;
205         if (i==size()||1ll*i*k>=m) return Poly(vector<Z>(m));
206         Z v=a[i];
207         auto f=divxk(i)*v.inv();
208         return (f.ln(m-i*k)*k).exp(m-i*k).mulxk(i*k)*power(v,k);
209     }
210
211     Poly sqrt(int m) const
212     {
213         Poly x{1};
214         int k=1;
215         while (k<m)

```

```

216     {
217         k<=1;
218         x=(x+(modxk(k)*x.inv(k)).modxk(k))*((P+1)/2);
219     }
220     return x.modxk(m);
221 }
222 Poly mult(Poly b) const
223 {
224     if (b.size()==0) return Poly();
225     int n=b.size();
226     reverse(b.a.begin(),b.a.end());
227     return ((*this)*b).divxk(n-1);
228 }
229
230 vector<Z> eval(vector<Z> x) const
231 {
232     if (size()==0) return vector<Z>(x.size(),0);
233     const int n=max(int(x.size()),size());
234     vector<Poly> q(n<<2);
235     vector<Z> ans(x.size());
236     x.resize(n);
237     function<void(int,int,int)> build=[&](int p,int l,int r)
238     {
239         if (r-l==1) q[p]=Poly{1,-x[l]};
240         else
241         {
242             int m=(l+r)>>1;
243             build(p<<1,l,m);
244             build(p<<1|1,m,r);
245             q[p]=q[p<<1]*q[p<<1|1];
246         }
247     };
248     function<void(int,int,int,const Poly&)> work=[&](int p,int l,int r,const Poly &num)
249     {
250         if (r-l==1)
251         {
252             if (l<int(ans.size())) ans[l]=num[0];
253         }
254         else
255         {
256             int m=(l+r)>>1;
257             work(p<<1,l,m,num.mult(q[p<<1|1]).modxk(m-l));
258             work(p<<1|1,m,r,num.mult(q[p<<1]).modxk(r-m));
259         }
260     };
261     build(1,0,n);
262     work(1,0,n,mult(q[1].inv(n)));
263     return ans;
264 }
265 };

```

线性基

```

1 struct LB
2 {
3     static constexpr int L=60;
4     array<i64,L+1> a{};
5
6     LB(){}
7
8     LB(const vector<i64> &v) { init(v); }
9
10    bool insert(i64 t)
11    {
12        for (int i=L;i>=0;i--)
13            if (t&(1ll<<i))
14            {
15                if (!a[i])
16                {
17                    a[i]=t;
18                    return 1;

```

```

19         }
20         else t^=a[i];
21     }
22     return 0;
23 }
24
25 void init(const vector<i64> &v) { for (auto x:v) insert(x); }
26
27 bool check(i64 t)
28 {
29     for (int i=L;i>=0;i--)
30         if (t&(1ll<<i))
31             if (!a[i]) return 0;
32             else t^=a[i];
33     return 1;
34 }
35
36 i64 QueryMax()
37 {
38     i64 res=0;
39     for (int i=L;i>=0;i--)
40         res=max(res,res^a[i]);
41     return res;
42 }
43
44 i64 QueryMin()
45 {
46     for (int i=0;i<=L;i++)
47         if (a[i]) return a[i];
48     return 0;
49 }
50
51 i64 QueryKth(int k)
52 {
53     i64 res=0;
54     int cnt=0;
55     array<i64,L+1> tmp{};
56     for (int i=0;i<=L;i++)
57     {
58         for (int j=i-1;j>=0;j--)
59             if (a[i]&(1ll<<j)) a[i]^=a[j];
60         if (a[i]) tmp[cnt++]=a[i];
61     }
62     if (k>=(1ll<<cnt)) return -1;
63     for (int i=0;i<cnt;i++)
64         if (k&(1ll<<i)) res^=tmp[i];
65     return res;
66 }
67 };

```

原根表

	prime	r	k	g
1	3	1	1	2
2	5	1	2	2
3	17	1	4	3
4	97	3	5	5
5	193	3	6	5
6	257	1	8	3
7	7681	15	9	17
8	12289	3	12	11
9	40961	5	13	3
10	65537	1	16	3
11	786433	3	18	10
12	5767169	11	19	3
13	7340033	7	20	3
14	23068673	11	21	3
15	104857601	25	22	3
16	167772161	5	25	3
17	469762049	7	26	3
18	1004535809	479	21	3

```

20 2013265921      15 27 31
21 2281701377      17 27 3
22 3221225473      3 30 5
23 75161927681     35 31 3
24 77309411329     9 33 7
25 206158430209    3 36 22
26 2061584302081   15 37 7
27 2748779069441   5 39 3
28 6597069766657   3 41 5
29 39582418599937   9 42 5
30 79164837199873   9 43 5
31 263882790666241 15 44 7
32 1231453023109121 35 45 3
33 1337006139375617 19 46 3
34 3799912185593857 27 47 5
35 4222124650659841 15 48 19
36 7881299347898369 7 50 6
37 31525197391593473 7 52 3
38 180143985094819841 5 55 6
39 1945555039024054273 27 56 5
40 4179340454199820289 29 57 3
41

```

数据结构

并查集（启发式合并 + 带撤销）

```

1 struct DSU
2 {
3     int n=0;
4     vector<int> fa,siz;
5     stack<int> s;
6
7     DSU(int n) { init(n); }
8
9     void init(int n)
10    {
11        fa.resize(n);
12        iota(fa.begin(),fa.end(),0);
13        siz.assign(n,1);
14        while (!s.empty()) s.pop();
15    }
16
17    int get(int x) { return fa[x]==x?x:get(fa[x]); }
18
19    void merge(int x,int y)
20    {
21        x=get(x),y=get(y);
22        if (x==y) return;
23        if (siz[x]<siz[y]) swap(x,y);
24        s.push(y),fa[y]=x,siz[x]+=siz[y];
25    }
26
27    void undo()
28    {
29        if (s.empty()) return;
30        int y=s.top();
31        s.pop();
32        siz[fa[y]]-=siz[y];
33        fa[y]=y;
34    }
35
36    void back(int t=0) { while (s.size()) undo(); }
37 };

```

状压 RMQ

```

1 template <class T,class Cmp=less<T>>
2 struct RMQ

```

```

3 {
4     const Cmp cmp=Cmp();
5     static constexpr unsigned B=64;
6     using u64=unsigned long long;
7     int n;
8     vector<vector<T>> a;
9     vector<T> pre,suf,ini;
10    vector<u64> stk;
11
12    RMQ() {}
13    RMQ(const vector<T> &v) { init(v); }
14
15    void init(const vector<T> &v)
16    {
17        n=v.size();
18        pre=suf=ini=v;
19        stk.resize(n);
20        if (!n) return;
21        const int M=(n-1)/B+1;
22        const int lg=__lg(M);
23        a.assign(lg+1,vector<T>(M));
24        for (int i=0;i<M;i++)
25        {
26            a[0][i]=v[i*B];
27            for (int j=1;j<B&& i*B+j<n;j++)
28                a[0][i]=min(a[0][i],v[i*B+j],cmp);
29        }
30        for (int i=1;i<n;i++)
31            if (i%B) pre[i]=min(pre[i],pre[i-1],cmp);
32        for (int i=n-2;i>=0;i--)
33            if (i%B!=B-1) suf[i]=min(suf[i],suf[i+1],cmp);
34        for (int j=0;j<lg;j++)
35            for (int i=0;i+(2<<j)<=M;i++)
36                a[j+1][i]=min(a[j][i],a[j][i+(1<<j)],cmp);
37        for (int i=0;i<M;i++)
38        {
39            const int l=i*B;
40            const int r=min(1U*n,l+B);
41            u64 s=0;
42            for (int j=l;j<r;j++)
43            {
44                while (s&&cmp(v[j],v[__lg(s)+l])) s|=1ULL<<__lg(s);
45                s|=1ULL<<(j-l);
46                stk[j]=s;
47            }
48        }
49    }
50
51    //查询区间 [l,r) 的 RMQ
52    T operator()(int l,int r)
53    {
54        if (l/B!=(r-1)/B)
55        {
56            T ans=min(suf[l],pre[r-1],cmp);
57            l=l/B+1,r=r/B;
58            if (l<r)
59            {
60                int k=__lg(r-l);
61                ans=min({ans,a[k][l],a[k][r-(1<<k)]},cmp);
62            }
63            return ans;
64        }
65        else
66        {
67            int x=B*(l/B);
68            return ini[__builtin_ctzll(stk[r-1]>>(l-x))+l];
69        }
70    }
71 };

```

树状数组

```
1  template <class T>
2  struct BIT
3  {
4      int n;
5      vector<T> a;
6
7      BIT(int n_=0) { init(n_); }
8
9      void init(int n_)
10     {
11         n=n_;
12         a.assign(n,T{});
13     }
14
15     void add(int x,const T &v)
16     {
17         for (int i=x+1;i<=n;i+=i&-i)
18             a[i-1]=a[i-1]+v;
19     }
20
21     //查询区间 [0,x)
22     T sum(int x)
23     {
24         T ans{};
25         for (int i=x;i>0;i-=i&-i)
26             ans=ans+a[i-1];
27         return ans;
28     }
29
30     //查询区间 [l,r)
31     T rangeSum(int l,int r) { return sum(r)-sum(l); }
32
33     int select(const T &k)
34     {
35         int x=0;
36         T cur{};
37         for (int i=1<<__lg(n);i;i>>=1)
38         {
39             if (x+i<=n&&cur+a[x+i-1]<=k)
40             {
41                 x+=i;
42                 cur=cur+a[x-1];
43             }
44         }
45         return x;
46     }
47 };
```

线段树

```
1  template <class Info,class Tag>
2  struct SGT
3  {
4      int n;
5      vector<Info> info;
6      vector<Tag> tag;
7
8      SGT():n(0) {}
9      SGT(int n_,Info v_=Info()) { init(n_,v_); }
10
11     template <class T>
12     SGT(vector<T> init_) { init(init_); }
13
14     void init(int n_,Info v_=Info()) { init(vector(n_,v_)); }
15
16     template <class T>
17     void init(vector<T> init_)
18     {
19         n=init_.size();
```

```

20     info.assign(4<<__lg(n),Info());
21     tag.assign(4<<__lg(n),Tag());
22     function<void(int,int,int)> build=[&](int p,int l,int r)
23     {
24         if (r-l==1)
25         {
26             info[p]=init_[l];
27             return;
28         }
29         int m=(l+r)>>1;
30         build(p<<1,l,m);
31         build(p<<1|1,m,r);
32         pushup(p);
33     };
34     build(1,0,n);
35 }
36
37 void pushup(int p) { info[p]=info[p<<1]+info[p<<1|1]; }
38
39 void apply(int p,const Tag &v)
40 {
41     info[p].apply(v);
42     tag[p].apply(v);
43 }
44
45 void pushdown(int p)
46 {
47     apply(p<<1,tag[p]);
48     apply(p<<1|1,tag[p]);
49     tag[p]=Tag();
50 }
51
52 void modify(int p,int l,int r,int x,const Info &v)
53 {
54     if (r-l==1)
55     {
56         info[p]=v;
57         return;
58     }
59     int m=(l+r)>>1;
60     pushdown(p);
61     if (x<m) modify(p<<1,l,m,x,v);
62     else modify(p<<1|1,m,r,x,v);
63     pushup(p);
64 }
65
66 //O(log n) 单点修改
67 void modify(int p,const Info &v) { modify(1,0,n,p,v); }
68
69 Info rangeQuery(int p,int l,int r,int x,int y)
70 {
71     if (l>=y||r<=x) return Info();
72     if (l>=x&&r<=y) return info[p];
73     int m=(l+r)>>1;
74     pushdown(p);
75     return rangeQuery(p<<1,l,m,x,y)+rangeQuery(p<<1|1,m,r,x,y);
76 }
77
78 //O(log n) 区间查询 [l,r)
79 Info rangeQuery(int l,int r) { rangeQuery(1,0,n,l,r); }
80
81 void rangeApply(int p,int l,int r,int x,int y,const Tag &v)
82 {
83     if (l>=y||r<=x) return;
84     if (l>=x&&r<=y)
85     {
86         apply(p,v);
87         return;
88     }
89     int m=(l+r)>>1;
90     pushdown(p);

```

```

91     rangeApply(p<<1,l,m,x,y,v);
92     rangeApply(p<<1|1,m,r,x,y,v);
93     pushup(p);
94 }
95
96 //O(log n) 区间操作 [l,r)
97 void rangeApply(int l,int r,const Tag &v) { rangeApply(1,0,n,l,r,v); }
98
99 //O(log n) 区间 [l,r) 内查找第一个合法位置
100 template <class F>
101 int findFirst(int p,int l,int r,int x,int y,F pred)
102 {
103     if (l>=y||r<=x||!pred(info[p])) return -1;
104     if (r-l==1) return l;
105     int m=(l+r)>>1;
106     pushdown(p);
107     int res=findFirst(p<<1,l,m,x,y,pred);
108     if (res==-1) res=findFirst(p<<1|1,m,r,x,y,pred);
109     return res;
110 }
111
112 template <class F>
113 int findFirst(int l,int r,F pred) { return findFirst(1,0,n,l,r,pred); }
114
115 template <class F>
116 int findLast(int p,int l,int r,int x,int y,F pred)
117 {
118     if (l>=y||r<=x||!pred(info[p])) return -1;
119     if (r-l==1) return l;
120     int m=(l+r)>>1;
121     pushdown(p);
122     int res=findFirst(p<<1|1,m,r,x,y,pred);
123     if (res==-1) res=findFirst(p<<1,l,m,x,y,pred);
124     return res;
125 }
126
127 template <class F>
128 int findLast(int l,int r,F pred) { return findLast(1,0,n,l,r,pred); }
129 };
130
131 //这里默认乘法优先 (x*a+b)*c+d=x*(a*c)+(b*c+d)
132 struct Tag
133 {
134     i64 a=1,b=0;
135     void apply(Tag t)
136     {
137         a*=t.a;
138         b=b*t.a+t.b;
139     }
140 };
141
142 struct Info
143 {
144     i64 x=0,l=0,r=0;
145     void apply(Tag t)
146     {
147         int len=r-l+1;
148         x=x*t.a+len*t.b;
149     }
150 };
151
152 Info operator + (Info a,Info b)
153 {
154     return {a.x+b.x,min(a.l,b.l),max(a.r,b.r)};
155 }

```


字符串

字符串哈希（随机模数）

```
1 bool isPrime(int n)
2 {
3     if (n<=1) return 0;
4     for (int i=2;i*i<=n;i++)
5         if (n%i==0) return 0;
6     return 1;
7 }
8
9 int findPrime(int n)
10 {
11     while (!isPrime(n)) n++;
12     return n;
13 }
14
15 mt19937 rng(time(0));
16 const int P=findPrime(rng()%9000000000+1000000000);
17 struct StrHash
18 {
19     int n;
20     vector<int> h,p;
21
22     StrHash(const string &s){ init(s); }
23
24     void init(const string &s)
25     {
26         n=s.size();
27         h.resize(n+1);
28         p.resize(n+1);
29         p[0]=1;
30         for (int i=0;i<n;i++) h[i+1]=(10ll*h[i]+s[i]-'a')%P;
31         for (int i=0;i<n;i++) p[i+1]=10ll*p[i]%P;
32     }
33
34     //查询 [l,r) 的区间哈希
35     int get(int l,int r) { return (h[r]+1ll*(P-h[l])*p[r-l])%P; }
36 };
```

KMP

```
1 vector<int> KMP(const string &s)
2 {
3     int now=0;
4     vector<int> pre(s.size(),0);
5     for (int i=1;i<s.size();i++)
6     {
7         while (now&& s[i]!=s[now]) now=pre[now-1];
8         if (s[i]==s[now]) now++;
9         pre[i]=now;
10    }
11    return pre;
12 }
```

Z函数

```
1 vector<int> zFunction(string s)
2 {
3     int n=s.size();
4     vector<int> z(n);
5     z[0]=n;
6     for (int i=1,j=1;i<n;i++)
7     {
8         z[i]=max(0,min(j+z[j]-i,z[i-j]));
9         while (i+z[i]<n&& s[z[i]]==s[i+z[i]]) z[i]++;
10        if (i+z[i]>j+z[j]) j=i;
11    }
```

```

12     return z;
13 }

```

AC 自动机

```

1  struct ACAM
2  {
3      static constexpr int ALPHABET=26;
4      struct Node
5      {
6          int len;
7          int link;
8          array<int,ALPHABET> next;
9          Node():len{0},link{0},next{}{}
10     };
11
12     vector<Node> t;
13
14     ACAM() { init(); }
15
16     void init()
17     {
18         t.assign(2,Node());
19         t[0].next.fill(1);
20         t[0].len=-1;
21     }
22
23     int newNode()
24     {
25         t.emplace_back();
26         return t.size()-1;
27     }
28
29     int add(const string &a)
30     {
31         int p=1;
32         for (auto c:a)
33         {
34             int x=c-'a';
35             if (t[p].next[x]==0)
36             {
37                 t[p].next[x]=newNode();
38                 t[t[p].next[x]].len=t[p].len+1;
39             }
40             p=t[p].next[x];
41         }
42         return p;
43     }
44
45     void work()
46     {
47         queue<int> q;
48         q.push(1);
49         while (!q.empty())
50         {
51             int x=q.front();
52             q.pop();
53             for (int i=0;i<ALPHABET;i++)
54             {
55                 if (t[x].next[i]==0) t[x].next[i]=t[t[x].link].next[i];
56                 else
57                 {
58                     t[t[x].next[i]].link=t[t[x].link].next[i];
59                     q.push(t[x].next[i]);
60                 }
61             }
62         }
63     }
64
65     int next(int p,int x) { return t[p].next[x]; }
66

```

```

67     int link(int p) { return t[p].link; }
68
69     int size() { return t.size(); }
70 };

```

后缀数组

```

1  struct SA
2  {
3      int n;
4      vector<int> sa,rk,lc;
5      SA(const string &s)
6      {
7          n=s.length();
8          sa.resize(n);
9          rk.resize(n);
10         lc.resize(n-1);
11         iota(sa.begin(),sa.end(),0);
12         sort(sa.begin(),sa.end(),[&](int a,int b){ return s[a]<s[b]; });
13         rk[sa[0]]=0;
14         for (int i=1;i<n;i++) rk[sa[i]]=rk[sa[i-1]]+(s[sa[i]]!=s[sa[i-1]]);
15         int k=1;
16         vector<int> tmp,cnt(n);
17         tmp.reserve(n);
18         while (rk[sa[n-1]]<n-1)
19         {
20             tmp.clear();
21             for (int i=0;i<k;i++) tmp.push_back(n-k+i);
22             for (auto i:sa)
23                 if (i>=k) tmp.push_back(i-k);
24             fill(cnt.begin(),cnt.end(),0);
25             for (int i=0;i<n;i++) cnt[rk[i]]++;
26             for (int i=1;i<n;i++) cnt[i]+=cnt[i-1];
27             for (int i=n-1;i>=0;i--) sa[--cnt[rk[tmp[i]]]]=tmp[i];
28             swap(rk,tmp);
29             rk[sa[0]]=0;
30             for (int i=1;i<n;i++)
31                 rk[sa[i]]=rk[sa[i-1]]+(tmp[sa[i-1]]<tmp[sa[i]] || sa[i-1]+k==n || tmp[sa[i-1]+k]<tmp[sa[i]+k]);
32             k<=1;
33         }
34         for (int i=0,j=0;i<n;i++)
35         {
36             if (rk[i]==0) j=0;
37             else
38             {
39                 for (j=j>0;i+j<n&&sa[rk[i]-1]+j<n&&s[i+j]==s[sa[rk[i]-1]+j];) j++;
40                 lc[rk[i]-1]=j;
41             }
42             //lc[i]:lcp(sa[i],sa[i+1]),lcp(sa[i],sa[j])=min{lc[i...j-1]}
43         }
44 };

```

(广义) 后缀自动机

```

1  struct SAM
2  {
3      static constexpr int ALPHABET=26;
4      struct Node
5      {
6          int len;
7          int link;
8          array<int,ALPHABET> next;
9          Node():len{},link{},next{} {}
10     };
11
12     vector<Node> t;
13
14     SAM() { init(); }
15
16     void init()

```

```

17     {
18         t.assign(2,Node());
19         t[0].next.fill(1);
20         t[0].len=-1;
21     }
22
23     int newNode()
24     {
25         t.emplace_back();
26         return t.size()-1;
27     }
28
29     int extend(int lst,int c)
30     {
31         if (t[lst].next[c]&& t[t[lst].next[c]].len==t[lst].len+1)
32             return t[lst].next[c];
33         int p=lst,np=newNode(),flag=0;
34         t[np].len=t[p].len+1;
35         while (!t[p].next[c])
36         {
37             t[p].next[c]=np;
38             p=t[p].link;
39         }
40         if (!p)
41         {
42             t[np].link=1;
43             return np;
44         }
45         int q=t[p].next[c];
46         if (t[q].len==t[p].len+1)
47         {
48             t[np].link=q;
49             return np;
50         }
51         if (p==lst) flag=1,np=0,t.pop_back();
52         int nq=newNode();
53         t[nq].link=t[q].link;
54         t[nq].next=t[q].next;
55         t[nq].len=t[p].len+1;
56         t[q].link=t[np].link=nq;
57         while (p&& t[p].next[c]==q)
58         {
59             t[p].next[c]=nq;
60             p=t[p].link;
61         }
62         return flag?nq:np;
63     }
64
65     int add(const string &a)
66     {
67         int p=1;
68         for (auto c:a) p=extend(p,c-'a');
69         return p;
70     }
71
72     int next(int p,int x) { return t[p].next[x]; }
73
74     int link(int p) { return t[p].link; }
75
76     int len(int p) { return t[p].len; }
77
78     int size() { return t.size(); }
79 };

```

Manacher

```

1 vector<int> manacher(vector<int> s)
2 {
3     vector<int> t{0};
4     for (auto c:s)
5     {

```

```

6         t.push_back(c);
7         t.push_back(0);
8     }
9     int n=t.size();
10    vector<int> r(n);
11    for (int i=0,j=0;i<n;i++)
12    {
13        if (j*2-i>=0&&j+r[j]>i) r[i]=min(r[j*2-i],j+r[j]-i);
14        while (i-r[i]>=0&&i+r[i]<n&&t[i-r[i]]==t[i+r[i]]) r[i]++;
15        if (i+r[i]>j+r[j]) j=i;
16    }
17    return r;
18 }

```

回文自动机

```

1  struct PAM
2  {
3      static constexpr int ALPHABET_SIZE=28;
4      struct Node
5      {
6          int len,link,cnt;
7          array<int,ALPHABET_SIZE> next;
8          Node():len{},link{},cnt{},next{}{}
9      };
10     vector<Node> t;
11     int suff;
12     string s;
13
14     PAM() { init(); }
15
16     void init()
17     {
18         t.assign(2,Node());
19         t[0].len=-1;
20         suff=1;
21         s.clear();
22     }
23
24     int newNode()
25     {
26         t.emplace_back();
27         return t.size()-1;
28     }
29
30     bool add(char c,char offset='a')
31     {
32         int pos=s.size();
33         s+=c;
34         int let=c-offset;
35         int cur=suff,curlen=0;
36         while (1)
37         {
38             curlen=t[cur].len;
39             if (pos-curlen-1>=0&&s[pos-curlen-1]==s[pos]) break;
40             cur=t[cur].link;
41         }
42         if (t[cur].next[let])
43         {
44             suff=t[cur].next[let];
45             return 0;
46         }
47         int num=newNode();
48         suff=num;
49         t[num].len=t[cur].len+2;
50         t[cur].next[let]=num;
51         if (t[num].len==1)
52         {
53             t[num].link=t[num].cnt=1;
54             return 1;
55         }

```

```

56     while (1)
57     {
58         cur=t[cur].link;
59         curlen=t[cur].len;
60         if (pos-curlen-1>=0&&s[pos-curlen-1]==s[pos])
61         {
62             t[num].link=t[cur].next[let];
63             break;
64         }
65     }
66     t[num].cnt=t[t[num].link].cnt+1;
67     return 1;
68 }
69 };

```

图论

强连通分量

```

1  struct SCC
2  {
3      int n,cur,cnt;
4      vector<vector<int>> adj;
5      vector<int> stk,dfn,low,bel;
6
7      SCC() {}
8      SCC(int n) { init(n); }
9
10     void init(int n)
11     {
12         this->n=n;
13         adj.assign(n,{});
14         stk.clear();
15         dfn.assign(n,-1);
16         low.resize(n);
17         bel.assign(n,-1);
18         cur=cnt=0;
19     }
20
21     void add(int u,int v) { adj[u].push_back(v); }
22
23     void dfs(int x)
24     {
25         dfn[x]=low[x]=cur++;
26         stk.push_back(x);
27         for (auto y:adj[x])
28         {
29             if (dfn[y]==-1)
30             {
31                 dfs(y);
32                 low[x]=min(low[x],low[y]);
33             }
34             else if (bel[y]==-1) low[x]=min(low[x],dfn[y]);
35         }
36         if (dfn[x]==low[x])
37         {
38             int y;
39             do
40             {
41                 y=stk.back();
42                 bel[y]=cnt;
43                 stk.pop_back();
44             } while (y!=x);
45             cnt++;
46         }
47     }
48
49     vector<int> work()
50     {
51         for (int i=0;i<n;i++)

```

```

52         if (dfn[i]==-1) dfs(i);
53         return bel;
54     }
55
56     struct Graph
57     {
58         int n;
59         vector<pair<int,int>> edges;
60         vector<int> siz,cnt;
61     };
62
63     Graph compress()
64     {
65         Graph G;
66         G.n=cnt;
67         G.siz.resize(cnt);
68         G.cnt.resize(cnt);
69         for (int i=0;i<n;i++)
70         {
71             G.siz[bel[i]]++;
72             for (auto j:adj[i])
73                 if (bel[i]!=bel[j])
74                     G.edges.emplace_back(bel[j],bel[i]);
75         }
76         return G;
77     };
78 };

```

边双连通分量

```

1  struct EBCC
2  {
3      int n;
4      vector<vector<int>> adj;
5      vector<int> stk,dfn,low,bel;
6      int cur,cnt;
7
8      EBCC() {}
9      EBCC(int n) { init(n); }
10
11     void init(int n)
12     {
13         this->n=n;
14         adj.assign(n,{});
15         dfn.assign(n,-1);
16         low.resize(n);
17         bel.assign(n,-1);
18         stk.clear();
19         cur=cnt=0;
20     }
21
22     void add(int u,int v)
23     {
24         adj[u].push_back(v);
25         adj[v].push_back(u);
26     }
27
28     void dfs(int x,int p)
29     {
30         dfn[x]=low[x]=cur++;
31         stk.push_back(x);
32         for (auto y:adj[x])
33         {
34             if (y==p) continue;
35             if (dfn[y]==-1)
36             {
37                 dfs(y,x);
38                 low[x]=min(low[x],low[y]);
39             }
40             else if (bel[y]==-1&&dfn[y]<dfn[x]) low[x]=min(low[x],dfn[y]);
41         }

```

```

42     if (dfn[x]==low[x])
43     {
44         int y;
45         do
46         {
47             y=stk.back();
48             bel[y]=cnt;
49             stk.pop_back();
50         } while (y!=x);
51         cnt++;
52     }
53 }
54
55 vector<int> work()
56 {
57     dfs(0,-1);
58     return bel;
59 }
60
61 struct Graph
62 {
63     int n;
64     vector<pair<int,int>> edges;
65     vector<int> siz,cnt;
66 };
67
68 Graph compress()
69 {
70     Graph G;
71     G.n=cnt;
72     G.siz.resize(cnt);
73     G.cnt.resize(cnt);
74     for (int i=0;i<n;i++)
75     {
76         G.siz[bel[i]]++;
77         for (auto j:adj[i])
78         {
79             if (bel[i]<bel[j]) G.edges.emplace_back(bel[i],bel[j]);
80             else if (i<j) G.cnt[bel[i]]++;
81         }
82     }
83     return G;
84 };
85 };

```

轻重链剖分

```

1  struct HLD
2  {
3      int n;
4      vector<int> siz,top,dep,pa,in,out,seq;
5      vector<vector<int>> adj;
6      int cur;
7
8      HLD(){}
9      HLD(int n) { init(n); }
10
11     void init(int n)
12     {
13         this->n=n;
14         siz.resize(n);
15         top.resize(n);
16         dep.resize(n);
17         pa.resize(n);
18         in.resize(n);
19         out.resize(n);
20         seq.resize(n);
21         cur=0;
22         adj.assign(n,{});
23     }
24

```



```

25 void addEdge(int u,int v)
26 {
27     adj[u].push_back(v);
28     adj[v].push_back(u);
29 }
30
31 void work(int rt=0)
32 {
33     top[rt]=rt;
34     dep[rt]=0;
35     pa[rt]=-1;
36     dfs1(rt);
37     dfs2(rt);
38 }
39
40 void dfs1(int u)
41 {
42     if (pa[u]!=-1) adj[u].erase(find(adj[u].begin(),adj[u].end(),pa[u]));
43     siz[u]=1;
44     for (auto &v:adj[u])
45     {
46         pa[v]=u;
47         dep[v]=dep[u]+1;
48         dfs1(v);
49         siz[u]+=siz[v];
50         if (siz[v]>siz[adj[u][0]])
51             swap(v,adj[u][0]);
52     }
53 }
54
55 void dfs2(int u)
56 {
57     in[u]=cur++;
58     seq[in[u]]=u;
59     for (auto v:adj[u])
60     {
61         top[v]=(v==adj[u][0])?top[u]:v;
62         dfs2(v);
63     }
64     out[u]=cur;
65 }
66
67 int lca(int u,int v)
68 {
69     while (top[u]!=top[v])
70     {
71         if (dep[top[u]]>dep[top[v]]) u=pa[top[u]];
72         else v=pa[top[v]];
73     }
74     return dep[u]<dep[v]?u:v;
75 }
76
77 int dist(int u,int v) { return dep[u]+dep[v]-(dep[lca(u,v)]<<1); }
78
79 int jump(int u,int k)
80 {
81     if (dep[u]<k) return -1;
82     int d=dep[u]-k;
83     while (dep[top[u]]>d) u=pa[top[u]];
84     return seq[in[u]-dep[u]+d];
85 }
86
87 bool isAncestor(int u,int v) { return in[u]<=in[v]&&in[v]<out[u]; }
88
89 int rootedParent(int u,int v)//u->root,v->point
90 {
91     if (u==v) return u;
92     if (!isAncestor(v,u)) return pa[v];
93     auto it=upper_bound(adj[v].begin(),adj[v].end(),u,[&](int x,int y){ return in[x]<in[y]; })-1;
94     return *it;
95 }

```

```

96
97     int rootedSize(int u,int v)//same as rootedParent
98     {
99         if (u==v) return n;
100        if (!isAncestor(v,u)) return siz[v];
101        return n-siz[rootedParent(u,v)];
102    }
103
104    int rootedLca(int a,int b,int c) { return lca(a,b)^lca(b,c)^lca(c,a); }
105 };

```

2-SAT

```

1  struct TwoSat
2  {
3      int n;
4      vector<vector<int>> e;
5      vector<bool> ans;
6
7      TwoSat(int n):n(n),e(n<<1),ans(n){}
8
9      void addClause(int u,bool f,int v,bool g)
10     {
11         e[u*2+!f].push_back(v*2+g);
12         e[v*2+!g].push_back(u*2+f);
13     }
14
15     bool satisfiable()
16     {
17         vector<int> id(n*2,-1),dfn(n*2,-1),low(n*2,-1),stk;
18         int now=0,cnt=0;
19         function<void(int)> tarjan=[&](int u)
20         {
21             stk.push_back(u);
22             dfn[u]=low[u]=now++;
23             for (auto v:e[u])
24             {
25                 if (dfn[v]==-1)
26                 {
27                     tarjan(v);
28                     low[u]=min(low[u],low[v]);
29                 }
30                 else if (id[v]==-1)
31                     low[u]=min(low[u],dfn[v]);
32             }
33             if (dfn[u]==low[u])
34             {
35                 int v;
36                 do
37                 {
38                     v=stk.back();
39                     stk.pop_back();
40                     id[v]=cnt;
41                 } while (v!=u);
42                 cnt++;
43             }
44         };
45         for (int i=0;i<n*2;i++)
46             if (dfn[i]==-1)
47                 tarjan(i);
48         for (int i=0;i<n;i++)
49         {
50             if (id[i*2]==id[i*2+1]) return 0;
51             ans[i]=id[i*2]>id[i*2+1];
52         }
53         return 1;
54     }
55     vector<bool> answer() { return ans; }
56 };

```

计算几何

EPS

```
1  const double eps=1e-8;
2  int sgn(double x)
3  {
4      if (fabs(x)<eps) return 0;
5      if (x>0) return 1;
6      return -1;
7  }
```

Point

```
1  template <class T>
2  struct Point
3  {
4      T x,y;
5      Point(T x_=0,T y_=0):x(x_),y(y_) {}
6
7      Point &operator += (Point p) &
8      {
9          x+=p.x;
10         y+=p.y;
11         return *this;
12     }
13
14     Point &operator -= (Point p) &
15     {
16         x-=p.x;
17         y-=p.y;
18         return *this;
19     }
20
21     Point &operator *= (T v) &
22     {
23         x*=v;
24         y*=v;
25         return *this;
26     }
27
28     Point operator - () const { return Point(-x,-y); }
29
30     friend Point operator + (Point a,Point b) { return a+=b; }
31     friend Point operator - (Point a,Point b) { return a-=b; }
32     friend Point operator * (Point a,T b) { return a*=b; }
33     friend Point operator * (T a,Point b) { return b*=a; }
34
35     friend bool operator == (Point a,Point b) { return a.x==b.x&& a.y==b.y; }
36
37     friend istream &operator >> (istream &is,Point &p) { return is>>p.x>>p.y; }
38
39     friend ostream &operator << (ostream &os,Point p) { return os<<'('<<p.x<<','<<p.y<<')'; }
40 };
41
42 template <class T>
43 T dot(Point<T> a,Point<T> b) { return a.x*b.x+a.y*b.y; }
44
45 template <class T>
46 T cross(Point<T> a,Point<T> b) { return a.x*b.y-a.y*b.x; }
47
48 template <class T>
49 T square(Point<T> p) { return dot(p,p); }
50
51 template <class T>
52 double length(Point<T> p) { return sqrt(double(square(p))); }
53
54 long double length(Point<long double> p) { return sqrt(square(p)); }
```

Line

```
1 template <class T>
2 struct Line
3 {
4     Point<T> a,b;
5     Line(Point<T> a_=Point<T>(),Point<T> b_=Point<T>()):a(a_),b(b_) {}
6 };
```

距离

```
1 template <class T>
2 double dis_PP(Point<T> a,Point<T> b) { return length(a-b); }
3
4 template <class T>
5 double dis_PL(Point<T> a,Line<T> l) { return fabs(cross(a-l.a,a-l.b))/dis_PP(l.a,l.b); }
6
7 template <class T>
8 double dis_PS(Point<T> a,Line<T> l)
9 {
10     if (dot(a-l.a,l.b-l.a)<0) return dis_PP(a,l.a);
11     if (dot(a-l.b,l.a-l.b)<0) return dis_PP(a,l.b);
12     return dis_PL(a,l);
13 }
```

点绕中心旋转

```
1 template <class T>
2 Point<T> rotate(Point<T> a,double alpha)
3 { return Point<T>(a.x*cos(alpha)-a.y*sin(alpha),a.x*sin(alpha)+a.y*cos(alpha)); }
```

关于线的对称点

```
1 template <class T>
2 Point<T> lineRoot(Point<T> a,Line<T> l)
3 {
4     Point<T> v=l.b-l.a;
5     return l.a+v*(dot(a-l.a,v)/dot(v,v));
6 }
7
8 template <class T>
9 Point<T> symmetry_PL(Point<T> a,Line<T> l) { return a+(lineRoot(a,l)-a)*2; }
```

位置关系判断

```
1 template <class T>
2 bool pointOnSegment(Point<T> a,Line<T> l)
3 { return (sgn(cross(a-l.a,a-l.b))==0)&&(sgn(dot(a-l.a,a-l.b))<=0); }
4
5 template <class T>
6 bool lineCrossLine(Line<T> a,Line<T> b)
7 {
8     double f1=cross(b.a-a.a,a.b-a.a),f2=cross(b.b-a.a,a.b-a.a);
9     double g1=cross(a.a-b.a,b.b-b.a),g2=cross(a.b-b.a,b.b-b.a);
10    return ((f1<0)^(f2<0))&&((g1<0)^(g2<0));
11 }
12
13 template <class T>
14 bool pointOnLineLeft(Point<T> a,Line<T> l) { return cross(l.b-l.a,a-l.a)>0; }
15
16 //适用任意多边形,0(n)
17 template <class T>
18 bool pointInPolygon(Point<T> a,const vector<Point<T>> &p)
19 {
20     int n=p.size();
21     for (int i=0;i<n;i++)
22         if (pointOnSegment(a,Line<T>(p[i],p[(i+1)%n])))
23             return 1;
24     bool t=0;
```

```

25     for (int i=0;i<n;i++)
26     {
27         Point<T> u=p[i],v=p[(i+1)%n];
28         if (u.x<a.x&&v.x>=a.x&&pointOnLineLeft(a,Line<T>(v,u))) t^=1;
29         if (u.x>=a.x&&v.x<a.x&&pointOnLineLeft(a,Line<T>(u,v))) t^=1;
30     }
31     return t;
32 }
33
34 //适用凸多边形,  $O(\log n)$ 
35 template <class T>
36 bool pointInPolygon_(Point<T> a,const vector<Point<T>> &p)
37 {
38     int n=p.size();
39     if (cross(a-p[0],p[1]-p[0])<0||cross(a-p[0],p[n-1]-p[0])>0) return 0;
40     if (pointOnSegment(a,Line<T>(p[0],p[1]))||pointOnSegment(a,Line<T>(p[n-1],p[0]))) return 1;
41     int l=1,r=n-1;
42     while (l+1<r)
43     {
44         int mid=(l+r)>>1;
45         if (cross(a-p[l],p[mid]-p[l])<0) l=mid;
46         else r=mid;
47     }
48     if (cross(a-p[l],p[r]-p[l])>0) return 0;
49     if (pointOnSegment(a,Line<T>(p[l],p[r]))) return 1;
50     return 1;
51 }

```

线段交点

```

1 //小心平行
2 template <class T>
3 Point<T> lineIntersection(Line<T> a,Line<T> b)
4 {
5     Point<T> u=a.a-b.a,v=a.b-a.a,w=b.b-b.a;
6     double t=cross(u,w)/cross(w,v);
7     return a.a+t*v;
8 }

```

过定点做圆的切线

```

1 template <class T>
2 vector<Line<T>> tan_PC(Point<T> a,Point<T> c,T r)
3 {
4     Point<T> v=c-a;
5     vector<Line<T>> res;
6     int dis=dis_PP(a,c);
7     if (sgn(dis-r)==0) res.push_back(rotate(v,acos(-1)/2));
8     else if (dis>r)
9     {
10         double alpha=asin(r/dis);
11         res.push_back(rotate(v,alpha));
12         res.push_back(rotate(v,-alpha));
13     }
14     return res;
15 }

```

两圆交点

```

1 template <class T>
2 vector<Point<T>> circleIntersection(Point<T> c1,T r1,Point<T> c2,T r2)
3 {
4     auto get=[&](Point<T> c,T r,double alpha)->Point<T>
5     { return Point<T>(c.x+cos(alpha)*r,c.y+sin(alpha)*r); };
6
7     auto angle=[&](Point<T> a)->double { return atan2(a.x,a.y); };
8
9     vector<Point<T>> res;
10    double d=dis_PP(c1,c2);

```

```

11     if (sgn(d)==0) return res;
12     if (sgn(r1+r2-d)<0) return res;
13     if (sgn(fabs(r1-r2)-d)>0) return res;
14     double alpha=angle(c2-c1);
15     double beta=acos((r1*r1-r2*r2+d*d)/(r1*d*2));
16     Point<T> p1=get(c1,r1,alpha-beta),p2=get(c1,r1,alpha+beta);
17     res.push_back(p1);
18     if (p1!=p2) res.push_back(p2);
19     return res;
20 }

```

多边形面积

```

1 template <class T>
2 double polygonArea(const vector<Point<T>> &p)
3 {
4     int n=p.size();
5     double res=0;
6     for (int i=1;i<n-1;i++) res+=cross(p[i]-p[0],p[i+1]-p[0]);
7     return fabs(res/2);
8 }

```

自适应辛普森法

```

1 //注意边界函数值不能小于 eps
2 double f(double x) { return pow(x,0.5); }
3 double calc(double l,double r)
4 {
5     double mid=(l+r)/2.0;
6     return (r-l)*(f(l)+f(r)+f(mid)*4.0)/6.0;
7 }
8 double simpson(double l,double r,double lst)
9 {
10     double mid=(l+r)/2.0;
11     double fl=calc(l,mid),fr=calc(mid,r);
12     if (sgn(fl+fr-lst)==0) return fl+fr;
13     else return simpson(l,mid,fl)+simpson(mid,r,fr);
14 }

```

静态凸包

```

1 template <class T>
2 vector<Point<T>> getHull(vector<Point<T>> p)
3 {
4     vector<Point<T>> h,l;
5     sort(p.begin(),p.end(),[&](auto a,auto b)
6     {
7         if (a.x!=b.x) return a.x<b.x;
8         else return a.y<b.y;
9     });
10    p.erase(unique(p.begin(),p.end()),p.end());
11    if (p.size()<=1) return p;
12    for (auto a:p)
13    {
14        while (h.size()>1&&sgn(cross(a-h.back(),a-h[h.size()-2]))<=0) h.pop_back();
15        while (l.size()>1&&sgn(cross(a-l.back(),a-l[l.size()-2]))>=0) l.pop_back();
16        l.push_back(a);
17        h.push_back(a);
18    }
19    l.pop_back();
20    reverse(h.begin(),h.end());
21    h.pop_back();
22    l.insert(l.end(),h.begin(),h.end());
23    return l;
24 }

```

旋转卡壳求直径

```
1  template <class T>
2  double getDiameter(vector<Point<T>> p)
3  {
4      double res=0;
5      if (p.size()==2) return dis_PP(p[0],p[1]);
6      int n=p.size();
7      p.push_back(p.front());
8      int j=2;
9      for (int i=0;i<n;i++)
10     {
11         while (sgn(cross(p[i+1]-p[i],p[j]-p[i])-cross(p[i+1]-p[i],p[j+1]-p[i]))<0)
12             j=(j+1)%n;
13         res=max({res,dis_PP(p[i],p[j]),dis_PP(p[i+1],p[j])});
14     }
15     return res;
16 }
```