Assignment2_9628_jzeiders

September 22, 2024

1 John Zeiders (jzeiders) - Assignment 2

[ ]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt

np.random.seed(9628)
```

Part 1

[ ]:
```
def optimizer(a, eta):
    if a > eta/2:
        return a - eta/2
    if np.abs(a) <= eta/2:
        return 0
    return a + eta/2

def one_var_lasso(r, x, lam):
    b_hat = np.dot(r.T,x) / np.sum(x**2)

    n = np.shape(x)[0]
    result = optimizer(b_hat, 2*n*lam / np.sum(x**2))
    return result

def MyLasso(X, y, lam_seq, maxit = 100):
    _, p = X.shape
    nlam = len(lam_seq)
    B = np.zeros((p+1, nlam))


    newX = (X - np.mean(X, axis=0)) / np.std(X, axis=0)

    b = np.zeros(p)
    r = y

      # Triple nested loop
    for m in range(nlam):
        for _ in range(maxit):
```

```
            for j in range(p):
                X_j = newX[:, j].reshape(-1,1)
                r = r + X_j * b[j]
                b[j] = one_var_lasso(r, X_j, lam_seq[m])
                r = r - X_j * b[j]
        B[1:, m] = b

    ##############################
    # YOUR CODE:
    # Scale back the coefficients;
    # Update the intercepts stored in B[, 0]
    ##############################

    B[1:, :] = B[1:, :] / np.std(X, axis=0)[:, np.newaxis]
    B[0, :] = np.mean(y) - np.mean(X, axis=0) @ B[1:, :]


    return(B)

myData = pd.read_csv("https://liangfgithub.github.io/Data/Coding2_Data0.csv")
var_names = myData.columns
y = myData[['Y']].to_numpy()
X = myData.drop(['Y'], axis = 1).to_numpy()

log_lam_seq = np.linspace(-1, -8, num = 80)
lam_seq = np.exp(log_lam_seq)
myout = MyLasso(X, y, lam_seq, maxit = 100)
```

```
/var/folders/fc/z903c2bn73s9_l02mz68j63h0000gn/T/ipykernel_8493/1744841771.py:32
: DeprecationWarning: Conversion of an array with ndim > 0 to a scalar is
deprecated, and will error in future. Ensure you extract a single element from
your array before performing this operation. (Deprecated NumPy 1.25.)
  b[j] = one_var_lasso(r, X_j, lam_seq[m])
```

[ ]:
```
p, _ = myout.shape
plt.figure(figsize = (12,8))

for i in range(p-1):
    plt.plot(log_lam_seq, myout[i+1, :], label = var_names[i])

plt.xlabel('Log Lambda')
plt.ylabel('Coefficients')
plt.title('Lasso Paths - Numpy implementation')
plt.legend()
plt.axis('tight')
```

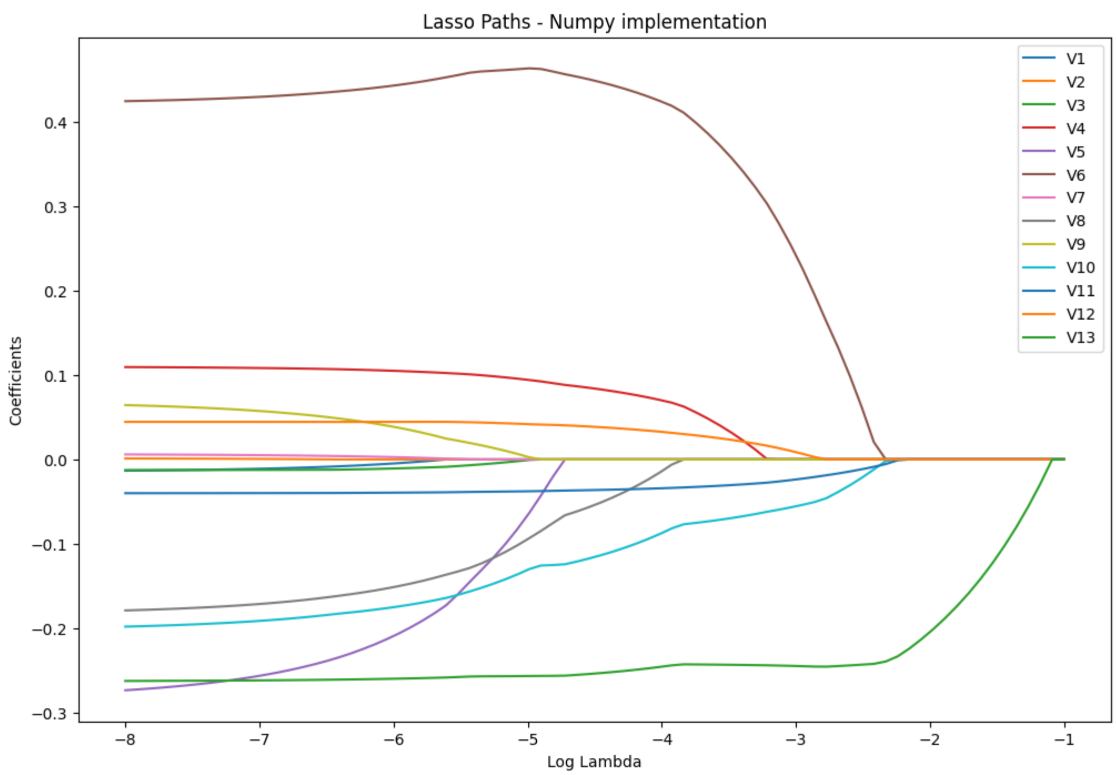
```
true_coefs_df = pd.read_csv("https://liangfgithub.github.io/Data/
↪Coding2_lasso_coefs.csv")
true_B = true_coefs_df.to_numpy()

# Compute the absolute differences
differences = np.abs(myout - true_B)

# Find the maximum difference
max_diff = np.max(differences)

print(f"Maximum difference between estimated and true coefficients: {max_diff:.
↪6f}")
```

```
Maximum difference between estimated and true coefficients: 0.004645
```



[ ]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt

from sklearn.metrics import mean_squared_error
from sklearn.model_selection import train_test_split
from sklearn.linear_model import LinearRegression as lm
```

```python
from sklearn.linear_model import Ridge, RidgeCV, Lasso, LassoCV
import os

from sklearn.decomposition import PCA
from sklearn.preprocessing import StandardScaler
from sklearn.pipeline import Pipeline
import warnings
from sklearn.exceptions import ConvergenceWarning
warnings.filterwarnings("ignore", category=ConvergenceWarning)

# Load the data
url = "https://liangfgithub.github.io/Data/Coding2_Data1.csv"
myData = pd.read_csv(url)

class PCR(object):

    def __init__(self, num_folds=10):
        self.folds = num_folds

    def fit(self, X, Y):
        n, p = X.shape
        indices = np.arange(n)
        np.random.shuffle(indices)
        index_sets = np.array_split(indices, self.folds)
        ncomp = min(p, n - 1 - max([len(i) for i in index_sets]))
        cv_err = np.zeros(ncomp)

        for ifold in range(self.folds):
            train_inds = np.concatenate(index_sets[:ifold] + index_sets[ifold+1:
↪])
            test_inds = index_sets[ifold]

            X_train = X.iloc[train_inds, :]
            pipeline = Pipeline([('scaling', StandardScaler()), ('pca', PCA())])
            pipeline.fit(X_train)
            X_train = pipeline.transform(X_train)
            coefs = Y[train_inds].T @ X_train / np.sum(X_train**2, axis=0)
            b0 = np.mean(Y[train_inds])

            X_test = pipeline.transform(X.iloc[test_inds, :])

            for k in np.arange(ncomp):
                preds = X_test[:, :k] @ coefs.T[:k] + b0
                cv_err[k] += cv_err[k] + np.sum((Y[test_inds]-preds)**2)

        min_ind = np.argmin(cv_err)
        self.ncomp = min_ind+1
```

```
        pipeline = Pipeline([('scaling', StandardScaler()), ('pca',␣
 ↪PCA(n_components=self.ncomp))])
        self.transform = pipeline.fit(X)
        self.model = lm().fit(self.transform.transform(X), Y)

    def predict(self, X):
        X_ = self.transform.transform(X)
        return self.model.predict(X_)


# Separate response and predictors
Y = myData['Y'].values
X = myData.drop(['Y'], axis=1).values  # Convert to NumPy arrays for efficiency

# Number of predictors
n_samples, n_features = X.shape
n_simulations = 50

# Define model functions

def lasso_alphas(X_train, Y_train):
    lasso_alphas = np.logspace(-10, 1, 100)
    lassocv = LassoCV(alphas = lasso_alphas, cv = 10, n_jobs=-1)
    lassocv.fit(X_train, Y_train)
    lassocv.alpha_

    mean_mse = np.mean(lassocv.mse_path_, axis=1)
    std_mse = np.std(lassocv.mse_path_, axis=1) / np.sqrt(10)

    cv_alphas = lassocv.alphas_
    min_idx = np.argmin(mean_mse)

    alpha_min = cv_alphas[min_idx]

    threshold = mean_mse[min_idx] + std_mse[min_idx]
    alpha_1se = max(cv_alphas[np.where(mean_mse <= threshold)])

    return alpha_min, alpha_1se


def full_model(X_train, Y_train, X_test):
    full = lm().fit(X_train, Y_train)
    return mean_squared_error(Y_test, full.predict(X_test))

def ridge_min_model(X_train, Y_train, X_test):
    ridge_alphas = np.logspace(-10, 1, 100)
    ridgecv = RidgeCV(alphas = ridge_alphas, cv = 10,
```

```
                  scoring = 'neg_mean_squared_error')
    ridgecv.fit(X_train, Y_train)
    ridge_model = Ridge(alpha = ridgecv.alpha_)
    ridge_model.fit(X_train, Y_train)
    return mean_squared_error(Y_test, ridge_model.predict(X_test))

def lasso_model(X_train, Y_train, X_test, alpha):
    lasso_model_min = Lasso(alpha = alpha, max_iter=10000)
    lasso_model_min.fit(X_train, Y_train)
    return mean_squared_error(Y_test, lasso_model_min.predict(X_test)),␣
 ↪lasso_model_min

def lasso_model_refit(X_train, Y_train, X_test, model):
    nonzero_indices = np.where(model.coef_ != 0)[0]
    lm_refit = lm()
    lm_refit.fit(X_train.iloc[:, nonzero_indices], Y_train)
    return mean_squared_error(Y_test, lm_refit.predict(X_test.iloc[:,␣
 ↪nonzero_indices]))


def pcr_model(X_train, Y_train, X_test):
    pcr = PCR()
    pcr.fit(X_train, Y_train)
    return mean_squared_error(Y_test, pcr.predict(X_test))

# Initialize a list to store MSPE results
results_list = []

cache_file = "/Users/jzeiders/Documents/Code/Learnings/GraduateML/src/Coding2/
 ↪part2_a_cache.csv"


if os.path.exists(cache_file):
    print(f"Loading cached results from {cache_file}")
    results = pd.read_csv(cache_file, index_col=0)
else:
    # Number of simulations

    for i in range(n_simulations):
        # Split the data: 75% training, 25% testing
        X_train, X_test, Y_train, Y_test = train_test_split(
            X, Y, test_size=0.25, random_state=i
        )

        scaler = StandardScaler(with_mean=True, with_std=True)
        X_train = scaler.fit_transform(X_train)
        X_test = scaler.transform(X_test)
```

```python
        X_train = pd.DataFrame(X_train)
        X_test = pd.DataFrame(X_test)



        # Full Model
        mse_full = full_model(X_train, Y_train, X_test)

        # Ridge.min
        mse_ridge = ridge_min_model(X_train, Y_train, X_test)

        alpha_min, alpha_1se = lasso_alphas(X_train, Y_train)

        # Lasso.min and Lasso.1se
        mse_lasso_min, _ = lasso_model(X_train, Y_train, X_test, alpha_min)
        mse_lasso_1se, lasso_model_1se= lasso_model(X_train, Y_train, X_test,␣
↪alpha_1se)


        # L.Refit
        mse_l_refit = lasso_model_refit(X_train, Y_train, X_test,␣
↪lasso_model_1se)

        # PCR
        mse_pcr = pcr_model(X_train, Y_train, X_test)

        # Append the results
        results_list.append({
            'Full': mse_full,
            'Ridge.min': mse_ridge,
            'Lasso.min': mse_lasso_min,
            'Lasso.1se': mse_lasso_1se,
            'L.Refit': mse_l_refit,
            'PCR': mse_pcr
        })

        print(f"Simulation {i+1}/{n_simulations} completed.")

    # Convert the list of dictionaries to a DataFrame
    results = pd.DataFrame(results_list)

    results.to_csv(cache_file)

    # Display the first few rows of the results
    print(results.head())
```

```
Loading cached results from /Users/jzeiders/Documents/Code/Learnings/GraduateML/
src/Coding2/part2_a_cache.csv
```

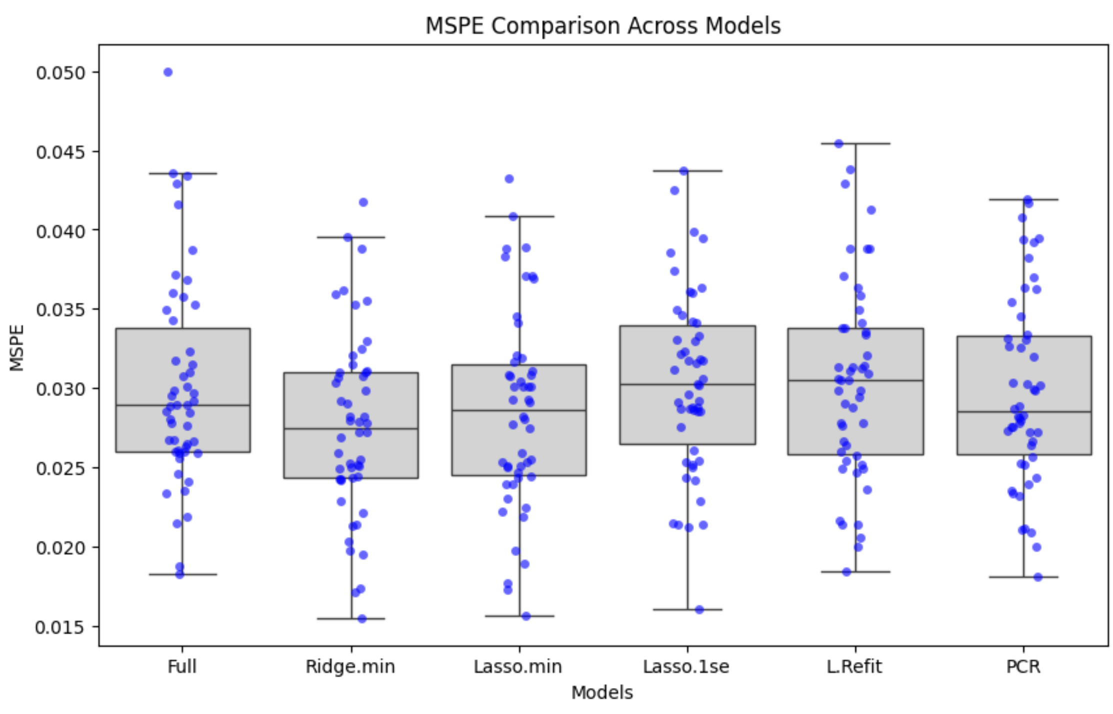
[ ]:
```python
import seaborn as sns
# Set up the figure and axis
plt.figure(figsize=(10, 6))

# Create the strip chart
sns.stripplot(data=results, jitter=True, color='blue', alpha=0.6)

# Overlay a boxplot for additional insights
sns.boxplot(data=results, whis=1.5, color='lightgray', fliersize=0)

# Label the chart
plt.title('MSPE Comparison Across Models')
plt.xlabel('Models')
plt.ylabel('MSPE')

# Show the plot
plt.show()
```



1.0.1 Which procedure or procedures yield the best performance in terms of MSPE?

The best procedure is Ridge.min

1.0.2 Conversely, which procedure or procedures show the poorest performance?

The wrost is Lassso refit

1.0.3 In the context of Lasso regression, which procedure, Lasso.min or Lasso.1se, yields a better MSPE?

Lasso Min yields a better results

1.0.4 Is refitting advantageous in this case? In other words, does L.Refit outperform Lasso.1se?

Refitting performs slightly worse. ### Is variable selection or shrinkage warranted for this particular dataset? To clarify, do you find the performance of the Full model to be comparable to, or divergent from, the best-performing procedure among the other five?

Shrinkage is helpful, particularly ridge min regression, it meaninfuly outperforms the full regression.

[ ]:
```
results_list = []


cache_file = "/Users/jzeiders/Documents/Code/Learnings/GraduateML/src/Coding2/
↪part2_b_cache.csv"
url = "https://liangfgithub.github.io/Data/Coding2_Data2.csv"
myData = pd.read_csv(url)

# Separate response and predictors
Y = myData['Y'].values
X = myData.drop(['Y'], axis=1).values  # Convert to NumPy arrays for efficiency

# Number of predictors
n_samples, n_features = X.shape

if os.path.exists(cache_file):
    print(f"Loading cached results from {cache_file}")
    results = pd.read_csv(cache_file, index_col=0)
else:
    # Number of simulations
    for i in range(n_simulations):
        # Split the data: 75% training, 25% testing
        X_train, X_test, Y_train, Y_test = train_test_split(
            X, Y, test_size=0.25, random_state=i
        )

        scaler = StandardScaler(with_mean=True, with_std=True)
        X_train = scaler.fit_transform(X_train)
        X_test = scaler.transform(X_test)

        X_train = pd.DataFrame(X_train)
```

```
        X_test = pd.DataFrame(X_test)

        # Ridge.min
        mse_ridge = ridge_min_model(X_train, Y_train, X_test)

        alpha_min, alpha_1se = lasso_alphas(X_train, Y_train)

        # Lasso.min and Lasso.1se
        mse_lasso_min, _ = lasso_model(X_train, Y_train, X_test, alpha_min)
        mse_lasso_1se, lasso_model_1se= lasso_model(X_train, Y_train, X_test,␣
 ↪alpha_1se)


        # L.Refit
        mse_l_refit = lasso_model_refit(X_train, Y_train, X_test,␣
 ↪lasso_model_1se)

        # PCR
        mse_pcr = pcr_model(X_train, Y_train, X_test)

        # Append the results
        results_list.append({
            'Ridge.min': mse_ridge,
            'Lasso.min': mse_lasso_min,
            'Lasso.1se': mse_lasso_1se,
            'L.Refit': mse_l_refit,
            'PCR': mse_pcr
        })

        print(f"Simulation {i+1}/{n_simulations} completed.")

    # Convert the list of dictionaries to a DataFrame
    results = pd.DataFrame(results_list)

    results.to_csv(cache_file)

    # Display the first few rows of the results
    print(results.head())
```

Loading cached results from /Users/jzeiders/Documents/Code/Learnings/GraduateML/
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[ ]:
```
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```
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plt.show()
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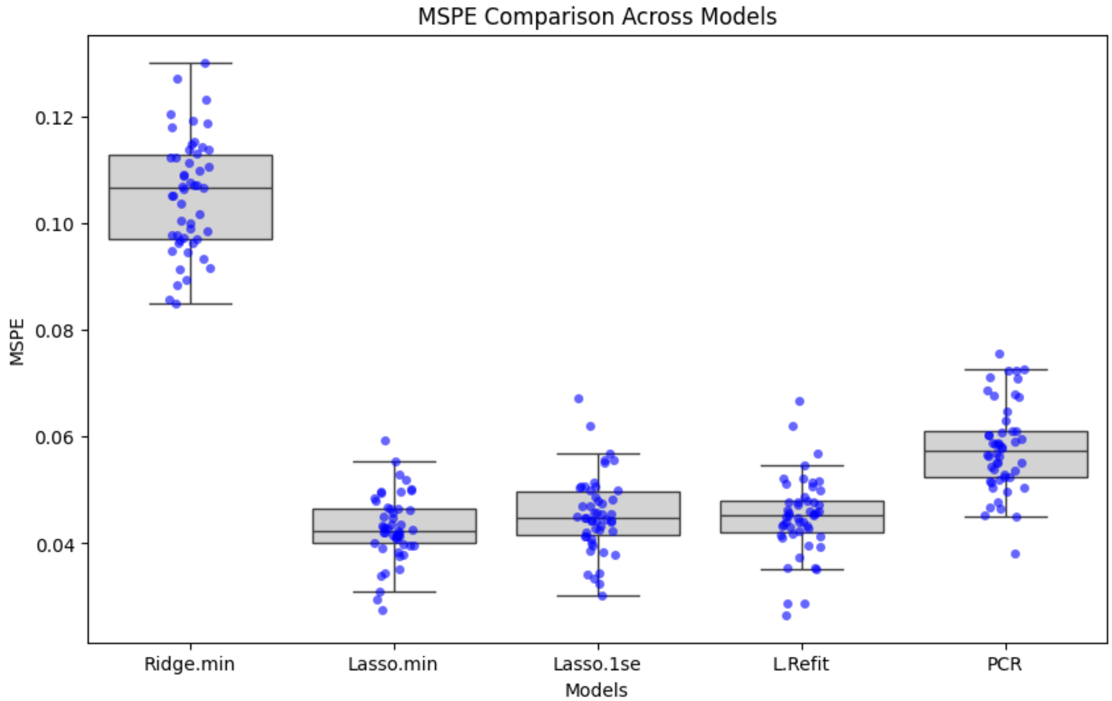

1.0.5 Which procedure or procedures yield the best performance in terms of MSPE?

Lasoso Min performs the best, although all the Lassos do quite well.

1.0.6 Conversely, which procedure or procedures show the poorest performance?

The worst performance is Ridge Min.

1.0.7 Have you observed any procedure or procedures that performed well in Case I but exhibited poorer performance in Case II, or vice versa? If so, please offer an explanation.

Ridge Min performs worse on a relative basis then in Case I. This makes sense as it has trouble removing the noisy features, where as Lasso is able to clamp them to 0.

1.0.8 Given that Coding2_Data2.csv includes all features found in Coding2_Data1.csv, one might anticipate that the best MSPE in Case II would be equal to or lower than the best MSPE in Case I. Do your simulation results corroborate this expectation? If not, please offer an explanation.

The MSPE is worse for all regression in Case II. Despite having the same true predictors, the additional noise which reduce the ability of regression to deduce the true coefficeints. However, given enough data we would expect the difference to decrease.